Matter Effects in $P_{\mu\tau}$ at long baselines

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Neutrino oscillations

\[
\frac{CC}{NC} = \frac{\nu_e}{\nu_e + \nu_\mu + \nu_\tau} < 1
\]
Neutrino oscillations

Based on measuring the appearance and disappearance of muon and electron neutrinos
Atmospheric $\nu_\tau$ in Super-Kamiokande

Not event by event but statistical

SK excludes no $\nu_\tau$ appearance at $4.6\sigma$

From: Y. Nakano, ICNFP 2018
SK collaboration, PRD 2018
Three neutrino paradigm

- Measurement of non-zero $\theta_{13}$ in reactor experiments $\rightarrow$ three neutrino picture

\[
\begin{pmatrix}
  v_e \\
  v_\mu \\
  v_\tau
\end{pmatrix} =
\begin{pmatrix}
  1 \\
  C_{23} & S_{23} \\
  -S_{23} & C_{23}
\end{pmatrix}
\begin{pmatrix}
  C_{13} & \frac{1}{2}i\delta S_{13} \\
  -i\delta S_{13} & C_{13}
\end{pmatrix}
\begin{pmatrix}
  C_{12} & S_{12} \\
  -S_{12} & C_{12}
\end{pmatrix}
\begin{pmatrix}
  v_1 \\
  v_2 \\
  v_3
\end{pmatrix}
\]

$C_{12} = \cos \theta_{12}$ etc., $\delta$ CP-violating phase

- $\Delta m_{21}^2$, $\theta_{12}$, $\theta_{13}$ Solar + KamLAND
- $\Delta m_{31}^2$, $\theta_{13}$ Reactor
- $\Delta m_{31}^2$, $\theta_{23}$, $\theta_{13}$, $\delta_{CP}$ Atm + LBL

Interplay among different sectors because of $\theta_{13}$
### Three generation oscillation parameters

<table>
<thead>
<tr>
<th></th>
<th>Normal Ordering (best fit)</th>
<th>Inverted Ordering ($\Delta \chi^2 = 6.4$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>bfp ±1σ</td>
<td>3σ range</td>
</tr>
<tr>
<td>$\sin^2 \theta_{12}$</td>
<td>0.303$^{+0.012}_{-0.012}$</td>
<td>0.270 → 0.341</td>
</tr>
<tr>
<td>$\theta_{12} / ^\circ$</td>
<td>33.41$^{+0.75}_{-0.72}$</td>
<td>31.31 → 35.74</td>
</tr>
<tr>
<td>$\sin^2 \theta_{23}$</td>
<td>0.451$^{+0.019}_{-0.016}$</td>
<td>0.408 → 0.603</td>
</tr>
<tr>
<td>$\theta_{23} / ^\circ$</td>
<td>42.2$^{+1.1}_{-0.9}$</td>
<td>39.7 → 51.0</td>
</tr>
<tr>
<td>$\sin^2 \theta_{13}$</td>
<td>0.02225$^{+0.00056}_{-0.00059}$</td>
<td>0.02052 → 0.02398</td>
</tr>
<tr>
<td>$\theta_{13} / ^\circ$</td>
<td>8.58$^{+0.11}_{-0.11}$</td>
<td>8.23 → 8.91</td>
</tr>
<tr>
<td>$\delta_{\text{CP}} / ^\circ$</td>
<td>232$^{+136}_{-26}$</td>
<td>144 → 350</td>
</tr>
<tr>
<td>$\Delta m_{21}^2 / 10^{-5} \text{ eV}^2$</td>
<td>7.41$^{+0.21}_{-0.20}$</td>
<td>6.82 → 8.03</td>
</tr>
<tr>
<td>$\Delta m_{3\ell}^2 / 10^{-3} \text{ eV}^2$</td>
<td>+2.507$^{+0.026}_{-0.027}$</td>
<td>+2.427 → +2.590</td>
</tr>
</tbody>
</table>
Neutrino oscillation (in vacuum)

\[ v_\alpha = \sum_i U_{\alpha i} v_i \]

\[ P_{\alpha \beta} = \delta_{\alpha \beta} - 4 \sum_{i < j} \text{Re}(U_{\alpha i}^* U_{\beta j}^* U_{\alpha j} U_{\beta i}) \sin^2 \Delta_{ij} \]

\[ + 2 \sum_{i > j} \text{Im}(U_{\alpha i}^* U_{\beta j}^* U_{\alpha j} U_{\beta i}) \sin 2\Delta_{ij} \]

\[ \Delta_{ij} = \Delta m_{ij}^2 L / 4E \]

\[ \Delta m_{ij}^2 = m_i^2 - m_j^2 \]

\[ \bar{\nu} : U \rightarrow U^* \]
Matter effect

The propagation is different in matter due to interactions

- Mixing angle in matter is defined with respect to the matter eigenstates.
- Mixing angle and masses in matter are determined by diagonalizing the effective Hamiltonian in matter

