Matter Effects in $P_{\mu\tau}$ at long baselines

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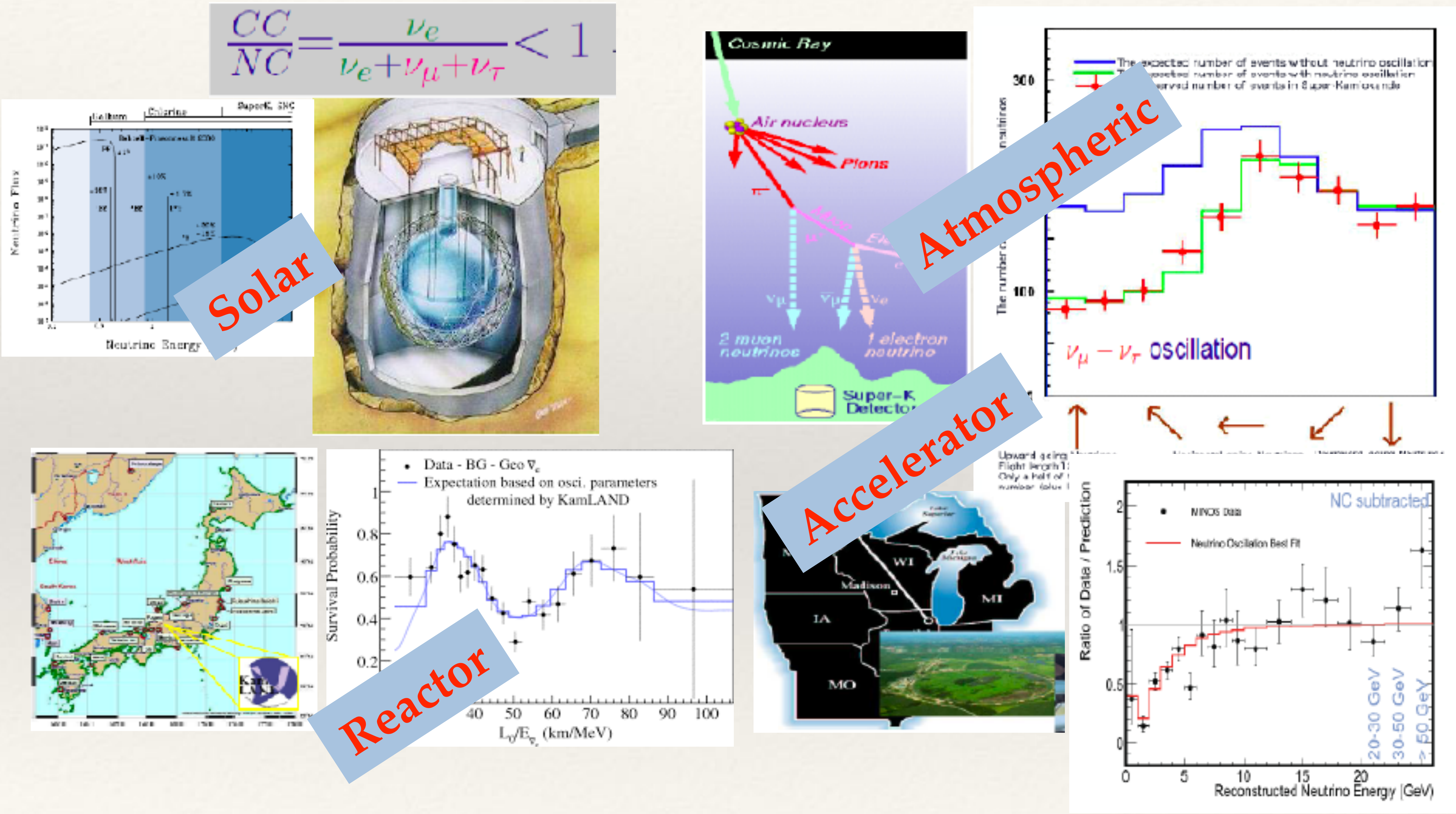
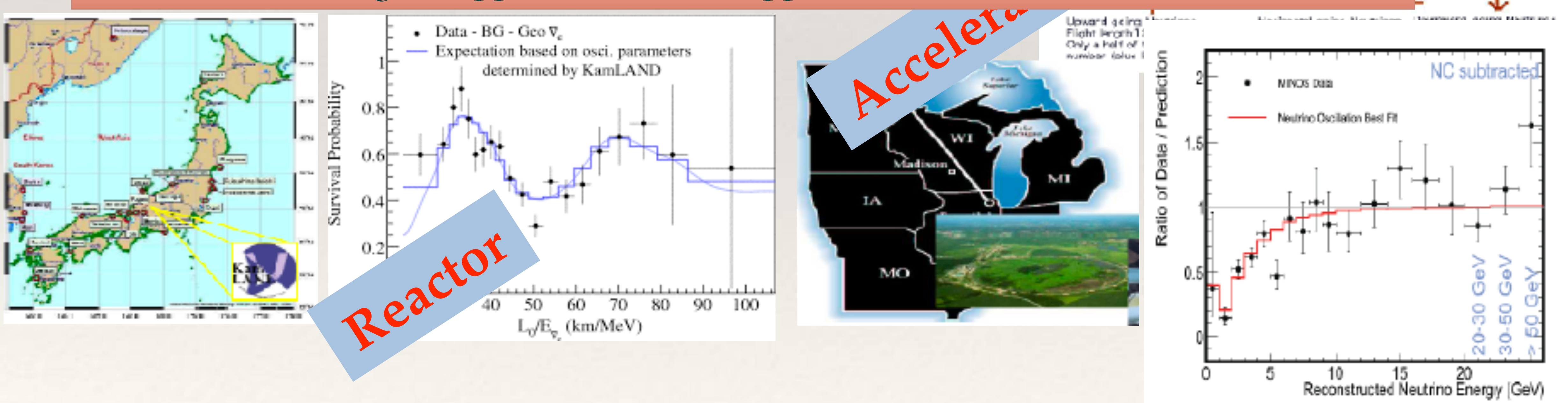
Neutrino oscillations



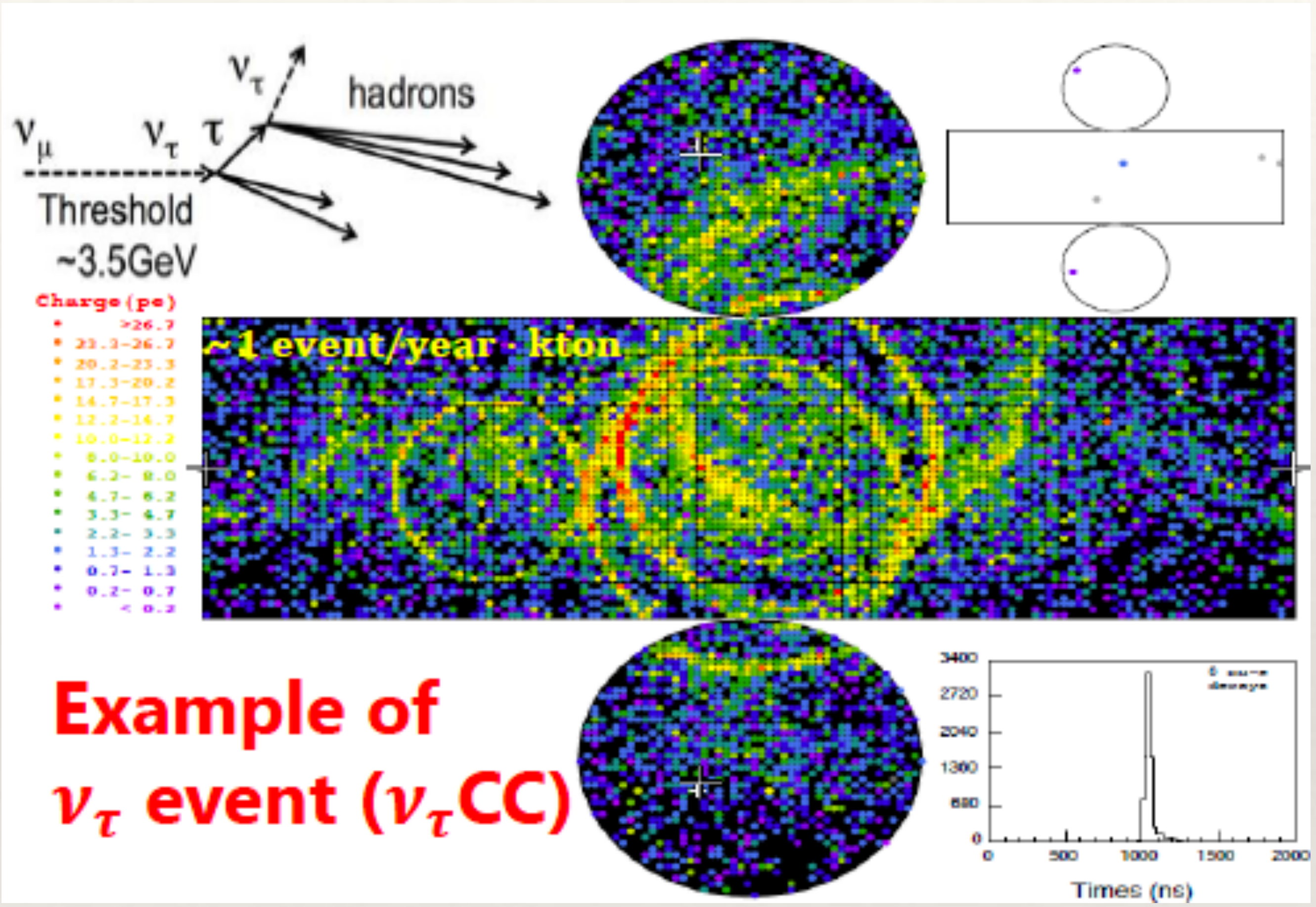
Neutrino oscillations

$$\frac{CC}{NC} = \frac{\nu_e}{\nu_e + \nu_\mu + \nu_\tau} < 1$$

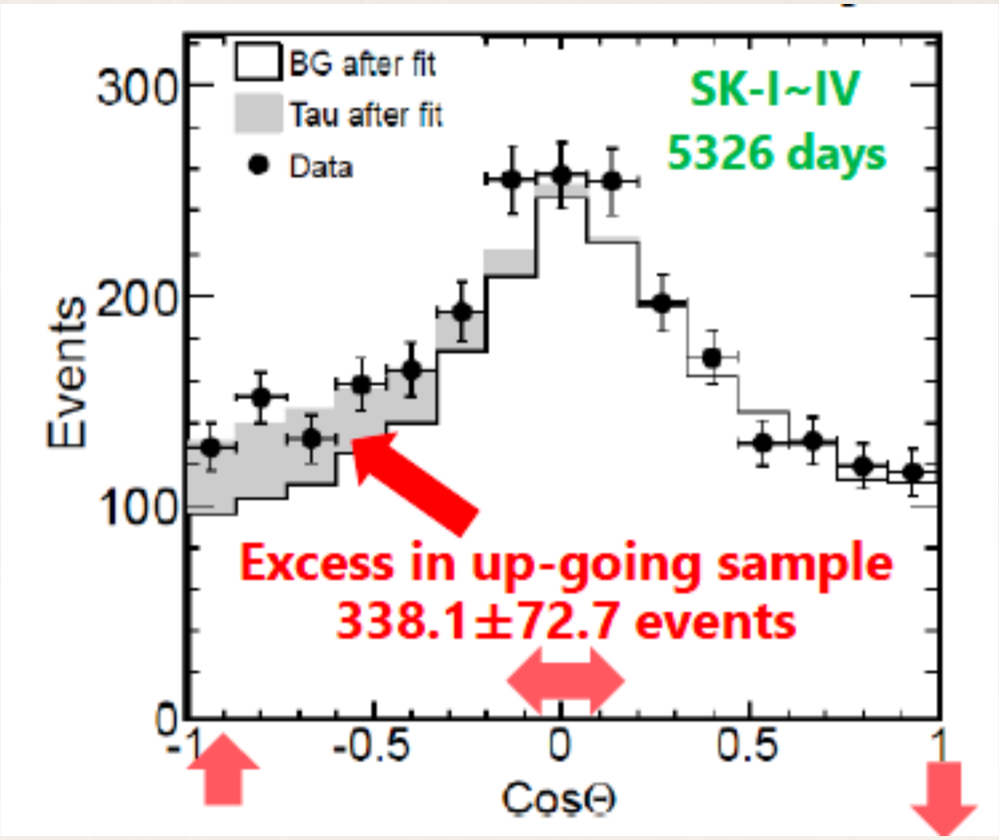
Based on measuring the appearance and disappearance of muon and electron neutrinos



Atmospheric ν_τ in Super-Kamiokande



Not event by event but statistical



SK excludes no ν_τ appearance at 4.6σ

From : Y. Nakano, ICNFP 2018

SK collaboration, PRD 2018

Three neutrino paradigm

- Measurement of non-zero θ_{13} in reactor experiments → three neutrino picture

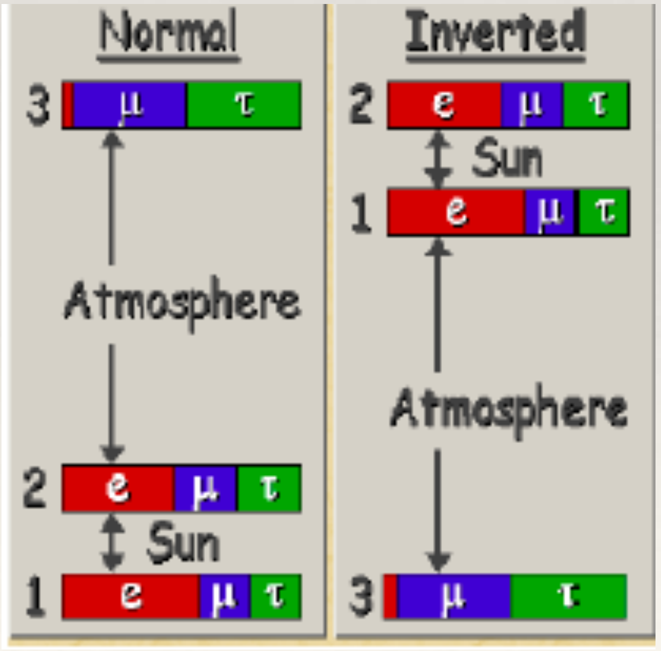
Atm +LBL | Sol+KL

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} 1 & & \\ & c_{23} & s_{23} \\ & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & & e^{-i\delta} s_{13} \\ & 1 & \\ -e^{i\delta} s_{13} & & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & \\ -s_{12} & c_{12} & \\ & & 1 \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

$c_{12} = \cos\theta_{12}$ etc., δ CP-violating phase

- Δm^2_{21}, θ_{12}, θ_{13} Solar + KamLND
- Δm^2_{31}, θ_{13} Reactor
- Δm^2_{31}, θ_{23}, θ_{13}, δ_{CP} Atm + LBL

Interplay among different sectors because of θ_{13}



Three generation oscillation parameters

NuFIT 5.2 (2022)

