

NAGOYA UNIVERSITY

Status and prospects of measuring Electric Dipole Moment of tau lepton

Kenji Inami (Nagoya university/KEK)

2023/12 at international workshop on tau physics

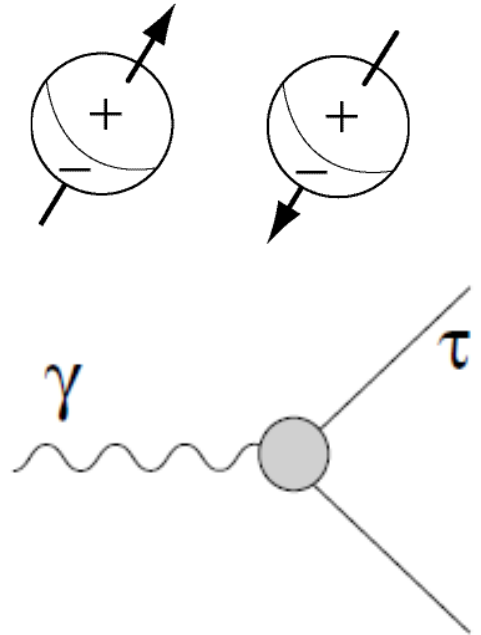
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Electric dipole moment of τ lepton

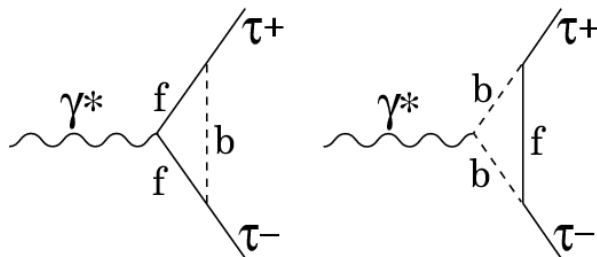
- Charge asymmetry along spin direction
- CP/T violating effect in the interaction with electric field

$$\mathcal{H}_{\text{int}} = \rho_m \boldsymbol{\sigma} \cdot \mathbf{H} + \rho_e \boldsymbol{\sigma} \cdot \mathbf{E}$$

- Non zero EDM indicates P and T violation
- CP violation parameter in $\gamma\tau\tau$ vertex
- Standard Model prediction: $O(10^{-37})$ ecm
 - Far below the current sensitivity
- A non-zero EDM may arise from new physics
 - e.g. new particles in a loop diagram



$$\mathcal{L}_{CP} = -\frac{i}{2} \bar{\tau} \sigma^{\mu\nu} \gamma_5 \tau d_\tau(s) F_{\mu\nu}$$



PDG2023 Prog.Theor.Exp.Phys. 2022, 083C01 (2022) and 2023 update

τ ELECTRIC DIPOLE MOMENT (d_τ)

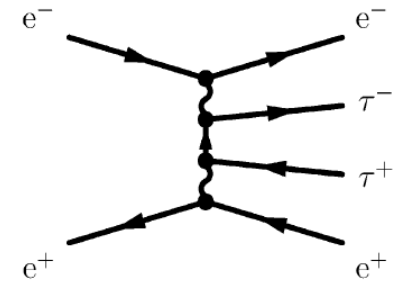
A nonzero value is forbidden by both T invariance and P invariance.

The q^2 dependence is expected to be small providing no thresholds are nearby.

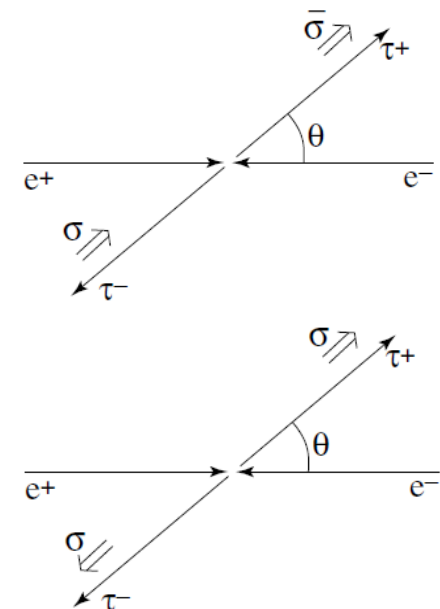
$Re(d_\tau)$

VALUE ($10^{-16} e\text{cm}$)	CL%	DOCUMENT ID	TECN	COMMENT
- 0.185 to 0.061	95	¹ INAMI	22 BELL	$E_{cm}^{ee} = 10.6 \text{ GeV}$
● ● ● We do not use the following data for averages, fits, limits, etc. ● ● ●				
< 2.3	90	² GROZIN	09A RVUE	From e EDM limit
< 3.7	95	³ ABDALLAH	04K DLPH	$e^+e^- \rightarrow e^+e^-\tau^+\tau^-$ at LEP2
< 11.4	95	⁴ ACHARD	04G L3	$e^+e^- \rightarrow e^+e^-\tau^+\tau^-$ at LEP2
- 0.22 to 0.45	95	⁵ INAMI	03 BELL	$E_{cm}^{ee} = 10.6 \text{ GeV}$
< 4.6	95	⁶ ALBRECHT	00 ARG	$E_{cm}^{ee} = 10.4 \text{ GeV}$

- Production cross-section
 - LEP2: $e^+e^- \rightarrow e^+e^-\tau^+\tau^-$
 - LHC: $qq \rightarrow \tau^+\tau^-$ (arXiv:2307.14133 [hep-ph])
- Spin correlation at low energy e^+e^- collision
 - ARGUS, Belle ($\sqrt{s} = \sim 10 \text{ GeV}$)
- Restriction from electron EDM



