Wishlist of $\tau$ results for $\tau$2025

08/12/2023
Gianluca Inguglia
Disclaimer

This is a personal selection of important topics that might not fully include all aspects for the development of the field

so...if I missed your (expected) results, apologies!
Disclaimer II

I am (proudly) a member of Belle II, but will not talk on its behalf
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I am (proudly) a member of Belle II, but will not talk on its behalf
In principle, I could just provide you with a list of items that I select as wishlist for 2025 and end the talk here.

I will try to instead tell you first why these items are important.
τ mass, it’s not just for the sake to know its mass..

- τ mass poorly known compared to e or μ (a few orders of magnitude less precise)

\[
M_e = 0.51099895000 \pm 0.00000000015 \text{ MeV/c}^2 \\
M_\mu = 105.6583755 \pm 0.0000023 \text{ MeV/c}^2 \\
M_\tau = 1776.86 \pm 0.12 \text{ MeV/c}^2
\]
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\[ M_\tau = 1776.86\pm0.09 \text{ MeV/c}^2 \]

See Alberto Lusiani’s slides
\[ \tau \text{ mass, it’s not just for the sake to know its mass..} \]

- \( \tau \) mass poorly known compared to \( e \) or \( \mu \) (a few orders of magnitude less precise)

- Both BES III (~x5) and Belle II (~x2) have accumulated more data in the past, can we imagine to have a more precise determination of the \( \tau \) mass by 2025?
Experimental absolute leptonic $B_f$ determination

\[ \frac{\Gamma(\mu^-\bar{\nu}_\mu\nu_T)}{\Gamma_{\text{total}}} \]  
\[ \frac{\Gamma_3}{\Gamma} \]

To minimize the effect of experiments with large systematic errors, we exclude experiments which together would contribute 5% of the weight in the average.

<table>
<thead>
<tr>
<th>VALUE (%)</th>
<th>EVTS</th>
<th>DOCUMENT ID</th>
<th>TECN</th>
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</tr>
</thead>
<tbody>
<tr>
<td>17.39 ± 0.04</td>
<td>OUR FIT</td>
<td>SCHAELE 05C</td>
<td>ALEP</td>
<td>1991-1995 LEP runs</td>
</tr>
<tr>
<td>17.33 ± 0.05</td>
<td>OUR AVERAGE</td>
<td>17.319 ± 0.070 ± 0.032</td>
<td>54k</td>
<td></td>
</tr>
<tr>
<td>17.34 ± 0.09 ± 0.06</td>
<td>ABBIENDI 03</td>
<td>OPAL</td>
<td>1990-1995 LEP runs</td>
<td></td>
</tr>
<tr>
<td>17.342 ± 0.110 ± 0.067</td>
<td>2</td>
<td>17.325 ± 0.095 ± 0.077</td>
<td>27.7k</td>
<td>1991-1995 LEP runs</td>
</tr>
<tr>
<td>17.37 ± 0.08 ± 0.018</td>
<td>3</td>
<td>ANASTASSOV 97</td>
<td>CLEO</td>
<td>$E^{\text{ee}}_{\text{cm}} = 10.6$ GeV</td>
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- We use the following data for averages but not for fits.

\[ \frac{\Gamma(e^-\bar{\nu}_e\nu_T)}{\Gamma_{\text{total}}} \]  
\[ \frac{\Gamma_5}{\Gamma} \]

To minimize the effect of experiments with large systematic errors, we exclude experiments which together would contribute 5% of the weight in the average.

