The Euclidean Adler function and its interplay with low-energy data and α_s

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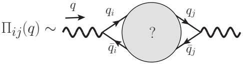
SISSA

The Euclidean Adler function, $D(Q^2)$

- Two point correlation functions of neutral quark current

$$\Pi^{\mu\nu}_{ij}(q) \equiv i \int d^4x \, e^{-iqx} \langle 0| \, T \left(\bar{q}_i(x)\gamma^\mu q_i(x) \, \bar{q}_j(0)\gamma^\nu q_j(0) \right) |0\rangle \, .$$



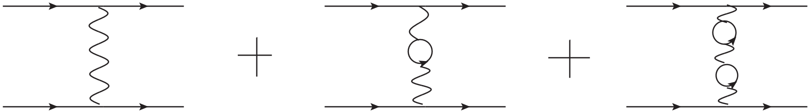
- Asymptotic limit, $Q^2 = -q^2 \gg \Lambda_{\mathrm{QCD}}$

$$\Pi^{\mathrm{part}}_{ij}(Q^2) \sim \frac{N_C}{\pi^2}\delta_{ij}\left[\log(Q^2) + C_{\mathrm{subs}}\right] \to D_{ij}(Q^2) \sim \pi^2 Q^2 \frac{d\Pi_{ij}(Q)}{dQ^2} \sim N_C \delta_{ij}$$

- Separation from the asymptotic regime given by $\delta D_{ij} \sim N_C \delta_{ij} \alpha_s(Q^2)$
- Perturbative QCD (pQCD) breaks down at low Q. $D_{ij}(Q^2 \to 0) = 0$
- EM Adler function is simply given by $D_{\mathrm{EM}} = \sum_{i,j} Q_i Q_j D_{ij}$

Interplay with $\alpha_{\rm QED}$ and low-energy data

- Hadronic running of $\alpha_{\rm QED}$. $\Delta\alpha_{\rm had}(Q^2) \sim (\Pi(Q^2) - \Pi(0)) \sim \int_0^{Q^2} dQ'^2 \frac{D(Q'^2)}{Q'^2}$



$$F(q^2) = \frac{1}{q^2}\left[\frac{\alpha_0}{1-\Pi_0(q^2)}\right] = \frac{1}{q^2}\left[\frac{\alpha_R}{1-\Pi_R(q^2)}\right]$$

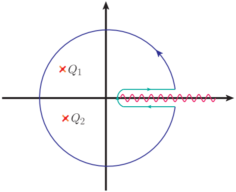
$$q^2F(0) = \alpha_{OS} \to F(q^2) = \frac{1}{q^2}\frac{\alpha_{OS}}{1-(\Pi(q^2)-\Pi(0))} = \frac{\alpha_{\rm QED}(q^2)}{q^2}\,.$$

- Precise lattice data results at low Q BMW, Nature 593 (2021) 51. Mainz, JHEP 08 (2022) 220. Driven by muon g-2 program

- Connection to experimental data (optical theorem)

$$R(q^2) \sim q^2\,\sigma(e^+e^- \to \text{hadrons}) \sim \,\text{Im}\Pi(q^2)$$

Interplay with α_{QED} and low-energy data.



$$W(Q^2; Q_1^2, Q_2^2) = \frac{1}{(Q^2 - Q_1^2)(Q^2 - Q_2^2)}$$

$$\frac{1}{2\pi i}\oint_{|Q^2|=Q_0^2} dQ^2 \frac{\Pi_{ij}(Q^2)}{(Q^2 - Q_1^2)(Q^2 - Q_2^2)} + \frac{1}{2\pi i}\left(\int_{|Q_{th}^2|e^{-i\pi}}^{|Q_0^2|e^{-i\pi}} - \int_{|Q_{th}^2|e^{i\pi}}^{|Q_0^2|e^{i\pi}}\right) dQ^2 \frac{\Pi_{ij}(Q^2)}{(Q^2 - Q_1^2)(Q^2 - Q_2^2)}$$
$$= \frac{\Pi_{ij}(Q_2^2)}{Q_2^2 - Q_1^2} + \frac{\Pi_{ij}(Q_1^2)}{Q_1^2 - Q_2^2}$$