	Normal Ordering (best fit)		Inverted Ordering ($\Delta\chi^2 = 6.4$)	
	bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
$\sin^2\theta_{12}$	$0.303^{+0.012}_{-0.012}$	$0.270 \to 0.341$	$0.303^{+0.012}_{-0.011}$	$0.270 \to 0.341$
$\theta_{12}/^\circ$	$33.41^{+0.75}_{-0.72}$	$31.31 \to 35.74$	$33.41^{+0.75}_{-0.72}$	$31.31 \to 35.74$
$\sin^2\theta_{23}$	$0.451^{+0.019}_{-0.016}$	$0.408 \to 0.603$	$0.569^{+0.016}_{-0.021}$	$0.412 \to 0.613$
$\theta_{23}/^\circ$	$42.2^{+1.1}_{-0.9}$	$39.7 \to 51.0$	$49.0^{+1.0}_{-1.2}$	$39.9 \to 51.5$
$\sin^2\theta_{13}$	$0.02225^{+0.00056}_{-0.00059}$	$0.02052 \to 0.02398$	$0.02223^{+0.00058}_{-0.00058}$	$0.02048 \to 0.02416$
$\theta_{13}/^\circ$	$8.58^{+0.11}_{-0.11}$	$8.23 \to 8.91$	$8.57^{+0.11}_{-0.11}$	$8.23 \to 8.94$
$\delta_{CP}/^\circ$	232^{+36}_{-26}	$144 \to 350$	276^{+22}_{-29}	$194 \to 344$
$\frac{\Delta m^2_{21}}{10^{-5}\ \mathrm{eV}^2}$	$7.41^{+0.21}_{-0.20}$	$6.82 \to 8.03$	$7.41^{+0.21}_{-0.20}$	$6.82 \to 8.03$
$\frac{\Delta m^2_{3\ell}}{10^{-3}\ \mathrm{eV}^2}$	$+2.507^{+0.026}_{-0.027}$	$+2.427 \to +2.590$	$-2.486^{+0.025}_{-0.028}$	$-2.570 \to -2.406$

Neutrino oscillation (in vacuum)

Flavour states $\nu_\alpha = \Sigma_i U_{\alpha i} \nu_i$ Mass states

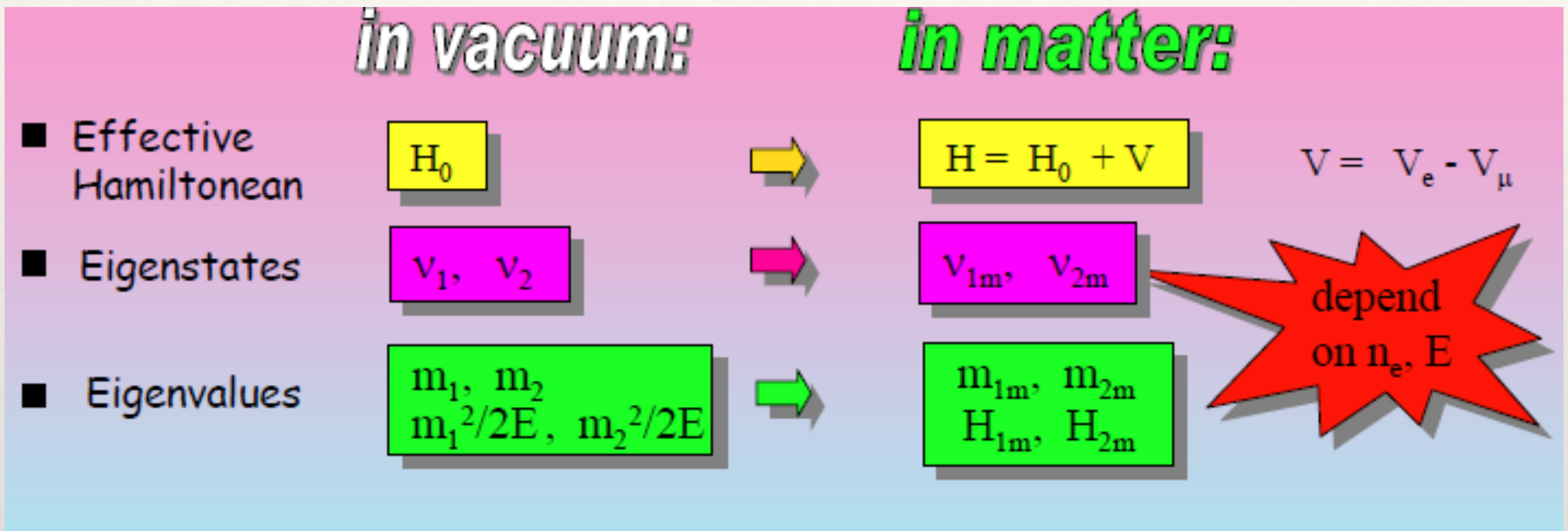
$$P_{\alpha\beta} = \delta_{\alpha\beta} - 4\sum_{i<j} \mathrm{Re}\left(U^*_{\alpha i} U^*_{\beta j} U_{\alpha j} U_{\beta i}\right) \sin^2 \Delta_{ij} + 2\sum_{i>j} \mathrm{Im}\left(U^*_{\alpha i} U^*_{\beta j} U_{\alpha j} U_{\beta i}\right) \sin 2\Delta_{ij}$$

$\Delta_{ij} = \Delta m^2_{ij} L / 4E$ $\Delta m^2_{ij} = m^2_i - m^2_j$

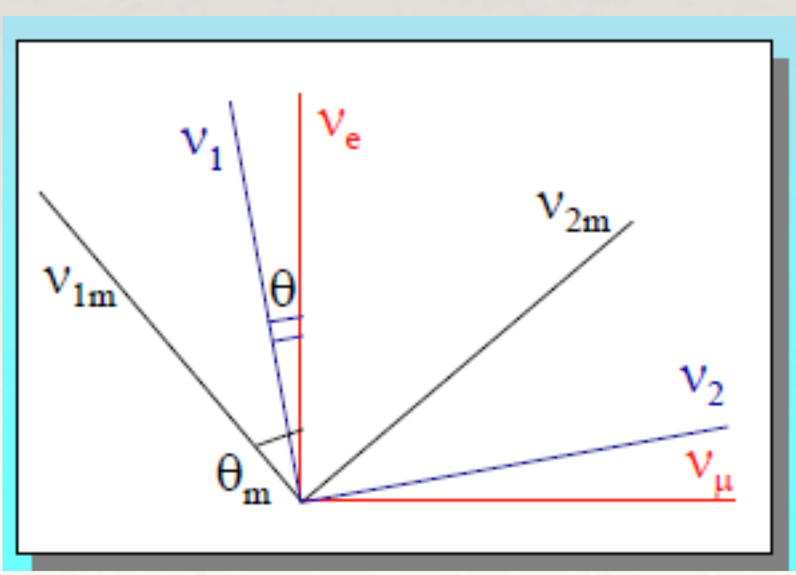
$\bar{\nu} : U \rightarrow U^*$

Matter effect

The propagation is different in matter due to interactions



Courtsey: A. Yu. Smirnov



- Mixing angle in matter is defined with respect to the matter eigenstates.
- Mixing angle and masses in matter are determined by diagonalizing the effective Hamiltonian in matter

Two flavour case

The propagation equation

$$\iota \frac{d}{dt}\begin{pmatrix}\nu_e \\ \nu_\mu\end{pmatrix} = H_F^{mat}\begin{pmatrix}\nu_e \\ \nu_\mu\end{pmatrix}$$

$$H_F^{mat} = E\mathbf{I} + \frac{1}{2E}U\begin{pmatrix}m_1^2 & 0 \\ 0 & m_2^2\end{pmatrix}U^\dagger + \frac{1}{2E}\begin{pmatrix}A & 0 \\ 0 & 0\end{pmatrix}$$

$$A = 2EV_{CC}$$
$$V_{CC} = \sqrt{2}G_F n_e$$

Due to charge current interaction of ν_e with electrons

ν_μ , ν_τ → only neutral current interaction with electrons → same matter potential

No matter effect for two generations in the $\nu_\mu - \nu_\tau$ channel to the leading order

MSW resonance

- In matter, only ν_e's undergoes Charged current interaction $\longrightarrow$ an effective potential of $\sqrt{2}G_F N_e$

Effective mixing angle θ_M in matter

$$\tan 2\theta_M = \frac{\Delta m^2 \sin 2\theta}{\Delta m^2 \cos 2\theta - 2\sqrt{2} G_F n_e E}$$

$$\Delta m^2 \cos 2\theta = 2\sqrt{2} G_F n_e E,\ \theta_M \to \pi/4$$ MSW Resonance

L. Wolfenstein, PRD **17**, 1978 S.P. Mikhyev, A.Yu. Smirnov, SJNP **42**, 1985

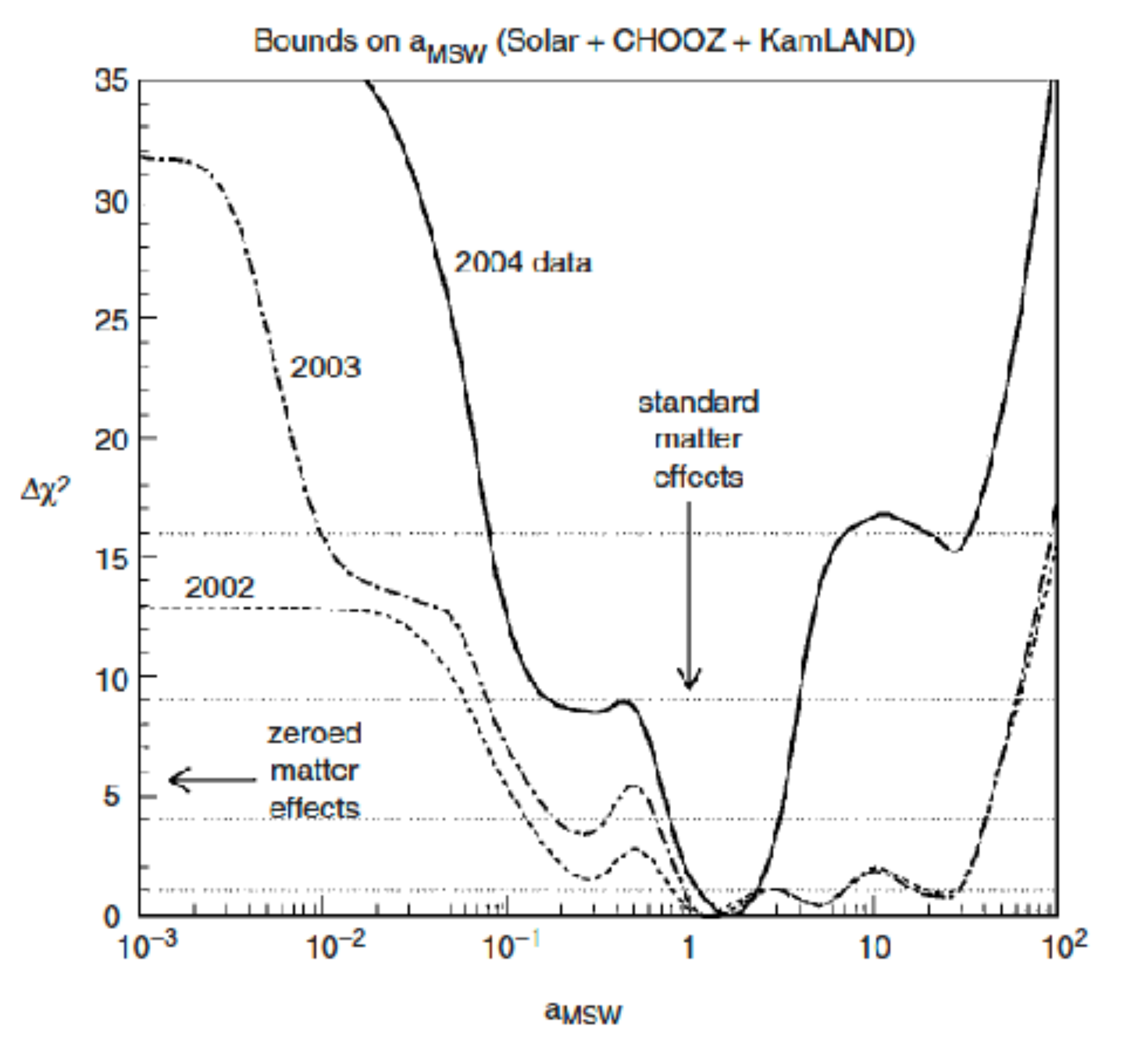
$$\Delta m_m^2 = [(\Delta m^2 \cos 2\theta - 2\sqrt{2} G_F n_e E)^2 + \Delta m^2 \sin^2 2\theta]^{1/2}$$

For antineutrinos the potential changes sign

Resonance occurs for $\Delta m^2 < 0$

Matter effect is sensitive to the ordering of the mass states

Evidence of MSW effect from solar data



Fogli and Lisi, New Journal of Physics, 2004

Matter Effect : Three flavours

- The propagation equation in matter for three flavours

$$\tilde{H} = \frac{1}{2E}[U \begin{pmatrix} 0 & 0 & 0 \\ 0 & \Delta m^2_{21} & 0 \\ 0 & 0 & \Delta m^2_{31} \end{pmatrix} U^\dagger + \begin{pmatrix} A & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$\Delta_{21} = 0$ Approximation

As $\Delta_{21} << \Delta_{31}$

Validity: $L/E << 1/\Delta_{21}$

Resonance in the 1-3 sector

Better for resonance energies, higher baselines

$\alpha - s_{13}$ Approximation

$\alpha = \Delta_{21}/\Delta_{31} \sim 0.03$

$\sin\theta_{13} \sim 0.15$

Series expansion in terms of α, s_{13}

Better for away from resonance, lower baselines

Matter resonance

$$\tan 2\theta_{13}{}^{m} = \frac{\Delta m^2_{31} \sin 2\theta_{13}}{\Delta m^2_{31} \cos 2\theta_{13} \pm 2\sqrt{2} G_F n_e E}$$

$$E_{res} = \frac{|\Delta m^2_{atm}| \cos 2\theta_{13}}{2\sqrt{2} G_F N_e}$$

$$(\Delta m^2_{31})^m = \sqrt{(\Delta m^2_{31} \cos 2\theta_{13} - A)^2 + (\Delta m^2_{31} \sin 2\theta_{13})^2}$$

- For $\Delta m^2_{31} > 0$ resonance in neutrinos
- For $\Delta m^2_{31} < 0$ resonance in antineutrinos

L (km)	ρ_{avg} (g/cc)	E_{res} (GeV)
1000	3.00	9.9
3000	3.32	9.4
5000	3.59	8.7
7000	4.15	7.5
10000	4.76	6.6

Hierarchy sensitivity

Is matter effect maximum at resonance ?

- $\mathcal{P}^m_{\nu_\mu \to \nu_e} = s^2_{23} \sin^2 2\theta_{13}{}^m \sin^2[\Delta^m_{31} L/E]$

- Matter effect is observed near $E \sim E_{res}$, where the amplitude is large, but we also require large phase.