Courtsey: A. Yu. Smirnov
Two flavour case

The propagation equation

\[
\frac{d}{dt} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = H_{F}^{\text{mat}} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}
\]

\[
H_{F}^{\text{mat}} = E I + \frac{1}{2E} U \begin{pmatrix} m_1^2 & 0 \\ 0 & m_2^2 \end{pmatrix} U^\dagger + \frac{1}{2E} \begin{pmatrix} A & 0 \\ 0 & 0 \end{pmatrix}
\]

Due to charge current interaction of \( \nu_e \) with electrons

\[
A = 2EV_{CC} \\
V_{CC} = \sqrt{2}G_Fn_e
\]

\( \nu_\mu, \nu_\tau \) only neutral current interaction with electrons same matter potential

No matter effect for two generations in the \( \nu_\mu - \nu_\tau \) channel to the leading order
MSW resonance

In matter, only $\nu_e$'s undergoes Charged current interaction $\rightarrow$ an effective potential of $\sqrt{2}G_F N_e$

**Effective mixing angle $\theta_M$ in matter**

$$\tan 2\theta_M = \frac{\Delta m^2 \sin 2\theta}{\Delta m^2 \cos 2\theta - 2\sqrt{2}G_F n_e E}$$

$$\Delta m^2 \cos 2\theta = 2\sqrt{2}G_F n_e E, \quad \theta_M \rightarrow \pi/4 \quad \text{MSW Resonance}$$


For antineutrinos the potential changes sign

Resonance occurs for $\Delta m^2 < 0$

Matter effect is sensitive to the ordering of the mass states
Evidence of MSW effect from solar data

Fogli and Lisi, New Journal of Physics, 2004
Matter Effect: Three flavours

The propagation equation in matter for three flavours

\[ \tilde{H} = \frac{1}{2E} [U \begin{pmatrix} 0 & 0 & 0 \\ 0 & \Delta m_{21}^2 & 0 \\ 0 & 0 & \Delta m_{31}^2 \end{pmatrix} U^\dagger + \begin{pmatrix} A & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ] \]

\( \Delta_{21} = 0 \) Approximation
As \( \Delta_{21} < < \Delta_{31} \)
Validity: \( L/E < < 1/\Delta_{21} \)

Resonance in the 1-3 sector

Better for resonance energies, higher baselines

\( \alpha - s_{13} \) Approximation

\[ \alpha = \Delta_{21}/\Delta_{31} \sim 0.03 \]
\[ \sin \theta_{13} \sim 0.15 \]
Series expansion in terms of \( \alpha, s_{13} \)

Better for away from resonance, lower baselines
Matter resonance

\[ \tan 2\theta_{13}^m = \frac{\Delta m_{31}^2 \sin 2\theta_{13}}{\Delta m_{31}^2 \cos 2\theta_{13} \pm 2\sqrt{2} G_F n_e E} \]

\[ (\Delta m_{31}^2)^m = \sqrt{(\Delta m_{31}^2 \cos 2\theta_{13} - A)^2 + (\Delta m_{31}^2 \sin 2\theta_{13})^2} \]

For \( \Delta m_{31}^2 > 0 \) resonance in neutrinos

For \( \Delta m_{31}^2 < 0 \) resonance in antineutrinos

\[ E_{res} = \frac{|\Delta m_{atm}^2| \cos 2\theta_{13}}{2\sqrt{2} G_F N_e} \]

Hierarchy sensitivity

<table>
<thead>
<tr>
<th>L (km)</th>
<th>( \rho_{avg} ) (g/cc)</th>
<th>( E_{res} ) (GeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1000</td>
<td>3.00</td>
<td>9.9</td>
</tr>
<tr>
<td>3000</td>
<td>3.32</td>
<td>9.4</td>
</tr>
<tr>
<td>5000</td>
<td>3.59</td>
<td>8.7</td>
</tr>
<tr>
<td>7000</td>
<td>4.15</td>
<td>7.5</td>
</tr>
<tr>
<td>10000</td>
<td>4.76</td>
<td>6.6</td>
</tr>
</tbody>
</table>
Is matter effect maximum at resonance?

\[ \mathcal{P}_{\nu_\mu \rightarrow \nu_e}^m = s_{23}^2 \sin^2 2\theta_{13} \sin^2 [\Delta_{31}^m L/E] \]