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<td>17.82 ± 0.04</td>
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<td>17.837 ± 0.072 ± 0.036</td>
<td>56k</td>
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<td>17.806 ± 0.104 ± 0.076</td>
<td>2</td>
<td>17.81 ± 0.09 ± 0.06</td>
<td>24.7k</td>
<td>1991-1995 LEP runs</td>
</tr>
<tr>
<td>17.877 ± 0.109 ± 0.110</td>
<td>3</td>
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<td>33.1k</td>
<td>1991-1995 LEP runs</td>
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- Some of you weren’t even born when the last measurements of the absolute leptonic $B_f$ took place.
- What were you doing, say, in 1990?
- Anyone else thinking it’s time for an update?
## Mean lifetime of tau lepton

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<tr>
<td>290.17 ± 0.53 ± 0.33</td>
<td>1.1M</td>
<td>BELOUS 14</td>
<td>BELL</td>
<td>$711 \text{ fb}^{-1} E_{\text{CM}}^{ee} = 10.6 \text{ GeV}$</td>
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<td>290.9 ± 1.4 ± 1.0</td>
<td></td>
<td>ABDALLAH 04T</td>
<td>DLPH</td>
<td>1991-1995 LEP runs</td>
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<td>293.2 ± 2.0 ± 1.5</td>
<td></td>
<td>ACCIARI 00B</td>
<td>L3</td>
<td>1991–1995 LEP runs</td>
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<td>290.1 ± 1.5 ± 1.1</td>
<td></td>
<td>BARATE 97R</td>
<td>ALEP</td>
<td>1989–1994 LEP runs</td>
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<tr>
<td>289.2 ± 1.7 ± 1.2</td>
<td></td>
<td>ALEXANDER 96E</td>
<td>OPAL</td>
<td>1990–1994 LEP runs</td>
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<tr>
<td>289.0 ± 2.8 ± 4.0</td>
<td>57.4k</td>
<td>BALEST 96</td>
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Mean lifetime of tau lepton

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\begin{align*}
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289.2 & \pm 1.7 \pm 1.2 \\
289.0 & \pm 2.8 \pm 4.0 \quad 57.4k
\end{align*}
\]

\[n^+ = n^+_0(x^0, y^0, z^0)\]

\[
x^* \cdot P_{1x} + y^* \cdot P_{1y} + z^* \cdot P_{1z} = |P_1| \cos \theta_1^* \\
x^* \cdot P_{2x} + y^* \cdot P_{2y} + z^* \cdot P_{2z} = -|P_2| \cos \theta_2^* \\
(x^*)^2 + (y^*)^2 + (z^*)^2 = 1
\]

\[n^+_{\pm} \text{ converted to a 4-momentum using } e^\pm \text{ beam energy and } \tau \text{ mass} \]

\[
|\langle \tau_+ \rangle - \langle \tau_- \rangle|/\langle \tau_+ \rangle < 7.0 \times 10^{-3} \text{ at 90% CL}
\]

Phys. Rev. Lett. 112, 031801

Belle II measured with world's best precision the lifetime of D^0, D^+ and \Lambda_c, can we expect also the \tau lifetime to be measured in the near future?
τ mass, it’s not just for the sake to know its mass..

- τ mass poorly known compared to e or μ (a few orders of magnitude less precise)
- Important parameter in lepton universality tests

\[ B_{\tau \to l}^{SM} \propto B_{\mu \to e} \frac{\tau_\tau}{\tau_\mu} \frac{m_\tau^5}{m_\mu^5} \]
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\[ B_{\tau \to l}^{SM} \propto B_{\mu \to e} \frac{\tau}{\mu} \frac{m_\tau}{m_\mu} \]

Private plot
Credits: A. Rostomyan, N. Rad et al.
Using $\tau$ for LFU tests, LEP and hadron colliders

Two reference papers, of experimental measurements and their combinations

$R_{\tau/(e+\mu)} = \frac{2 \mathcal{B}(W \rightarrow \tau \bar{\nu}_\tau)}{\mathcal{B}(W \rightarrow e \bar{\nu}_e) + \mathcal{B}(W \rightarrow \mu \bar{\nu}_\mu)} = 1.066 \pm 0.025$

LEP results from $W$ boson decays to leptons indicated a $\sim 2.5\sigma$ tension with the predicted value of $R_{\tau/l} = 0.9996$ (0709.1075, 0005060)
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LEP results from $W$ boson decays to leptons indicated a $\sim 2.5\sigma$ tension with the predicted value of $R_{\tau/l} = 0.9996$ (0709.1075, 0005060)

