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Electromagnetic corrections in hadronic tau decays

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Introduction

- The tau lepton is the only known lepton that is heavy enough to decay into hadrons.
- Semileptonic tau decays are a valuable tool for studying QCD hadronization at low energies.

H^-	Precision $[\mathcal{B}_H]$ PDG 2022	Rad. Corr.	Application
π^-	0.5%	✓ ✦	LFU, NP
K^-	1.4%	✓ ✦	V_{us}, LFU, NP
$\pi^-\pi^0$	0.4%	✓ ✦	$\rho,\ \rho',\ \cdots,\ (g-2)_\mu$, NP
K^-K^0	2.3%	✘	$\rho',\ \cdots$, NP
$\bar{K}^0\pi^-$	1.7%	✓ ✦	K^*, V_{us}, $\not{CP}$, NP
$K^-\pi^0$	3.5%	✓ ✦	K^*, V_{us}, NP
$K^-\eta$	5.2%	✘	K^*, NP
$\pi^-\pi^+\pi^-$	0.5%	✘	a_1
$\pi^-2\pi^0$	1.1%	✘	a_1

✦ Decker and Fikemeier '95, Arroyo-Ureña et al '21
✦ Cirigliano et al '01, Flores-Tlalpa et al '06, Miranda and Roig '20
✦ Antonelli et al '13, Flores-Baéz and Morones-Ibarra '13

Short-Distance corrections: Sirlin '78; Marciano-Sirlin '93

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π^-	0.5%	✓		LFU, NP
K^-	1.4%	✓		V_{us}, LFU, NP
$\pi^-\pi^0$	0.4%	✓	✓	ρ, ρ', $\cdots$, $(g-2)_\mu$, NP
K^-K^0	2.3%	✗	✓	ρ', $\cdots$, NP
$\bar{K}^0\pi^-$	1.7%	✓	✓	K^*, V_{us}, $\not{C}\not{P}$, NP
$K^-\pi^0$	3.5%	✓	✓	K^*, V_{us}, NP
$K^-\eta$	5.2%	✗	✓	K^*, NP
$\pi^-\pi^+\pi^-$	0.5%	✗		a_1
$\pi^-2\pi^0$	1.1%	✗		a_1

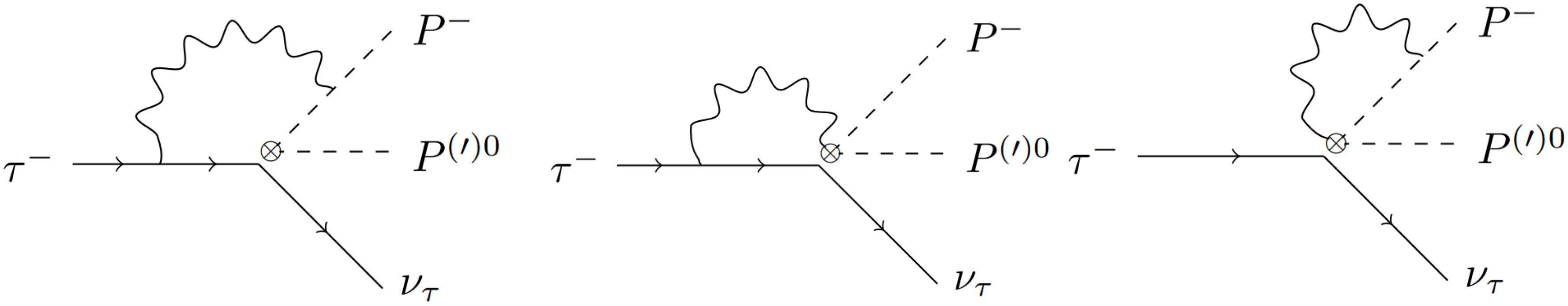
This work

Introduction

- The EM radiative corrections require the inclusion of virtual and real photons.
- The SI contributions for the Kπ channel were studied in Phys. Rev. D 88 (2013) 7, 073009 and JHEP 10 (2013) 070 .
- The virtual-photon corrections are IR divergent. The virtual loops, which treat the K and π as point-like, induce a shift to the form factors:

$$\bar{f}_{\pm,0}^{K\pi}(s) \to \bar{f}_{\pm,0}^{K\pi}(s) + \delta\bar{f}_{\pm,0}^{K\pi}(s,u)$$

- In ChPT, three diagrams contribute to this effect.



Phys. Lett. B 513 (2001) 361-370

Introduction

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- The **SI** contributions for the **Kπ** channel were studied in **Phys. Rev. D 88 (2013) 7, 073009** and **JHEP 10 (2013) 070** .
- The SI part of the radiative process is introduced by means of the Low theorem.

$$\mathcal{M}^{\gamma} = \boxed{\frac{\mathcal{M}}{k}} + \mathcal{M}_1 k^0 + k\mathcal{M}_2 + ...$$

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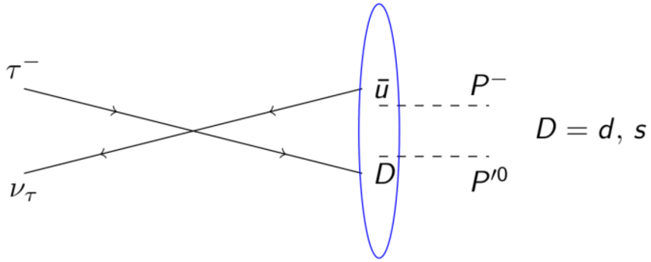
$$\mathcal{M}_\tau^\gamma = \mathcal{M}_\tau^{(0)\prime} e \left[\frac{p^+ \cdot \epsilon(k)}{p^+ \cdot k} - \frac{p_\tau \cdot \epsilon(k)}{p_\tau \cdot k} \right]$$

- The leading Low-term is fully determined by the amplitude of the non-radiative decay.



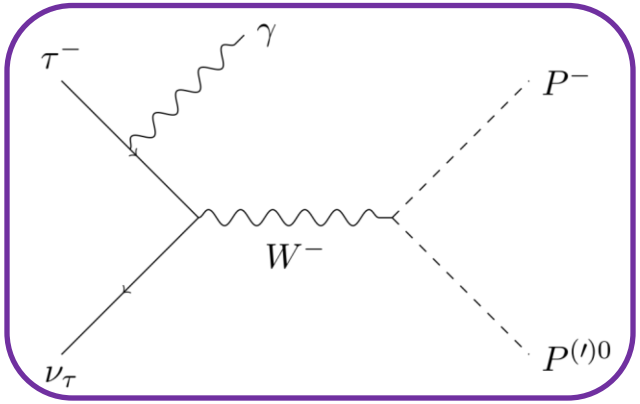
Amplitude

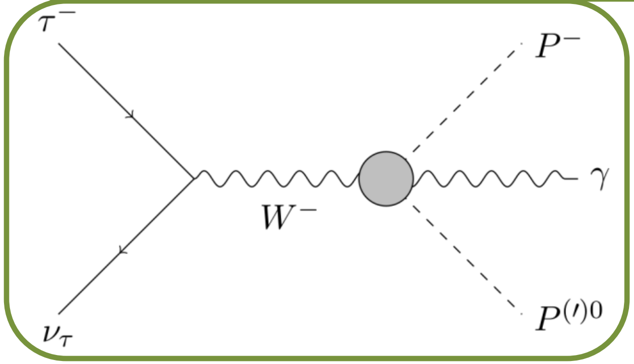
JHEP 08 (2002) 002
Phys. Rev. D 95 (2017) 5, 054015
Phys. Rev. D 102 (2020) 114017

- The most general structure is given by

$$\mathcal{M} = \frac{eG_F V_{ud}^*}{\sqrt{2}} \epsilon_\mu^* \left\{ \boxed{\frac{H_\nu(p_-, p_0)}{k^2 - 2k\cdot P} \bar{u}(q)\gamma^\nu(1-\gamma^5)(m_\tau + \not{P} - \not{k})\gamma^\mu u(P)} \right.$$

$$\left. + \boxed{(V^{\mu\nu} - A^{\mu\nu})\bar{u}(q)\gamma_\nu(1-\gamma^5)u(P)} \right\}$$





- The hadronic matrix elements is

$$H^\nu(p_-, p_0) = C_V F_+(t) Q^\nu + C_S \frac{\Delta_{-0}}{t} q^\nu F_0(t), \qquad t = q^2$$

where

$$q^\nu = (p_- + p_0)^\nu,\ Q^\nu = (p_- - p_0)^\nu - \frac{\Delta_{-0}}{t} q^\nu \text{ and } \Delta_{ij} = m_i^2 - m_j^2$$

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$$\left. + (V^{\mu\nu} - A^{\mu\nu})\bar{u}(q)\gamma_\nu(1-\gamma^5)u(P) \right\}$$

- The vector and axial-vector terms can be split into two parts, structure-independent (**SI**) and structure-dependent (**SD**), according to the Low and Burnett-Kroll theorems.

JHEP 08 (2002) 002

$$V^{\mu\nu} = V^{\mu\nu}_{SI} + V^{\mu\nu}_{SD}$$
$$A^{\mu\nu} = A^{\mu\nu}_{SD}$$

- At low-energies, the SM of EW and strong interactions is described by an EFT known as Chiral Perturbation Theory (**ChPT**).