$$\Delta\alpha_{\text{had}}(Q^2) = \frac{\alpha Q^2}{3\pi}\int_{s_{th}}^{\infty} ds \frac{R(s)}{s(s+Q^2)} \qquad D(Q^2) \sim Q^2 \int_{s_{th}}^{\infty} ds \frac{R(s)}{(s+Q^2)^2}$$

pQCD EM Adler function: state-of-art and approach

- Previous assessments of D_{EM} made in MOM scheme hep-ph/9812521, CERN Yellow Reports: Monographs 3 (2020) 9
- Decoupling scheme, but less known perturbative expansion (up to α_s^2). Ill-defined pole masses as inputs.
- In the limit of n_f massless quarks, D_{ii} known in the $\overline{\mathrm{MS}}$ at 5 loops.
- $m_{u,d} \ll m_s \ll \Lambda_{\mathrm{QCD}} \ll m_c \ll m_b \ll m_t$. Interested in $Q < 2m_c$
- Improved precision if instead one starts from $n_f = 3$ massless and systematically adds expansions in $\frac{Q^2}{(2m_c)^2}$. Inputs from FLAG
- Study the different two-point functions separately $D_{\mathrm{EM}} = \sum_{i,j} Q_i Q_j D_{ij}$

pQCD Adler function. Light quarks, massless

$$D_{ii}^{L,(0)}(Q^2)=N_C\left\{1+\sum_{n=1}^{\infty}\sum_{p=0}^{n-1}K_{n,p}\left(\frac{\alpha_s(\mu^2)}{\pi}\right)^n\log^p\left(Q^2/\mu^2\right)\right\},$$

All coefficients known up to (and including) $K_{4,n}$

			$D_{ii}^{L,(0)}(Q^2)$					
$\alpha_s^{(n_f=5)}(M_Z^2)$	Q	$\alpha_s(Q^2)$	0	1	2	3	4	5
0.115	1.0	0.4227	3	3.4036	3.4927	3.5392	3.5874	3.6238
	1.5	0.3197	3	3.3053	3.3562	3.3764	3.3921	3.4011
	2.0	0.2751	3	3.2627	3.3005	3.3133	3.3219	3.3262
0.120	1.0	0.5277	3	3.5039	3.6427	3.7332	3.8504	3.9606
	1.5	0.3681	3	3.3515	3.4191	3.4498	3.4776	3.4958
	2.0	0.3085	3	3.2946	3.3420	3.3601	3.3738	3.3813

Strange mass corrections also known (tiny, slow convergence)

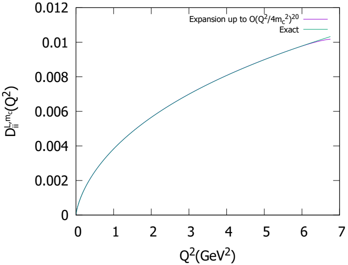
$$\Delta_{m_s}D_{33}^L(Q^2)=-3N_C\frac{m_s^2(Q^2)}{Q^2}\sum_n(2c_n^{L+T}+e_n^{L+T}+f_n^{L+T})\left(\frac{\alpha_s(Q^2)}{\pi}\right)^n+\mathcal{O}\left(\frac{m_s^4}{Q^4}\right)$$

pQCD Adler function. Light quarks. Charm corrections

- Heavy quark loops inside light quarks correlators induce corrections suppressed by $\sim \alpha_s^2 \frac{Q^2}{(2m_c)^2}$
- Reconstructed from analyticity and known contribution on δR_q Phys.Lett.B 338 (1994) 330-335

$$D_{ii}^{L,m_c}(Q^2) = Q^2 \int_{s_{th}}^{\infty} ds\, \frac{\delta R_q(s)}{(s+Q^2)^2}$$