Table 5: Ratios of different leptonic branching fractions, $R_{\mu/e} = \mathcal{B}(W \rightarrow \mu \bar{\nu}_\mu) / \mathcal{B}(W \rightarrow e \bar{\nu}_e)$, $R_{\tau/e} = \mathcal{B}(W \rightarrow \tau \bar{\nu}_\tau) / \mathcal{B}(W \rightarrow e \bar{\nu}_e)$, and $R_{\tau/\mu} = \mathcal{B}(W \rightarrow \tau \bar{\nu}_\tau) / \mathcal{B}(W \rightarrow \mu \bar{\nu}_\mu)$, measured here compared with the values obtained by other LEP [8], LHC [13, 16, 17], and Tevatron [14, 15] experiments.

<table>
<thead>
<tr>
<th></th>
<th>CMS</th>
<th>LEP</th>
<th>ATLAS</th>
<th>LHCb</th>
<th>CDF</th>
<th>D0</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_{\mu/e}$</td>
<td>1.009 ± 0.009</td>
<td>0.993 ± 0.019</td>
<td>1.003 ± 0.010</td>
<td>0.980 ± 0.012</td>
<td>0.991 ± 0.012</td>
<td>0.886 ± 0.121</td>
</tr>
<tr>
<td>$R_{\tau/e}$</td>
<td>0.994 ± 0.021</td>
<td>1.063 ± 0.027</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>$R_{\tau/\mu}$</td>
<td>0.985 ± 0.020</td>
<td>1.070 ± 0.026</td>
<td>0.992 ± 0.013</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>$R_{\tau/\ell}$</td>
<td>1.002 ± 0.019</td>
<td>1.066 ± 0.025</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
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Test of lepton flavour universality

It is in principle a very simple test: compare the rates of $\tau \rightarrow \mu \nu \nu$ vs $\tau \rightarrow e \nu \nu$

$$\left( \frac{g_\mu}{g_e} \right)_\tau = \sqrt{\frac{BF[\tau^- \rightarrow \mu^- \bar{\nu}_\mu \nu_\tau]}{BF[\tau^- \rightarrow e^- \bar{\nu}_e \nu_\tau]} \frac{f(m_e^2/m_\tau^2)}{f(m_\mu^2/m_\tau^2)}}$$

$$R_\mu = \frac{BF[\tau^- \rightarrow \mu^- \bar{\nu}_\mu \nu_\tau]}{BF[\tau^- \rightarrow e^- \bar{\nu}_e \nu_\tau]}$$

$$f(x) = -8x + 8x^3 - x^4 - 12x^2 \log x$$

Details in Paul Feichtinger’s slides
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$$R_\mu = \frac{BF[\tau^- \rightarrow \mu^- \bar{\nu}_\mu \nu_\tau]}{BF[\tau^- \rightarrow e^- \bar{\nu}_e \nu_\tau]}$$

$$f(x) = -8x + 8x^3 - x^4 - 12x^2 \log x$$

$$R^\text{SM}_\mu = 0.9726$$

- **CLEO (1997)**
  - $0.9777 \pm 0.0063 \pm 0.0087$

- **BaBar (2010)**
  - $0.9796 \pm 0.0016 \pm 0.0036$

- **Belle II Preliminary (2023)**
  - $0.9675 \pm 0.0007 \pm 0.0036$

**Combination**
- $0.9735 \pm 0.0026$

**Details in Paul Feichtinger’s slides**
Within this Abelian symmetry, $L_\mu - L_\tau$, LFV terms are allowed
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LFU tests in tau decays, implications on BSM

\[ \mathcal{L}_{Z'} = g'_L (\bar{\mu} \gamma^\alpha P_L \tau + \bar{\nu}_\mu \gamma^\alpha P_L \nu_\tau) Z'_\alpha + g'_R (\bar{\mu} \gamma^\alpha P_R \tau) Z'_\alpha + \text{H.c.} \]

\[ P_{L,R} = (1 \mp \gamma^5)/2 \]