Matter effect is observed near \( E \sim E_{\text{res}} \), where the amplitude is large, but we also require large phase.

\[ \mathcal{P}_{\nu_\mu \rightarrow \nu_e}^m \] is maximum when simultaneously

\[ \sin^2 (2\theta_{13})^m = 1 \]
\[ \sin^2 \Delta_{31}^m = 1 = \sin^2((2p + 1)\pi/2) \]

This gives the maximum matter effect condition for \( L \):

\[ [\rho L]_{\mu e}^{\text{max}} = \frac{(2p + 1)\pi \times 5.18 \times 10^3}{\tan 2\theta_{13}} \text{ km \ gm/cc} \]
Maximum matter effect

\[ P(\nu_\mu \rightarrow \nu_e) \]

For \( \sin^2 2\theta_{13} = 0.1, p=0 \), the maximum matter effect comes at \( L \sim 10,000 \text{ km} \)

Maximum matter effect in $P_{\nu_\mu \rightarrow \nu_\mu}$

$$P_{\mu\mu}^m = 1 - \cos^2 \theta_{13}^m \sin^2 2\theta_{23} \sin^2 [1.27(\Delta_{31} + A + \Delta_{31}^m) L/2E]$$
$$- \sin^2 \theta_{13}^m \sin^2 2\theta_{23} \sin^2 [1.27(\Delta_{31} + A - \Delta_{31}^m) L/2E]$$
$$- \sin^4 \theta_{23} \sin^2 2\theta_{13}^m \sin^2 (1.27\Delta_{31}^m L/E)$$

Condition for maximum matter effect in $P_{\mu\mu}$ is

- $E_{\nu_{\text{peak}}}^\nu = E_{\text{res}}$
- $1.27 \frac{\Delta m_{31}^2 L}{E_{\text{peak}}} = p \pi$

This gives

$$[\rho L]_{\mu\mu}^{\text{max,peak}} \simeq p \pi \times 10^4 \times \cos 2\theta_{13} \text{ km}$$

for $p=1$, $L \simeq 7000 \text{ km}$
Different behaviour at 9700 km

- Fall at peak but rise at dip
Matter effect in $P_{\nu_\mu} \rightarrow \nu_\tau$

Maximum matter effect when

$$E_{\text{res}} \sim E_{\text{peak}}$$

$$[\rho L]_{\mu\tau}^{\text{max}} \simeq (2p + 1) \pi 5.18 \times 10^3 (\cos 2\theta_{13}) \text{Km gm/cc.}$$

For $p = 1$ and $\sin^2 2\theta_{13} = 0.1$

$$L = 9700 \text{ km}$$

$$\Delta P_{\mu\tau} = P_{\mu\tau}^m - P_{\mu\tau}^v$$

$$\Delta P_{\mu\tau} \simeq \cos^4 \left[ \sin 2\theta_{13}(2p + 1) \frac{\pi}{4} \right] - 1$$

$$= -0.7$$
Matter effect in $P_{\nu_\mu \rightarrow \nu_\tau}$

The vacuum mass eigenstate $\nu_1$ is largely $\nu_e$, $\nu_3$ is largely $\nu_\mu$ & $\nu_\tau$ and $\nu_2$ has no $\nu_e$ component.

Matter effect (increasing $A$) causes $\nu_e$ in $\nu_1^m$ to decrease & $\nu_\mu, \nu_\tau$ to increase. At $A = A_{res}$ they are 50%. Similarly, $\nu_e$ in $\nu_3^m$ increases to 50%.

At resonance, all matter-dependent mass eigenstates $\nu_1^m, \nu_2^m$ & $\nu_3^m$ have significant $\nu_\mu$ & $\nu_\tau$ components.

$P(\nu_\mu \rightarrow \nu_\tau)$ depends on all 3 mass-squared differences.
Matter effect in $P_{\nu_\mu \rightarrow \nu_\tau}$

- No matter effect in two flavor $\nu_\mu - \nu_\tau$ oscillation since both interact via neutral current
- At 9700 km significant matter effect in $P_{\mu\tau}$
- 50% rise in $P_{\mu e}$, 20% rise in $P_{\mu\mu}$
- $P_{\mu\tau} = 1 - P_{\mu e} - P_{\mu\mu}$
- $\Delta P_{\mu\tau} = -(\Delta P_{\mu e} + \Delta P_{\mu\mu})$
- 70% matter induced fall in $P_{\mu\tau}$
- Genuine three flavour effect