SI contributions

JHEP 08 (2002) 002
Phys. Rev. D 95 (2017) 5, 054015
Phys. Rev. D 102 (2020) 114017

- The **SI** contribution reads

$$V_{\mathrm{SI}}^{\mu\nu} = \frac{H^{\nu}(p_- + k, p_0)(2p_- + k)^{\mu}}{2k \cdot p_- + k^2} + \left\{ -C_V F_+(t') - \frac{\Delta_{-0}}{t'} \left[C_S F_0(t') - C_V F_+(t') \right] \right\} g^{\mu\nu}$$

$$- C_V \frac{F_+(t') - F_+(t)}{k \cdot (p_- + p_0)} Q^{\nu} q^{\mu} + \frac{\Delta_{-0}}{tt'} \left\{ 2 \left[C_S F_0(t') - C_V F_+(t') \right] - \frac{C_S t'}{k \cdot (p_- + p_0)} \left[F_0(t') - F_0(t) \right] \right\} q^{\mu} q^{\nu}$$

$$\boxed{\tau^- \to K^- \pi^0 \gamma \nu_\tau}$$

$$p_- \to p_K$$
$$p_0 \to p_\pi$$
$$\Delta_{-0} \to \Delta_{K\pi}$$

$$C_V = \frac{1}{\sqrt{2}}$$
$$C_S = \frac{1}{\sqrt{2}}$$

$$\boxed{\tau^- \to \bar{K}^0 \pi^- \gamma \nu_\tau}$$

$$p_- \to p_\pi$$
$$p_0 \to p_K$$
$$\Delta_{-0} \to -\Delta_{K\pi}$$
$$C_{V,S} \to -C_{V,S}$$

$$C_V = 1$$
$$C_S = 1$$

$$\boxed{\tau^- \to K^- K^0 \gamma \nu_\tau}$$

$$C_V = -1$$
$$C_S = -1$$

We recover the usual definition of $H^{\nu}_{K\pi}$

Phys. Rev. D 99 (2019) 093005

SI contributions

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Phys. Rev. D 95 (2017) 5, 054015
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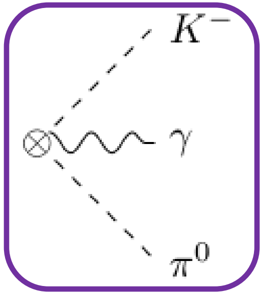
$$V_{\rm SI}^{\mu\nu} = \frac{H^\nu(p_- + k, p_0)(2p_- + k)^\mu}{2k\cdot p_- + k^2} + \left\{ \boxed{-C_V F_+(t')} - \frac{\Delta_{-0}}{t'}\left[C_S F_0(t') - C_V F_+(t')\right] \right\} g^{\mu\nu}$$

$$- C_V \frac{F_+(t') - F_+(t)}{k\cdot(p_- + p_0)} Q^\nu q^\mu + \frac{\Delta_{-0}}{tt'}\left\{ 2\left[C_S F_0(t') - C_V F_+(t')\right] - \frac{C_S t'}{k\cdot(p_- + p_0)}\left[F_0(t') - F_0(t)\right] \right\} q^\mu q^\nu$$

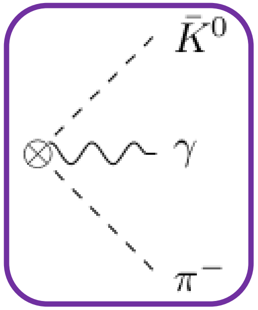
$$\boxed{\tau^- \to K^- \pi^0 \gamma \nu_\tau}$$

$p_- \to p_K$
$p_0 \to p_\pi$
$\Delta_{-0} \to \Delta_{K\pi}$



$$\boxed{\tau^- \to \bar{K}^0 \pi^- \gamma \nu_\tau}$$

$p_- \to p_\pi$
$p_0 \to p_K$
$\Delta_{-0} \to -\Delta_{K\pi}$
$\boxed{C_{V,S} \to -C_{V,S}}$



$$\boxed{\tau^- \to K^- K^0 \gamma \nu_\tau}$$

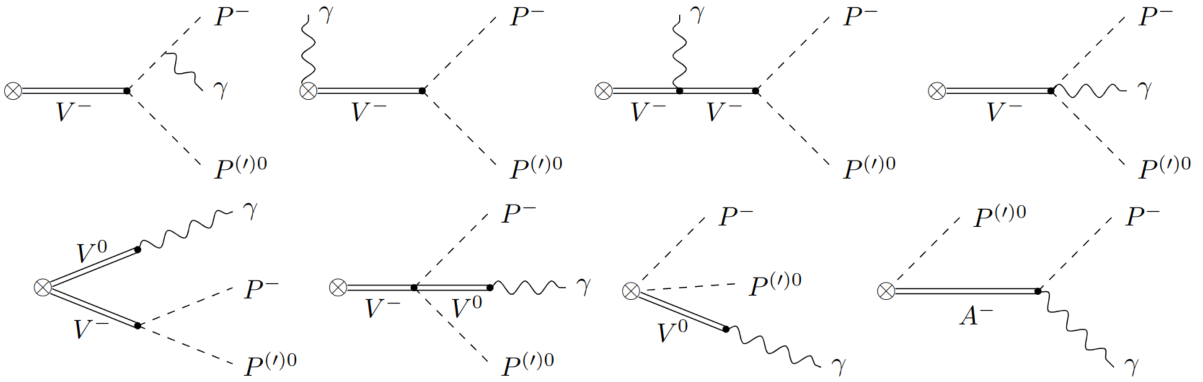
$C_V = -1$
$C_S = -1$

We recover the usual definition of $H_{K\pi}^\nu$

Phys. Rev. D 99 (2019) 093005

SD contributions

- At **O(p^4)** in ChPT with resonances (RChT), the **vector form factors** $V^{\mu\nu}$ are saturated by the exchange of vector and axial-vector resonances:



V^0 stands for
$\rho^0 \quad \omega \quad \phi$

$$\boxed{\tau^- \to K^-\pi^0\gamma\nu_\tau}$$

$$V^- = K^{*-}$$
$$A^- = K_1^-$$

$$\boxed{\tau^- \to \bar{K}^0\pi^-\gamma\nu_\tau}$$

$$V^- = K^{*-}$$
$$A^- = a_1^-$$

$$\boxed{\tau^- \to K^-K^0\gamma\nu_\tau}$$

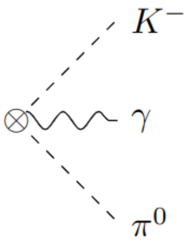
$$V^- = \rho^-$$
$$A^- = K_1^-$$

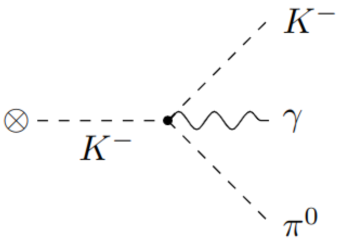
$$F_V = \sqrt{2}F, G_V = \frac{F}{\sqrt{2}}, F_A = F,$$
$$F_V = \sqrt{3}F, G_V = \frac{F}{\sqrt{3}}, F_A = \sqrt{2}F,$$

SD contributions

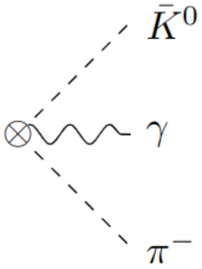
- At **O(p^4)** in ChPT, the **axial form factors,** $A^{\mu\nu}$, get contributions from the Wess-Zumino-Witten functional:

$$\boxed{\tau^- \to K^-\pi^0\gamma\nu_\tau}$$

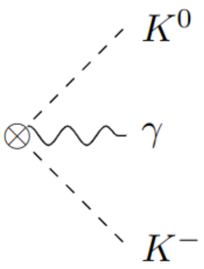




$$\boxed{\tau^- \to \bar{K}^0\pi^-\gamma\nu_\tau}$$



$$\boxed{\tau^- \to K^-K^0\gamma\nu_\tau}$$



Decay spectrum

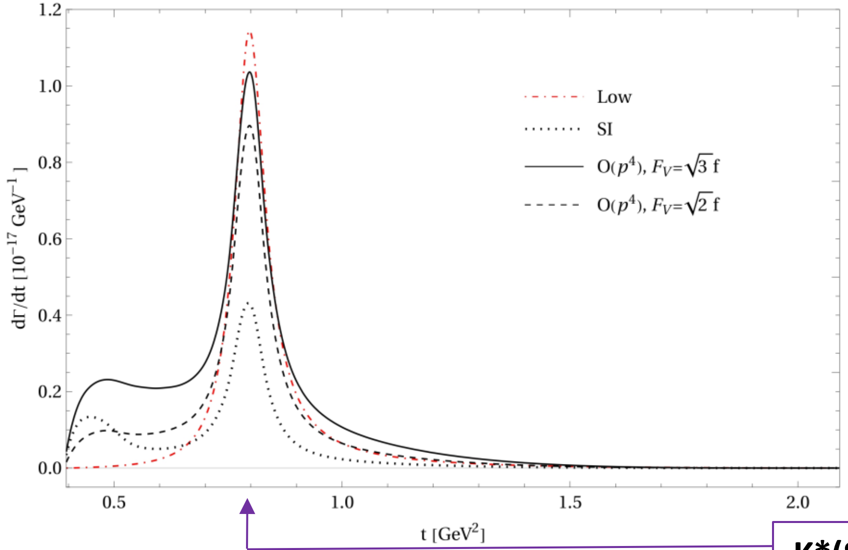
$\boxed{E_{\gamma}^{\text{cut}} = 300\,\text{MeV}}$

