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An alternative evaluation of the leading-order hadronic contribution to the muon $g-2$ with MUonE

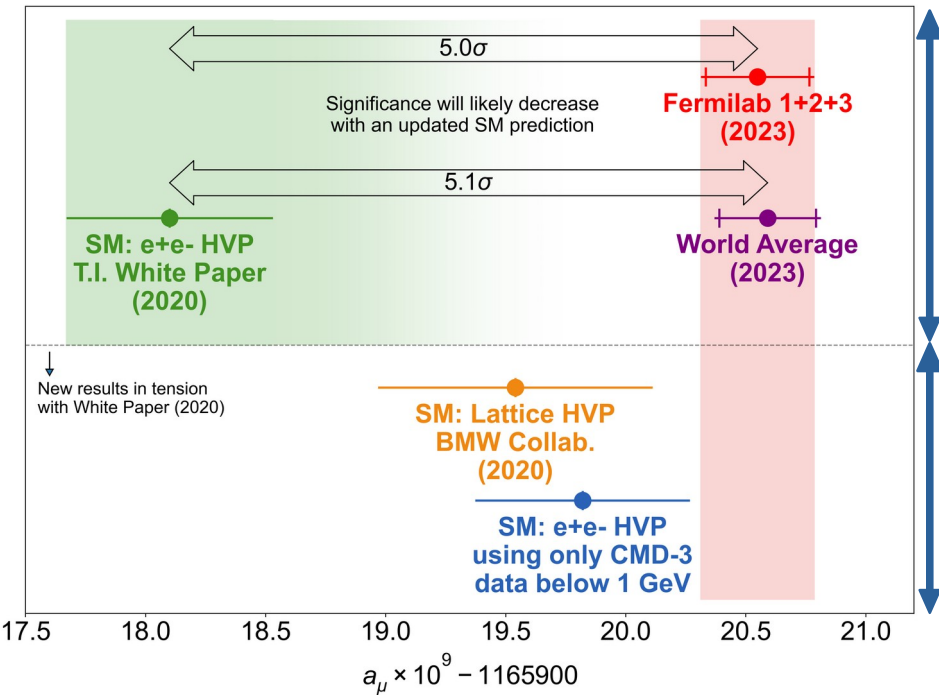
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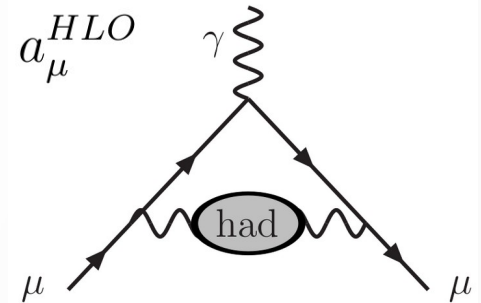
17th International Workshop on Tau Lepton Physics (TAU2023)
December 5th 2023

Muon g-2: current status



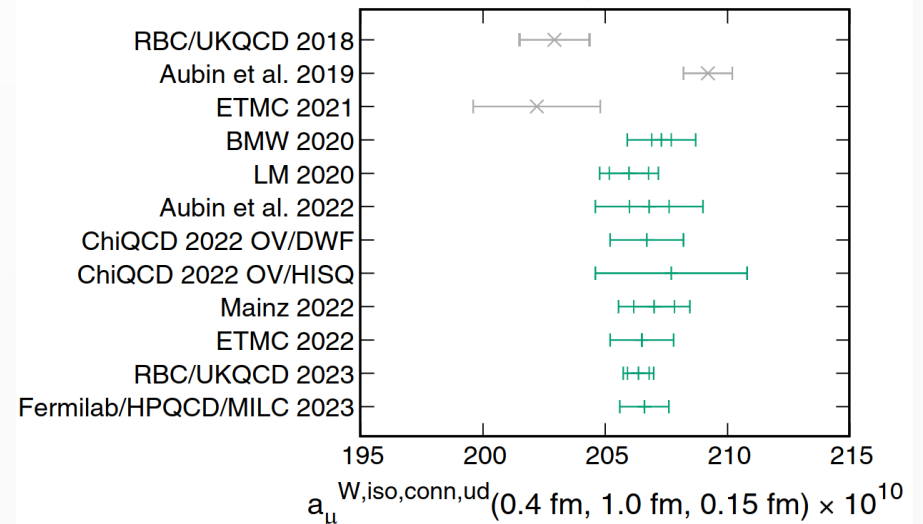
Comparison with WP20

New results after WP20



New lattice results in the intermediate window ($\sim 30\% a_\mu^{HLO}$):

RBC/UKQCD Phys.Rev.D 108 (2023)



A clarification of the theoretical prediction is needed.

The MUonE experiment

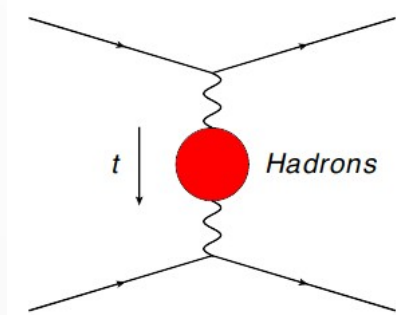


MUonE: a new independent evaluation of a_μ^{HLO}

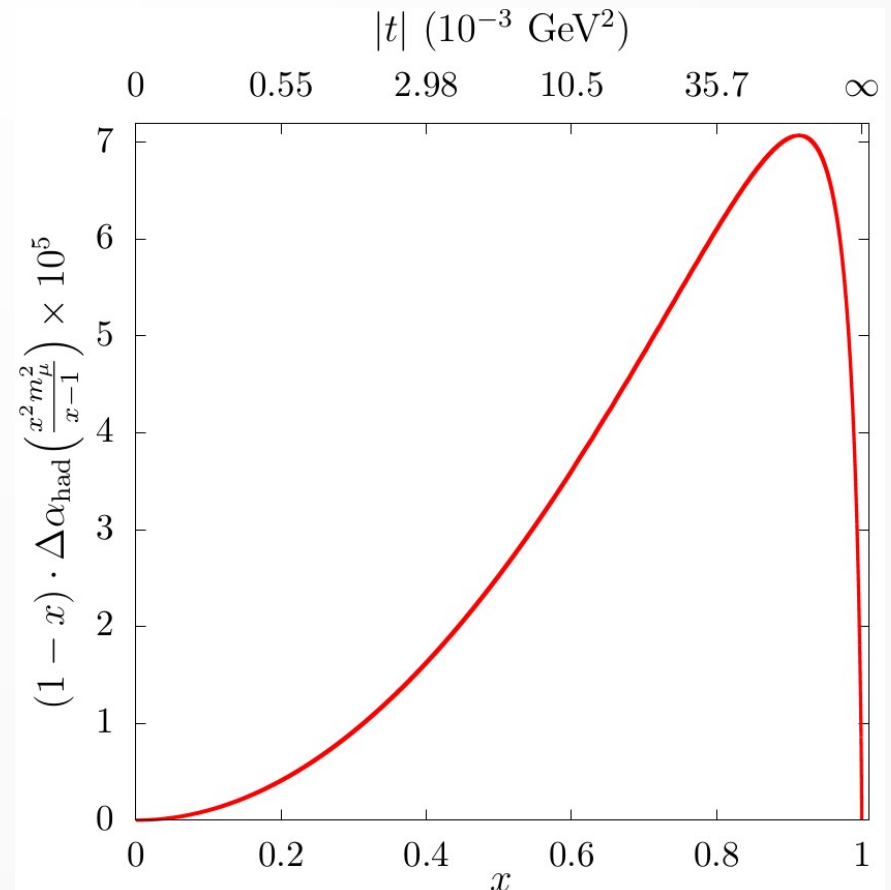
Phys. Rep. C 3 (1972), 193

$$a_\mu^{\text{HLO}} = \frac{\alpha_0}{\pi} \int_0^1 dx (1-x) \Delta\alpha_{\text{had}}[t(x)]$$

$$t(x) = \frac{x^2 m_\mu^2}{x-1} < 0$$



Based on the measurement of $\Delta\alpha_{\text{had}}(t)$:
hadronic contribution to the running of the
electromagnetic coupling constant.

