Status and plans of tau fits for HFLAV/PDG

Alberto Lusiani, for the *BABAR* collaboration
orcid.org/0000-0002-6876-3288

Introduction

HFLAV tau branching fraction fit

- HFLAV does a global fit of tau branching fractions
- preliminary measurements are included if judged sound and near publication
- published measurements adjusted for biases and uncertainties from updated external inputs
- correlations and dependencies from common nuisance parameters are taken into account
- PDG tau BR fit relies on HFLAV tau BR fit using just published results and BR unitarity constraint

HFLAV Tau group

Sw. Banerjee (co-convener)	M. Chrząszcz	K. Hayasaka
A. Lusiani (co-convener)	M. Roney	B. Shwartz

Introduction (2)

fit result elaborations

- lepton universality tests, determination of $|V_{us}|$ with tau measurements

recent updates, included in HFLAV 2022 report

- new radiative corrections for tau hadronic decays

recent updates, included in the incoming HFLAV report

- tau mass measurement by Belle II [PRD 108 (2023) 032006] updates universality tests
- fit now includes nuisance parameters constrained by measurements other than tau BR

HFLAV Tau BR fit basis modes linear combination supposed to sum to 1

2015, 31 basis modes

$e^-\overline{\nu}_e\nu_\tau$	17.83 ± 0.04
$\mu^-\overline{\nu}_\mu\nu_\tau$	17.41 ± 0.04
$\pi^-\nu_\tau$	10.83 ± 0.06
$\pi^-\pi^0\nu_\tau$	25.52 ± 0.09
$\pi^-2\pi^0\nu_\tau$ (ex. K^0)	9.30 ± 0.11
$\pi^-3\pi^0\nu_\tau$ (ex. K^0)	1.05 ± 0.07
$h^-4\pi^0\nu_\tau$ (ex. K^0,η)	0.11 ± 0.04
$K^-\nu_\tau$	0.700 ± 0.010
$K^-\pi^0\nu_\tau$	0.430 ± 0.015
$K^-2\pi^0\nu_\tau$ (ex. K^0)	0.069 ± 0.028
$K^-3\pi^0\nu_\tau$ (ex. K^0,η)	0.052 ± 0.027
$\pi^-\overline{K}^0\nu_\tau$	0.845 ± 0.028
$\pi^-\overline{K}^0\pi^0\nu_\tau$	0.388 ± 0.015
$\pi^-K^0_SK^0_S\nu_\tau$	0.0232 ± 0.0007
$\pi^-K^0_SK^0_L\nu_\tau$	0.12 ± 0.05
$K^-K^0\nu_\tau$	0.149 ± 0.005
$K^-K^0\pi^0\nu_\tau$	0.151 ± 0.007
$\pi^-\pi^+\pi^-\nu_\tau$ (ex. K^0,ω)	8.99 ± 0.06
$\pi^-\pi^+\pi^-\pi^0\nu_\tau$ (ex. K^0,ω)	2.70 ± 0.08
$K^-\pi^+\pi^-\nu_\tau$ (ex. K^0)	0.294 ± 0.015
$K^-\pi^+\pi^-\pi^0\nu_\tau$ (ex. K^0,η)	0.078 ± 0.012
$K^-K^+\pi^-\nu_\tau$	0.144 ± 0.005
$K^-K^+\pi^-\pi^0\nu_\tau$	0.0061 ± 0.0025
$h^-h^-h^+2\pi^0\nu_\tau$ (ex. K^0,ω,η)	0.10 ± 0.04
$h^-h^-h^+3\pi^0\nu_\tau$	0.023 ± 0.007
$3h^-2h^+\nu_\tau$ (ex. K^0)	0.0839 ± 0.0035
$3h^-2h^+\pi^0\nu_\tau$ (ex. K^0)	0.0178 ± 0.0027
$h^-\omega\nu_\tau$	2.00 ± 0.08
$h^-\omega\pi^0\nu_\tau$	0.41 ± 0.04
$\eta\pi^-\pi^0\nu_\tau$	0.139 ± 0.010
$\eta K^-\nu_\tau$	0.0152 ± 0.0008