A “standard” \( L_\mu - L_\tau \) \( Z' \)

The model is a new gauge boson, \( Z' \), which couples to \( L_\mu - L_\tau \). The interaction Lagrangian is

\[ \mathcal{L} = -g' \bar{\mu} \gamma^\mu Z'_{\mu \mu} + g' \bar{\tau} \gamma^\mu Z'_{\mu \tau} - g' \bar{\nu}_{\mu,L} \gamma^\mu Z'_{\mu \nu_{\mu,L}} + g' \bar{\nu}_{\tau,L} \gamma^\mu Z'_{\mu \nu_{\tau,L}}. \]

The equations for the partial widths are,

\[ \Gamma(Z' \rightarrow \ell^+ \ell^-) = \frac{(g')^2 M_{Z'}}{12\pi} \left(1 + \frac{2M_\ell^2}{M_{Z'}^2}\right) \sqrt{1 - \frac{4M_\ell^2}{M_{Z'}^2}} \theta(M_{Z'} - 2M_\ell), \]

\[ \Gamma(Z' \rightarrow \nu_\ell \bar{\nu}_\ell) = \frac{(g')^2 M_{Z'}}{24\pi}. \]
LFU tests in tau decays, implications on BSM

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$$P_{L,R} = (1 \mp \gamma^5)/2$$

\[ L_\mu \leftrightarrow L_\tau, \quad \mu_R \leftrightarrow \tau_R, \]
\[ B^\alpha \leftrightarrow B^\alpha, \quad Z'^\alpha \leftrightarrow -Z'^\alpha \]

A “standard” $L_\mu$-$L_\tau$ $Z'$

The model is a new gauge boson, $Z'$, which couples to $L_\mu - L_\tau$. The interaction Lagrangian is

$$\mathcal{L} = -g' \bar{\mu} \gamma^\mu Z'_\mu \mu + g' \bar{\tau} \gamma^\mu Z'_\mu \tau - g' \bar{\nu}_\mu, L \gamma^\mu Z'_\mu \nu_\mu, L + g' \bar{\nu}_\tau, L \gamma^\mu Z'_\mu \nu_\tau, L.$$ 

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LFU tests in tau decays, implications on BSM

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\[ \Gamma(Z' \to e_\ell \bar{e}_\ell) = \frac{(g')^2 M_{Z'}}{24\pi}. \]
The sensitivity to a LFV $Z'$ depends on the level of systematics in the test of LFU in tau decays.
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A 3x3 matrix is defined by 18 parameters, however

\( V_{\text{CKM}} \) is a unitary matrix \( VV^\dagger = V^\dagger V = I \) : 9 unitarity conditions.

Only 9 parameters are "free", and these are 3 angles and 6 phases, however

5 phases are non-physical (unobservable)

\( V_{\text{CKM}} \) can be parametrised by 4 parameters: 3 Euler angles and 1 complex phase. The complex phase in \( V_{\text{CKM}} \) violates \( CP \).
CKM matrix and the unitarity relations

\[ V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \]

- Column orthogonality
  \[ \sum_i V_{ij} V_{ik}^* = \delta_{jk} \]

- Row orthogonality
  \[ \sum_j V_{ij} V_{kj}^* = \delta_{ik} \]

\[ |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 \]
\[ |V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 = 1 \]
\[ |V_{td}|^2 + |V_{ts}|^2 + |V_{tb}|^2 = 1 \]
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\[ V_{ud} V_{cd} + V_{us} V_{cs} + V_{ub} V_{cb} = 0 \]
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\end{pmatrix}
\]

\[
\sum_i V_{ij} V_{ik}^* = \delta_{jk} \quad \text{Column orthogonality}
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\]
CKM matrix and the unitarity relations

\[ V_{\text{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \]

\[ \sum_i V_{ij} V_{ik}^* = \delta_{jk} \quad \text{Column orthogonality} \]

\[ \sum_j V_{ij} V_{kj}^* = \delta_{ik} \quad \text{row orthogonality} \]

\[ |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 \]