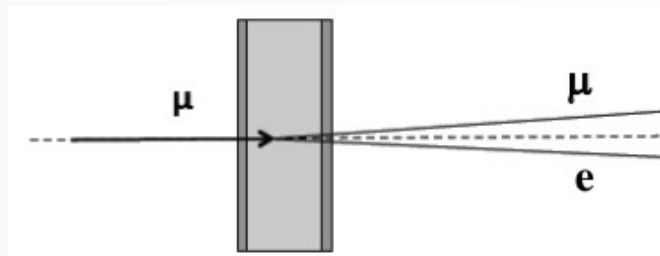


Phys. Lett. B 746 (2015), 325

The MUonE experiment



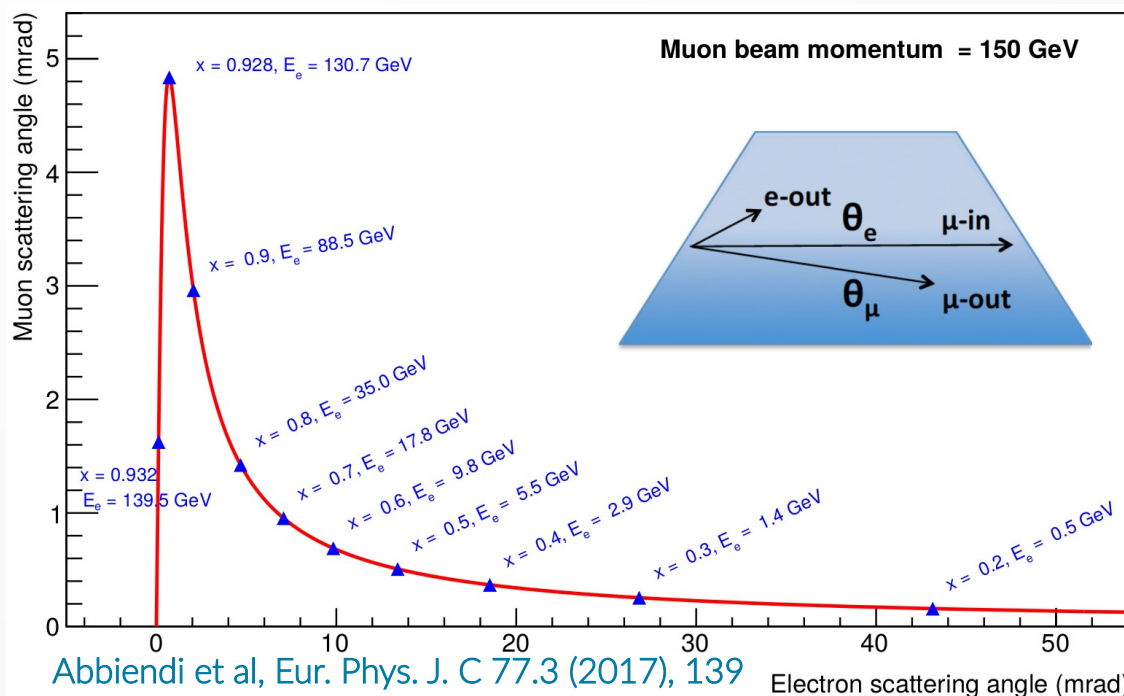
Extraction of $\Delta\alpha_{\text{had}}(t)$ from the *shape* of the $\mu e \rightarrow \mu e$ differential cross section



$$\frac{d\sigma_{\text{data}}(\Delta\alpha_{\text{had}})}{d\sigma_{\text{MC}}(\Delta\alpha_{\text{had}} = 0)} \sim 1 + \frac{2\Delta\alpha_{\text{had}}(t)}{\text{To be measured}}$$

From theoretical calculation

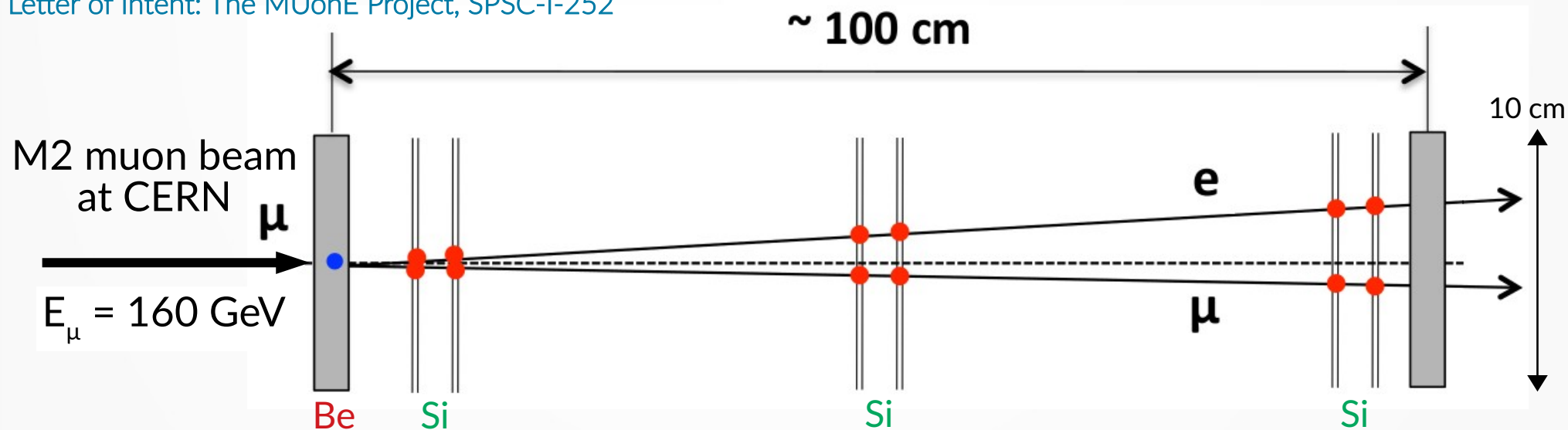
- Compute a_{μ}^{HLO} using data from one single experiment.
- Correlation between muon and electron angles allows to select elastic events and reject background ($\mu N \rightarrow \mu N e^+e^-$).
- Boosted kinematics: $\theta_{\mu} < 5 \text{ mrad}$, $\theta_e < 32 \text{ mrad}$.



The experimental apparatus

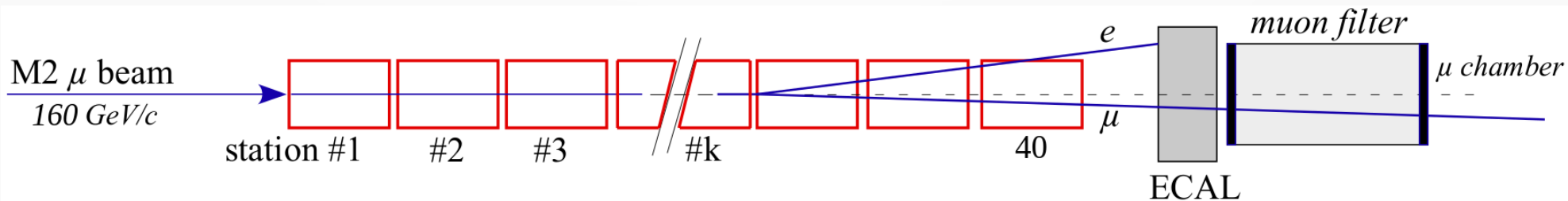


Letter of Intent: The MUonE Project, SPSC-I-252



Be (or C) target
1.5 cm thickness

Tracking system:
3 pairs of silicon strip detectors (CMS 2S modules)



Achievable accuracy



40 stations
(60 cm Be) + 3 years of data taking
($\sim 4 \times 10^7$ s)
($I_\mu \sim 10^7 \mu^+/\text{s}$)
 $\sim 4 \times 10^{12}$ events
with $E_e > 1$ GeV

=

$\sim 0.3\%$ statistical
accuracy on a_μ^{HLO}

Competitive with the latest
theoretical predictions.

$$\Delta a_\mu^{\text{HLO}}(\text{WP20}) \sim 0.6\%$$

$$\Delta a_\mu^{\text{HLO}}(\text{BMW}) \sim 0.8\%$$

Main challenge:
keep systematic accuracy
at the same level
of the statistical one.



