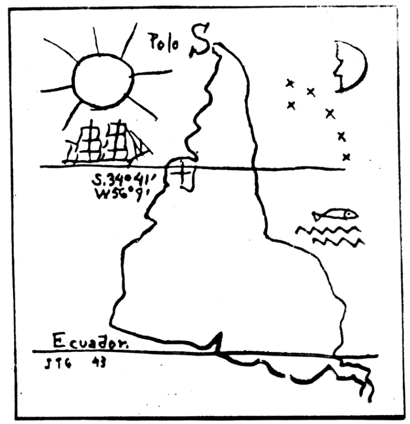
Magnetic and Electric Tau Dipole Moments Revisited



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1. **Tau lepton dipole moments**
2. **Electric DM / conclusions**
3. **Magnetic DM / conclusions**

1. Tau lepton DM

Why are we interested in Tau dipole moments?

- m = 1777 MeV
- Lifetime = 290 x 10^{-15} sec
- Tau lepton decays into hadrons
- Lepton universality

- Sensitivity to NP $(m_\tau/\Lambda)^n$
- $\left(\frac{m_\tau}{m_e}\right)^2 \approx 10^7$; $\left(\frac{m_\tau}{m_\mu}\right)^2 \approx 3 \cdot 10^2$

Magnetic and Electric Tau Dipole Moments Revisited

1. Tau lepton DM

PDG 2022/2023

τ MAGNETIC MOMENT ANOMALY (a_τ)

$\mu_\tau/(e\hbar/2m_\tau)-1 = (g_\tau-2)/2$

>-0.052 and <0.013 CL=95.0%

ABDALLAH 04K DLPH $e^+e^- \to e^+e^-\tau^+\tau^-$ at LEP2

τ ELECTRIC DIPOLE MOMENT (d_τ)

$\mathrm{Re}(d_\tau)$	-1.85×10^{-17} to 6.1×10^{-18} e cm
$\mathrm{Im}(d_\tau)$	-1.03×10^{-17} to 2.30×10^{-18} e cm

CL=95%

INAMI 2022 Belle

Magnetic and Electric Tau Dipole Moments Revisited

1. Tau lepton DM

τ MAGNETIC MOMENT ANOMALY

$\mu_\tau/(e\hbar/2m_\tau)-1 = (g_\tau-2)/2$ >-0.052 and <0.013 CL=95.0%

For a theoretical calculation [$(g_\tau-2)/2 = 117\ 721(5)\times10^{-8}$], see EIDELMAN 2007 .

VALUE	CL%	DOCUMENT ID		TECN	COMMENT
>-0.052 and <0.013 OUR LIMIT					
>-0.052 and <0.013	95	[1] ABDALLAH	2004K	DLPH	$e^+ e^- \to e^+ e^- \tau^+ \tau^-$ at LEP2
• • We do not use the following data for averages, fits, limits, etc. • •					
< 0.107	95	[2] ACHARD	2004G	L3	$e^+ e^- \to e^+ e^- \tau^+ \tau^-$ at LEP2
>-0.007 and <0.005	95	[3] GONZALEZ-SPRI..	2000	RVUE	$e^+ e^- \to \tau^+ \tau^-$ and $W \to \tau \nu_\tau$
>-0.052 and <0.058	95	[4] ACCIARRI	1998E	L3	1991--1995 LEP runs
>-0.068 and <0.065	95	[5] ACKERSTAFF	1998N	OPAL	1990--1995 LEP runs
>-0.004 and <0.006	95	[6] ESCRIBANO	1997	RVUE	$Z \to \tau^+ \tau^-$ at LEP
< 0.01	95	[7] ESCRIBANO	1993	RVUE	$Z \to \tau^+ \tau^-$ at LEP
< 0.12	90	GRIFOLS	1991	RVUE	$Z \to \tau \tau \gamma$ at LEP
< 0.023	95	[8] SILVERMAN	1983	RVUE	$e^+ e^- \to \tau^+ \tau^-$ at PETRA

Magnetic and Electric Tau Dipole Moments Revisited

1. Tau lepton DM

EDM real part

τ ELECTRIC DIPOLE MOMENT (d_τ)