- The differential rate for the radiative decays in the tau rest frame is given by

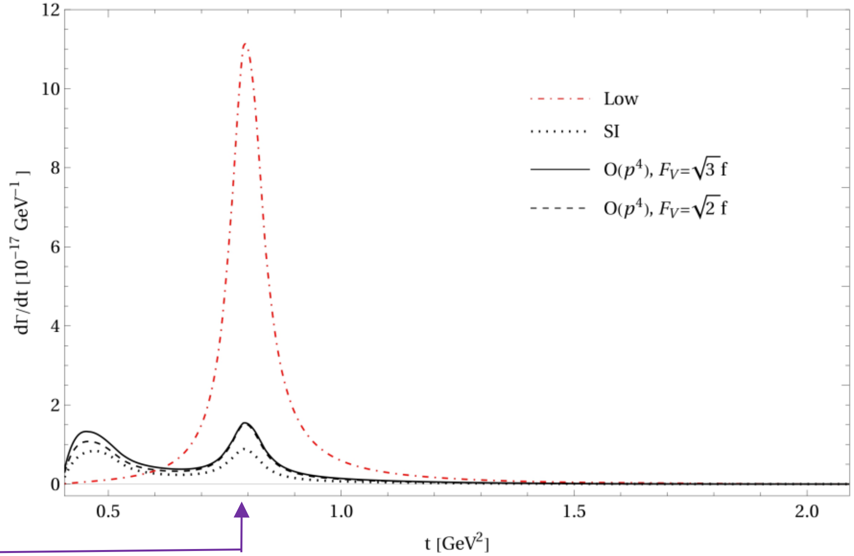
$$d\Gamma = \frac{(2\pi)^4}{4m_\tau}\sum_{\text{spin}}\overline{|\mathcal{M}|^2}d\Phi_4$$

- To be sensitive to the dynamics of the radiative transition, a cut on photon energies is necessary.

$\boxed{\tau^- \to K^-\pi^0\gamma\nu_\tau}$



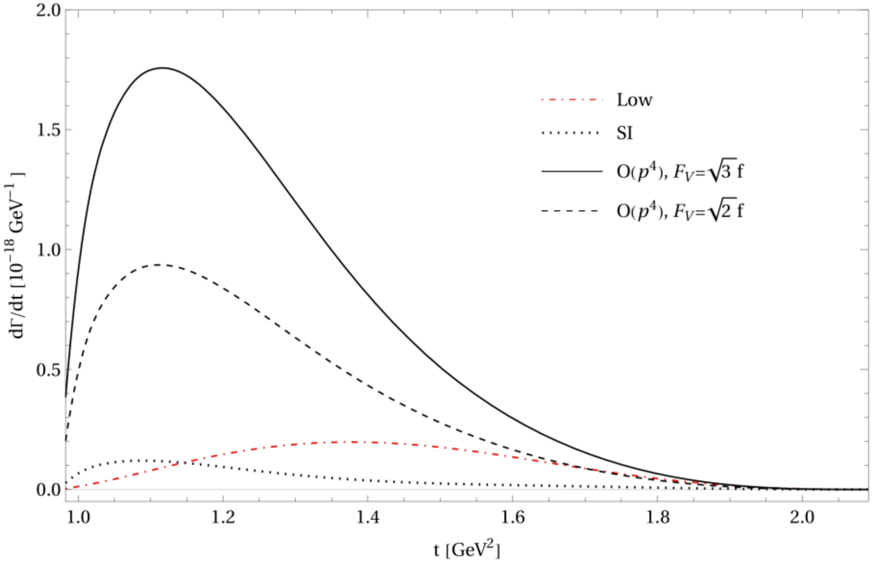
$\boxed{\tau^- \to \bar{K}^0\pi^-\gamma\nu_\tau}$



K*(892)

$$\boxed{E_\gamma^{\text{cut}} = 300\,\text{MeV}}$$

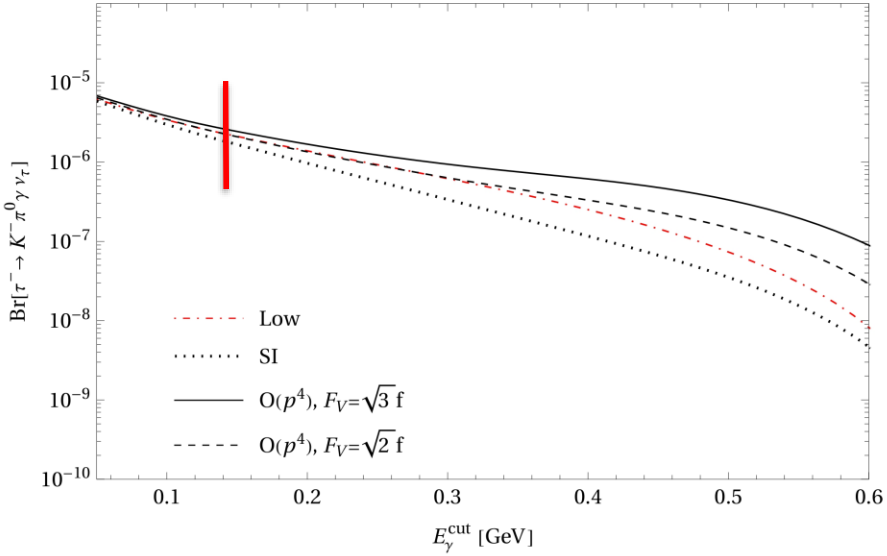
$$\boxed{\tau^- \to K^- K^0 \gamma \nu_\tau}$$



No **ρ(1450)**

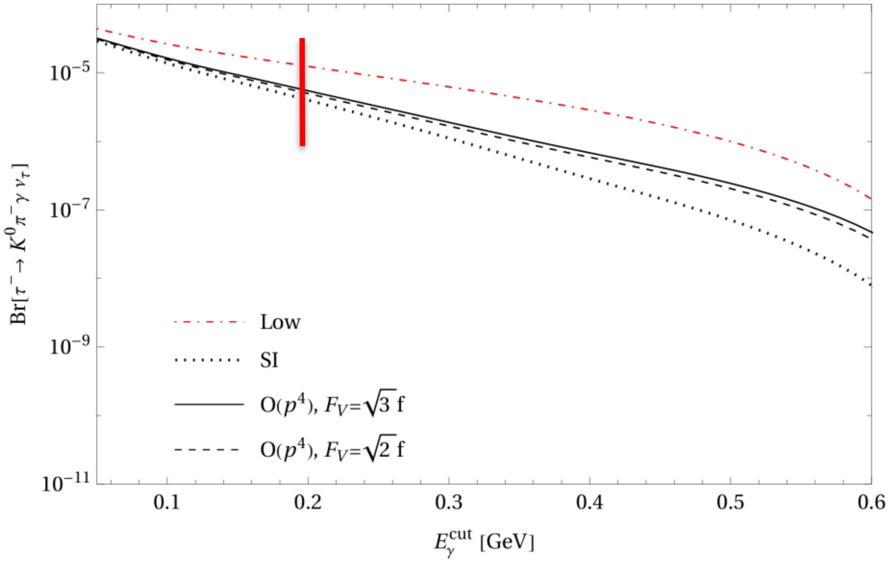
Branching ratio

$\tau^- \to K^- \pi^0 \gamma \nu_\tau$

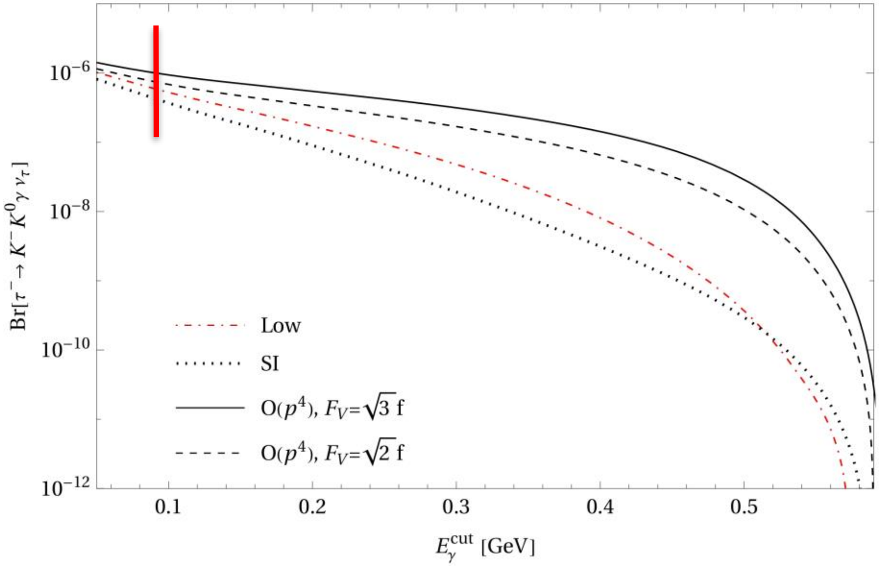


- The **SD contributions** become relevant above 300 MeV.

