Study of additional radiation in the initial-state-radiation processes $e^+e^- \to \mu^+\mu^-(\gamma)$ and $e^+e^- \to e^+e^-(\gamma)$ in the *BaBar* experiment

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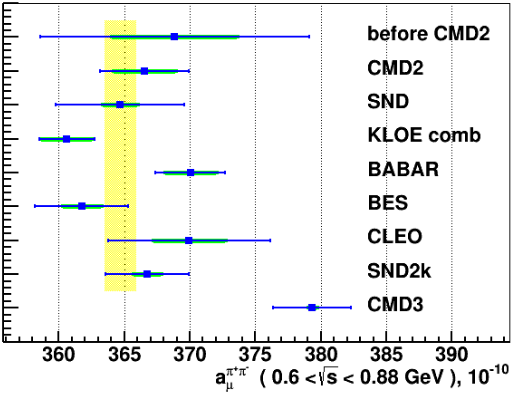
Introduction

- muon magnetic anomaly discrepancy at 5.1 σ between experiment and theory prediction [FNAL Muon $g-2$ 2023 (PRL 131 (2023) 161802)]
- discrepancy between dispersive and lattice QCD calculations of HVP,LO contribution to muon magnetic anomaly [BMW 2020 (Nature 593 (2021) 51)]
- experimental tensions between measurements of $\sigma[e^+e^- \to \pi^+\pi^-(\gamma)]$ [WP 2020, CMD3, SND2k]

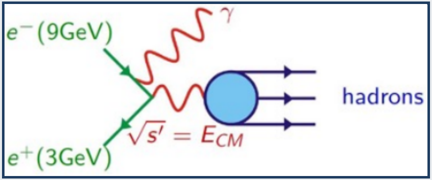


[from Fedor Ignatov, Bern, Sep 2023]

Introduction (2)

- *BABAR* measured $\sigma[e^+e^- \to \pi^+\pi^-(\gamma)]$ in 2009 with ISR technique and is working on $\sigma]e^+e^- \to \pi^+\pi^-(\gamma)]/\sigma]e^+e^- \to \mu^+\mu^-(\gamma)]$ with μ/π separation without PID

ISR technique



- measure $\sigma(e^+e^- \to \text{hadrons})(s)$ by measuring $\sigma(e^+e^- \to \gamma_{\text{ISR}} + \text{hadrons})[\Upsilon(4s)]$
- $$\frac{d\sigma(s,x,\theta_\gamma)}{dx\, d\cos\theta_\gamma} = W(s,x,\theta_\gamma)\,\sigma_0[s' = s(1-x)]$$
  - $\sqrt{s} = e^+e^-$ center-of-mass (CM) energy
  - $x \equiv 2E_\gamma/\sqrt{s}$
  - $\sigma_0 =$ Born cross section

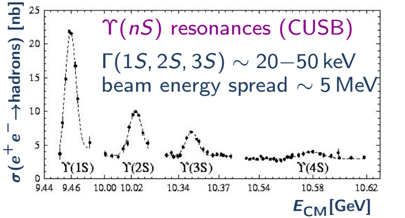
this presentation

- study of radiative corrections in data and simulation, related to above on-going measurement [arXiv:2308.05233 [hep-ex], accepted by PRD]

BABAR, asymmetric-beam-energies B-factory, 1999-2008

- primary purpose: measure time-dependent CP violation on coherent B pairs
- secondarily: general purpose e^+e^- collider & heavy flavour factory with well-defined e^+e^- initial state

center-of-mass energies



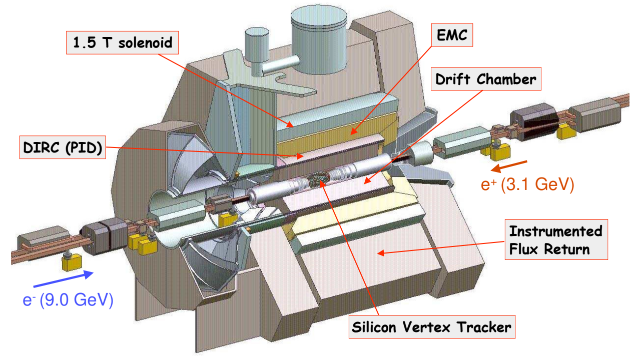
integrated $\mathcal{L}$

energy	$\mathcal{L}(\text{fb}^{-1})$
$\Upsilon(4s)$	430
$\Upsilon(3s)$	30.2
$\Upsilon(2s)$	14.5
off-peak	54

yields

flavour	events
$B\bar{B}$	$470 \cdot 10^6$
$c\bar{c}$	$690 \cdot 10^6$
$\tau^+\tau^-$	$485 \cdot 10^6$

BABAR general purpose 4π hermetic detector



Data and Monte Carlo simulation samples

Data

- *BaBar* Runs 1-6 [full data sample at $\Upsilon(4S)$]
- $424.2\,\text{fb}^{-1}$at $\Upsilon(4S)$, $43.9\,\text{fb}^{-1}$slightly below $\Upsilon(4S)$