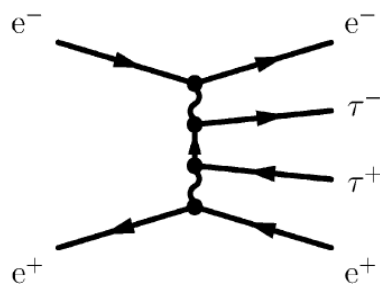
Physics Letters B 585 (2004) 53





LEP2/DELPHI and L3

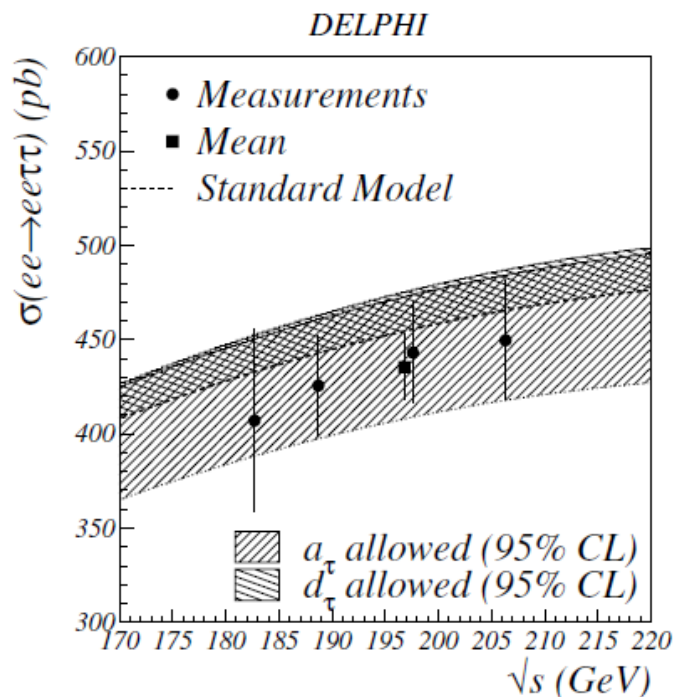
- Evaluate from the cross-section of $e^+e^- \rightarrow e^+e^- \tau^+\tau^-$



DELPHI

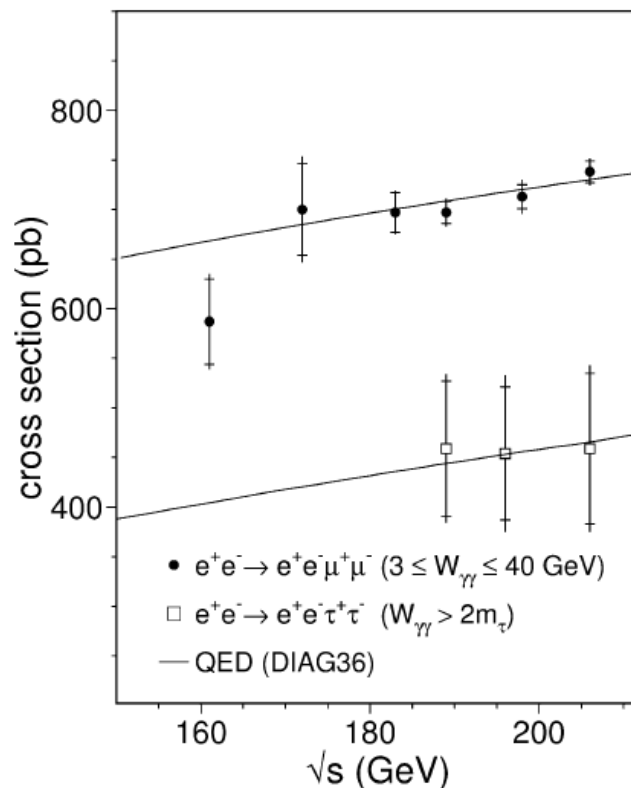
[Eur. Phys. J. C 35, 159–170 (2004)]