Re(d_τ) -1.85×10^{-17} to 6.1×10^{-18} e cm

VALUE (10^{-16} e cm)	*CL%*	*DOCUMENT ID*		*TECN*	*COMMENT*
-0.185 to 0.061	95	[1] INAMI	2022	BELL	E^{ee}_{cm} = 10.6 GeV
• • We do not use the following data for averages, fits, limits, etc. • •					
< 2.3	90	[2] GROZIN	2009A	RVUE	From e EDM limit
< 3.7	95	[3] ABDALLAH	2004K	DLPH	$e^+ e^- \to e^+ e^- \tau^+ \tau^-$ at LEP2
< 11.4	95	[4] ACHARD	2004G	L3	$e^+ e^- \to e^+ e^- \tau^+ \tau^-$ at LEP2
-0.22 to 0.45	95	[5] INAMI	2003	BELL	E^{ee}_{cm} = 10.6 GeV
< 4.6	95	[6] ALBRECHT	2000	ARG	E^{ee}_{cm} = 10.4 GeV
>-3.1 and <3.1	95	ACCIARRI	1998E	L3	1991--1995 LEP runs
>-3.8 and <3.6	95	[7] ACKERSTAFF	1998N	OPAL	1990--1995 LEP runs
< 0.11	95	[8, 9] ESCRIBANO	1997	RVUE	$Z \to \tau^+ \tau^-$ at LEP
< 0.5	95	[10] ESCRIBANO	1993	RVUE	$Z \to \tau^+ \tau^-$ at LEP
< 7	90	GRIFOLS	1991	RVUE	$Z \to \tau \tau \gamma$ at LEP
< 1.6	90	DELAGUILA	1990	RVUE	$e^+ e^- \to \tau^+ \tau^-$ E^{ee}_{cm} = 35 GeV

Magnetic and Electric Tau Dipole Moments Revisited

1. Tau lepton DM

EDM imaginary part

$\text{Im}(d_\tau)$	-1.03×10^{-17} to 2.30×10^{-18} e cm

VALUE (10^{-16} e cm)	*CL%*	*DOCUMENT ID*		*TECN*	*COMMENT*
−0.103 to 0.023	95	[1] INAMI	2022	BELL	E^{ee}_{cm} = 10.6 GeV
• • We do not use the following data for averages, fits, limits, etc. • •					
−0.25 to 0.008	95	[2] INAMI	2003	BELL	E^{ee}_{cm} = 10.6 GeV
< 1.8	95	[3] ALBRECHT	2000	ARG	E^{ee}_{cm} = 10.4 GeV

[1] INAMI 2022 use $e^+ e^- \to \tau^+\tau^-$ events from 833 fb^{-1} of data. Also report a measurement of $\text{Im}(d_\tau) = (-0.40 \pm 0.32) \times 10^{-17}$ ecm.

[2] INAMI 2003 use $e^+ e^- \to \tau^+\tau^-$ events.

[3] ALBRECHT 2000 use $e^+ e^- \to \tau^+\tau^-$ events. Limit is on the absolute value of $\text{Im}(d_\tau)$.

1. Tau lepton DM

τ $J = \frac{1}{2}$

Mass $m = 1776.86 \pm 0.12$ MeV

$(m_{\tau^+} - m_{\tau^-})/m_{\text{average}} < 2.8 \times 10^{-4}$, CL = 90%

Mean life $\tau = (290.3 \pm 0.5) \times 10^{-15}$ s

$c\tau = 87.03\ \mu$m

Magnetic moment anomaly > -0.052 and < 0.013, CL = 95%

γ $\text{Re}(d_\tau) = -0.220$ to 0.45×10^{-16} e cm, CL = 95%

$\text{Im}(d_\tau) = -0.250$ to 0.0080×10^{-16} e cm, CL = 95%

Z → **Weak dipole moment**

$\text{Re}(d_\tau^w) < 0.50 \times 10^{-17}$ e cm, CL = 95%

$\text{Im}(d_\tau^w) < 1.1 \times 10^{-17}$ e cm, CL = 95%

Z → **Weak anomalous magnetic dipole moment**

$\text{Re}(\alpha_\tau^w) < 1.1 \times 10^{-3}$, CL = 95%

$\text{Im}(\alpha_\tau^w) < 2.7 \times 10^{-3}$, CL = 95%

2. Electric dipole moment

FIRST ORDER INTERACTION WITH ELECTRIC FIELD

$$H_{\mathrm{EDM}} = -\vec{\mathrm{d}} \cdot \vec{\mathrm{E}} \quad ; \quad \vec{\mathrm{d}} = \mathrm{d}\,\vec{\mathrm{s}}$$