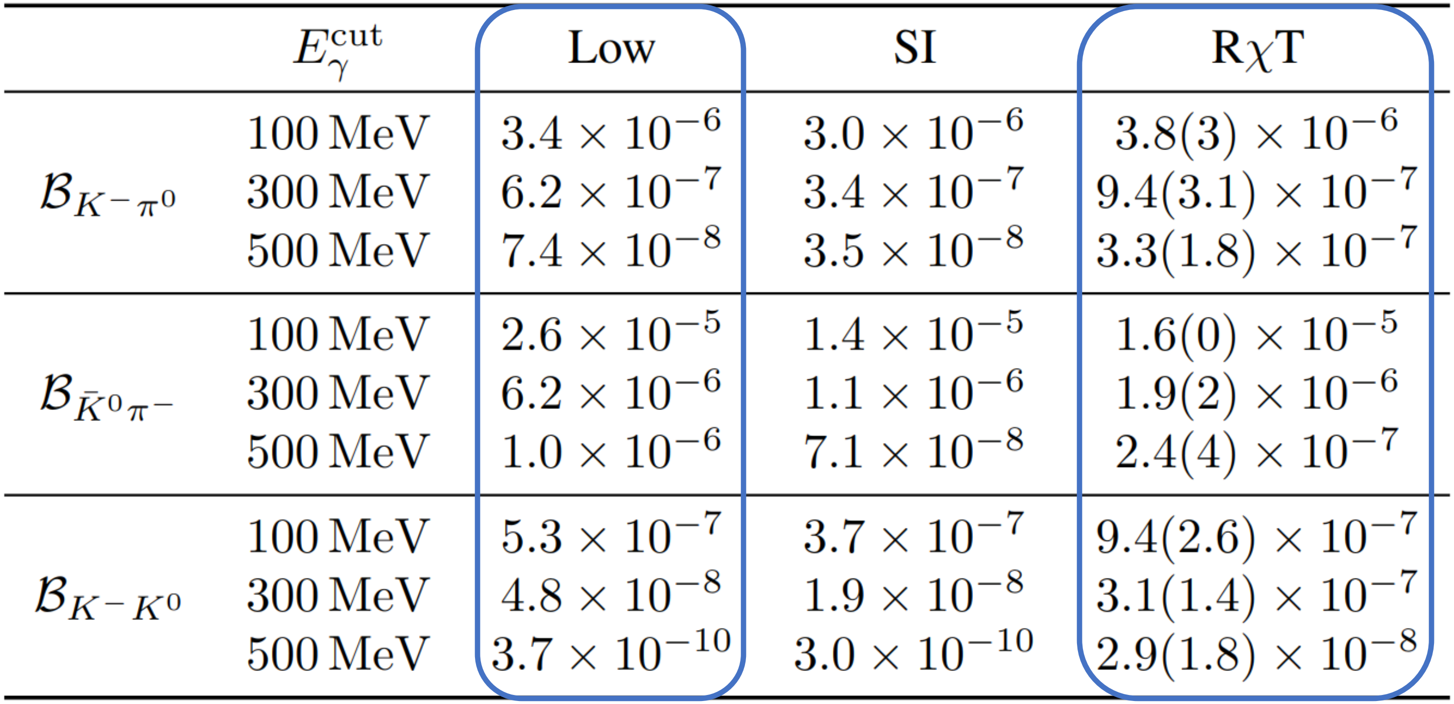
$\tau^- \to \bar{K}^0 \pi^- \gamma \nu_\tau$



$\tau^- \rightarrow K^- K^0 \gamma \nu_\tau$

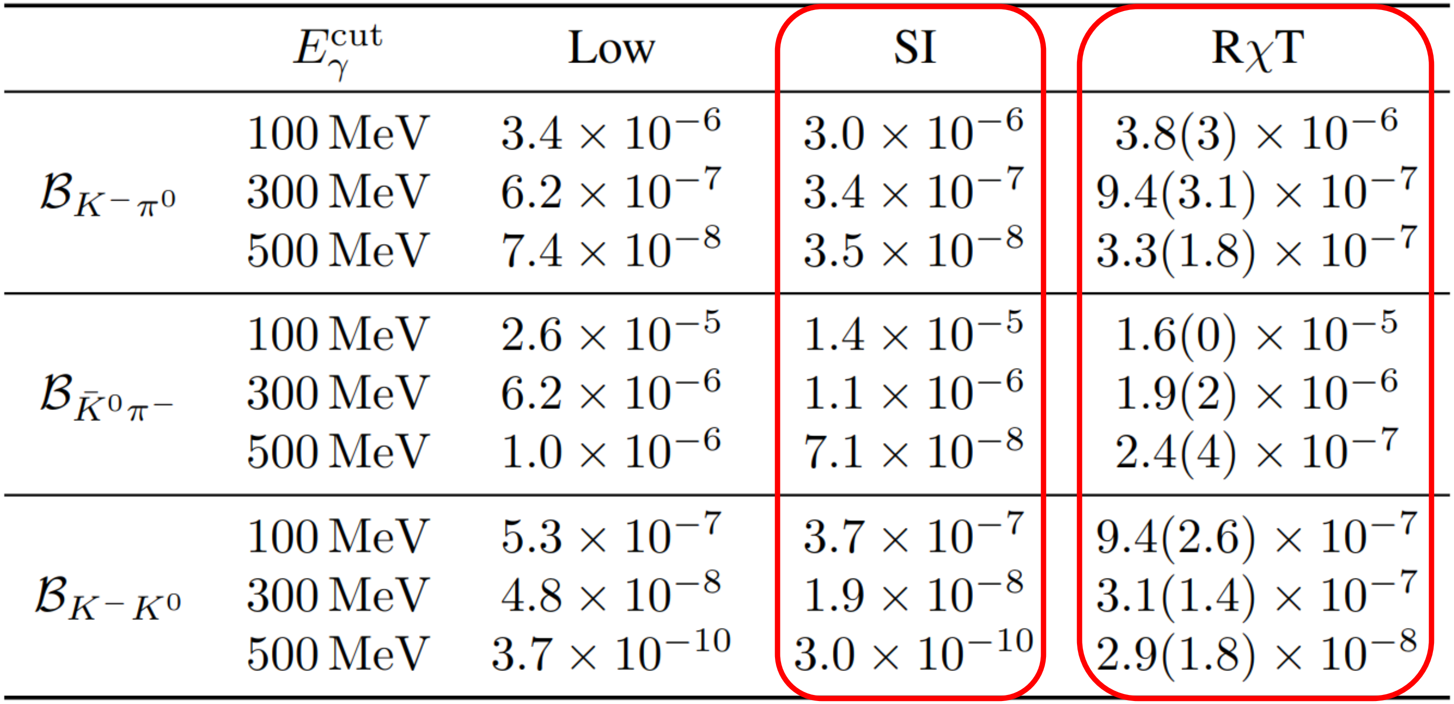


- The **SD contributions** are important even below 100 MeV.



	E_γ^{cut}	Low	SI	RχT	
$\mathcal{B}_{K^-\pi^0}$	100 MeV	3.4×10^{-6}	3.0×10^{-6}	$3.8(3) \times 10^{-6}$	~ 10 %
	300 MeV	6.2×10^{-7}	3.4×10^{-7}	$9.4(3.1) \times 10^{-7}$	~ 34 %
	500 MeV	7.4×10^{-8}	3.5×10^{-8}	$3.3(1.8) \times 10^{-7}$	~ 78 %
$\mathcal{B}_{\bar{K}^0\pi^-}$	100 MeV	2.6×10^{-5}	1.4×10^{-5}	$1.6(0) \times 10^{-5}$	~ 63 %
	300 MeV	6.2×10^{-6}	1.1×10^{-6}	$1.9(2) \times 10^{-6}$	~ 226 %
	500 MeV	1.0×10^{-6}	7.1×10^{-8}	$2.4(4) \times 10^{-7}$	~ 317 %
$\mathcal{B}_{K^-K^0}$	100 MeV	5.3×10^{-7}	3.7×10^{-7}	$9.4(2.6) \times 10^{-7}$	~ 44 %
	300 MeV	4.8×10^{-8}	1.9×10^{-8}	$3.1(1.4) \times 10^{-7}$	~ 85 %
	500 MeV	3.7×10^{-10}	3.0×10^{-10}	$2.9(1.8) \times 10^{-8}$	~ 99 %

- The Low's approximation is not sufficient to describe these decays for photon energies above ~100 MeV.



	E_γ^{cut}	Low	SI	RχT	
$\mathcal{B}_{K^-\pi^0}$	100 MeV	3.4×10^{-6}	3.0×10^{-6}	$3.8(3) \times 10^{-6}$	~ 21 %
	300 MeV	6.2×10^{-7}	3.4×10^{-7}	$9.4(3.1) \times 10^{-7}$	~ 64 %
	500 MeV	7.4×10^{-8}	3.5×10^{-8}	$3.3(1.8) \times 10^{-7}$	~ 89 %
$\mathcal{B}_{\bar{K}^0\pi^-}$	100 MeV	2.6×10^{-5}	1.4×10^{-5}	$1.6(0) \times 10^{-5}$	~ 12 %
	300 MeV	6.2×10^{-6}	1.1×10^{-6}	$1.9(2) \times 10^{-6}$	~ 42 %
	500 MeV	1.0×10^{-6}	7.1×10^{-8}	$2.4(4) \times 10^{-7}$	~ 70 %
$\mathcal{B}_{K^-K^0}$	100 MeV	5.3×10^{-7}	3.7×10^{-7}	$9.4(2.6) \times 10^{-7}$	~ 61 %
	300 MeV	4.8×10^{-8}	1.9×10^{-8}	$3.1(1.4) \times 10^{-7}$	~ 94 %
	500 MeV	3.7×10^{-10}	3.0×10^{-10}	$2.9(1.8) \times 10^{-8}$	~ 99 %

- The Low's approximation is not sufficient to describe these decays for photon energies above ~100 MeV.
- These decays are an excellent probe for testing SD effects.