Cabibbo Angle Anomaly

- \[ |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9985 \pm 0.0005 \]
- 3 sigma tension with SM!
- This is the CAA

See Matthew Kirk slides at Anomalies and precision in the Belle II era workshop
Or Andreas Crivellin, explaining the Cabibbo Angle Anomaly, 2207.02507
Kaon decays provide a very precise determination of $V_{us}$ and indicate a tension with CKM unitarity.

Hadronic $\tau$ decays provide a less precise determination of $V_{us}$, but its an independent approach that provides a consistency check and indicate a tension with CKM unitarity.

See Alberto Lusiani’s slides.
LFU in $\tau \to h\nu$ and $V_{us}$

BaBar has performed a precision test using hadronic $\tau$ decays. Phys. Rev. Lett. 105 051602

$$\left(\frac{g_{\tau}}{g_{\mu}}\right)_{h}^{2} = \frac{B(\tau \to h\nu_{\tau})}{B(h \to \mu\nu_{\mu})} \left[\frac{2m_{h}m_{\mu}^{2}\tau_{h}}{(1 + \delta_{h})m_{\tau}^{3}\tau_{\tau}} \left(1 - \frac{m_{\mu}^{2}}{m_{h}^{2}}\right)\right]^{2},$$

where the radiative corrections are $\delta_{\pi} = (0.16 \pm 0.14)\%$ and $\delta_{K} = (0.90 \pm 0.22)\%$ [23]. Using the world averaged mass and lifetime values and meson decay rates [3], we determine $\left(\frac{g_{\tau}}{g_{\mu}}\right)_{\pi(K)} = 0.9856 \pm 0.0057 (0.9827 \pm 0.0086)$ and $\left(\frac{g_{\tau}}{g_{\mu}}\right)_{h} = 0.9850 \pm 0.0054$ when combining these results; this is $2.8\sigma$ below the SM expectation and within $2\sigma$ of the world average.

$$B(\tau^{-} \to K^{-}\nu_{\tau}) = \frac{G_{F}^{2}f_{K}^{2}(V_{us})^{2}m_{\tau}^{3}\tau_{\tau}}{16\pi^{2}} \left(1 - \frac{m_{K}^{2}}{m_{\tau}^{2}}\right)^{2}S_{EW}$$

$$R_{K/\pi} = \frac{f_{K}^{2}(V_{us})^{2}}{f_{\pi}^{2}|V_{ud}|^{2}} \left(1 - \frac{m_{\pi}^{2}}{m_{\tau}^{2}}\right)^{2} \left(1 + \delta_{LD}\right),$$

$|V_{us}| = 0.2193 \pm 0.0032$ $|V_{us}| = 0.2255 \pm 0.0024$

Systematic uncertainties here are much larger than in the case of leptonic decays.

Is it foreseeable that by 2025 we see an update of these measurements, from Belle II?

New physics?


Lepton flavour violation

See Alberto Martini’s slides

• Due to their large mass, $\tau$ leptons provide a wide variety of LFV (and LNV) decay modes to study:
  - radiative: $\tau \rightarrow \ell \gamma$
  - leptonic: $\tau \rightarrow \ell \ell \\
  - semileptonic: $\tau \rightarrow \ell h(h)$

  – “golden channels” for discovery: $\tau \rightarrow \mu \gamma$, $\tau \rightarrow \mu \mu \\
  – complementary: semileptonic modes allow us to test LFV couplings b/w quarks and leptons, and better discriminate b/w NP models

• So far, searches for $\tau$ LFV decays mostly occurred at last-gen $B$ factories

• Upper limits had approached the regime sensitive to NP ($10^{-10}$-$10^{-7}$)

Extrapolating from Belle results (50 ab$^{-1}$):

Belle II will push the current bounds forward by at least one order of magnitude!