$$|d_\tau| < 3.7 \cdot 10^{-16} e \cdot \text{cm}$$

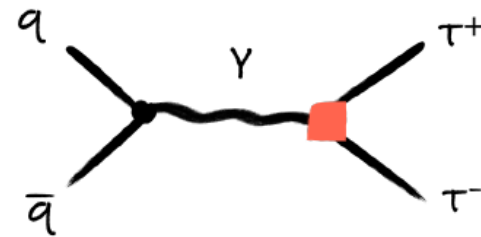
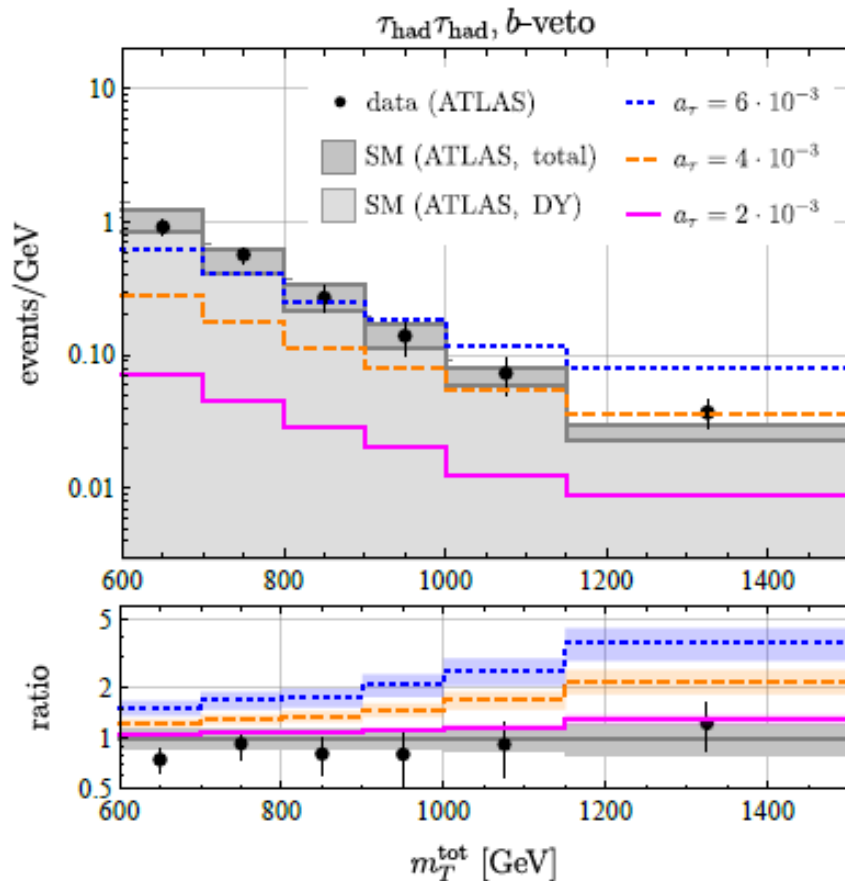


L3 [Physics Letters B 585 (2004) 53–62]

$$|d_\tau| \leq 1.14 \times 10^{-15} e \text{ cm}$$



- From the result of measurement of $H \rightarrow \tau\tau$
 - [arXiv:2307.14133 [hep-ph]]



$$|a_\tau| < 1.8 \cdot 10^{-3},$$

$$|d_\tau| < 1.0 \cdot 10^{-17} \text{ ecm.}$$

- Similar strength with the current limit
- 2.8 times better sensitivity by HL-LHC data (3ab^{-1})

- Effective Lagrangian with EDM term for $e^+e^- \rightarrow \tau^+\tau^-$

$$\mathcal{L}_{\text{eff}} = \bar{\psi}(i \not{\partial} - eQ \not{A})\psi - id_{\tau}\bar{\psi}\sigma^{\mu\nu}\gamma_5\psi\partial_{\mu}A_{\nu}$$

The image shows two Feynman diagrams for the process $e^+e^- \rightarrow \tau^+\tau^-$. The first diagram is the Standard Model contribution, where an electron-positron pair annihilates into a virtual photon (γ^*), which then decays into a tau-antitau pair. The second diagram is the contribution from the EDM term, which is identical to the first but includes an additional vertex labeled d_{τ} on the tau line, representing the interaction with the photon's magnetic field.

- Squared spin density matrix (proportional to cross section)

$$\mathcal{M}_{\text{prod}}^2 = \mathcal{M}_{\text{SM}}^2 + \underline{\text{Re}(d_{\tau})\mathcal{M}_{\text{Re}}^2} + \underline{\text{Im}(d_{\tau})\mathcal{M}_{\text{Im}}^2} + |d_{\tau}|^2\mathcal{M}_{d^2}^2$$

- Interference term between lowest order and EDM term affects spin-dependent cross-section



- For $e^+(\mathbf{p}) + e^-(-\mathbf{p}) \rightarrow \tau^+(\mathbf{k}, \mathbf{S}_+) + \tau^-(-\mathbf{k}, \mathbf{S}_-)$

$$\mathcal{M}_{\text{prod}}^2 = \mathcal{M}_{\text{SM}}^2 + \text{Re}(d_\tau)\mathcal{M}_{\text{Re}}^2 + \text{Im}(d_\tau)\mathcal{M}_{\text{Im}}^2 + |d_\tau|^2\mathcal{M}_{d^2}^2,$$

$$\begin{aligned} \mathcal{M}_{\text{SM}}^2 = & \frac{e^4}{k_0^2} [k_0^2 + m_\tau^2 + |\mathbf{k}^2|(\hat{\mathbf{k}} \cdot \hat{\mathbf{p}})^2 - \mathbf{S}_+ \cdot \mathbf{S}_- |\mathbf{k}|^2 (1 - (\hat{\mathbf{k}} \cdot \hat{\mathbf{p}})^2) \\ & + 2(\hat{\mathbf{k}} \cdot \mathbf{S}_+)(\hat{\mathbf{k}} \cdot \mathbf{S}_-)(|\mathbf{k}|^2 + (k_0 - m_\tau)^2(\hat{\mathbf{k}} \cdot \hat{\mathbf{p}})^2) + 2k_0^2(\hat{\mathbf{p}} \cdot \mathbf{S}_+)(\hat{\mathbf{p}} \cdot \mathbf{S}_-) \\ & - 2k_0(k_0 - m_\tau)(\hat{\mathbf{k}} \cdot \hat{\mathbf{p}})((\hat{\mathbf{k}} \cdot \mathbf{S}_+)(\hat{\mathbf{p}} \cdot \mathbf{S}_-) + (\hat{\mathbf{k}} \cdot \mathbf{S}_-)(\hat{\mathbf{p}} \cdot \mathbf{S}_+))], \end{aligned}$$