2016, 46 basis modes

decay mode	fit result (%)	coefficient
$\mu^-\bar{\nu}_\mu\nu_\tau$	17.3936 ± 0.0384	1.0000
$e^-\bar{\nu}_e\nu_\tau$	17.8174 ± 0.0399	1.0000
$\pi^-\nu_\tau$	10.8165 ± 0.0512	1.0000
$K^-\nu_\tau$	0.6964 ± 0.0096	1.0000
$\pi^-\pi^0\nu_\tau$	25.4940 ± 0.0893	1.0000
$K^-\pi^0\nu_\tau$	0.4329 ± 0.0148	1.0000
$\pi^-2\pi^0\nu_\tau$ (ex. K^0)	9.2595 ± 0.0964	1.0021
$K^-2\pi^0\nu_\tau$ (ex. K^0)	0.0648 ± 0.0218	1.0000
$\pi^-3\pi^0\nu_\tau$ (ex. K^0)	1.0428 ± 0.0707	1.0000
$K^-3\pi^0\nu_\tau$ (ex. K^0,η)	0.0478 ± 0.0212	1.0000
$h^-4\pi^0\nu_\tau$ (ex. K^0,η)	0.1119 ± 0.0391	1.0000
$\pi^-\bar{K}^0\nu_\tau$	0.8395 ± 0.0140	1.0000
$K^-K^0\nu_\tau$	0.1479 ± 0.0053	1.0000
$\pi^-\bar{K}^0\pi^0\nu_\tau$	0.3821 ± 0.0129	1.0000
$K^-\pi^0K^0\nu_\tau$	0.1503 ± 0.0071	1.0000
$\pi^-\bar{K}^0\pi^0\pi^0\nu_\tau$ (ex. K^0)	0.0263 ± 0.0226	1.0000
$\pi^-K^0_SK^0_S\nu_\tau$	0.0233 ± 0.0007	2.0000
$\pi^-K^0_SK^0_L\nu_\tau$	0.1080 ± 0.0241	1.0000
$\pi^-\pi^0K^0_SK^0_S\nu_\tau$	0.0018 ± 0.0002	2.0000
$\pi^-\pi^0K^0_SK^0_L\nu_\tau$	0.0325 ± 0.0119	1.0000
$\bar{K}^0h^-h^-h^+\nu_\tau$	0.0247 ± 0.0199	1.0000
$\pi^-\pi^-\pi^+\nu_\tau$ (ex. K^0,ω)	8.9870 ± 0.0514	1.0021
$\pi^-\pi^-\pi^+\pi^0\nu_\tau$ (ex. K^0,ω)	2.7404 ± 0.0710	1.0000
$h^-h^-h^+2\pi^0\nu_\tau$ (ex. K^0,ω,η)	0.0980 ± 0.0356	1.0000
$\pi^-K^-K^+\nu_\tau$	0.1435 ± 0.0027	1.0000
$\pi^-K^-K^+\pi^0\nu_\tau$	0.0061 ± 0.0018	1.0000
$\pi^-\pi^0\eta\nu_\tau$	0.1389 ± 0.0072	1.0000
$K^-\eta\nu_\tau$	0.0155 ± 0.0008	1.0000
$K^-\pi^0\eta\nu_\tau$	0.0048 ± 0.0012	1.0000
$\pi^-\bar{K}^0\eta\nu_\tau$	0.0094 ± 0.0015	1.0000
$\pi^-\pi^+\pi^-\eta\nu_\tau$ (ex. K^0)	0.0219 ± 0.0013	1.0000
$K^-\omega\nu_\tau$	0.0410 ± 0.0092	1.0000
$h^-\pi^0\omega\nu_\tau$	0.4085 ± 0.0419	1.0000
$K^-\phi\nu_\tau$	0.0044 ± 0.0016	0.8310
$\pi^-\omega\nu_\tau$	1.9494 ± 0.0645	1.0000
$K^-\pi^-\pi^+\nu_\tau$ (ex. K^0,ω)	0.2927 ± 0.0068	1.0000
$K^-\pi^-\pi^+\pi^0\nu_\tau$ (ex. K^0,ω,η)	0.0394 ± 0.0142	1.0000
$\pi^-2\pi^0\omega\nu_\tau$ (ex. K^0)	0.0071 ± 0.0016	1.0000
$2\pi^-\pi^+3\pi^0\nu_\tau$ (ex. K^0,η,ω,f_1)	0.0014 ± 0.0027	1.0000
$3\pi^-2\pi^+\nu_\tau$ (ex. K^0,ω,f_1)	0.0769 ± 0.0030	1.0000
$K^-2\pi^-2\pi^+\nu_\tau$ (ex. K^0)	0.0001 ± 0.0001	1.0000
$2\pi^-\pi^+\omega\nu_\tau$ (ex. K^0)	0.0084 ± 0.0006	1.0000
$3\pi^-2\pi^+\pi^0\nu_\tau$ (ex. K^0,η,ω,f_1)	0.0038 ± 0.0009	1.0000
$K^-2\pi^-2\pi^+\pi^0\nu_\tau$ (ex. K^0)	0.0001 ± 0.0001	1.0000
$\pi^-f_1\nu_\tau$ ($f_1 \to 2\pi^-2\pi^+$)	0.0052 ± 0.0004	1.0000
$\pi^-2\pi^0\eta\nu_\tau$	0.0194 ± 0.0038	1.0000