Classical electromagnetism
Non relativistic quantum mechanics

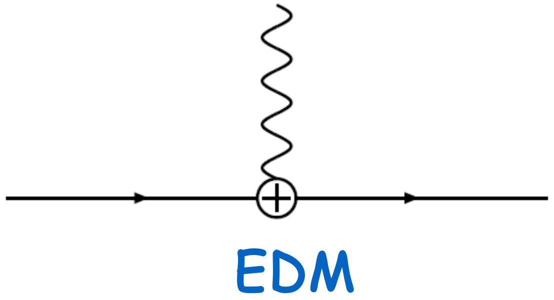
Non-relativistic limit

Relativistic quantum mechanics: Dirac equation

$$H = \bar{\Psi}\left(i(\partial\!\!\!/ + e A\!\!\!/\,) - m\right)\Psi + \frac{i}{2} d\, \bar{\Psi}\gamma^5\sigma^{\mu\upsilon}\Psi F_{\mu\upsilon}$$

2. Electric dipole moment

Time reversal T-odd, Parity P-odd (Landau 1957) / CPV



SM :
- **vertex corrections**
- **at least 4-loops for leptons (Shabalin '78, Hoogeveen '90)**

Beyond SM:
- **one loop effect (2HDM,SUSY, ...)**
- **dimension six effective operator**

Besides:

- **EDM tensor structure is chirality flipping magnitude.**
- **EDM generated by physics at Λ– scale may have ~ $(m/\Lambda)^n$ factors**
- **τ – EDM depends on the underlying mechanisms of CP violation**

2. Electric dipole moment

Experiments

$|d_\gamma^e| < 1.1 \times 10^{-29}$ e cm CL 90.0 %

$|d_\gamma^\mu| < 1.8 \times 10^{-19}$ e cm CL 95.0 %

$|d_\gamma^n| < 1.8 \times 10^{-26}$ e cm CL 90.0 %

τ

$\mathrm{Re}(d_\tau)$	-1.85×10^{-17} to 6.1×10^{-18} e cm
$\mathrm{Im}(d_\tau)$	-1.03×10^{-17} to 2.30×10^{-18} e cm

2. Electric dipole moment

SM/EDM is well below within present experimental limits:

$$d_\gamma^q \approx 10^{-32} - 10^{-34}\, ecm$$

CKM 3-loops

(SM)

$$d_\gamma^e \approx 10^{-38}\, ecm$$

CKM 4-loops

Naively, for the SM Tau EDM

$$d_\gamma^\tau \approx \frac{m_\tau}{m_e} d_\gamma^e \approx 10^{-33} - 10^{-34}\, ecm$$

J.F.Donoghue '78
I.B.Khriplovich,M.E.Pospelov '90
A.Czarnecki,B.Krause '97
M.Pospelov, A.Ritz '14

…16 orders of magnitude below experiments...

2. Electric dipole moment

τ-EDM computed in many BSM:

ext. Higgs sector / SUSY / Leptoquarks / LR …

BSM τ-EDM not far from the bounds

Non-vanishing signal for a τ—EDM CPV observable

⇕

NEW PHYSICS

2. Electric dipole moment

Experiments/bounds:

$$\left.\begin{array}{l} \Delta\sigma\,(e^+e^- \to \tau^+\tau^-) \\ \Delta\Gamma\,(Z \to \tau^+\tau^-) \\ \Delta\sigma\,(e^+e^- \to \tau^+\tau^-\gamma\,\gamma) \\ \Delta\Gamma(Z \to \tau^+\tau^-\,\gamma) \end{array}\right\} \sim |d^{\tau}_{\Upsilon}|^2$$