$\mathcal{P}^m_{\nu_\mu \to \nu_e}$ is maximum when simultaneously

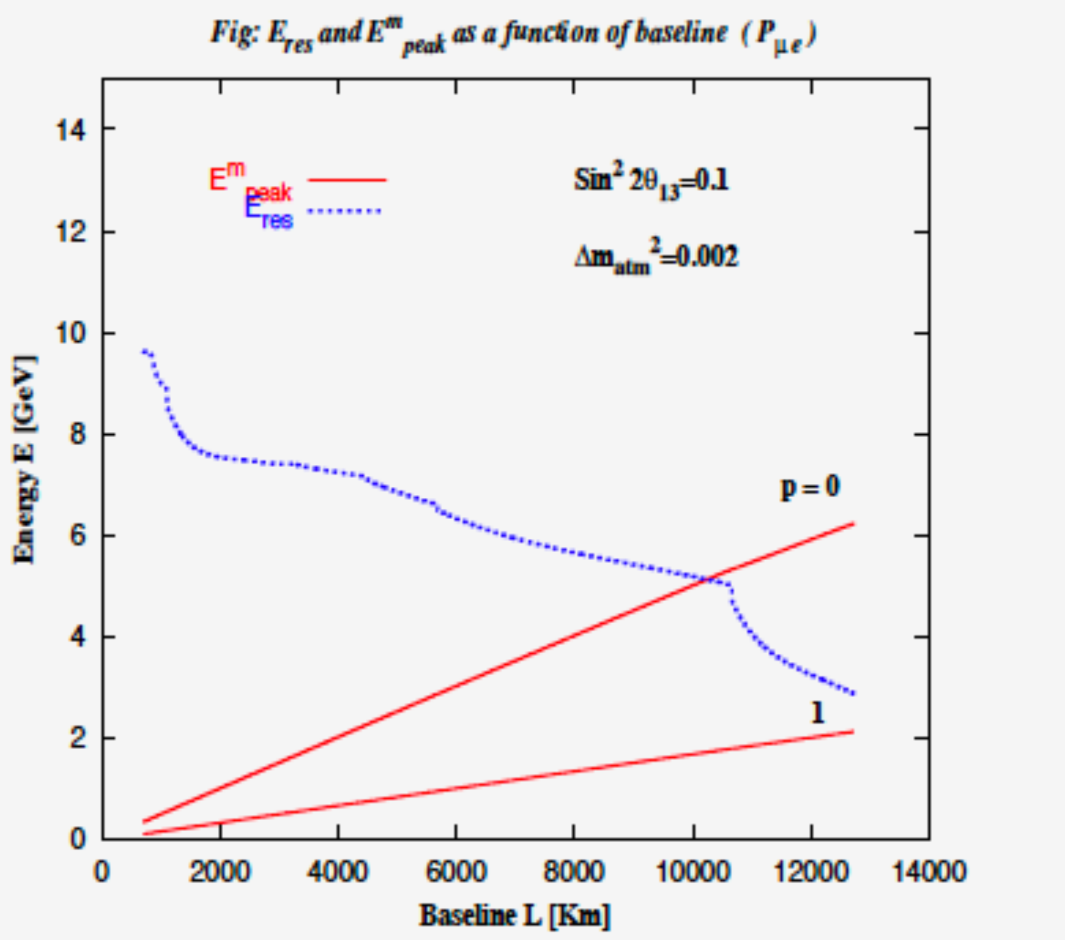
$$\sin^2(2\theta_{13})^m = 1$$
$$\sin^2 \Delta^m_{31} = 1 = \sin^2((2p+1)\pi/2)$$

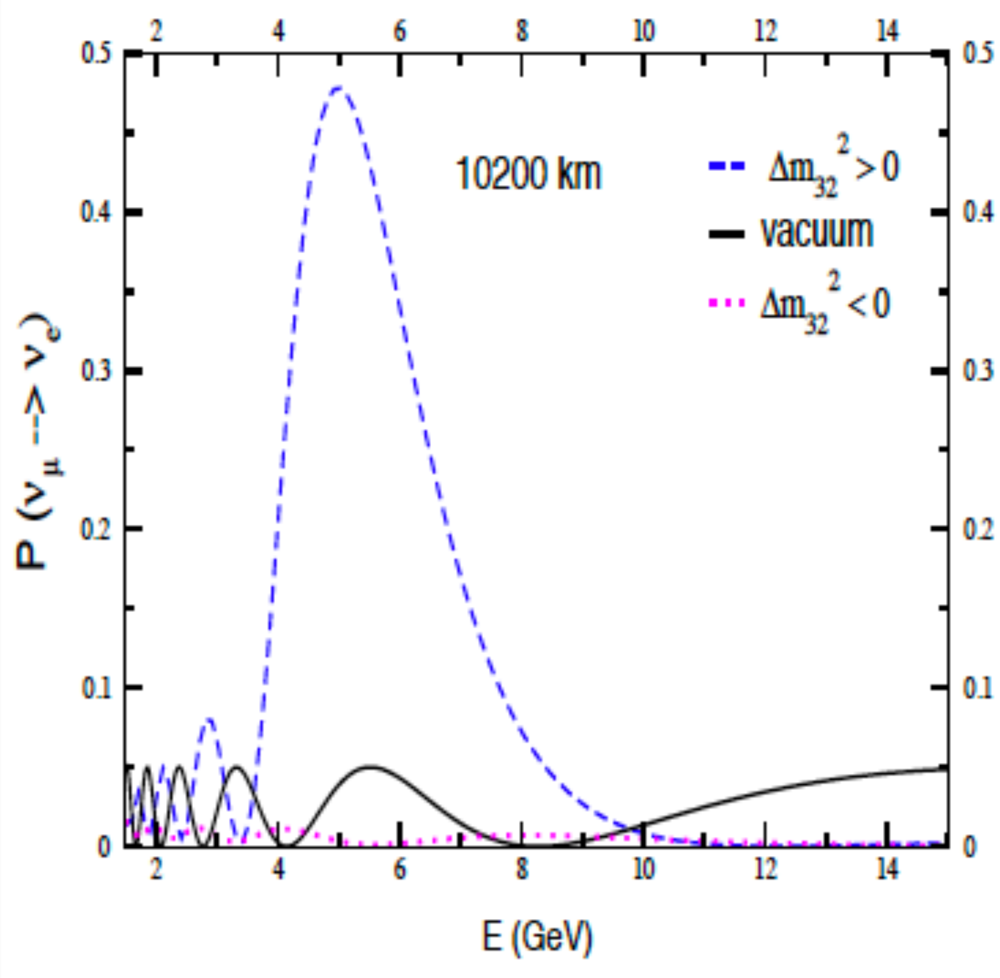
- This gives the maximum matter effect condition for L:

$$[\rho L]^{max}_{\mu e} = \frac{(2p+1)\pi \times 5.18 \times 10^3}{\tan 2\theta_{13}} \; km \; gm/cc$$

Maximum matter effect

$P(\nu_\mu \to \nu_e)$





- For $\sin^2 2\theta_{13} = 0.1$, p=0, the maximum matter effect comes at $L \sim$ 10,000 km

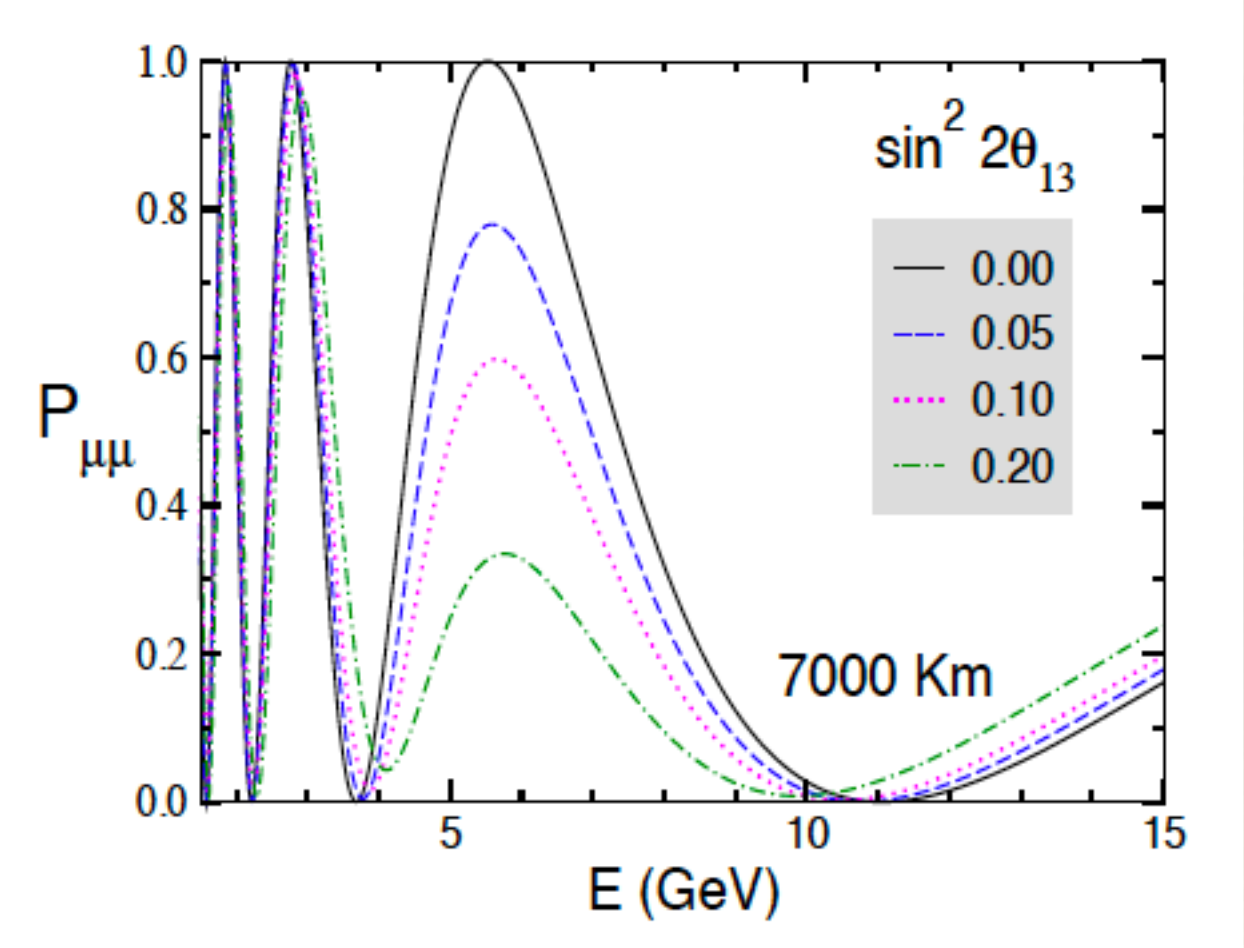
R. Gandhi, P. Ghoshal, S.G., P. Mehta, S. Umashanakar PRD 2005

Maximum matter effect in $P_{\nu_\mu \to \nu_\mu}$

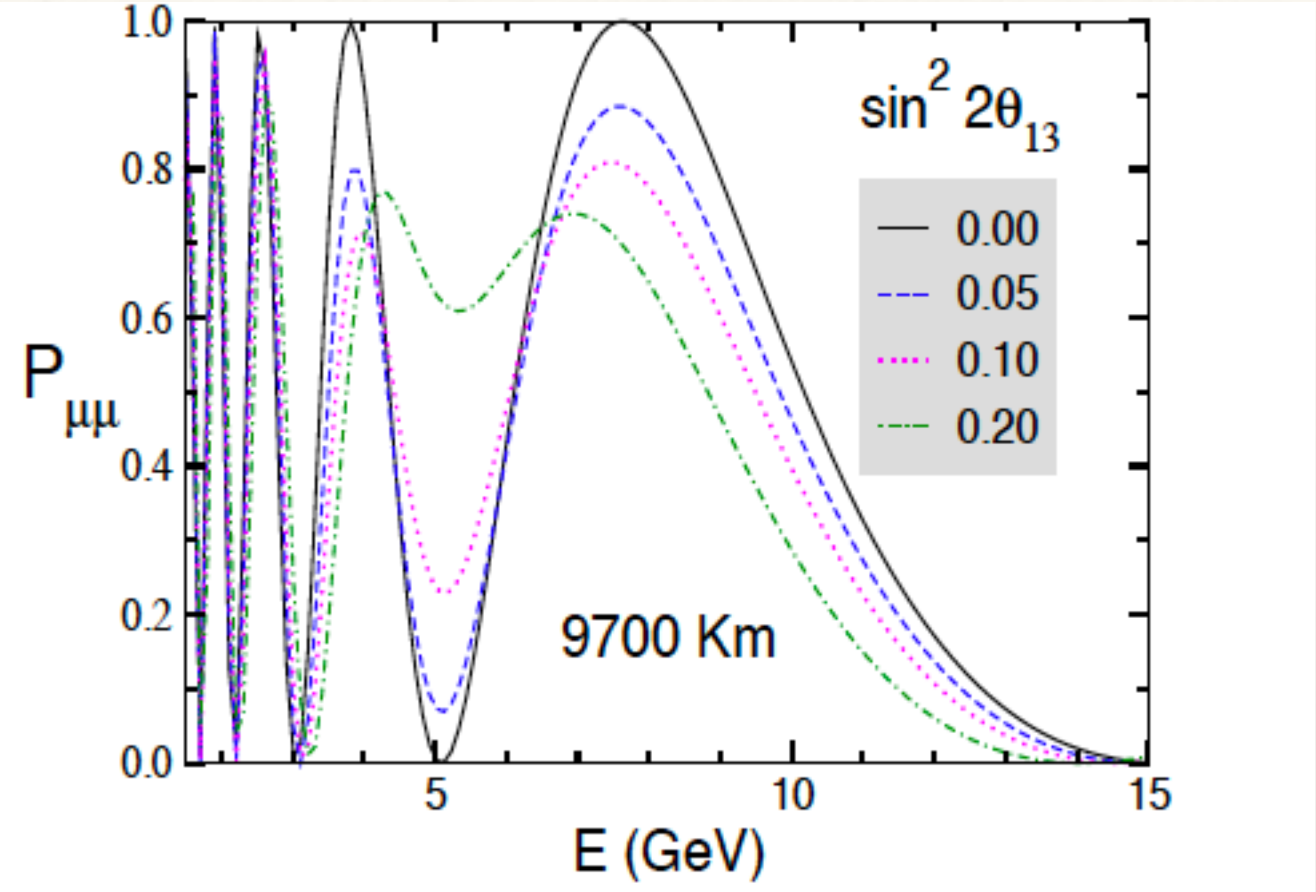
$$P^m_{\mu\mu} = 1 - \cos^2\theta^m_{13}\sin^2 2\theta_{23}\sin^2\left[1.27(\Delta_{31} + A + \Delta^m_{31})\mathrm{L}/2\mathrm{E}\right]$$
$$-\sin^2\theta^{\mathrm{m}}_{13}\sin^2 2\theta_{23}\sin^2\left[1.27(\Delta_{31} + \mathrm{A} - \Delta^{\mathrm{m}}_{31})\mathrm{L}/2\mathrm{E}\right]$$
$$-\sin^4\theta_{23}\sin^2 2\theta^{\mathrm{m}}_{13}\sin^2\left(1.27\Delta^{\mathrm{m}}_{31}\mathrm{L}/\mathrm{E}\right)$$