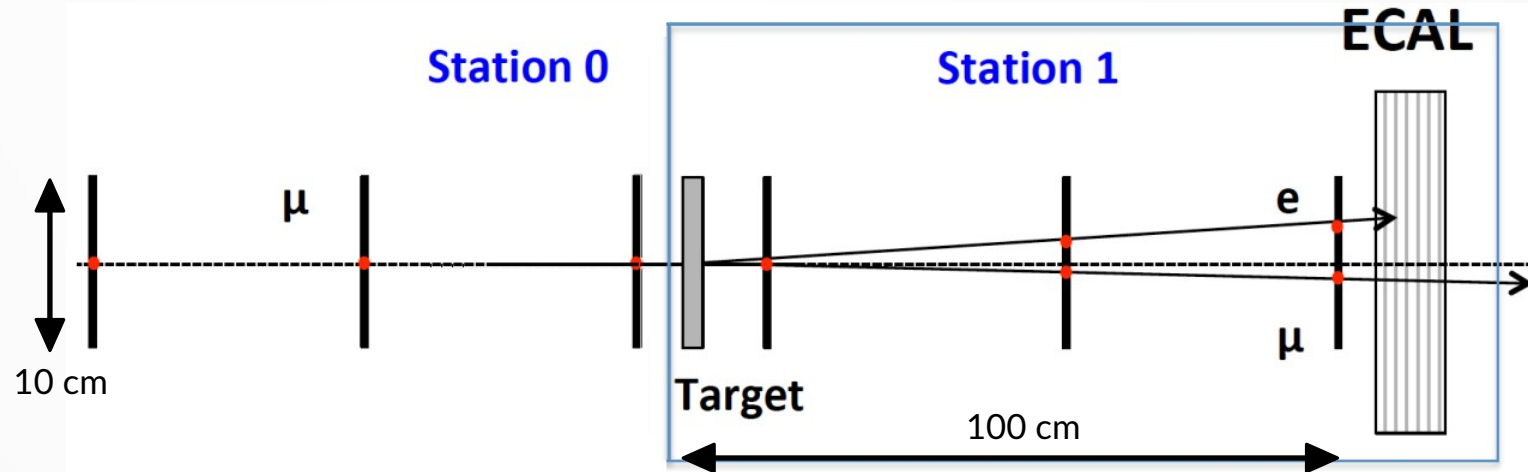
Systematic uncertainty of 10 ppm
in the signal region
(low θ_e , large θ_μ).

Test Run 2023 (21 Aug – 10 Sept)



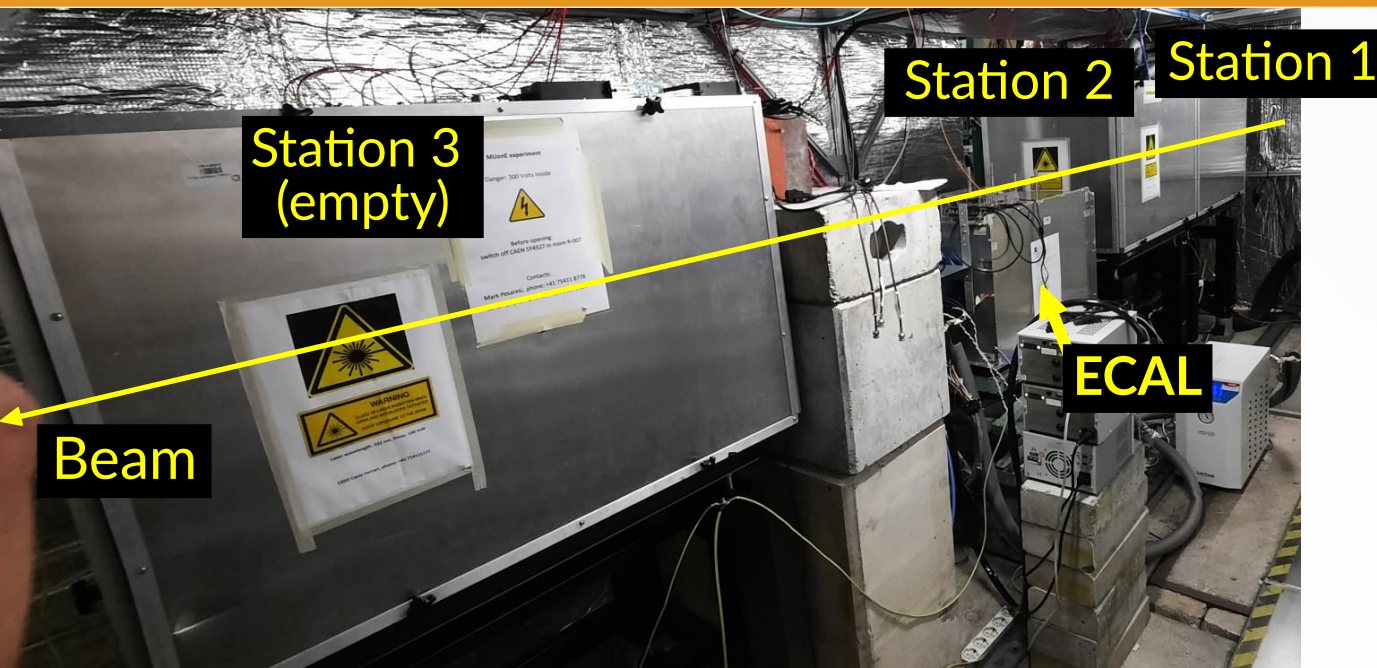
A 3 weeks Test Run with a reduced detector has been approved by SPSC, to validate our proposal.

- Pretracker +
- 1 station +
- ECAL

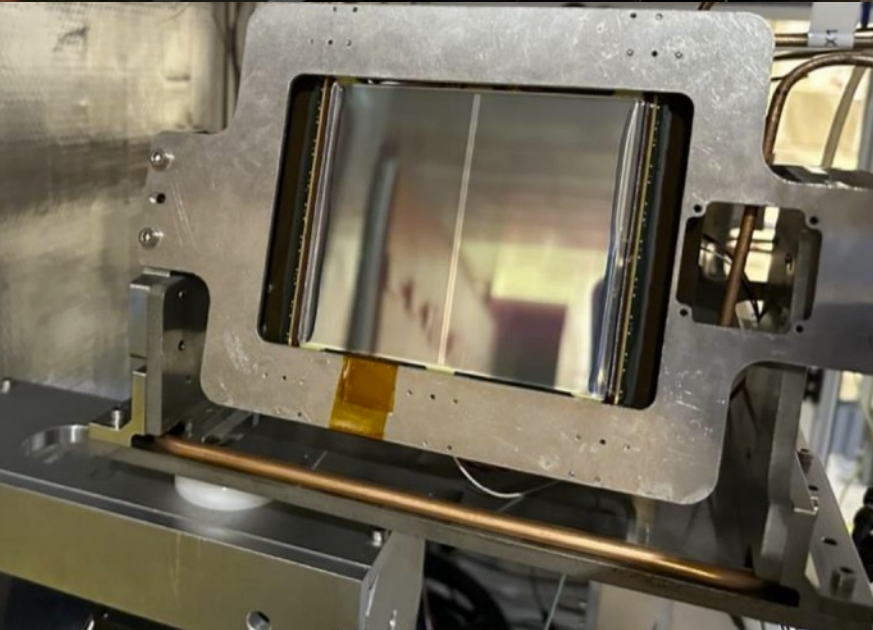


Main goals:

- Confirm the system engineering.
- Test the detector performance.
- Test the reconstruction algorithms and event selection.
- Study the background processes and the main sources of systematic error.
- Demonstration measurement: $\Delta\alpha_{\text{lep}}(t)$ with O(5-10%) precision.