CPV-observables in Tau–pair production: linear EDM terms

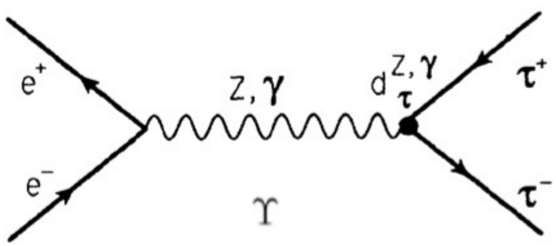
SPIN TERMS **and/or** **TRIPLE PRODUCTS**

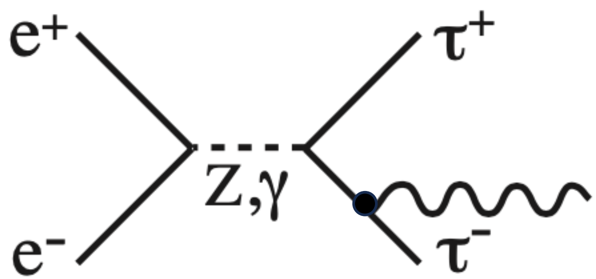
(angular distributions) **(triple spin-momentum products)**

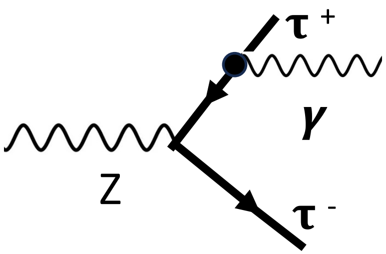
2. Electric dipole moment

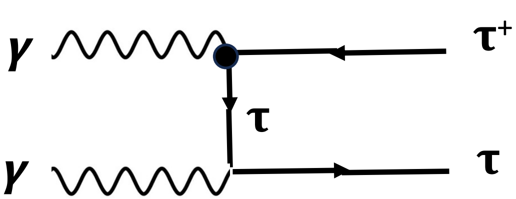
Experiments/colliders:

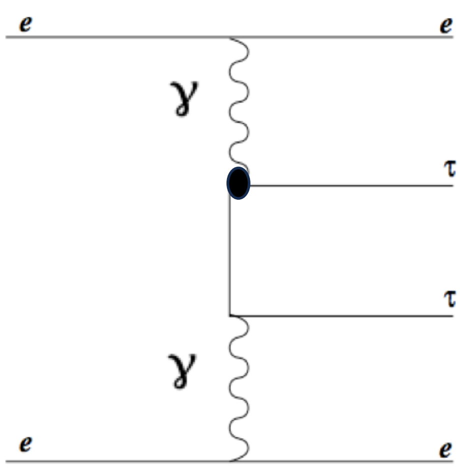
LEP LEP2 LHC BABAR BELLE BELLE2...











Magnetic and Electric Tau Dipole Moments Revisited

2. Electric dipole moment

$\not{CP}$:
- **Spin terms**
- **Expectation values of tensor observables**

a. Spin terms : **Linear polarizations**

Spin-spin correlations

Which means measurements of the angular distribution of Tau decay products

b. Expectation values of tensor spin-momentum obs.

Triple spin-momentum correlations

2. Electric dipole moment

a. Spin terms in $e^{\pm}$ unpol prod. of tau pairs

Polarizations	P	CP	T
$(\mathbf{s}_1 + \mathbf{s}_2)_{x,z}$	$-$	$+$	$+$
$(\mathbf{s}_1 + \mathbf{s}_2)_y$	$+$	$+$	$-$
• $(\mathbf{s}_1 - \mathbf{s}_2)_y$	$+$	$-$	$-$
• $(\mathbf{s}_1 - \mathbf{s}_2)_{x,z}$	$-$	$-$	$+$