- Condition for maximum matter effect in $P_{\mu\mu}$ is
  - $\mathrm{E}^{\mathrm{v}}_{\mathrm{peak}} = \mathrm{E}_{\mathrm{res}}$
  - $1.27\,\frac{\Delta \mathrm{m}^2_{31}\mathrm{L}}{\mathrm{E}^{\mathrm{v}}_{\mathrm{peak}}} = \mathrm{p}\,\pi$
- This gives
  $[\rho \mathrm{L}]^{\mathrm{max,peak}}_{\mu\mu} \simeq \mathrm{p}\,\pi \times 10^4 \times \cos 2\theta_{13}$ km
  for p =1, L $\simeq$ 7000 km

Fall in $P_{\mu\mu}$ in matter

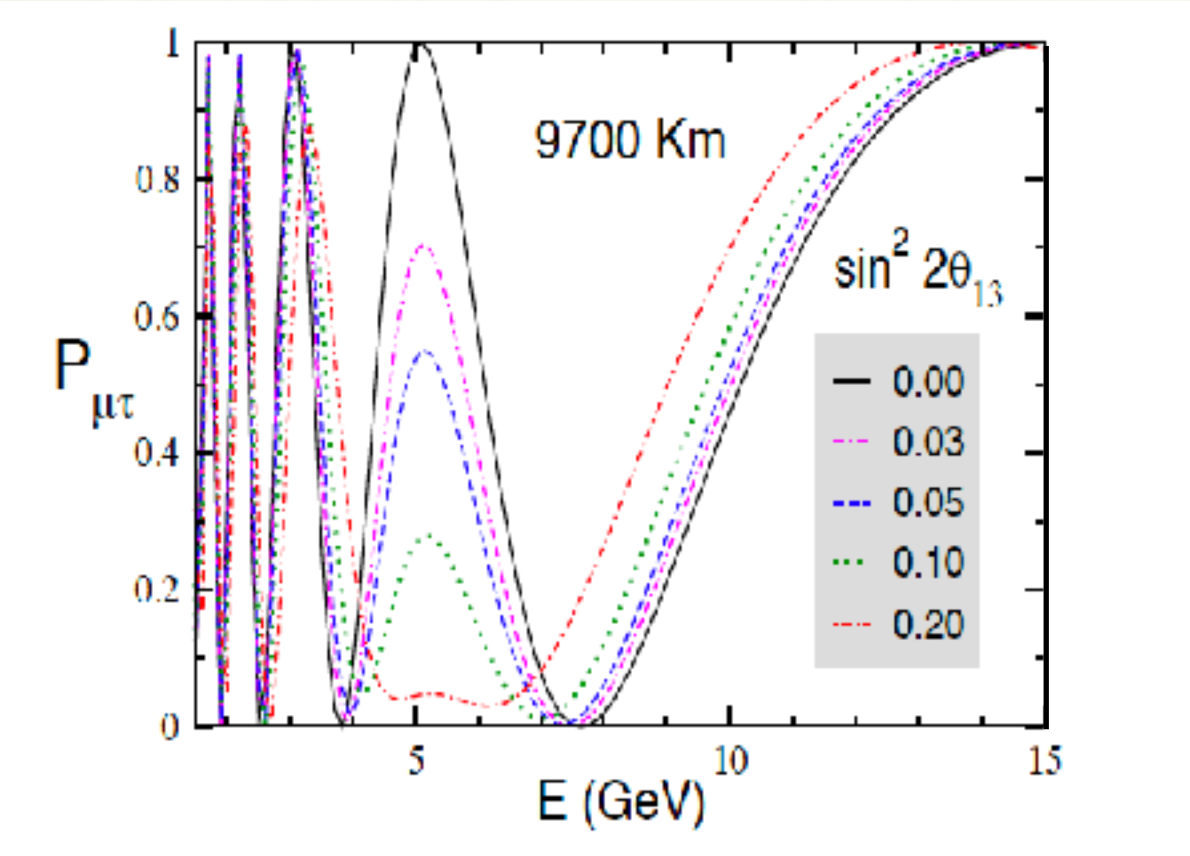


Different behaviour at 9700 km



- **Fall at peak but rise at dip**

Matter effect in $P_{\nu_\mu \to \nu_\tau}$



$$
\begin{aligned}
P^{m}_{\mu\tau} &= \cos^2\theta^{m}_{13}\sin^2 2\theta_{23}\sin^2\left[1.27(\Delta_{31} + A + \Delta^{m}_{31})L/2E\right] \\
&+ \sin^2\theta^{m}_{13}\sin^2 2\theta_{23}\sin^2\left[1.27(\Delta_{31} + A - \Delta^{m}_{31})L/2E\right] \\
&- \cos^2\theta_{23} P^{m}_{\mu e}
\end{aligned}
$$

Maximum matter effect when

$$E_{res} \simeq E^{v}_{peak}$$

$$[\rho L]^{max}_{\mu\tau} \simeq (2p+1)\,\pi\, 5.18 \times 10^3\,(\cos 2\theta_{13})\ \text{Km gm/cc.}$$

For $p = 1$ and $\sin^2 2\theta_{13} = 0.1$

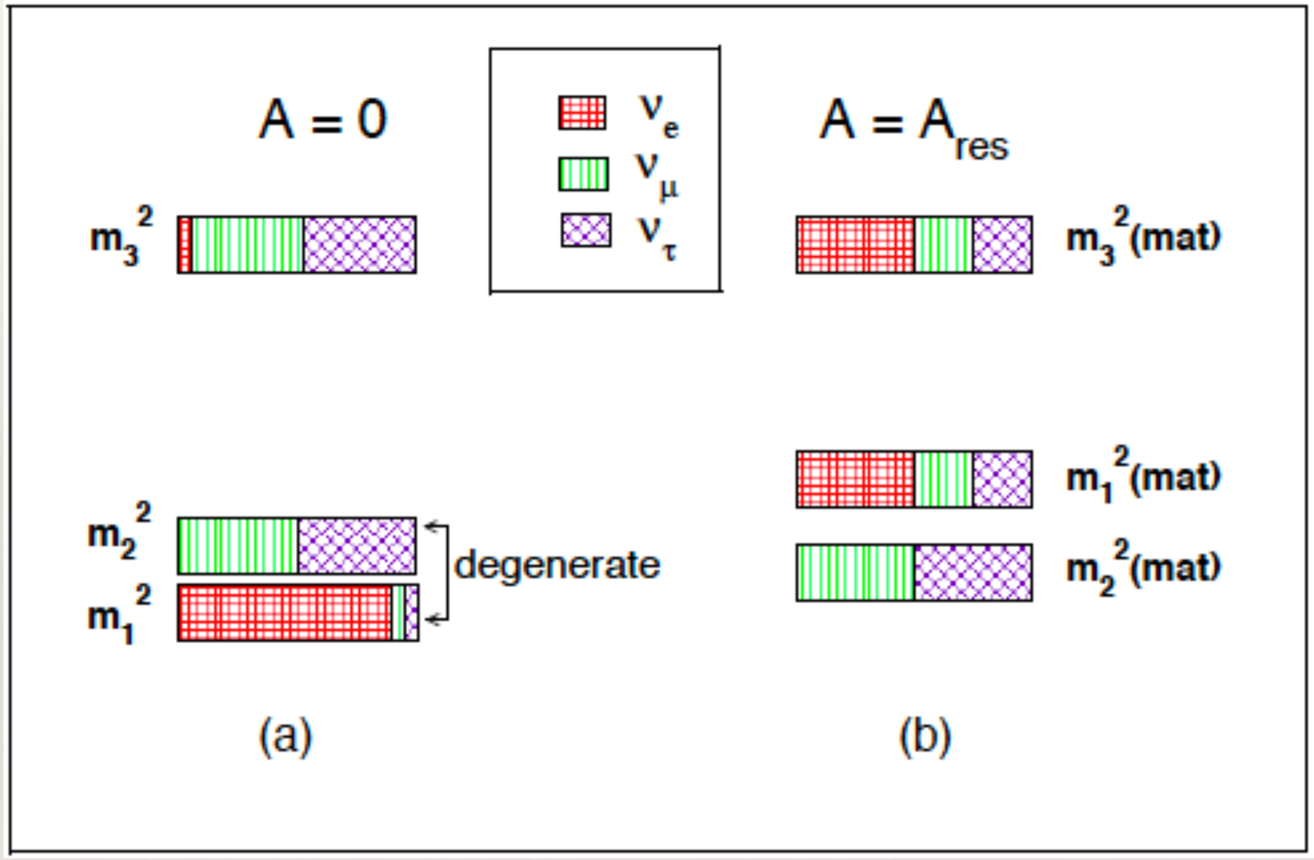
$$\Delta P_{\mu\tau} = P^{m}_{\mu\tau} - P^{v}_{\mu\tau}$$

L = 9700 km

$$\Delta P_{\mu\tau} \simeq \cos^4\left[\sin 2\theta_{13}(2p+1)\frac{\pi}{4}\right] - 1$$

= -0.7

Matter effect in $P_{\nu_\mu \to \nu_\tau}$



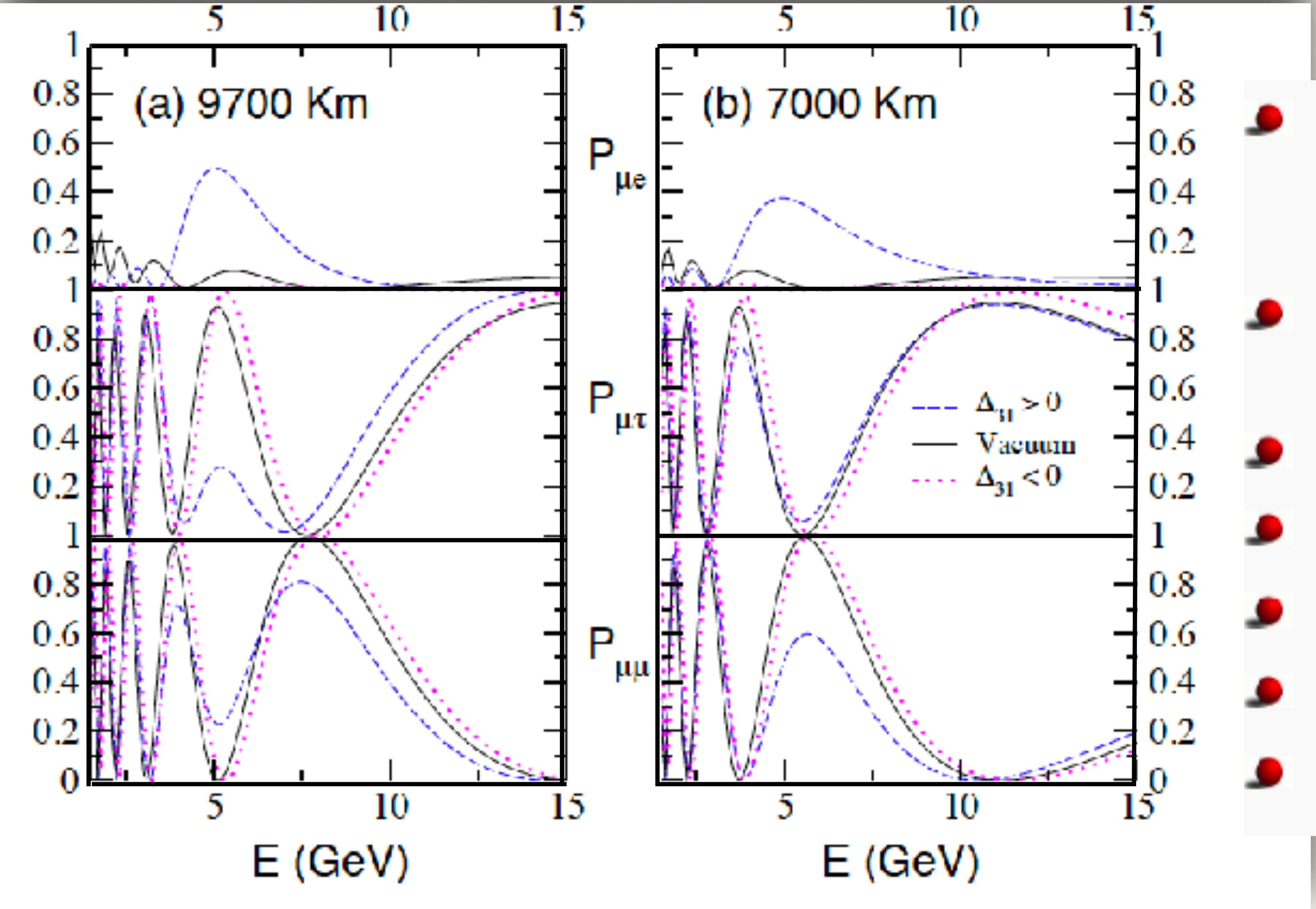
The vacuum mass eigenstate ν_1 is largely ν_e, ν_3 is largely ν_μ & ν_τ and ν_2 has no ν_e component.

Matter effect (increasing A) causes ν_e in ν_1^m to decrease & ν_μ, ν_τ to increase. At $A = A_{res}$ they are 50%. Similarly, ν_e in ν_3^m increases to 50%.

At resonance, all matter-dependent mass eigenstates ν_1^m, ν_2^m & ν_3^m have significant ν_μ & ν_τ components.

$P(\nu_\mu \to \nu_\tau)$ depends on all 3 mass-squared differences.