$\mathcal{B}(\tau \to a_1(\pi\gamma)\nu)$ pseudo-measurement

- $\mathcal{B}(\tau \to a_1(\pi\gamma)\nu)$ is ALEPH estimate, not a measurement
  - $\mathcal{B}(\tau \to a_1\gamma\nu) = \mathcal{B}(\tau \to 3\pi\nu)_{\text{EXP, ALEPH}} \cdot \mathcal{B}(a_1 \to \pi\gamma)_{\text{EXP}}$
- required to close the measured / estimated tau BR unitarity

past HFLAV editions

- HFLAV
  - use ALEPH pseudo-measurement, based on $\mathcal{B}(\tau \to 3\pi\nu)_{\text{EXP, ALEPH}}$
  - con: ALEPH pseudo-measurement does not use updated value of $\mathcal{B}(\tau \to 3\pi\nu)$
- PDG (HFLAV fit)
  - use constraint $\mathcal{B}(\tau \to a_1\gamma\nu) = \mathcal{B}(\tau \to 3\pi\nu)_{\text{current fit}} \cdot \mathcal{B}(a_1 \to \pi\gamma)_{\text{EXP}}$
  - pro: uses updated value of $\mathcal{B}(\tau \to 3\pi\nu)$
  - con: uncertainty of $\mathcal{B}(a_1 \to \pi\gamma)$ not taken into account
    $\Rightarrow$ resulting $\mathcal{B}(\tau \to a_1\gamma\nu)$ uncertainty underestimated

since 2023

- add nuisance fit parameter $\mathcal{B}(a_1 \to \pi\gamma)$ with its own χ^2 contribution
- after fit convergence, χ^2 restricted to tau measurements is computed
  $\Rightarrow$ consistency checks remain limited to tau measurements
- same treatment for PDG and HFLAV variants

Radiative corrections notation

not including universal EW correction $S_{EW}^{m_\tau}$

- $\Gamma(\tau^- \to \pi^- \nu_\tau) = \Gamma_{\text{th,LO}}(\tau^- \to \pi^- \nu_\tau) S_{EW}^{m_\tau} (1 + \delta R_{\tau\pi})$
- $\Gamma(\tau^- \to K^- \nu_\tau) = \Gamma_{\text{th,LO}}(\tau^- \to K^- \nu_\tau) S_{EW}^{m_\tau} (1 + \delta R_{\tau K})$
- $\dfrac{\Gamma(\tau^- \to K^- \nu_\tau)}{\Gamma(\tau^- \to \pi^- \nu_\tau)} = \dfrac{\Gamma_{\text{th,LO}}(\tau^- \to K^- \nu_\tau)}{\Gamma_{\text{th,LO}}(\tau^- \to \pi^- \nu_\tau)} \dfrac{S_{EW}^{m_\tau}(1 + \delta R_{\tau K})}{S_{EW}^{m_\tau}(1 + \delta R_{\tau\pi})} = \dfrac{\Gamma_{\text{th,LO}}(\tau^- \to K^- \nu_\tau)}{\Gamma_{\text{th,LO}}(\tau^- \to \pi^- \nu_\tau)} (1 + \delta R_{\tau K/\tau\pi})$