Correlations	P	CP	T
s_{xx}, s_{yy}, s_{zz}, $(s_{xz} + s_{zx})$	$+$	$+$	$+$
$(s_{xy} + s_{yx})$, $(s_{yz} + s_{zy})$	$-$	$+$	$-$
• $(\mathbf{s}_1 \times \mathbf{s}_2)_{x,z}$	$-$	$-$	$-$
$(\mathbf{s}_1 \times \mathbf{s}_2)_y$	$+$	$-$	$+$

x: Transverse y: Normal z: Longitudinal

x,z production plane

2. Electric dipole moment

a. Spin terms in $e^{\pm}$ unpol prod. of tau pairs

Normal polarization/spin-correlations

P_N^{τ} / $(s_1 \times s_2)_{x,z}$

NORMAL-TRANSVERSE CORRELATION

C^-_{xy} term in $d\sigma(e^+e^- \to \gamma \to \tau^+\tau^- \to h^+\bar{\nu}h'^-\nu)$

$$\left.\frac{d\sigma^8}{d\Omega_\tau d^3q^*_- d^3q^*_+}\right|_{C^-_{xy}} = \frac{\alpha^2\beta^2}{128\pi^3 s^2} Br_+ Br_- \, d^\gamma_\tau \, \sin^2\theta \, (n^*_{+x}n^*_{-y} - n^*_{+y}n^*_{-x}) \, \delta(q^*_- - P_-)\delta(q^*_+ - P_+)$$

$n^*_\pm = \pm\alpha_\pm \hat{q}^*_\pm$

$q_\pm$ are the momentum of the hadrons

$P_\pm = \frac{m_\tau^2 - m_\pm}{2m_\tau}$

2. Electric dipole moment

b. Tensor observables:

$$T_{ij} = (s_+ \times s_-)_i \cdot q_j$$

W.Bernreuther,O.Nachtmann etal

since'89 and in 2022

More on this:

K. Inami talk in a few minutes…

2. Electric dipole moment

Some conclusions

- **Linear polarization / correlations / triple products CP-odd observables at low/high energy may provide stringent Tau-EDM bounds**
- **Recent results have recently improve the limits on the Tau EDM**
- **Tau-EDM bounds might become competitive with other EDM bounds if bounds improved**
- **High statistics data may contribute to this limits**
- **New data and new ideas are coming**

3. Magnetic dipole moment

FIRST ORDER INTERACTION WITH MAGNETIC FIELD

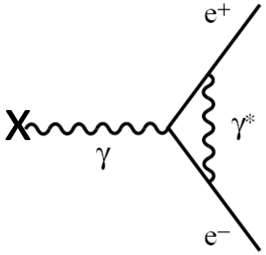
$$H_{MDM} = -\vec{\mu} \cdot \vec{B} \qquad \vec{\mu} = \mu \vec{s}$$

$$H_{Dirac} = \bar{\psi}\,(\,i\,(\partial + eA) - m)\,\psi + \frac{i}{2}\,a\,\bar{\psi}\sigma^{\mu\upsilon}\psi\,F_{\mu\upsilon}$$

$$\mu = 2(1+a)\,\frac{e\hbar}{2mc}$$

g

Schwinger 1948 QED:



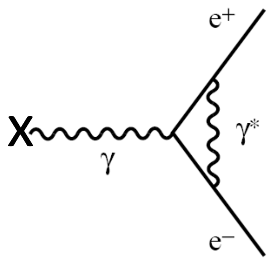
$a = \alpha/2\pi \approx 0.00116141$

Flavor and mass-independent at first order

3. Magnetic dipole moment

a = $\alpha/2\pi \approx 0.00116141$

a_γ^τ



>-0.052 and <0.013 CL=95.0%

ABDALLAH 04K DLPH $e^+e^- \to e^+e^-\tau^+\tau^-$ at LEP2

- Close to $\alpha/2\pi$ in the SM
- Finite 1-loop contr. SM / BSM (finite: tensor structure)
- chirality-flipping magnitude
- may depend on the underlying mass generation mechanism