Matter effect in $P_{\nu_\mu \to \nu_\tau}$



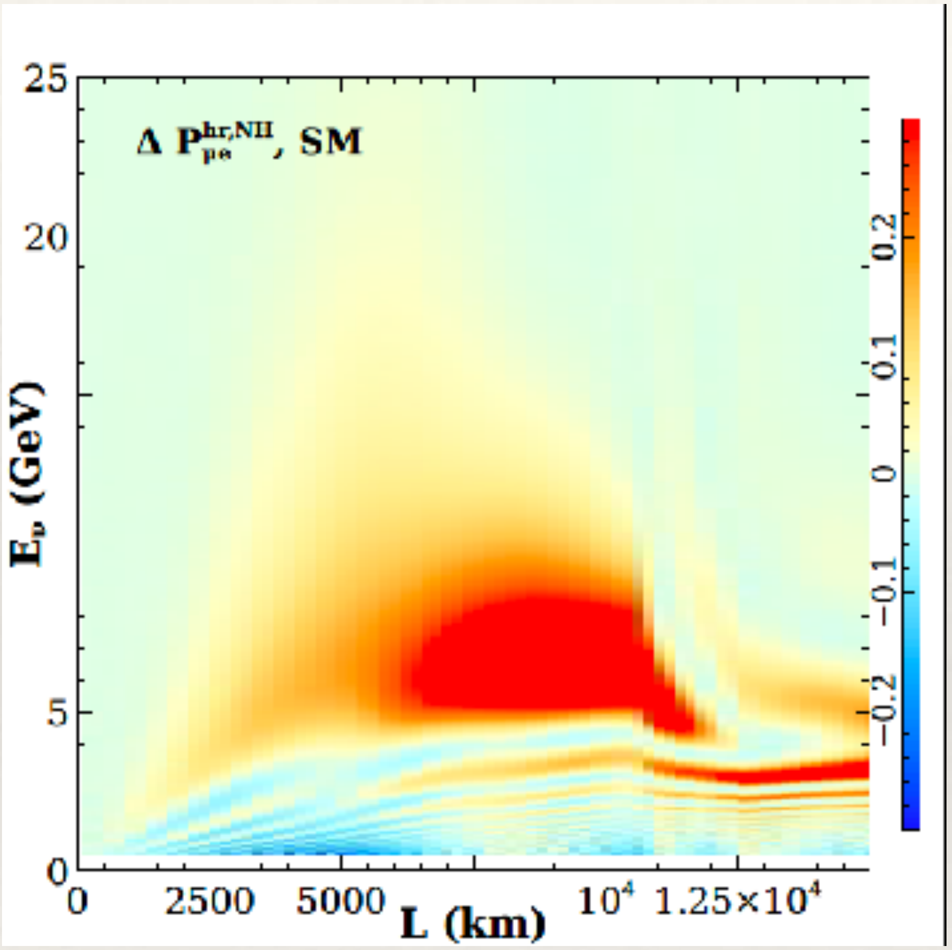
- No matter effect in two flavor $\nu_\mu - \nu_\tau$ oscillation since both interact via neutral current
- At 9700 km significant matter effect in $P_{\mu\tau}$
- 50% rise in $P_{\mu e}$, 20% rise in $P_{\mu\mu}$
- $P_{\mu\tau} = 1 - P_{\mu e} - P_{\mu\mu}$
- $\Delta P_{\mu\tau} = -(\Delta P_{\mu e} + \Delta P_{\mu\mu})$
- 70% matter induced fall in $P_{\mu\tau}$
- Genuine three flavour effect

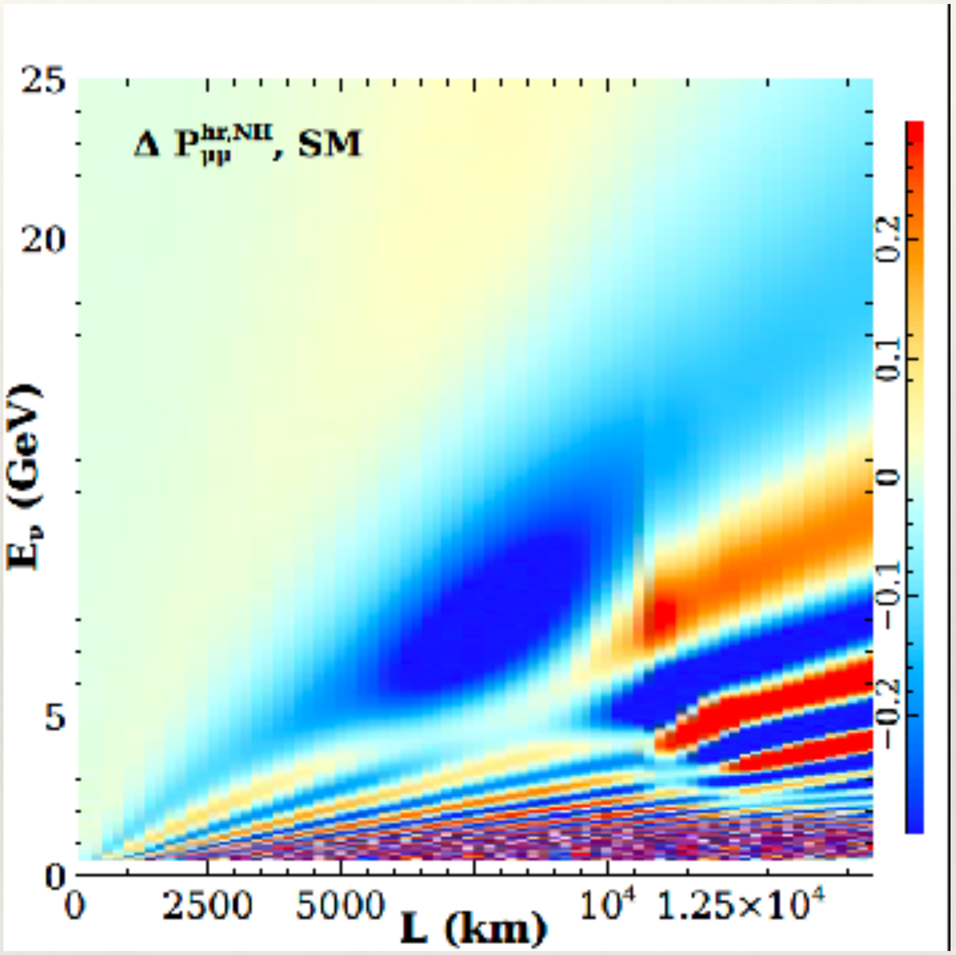
R. Gandhi, P. Ghoshal, S.G., P. Mehta, S. UmaShanakar , PRL, 2005

Baselines covered by atmospheric neutrinos

Hierarchy sensitivity

$$\Delta P_{\alpha\beta} = P_{\alpha\beta}^{NH} - P_{\alpha\beta}^{IH}$$



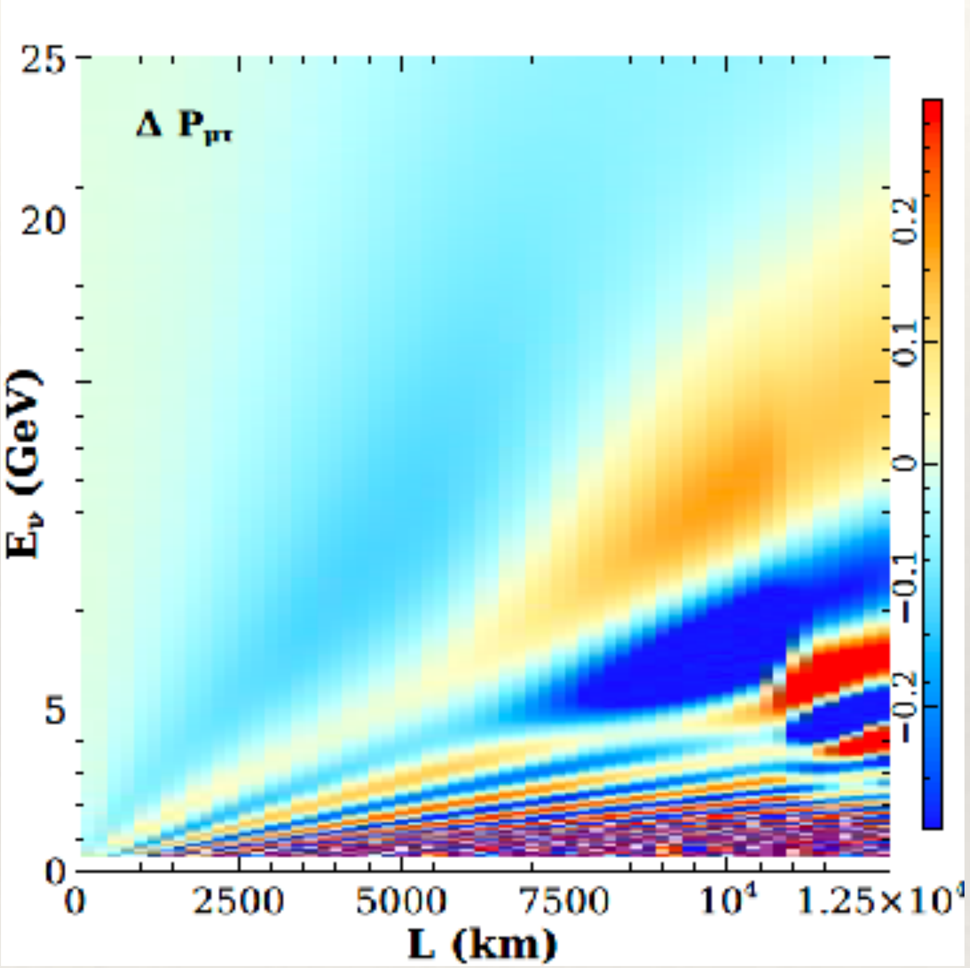


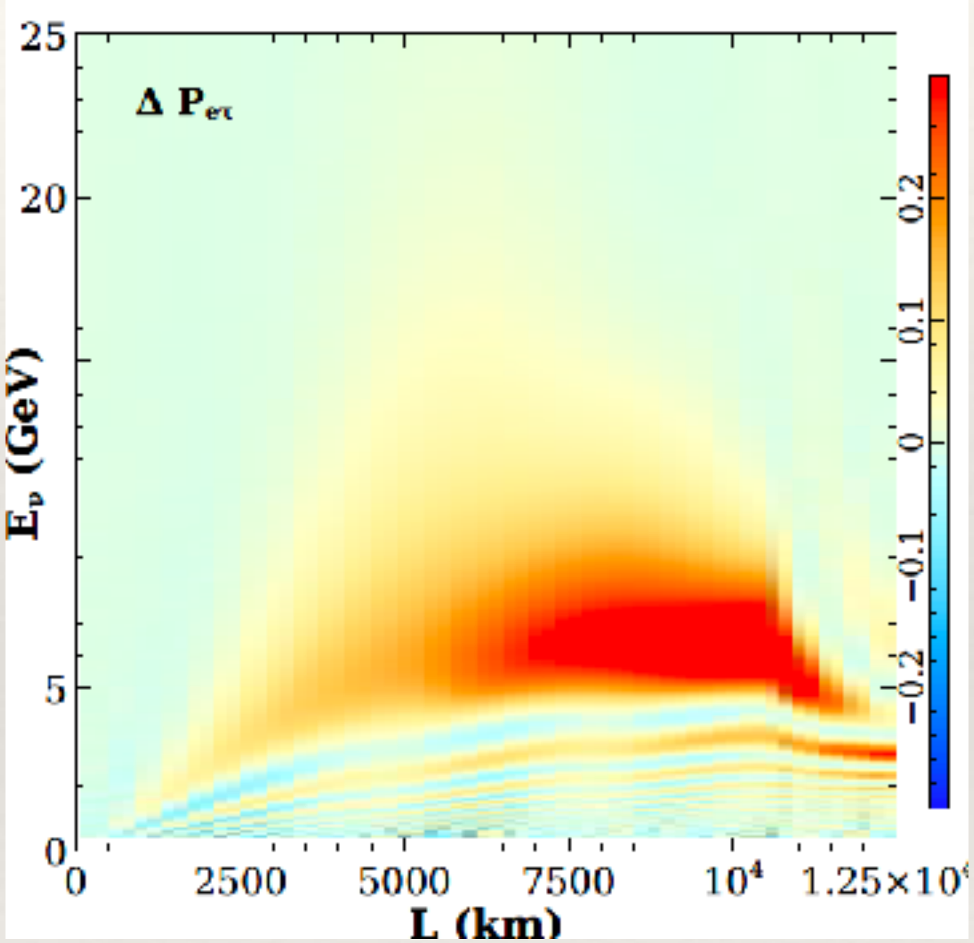
$$N_\mu = N_\mu^0 \ P_{\mu\mu} + N_e^0 \ P_{e\mu}$$

$$\Delta N_\mu = N_\mu^0 \ \Delta P_{\mu\mu} + N_e^0 \ \Delta P_{e\mu}$$

Hierarchy sensitivity

$$\Delta P_{\alpha\beta} = P_{\alpha\beta}^{NH} - P_{\alpha\beta}^{IH}$$





$$N_\tau = N_\mu^0 \; P_{\mu\tau} + N_e^0 \; P_{e\tau}$$

$$\Delta N_\tau = N_\mu^0 \; \Delta P_{\mu\tau} + N_e^0 \; \Delta P_{e\tau}$$

Hierarchy sensitivity from higher baselines and higher energies in tau events

What happens in presence of non-standard interactions ?

With Animesh Chatterjee and Supriya Pan ….., ongoing

Non-standard interactions

Standard-NC interaction

$$\nu_\alpha + f \to \nu_\alpha + f$$

Non-Standard NC interaction

$$\nu_\alpha + f \to \nu_\beta + f$$

$$\mathcal{L} = -G^{\alpha\beta}\epsilon^f_{\alpha\beta}\bar{\nu}_\alpha\gamma^\mu\nu_\beta\bar{f}\gamma_\mu f$$

$$\epsilon_{\alpha\beta} = \sum_{f=e,u,d}\frac{N_f}{N_e}\epsilon^f_{\alpha\beta}$$

$$H = \frac{1}{2E}\left[U\mathrm{diag}(0, \Delta m^2_{21}, \Delta m^2_{31})U^\dagger + V\right]$$

For earth's matter

$$N_u \approx N_d \approx 3N_e$$

$$\varepsilon_{\alpha\beta} \approx \varepsilon^e_{\alpha\beta} + 3\,\varepsilon^u_{\alpha\beta} + 3\,\varepsilon^d_{\alpha\beta}$$

$V \Rightarrow$ matter potential in presence of NSI,

$$V = A\begin{pmatrix} 1+\epsilon_{ee} & \epsilon_{e\mu}e^{i\phi_{e\mu}} & \epsilon_{e\tau}e^{i\phi_{e\tau}} \\ \epsilon_{e\mu}e^{-i\phi_{e\mu}} & \epsilon_{\mu\mu} & \epsilon_{\mu\tau}e^{i\phi_{\mu\tau}} \\ \epsilon_{e\tau}e^{-i\phi_{e\tau}} & \epsilon_{\mu\tau}e^{-i\phi_{\mu\tau}} & \epsilon_{\tau\tau} \end{pmatrix}.$$

Here, $A \equiv 2\sqrt{2}G_F N_e E$ and $\epsilon_{\alpha\beta}e^{i\phi_{\alpha\beta}} \equiv \sum_{f,C}\epsilon^{fC}_{\alpha\beta}\frac{N_f}{N_e}$

Effect of $\epsilon_{\mu\tau}$

$$H_{mat} = \frac{A}{2E}\begin{bmatrix}1 & 0 & 0\\ 0 & 0 & \epsilon_{\mu\tau}e^{\imath\phi_{\mu\tau}}\\ 0 & \epsilon_{\mu\tau}e^{-\imath\phi_{\mu\tau}} & 0\end{bmatrix}$$

Current bound ;

$-0.07 < \epsilon_{\mu\tau} < 0.04$

Coloma et al. JHEP 2023

$$H_F = \frac{1}{2E}\left[Udiag(0,0,\Delta_{31})U^\dagger\right] + \frac{A}{2E}\begin{bmatrix}1 & 0 & 0\\ 0 & 0 & \epsilon_{\mu\tau}e^{\imath\phi_{\mu\tau}}\\ 0 & \epsilon_{\mu\tau}e^{-\imath\phi_{\mu\tau}} & 0\end{bmatrix}$$