including universal EW correction $S_{EW}^{m_\tau}$

- $\Gamma(\tau^- \to \pi^- \nu_\tau) = \Gamma_{\text{th,LO}}(\tau^- \to \pi^- \nu_\tau)(1 + \delta R'_{\tau\pi})$
- $\Gamma(\tau^- \to K^- \nu_\tau) = \Gamma_{\text{th,LO}}(\tau^- \to K^- \nu_\tau)(1 + \delta R'_{\tau K})$
- $\dfrac{\Gamma(\tau^- \to K^- \nu_\tau)}{\Gamma(\tau^- \to \pi^- \nu_\tau)} = \dfrac{\Gamma_{\text{th,LO}}(\tau^- \to K^- \nu_\tau)}{\Gamma_{\text{th,LO}}(\tau^- \to \pi^- \nu_\tau)} \dfrac{(1 + \delta R'_{\tau K})}{(1 + \delta R'_{\tau\pi})} = \dfrac{\Gamma_{\text{th,LO}}(\tau^- \to K^- \nu_\tau)}{\Gamma_{\text{th,LO}}(\tau^- \to \pi^- \nu_\tau)} (1 + \delta R'_{\tau K/\tau\pi})$

note

- $\delta R_{\tau K/\tau\pi} = \delta R'_{\tau K/\tau\pi}$

Radiative corrections in tau fit results elaborations

lepton universality

$$\left(\frac{g_\tau}{g_\mu}\right)^2 = \frac{\mathcal{B}(\tau \to h\nu_\tau)}{\mathcal{B}(h \to \mu\bar{\nu}_\mu)} \frac{2 m_h m_\mu^2 \tau_h}{(1+\delta R_{\tau h/h\mu}) m_\tau^3 \tau_\tau} \left(\frac{1 - m_\mu^2/m_h^2}{1 - m_h^2/m_\tau^2}\right)^2 \qquad (h = \pi \text{ or } K)$$

$|V_{us}|$ from tau exclusive decays

- $$\frac{\mathcal{B}(\tau^- \to K^-\nu_\tau)}{\mathcal{B}(\tau^- \to \pi^-\nu_\tau)} = \left(\frac{f_{K\pm}}{f_{\pi\pm}}\right)^2 \frac{|V_{us}|^2_{\tau K/\pi}}{|V_{ud}|^2} \frac{\left(m_\tau^2 - m_K^2\right)^2}{\left(m_\tau^2 - m_\pi^2\right)^2} (1 + \delta R_{\tau K/\tau\pi})$$ $\tau\to K/\tau\to\pi$
- $$\mathcal{B}(\tau^- \to K^-\nu_\tau) = \frac{1}{16\pi}\left(\frac{G_F}{\hbar^3 c^3}\right)^2 |V_{us}|^2_{\tau K} f_{K\pm}^2 \frac{\tau_\tau}{\hbar} m_\tau^3 c^3 \left(1 - \frac{m_K^2}{m_\tau^2}\right)^2 (1 + \delta R'_{\tau K})$$ $\tau\to K$

Radiative corrections in tau fit results elaborations

until HFLAV 2018

- $\delta R_{\tau\pi/\pi\mu} = (0.16 \pm 0.14)\%, \quad \delta R_{\tau K/K\mu} = (0.90 \pm 0.22)\%$ [Decker, Finkemeier 1994]
- $\delta R_{\tau\pi/\pi\mu} = \delta R_{\tau K/K\mu} \cdot \delta R_{K\mu}$
- $\delta R_{K\mu} = (1.07 \pm 0.21)\%$ [Cirigliano, Neufeld 2011] (updated)