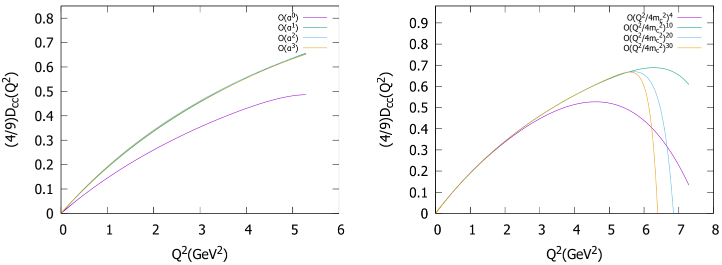
$$= N_C C_F T_F\, Q^2 \left(\int_0^{4m_c^2} ds\, \frac{\rho_V(s)}{(s+Q^2)^2} + \int_{4m_c^2}^{\infty} ds\, \frac{\rho_R(s)+\rho_V(s)}{(s+Q^2)^2} \right) \left(\frac{\alpha_s(\mu^2)}{\pi} \right)^2$$



pQCD Adler function. Heavy quarks. Charm corrections

$$D_{ii}(Q^2) = -\frac{9}{4} \sum_j (-1)^j \, j \, \overline{C}_j(\mu) \left(\frac{Q^2}{4m_i^2(\mu^2)} \right)^j$$

Excellent knowledge of coefficients thanks to cumulative efforts of many works ($\sim \mathcal{L}_{\mathrm{EFT}}^{D=60}$ up to 4 loops!)



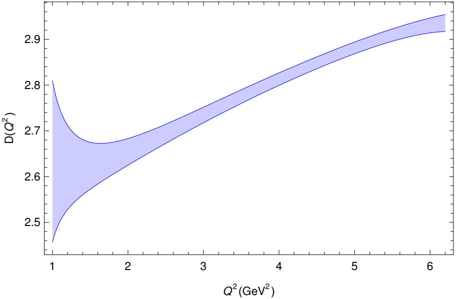
pQCD Adler function. Results

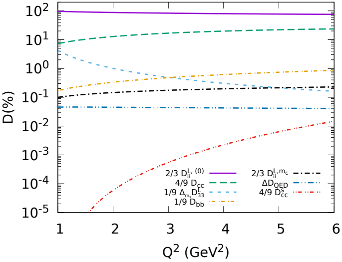
$$D(Q^2) = \sum_{i,j} Q_i Q_j D_{ij}(Q^2) = \frac{2}{3}\, D_{ii}^{L,(0)}(Q^2) + \frac{1}{9}\, \Delta_{m_s} D_{33}^{L}(Q^2) + \frac{2}{3}\, D_{ii}^{L,m_c}(Q^2)$$

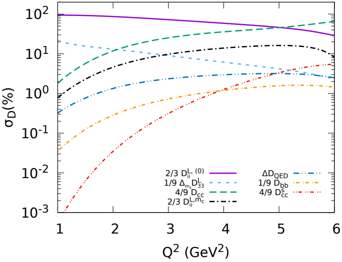
$$+ \frac{4}{9}\, D_{cc}(Q^2) + \frac{4}{9}\, D_{cc}^{s}(Q^2) + \frac{1}{9}\, D_{bb}(Q^2) + \Delta D_{\mathrm{QED}}(Q^2)$$