Test Run Analysis



- Determine selection algorithms to be applied on FPGA.
- Beam rate measurements.
- Hardware metrology.
- Software alignment.
- Detector performance.

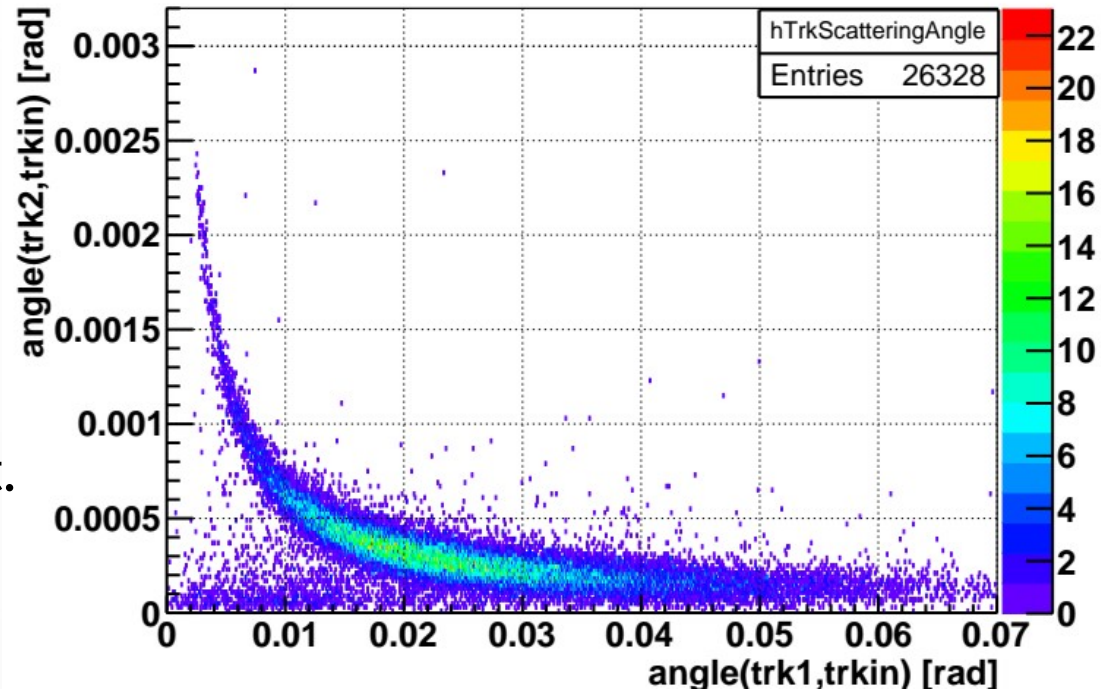
Elastic events

Golden event selection:

- 1 hit/module in 1st station.
- 2 hits/module in 2nd station.
- 3 reconstructed tracks with good χ^2 .
- Loose cut on z_{vertex} to select interactions in the target.

Ongoing work:

- Include a cut on the acoplanarity of the 3 tracks (elastic events are planar).
- Identify and study background events.



Extraction of $\Delta\alpha_{\text{had}}(t)$



$\Delta\alpha_{\text{had}}(t)$ parameterization:

inspired from the 1 loop QED contribution of lepton pairs and t -quark at $q^2 < 0$

$$\Delta\alpha_{\text{had}}(t) = KM \left\{ -\frac{5}{9} - \frac{4M}{3t} + \left(\frac{4M^2}{3t^2} + \frac{M}{3t} - \frac{1}{6} \right) \frac{2}{\sqrt{1 - \frac{4M}{t}}} \ln \left| \frac{1 - \sqrt{1 - \frac{4M}{t}}}{1 + \sqrt{1 - \frac{4M}{t}}} \right| \right\}$$

2 parameters:
K, M

Extraction of $\Delta\alpha_{\text{had}}(t)$

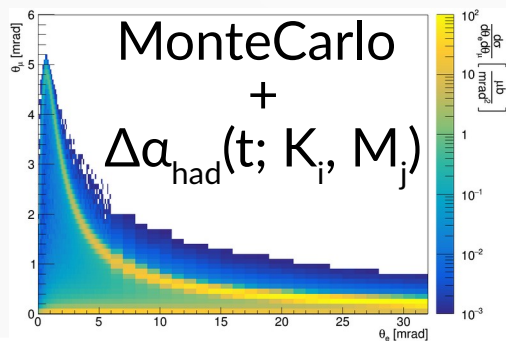
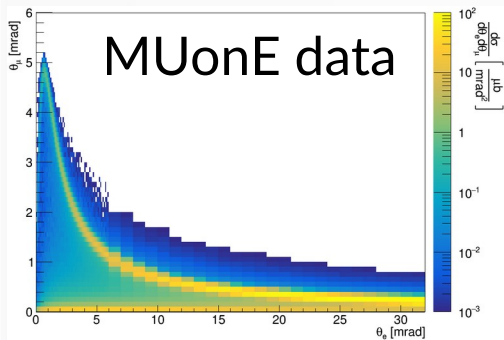


$\Delta\alpha_{\text{had}}(t)$ parameterization:

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$$\Delta\alpha_{\text{had}}(t) = KM \left\{ -\frac{5}{9} - \frac{4M}{3t} + \left(\frac{4M^2}{3t^2} + \frac{M}{3t} - \frac{1}{6} \right) \frac{2}{\sqrt{1 - \frac{4M}{t}}} \ln \left| \frac{1 - \sqrt{1 - \frac{4M}{t}}}{1 + \sqrt{1 - \frac{4M}{t}}} \right| \right\} \quad \text{2 parameters: } K, M$$

Extraction of $\Delta\alpha_{\text{had}}(t)$ through a template fit to the 2D (θ_e, θ_μ) distribution:



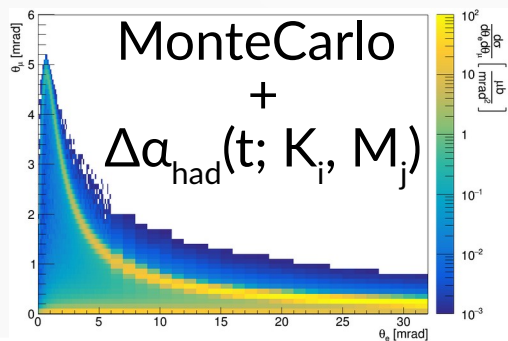
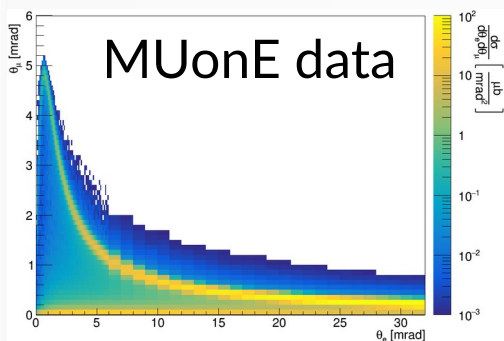
Extraction of $\Delta\alpha_{had}(t)$

$\Delta\alpha_{had}(t)$ parameterization:

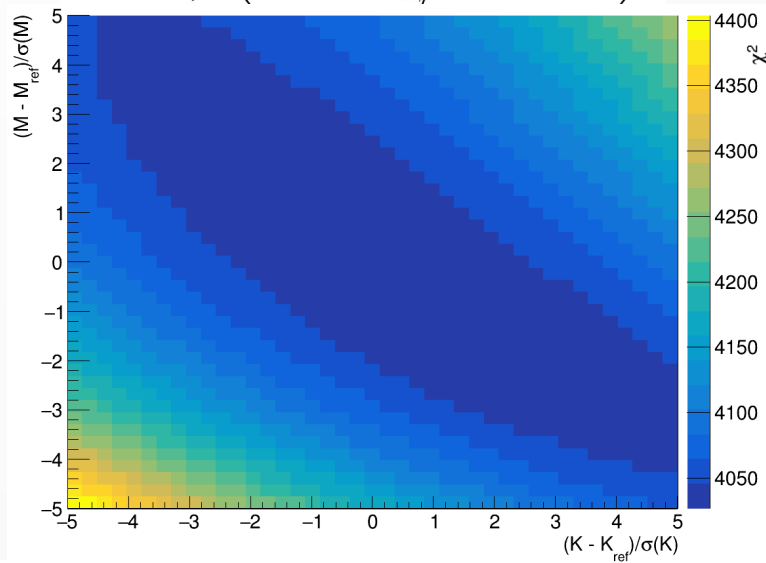
inspired from the 1 loop QED contribution of lepton pairs and t -quark at $q^2 < 0$

$$\Delta\alpha_{had}(t) = KM \left\{ -\frac{5}{9} - \frac{4M}{3t} + \left(\frac{4M^2}{3t^2} + \frac{M}{3t} - \frac{1}{6} \right) \frac{2}{\sqrt{1 - \frac{4M}{t}}} \ln \left| \frac{1 - \sqrt{1 - \frac{4M}{t}}}{1 + \sqrt{1 - \frac{4M}{t}}} \right| \right\} \quad \text{2 parameters: } K, M$$

Extraction of $\Delta\alpha_{had}(t)$ through a template fit to the 2D (θ_e, θ_μ) distribution:



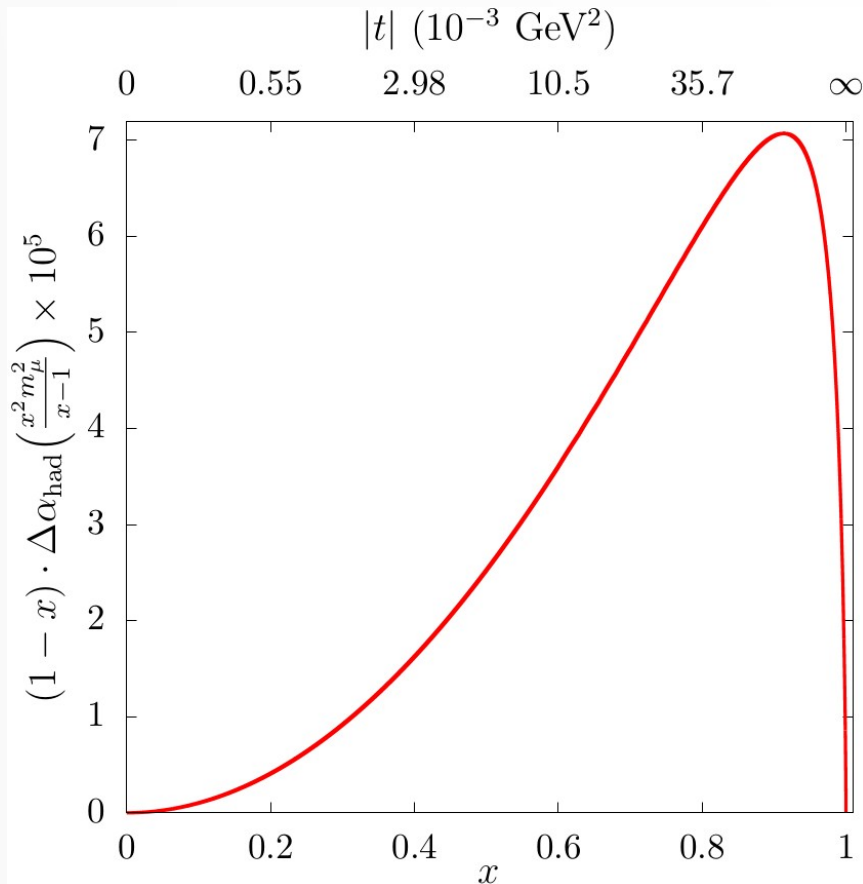
$$\chi^2 = \sum_{\text{bins}} \left(\frac{\text{data}_i - \text{templ}(K, M)_i}{\sigma_i^{\text{data}}} \right)^2$$



Compute a_μ^{HLO}



Input the best fit parameters
in the MUonE master integral



$$a_\mu^{\text{HLO}} = \frac{\alpha_0}{\pi} \int_0^1 dx (1-x) \Delta\alpha_{\text{had}}[t(x)]$$

($K_{\text{best}}, M_{\text{best}}$)
↓

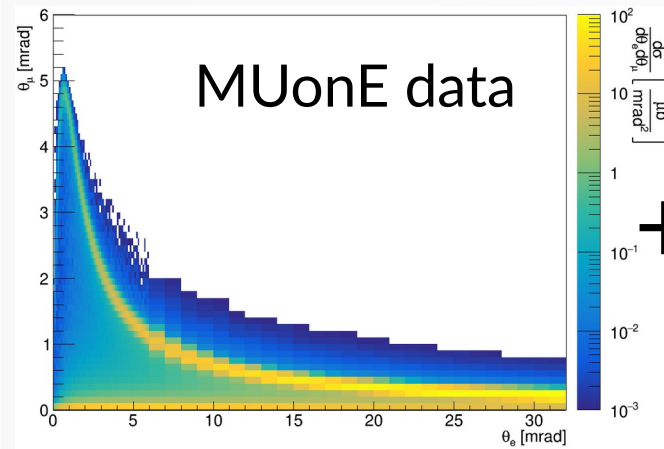
Results from a
simulation with the
expected final statistics
(4×10^{12} elastic events):

$$a_\mu^{\text{HLO}} = (688.8 \pm 2.4) \times 10^{-10}$$

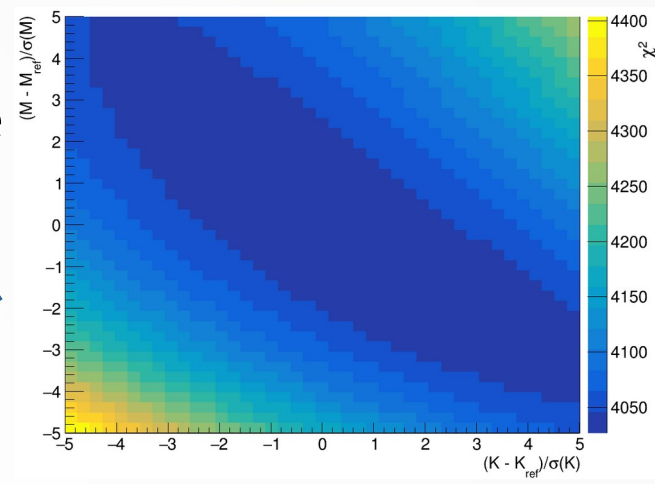
(0.35% accuracy)

Input value

$$a_\mu^{\text{HLO}} = 688.6 \times 10^{-10}$$

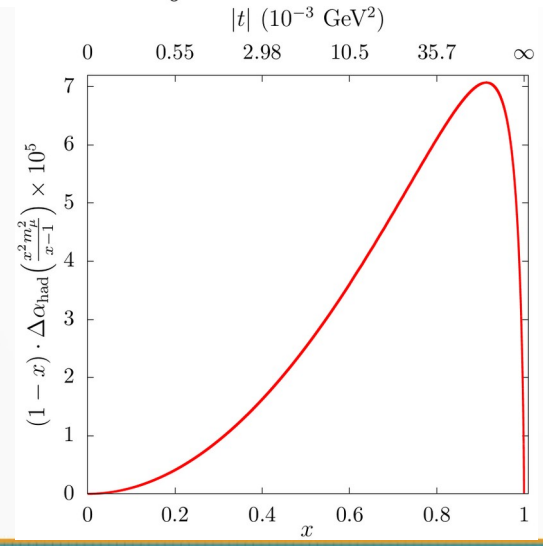


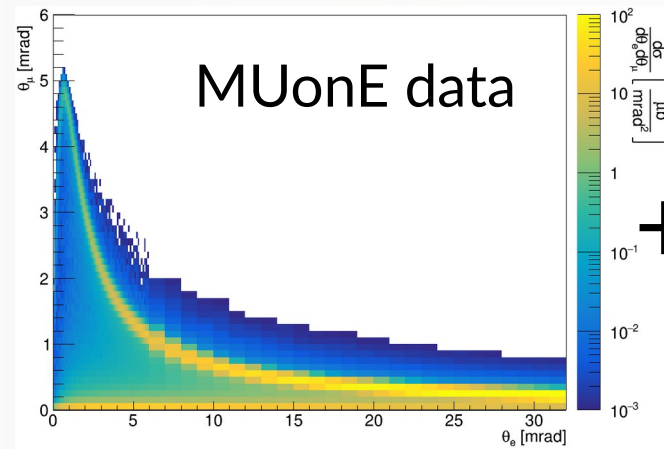
$$+ \Delta\alpha_{\text{had}}(t; K, M) \xrightarrow{\text{Template fit}}$$