Main channels affected are $P_{\mu\mu}$, $P_{\mu\tau}$

$P_{\nu_\mu \to \nu_\tau}$ in presence of NSI

$$\begin{aligned} P^{m}_{\mu\tau} &= \cos^2\theta^{m}_{13}\sin^2 2\theta_{23}\sin^2[1.27(\Delta_{31}+A+\Delta^{m}_{31})L/2E] \\ &+ \sin^2\theta^{m}_{13}\sin^2 2\theta_{23}\sin^2[1.27(\Delta_{31}+A-\Delta^{m}_{31})L/2E] \\ &- \cos^2\theta_{23}P^{m}_{\mu e} \end{aligned}$$

In absence of NSI, OMSD

$$P^{NSI}_{\mu\tau} = -\sin^2 2\theta^M_{13}\sin^2\theta^M_{23}\cos^2\theta^M_{23}\sin^2[\frac{1.27\Delta^M_{31}L}{E}] + \sin^2 2\theta^M_{23}\{\cos^2\theta^M_{13}\sin^2[\frac{1.27\Delta^M_{32}L}{E}] + \sin^2\theta^M_{13}\sin^2[\frac{1.27\Delta^M_{21}L}{E}]\}$$

In presence of NSI, OMSD

$$\sin 2\theta^M_{13} = \sin 2\theta^m_{13}\left[1 - \frac{A\epsilon_{\mu\tau}\cos 2\theta^m_{13}\sin 2\theta_{23}}{\Delta^m_{31}}\right]$$

Resonance in absence of NSI

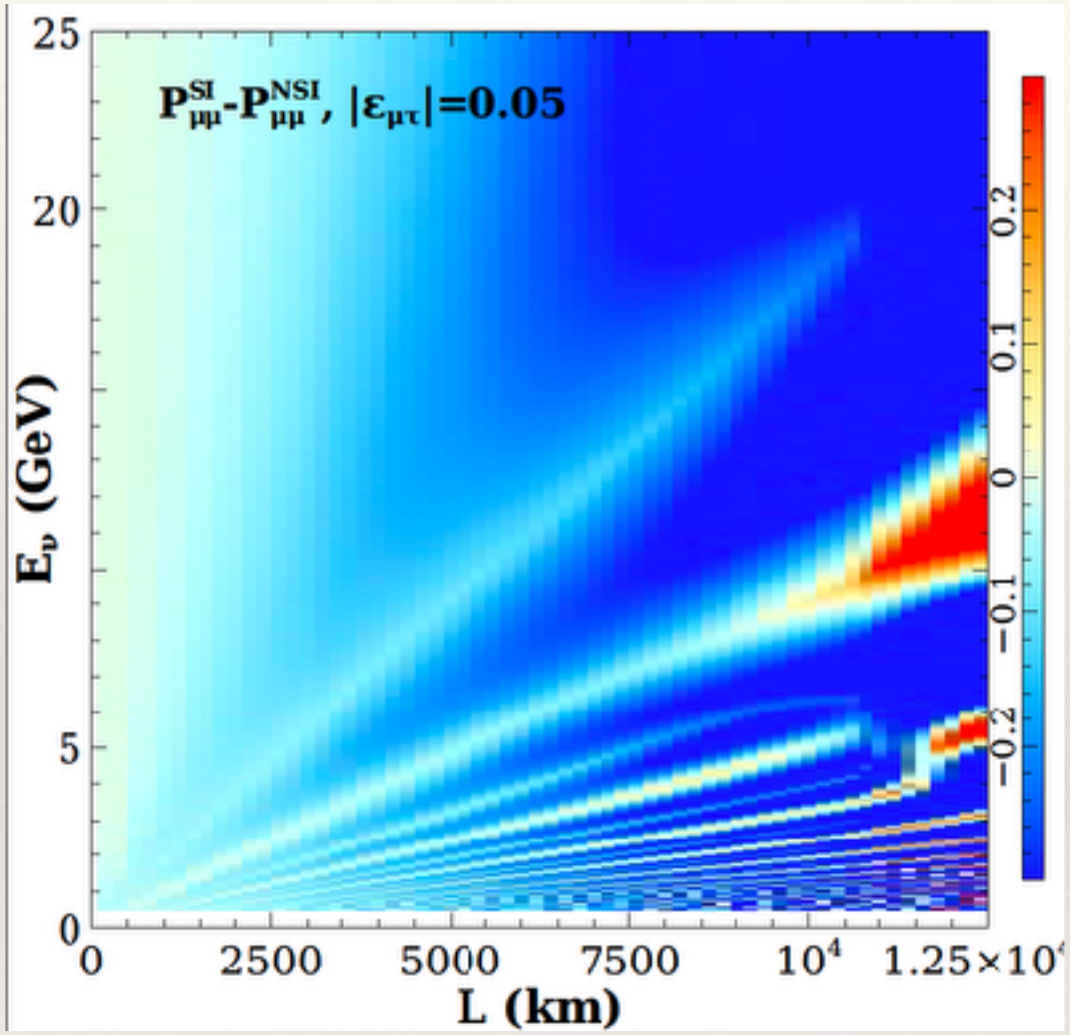
$$\implies \sin 2\theta^m_{13} = 1 \implies \sin 2\theta^M_{13} = 1$$

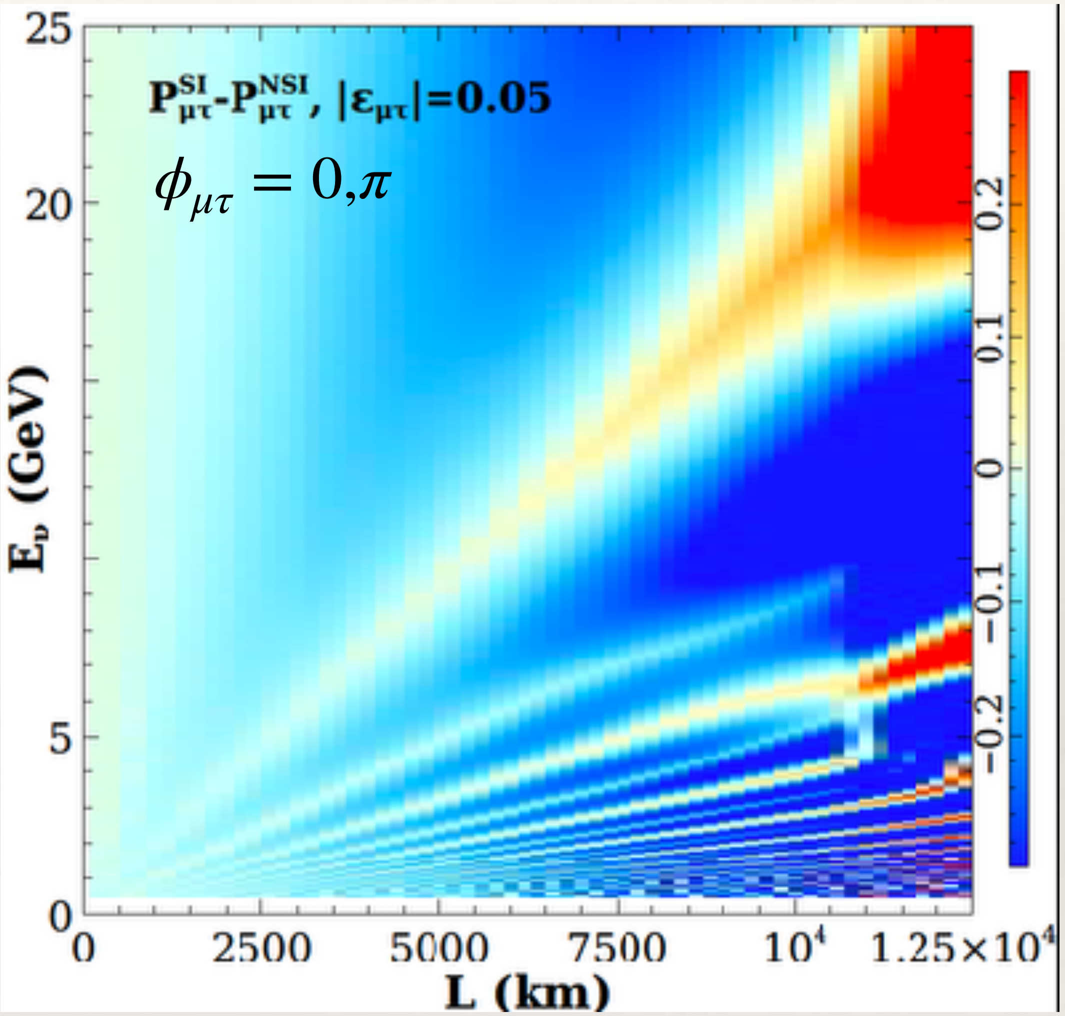
$$E^M_3 - E^M_1 = \Delta^m_{31} + A\epsilon_{\mu\tau}\cos\phi_{\mu\tau}\cos 2\theta^m_{13}\sin 2\theta_{23}$$

$$E^M_2 - E^M_1 = \Delta^m_{21} - A\epsilon_{\mu\tau}\cos\phi_{\mu\tau}(1+\sin^2\theta^m_{13})\sin 2\theta_{23}$$

$$E^M_3 - E^M_2 = \Delta^m_{32} + A\epsilon_{\mu\tau}\cos\phi_{\mu\tau}\sin 2\theta_{23}(1+\cos^2\theta^m_{13})$$