$\alpha_s^{(n_f=5)}(M_Z^2)$	Q^2	$\frac{2}{3} D_{ii}^{L,(0)}$	$\frac{1}{9} \Delta_{m_s} D_{33}^{L}$	$\frac{2}{3} D_{ii}^{L,m_c}$	$\frac{4}{9} D_{cc}$	$\frac{4}{9} D_{cc}^{s}$	$\frac{1}{9} D_{bb}$	ΔD_{QED}	D
0.115	3	2.2395(77)	−0.0123(12)	0.0039(10)	0.4484(21)(24)	0.0000(00)	0.0130(01)	0.0012(04)	2.694(09)
	4	2.2175(52)	−0.0080(07)	0.0045(12)	0.5435(24)(26)	0.0000(01)	0.0171(02)	0.0012(04)	2.776(07)
	5	2.2033(39)	−0.0058(04)	0.0050(13)	0.6197(31)(26)	0.0001(03)	0.0212(02)	0.0012(04)	2.845(06)
0.120	3	2.2866(156)	−0.0141(17)	0.0053(21)	0.4649(47)(24)	0.0000(01)	0.0132(01)	0.0012(04)	2.757(17)
	4	2.2542(98)	−0.0089(09)	0.0062(24)	0.5629(53)(26)	0.0001(02)	0.0174(02)	0.0012(04)	2.833(12)
	5	2.2343(70)	−0.0063(06)	0.0069(26)	0.6429(65)(27)	0.0002(05)	0.0215(02)	0.0012(04)	2.900(11)
0.1184(8)	3	2.2699(124)	−0.0134(15)	0.0048(17)	0.4591(36)(24)	0.0000(01)	0.0131(01)	0.0012(04)	2.735(17)
	4	2.2414(79)	−0.0086(08)	0.0055(19)	0.5561(41)(26)	0.0001(02)	0.0173(02)	0.0012(04)	2.813(14)
	5	2.2236(58)	−0.0061(05)	0.0062(20)	0.6348(51)(27)	0.0002(04)	0.0214(02)	0.0012(04)	2.881(13)

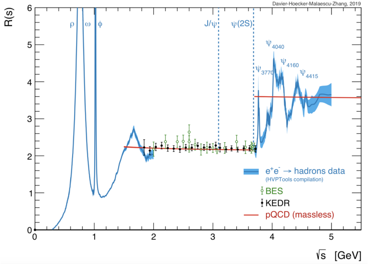
pQCD Adler function. Results



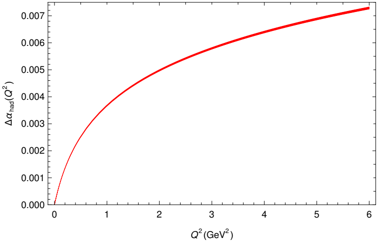


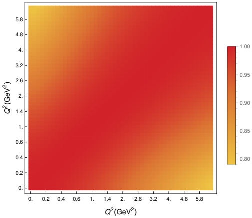


Experimental input for the EM Adler function



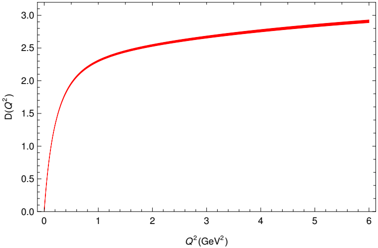
$$\Delta\alpha_{\rm had}(Q^2) = \frac{\alpha Q^2}{3\pi}\int_{s_{th}}^{\infty} ds\, \frac{R(s)}{s(s+Q^2)}$$

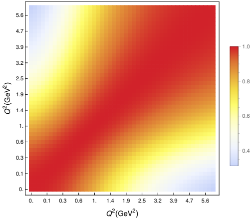




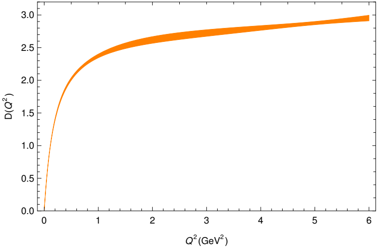
Alternative evaluations of the EM Adler function

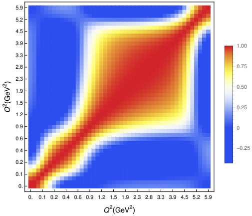
- R-ratio based evaluation (DHMZ)



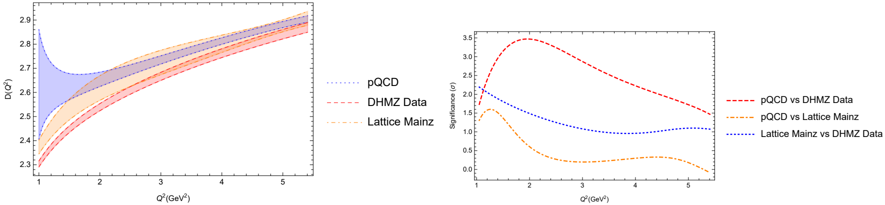


- Lattice-based evaluation (From rational approximation in JHEP 08 (2022) 220)