While luminosities are integrated, could we know something in between? @Belle, what about the $\sim$2ab$^{-1}$ of Belle+Belle II data expected by next summer?

see here, from P. Rados
CP violation in $\tau$ decays

- Two different analysis techniques adopted by BaBar and Belle.
- BaBar looked at the asymmetry in decays to a specific final state (and reported the observation of CP violation with a significance of $2.8\sigma$).
- Belle looked instead into possible effects in angular distributions (and reported basically no CP violation).
- What about Belle II? A measurement to be presented at $\tau$2025?

Due to CP violation in the kaon sector, $\tau \to K_\pi \nu_\tau$ decays in the SM have a nonzero decay-rate asymmetry:

$$A_\tau = \frac{\Gamma(\tau^+ \to K^+\pi^-\nu_\tau) - \Gamma(\tau^- \to K^0\pi^0\nu_\tau)}{\Gamma(\tau^+ \to K^+\pi^-\nu_\tau) + \Gamma(\tau^- \to K^0\pi^0\nu_\tau)}$$

- SM prediction: $(3.6 \pm 0.1) \times 10^{-3}$
- BaBar measurement: $(-3.6 \pm 2.3 \pm 1.1) \times 10^{-3}$ ($2.8\sigma$)
- An improved $A_\tau$ measurement is a priority at Belle II.

CP violation could also arise from a charged scalar boson exchange. It would be detected as a difference in the decay angular distributions:

$$A_{\tau}^{CP} = \frac{\int \frac{dQ^2}{Q^2} \cos \beta \cos \psi (dN_{+}/d\omega - dN_{-}/d\omega) d\omega}{\frac{1}{2} \int \frac{dQ^2}{Q^2} (dN_{+}/d\omega + dN_{-}/d\omega) d\omega} \approx \langle \cos \beta \cos \psi \rangle_{+} - \langle \cos \beta \cos \psi \rangle_{-}$$

$$d\omega = dQ^2 d\cos \theta d\cos \beta$$

With 50 ab$^{-1}$ of data, Belle II is expected to provide a $7\sigma$ more precise measurement:

$$|A_{\tau}^{CP}| < (0.5-3.8) \times 10^{-4}$$

(assuming central value $A_{\tau}^{CP} = 0$)
Abdollah Mohammadi

- Polarization of $\tau^-$ leptons in the decay of Z bosons produced in pp collisions using CMS detector is presented to an integrated luminosity of 36.3 fb$^{-1}$.
- The measured $\tau^-$ lepton polarization, $P_{\tau}(Z) = -0.144 \pm 0.015$, is in good agreement with the SLD, LEP and ATLAS results.
- The measured polarization constrains the effective couplings of $\tau^-$ leptons to the Z boson and determines the effective weak mixing angle to be $\sin^2 \theta_{\text{eff}}^W = 0.2319 \pm 0.0019$
- No deviation from SM! Improving the sensitivity requires both more data and more importantly, better understanding/reducing the systematics.

Caleb Miller

**Conclusions**

- $B\bar{A}B\bar{A}R$ has implemented the first application of the new Tau Polarimetry technique to measure the PEP-II average beam polarization
  \[ \langle P \rangle = 0.0035 \pm 0.0024_{\text{stat}} + 0.0029_{\text{sys}} \]
- Identified 21 sources of systematic uncertainty
- Modelling/Understanding of neutral processes dominates the largest systematics
- Tau Polarimetry could be applied at other $e^+e^-$ colliders interested in polarization
- Final uncertainty exceeds Chiral Belle assumptions suggesting the experiment could make even more precise measurements

Left, CMS, used only partial data samples, much more data available that might help shrinking Systematics → hope for better precision by 2025

Below, BaBar new tau polarimetry technique allowed to measure “nothing” with good precision! Can you try to measure “nothing” at Belle II by 2025?
B decays involving missing energy