$$\begin{aligned} \mathcal{M}_{\text{Re}}^2 = & 4\frac{e^3}{k_0} |\mathbf{k}| [- (m_\tau + (k_0 - m_\tau)(\hat{\mathbf{k}} \cdot \hat{\mathbf{p}})^2) \underline{(\mathbf{S}_+ \times \mathbf{S}_-) \cdot \hat{\mathbf{k}}} \\ & + k_0(\hat{\mathbf{k}} \cdot \hat{\mathbf{p}}) \underline{(\mathbf{S}_+ \times \mathbf{S}_-) \cdot \hat{\mathbf{p}}}], \end{aligned}$$

$$\begin{aligned} \mathcal{M}_{\text{Im}}^2 = & 4\frac{e^3}{k_0} |\mathbf{k}| [- (m_\tau + (k_0 - m_\tau)(\hat{\mathbf{k}} \cdot \hat{\mathbf{p}})^2) \underline{(\mathbf{S}_+ - \mathbf{S}_-) \cdot \hat{\mathbf{k}}} \\ & + k_0(\hat{\mathbf{k}} \cdot \hat{\mathbf{p}}) \underline{(\mathbf{S}_+ - \mathbf{S}_-) \cdot \hat{\mathbf{p}}}], \end{aligned}$$

$$\mathcal{M}_{d^2}^2 = 4e^2 |\mathbf{k}|^2 \cdot (1 - (\hat{\mathbf{k}} \cdot \hat{\mathbf{p}})^2)(1 - \mathbf{S}_+ \cdot \mathbf{S}_-),$$

\mathbf{S}_\pm : Spin vectors of τ^\pm
 \mathbf{k}, \mathbf{p} : Momenta of τ^+ and e^+ beam





- Spin direction can be obtained from the information of tau decay products and tau direction

$$\tau \rightarrow l\nu_l\nu_\tau$$

$$\mathbf{S}_\pm = \frac{4c_\pm - m_\tau^2 - 3m_l^2}{3m_\tau^2 c_\pm - 4c_\pm^2 - 2m_l^2 m_\tau^2 + 3c_\pm m_l^2} \left(\pm m_\tau \mathbf{p}_{l\pm} - \frac{c_\pm + E_{l\pm} m_\tau}{k_0 + m_\tau} \mathbf{k} \right)$$
$$c_\pm = k_0 E_{l\pm} \mp \mathbf{k} \cdot \mathbf{p}_{l\pm}$$

$$\tau \rightarrow \pi\nu_\tau$$

$$\mathbf{S}_\pm = \frac{2}{m_\tau^2 - m_\pi^2} \left(\mp m_\tau \mathbf{p}_{\pi\pm} + \frac{m_\tau^2 + m_\pi^2 + 2m_\tau E_{\pi\pm}}{2(E_\tau + m_\tau)} \mathbf{k} \right)$$

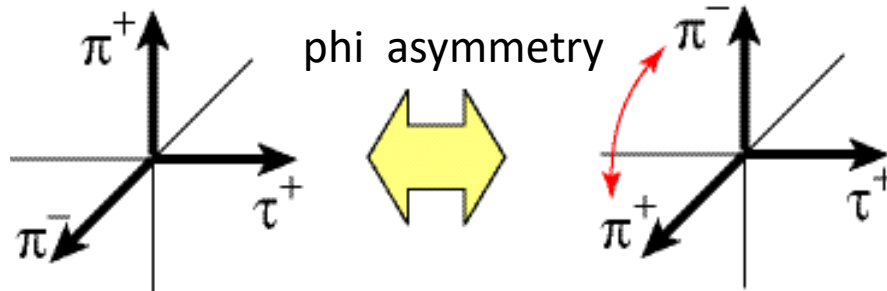
$$\tau \rightarrow \rho\nu_\tau \rightarrow \pi\pi^0\nu_\tau$$

$$\mathbf{S}_\pm = \mp \frac{1}{(k_\pm H_\pm) - m_\tau^2 (p_{\pi^\pm} - p_{\pi^0})^2} \left(\mp H_0^\pm \mathbf{k} + m_\tau \mathbf{H}^\pm + \frac{\mathbf{k}(\mathbf{k} \cdot \mathbf{H}^\pm)}{(E_\tau + m_\tau)} \right)$$
$$(H^\pm)^\nu = 2(p_{\pi^\pm} - p_{\pi^0})^\nu (p_{\pi^\pm} - p_{\pi^0})^\mu (k_\pm)_\mu + (p_{\pi^\pm} + p_{\pi^0})^\nu (p_{\pi^\pm} - p_{\pi^0})^2$$