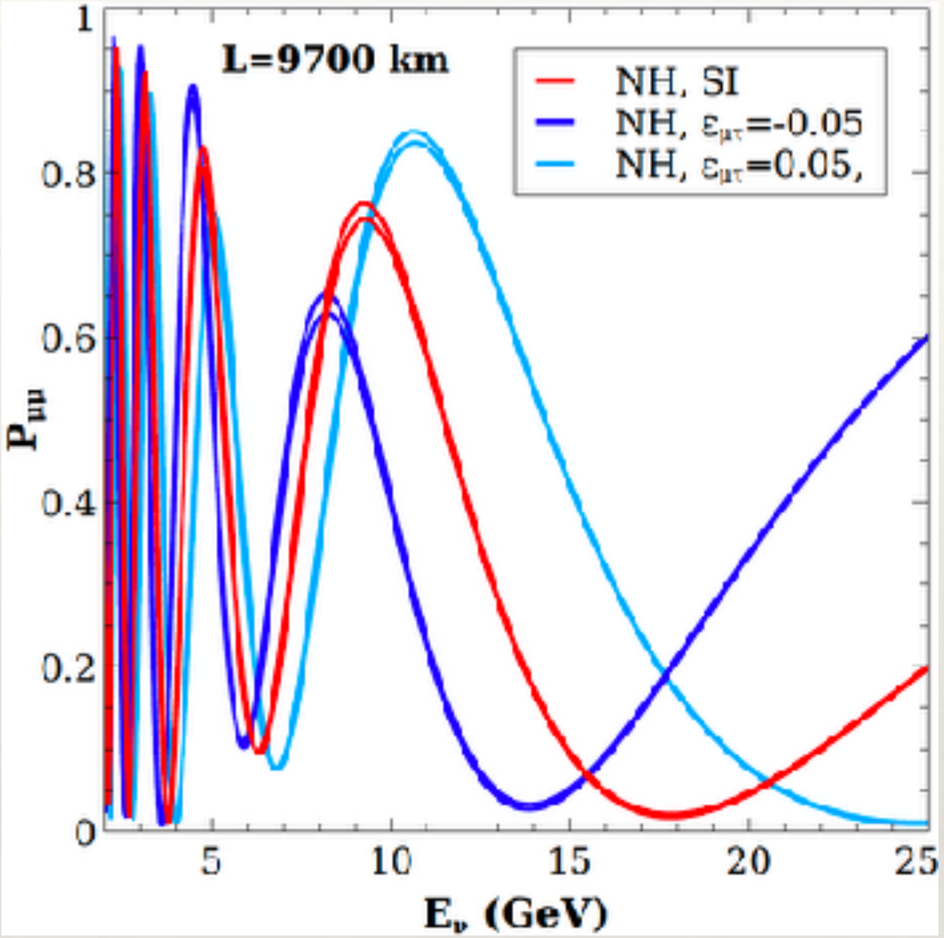
Matter effect due to NSI

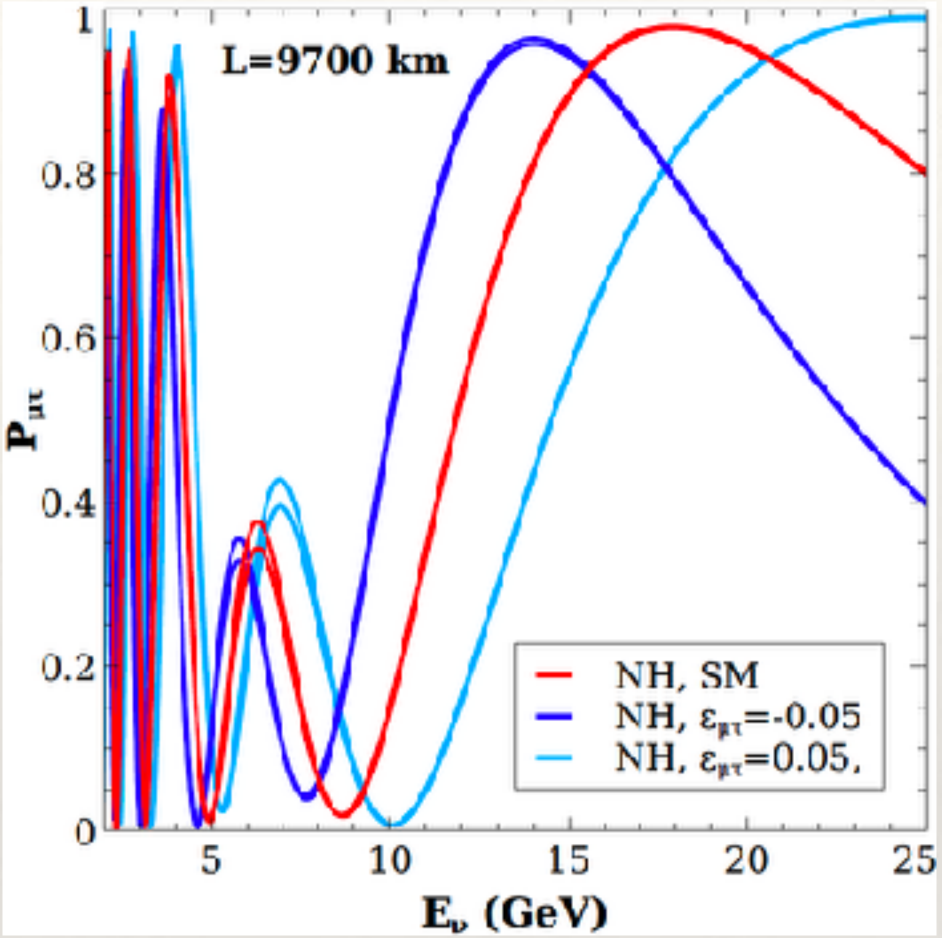




Large effects in higher energies and baselines

Probabilities at 9700 km





Larger differences between SI, NSI at higher energies

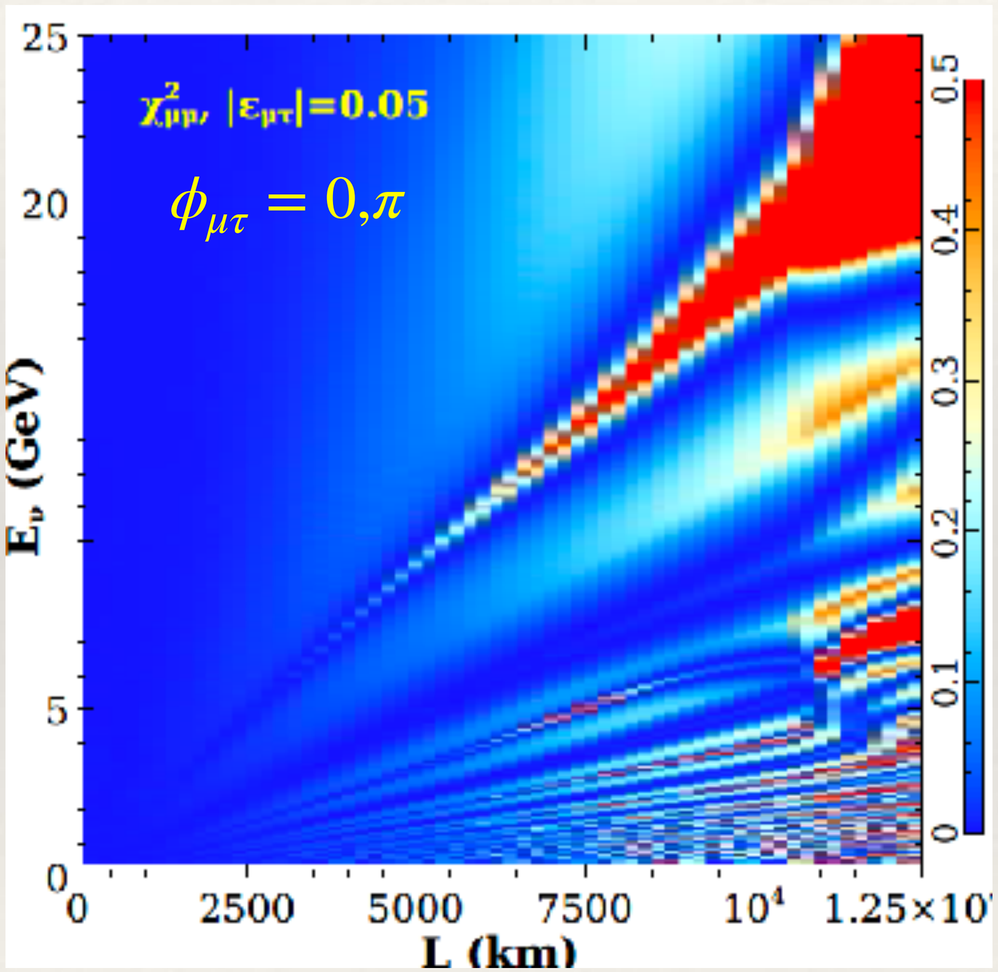
NSI probabilities higher or lower than the standard probability depending on the sign of $\epsilon_{\mu\tau}$

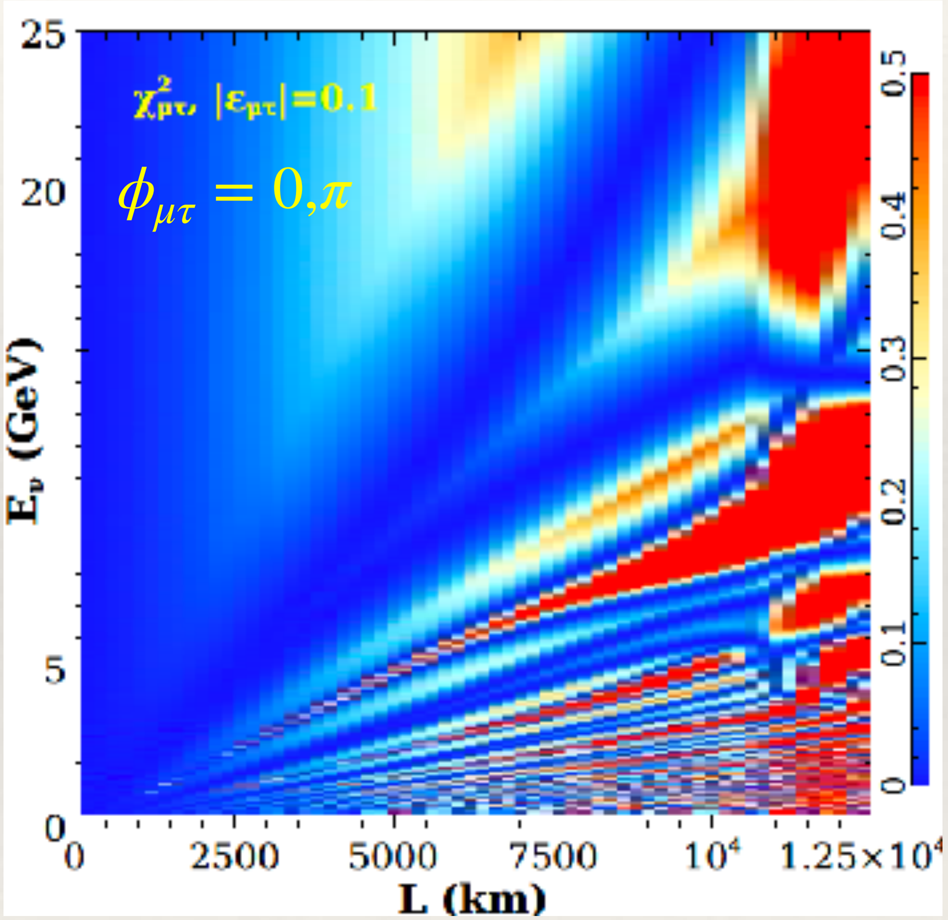
There are energies where $+\epsilon_{\mu\tau}$, $-\epsilon_{\mu\tau}$ give same probability

Probability level analysis

$$\chi^2 = \frac{(P(\epsilon_{\mu\tau} = x) - P(\epsilon_{\mu\tau} = 0))^2}{P(\epsilon_{\mu\tau} = 0)}$$