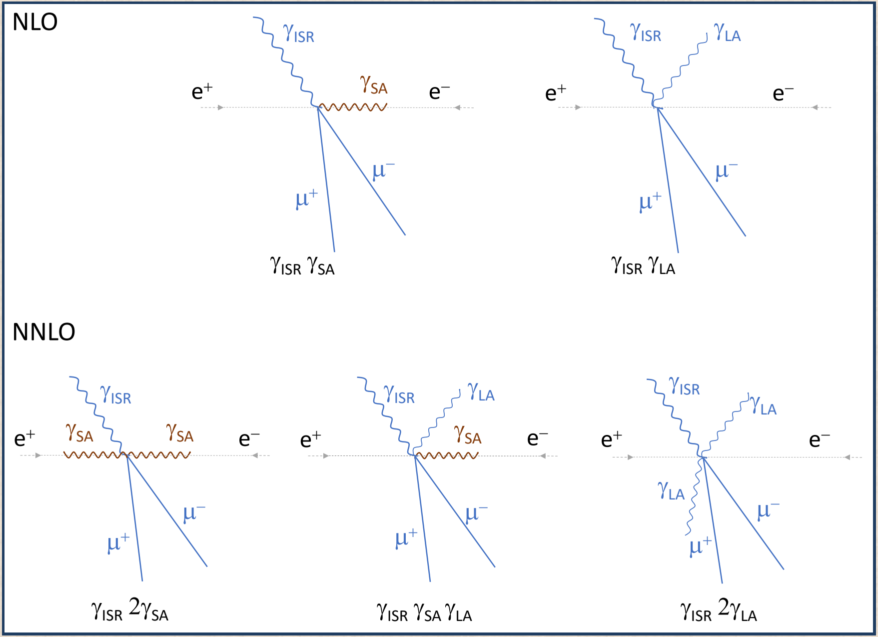
Monte Carlo signal

- Phokhara 9.1 (ISR up to full NLO matrix element) ($10\times$ data)
- AfkQED ($\simeq$ 50% data)
  - 1 additional FSR photon generated with Photos
  - additional ISR beam-collinear γ's with structure function method [Nuovo Cim. A 110 (1997) 515]
  - cutoff $m(X\gamma) > 8\,\text{GeV} \quad \Rightarrow \quad E_{\text{additional}\ \gamma} < 2.3\,\text{GeV}$

Monte Carlo background

- Phokhara 9.1: $KK(\gamma)$
- AfkQED: $e^+e^- \to \pi^+\pi^-\pi^0$, $\pi^+\pi^-2\pi^0$, ...
- Jetset: $e^+e^- \to q\bar{q}$ ($q = u, d, s, c$)
- KK2f: $e^+e^- \to \tau^+\tau^-$
- all backgrounds are normalized to match the *BaBar* data sample integrated luminosity except for $3\pi\gamma$, $q\bar{q}$ where dedicated studies have been performed

Large angle (LA) and small angle (SA) radiation in NLO and NNLO processes



Pre-selection

- similar to past analyses, but looser (removed track momentum >1 GeV requirement)
- two tracks with opposite charges, each with
  - $0.4 < \theta < 2.45\,\text{rad}$, $p_T > 0.1\,\text{GeV}$, $\text{doca}_{xy} < 5\,\text{mm}$, $|\Delta z| < 6\,\text{cm}$
  - either both tight-PID muons or both tight-PID pions
- ISR photon candidate
  - $0.35 < \theta < 2.4\,\text{rad}$
  - $E^*_\gamma > 4\,\text{GeV}$
- additional radiation photon candidates
  - $0.35 < \theta < 2.4\,\text{rad}$
  - $E_\gamma > 50\,\text{MeV}$
- extra tracks and photons are allowed

Fitting

- each events is fit as candidate for several categories, constraint that total 4-momentum matches well-known e^+e^- initial state 4-momentum
- small-angle (SA) photons collinear with beams cannot be detected
  - direction approximated to be parallel to one of the beams
  - energy determined by constrained fit
- large-angle (LA) photons in detector acceptance have measured energy and direction