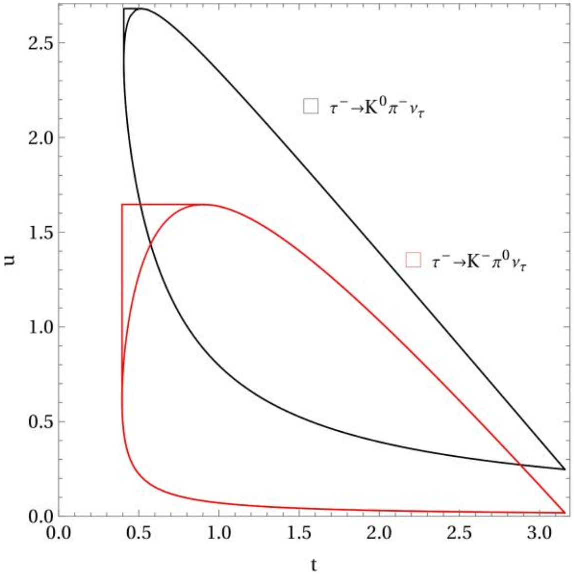
Radiative corrections

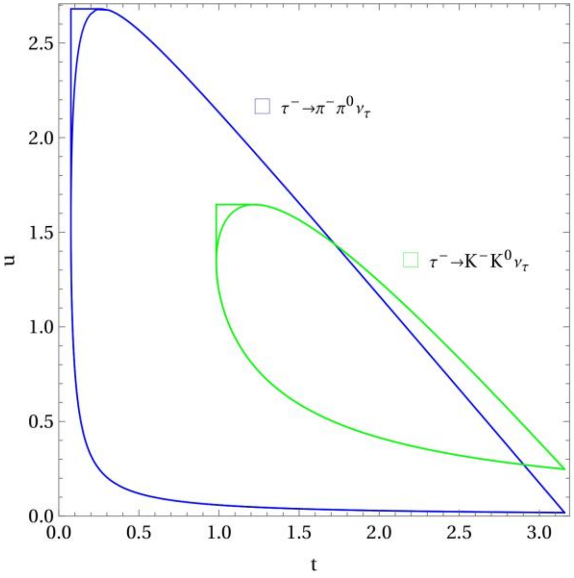
- The photon-inclusive double differential rate can be written as

$$\left.\frac{d\Gamma}{dt}\right|_{PP(\gamma)} = \frac{G_F^2\,|V_{uD}\,F_+(0)|^2\,S_{EW}\,m_\tau^3}{768\pi^3 t^3}\left(1-\frac{t}{m_\tau^2}\right)^2 \lambda^{1/2}(t,m_-^2,m_0^2)\left[C_V^2|\tilde{F}_+(t)|^2\left(1+\frac{2t}{m_\tau^2}\right)\lambda(t,m_-^2,m_0^2) + 3C_S^2\Delta_{-0}^2|\tilde{F}_0(t)|^2\right]\boxed{G_{EM}(t)}$$

- For simplicity, we split the contributions to the decay width as

$$\left.\frac{d\Gamma}{dt}\right|_{PP(\gamma)} = \underbrace{\left.\frac{d\Gamma}{dt}\right|_{PP} + \left.\frac{d\Gamma}{dt}\right|_{III}}_{G^{(0)}_{\rm EM}(t)} + \underbrace{\left.\frac{d\Gamma}{dt}\right|_{IV/III}}_{\rm Negligible} + \underbrace{\left.\frac{d\Gamma}{dt}\right|_{rest}}_{\rm SI+SD}, \qquad G_{EM}(t) = G^{(0)}_{EM}(t) + \delta G_{EM}(t)$$





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$$F_{+/0}(t,u) = F_{+/0}(t) + \delta F_{+/0}(t,u),$$