3. Magnetic dipole moment

Theory SM: Eidelman/Passera 2007

$$a_f = a_f^{QED} + a_f^{Weak} + a_f^{Strong} + \ldots$$

$$a_l^{\rm QED} = A_1 + A_2\left(\frac{m_l}{m_j}\right) + A_2\left(\frac{m_l}{m_k}\right) + A_3\left(\frac{m_l}{m_j}, \frac{m_l}{m_k}\right)$$

$$A_i = A_i^{(2)}\left(\frac{\alpha}{\pi}\right) + A_i^{(4)}\left(\frac{\alpha}{\pi}\right)^2 + A_i^{(6)}\left(\frac{\alpha}{\pi}\right)^3 + \cdots$$

$$\left(A_1^{(2)} = {}^1/_2\right)$$

Schwinger

$$a_\tau^{SM} = 0.001\,177\,21(5)$$

3 50	HADRONIC 3-LOOPS
47	WEAK 2-LOOPS

$$a_\tau^{\rm SM} = 117\,721\,(5) \times 10^{-8}$$

Samuel/Li/Mendel '91 $117300(300)\mathrm{x}10^{-8}$
260 HAD 56 EW

3. Magnetic dipole moment

PDG 2022/3

e Magnetic moment anomaly

$$(g-2)/2 = (1159.65218062 \pm 0.00000012) \times 10^{-6}$$

μ Magnetic moment anomaly $(g-2)/2 = (11659206 \pm 4) \times 10^{-10}$

τ Magnetic moment anomaly > -0.052 and < 0.013, CL $= 95\%$

3. Magnetic dipole moment

BSM effective Lagrangian approach

$$L_{eff} = L_0 + \frac{1}{\Lambda^2}\sum_i C_i O_i + O\left(\frac{1}{\Lambda^4}\right)$$

$$L_{NP} = \alpha_B O_B + \alpha_W O_W + \text{h.c.}$$

Operators

$$O_W = \frac{g}{2\Lambda^2}\bar{L}_L \vec{\sigma}\varphi\sigma_{\mu\nu}\tau_R W^{\mu\nu}$$

$$O_B = \frac{g'}{2\Lambda^2}\bar{L}_L \sigma\varphi\sigma_{\mu\nu}\tau_R B^{\mu\nu}$$

$$B^\mu, \vec{W}^\mu \rightarrow A^\mu, Z^\mu, W_+{}^\mu, W_-{}^\mu$$

$$\varphi \rightarrow v/\sqrt{2}$$

3. Magnetic dipole moment

SSB:

$$\mathcal{L}_{eff} = \epsilon_\gamma \frac{e}{2m_Z}\overline{\tau}\sigma_{\mu\nu}\tau F^{\mu\nu} + \epsilon_Z \frac{e}{2m_Z s_W c_W}\overline{\tau}\sigma_{\mu\nu}\tau Z^{\mu\nu} + \left(\epsilon_W \frac{e}{2m_Z s_W}\overline{\nu_{\tau L}}\sigma_{\mu\nu}\tau_R W_+^{\mu\nu} + \text{h.c.}\right) \;,$$

$$\varepsilon_\gamma = (\alpha_B - \alpha_W)\frac{v m_Z}{\sqrt{2}\Lambda^2} \;;$$

$$\varepsilon_Z = -\left(\alpha_B s^2{}_W + \alpha_W c^2{}_W\right)\frac{v m_Z}{\sqrt{2}\Lambda^2} \;;$$

$$\varepsilon_W = \alpha_W \frac{v m_Z}{\Lambda^2} = -\sqrt{2}\left(\varepsilon_Z + \varepsilon_\gamma s_W^2\right)$$

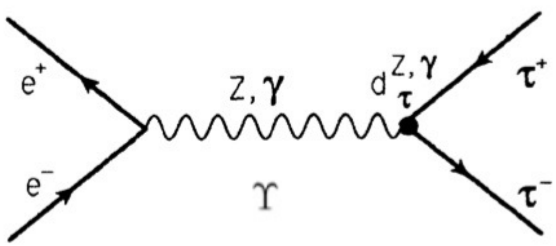
$$-0.005 < a_\gamma < 0.002,$$
$$-0.0007 < a_Z < 0.0019,$$
$$-0.06 < \epsilon_W < 0.07$$