assignement to event categories

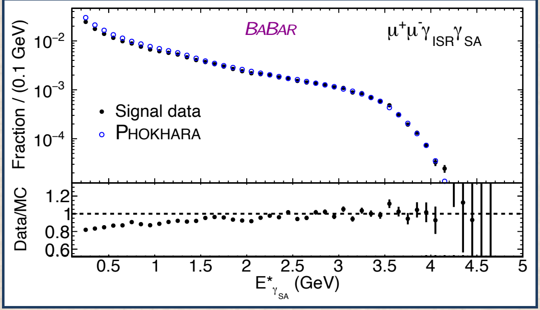
- LO, ISR event with no additional radiation
- NLO, $\gamma_{\text{ISR}} + \gamma_{\text{SA}}$
- NLO, $\gamma_{\text{ISR}} + \gamma_{\text{LA}}$
- NNLO, $\gamma_{\text{ISR}} + 2\gamma_{\text{SA}}$ (assume one γ collinear to one beam, other γ collinear to other beam)
- NNLO, $\gamma_{\text{ISR}} + \gamma_{\text{SA}} + \gamma_{\text{LA}}$
- NNLO, $\gamma_{\text{ISR}} + 2\gamma_{\text{LA}}$ (no acceptance correction since not simulated by PHOKHARA or AFKQED)
- after fit, category-dependent requirements on additional photon energies $E_\gamma > 100$ or $200\,\text{MeV}$
- events assigned to category with lowest fit χ^2
- suppress backgrounds to $\pi^+\pi^-$ requiring BDT classifier and $m_{\pi^-+\pi^-}$ in $[0.6-0.9]$ GeV interval

Acceptance, backgrounds, feed-throughs, ISR vs. FSR radiative contributions

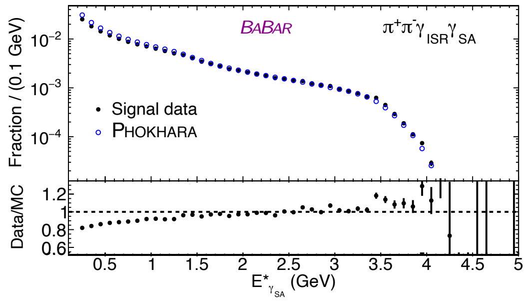
- selection efficiencies and external background subtraction estimated with Monte Carlo simulations
- background from fake photons due to pion interactions in calorimeter is measured in data and MC relative data excess of $(21.5 \pm 3.5)\%$ in data is corrected for
- feed-through backgrounds (from other event categories in this study)
  - mostly relying on Monte Carlo simulations
  - NNLO $\gamma_{\mathrm{ISR}}2\gamma_{\mathrm{SA}}$ with both γ_{SA} collinear to same beam feed-through to $1\gamma_{\mathrm{SA}}$ from data (see later)
  - complement with data-driven spectrum AFKQED simulation limited to additional $E_\gamma < 2.3\,\mathrm{GeV}$
  - simulation rates and feed-through corrected with data rates (iterative procedure)
- FSR and LA ISR separation using template fits (backup slide)

NLO SA events ($E^*_{SA} > 200$ MeV)

$\mu^+\mu^-$



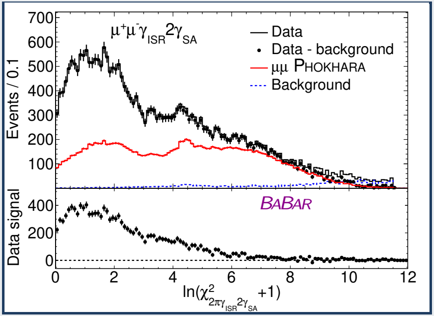
$\pi^+\pi^-$



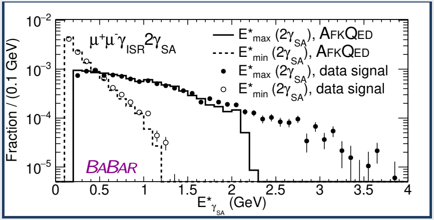
- both $\mu^+\mu^-$ and $\pi^+\pi^-$ measure data / PHOKHARA significant deficit and slope vs. energy
- also confirmed with alternative analysis (backup slide)
- bias due to collinear approximation small and well simulated (backup slide)

NNLO 2SA events ($E^*_{SA1} > 200$ MeV, $E^*_{SA2} > 100$ MeV)

$\mu^+\mu^-$

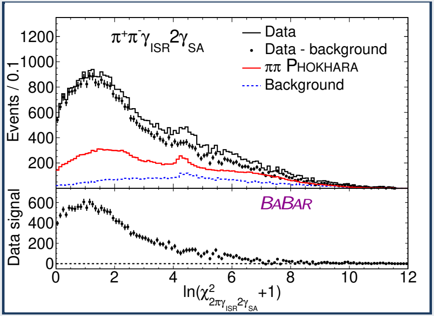


PHOKHARA only predicts NLO feed-through (dominant)