Baselines covered by atmospheric neutrinos
Hierarchy sensitivity

\[ \Delta P_{\alpha\beta} = P_{\alpha\beta}^{NH} - P_{\alpha\beta}^{IH} \]

\[ N_\mu = N_\mu^0 P_{\mu\mu} + N_e^0 P_{e\mu} \]

\[ \Delta N_\mu = N_\mu^0 \Delta P_{\mu\mu} + N_e^0 \Delta P_{e\mu} \]
Hierarchy sensitivity

\[ \Delta P_{\alpha\beta} = P_{\alpha\beta}^{NH} - P_{\alpha\beta}^{IH} \]

\[ N_\tau = N_\mu^0 \ P_{\mu\tau} + N_e^0 \ P_{e\tau} \]

\[ \Delta N_\tau = N_\mu^0 \ \Delta P_{\mu\tau} + N_e^0 \ \Delta P_{e\tau} \]

Hierarchy sensitivity from higher baselines and higher energies in tau events
What happens in presence of non-standard interactions?

With Animesh Chatterjee and Supriya Pan ..... ongoing
Non-standard interactions

Standard-NC interaction

\[ \nu_\alpha + f \rightarrow \nu_\alpha + f \]

Non-Standard NC interaction

\[ \nu_\alpha + t \rightarrow \nu_\beta + t \]

\[ \mathcal{L} = -G^{\alpha\beta} \varepsilon^{f}_{\alpha\beta} \bar{\nu}_\alpha \gamma^\mu \nu_\beta \bar{f} \gamma_\mu f \]

\[ \varepsilon^{\alpha\beta} = \sum_{f=e,u,d} \frac{N_f}{N_e} \varepsilon^{f}_{\alpha\beta} \]

For earth’s matter

\[ N_u \approx N_d \approx 3N_e \]

\[ \varepsilon^{e}_{\alpha\beta} \approx \varepsilon^{e}_{\alpha\beta} + 3 \varepsilon^{u}_{\alpha\beta} + 3 \varepsilon^{d}_{\alpha\beta} \]

\[ H = \frac{1}{2E} \left[ U \text{diag}(0, \Delta m^{2}_{21}, \Delta m^{2}_{31}) U^\dagger + V \right] \]

\[ V \Rightarrow \text{matter potential in presence of NSI,} \]

\[ V = A \begin{pmatrix} 1 + \varepsilon^{ee} & \varepsilon^{e\mu} e^{i\phi_{e\mu}} & \varepsilon^{e\tau} e^{i\phi_{e\tau}} \\ \varepsilon^{e\mu} e^{-i\phi_{e\mu}} & \varepsilon^{\mu\mu} & \varepsilon^{\mu\tau} e^{i\phi_{\mu\tau}} \\ \varepsilon^{e\tau} e^{-i\phi_{e\tau}} & \varepsilon^{\mu\tau} e^{-i\phi_{\mu\tau}} & \varepsilon^{\tau\tau} \end{pmatrix} \]

Here, \[ A \equiv 2\sqrt{2} G_F N_e E \] and \[ \varepsilon^{\alpha\beta} e^{i\phi_{\alpha\beta}} \equiv \sum_{f} \varepsilon_{\alpha\beta}^{fc} \frac{N_f}{N_e} \]
Effect of $\epsilon_{\mu\tau}$

$$H_{\text{mat}} = \frac{A}{2E} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & \epsilon_{\mu\tau} e^{i\phi_{\mu\tau}} \\ 0 & \epsilon_{\mu\tau} e^{-i\phi_{\mu\tau}} & 0 \end{bmatrix}$$

$$H_F = \frac{1}{2E} \left[ U \text{diag}(0,0,\Delta_{31}) U^\dagger \right] + \frac{A}{2E} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & \epsilon_{\mu\tau} e^{i\phi_{\mu\tau}} \\ 0 & \epsilon_{\mu\tau} e^{-i\phi_{\mu\tau}} & 0 \end{bmatrix}$$

Main channels affected are $P_{\mu\mu}, P_{\mu\tau}$

Current bound:

$-0.07 < \epsilon_{\mu\tau} < 0.04$

Coloma et al. JHEP 2023
\[ P_{\nu_{\mu} \rightarrow \nu_{\tau}} \text{ in presence of NSI} \]

\[
P_{\mu \tau}^{\text{NSI}} = -\sin^2 2\theta_{13}^M \sin^2 2\theta_{23}^M \cos^2 \theta_{23}^M \sin^2 \left[ \frac{1.27 \Delta_{31}^M L}{E} \right] + \sin^2 2\theta_{23}^M \{ \cos^2 \theta_{13}^M \sin^2 \left[ \frac{1.27 \Delta_{32}^M L}{E} \right] + \sin^2 \theta_{13}^M \sin^2 \left[ \frac{1.27 \Delta_{21}^M L}{E} \right] \} \]

\[
\sin 2\theta_{13}^M = \sin 2\theta_{13}^m \left[ 1 - \frac{A \epsilon_{\mu \tau} \cos 2\theta_{13}^m \sin 2\theta_{23}^m}{\Delta_{31}^m} \right] \]

\[
\begin{align*}
E_3^M - E_1^M &= \Delta_{31}^m + A \epsilon_{\mu \tau} \cos \phi_{\mu \tau} \cos 2\theta_{13}^m \sin 2\theta_{23}^m \\
E_2^M - E_1^M &= \Delta_{21}^m - A \epsilon_{\mu \tau} \cos \phi_{\mu \tau} (1 + \sin^2 \theta_{13}^m) \sin 2\theta_{23}^m \\
E_3^M - E_2^M &= \Delta_{32}^m + A \epsilon_{\mu \tau} \cos \phi_{\mu \tau} \sin 2\theta_{23}^m (1 + \cos^2 \theta_{13}^m) 
\end{align*}
\]

In absence of NSI, OMSD

\[
P_{\mu \tau}^{\text{NSI}} = \cos^2 \theta_{13}^m \sin^2 2\theta_{23}^m [1.27 (\Delta_{31} + A + \Delta_{31}^m) L/2E] + \sin^2 \theta_{13}^m \sin^2 2\theta_{23}^m [1.27 (\Delta_{31} + A - \Delta_{31}^m) L/2E] - \cos^2 \theta_{23} P_{\mu e}^m
\]

In presence of NSI, OMSD

Resonance in absence of NSI

\[
\implies \sin 2\theta_{13}^m = 1 \implies \sin 2\theta_{13}^M = 1
\]
Matter effect due to NSI

Large effects in higher energies and baselines
Probabilities at 9700 km

Larger differences between SI, NSI at higher energies

NSI probabilities higher or lower than the standard probability depending on the sign of $\epsilon_{\mu\tau}$

There are energies where $+\epsilon_{\mu\tau}$, $-\epsilon_{\mu\tau}$ give same probability
Probability level analysis

\[ \chi^2 = \left( \frac{P(c_{\mu\tau} = x) - P(c_{\mu\tau} = 0)}{P(\epsilon_{\mu\tau} = 0)} \right)^2 \]

Marginalized over \( \delta_{CP}, \theta_{23}, \phi_{\mu\tau}(0,\pi) \)

Higher sensitivity at larger baselines and energies

Sensitivities seen in the \( P_{\mu\tau} \) channel
Bound on $\epsilon_{\mu\tau}$ using muon events

Using atmospheric muon events for a liquid argon detector

True neutrino energy up to 20 GeV
Impact of including the $\nu_\tau$ channel

$\nu_\tau$ Detection efficiency 30%

$\nu_\tau$ Detection efficiency 100%

Atmospheric neutrinos in DUNE 400 kt years, $E > 15$ - 200 GeV

Improvement in the bound by one order of magnitude by including higher energy events

The effect of the $\nu_\tau$ channel depends on the detection efficiency

Bakhti, Rajaee, Shin, PRD, 2022
Simulation of atmospheric $\nu_\tau$ events

Hadronic decay channels

Leptonic decay channels

Conrad, de Gouvea, Shalgar, Spitz, PRD 2010

Talk by Adam Aurisano
Talk by Barbara Yaggey Alvarez
NSI and beam $\nu_\tau$ events

De Gouvea, Kelly, Stenico, Pasquini, PRD, 2019
Summary

- In two generation no matter effect in $\nu_\mu - \nu_\tau$ channel
- In three generation appreciable matter effect around 9700 km
- Genuine three generation effect
- Contribution to hierarchy sensitivity from $\nu_\tau$ events?
- Considered NSI driven by $\epsilon_{\mu\tau}$
- Enhanced sensitivity due to $\nu_\tau$ events
Future plans

- Introduce other relevant NSI parameters
- Event level analysis of the $\nu_\tau$ events for atmospheric neutrinos
- Other new physics through $\nu_\tau$ channel and matter effect in atmospheric neutrinos
- Ultimate wish list — exploring combined sensitivity of beam and atmospheric $\nu_\tau$ events to explore possible synergies
Impossible is not a fact.
It’s an opinion.
Impossible is not a declaration.
It’s a dare.
Impossible is potential.
Impossible is temporary.
Impossible is nothing.

- Muhammad Ali
Thank you!!