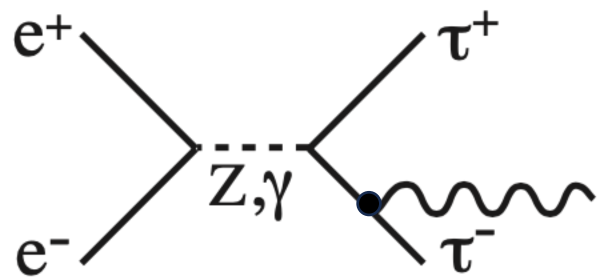
LEP1/SLD: Z decay rates + pol asymm
LEP2: xsections + W DR
CDF/D0: W DR

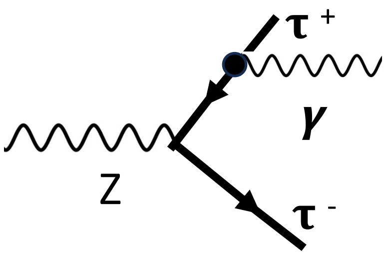
3. Magnetic dipole moment

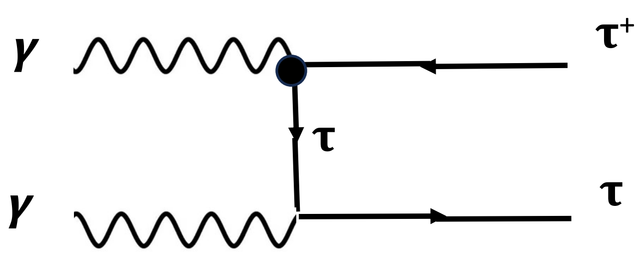
Experiments/colliders:

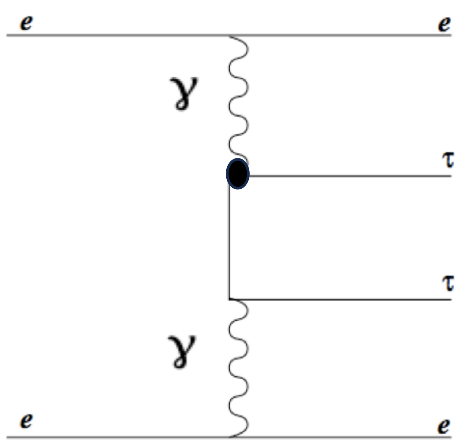
LEP LEP2 LHC BABAR BELLE BELLE2…







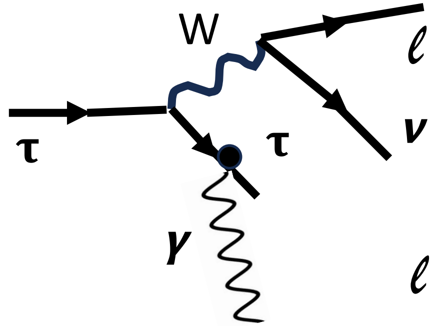


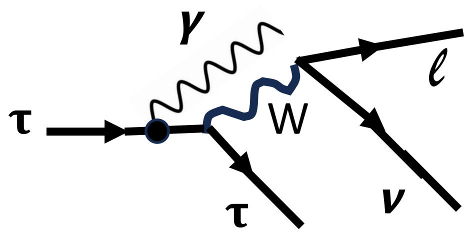


3. Magnetic dipole moment

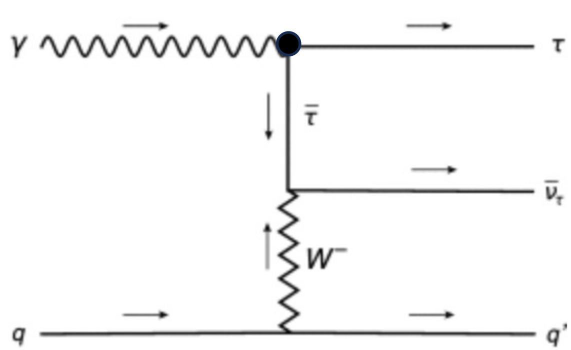
Experiments/colliders:

LEP LEP2 LHC BABAR BELLE BELLE2…





$\ell = e\ ,\ \mu$



3. Magnetic dipole moment

Experiments

OBSERVABLES

Cross sections

Linear polarizations

Spin correlations

Decay distributions

3. Magnetic dipole moment

The anomaluos magnetic moment is defined by

$a_\tau = F_2 (q^2 = 0)$. Both taus also on-shell.