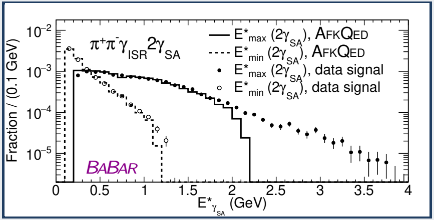


AFKQED matches bkg-subtracted data up to 2.3 GeV

$\pi^+\pi^-$



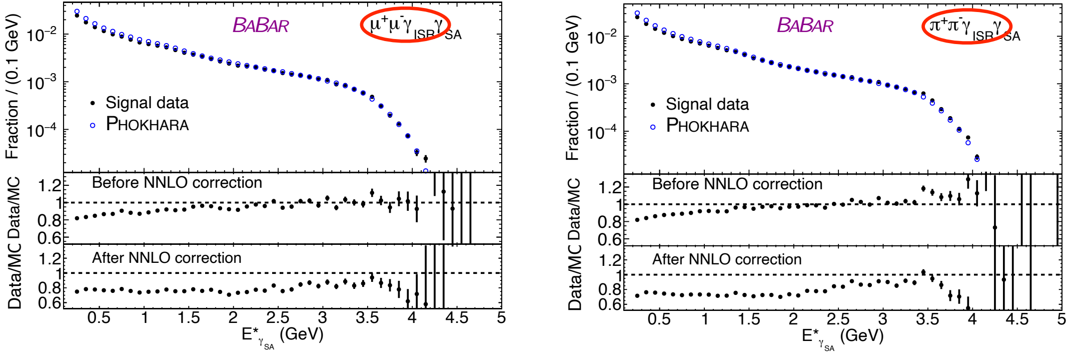
Non-NLO background on $\pi^+\pi^-$ suppressed by BDT



AFKQED matches bkg-subtracted data up to 2.3 GeV

NLO SA events distributions after correcting for NNLO events feed-through

- NLO events with 1 additional SA photon have indistinguishable background from NNLO events with 2 SA hard photons collinear to same beam
  - no simulation available
- subtract using data-measured NNLO events with 2 SA hard photons collinear to different beams
  - expected similar distributions since mostly relatively soft photons



- NNLO feed-through subtraction improves shape, but data / PHOKHARA remains significantly < 1

Efficiency and feed-through probabilities

➜ Fitted categories

➜ True categories (same energy thresholds)

%	LO	γISRγSA	γISRγLA $(< 20°)$	γISRγLA $(> 20°)$	γISR2γSA	γISRγSAγLA $(< 20°)$	γISRγSAγLA $(> 20°)$
	$\mu\mu$ $(m_{\pi\pi} < 1.4\,\mathrm{GeV/c^2})$						
LO	**99.64(1)** **99.439(3)**	*0.223(9)* 0.289(2)	*0.008(2)* 0.0140(4)	*0.033(3)* 0.0756(9)	*0.139(7)* 0.136(1)	*0.0004(4)* 0.00011(3)	*0.005(1)* 0.0073(3)
γISRγSA	*0.327(1)* 0.325(2)	**97.9(5)** **94.28(2)**	*0.006(1)* 0.0169(4)	*0.008(2)* 0.212(1)	*0.254(9)* 0.838(3)	*0.0021(9)* 0.0077(3)	*0.010(2)* 0.127(1)
γISRγLA $(< 20°)$	*0.096(6)* 0.118(1)	*0.008(2)* 0.0113(3)	**74.8(8)** **78.8(1)**	*0.0017(7)* 0.0057(2)	*0.103(6)* 0.138(1)	*0.005((1)* 0.0008(1)	*0.0014(7)* 0.0007(1)
γISRγLA $(> 20°)$	*0.003(1)* 0.099(1)	*0.0016(7)* 0.0645(8)	*0.0023(9)* 0.0045(2)	*0.181(8)* **83.89(6)**	*0.003(1)* 0.0517(7)	*0(0)* 0.00021(5)	*0.0010(6)* 0.0135(4)
γISR2γSA	0.19(2)	1.69(3)	0.014(2)	0.003(3)	**90.8(5)**	0.0010(5)	0.0015(5)
γISRγSAγLA $(< 20°)$	0.010(1)	0.074(4)	0.005(1)	0.0007(5)	0.035(4)	**72.2(1.6)**	0.0004(4)
γISRγSAγLA $(> 20°)$	0.0007(5)	0.010(2)	0(0)	0.0013(7)	0.004(1)	0.0007(5)	*0.042(4)*
Rest	0.0011(6)	0.003(1)	0.0011(6)	0(0)	0.011(2)	0.007(2)	0.003(1)

- diagonal = selection efficiencies from AfkQED (1st row) in each block) and Phokhara (2nd row)
- non-diagonal = feed-through probabilities normalized to total event yields
- data vs. MC difference especially in LA categories taken into account
- not shown: di-pion selections using BDT have lower efficiencies

Measured rates of all event categories

TABLE I: Event fractions in data for the $\mu\mu$ and $\pi\pi$ processes in all fit categories. The numbers in parentheses represent uncertainties, where the first is statistical and the second systematic. The results, except for NNLO 2LA (which is not simulated by any generator currently available) are corrected using efficiencies that vary category-by-category between 99% and 72%, except for NLO FSR $\pi\pi$ (40%) and NNLO FSR $\pi\pi$ (22% due to BDT selection.)