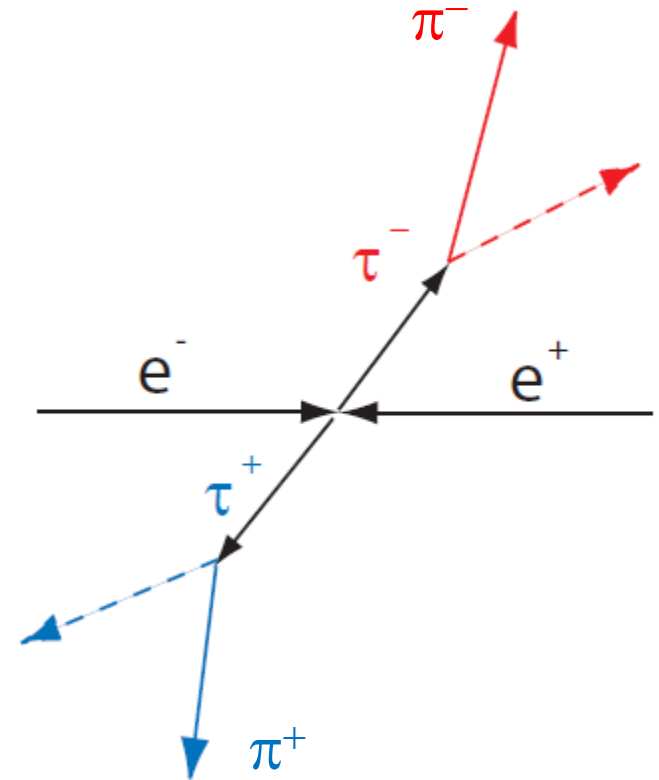
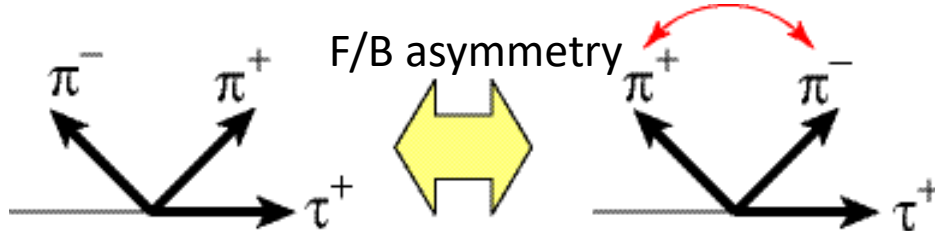


Asymmetry in event shape

$$\mathcal{M}_{Re}^2 \sim (\mathbf{S}_+ \times \mathbf{S}_-) \hat{\mathbf{k}} \quad , \quad (\mathbf{S}_+ \times \mathbf{S}_-) \hat{\mathbf{p}}$$



$$\mathcal{M}_{Im}^2 \sim (\mathbf{S}_+ - \mathbf{S}_-) \hat{\mathbf{k}} \quad , \quad (\mathbf{S}_+ - \mathbf{S}_-) \hat{\mathbf{p}}$$



- $\text{Re}(d_\tau)$: phi asymmetry
- $\text{Im}(d_\tau)$: forward/backward asymmetry



- Electron-positron collider experiment at KEK Tsukuba Japan
 - Collected $\sim 10^9$ τ pairs (1ab^{-1}) at $\sqrt{s} = 10.58$ GeV
- Select 8 final modes from 833 fb^{-1} data ($\sim 7.6 \times 10^8$ τ pairs)

$$\tau\tau \rightarrow (e\nu\bar{\nu})(\mu\nu\bar{\nu}), (e\nu\bar{\nu})(\pi\nu), (\mu\nu\bar{\nu})(\pi\nu), (e\nu\bar{\nu})(\rho\nu),$$

$$(\mu\nu\bar{\nu})(\rho\nu), (\pi\nu)(\rho\nu), (\rho\nu)(\rho\bar{\nu}), \text{ and } (\pi\nu)(\pi\bar{\nu})$$
- Total yield : 3.1×10^7 events, Averaged purity : 88.5%
- Background
 - Main : from tau decay : Multi- π^0 and mis-PID
 - Non- τ process: negligibly small

Mode	Yield	Purity(%)	Background (%)
$e\mu$	6434268	95.8	two-photon process ($ee\mu\mu$) [2.5], $\tau\tau \rightarrow (e\nu\nu)(\pi\nu)$ [1.3]
$e\pi$	2644971	85.7	$\tau\tau \rightarrow (e\nu\nu)(\rho\nu)$ [6.5], $(e\nu\nu)(\mu\nu\nu)$ [5.1], $(e\nu\nu)(K^*\nu)$ [1.3]
$\mu\pi$	2503936	80.5	$\tau\tau \rightarrow (\mu\nu\nu)(\rho\nu)$ [6.4], $(\mu\nu\nu)(\mu\nu\nu)$ [4.9], $(\mu\nu\nu)(K^*\nu)$ [1.3], two-photon process ($ee\mu\mu$) [3.1]
$e\rho$	7218823	91.7	$\tau\tau \rightarrow (e\nu\nu)(\pi\pi^0\pi^0\nu)$ [4.6], $(e\nu\nu)(K^*\nu)$ [1.7]
$\mu\rho$	6203489	91.0	$\tau\tau \rightarrow (\mu\nu\nu)(\pi\pi^0\pi^0\nu)$ [4.3], $(\mu\nu\nu)(K^*\nu)$ [1.6], $(\pi\nu)(\rho\nu)$ [1.1]
$\pi\rho$	2655696	77.0	$\tau\tau \rightarrow (\rho\nu)(\rho\nu)$ [6.7], $(\pi\nu)(\pi\pi^0\pi^0\nu)$ [3.9], $(\mu\nu\nu)(\rho\nu)$ [5.1], $(\rho\nu)(K^*\nu)$ [1.4], $(\pi\nu)(K^*\nu)$ [1.4]
$\rho\rho$	3277001	82.4	$\tau\tau \rightarrow (\rho\nu)(\pi\pi^0\pi^0\nu)$ [9.4], $(\rho\nu)(K^*\nu)$ [3.1]
$\pi\pi$	460288	71.9	$\tau\tau \rightarrow (\pi\nu)(\rho\nu)$ [11.3], $(\pi\nu)(\mu\nu\nu)$ [8.8], $(\pi\nu)(K^*\nu)$ [2.5]