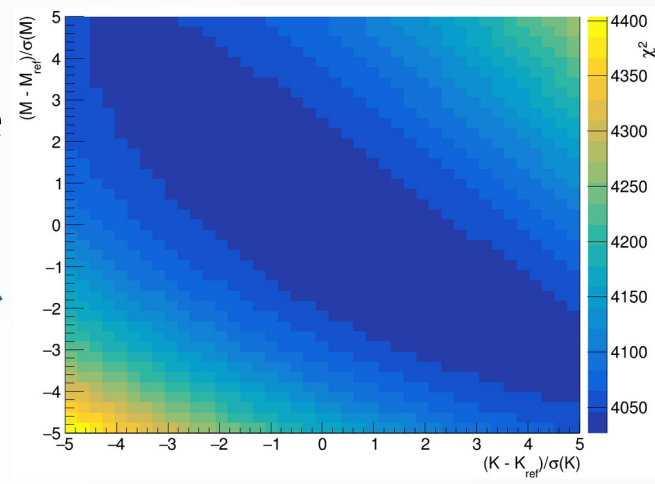
$$\Delta\alpha_{\text{had}}(t; K_{\text{best}}, M_{\text{best}})$$

$$a_{\mu}^{\text{HLO}} = \frac{\alpha_0}{\pi} \int_0^1 dx (1-x) \Delta\alpha_{\text{had}}[t(x)]$$



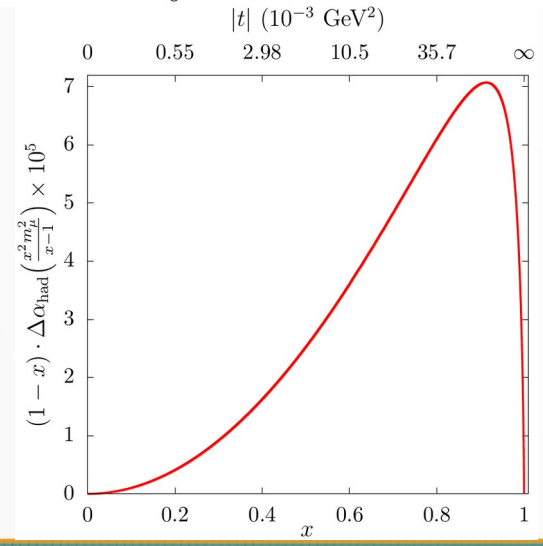


$$+ \Delta\alpha_{\text{had}}(t; K, M) \xrightarrow{\text{Template fit}}$$



$$\Delta\alpha_{\text{had}}(t; K_{\text{best}}, M_{\text{best}})$$

$$a_{\mu}^{\text{HLO}} = \frac{\alpha_0}{\pi} \int_0^1 dx (1-x) \Delta\alpha_{\text{had}}[t(x)]$$



Can we compute a_{μ}^{HLO} in a different way using MUonE data?



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Letter

An alternative evaluation of the leading-order hadronic contribution to the muon $g-2$ with MUonE

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ARTICLE INFO

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ABSTRACT

We propose an alternative method to extract the leading-order hadronic contribution to the muon $g-2$, a_{μ}^{HLO} , with the MUonE experiment. In contrast to the traditional method based on the integral of the hadronic contribution to the running of the electromagnetic coupling, $\Delta\alpha_{\text{had}}$, in the space-like region, our approach relies on the computation of the derivatives of $\Delta\alpha_{\text{had}}(t)$ at zero squared momentum transfer t . We show that this approach allows to extract $\sim 99\%$ of the total value of a_{μ}^{HLO} from the MUonE data, while the remaining $\sim 1\%$ can be computed combining perturbative QCD and data on $e^{+}e^{-}$ annihilation to hadrons. This leads to a competitive evaluation of a_{μ}^{HLO} which is robust against the parameterization used to model $\Delta\alpha_{\text{had}}(t)$ in the MUonE kinematic region, thanks to the analyticity properties of $\Delta\alpha_{\text{had}}(t)$, which can be expanded as a polynomial at $t \sim 0$.

An alternative method to compute a_μ^{HLO} with MUonE

Based on:

S. Bodenstein et al, Phys. Rev. D 85 (2012)

C.A. Dominguez et al, Phys. Rev. D 96 (2017)

Start from traditional dispersive integral:

$$a_\mu^{\text{HLO}} = \frac{\alpha^2}{3\pi^2} \int_{s_{\text{th}}}^{\infty} \frac{ds}{s} K(s) R(s)$$

$$K(s) = \int_0^1 dx \frac{x^2(1-x)}{x^2 + (1-x)s/m_\mu^2}$$

$$R(s) \propto \sigma(e^+e^- \rightarrow \text{hadrons})$$

An alternative method

to compute a_μ^{HLO} with MUonE



Based on:

S. Bodenstein et al, Phys. Rev. D 85 (2012)

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Start from traditional dispersive integral:

$$a_\mu^{\text{HLO}} = \frac{\alpha^2}{3\pi^2} \int_{s_{\text{th}}}^{\infty} \frac{ds}{s} K(s) R(s)$$

$$s_{\text{th}} = m_{\pi^0}^2$$

$$s_0 \gtrsim (2 \text{ GeV})^2$$

$$K(s) = \int_0^1 dx \frac{x^2(1-x)}{x^2 + (1-x)s/m_\mu^2}$$

$$R(s) \propto \sigma(e^+e^- \rightarrow \text{hadrons})$$

$$\frac{\alpha^2}{3\pi^2} \int_{s_{\text{th}}}^{s_0} \frac{ds}{s} K(s) R(s) \quad + \quad \frac{\alpha^2}{3\pi^2} \int_{s_0}^{\infty} \frac{ds}{s} K(s) R(s)$$

pQCD

$$-\text{Im}\Pi_{\text{had}}(s) = \frac{\alpha}{3} R(s)$$

Low energy integral



$$\int_{s_{\text{th}}}^{s_0} \frac{ds}{s} K(s) \frac{\text{Im}\Pi_{had}(s)}{\pi} =$$

$$\int_{s_{\text{th}}}^{s_0} \frac{ds}{s} [K(s) - K_1(s)] \frac{\text{Im}\Pi_{had}(s)}{\pi} + \int_{s_{\text{th}}}^{s_0} \frac{ds}{s} K_1(s) \frac{\text{Im}\Pi_{had}(s)}{\pi}$$

Low energy integral



$$\int_{s_{th}}^{s_0} \frac{ds}{s} K(s) \frac{\text{Im}\Pi_{had}(s)}{\pi} =$$
$$\int_{s_{th}}^{s_0} \frac{ds}{s} [K(s) - K_1(s)] \frac{\text{Im}\Pi_{had}(s)}{\pi} + \int_{s_{th}}^{s_0} \frac{ds}{s} K_1(s) \frac{\text{Im}\Pi_{had}(s)}{\pi}$$

$$K_1(s) = a_0 s + \sum_{n=1}^3 \frac{a_n}{s^n}$$

$K_1(s)$ approximates $K(s)$ for $s < s_0$.
Meromorphic function:
no cuts, poles in $s = 0$.