Category	$\mu\mu$ $m_{\pi\pi} < 1.4\,\mathrm{GeV}/c^2$	$\pi\pi$ $0.6 < m_{\pi\pi} < 0.9\,\mathrm{GeV}/c^2$
LO	0.7716(4)(14)	0.7839(5)(12)
NLO SA-ISR	0.1469(3)(36)	0.1401(2)(16)
NLO LA-ISR	0.0340(2)(9)	0.0338(2)(9)
NLO ISR	0.1809(4)(35)	0.1739(3)(20)
NLO FSR	0.0137(2)(7)	0.0100(1)(16)
NNLO ISR [a]	0.0309(2)(38)	0.0310(2)(39)
NNLO FSR [b]	0.00275(6)(9)	0.00194(12)(50)
NNLO 2LA [c]	0.00103(3)(1)	0.00066(4)(4)

[a] NNLO ISR = 2SA-ISR or SA-ISR + LA-ISR
[b] NNLO FSR = SA-ISR + LA-FSR
[c] NNLO 2LA = 2LA-ISR, LA-ISR + LA-FSR or 2LA-FSR

systematic uncertainties

- (minor) efficiency corrections
- (dominant) background and feed-through subtraction

Summary

- NNLO radiation with ISR photon plus two additional radiation photons observed for the first time fraction $= (3.47 \pm 0.38)\%$ for muons and $(3.36 \pm 0.39)\%$ for pions
  - permits subtracting NNLO feed-through contributions to NLO samples
- PHOKHARA NLO γ_{SA} prediction significantly higher than data, data/MC ratios 0.763 ± 0.019 for dimuons and 0.750 ± 0.008 for dipions
- PHOKHARA NLO γ_{LA} prediction consistent with data, data/MC ratios 0.96 ± 0.03 for dimuons and 0.98 ± 0.03 for dipions
- PHOKHARA has significant excess of radiation at small angles w.r.t. the beams
- AFKQED reasonably simulates NLO and NNLO rates and energy distribution for $E_\gamma < 2.3\,\text{GeV}$, data/MC ratios 1.061 ± 0.015 for dimuons and 1.043 ± 0.010 for dipions
- PHOKHARA FSR NLO prediction higher than data, data/MC ratios 0.86 ± 0.05 for muons and 0.76 ± 0.12 for pions
- AFKQED FSR NLO prediction consistent than data, data/MC ratios 1.10 ± 0.06 for muons and 1.08 ± 0.10 for pions (pions point-like)
  - supersede previous measurements of 0.96 ± 0.06 for muons and 1.21 ± 0.05 for pions [*BABAR* 2009 (PRL 103 (2009) 231801)]

- [arXiv:2308.05233 [hep-ex], accepted by PRD]

Conclusions

- *BABAR* analyses designed to be resilient w.r.t. imperfect simulation of higher orders radiation
  - selection efficiency measured on dimuon data samples
  - acceptance uncertainty (only dependence on PHOKHARA) small $(0.03 \pm 0.01)\%$ compared 0.5% total
- other experiments with ISR techniques may be affected because of
  - dependence from missing higher order radiation in PHOKHARA simulation
  - excess of PHOKHARA simulated radiation at small angles w.r.t. beams

Future

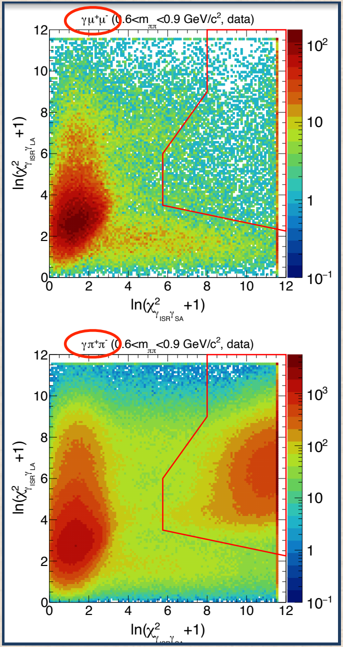
- these findings will help improving reliability of systematics estimations of all ISR measurements
- expect to publish *BABAR* pions / muons measurement next year

– end –

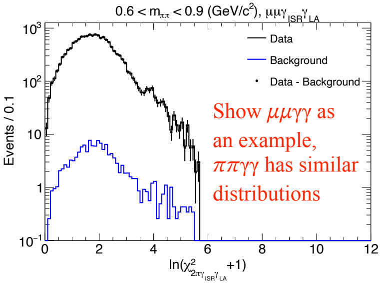
Backup Slides

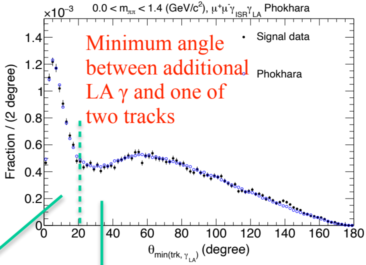
BDT requirement to suppress dipion backgrounds

- dipion processes: much larger background than dimuon processes
- bottom-right plot shows BDT cut against dipion backgrounds