Model 1

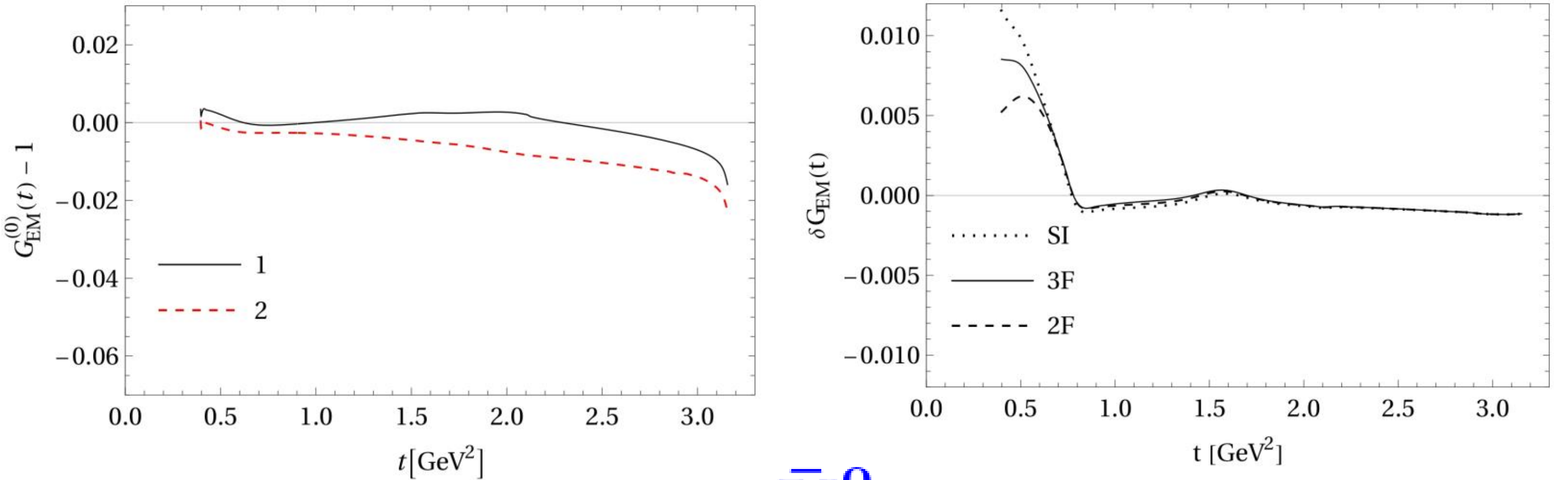
JHEP 08 (2002) 002

Model 2

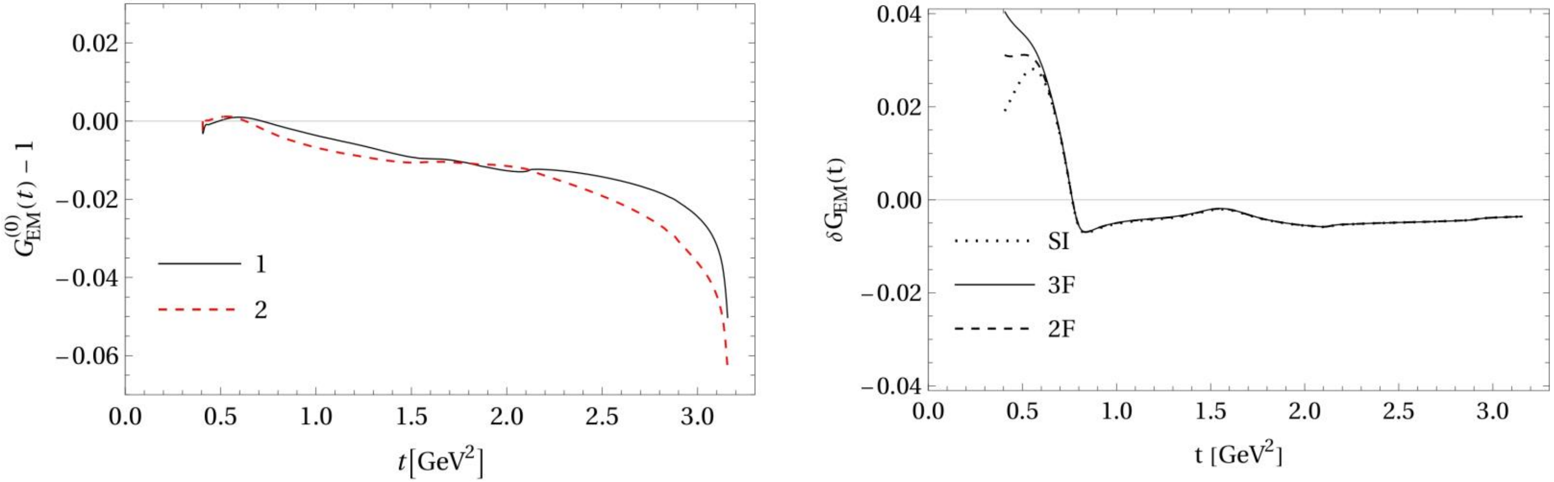
JHEP 10 (2013) 070

Radiative corrections

$\tau^- \to K^- \pi^0 \nu_\tau$

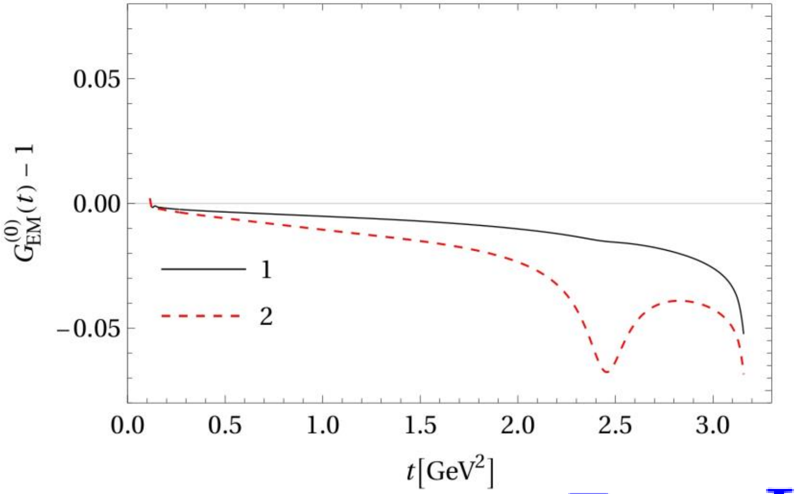


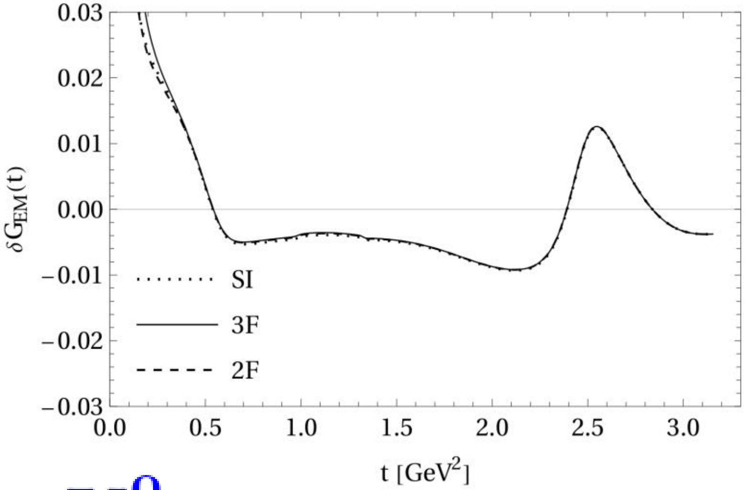
$\tau^- \to \bar{K}^0 \pi^- \nu_\tau$



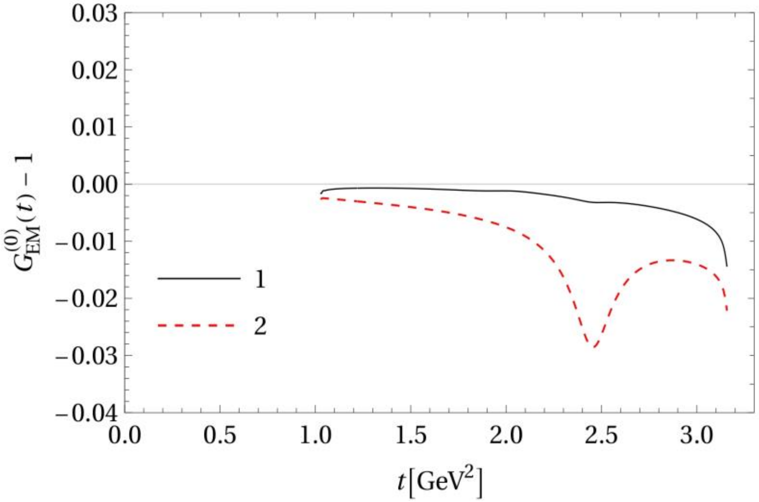
Radiative corrections

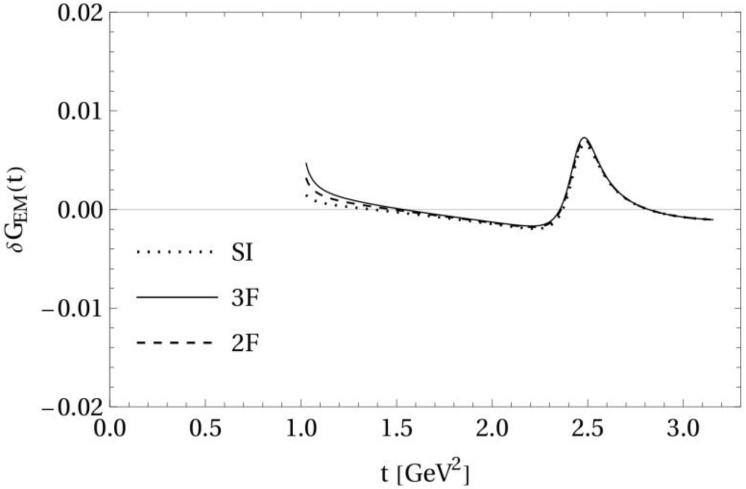
$\tau^- \to \pi^- \pi^0 \nu_\tau$





$\tau^- \to K^- K^0 \nu_\tau$





Radiative corrections

- Integrating upon t, we get

$$\Gamma_{PP(\gamma)} = \frac{G_F^2 S_{EW} m_\tau^5}{96\pi^3} \left|V_{uD}\, F_+(0)\right|^2 I_h^\tau \left(1+\delta_{EM}^{hh}\right)^2$$

where

$$I_h^\tau = \frac{1}{8m_\tau^2}\int_{t_{thr}}^{m_\tau^2}\frac{dt}{t^3}\left(1-\frac{t}{m_\tau^2}\right)^2 \lambda^{1/2}(t,m_-^2,m_0^2)\left[C_V^2|\tilde{F}_+(t)|^2\left(1+\frac{2t}{m_\tau^2}\right)\lambda(t,m_-^2,m_0^2)+3C_S^2\Delta_{-0}^2|\tilde{F}_0(t)|^2\right].$$

Electromagnetic corrections to hadronic tau decays in %

$\delta_{\rm EM}$	Ref. [33]	$G_{EM}^{(0)}(t)$		$\delta G_{EM}(t)$		
		Model 1	Model 2	SI	SI + 2F	SI + 3F
$K^-\pi^0$	$-0.20(20)$	-0.019	-0.137	-0.001	$+0.006$	$+0.010$
$\bar{K}^0\pi^-$	$-0.15(20)$	-0.086	-0.208	-0.098	-0.085	-0.080
K^-K^0	–	-0.046	-0.223	-0.012	$+0.003$	$+0.016$
$\pi^-\pi^0$	–	-0.196	-0.363	-0.010	-0.002	$+0.010$

$$\delta_{\rm EM}^{K^-\pi^0} = -\left(0.009^{+0.010}_{-0.118}\right)\%$$

$$\delta_{\rm EM}^{\bar{K}^0\pi^-} = -\left(0.166^{+0.100}_{-0.157}\right)\%$$

$$\delta_{\rm EM}^{K^-K^0} = -\left(0.030^{+0.032}_{-0.180}\right)\%$$

$$\delta_{\rm EM}^{\pi^-\pi^0} = -\left(0.186^{+0.114}_{-0.203}\right)\%$$

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$$\delta_{\rm EM}^{K^-\eta} = -\left(0.026^{+0.029}_{-0.163}\right)\% \quad \delta_{\rm EM}^{K^-\eta'} = -\left(0.304^{+0.422}_{-0.185}\right)\%$$

Impact of radiative corrections on NP

- The low-energy Lagrangian that describes the semileptonic strangeness-conserving **(ΔS=0)** or -changing **(ΔS=1)** charge current transitions reads,

$$(D = d, s)$$

$$\mathcal{L}_{eff} = -\frac{G_F V_{uD}}{\sqrt{2}}\Big[(1+\epsilon_L^\tau)\bar{\tau}\gamma_\mu(1-\gamma_5)\nu_\tau \cdot \bar{u}\gamma^\mu(1-\gamma_5)D + \epsilon_R^\tau\bar{\tau}\gamma_\mu(1-\gamma_5)\nu_\tau \cdot \bar{u}\gamma^\mu(1+\gamma_5)D$$

$$+\bar{\tau}(1-\gamma_5)\nu_\tau \cdot \bar{u}(\epsilon_S^\tau - \epsilon_P^\tau\gamma_5)D + \epsilon_T^\tau\bar{\tau}\sigma_{\mu\nu}(1-\gamma_5)\nu_\tau \cdot \bar{u}\sigma^{\mu\nu}(1-\gamma_5)D\Big] + \text{h.c.}$$

The one meson decay width is given by

$$G_F \tilde{V}_{uD}^e = G_F\left(1+\epsilon_L^e+\epsilon_R^e\right)V_{uD}$$

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$$\Gamma(\tau^- \to P^-\nu_\tau) = \frac{G_F^2|\tilde{V}_{uD}^e|^2 f_\pi^2 m_\tau^3}{16\pi}\left(1-\frac{m_\pi^2}{m_\tau^2}\right)^2 (1+\delta_{\text{em}}^{\tau P}+2\Delta^{\tau P}),$$

$$\delta_{\tau\pi} = -0.24(56)\%$$

$$\delta_{\tau K} = -0.15(57)\%$$

$$\Delta^{\tau P} = \epsilon_L^\tau - \epsilon_L^e - \epsilon_R^\tau - \epsilon_R^e - \frac{m_P^2}{m_\tau(m_u+m_D)\epsilon_P^\tau} \Longrightarrow \boxed{\begin{aligned}\Delta^{\tau\pi} &= -0.14(72)\cdot 10^{-2},\\ \Delta^{\tau K} &= -1.02(86)\cdot 10^{-2},\end{aligned}}$$