Two different techniques to get $K_1(s)$:

1) Least squares minimization

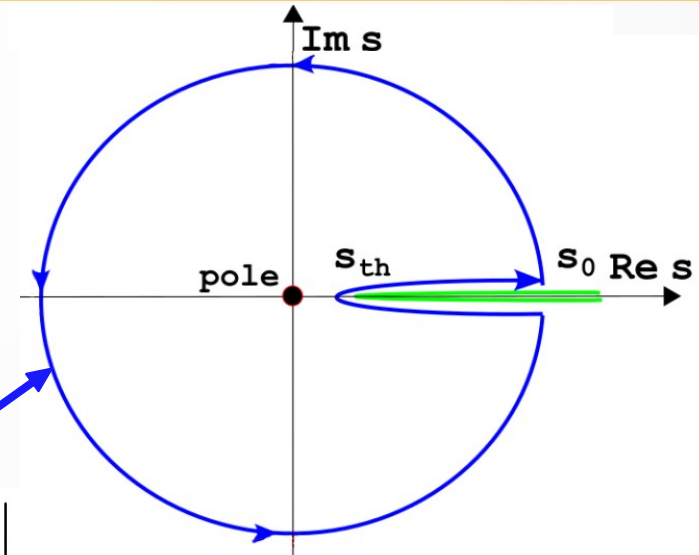
2) Minimize $\int_{s_{th}}^{s_0} \frac{ds}{s} |K(s) - K_1(s)| R(s)$

Low energy integral

Use Cauchy's theorem

$$\int_{s_{th}}^{s_0} \frac{ds}{s} K_1(s) \frac{\text{Im} \Pi_{had}(s)}{\pi} =$$

$$\text{Res} \left[\Pi_{had}(s) \frac{K_1(s)}{s} \right]_{s=0} - \frac{1}{2\pi i} \oint_{|s|=s_0} \frac{ds}{s} K_1(s) \Pi_{had}(s) \Big|_{\text{pQCD}}$$



$$\text{Res} \left[\Pi_{had}(s) \frac{K_1(s)}{s} \right]_{s=0} = \sum_{n=1}^3 \frac{c_n}{n!} \frac{d^{(n)}}{ds^n} \Pi_{had}(s) \Big|_{s=0} = \sum_{n=1}^3 \frac{c_n}{n!} \frac{d^{(n)}}{dt^n} \Delta \alpha_{had}(t) \Big|_{t=0}$$

From MUonE

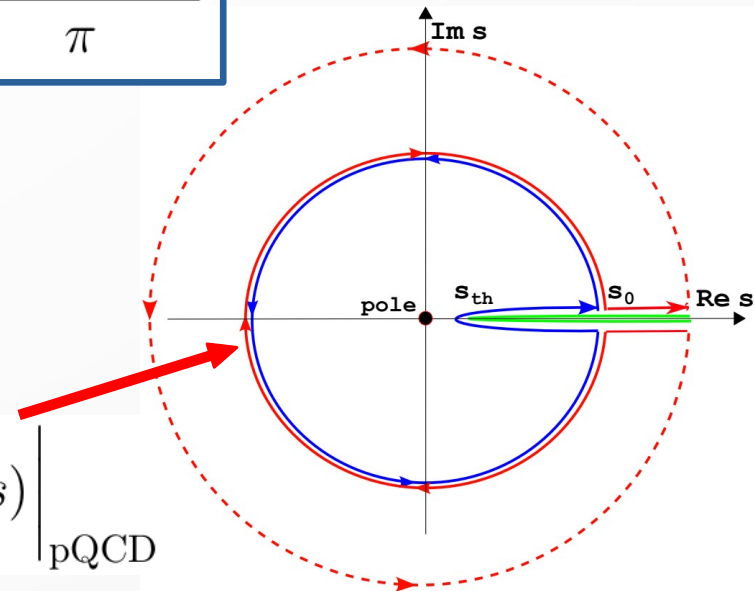
High energy integral

Similar strategy for the high energy part

$$\int_{s_0}^{\infty} \frac{ds}{s} K(s) \frac{\text{Im}\Pi_{had}(s)}{\pi} = \int_{s_0}^{\infty} \frac{ds}{s} [K(s) - \tilde{K}_1(s)] \frac{\text{Im}\Pi_{had}(s)}{\pi} + \int_{s_0}^{\infty} \frac{ds}{s} \tilde{K}_1(s) \frac{\text{Im}\Pi_{had}(s)}{\pi}$$

$$\tilde{K}_1(s) = K_1(s) - c_0 s$$

$$\int_{s_0}^{\infty} \frac{ds}{s} \tilde{K}_1(s) \frac{\text{Im}\Pi_{had}(s)}{\pi} = \frac{1}{2\pi i} \oint_{|s|=s_0} \frac{ds}{s} \tilde{K}_1(s) \Pi_{had}(s) \Big|_{\text{pQCD}}$$



Compute a_μ^{HLO}



Rearranging the previous equations...

$$a_\mu^{\text{HLO}} = a_\mu^{\text{HLO (I)}} + a_\mu^{\text{HLO (II)}} + a_\mu^{\text{HLO (III)}} + a_\mu^{\text{HLO (IV)}}$$

$$a_\mu^{\text{HLO (I)}} = -\frac{\alpha}{\pi} \sum_{n=1}^3 \frac{c_n}{n!} \frac{d^{(n)}}{dt^n} \Delta\alpha_{had}(t) \Big|_{t=0}$$

$$a_\mu^{\text{HLO (II)}} = \frac{\alpha}{\pi} \frac{1}{2\pi i} \oint_{|s|=s_0} \frac{ds}{s} c_0 s \Pi_{had}(s) \Big|_{\text{pQCD}}$$

$$a_\mu^{\text{HLO (III)}} = \frac{\alpha^2}{3\pi^2} \int_{s_{\text{th}}}^{s_0} \frac{ds}{s} [K(s) - K_1(s)] R(s)$$

$$a_\mu^{\text{HLO (IV)}} = \frac{\alpha^2}{3\pi^2} \int_{s_0}^{\infty} \frac{ds}{s} [K(s) - \tilde{K}_1(s)] R(s)$$

Compute a_μ^{HLO}



Rearranging the previous equations...

$$a_\mu^{\text{HLO}} = a_\mu^{\text{HLO (I)}} + a_\mu^{\text{HLO (II)}} + a_\mu^{\text{HLO (III)}} + a_\mu^{\text{HLO (IV)}}$$

99%

$$a_\mu^{\text{HLO (I)}} = -\frac{\alpha}{\pi} \sum_{n=1}^3 \frac{c_n}{n!} \frac{d^{(n)}}{dt^n} \Delta\alpha_{had}(t) \Big|_{t=0}$$

MUnE

$$a_\mu^{\text{HLO (II)}} = \frac{\alpha}{\pi} \frac{1}{2\pi i} \oint_{|s|=s_0} \frac{ds}{s} c_0 s \Pi_{had}(s) \Big|_{\text{pQCD}}$$

$$a_\mu^{\text{HLO (III)}} = \frac{\alpha^2}{3\pi^2} \int_{s_{\text{th}}}^{s_0} \frac{ds}{s} [K(s) - K_1(s)] R(s)$$

$$a_\mu^{\text{HLO (IV)}} = \frac{\alpha^2}{3\pi^2} \int_{s_0}^{\infty} \frac{ds}{s} [K(s) - \tilde{K}_1(s)] R(s)$$

Compute a_μ^{HLO}



Rearranging the previous equations...

$$a_\mu^{\text{HLO}} = a_\mu^{\text{HLO (I)}} + a_\mu^{\text{HLO (II)}} + a_\mu^{\text{HLO (III)}} + a_\mu^{\text{HLO (IV)}}$$

99%

$$a_\mu^{\text{HLO (I)}} = -\frac{\alpha}{\pi} \sum_{n=1}^3 \frac{c_n}{n!} \frac{d^{(n)}}{dt^n} \Delta\alpha_{had}(t) \Big|_{t=0}$$

MUnE

1%

$$a_\mu^{\text{HLO (II)}} = \frac{\alpha}{\pi} \frac{1}{2\pi i} \oint_{|s|=s_0} \frac{ds}{s} c_0 s \Pi_{had}(s) \Big|_{\text{pQCD}}$$

$$a_\mu^{\text{HLO (III)}} = \frac{\alpha^2}{3\pi^2} \int_{s_{\text{th}}}^{s_0} \frac{ds}{s} [K(s) - K_1(s)] R(s)$$

$$a_\mu^{\text{HLO (IV)}} = \frac{\alpha^2}{3\pi^2} \int_{s_0}^{\infty} \frac{ds}{s} [K(s) - \tilde{K}_1(s)] R(s)$$

Time-like data
+
pQCD

$a_{\mu}^{\text{HLO (I)}}$ from MUonE data



$$a_{\mu}^{\text{HLO (I)}} = -\frac{\alpha}{\pi} \sum_{n=1}^3 \frac{c_n}{n!} \frac{d^{(n)}}{dt^n} \Delta\alpha_{had}(t) \Big|_{t=0}$$

The relevant quantities are the derivatives of $\Delta\alpha_{had}(t)$ at $t = 0$.