Marginalized over
$\delta_{CP}, \theta_{23}, \phi_{\mu\tau}(0,\pi)$

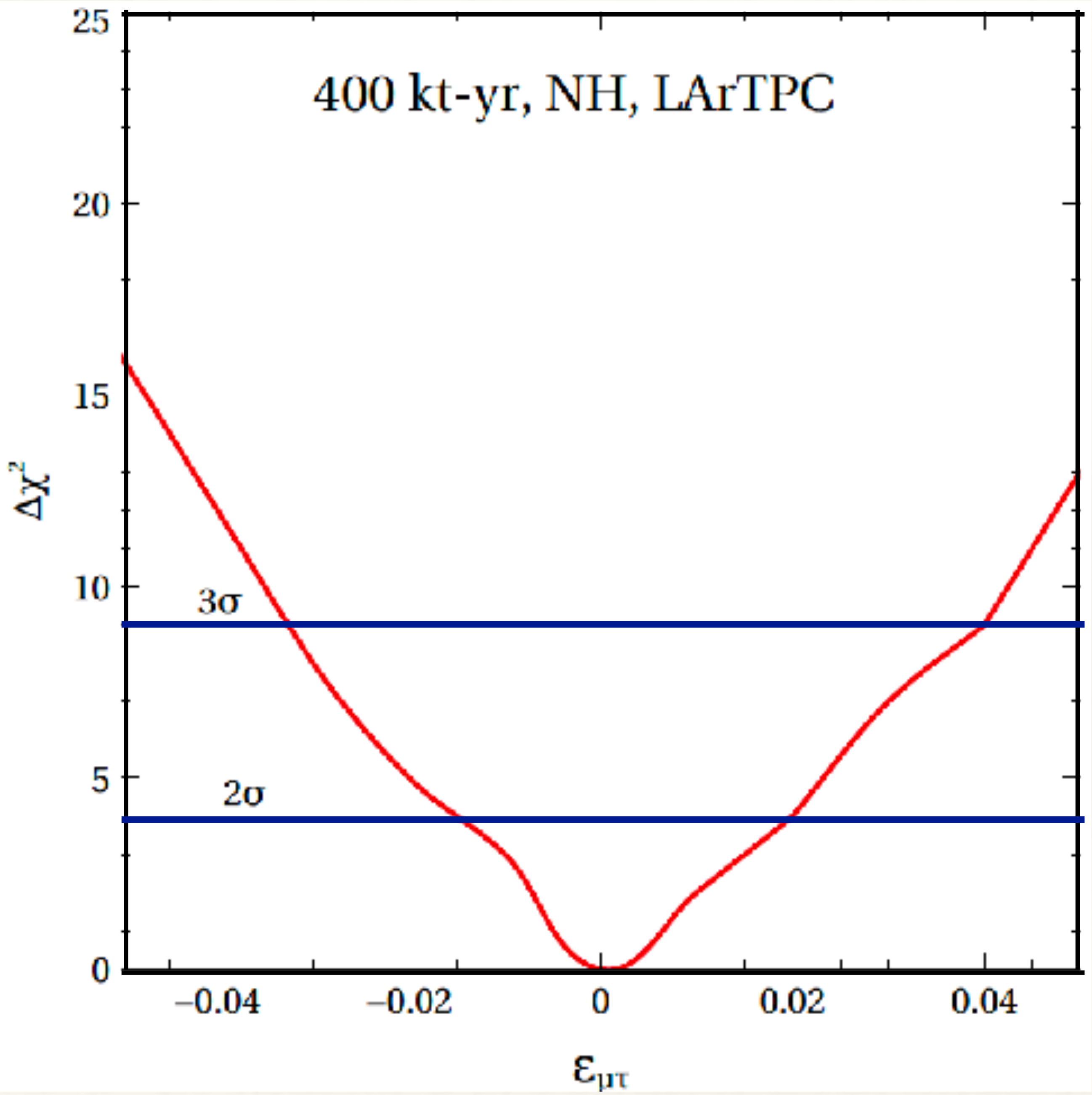




Higher sensitivity at larger baselines and energies

Sensitivities seen in the $P_{\mu\tau}$ channel

Bound on $\epsilon_{\mu\tau}$ using muon events

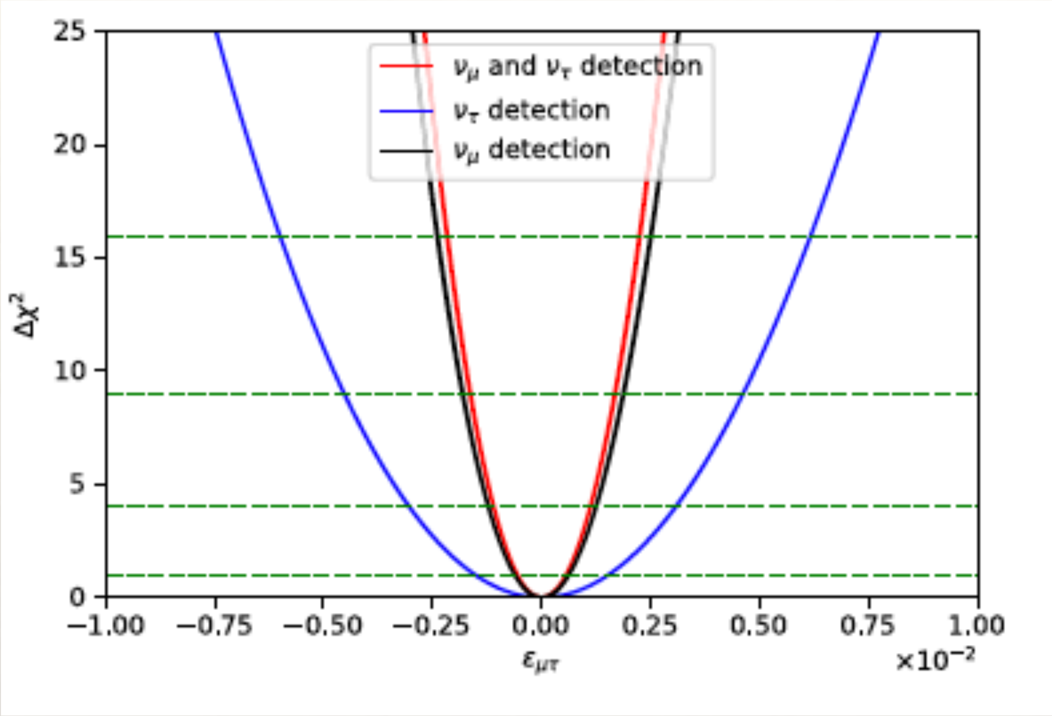


Using atmospheric muon events for a liquid argon detector

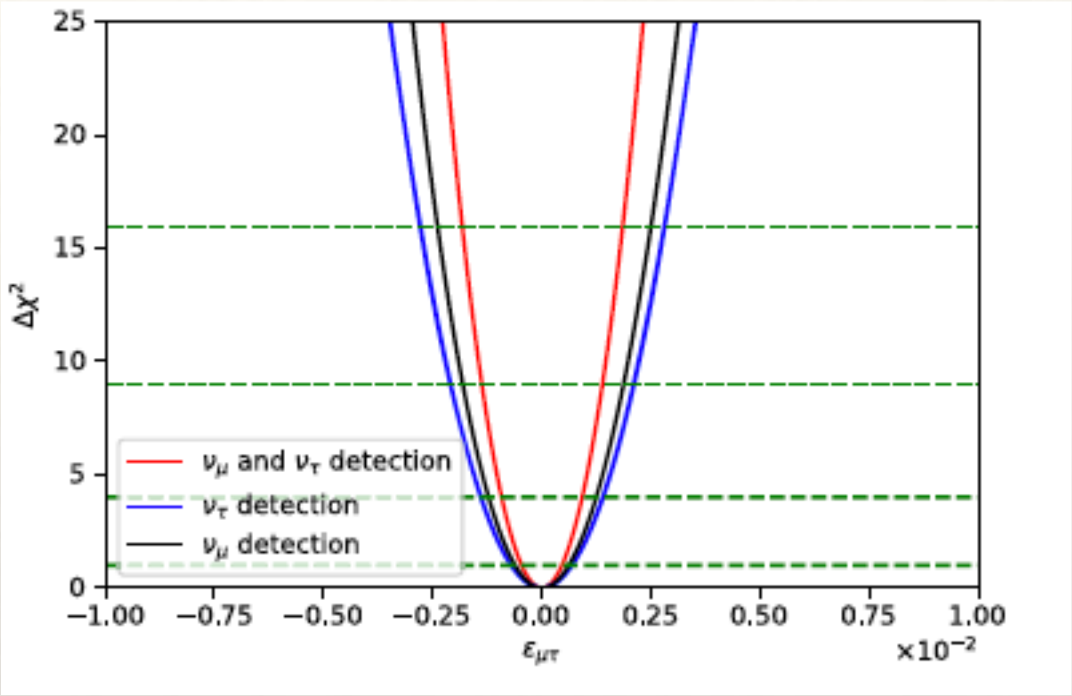
True neutrino energy upto 20 GeV

Impact of including the ν_τ channel

ν_τ Detection efficiency 30%



ν_τ Detection efficiency 100%



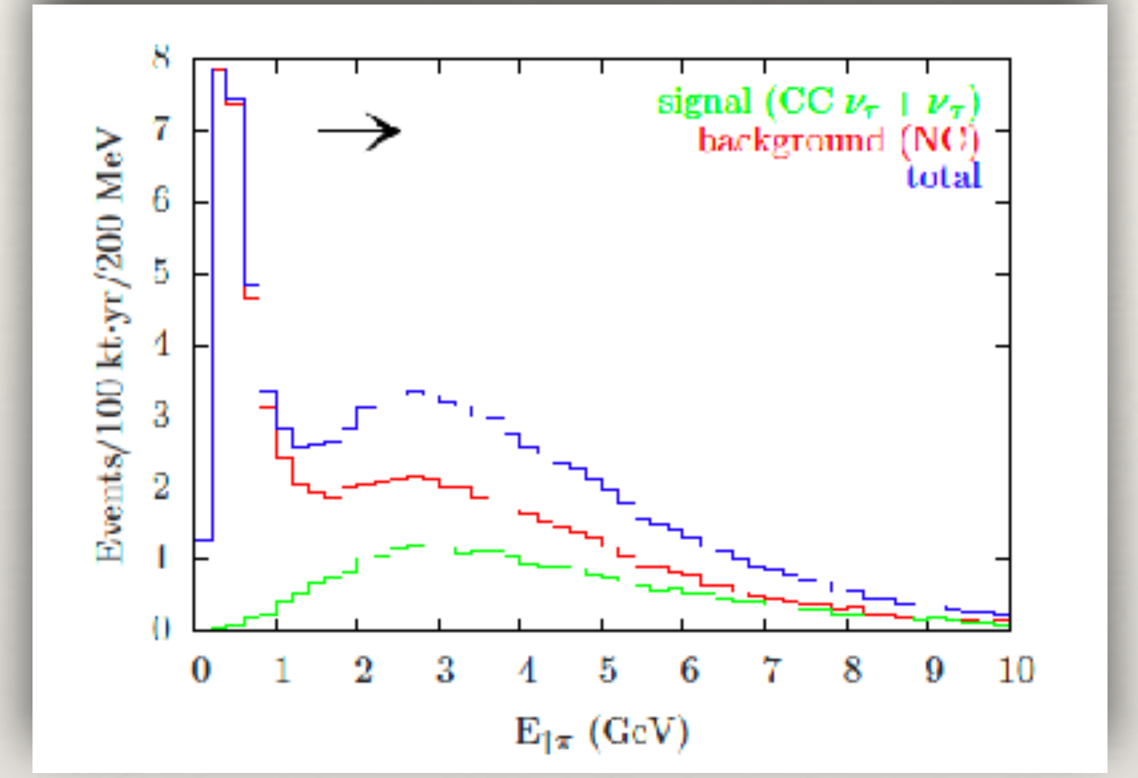
Atmospheric neutrinos in DUNE 400 kt years, E > 15 -200 GeV

Improvement in the bound by one order of magnitude by including higher energy events

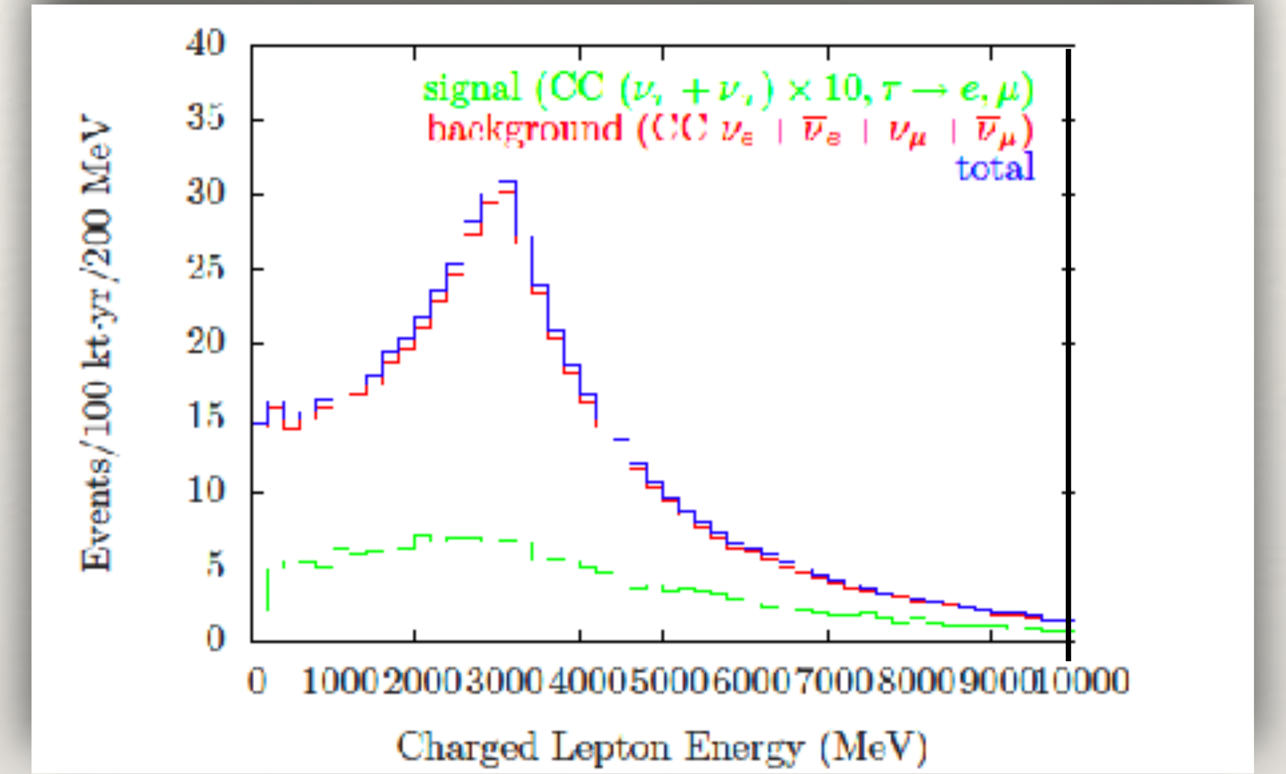
The effect of the ν_τ channel depends on the detection efficiency

Bakhti, Rajaee, Shin, PRD, 2022

Simulation of atmospheric ν_τ events



Hadronic decay channels

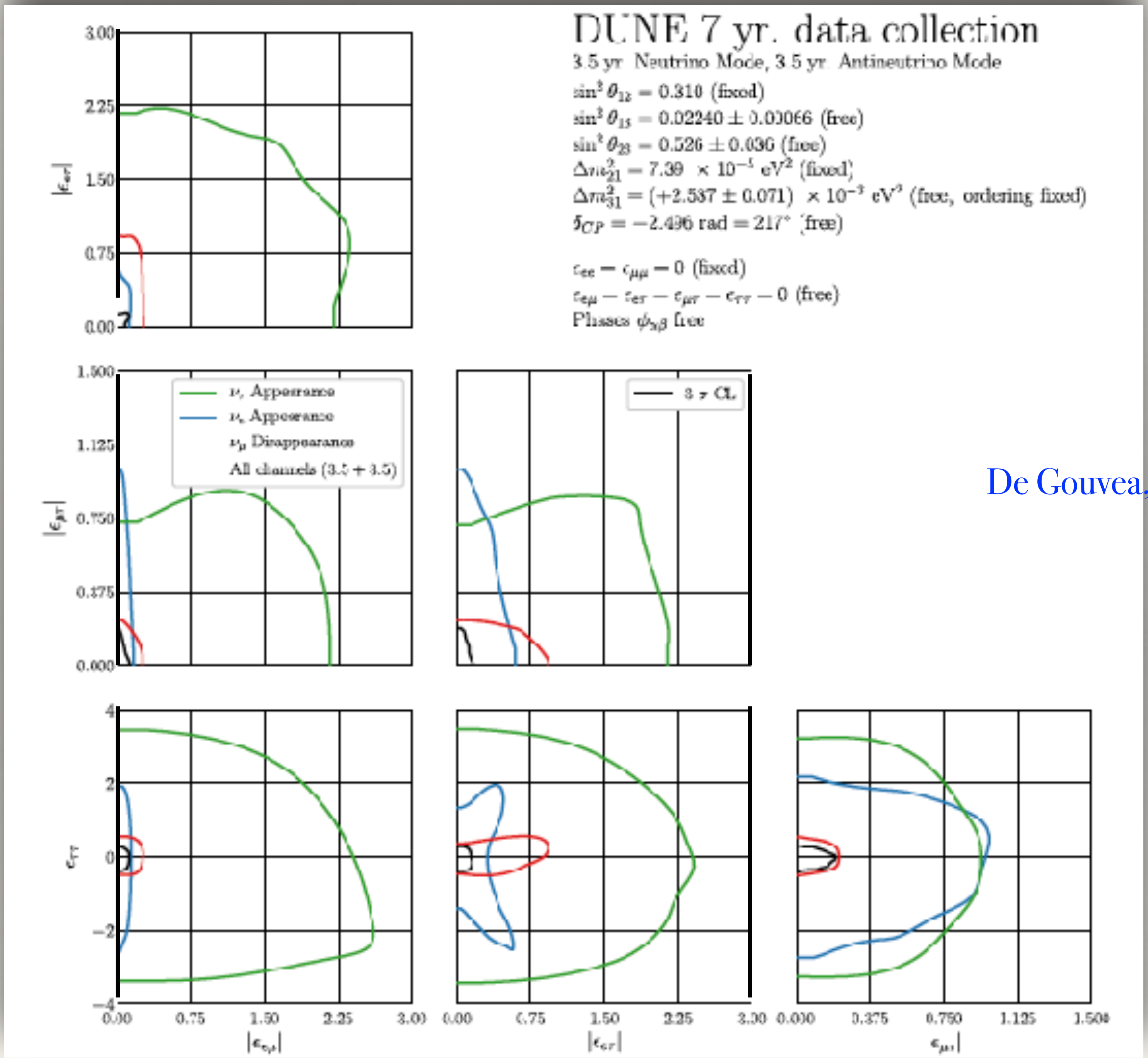


Leptonic decay channels

Conrad, de Gouvea, Shalgar, Spitz, PRD 2010

Talk by Adam Aurisano
Talk by Barbara Yaggey Alvarez

NSI and beam ν_τ events



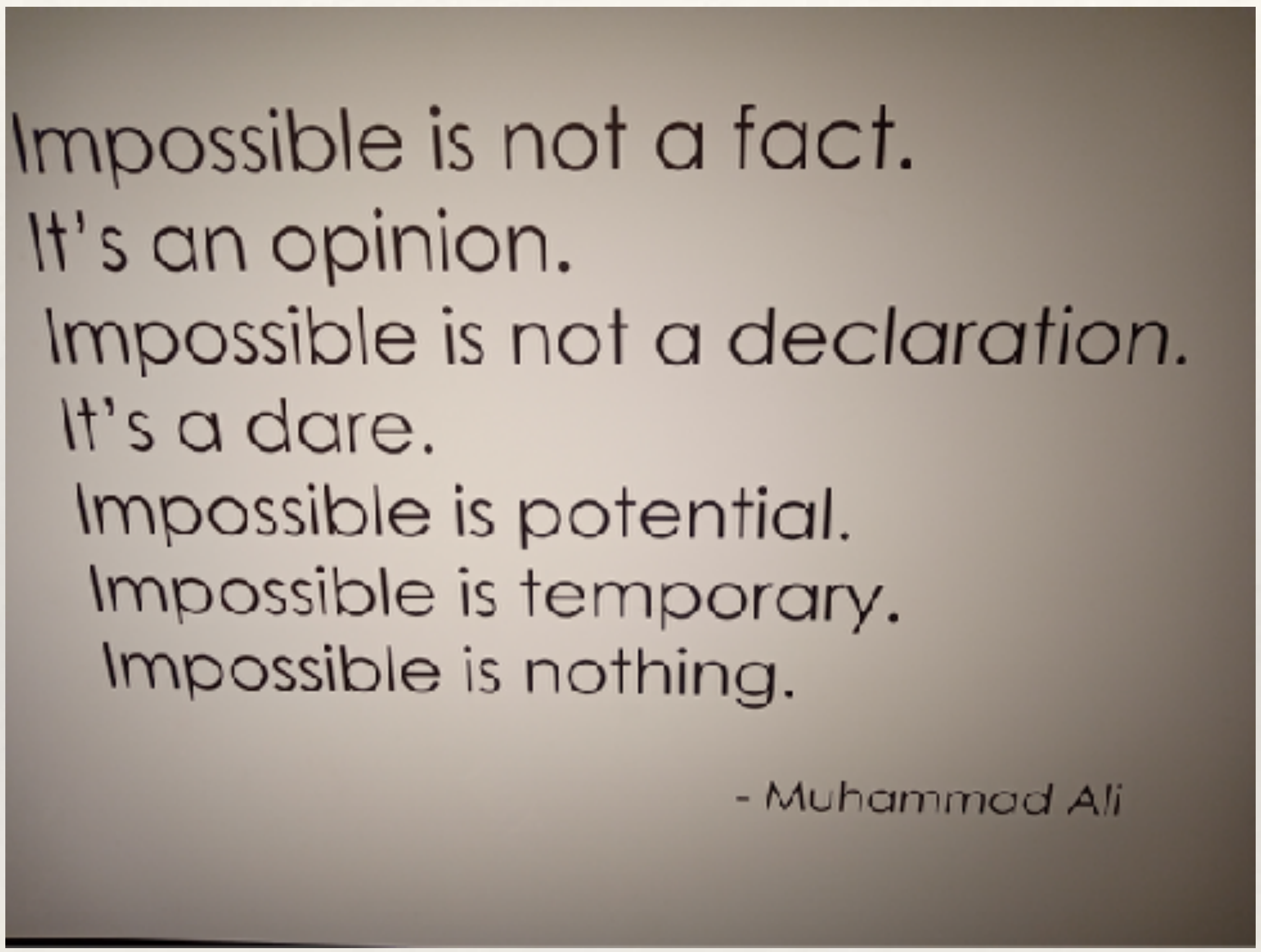
De Gouvea, Kelly, Stenico, Pasquini, PRD, 2019

Summary

- In two generation no matter effect in $\nu_\mu - \nu_\tau$ channel
- In three generation appreciable matter effect around 9700 km
- Genuine three generation effect
- Contribution to hierarchy sensitivity from ν_τ events ?
- Considered NSI driven by $\epsilon_{\mu\tau}$
- Enhanced sensitivity due to ν_τ events

Future plans

- Introduce other relevant NSI parameters
- Event level analysis of the ν_τ events for atmospheric neutrinos
- Other new physics through ν_τ channel and matter effect in atmospheric neutrinos
- Ultimate wish list — exploring combined sensitivity of beam and atmospheric ν_τ events to explore possible synergies


Impossible is not a fact.
It's an opinion.
Impossible is not a declaration.
It's a dare.
Impossible is potential.
Impossible is temporary.
Impossible is nothing.
- Muhammad Ali


Thank
you!!