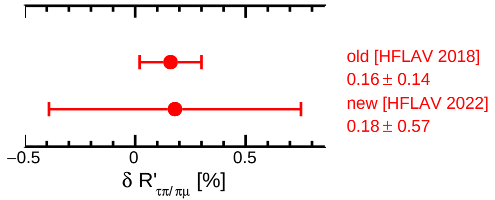
since HFLAV 2022

- $$\begin{aligned} \delta R'_{\tau\pi/\pi\mu} &= (0.18 \pm 0.57)\% \qquad & \delta R'_{\tau K/K\mu} &= (0.97 \pm 0.58)\% \\ \delta R_{\tau\pi} &= (1.77 \pm 0.57)\% \qquad & \delta R_{\tau K} &= (1.86 \pm 0.58)\% \end{aligned}$$

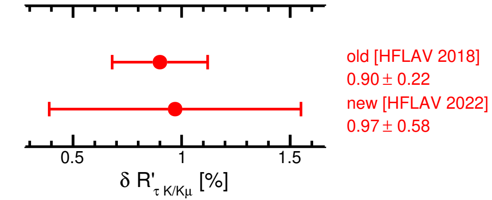
  [M.A.Arroyo-Ureña, G.Hernández-Tomé, G.López-Castro, P.Roig, I.Rosell, PRD 104 (2021) L091502]
- assuming 100% correlation between $\delta R'_{\tau\pi/\pi\mu}, \delta R_{\tau\pi}$ and $\delta R'_{\tau K/K\mu}, \delta R_{\tau K}$ because tau radiative corrections uncertainties dominate
- assuming no correlation between $\delta R'_{\tau\pi/\pi\mu}, \delta R'_{\tau K/K\mu}$ and $\delta R_{\tau\pi}, \delta R_{\tau K}$
- $(1 + \delta R_{\tau K/\tau\pi}) = \dfrac{(1 + \delta R_{\tau K})}{(1 + \delta R_{\tau\pi})}$
- $(1 + \delta R'_{\tau K}) = S_{EW}^{m_\tau}(1 + \delta R_{\tau K})$