- The partial decay width for two-meson decays is

$$\left.\frac{d\Gamma}{dt}\right|_{PP(\gamma)} = \frac{G_F^2|\tilde{V}_{uD}^e|^2 m_\tau^3 S_{EW}}{384\pi^3 t}\left(1-\frac{t}{m_\tau^2}\right)^2 \lambda^{1/2}(t, m_P^2, m_{P'}^2)$$

$$\times\Big[\left(G_{\text{EM}}(t)+2(\epsilon_L^\tau-\epsilon_L^e+\epsilon_R^\tau-\epsilon_R^e)\right)X_{VA} + \epsilon_S^\tau X_S + \epsilon_T^\tau X_T + (\epsilon_S^\tau)^2 X_{S^2} + (\epsilon_T^\tau)^2 X_{T^2}\Big]$$

Impact of radiative corrections on NP

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$$+\bar{\tau}(1-\gamma_5)\nu_\tau\cdot\bar{u}(\epsilon_S^\tau-\epsilon_P^\tau\gamma_5)D + \epsilon_T^\tau\bar{\tau}\sigma_{\mu\nu}(1-\gamma_5)\nu_\tau\cdot\bar{u}\sigma^{\mu\nu}(1-\gamma_5)D\Big] + \text{h.c.}$$

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$$\times\Big[(G_{\text{EM}}(t)+2(\epsilon_L^\tau-\epsilon_L^e+\epsilon_R^\tau-\epsilon_R^e))\,X_{VA}+\epsilon_S^\tau X_S+\epsilon_T^\tau X_T+(\epsilon_S^\tau)^2X_{S^2}+(\epsilon_T^\tau)^2X_{T^2}\Big]$$

We recover the **SM** case when $\epsilon_{L,R,S,P,T} = 0$

Global fit (ΔS=0)

- The chi-squared function to be minimized in our fits is

$$\chi^2 = \sum_k \left(\frac{\bar{N}_k^{\mathrm{th}} - \bar{N}_k^{\mathrm{exp}}}{\sigma_{\bar{N}_k^{\mathrm{exp}}}} \right)^2 + \left(\frac{BR_{\pi\pi}^{\mathrm{th}} - BR_{\pi\pi}^{\mathrm{exp}}}{\sigma_{BR_{\pi\pi}^{\mathrm{exp}}}} \right)^2 + \left(\frac{BR_{KK}^{\mathrm{th}} - BR_{KK}^{\mathrm{exp}}}{\sigma_{BR_{KK}^{\mathrm{exp}}}} \right)^2 + \left(\frac{BR_{\tau\pi}^{\mathrm{th}} - BR_{\tau\pi}^{\mathrm{exp}}}{\sigma_{BR_{\tau\pi}^{\mathrm{exp}}}} \right)^2 ,$$

The constraints for the non-standard interactions

$$\begin{pmatrix} \epsilon_L^\tau - \epsilon_L^e + \epsilon_R^\tau - \epsilon_R^e \\ \epsilon_R^\tau + \frac{m_\pi^2}{2m_\tau(m_u+m_d)}\epsilon_P^\tau \\ \epsilon_S^\tau \\ \epsilon_T^\tau \end{pmatrix} = \left(\boxed{\begin{array}{r} 0.0 \\ 0.1 \\ 10.3 \\ 0.4 \end{array}} \begin{array}{l} \pm 0.6 \ {}^{+\,6.8}_{-\,6.4} \pm 0.1 \pm 1.7 \ {}^{+\,0.0}_{-\,0.2} \\ \pm 0.5 \ {}^{+\,3.4}_{-\,3.3} \ {}^{+\,0.0}_{-\,0.1} \pm 0.9 \ \pm 0.1 \\ \pm 0.5 \ {}^{+\,1.2}_{-\,25.0} \ {}^{+\,0.0}_{-\,0.1} \pm 0.9 \ {}^{+\,6.2}_{-\,22.4} \\ \pm 0.2 \ {}^{+\,4.1}_{-\,4.4} \ {}^{+\,0.0}_{-\,0.1} \pm 1.1 \ {}^{+\,0.3}_{-\,0.2} \end{array} \right) \times 10^{-2} , \quad \rho_{ij} = \begin{pmatrix} 1 & 0.662 & -0.487 & -0.544 \\ & 1 & -0.323 & -0.360 \\ & & 1 & 0.452 \\ & & & 1 \end{pmatrix} .$$

at $\mu = 2$ GeV in the $\overline{\mathrm{MS}}$ scheme

$$\chi^2/d.o.f \sim 0.8$$

Stat. fit, VFF, m_q, TFF, Rad. Cors

$$\begin{pmatrix} \epsilon_L^\tau - \epsilon_L^e + \epsilon_R^\tau - \epsilon_R^e \\ \epsilon_R^\tau + \frac{m_\pi^2}{2m_\tau(m_u+m_d)}\epsilon_P^\tau \\ \epsilon_S^\tau \\ \epsilon_T^\tau \end{pmatrix} = \left(\boxed{\begin{array}{r} 0.5 \\ 0.3 \\ 9.7 \\ -0.1 \end{array}} \begin{array}{l} \pm 0.6^{+2.3\ +0.2}_{-1.8\ -0.1} \pm 0.4 \\ \pm 0.5^{+1.1\ +0.1}_{-0.9\ -0.0} \pm 0.2 \\ {}^{+0.5}_{-0.6} \pm 21.5 {}^{+0.0}_{-0.1} \pm 0.2 \\ \pm 0.2^{+1.1\ +0.0}_{-1.4\ -0.1} \pm 0.2 \end{array} \right) \times 10^{-2}$$

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Global fit (ΔS=1)

- The chi-squared function to be minimized in our fits is

$$\chi^2 = \sum_k \left(\frac{\bar{N}_k^{\rm th} - \bar{N}_k^{\rm exp}}{\sigma_{\bar{N}_k^{\rm exp}}} \right)^2 + \left(\frac{BR_{K\pi}^{\rm th} - BR_{K\pi}^{\rm exp}}{\sigma_{BR_{K\pi}^{\rm exp}}} \right)^2 + \left(\frac{BR_{K\eta}^{\rm th} - BR_{K\eta}^{\rm exp}}{\sigma_{BR_{K\eta}^{\rm exp}}} \right)^2 + \left(\frac{BR_{\tau K}^{\rm th} - BR_{\tau K}^{\rm exp}}{\sigma_{BR_{\tau K}^{\rm exp}}} \right)^2 ,$$

The constraints for the non-standard interactions

$$\begin{pmatrix} \epsilon_L^\tau - \epsilon_L^e + \epsilon_R^\tau - \epsilon_R^e \\ \epsilon_R^\tau + \frac{m_K^2}{2m_\tau(m_u+m_s)}\epsilon_P^\tau \\ \epsilon_S^\tau \\ \epsilon_T^\tau \end{pmatrix} = \begin{pmatrix} 0.4 \pm 1.5 \pm 0.4 \, {}^{+\,0.1}_{-\,0.0} \\ 0.7 \pm 0.9 \pm 0.2 \, {}^{+\,0.1}_{-\,0.0} \\ 0.8 \pm 0.9 \pm 0.2 \, {}^{+\,0.0}_{-\,0.1} \\ 0.5 \pm 0.7 \pm 0.4 \pm 0.0 \end{pmatrix} \times 10^{-2}, \qquad \rho_{ij} = \begin{pmatrix} 1 & 0.874 & -0.149 & 0.463 \\ & 1 & -0.130 & 0.404 \\ & & 1 & -0.057 \\ & & & 1 \end{pmatrix}.$$

at $\mu = 2$ GeV in the $\overline{\rm MS}$ scheme

$$\chi^2/d.o.f \sim 0.9$$

Stat. fit, TFF,
Rad. Cors

$$\begin{pmatrix} \epsilon_L^\tau - \epsilon_L^e + \epsilon_R^\tau - \epsilon_R^e \\ \epsilon_R^\tau + \frac{m_K^2}{2m_\tau(m_u+m_s)}\epsilon_P^\tau \\ \epsilon_S^\tau \\ \epsilon_T^\tau \end{pmatrix} = \begin{pmatrix} 0.5 \pm 1.5 \pm 0.3 \\ 0.4 \pm 0.9 \pm 0.2 \\ 0.8^{+0.8}_{-0.9} \pm 0.3 \\ 0.9 \pm 0.7 \pm 0.4 \end{pmatrix} \times 10^{-2}$$