- EDM results

Mode	$\text{Re}(d_\tau)(10^{-17} \text{ ecm})$	$\text{Im}(d_\tau)(10^{-17} \text{ ecm})$
$e\mu$	$-3.2 \pm 2.5 \pm 3.6$	$0.6 \pm 0.4 \pm 1.8$
$e\pi$	$0.7 \pm 2.3 \pm 4.8$	$2.4 \pm 0.5 \pm 2.2$
$\mu\pi$	$1.0 \pm 2.2 \pm 4.3$	$2.4 \pm 0.5 \pm 2.6$
$e\rho$	$-1.2 \pm 0.8 \pm 1.0$	$-1.1 \pm 0.3 \pm 0.6$
$\mu\rho$	$0.7 \pm 1.0 \pm 2.2$	$-0.5 \pm 0.3 \pm 0.8$
$\pi\rho$	$-0.6 \pm 0.7 \pm 1.0$	$0.4 \pm 0.3 \pm 1.2$
$\rho\rho$	$-0.4 \pm 0.5 \pm 0.9$	$-0.3 \pm 0.3 \pm 0.4$
$\pi\pi$	$-2.2 \pm 4.3 \pm 5.2$	$-0.9 \pm 0.9 \pm 1.2$

- Can obtain the weighted average of EDM and its error

$$\text{Re}(d_\tau) = (-0.62 \pm 0.63) \times 10^{-17} \text{ ecm},$$

$$\text{Im}(d_\tau) = (-0.40 \pm 0.32) \times 10^{-17} \text{ ecm}.$$

- Consistent with zero EDM
- Systematic errors are comparable with the statistical errors.

- Optimal observable [PRD 45(1992)2405]

$$\mathcal{O}_{Re} = \frac{\mathcal{M}_{Re}^2}{\mathcal{M}_{SM}^2}, \quad \mathcal{O}_{Im} = \frac{\mathcal{M}_{Im}^2}{\mathcal{M}_{SM}^2},$$

- Maximize sensitivity (S/N)
- Calculate event-by-event
 - Using tau flight direction and spin direction (from decay products)

$$\mathcal{M}_{\text{prod}}^2 = \mathcal{M}_{SM}^2 + \text{Re}(d_\tau)\mathcal{M}_{Re}^2 + \text{Im}(d_\tau)\mathcal{M}_{Im}^2 + |d_\tau|^2\mathcal{M}_{d^2}^2$$

$$\begin{aligned} \mathcal{M}_{SM}^2 = & \frac{e^4}{k_0^2} [k_0^2 + m_\tau^2 + |\mathbf{k}^2|(\hat{\mathbf{k}} \cdot \hat{\mathbf{p}})^2 - \mathbf{S}_+ \cdot \mathbf{S}_- |\mathbf{k}|^2 (1 - (\hat{\mathbf{k}} \cdot \hat{\mathbf{p}})^2) \\ & + 2(\hat{\mathbf{k}} \cdot \mathbf{S}_+)(\hat{\mathbf{k}} \cdot \mathbf{S}_-)(|\mathbf{k}|^2 + (k_0 - m_\tau)^2(\hat{\mathbf{k}} \cdot \hat{\mathbf{p}})^2) + 2k_0^2(\hat{\mathbf{p}} \cdot \mathbf{S}_+)(\hat{\mathbf{p}} \cdot \mathbf{S}_-) \\ & - 2k_0(k_0 - m_\tau)(\hat{\mathbf{k}} \cdot \hat{\mathbf{p}})((\hat{\mathbf{k}} \cdot \mathbf{S}_+)(\hat{\mathbf{p}} \cdot \mathbf{S}_-) + (\hat{\mathbf{k}} \cdot \mathbf{S}_-)(\hat{\mathbf{p}} \cdot \mathbf{S}_+))], \end{aligned}$$

$$\begin{aligned} \mathcal{M}_{Re}^2 = & 4 \frac{e^3}{k_0} |\mathbf{k}| [- (m_\tau + (k_0 - m_\tau)(\hat{\mathbf{k}} \cdot \hat{\mathbf{p}})^2) \underline{(\mathbf{S}_+ \times \mathbf{S}_-) \cdot \hat{\mathbf{k}}} \\ & + k_0(\hat{\mathbf{k}} \cdot \hat{\mathbf{p}}) \underline{(\mathbf{S}_+ \times \mathbf{S}_-) \cdot \hat{\mathbf{p}}}], \end{aligned}$$

$$\begin{aligned} \mathcal{M}_{Im}^2 = & 4 \frac{e^3}{k_0} |\mathbf{k}| [- (m_\tau + (k_0 - m_\tau)(\hat{\mathbf{k}} \cdot \hat{\mathbf{p}})^2) \underline{(\mathbf{S}_+ - \mathbf{S}_-) \cdot \hat{\mathbf{k}}} \\ & + k_0(\hat{\mathbf{k}} \cdot \hat{\mathbf{p}}) \underline{(\mathbf{S}_+ - \mathbf{S}_-) \cdot \hat{\mathbf{p}}}], \end{aligned}$$