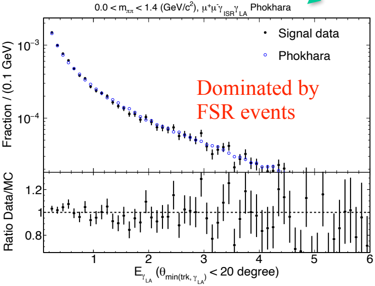
NLO LA events distribution in data and MC

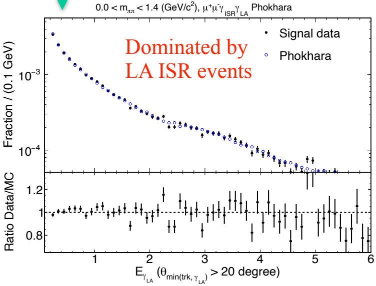




Fraction in data = event-yield of one fit category over those of all categories

Here, MC distributions normalized to data one



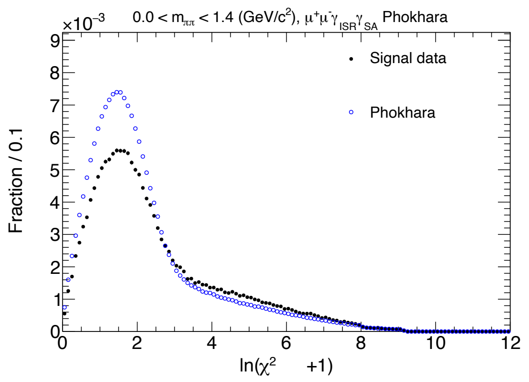


Good agreement data/Phokhara in shape for the angular and energy distributions

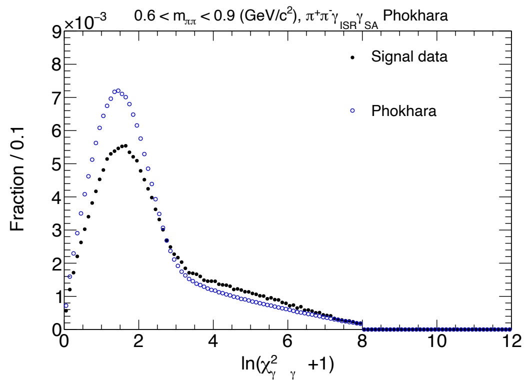
[from Zhiqing Zhang, Bern, Sep 2023]

Collinear approximation in NLO SA events ($E^*_{SA} > 200\,\text{MeV}$)

$\mu^+\mu^-$



$\pi^+\pi^-$

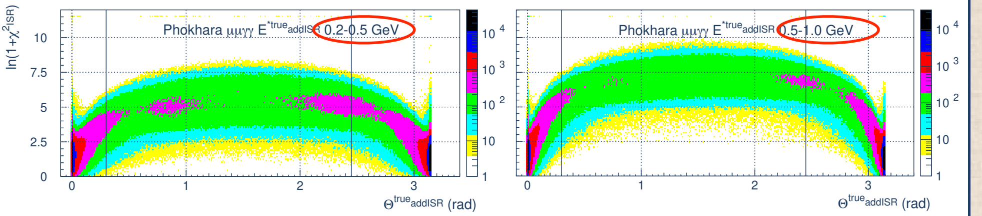


- collinear approximation causes long χ^2 tail, well modeled by PHOKHARA

NLO SA events, simulation of collinear approximation effect on χ^2

The long tail in the SA χ^2 distribution is checked with MC to be due to the collinear approximation: More central and larger energy ISR γ has longer tail

Similar bias expected in data



[from Zhiqing Zhang, Bern, Sep 2023]

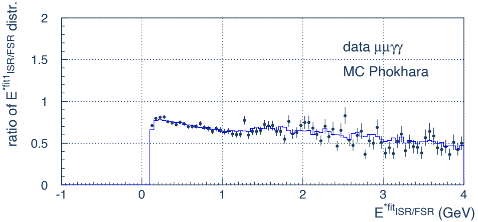
NLO SA events, simulation of collinear approximation effect on $E^*_{\gamma SA}$

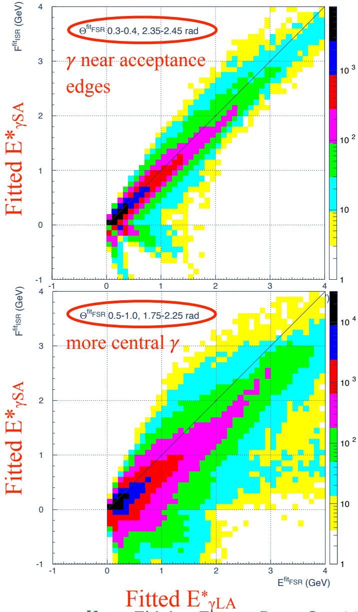
The bias on the fitted SA γ CM energy is checked using LA γ CM energy as reference