Comparison of new [HFLAV 2022] vs. old [HFLAV 2018] radiative corrections

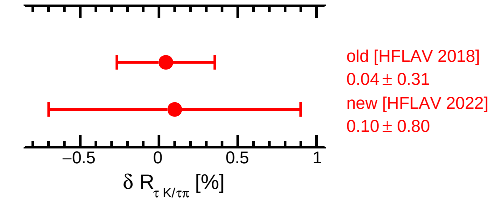
$\delta R'_{\tau\pi/\pi\mu}$



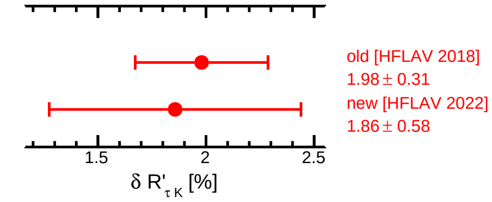
$\delta R'_{\tau K/K\mu}$



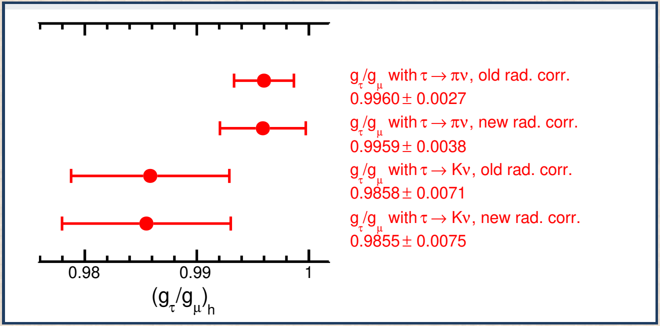
$\delta R_{\tau K/\tau\pi}$

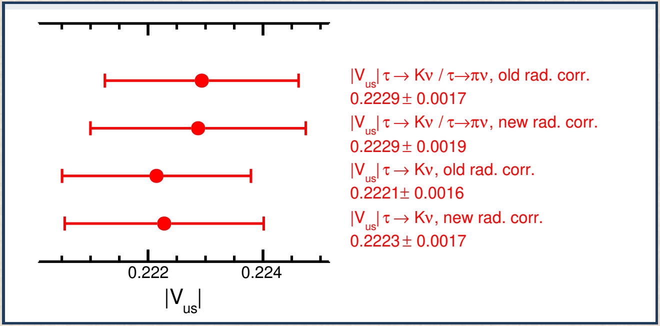


$\delta R'_{\tau K}$



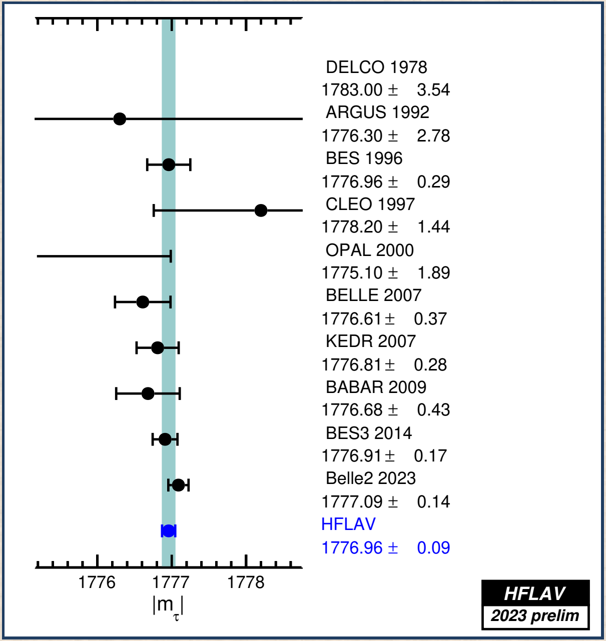
Comparison of new [HFLAV 2022] vs. old [HFLAV 2018] radiative corrections



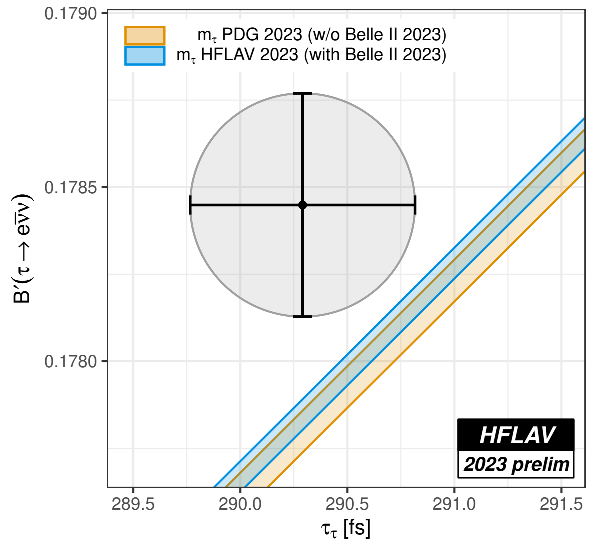


- lepton universality tests and $|V_{us}|$ all computed using the HFLAV 2022 tau branching ratio fit results and varying only the radiative corrections