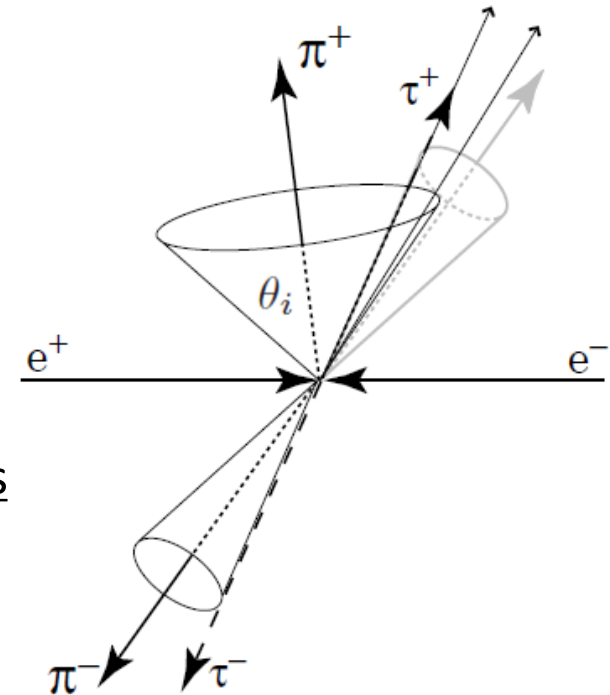
- Average value is proportional to EDM

$$\begin{aligned} \langle \mathcal{O}_{Re} \rangle & \propto \int \mathcal{O}_{Re} \mathcal{M}_{\text{prod}}^2 d\phi \\ & = \int \mathcal{M}_{Re}^2 d\phi + \text{Re}(d_\tau) \int \frac{(\mathcal{M}_{Re}^2)^2}{\mathcal{M}_{SM}^2} d\phi \end{aligned}$$

- Need tau flight direction : $\mathcal{M}_{Re}^2 \sim (\mathbf{S}_+ \times \mathbf{S}_-) \hat{\mathbf{k}}$, $(\mathbf{S}_+ \times \mathbf{S}_-) \hat{\mathbf{p}}$
 - Due to missing neutrinos from tau decays, there is uncertainty in the reconstructed tau direction
 - Two-fold ambiguity in case that both tau leptons decay hadronically

$$\cos \theta_i = \frac{2E_\tau E_i - m_i^2 - m_\tau^2}{2|\mathbf{k}||\mathbf{p}_i|}$$

- Additional ambiguity for leptonic decay
- Take an average over the possible tau directions
- Coefficient depends on the acceptance
 - Understanding of detection efficiency is important.



$$\begin{aligned} \langle \mathcal{O}_{Re} \rangle &\propto \int \mathcal{O}_{Re} \mathcal{M}_{\text{prod}}^2 d\phi \\ &= \int \mathcal{M}_{Re}^2 d\phi + \text{Re}(d_\tau) \int \frac{(\mathcal{M}_{Re}^2)^2}{\mathcal{M}_{SM}^2} d\phi \end{aligned}$$

- Statistical improvements
 - Belle II experiment is collecting more data, which will reach 50ab^{-1} .
 - ~ 60 times more data
 - Detection efficiency can be improved by machine-learning technique.
 - Adding other final state modes also improves the statistics.
- Observable improvement by vertexing
 - Averaging over the possible tau direction reduces the sensitivity.
 - Measuring the position of decay products solves the ambiguity.
 - Improve the sensitivity by a factor of ~ 2 for hadronic modes.
 - Improved vertex detector in Belle II make it possible.
- Expected statistical sensitivity
 - $\Delta\text{Re}(d_\tau) \sim 2 \times 10^{-19} \text{ ecm}$
(or better by improvement of efficiency)

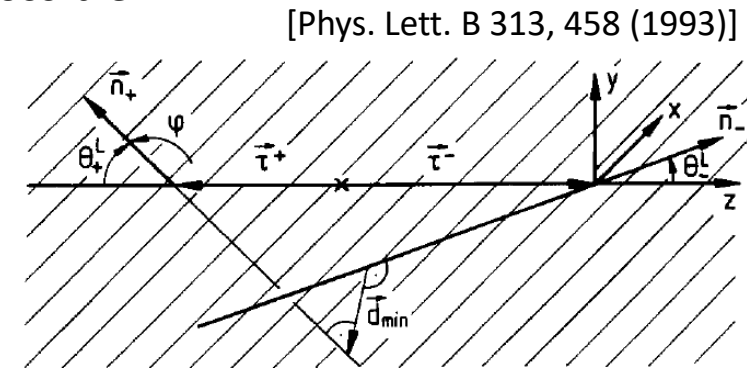


Fig. 1. Kinematic configuration indicating the relative orientation of the hadronic tracks, the τ directions and the vector \vec{d}_{\min} .



Systematic uncertainties

- Current statistical sensitivity is $\Delta\text{Re}(d_\tau) = 3.3 \times 10^{-18}$ ecm
- The systematic uncertainty from the detector modeling limits the result.
 - Understanding of detector response is the key for next analysis.
 - Tighter selection may improve, but trade off to detection efficiency.