For γ near the detector acceptance edges, the bias is found smaller than more central γ

For γ outside of the detector acceptance near the beam direction, the bias is expected to be smaller

The bias is well simulated by MC



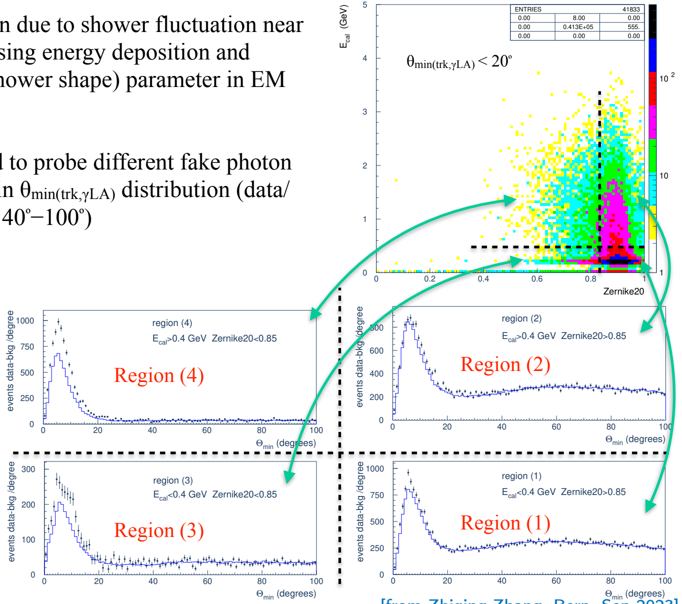


[from Zhiqing Zhang, Bern, Sep 2023]

Fake photon study

Fake photon contribution due to shower fluctuation near a pion track is studied using energy deposition and Zernike20 (transverse shower shape) parameter in EM calo

Four regions are defined to probe different fake photon contributions as shown in $\theta_{min(trk,\gamma LA)}$ distribution (data/Phokhara normalized at 40°−100°)



[from Zhiqing Zhang, Bern, Sep 2023]

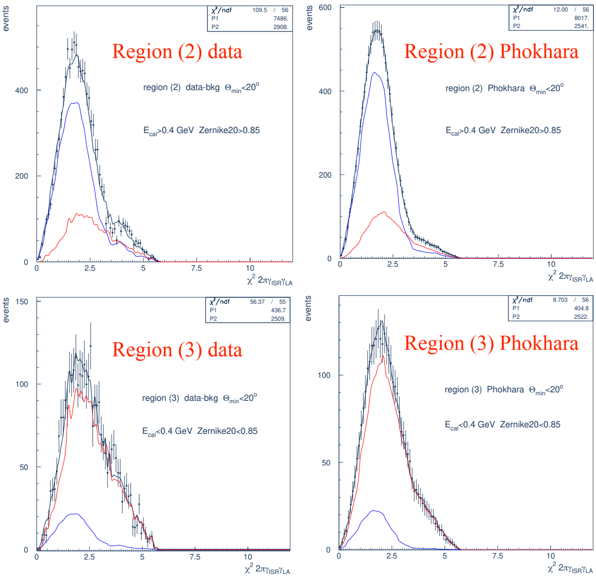
Fake photon study (2)

Template fits performed in χ^2 distributions in each of four regions using:

- FSR γ template from LA ISR γ $\theta_{min(trk,\gamma LA)} > 20^\circ$
- Fake γ template from region (4)

Two examples of the fits in regions (2) and (3) are shown comparing data (left) and Phokhara (right)

Fake fraction results:
Data: $0.458 \pm 0.004_{stat} \pm 0.010_{sys}$
Phok: $0.377 \pm 0.004_{stat} \pm 0.007_{sys}$



[from Zhiqing Zhang, Bern, Sep 2023]

"zero-constraint" (0c) analysis of SA and LA NLO events samples

measured quantities

- 4-momenta of the two tracks (pion mass assumed)
- direction of ISR photon
  - energy measurement discarded as it may be polluted with additional radiation

calculate using 4-momentum contraint to initial e^+e^- state

- energy of ISR photon (1 quantity)
- energy and direction of additional photon (3 quantities)

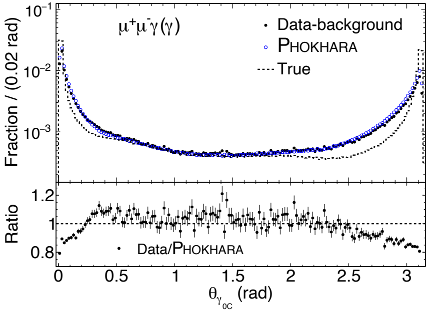
requirements

- additional photon calculated energy $E_\gamma > 200\,\text{MeV}$
- additional photon angle with ISR photon $> 0.5\,\text{rad}$

Angular and energy distribution of additional photon in 0c reconstrction

Calculated polar angle $\theta_{\gamma 0C}$ distribution

($E^*_{\gamma 0C} > 200$ MeV)

