Tau Polarization and Correlated Decays in Neutrino Experiments

Joshua Isaacson
In collaboration with: Stefan Höche, Frank Siegert, and Sherry Wang
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7 December 2023
Motivation

From Adam Aurisano’s Talk

Beam sample - expected counts/year:
- $130 \, \nu_e$ in low-energy neutrino mode
- $30 \, \bar{\nu}_e$ in low-energy antineutrino mode
- $800 \, \nu_e$ in high-energy neutrino mode

Motivation

How to do you extract these parameters from data?

From Kevin Kelly's Talk

de Gouvêa, Kelly, Pasquini, Stenico [1904.07265]
Motivation

\[
\frac{N_{FD}}{N_{ND}} \propto \int dE_\nu \frac{d\phi^{FD}_\alpha}{dE_\nu} P(\nu_\alpha \to \nu_\beta; E_\nu) \sigma_\beta(E_\nu) M^{FD}_\alpha(E_\nu, E_{reco})
\]

\[
\frac{d\phi^{ND}_\alpha}{dE_\nu} \sigma_\alpha(E_\nu) M^{ND}_\alpha(E_\nu, E_{reco})
\]
Motivation

\[ \frac{N_{FD}}{N_{ND}} \propto \frac{\int dE_\nu \frac{d\phi_{\alpha}^{FD}}{dE_\nu} P(\nu_\alpha \rightarrow \nu_\beta; E_\nu) \sigma_\beta(E_\nu) M^{FD}_{\alpha}(E_\nu, E_{reco})}{\int dE_\nu \frac{d\phi_{\alpha}^{ND}}{dE_\nu} \sigma_\alpha(E_\nu) M^{ND}_{\alpha}(E_\nu, E_{reco})} \]

- Number of events in near / far detector
Motivation

\[ \frac{N_{FD}}{N_{ND}} \propto \frac{\int dE_\nu \frac{d\phi^{FD}_\alpha}{dE_\nu} P(\nu_\alpha \rightarrow \nu_\beta; E_\nu) \sigma_\beta(E_\nu) \mathcal{M}_{\alpha}^{FD}(E_\nu, E_{reco})}{\int dE_\nu \frac{d\phi^{ND}_\alpha}{dE_\nu} \sigma_\alpha(E_\nu) \mathcal{M}_{\alpha}^{ND}(E_\nu, E_{reco})} \]

- Number of events in near / far detector
- Oscillation probability
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\]

- Number of events in near / far detector
- Oscillation probability
- Neutrino-nucleus cross section
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\[ \frac{N_{FD}}{N_{ND}} \propto \int dE_\nu \frac{d\phi_{FD}^\alpha}{dE_\nu} P(\nu_\alpha \rightarrow \nu_\beta; E_\nu) \sigma_\beta(E_\nu) M_{\alpha}^{FD}(E_\nu, E_{\text{reco}}) \]

\[ \int dE_\nu \frac{d\phi_{ND}^\alpha}{dE_\nu} \sigma_\alpha(E_\nu) M_{\alpha}^{ND}(E_\nu, E_{\text{reco}}) \]

- Number of events in near / far detector
- Oscillation probability
- Neutrino-nucleus cross section
- Migration matrix (Depends on topology of events)
Motivation

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\int dE_\nu \frac{d\phi^{ND}_\alpha}{dE_\nu} \sigma_\alpha(E_\nu) M^{ND}_\alpha(E_\nu, E_{reco})
\]

- Number of events in near / far detector
- Oscillation probability
- Neutrino-nucleus cross section
- Migration matrix (Depends on topology of events)
- **Conclusion**: Need theory driven neutrino event generators
What about tuning?

- Tuning can hide new physics if not handled correctly
- Even with tuning, theory uncertainty is dominant systematic

![Graph showing input parameters and uncertainties](image)

**Source of Uncertainty** | $\nu_e$ signal (%) | Total beam background (%)
---|---|---
Cross-section and FSI | 7.7 | 8.6
Normalization | 3.5 | 3.4
Calibration | 3.2 | 4.3
Detector response | 0.67 | 2.8
Neutrino flux | 0.63 | 0.43
$\nu_e$ extrapolation | 0.36 | 1.2
Total systematic uncertainty | 9.2 | 11
Statistical uncertainty | 15 | 22
Total uncertainty | 18 | 25


Achilles: A CHIcagoLand Lepton Event Simulator

Project Goals:

- Theory driven
- Leverage experiences from LHC event generators
- Develop modular neutrino event generator
- Provide automated BSM calculations for neutrino experiments
- Evaluate theory uncertainties
- Appropriately handle correlations within events
## Spin Correlations

<table>
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<tr>
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[JI et al. arXiv:2110.15319]
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![Diagram](image-url)
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[Ji et al. arXiv:2110.15319]  
[Ji et al. arXiv:2303.08104]
Spin Correlations: 2 to n-body scattering

- Full phase space → separation of Dirac and Majorana
- GENIE includes this model, but handles it with repeated decays → only can simulate Majorana case (by accident)

Example: Dark Neutrino explanation of MiniBooNE
Spin Correlations: Spin-Density Matrix

Step 1:
- Calculate the matrix elements tracking spin
- $\rho$ is the spin density matrix for incoming particles
- $D$ is the spin dependent decay matrix
- Initialize $D$ to be diagonal

$$\rho_{\kappa_1 \kappa'_1} \rho_{\kappa_2 \kappa'_2} \mathcal{M}_{\kappa_1 \kappa_2; \lambda_1 \ldots \lambda_n} \mathcal{M}^*_{\kappa_1' \kappa_2'; \lambda_1' \ldots \lambda_n'} \prod_{i=1,n} D_{\lambda_i \lambda_i'}$$
Spin Correlations: Spin-Density Matrix

Step 1:
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$$\rho_{\lambda_j \lambda_j'} = \frac{1}{N_P} \rho_{\kappa_1 \kappa_1'}^{1} \rho_{\kappa_2 \kappa_2'}^{2} M_{\kappa_1 \kappa_2; \lambda_1 \ldots \lambda_n} M^*_{\kappa_1' \kappa_2'; \lambda_1' \ldots \lambda_n'} \prod_{i \neq j} D^i_{\lambda_i \lambda_i'}$$

Step 2:
- Randomly select unstable particle
- Calculate the spin density matrix for the decay

$\prod_{i=1,n} D^i_{\lambda_i \lambda_i'}$
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Step 2:
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$$\rho_{\lambda_0 \lambda_0'} \mathcal{M}_{\lambda_0 \lambda_1 \ldots \lambda_k} \mathcal{M}_{\lambda_0' \lambda_1' \ldots \lambda_k'} \prod_{i=1,k} D_{\lambda_i \lambda_i'}$$

Step 3:
- Select decay channel based on branching ratio
- Generate momentum according to
- Continue down decay chain until only stable particles remain

[P. Richardson arxiv:hep-ph/0110108]
Spin Correlations: Spin-Density Matrix

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[Step 2:]
- Randomly select unstable particle
- Calculate the spin density matrix for the decay

$$\rho_{\lambda_0 \lambda_0'} M_{\lambda_0; \lambda_1 \ldots \lambda_k} M^*_{\lambda_0'; \lambda_1' \ldots \lambda_k'} \prod_{i=1,k} D^i_{\lambda_i \lambda_i'}$$

[Step 3:]
- Select decay channel based on branching ratio
- Generate momentum according to
- Continue down decay chain until only stable particles remain

$$D_{\lambda_0 \lambda_0'} = \frac{1}{N_D} M_{\lambda_0; \lambda_1 \ldots \lambda_k} M^*_{\lambda_0'; \lambda_1' \ldots \lambda_k'} \prod_{i=1,n} D^i_{\lambda_i \lambda_i'}$$

[Step 4:]
- Final decay matrix is obtained
- Repeat above steps until all particles are stable

[P. Richardson arxiv:hep-ph/0110108]
Collinear Limit

- Possible to calculate decays in collinear limit \((p_\tau \to \infty)\)
- Useful to validate predictions in the same limit

<table>
<thead>
<tr>
<th>Decay mode</th>
<th>Branching ratio (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Leptonic decays</td>
<td></td>
</tr>
<tr>
<td>(e^- \nu_\tau \bar{\nu}_e)</td>
<td>17.85</td>
</tr>
<tr>
<td>(\mu^- \nu_\tau \bar{\nu}_\mu)</td>
<td>17.36</td>
</tr>
<tr>
<td>Hadronic decays</td>
<td></td>
</tr>
<tr>
<td>(\pi^- \pi^0 \nu_\tau)</td>
<td>25.50</td>
</tr>
<tr>
<td>(\pi^- \nu_\tau)</td>
<td>10.90</td>
</tr>
<tr>
<td>(\pi^+ \pi^- \pi^- \nu_\tau)</td>
<td>9.32</td>
</tr>
<tr>
<td>(\pi^- \pi^0 \pi^0 \nu_\tau)</td>
<td>9.17</td>
</tr>
<tr>
<td>(\pi^+ \pi^- \pi^- \pi^0 \nu_\tau)</td>
<td>4.50</td>
</tr>
<tr>
<td>(\pi^- \pi^0 \pi^0 \pi^0 \nu_\tau)</td>
<td>1.04</td>
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<tr>
<td>(K^- \nu_\tau)</td>
<td>0.70</td>
</tr>
<tr>
<td>(\pi^+ \pi^- \pi^- \pi^0)</td>
<td>0.55</td>
</tr>
<tr>
<td>other</td>
<td>3.11</td>
</tr>
</tbody>
</table>