Belle result [JHEP. 2022, 110 (2022)]

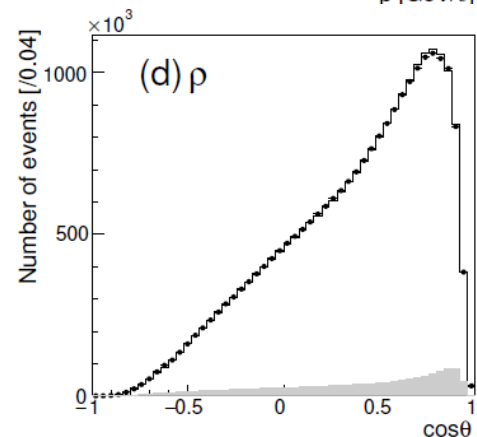
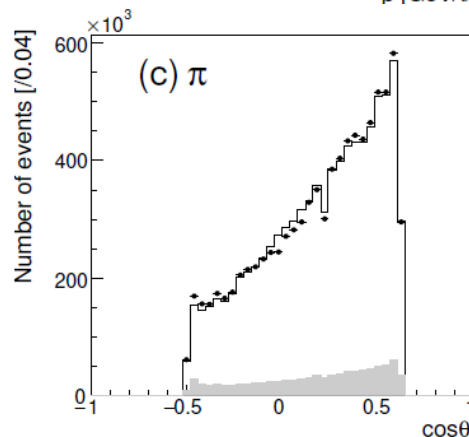
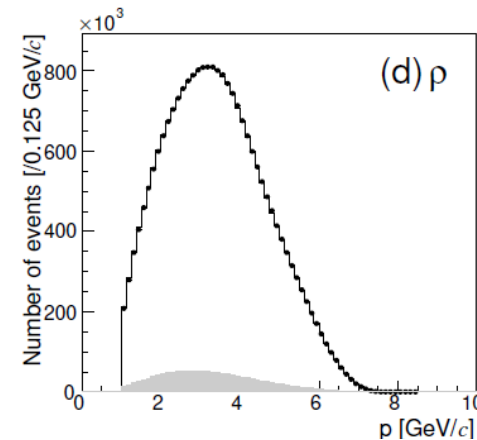
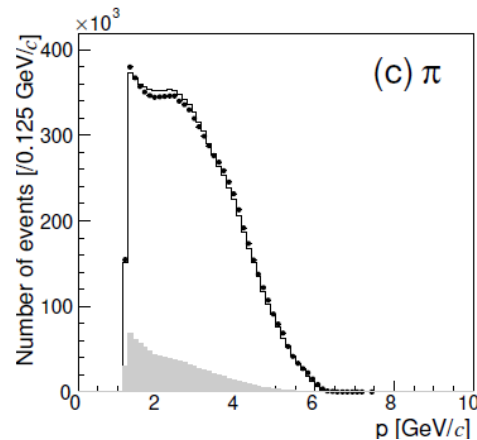
$$\text{Re}(d_\tau) = (-0.62 \pm 0.63) \times 10^{-17} \text{ ecm},$$

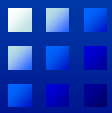
$$\text{Im}(d_\tau) = (-0.40 \pm 0.32) \times 10^{-17} \text{ ecm}.$$

Systematic uncertainties (10^{-17} ecm)

$\text{Re}(d_\tau)$	$e\mu$	$e\pi$	$\mu\pi$	$e\rho$	$\mu\rho$	$\pi\rho$	$\rho\rho$	$\pi\pi$
Detector alignment	0.2	0.2	0.1	0.1	0.2	0.1	0.1	0.3
Momentum reconstruction	0.1	0.6	0.5	0.1	0.3	0.2	0.1	1.5
Charge asymmetry	0.0	0.0	0.1	0.0	0.0	0.0	0.0	0.0
Mismatch of distribution	3.2	4.8	3.8	0.9	2.2	0.9	0.9	3.6
Background variation	1.6	0.3	1.7	0.4	0.2	0.2	0.2	3.5
Radiative effects	0.7	0.5	0.6	0.2	0.2	0.0	0.0	0.1
Total	3.6	4.8	4.3	1.0	2.2	1.0	0.9	5.2

$\text{Im}(d_\tau)$	$e\mu$	$e\pi$	$\mu\pi$	$e\rho$	$\mu\rho$	$\pi\rho$	$\rho\rho$	$\pi\pi$
Detector alignment	0.0	0.0	0.0	0.0	0.1	0.0	0.0	0.0
Momentum reconstruction	0.2	0.5	0.4	0.0	0.1	0.1	0.1	0.1
Charge asymmetry	0.2	2.0	2.4	0.1	0.1	1.1	0.0	0.0
Mismatch of distribution	1.0	0.9	0.6	0.5	0.8	0.4	0.4	1.2
Background variation	1.4	0.0	0.7	0.3	0.1	0.1	0.1	0.1
Radiative effects	0.1	0.1	0.1	0.1	0.1	0.0	0.0	0.0
Total	1.8	2.2	2.6	0.6	0.8	1.2	0.4	1.2





- Tau EDM has been tested in several ways.
 - Total cross-section, spin correlation and from electron EDM
- Current best result obtained by spin correlation at Belle experiment
$$\text{Re}(d_\tau) = (-0.62 \pm 0.63) \times 10^{-17} \text{ ecm},$$
$$\text{Im}(d_\tau) = (-0.40 \pm 0.32) \times 10^{-17} \text{ ecm}.$$
 - Utilizing information of decay products in optimal observable
 - Systematic error from detector modeling limits the sensitivity.
- More data will come from the current experiments.
 - Belle II experiment plans to collect 50 times larger amount of tau-pair events.
- Further improvements can be expected.
 - Detection efficiency, additional final states
 - Good vertex resolution can resolve tau direction, which will improve the observable sensitivity.
 - Reduction of systematic uncertainty from detector understanding, by large data.
- The sensitivity of $(1-2) \times 10^{-19}$ ecm can be achievable in near future.
 - Upgrade with beam polarization will improve the sensitivity further.