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Global fit

- Assuming MFV, we can perform a global fit that includes both, the strangeness-conserving and -changing sectors.
- The constraints for the non-standard interactions

$$\begin{pmatrix} \epsilon_L^\tau - \epsilon_L^e + \epsilon_R^\tau - \epsilon_R^e \\ \epsilon_R^\tau \\ \epsilon_P^\tau \\ \epsilon_S^\tau \\ \epsilon_T^\tau \end{pmatrix} = \begin{pmatrix} 2.7 \pm 0.5 \,{}^{+\,2.3}_{-\,3.1} \,{}^{+\,0.4}_{-\,0.5} \pm 0.0 \pm 0.3 \,{}^{+\,0.0}_{-\,1.3} \pm 0.0 \\ 7.1 \pm 4.7 \,{}^{+\,1.2}_{-\,1.6} \pm 0.9 \pm 1.8 \pm 0.2 \,{}^{+\,12.3}_{-\,3.6} \pm 0.0 \\ -7.7 \pm 6.1 \pm 0.0 \,{}^{+\,1.3}_{-\,1.2} \,{}^{+\,2.4}_{-\,2.3} \pm 0.0 \,{}^{+\,4.1}_{-\,17.0} \pm 0.0 \\ 5.3 \,{}^{+\,0.6}_{-\,0.7} \,{}^{+\,2.0}_{-\,14.9} \,{}^{+\,0.1}_{-\,0.0} \pm 0.0 \pm 0.1 \,{}^{+\,0.1}_{-\,15.9} \,{}^{+\,0.1}_{-\,0.0} \\ -0.2 \pm 0.2 \,{}^{+\,3.6}_{-\,2.9} \,{}^{+\,0.1}_{-\,0.0} \pm 0.0 \pm 0.4 \,{}^{+\,0.5}_{-\,0.0} \,{}^{+\,0.2}_{-\,0.0} \end{pmatrix} \times 10^{-2}, \quad \rho_{ij} = \begin{pmatrix} 1 & 0.056 & 0.000 & -0.270 & -0.402 \\ & 1 & -0.997 & -0.015 & -0.023 \\ & & 1 & 0.000 & 0.000 \\ & & & 1 & 0.235 \\ & & & & 1 \end{pmatrix}.$$

at $\mu = 2$ GeV in the $\overline{\text{MS}}$ scheme

$$\chi^2/d.o.f \sim 1.49$$

Stat. fit, VFF, CKM, Rad. Cors., TFF, m_q, Rad. Cors.

$$\begin{pmatrix} \epsilon_L^\tau - \epsilon_L^e + \epsilon_R^\tau - \epsilon_R^e \\ \epsilon_R^\tau \\ \epsilon_P^\tau \\ \epsilon_S^\tau \\ \epsilon_T^\tau \end{pmatrix} = \begin{pmatrix} 2.9 & \pm 0.6 & {}^{+1.0}_{-0.9} & \pm 0.6 & \pm 0.0 & \pm 0.4 & {}^{+0.2}_{-0.3} \\ 7.1 & \pm 4.9 & {}^{+0.5}_{-0.4} & {}^{+1.3}_{-1.5} & {}^{+1.2}_{-1.3} & \pm 0.2 & {}^{+40.9}_{-14.1} \\ -7.6 & \pm 6.3 & \pm 0.0 & {}^{+1.9}_{-1.6} & {}^{+1.7}_{-1.6} & \pm 0.0 & {}^{+19.0}_{-53.6} \\ 5.0 & {}^{+0.7}_{-0.8} & {}^{+0.8}_{-1.3} & {}^{+0.2}_{-0.1} & \pm 0.0 & \pm 0.2 & {}^{+1.1}_{-0.6} \\ -0.5 & \pm 0.2 & {}^{+0.8}_{-1.0} & \pm 0.0 & \pm 0.0 & \pm 0.6 & \pm 0.1 \end{pmatrix} \times 10^{-2}$$

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Conclusions

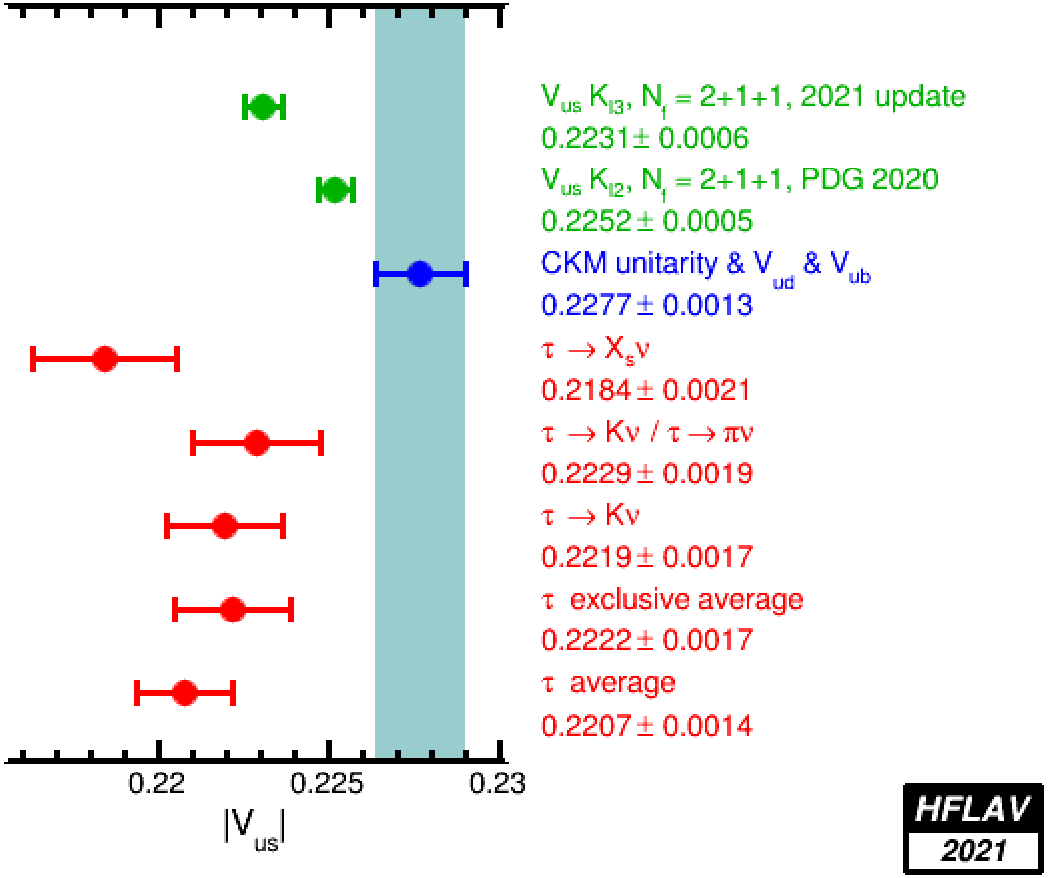
- At the current level of precision achieved in **semileptonic tau decays**, **radiative corrections** are required to test the **SM** and to extract information about **NP**.
- We study the electromagnetic corrections to the **hadronic tau decays**. Although the **model-independent contribution** for the **Kπ** modes was already available, a survey of the **structure-dependent** one was missing in the literature.
- **Our results** bridge this gap and **decrease the uncertainty** by a factor ~ 2, enabling more precise tests of **NP**.

References

- M. Antonelli, V. Cirigliano, A. Lusiani and E. Passemar. ¨Predicting the τ strange branching ratios and implications for V_{us}¨, JHEP 10 (2013) 070
- F.V. Flores-Baéz and J.R. Morones-Ibarra. ¨Model Independent Electromagnetic corrections in hadronic τ decays¨, Phys. Rev. D 88 (2013) 7, 073009
- R. Escribano, A. Miranda and P. Roig. e-Print: 2303.01362 [hep-ph]

$|V_{us}|$ from τ decays

- All $|V_{us}|$ determinations based on measured τ branching fractions are lower than both the kaon and the CKM-unitarity determinations.



Radiative corrections

- The differential decay width can be written as

$$\left.\frac{d\Gamma}{dt}\right|_{PP(\gamma)} = \frac{G_F^2\,|V_{uD}\,F_+(0)|^2\,S_{EW}\,m_\tau^3}{768\pi^3 t^3}\left(1-\frac{t}{m_\tau^2}\right)^2 \lambda^{1/2}(t,m_-^2,m_0^2)\left[C_V^2|\tilde{F}_+(t)|^2\left(1+\frac{2t}{m_\tau^2}\right)\lambda(t,m_-^2,m_0^2)\right.$$

$$\left. + 3C_S^2\Delta_{-0}^2|\tilde{F}_0(t)|^2\right]\boxed{G_{EM}(t)}$$

- For simplicity, we split the contributions to the decay width as

$$\left.\frac{d\Gamma}{dt}\right|_{PP(\gamma)} = \underbrace{\boxed{\left.\frac{d\Gamma}{dt}\right|_{PP} + \left.\frac{d\Gamma}{dt}\right|_{III}}}_{G_{\rm EM}^{(0)}(t)} + \underbrace{\left.\frac{d\Gamma}{dt}\right|_{IV/III}}_{\rm Negligible} + \underbrace{\left.\frac{d\Gamma}{dt}\right|_{rest}}_{\rm SI+SD},$$

$$F_{+/0}(t,u) = F_{+/0}(t) + \delta F_{+/0}(t,u),$$

Model 1

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$$\frac{\delta F_+(t,u)}{F_+(t)} = \frac{\alpha}{4\pi}\left[2(m_-^2+m_\tau^2-u)\mathcal{C}(u,M_\gamma) + 2\log\left(\frac{m_- m_\tau}{M_\gamma^2}\right)\right] + \delta\bar{f}_+(u),$$

Model 2

JHEP 10 (2013) 070

$$\frac{\delta F_+(t,u)}{F_+(t)} = \frac{\alpha}{4\pi}\left[2(m_-^2+m_\tau^2-u)\mathcal{C}(u,M_\gamma) + 2\log\left(\frac{m_- m_\tau}{M_\gamma^2}\right)\right] + \frac{\delta\bar{f}_+(u)}{F_+(t)},$$

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Model 1

JHEP 08 (2002) 002

Model 2

JHEP 10 (2013) 070

- The success of the standard model should not prevent us from exploring these questions to the fullest possible extent.
- Nelson and Tetradis. Phys. Lett. B 221 (1989), 80-84