\[ x_i = \frac{p_i}{p_\tau} \]

Monoenergetic Validation

\[ \nu_\tau A \rightarrow \tau^- (A - 1)p, \quad \tau^- \rightarrow \nu_\tau \pi^- \]

Correct handling of polarization

Left-handed only assumption
Monoenergetic Validation

\[ \nu_\tau A \to \tau^- (A - 1)p, \quad \tau^- \to \nu_\tau \pi^- \pi^0 \]

Correct handling of polarization  
Left-handed only assumption
DUNE Tau Optimized Flux

From Kevin Kelly’s talk yesterday

L. Fields, “DUNE Fluxes,” https://glaucus.crc.nd.edu/DUNEFluxes/
DUNE Tau Optimized Flux

Events with at least one pion

\[ \frac{1}{N} \frac{dN}{dp_{\pi_1}} |_{V_1/GeV} \]

- **Full Calculation**
- **Left-handed Only**

\[ p_{\pi_1} \text{ [GeV]} \]
DUNE Tau Optimized Flux

Events with at least one pion

$\frac{1}{N} \frac{dN}{dp_{\pi_2}} \left[ \frac{1}{\text{GeV}} \right]$

- Full Calculation
- Left-handed Only
DUNE Tau Optimized Flux

Events with at least one pion

\[
\frac{1}{N} \frac{dN}{dE_{\pi^3}} \left( \frac{1}{\text{GeV}} \right)
\]

- **Full Calculation**
- **Left-handed Only**

\(p_{\pi^3} \text{ [GeV]}\)
DUNE Tau Optimized Flux

Comparison to backgrounds

Highest energy $\pi^-$

- Signal
- $e$
- $\mu$
- $\tau$

$\frac{1}{N} \frac{dN}{dp_{\pi^-}}$ [h/GeV]

$\pi^-_{lead}$ [GeV]

$\frac{1}{N} \frac{dN}{dp_{\pi^-}}$ [h/GeV]

$\pi^-_{lead}$ [GeV]

DUNE Tau Optimized Flux

\[ \nu_\tau A \rightarrow \tau^- (A - 1)p, \quad \tau^- \rightarrow \nu_\tau \pi^- \]
DUNE Tau Optimized Flux

$$\nu_\tau A \rightarrow \tau^- (A - 1) p, \quad \tau^- \rightarrow \nu_\tau \pi^- \pi^0$$
Conclusions

- Extracting underlying physics parameters requires accurate modeling of the underlying theory
- Largest systematic uncertainty arises from event generator modeling of cross-sections
- Handling spin correlations will be vital for any process beyond 2→2 scattering
- Achilles only tool on the market that can handle this
- These effects are important for tau neutrino charged current interactions
- Future steps:
  - Investigate improvements on tau separation from background
  - Leverage spin-correlations for more accurate BSM studies

Codes can be found at:
- Achilles: [https://github.com/AchillesGen/Achilles/releases/tag/v0.2.0](https://github.com/AchillesGen/Achilles/releases/tag/v0.2.0)
- Sherpa: [https://gitlab.com/sherpa-team/sherpa/-/tree/achilles](https://gitlab.com/sherpa-team/sherpa/-/tree/achilles)
Backup
Tau Threshold

- Minimum neutrino energy depends on the nuclear structure.
- Nucleon momentum along z-axis
- Binding energy ($E_b$) impacts threshold
Monoenergetic Validation

\[ \nu_\tau A \rightarrow \tau^- (A - 1) p, \quad \tau^- \rightarrow \nu_\tau \pi^- \pi^0 \]

Correct handling of polarization

Left-handed only assumption

\[ z_{\pi} = \frac{p_{\pi^-}}{p_{\pi^- \pi^0}} \]