This quantity is gauge independent

AND

for QED, $F_2 (q^2 \neq 0)$ is also gauge independent

Besides, weak interactions gauge dependence goes to 0 for $q^2 > 0$

At one loop QED:

$$F_2(s) = \left(\frac{\alpha}{2\pi}\right)\frac{2m_\tau^2}{s}\frac{1}{\beta}\left(\log\frac{1+\beta}{1-\beta} - i\pi\right), \quad \text{for } q^2 = s > 4m_\tau^2,$$

3. Magnetic dipole moment

UNPOLARIZED BEAM

Normal polarization asymmetries: Imaginary part

$$\frac{d\sigma^S}{d\cos\theta_{\tau^-}} = \frac{\pi\alpha^2}{4s}\beta(s_- + s_+)_y Y_+ ,$$

$$Y_+ = \gamma\beta^2(\cos\theta_{\tau^-}\sin\theta_{\tau^-})\,\mathrm{Im}\{F_2(s)\}$$

Angular distribution:

$$\frac{d\sigma_{\mathrm{FB}}}{d\phi_\pm} = \mp\frac{\pi\alpha^2}{12s}\mathrm{Br}(\tau^+\to h^+\bar{\nu}_\tau)\,\mathrm{Br}(\tau^-\to h^-\nu_\tau)(\alpha_\pm)\beta^3\gamma\,\mathrm{Im}\{F_2(s)\}\sin\phi_\pm .$$

$$A_N^\pm = \frac{\sigma_L^\pm - \sigma_R^\pm}{\sigma} = \pm\alpha_\pm\frac{1}{2(3-\beta^2)}\beta^2\gamma\,\mathrm{Im}\{F_2(s)\}$$

3. Magnetic dipole moment

POLARIZED BEAM

Longitudinal and Transverse polarization asymmetries:

Real part

L / T pol. are P-odd , beam polarization is needed

$$\left.\frac{d\sigma^S}{d\cos_{\tau^-}}\right|_\lambda = \frac{\pi\alpha^2}{8\,s}\beta\Big\{(s_- + s_+)_y Y_+ + \lambda\big[(s_- + s_+)_x X_+ + (s_- + s_+)_z Z_+\big]\Big\},$$

$$X_+ = \sin\theta_{\tau^-}\big[|F_1|^2 + \big(2-\beta^2\big)\gamma^2\,\mathrm{Re}\{F_2\}\big]\frac{1}{\gamma},$$

$$Z_+ = \cos\theta_{\tau^-}\big[|F_1|^2 + 2\,\mathrm{Re}\{F_2\}\big],$$

3. Magnetic dipole moment

POLARIZED BEAM

Angular distribution L/T polarization asymmetries:

$$A_T^{\pm} = \frac{\sigma_R^{\pm}|_{\rm Pol} - \sigma_L^{\pm}|_{\rm Pol}}{\sigma} = \mp\alpha_{\pm}\frac{3\pi}{8(3-\beta^2)\gamma}\left[|F_1|^2 + \left(2-\beta^2\right)\gamma^2\,{\rm Re}\{F_2\}\right],$$

$$A_L^{\pm} = \frac{\sigma_{\rm FB}^{\pm}(+)|_{\rm Pol} - \sigma_{\rm FB}^{\pm}(-)|_{\rm Pol}}{\sigma} = \mp\alpha_{\pm}\frac{3}{4(3-\beta^2)}\left[|F_1|^2 + 2\,{\rm Re}\{F_2\}\right],$$

Combining both observables

$${\rm Re}\{F_2(s)\} = \mp\frac{8(3-\beta^2)}{3\pi\gamma\beta^2}\frac{1}{\alpha_{\pm}}\left(A_T^{\pm} - \frac{\pi}{2\gamma}A_L^{\pm}\right).$$

3. Magnetic dipole moment

Some conclusions

- **Still far from SM prediction…**
- **Enough room for BSM effects**
- **More statistics and experiments still ongoing**
- **New ideas/approaches are coming**

- **We may need to consider tau g-2 more seriously (first speaker)**

Thanks for your attention

Discussions/comments with

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