Tau mass fit including Belle II recent measurement

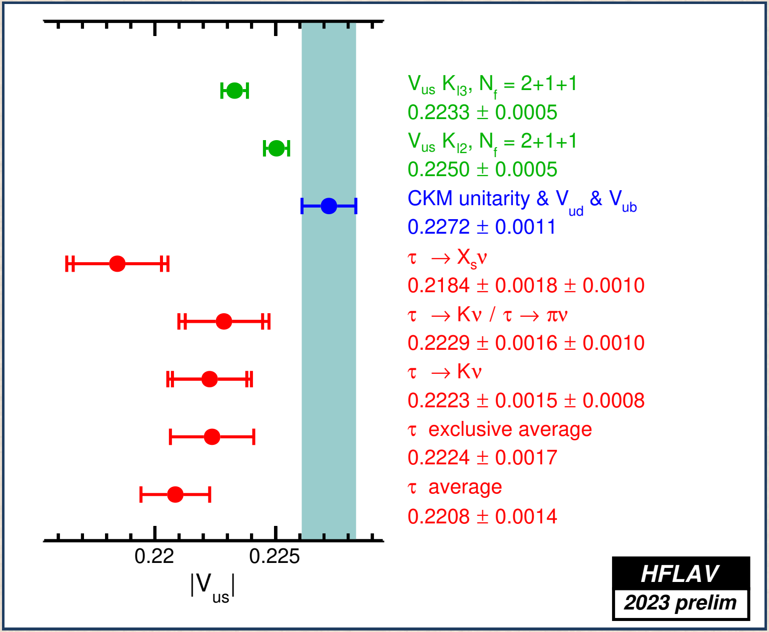


Canonical tau universality plot



- B'_e is the average of B_e and B_e predicted by the B_μ value (using HFLAV fit results complete of correlations)
- plot width and heigth proportional to B'_e and τ_τ central values to have respective plotted uncertainties of the same size when the relative uncertainties are the same

$|V_{us}|$ from tau measurements



- $|V_{ud}|$ and $|V_{ub}|$ from PDG 2023 reviews
- $|V_{us}|$ from Kaons from Cirigliano et al. PLB 838 (2023) 137748
- there are other determinations of $|V_{us}|$ from tau inclusive (by Kim Maltman *et al.*,) however they cannot be used in a simple way as the procedure by Gamiz, Pich *et al.*: this is the reason why HFLAV Tau continues to use the original calculation

$|V_{us}|$ from $\tau \to X_s \nu_\tau$ uncertainties budget

$\pi^- \bar{K}^0 2\pi^0 \nu_\tau$ (ex. K^0)	0.3933
$K^- 2\pi^0 \nu_\tau$ (ex. K^0)	0.3789
$K^- 3\pi^0 \nu_\tau$ (ex. K^0, η)	0.3715
$\bar{K}^0 h^- h^- h^+ \nu_\tau$	0.3452
$K^- \pi^0 \nu_\tau$	0.2561
$K^- \pi^- \pi^+ \pi^0 \nu_\tau$ (ex. K^0, ω, η)	0.2438
$\pi^- \bar{K}^0 \nu_\tau$	0.2373
$\pi^- \bar{K}^0 \pi^0 \nu_\tau$	0.2201
$K^- \nu_\tau$	0.1646
$K^- \omega \nu_\tau$	0.1573
$K^- \nu_\tau$	0.1453
$K^- \pi^- \pi^+ \nu_\tau$ (ex. K^0, ω)	0.1148
$\pi^- \bar{K}^0 \eta \nu_\tau$	0.0254
$K^- \pi^0 \eta \nu_\tau$	0.0198
$K^- \eta \nu_\tau$	0.0137
$K^- \phi \nu_\tau$ ($\phi \to K^+ K^-$)	0.0136
$K^- \phi \nu_\tau$ ($\phi \to K^0_S K^0_L$)	0.0094
$K^- 2\pi^- 2\pi^+ \nu_\tau$ (ex. K^0)	0.0021
$K^- 2\pi^- 2\pi^+ \pi^0 \nu_\tau$ (ex. K^0)	0.0010
$\tau \to$ non-strange	0.0855
$\mathcal{B}_e^{\text{univ}}$	0.0044
theory	0.4863

- to be updated, but negligible changes in HFLAB 2023

On-going work

LFV limits combinations

- compute tau LFV upper limit combinations averaging corresponding branching fraction mesurements and then calculating upper limit on the average
- can use HFLAV techniques to update measurements for updated external inputs and to take into account common systematics
- some papers may not document the measured branching fraction

SW infrasctructure for averaging and publishing

- improving automatization of averaging procedure and their publication
- export results in published electronic material
- access PDG information electronically (PDG functionality being improved)

Conclusion

- preparing for the Belle II tau measurements. . .

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