- Flavour-changing neutral currents forbidden at tree-level proceeds via loops

- Belle II studied

$$B^+ \rightarrow K^+ \nu \bar{\nu}$$

and reported the observation of the process with a significance of 3.5 $\sigma$, and a tension with SM predictions at the level of 2.8 $\sigma$.  

\[2311.14647, \text{submitted to PRD}\]
B decays involving missing energy

- Flavour-changing neutral currents forbidden at tree-level proceeds via loops

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\[ B^+ \rightarrow K^+ \nu \bar{\nu} \]

and reported the observation of the process with a significance of 3.5 \( \sigma \), and a tension with SM predictions at the level of 2.8 \( \sigma \).

- How does this relate to \( \tau \)?
  - because some models (ex. 2309.0005) assuming heavy NP, would predict enhancements of other channels, such us:

\[ B \rightarrow K^{(*)} \tau \tau \quad B_s \rightarrow \tau \tau \quad B \rightarrow K^{(*)} \tau \ell \]

See this presentation for details

Can we expect some updates (LHCb), or maybe a measurement from Belle II, by 2025?
Flavour-changing neutral currents forbidden at tree-level proceeds via loops

Belle II studied

$$B^+ \rightarrow K^+ \nu \bar{\nu}$$

and reported the observation of the process with a significance of 3.5 $\sigma$, and a tension with SM predictions at the level of 2.8 $\sigma$.

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→ because some models (ex. 2309.0005) assuming heavy NP, would predict enhancements of other channels, such as:

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See this presentation for details

Can we expect some updates (LHCb), or maybe a measurement from Belle II, by 2025?
Gravitational waves probing fundamental physics, leptophilic $Z'$

First-order phase transition if scalar sector is conformally invariant:

$$V_{\text{tree}} = \lambda_H (H^\dagger H)^2 + \lambda (\Phi^\dagger \Phi)^2 - \lambda' (\Phi^\dagger \Phi) (H^\dagger H).$$

- Heavy boson fields might be responsible for phase transitions, strong enough to generate gravitational waves → stochastic, not observed so far → set upper limits
- New, complementary, way with respect to typical HEP measurements with even more accessible parameter space.
- More analysis methodologies and results can be expected in the very near future.

 Enhanced sensitivity with already available new set of data

This data used  We are here
Take home message, my wishlist for 2025

• Understanding the properties of $\tau$ is a fundamental step towards gaining a more complete understanding of the standard model and beyond.

• Many players: hadron colliders, high/middle/low-energy electron-positron colliders.

• The $\tau$ mass is being measured with higher precision using different techniques, but absolute $B_f$ and lifetime need an update.
  • Current understanding is based on measurements performed in the 90s. Facilities that could update these measurements should do so.
  • Also new CP violation tests are needed.

• Precision tests of the SM from $\tau$ processes (i.e. LFU, CKM, LFV, EDM, etc.) can be used to discover or constrain new physics. It’s not always about adding more data, but we need to understand better the data → lower systematics should be a priority.

• If the excess reported by @Belle II in $B^+ \rightarrow K^+ \nu\bar{\nu}$ is due to NP, then we might observe something also in rare B decays involving $\tau$, updates would be welcome.

• Unconventional ways to study fundamental physics are being developed (gravitational waves, first-order phase transitions), let’s keep an eye on that and see what O4 brings that could be relevant to us.
Thank you!!
**Y(nS) → τμ decays at Belle 2**

Lepton flavor violating quarkonium decays

Derek E. Hazard and Alexey A. Petrov

Phys. Rev. D 94, 074023 – Published 17 October 2016

“Any new physics model that incorporates flavor and involves flavor-violating interactions at high energy scales can be cast in terms of the effective Lagrangian of Eq. (1) at low energies. We argued that Wilson coefficients of this Lagrangian could be effectively probed by studying decays of quarkonium states with different spin-parity quantum numbers, providing complementary constraints to those obtained from tau and mu decays”

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