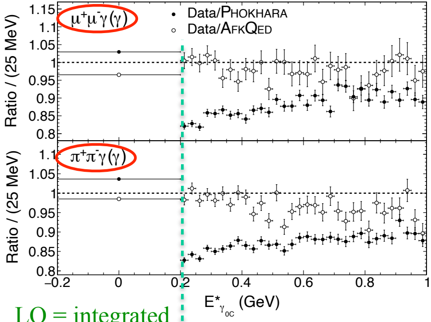


Phokhara NLO SA γ rate > data

Phokhara NLO LA γ rate ~ data

(the slight LA excess in data is due to larger resolution tails)

Calculated energy $E^*_{\gamma 0C}$ distribution



LO = integrated events < 0.2 GeV

NLO

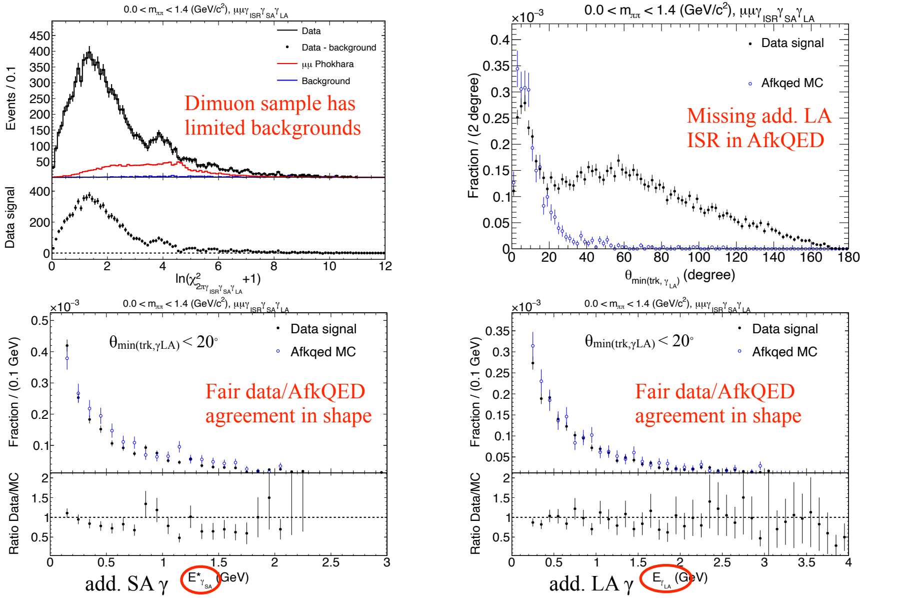
Data/Phokhara Mismatch LO-NLO,

NLO slope ~ NLO SA fit

Data/AfkQED similar rate LO & NLO

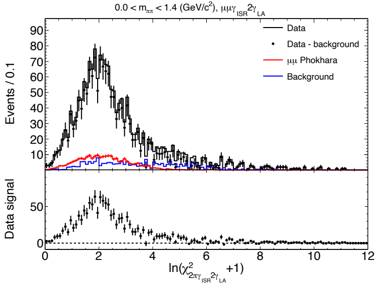
[from Zhiqing Zhang, Bern, Sep 2023]

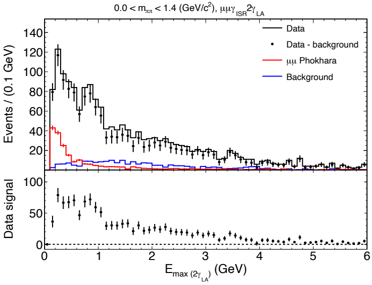
NNLO 1SA + 1LA events ($E_{LA} > 200$ MeV, $E^*_{SA} > 100$ MeV)



[from Zhiqing Zhang, Bern, Sep 2023]

NNLO 2LA events ($E_{\text{LA1, LA2}} > 100\,\text{MeV}$)

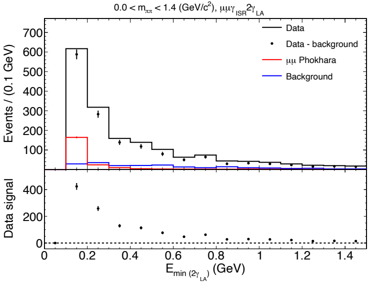




Background again small in dimuon process

Much larger background in dipion process suppressed by

- vetoing π^0 and η with 2 mass windows
- apply additional BDT selection



[from Zhiqing Zhang, Bern, Sep 2023]

Fit of FSR and LA-ISR contributions of NLO events with MC templates

- FSR template from AfkQED, ISR events with additional FSR with Photos and additional beam-collinear radiation but not LA radiation
- LA-ISR template from Phokhara, which simulates all cases at NLO, subtracted with the AfkQED-simulated FSR contribution normalized to data events with minimum-angle to track up to 10°