Comparison



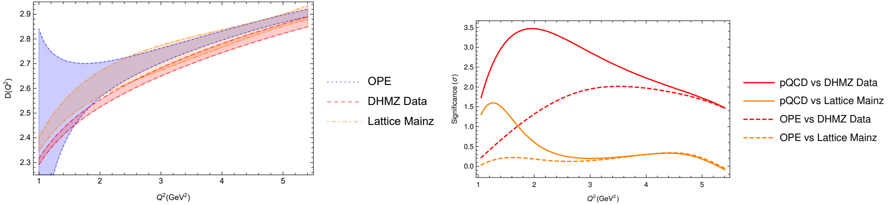
- Lattice curve above e^-e^+ data one, but with small significance.
- pQCD curve in good agreement with lattice but in tension with e^+e^- data

EM Adler function: estimates of power corrections

- Non-perturbative corrections scaling as $\Lambda^4_{\rm QCD}/Q^4$. Existence well established beyond pQCD
- Hard to separate from pQCD series (partially in pQCD uncertainties)
- Estimate based on limited knowledge

$$\delta D^{L,D=4}_{\rm em} \approx \frac{(0.10 \pm 0.18)\,{\rm GeV}^4}{Q^4} + \frac{-(0.36 \pm 0.36)\,{\rm GeV}^6}{Q^6}$$

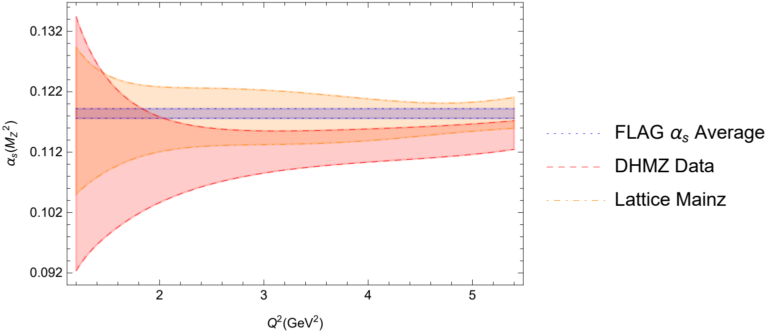
- They only become subleading source of uncertainties from $Q \gtrsim 2\,{\rm GeVs}$



- It may explain low-energy tension, but harder at high energies

EM Adler function: sensitivity to α_s

$$D^{\mathrm{OPE}}(Q^2, \alpha_s(Q^2)) - D^{\mathrm{data}}(Q^2) = 0 \to \alpha_s(Q^2)$$



Different fitting strategies analysed

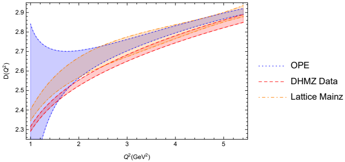
Fit to DHMZ compilation: $\alpha_s^{(n_f=5)}(M_Z^2) = 0.1136 \pm 0.0025$

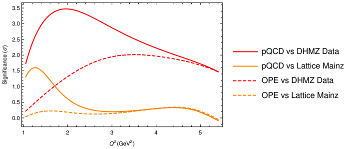
Larger central value is lattice is used, $\alpha_s^{(n_f=5)}(M_Z^2) = 0.1179 \pm 0.0025$

RGE could be tested, but more data at large s is needed

Conclusions

- Comprehensive study of the perturbative EM Adler function at $\mathcal{O}(2\,\mathrm{GeV})$
- Implementation within $\overline{\text{MS}}$ overcomes full-scheme limitations
- Comparison with e^+e^- and lattice data provides interesting insights





- Potential $\sim 1\%$ level $\alpha_s(M_Z)$ from data-based Euclidean Adler function
- Very important interplay to extract precise value of $\alpha_{\mathrm{EM}}(M_Z)$
- Important to extend this approach to higher energies and test regime of validity of pQCD/lattice for the different correlators