$a_\mu^{\text{HLO (I)}}$ from MUonE data



$$a_\mu^{\text{HLO (I)}} = -\frac{\alpha}{\pi} \sum_{n=1}^3 \frac{c_n}{n!} \frac{d^{(n)}}{dt^n} \Delta\alpha_{\text{had}}(t) \Big|_{t=0}$$

The relevant quantities are the derivatives of $\Delta\alpha_{\text{had}}(t)$ at $t = 0$.

Try different parameterizations to fit MUonE data
(max 3 fit parameters, due to the statistics collected by MUonE)

$$\Delta\alpha_{\text{had}}(t) = KM \left\{ -\frac{5}{9} - \frac{4M}{3t} + \left(\frac{4M^2}{3t^2} + \frac{M}{3t} - \frac{1}{6} \right) \frac{2}{\sqrt{1 - \frac{4M}{t}}} \ln \left| \frac{1 - \sqrt{1 - \frac{4M}{t}}}{1 + \sqrt{1 - \frac{4M}{t}}} \right| \right\} \quad \text{Lepton-like}$$

$$\Delta\alpha_{\text{had}}(t) = P_1 t \frac{1 + P_2 t}{1 + P_3 t}$$

Padé approximant

$$\Delta\alpha_{\text{had}}(t) = P_1 t + P_2 t^2 + P_3 t^3$$

3° polynomial

a_μ^{HLO} (I) from MUonE data



Reconstruction approximants

D. Greynat, E. de Rafael, JHEP 2022 (5)

$$\Delta\alpha_{\text{had}}(t) = \sum_{n=1}^N \mathcal{A}(n, \mathbf{L}) \left(\frac{\sqrt{1 - \frac{t}{t_0}} - 1}{\sqrt{1 - \frac{t}{t_0}} + 1} \right)^n + \sum_{p=1}^{\lfloor \frac{L+1}{2} \rfloor} \mathcal{B}(2p-1) \text{Li}_{2p-1} \left(\frac{\sqrt{1 - \frac{t}{t_0}} - 1}{\sqrt{1 - \frac{t}{t_0}} + 1} \right)$$

$$\Delta\alpha_{\text{had}}(t) = A_1 \mathcal{S}_1 + A_2 \mathcal{S}_2 + A_3 \mathcal{S}_3 + B_1 \mathcal{L}_1$$

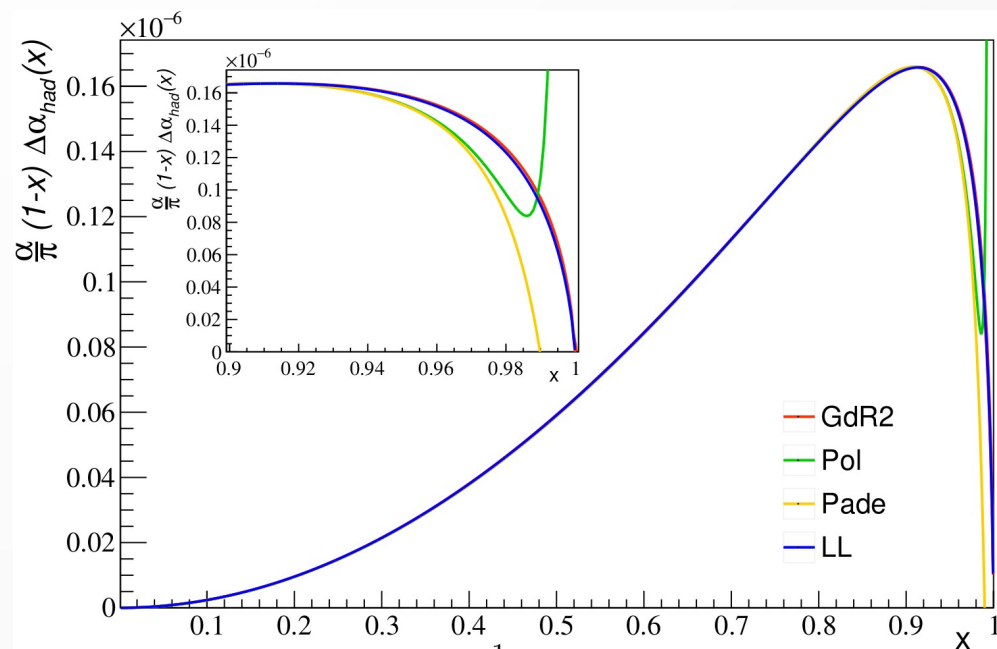
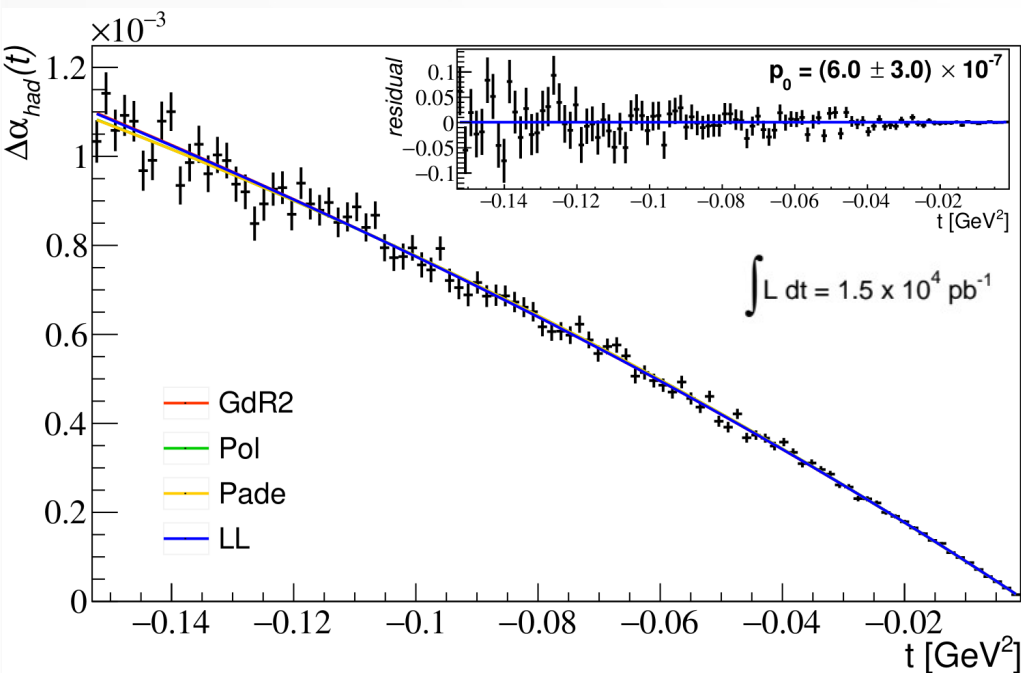
$$\mathcal{S}_i = \left(\frac{\sqrt{1 - \frac{t}{t_0}} - 1}{\sqrt{1 - \frac{t}{t_0}} + 1} \right)^i; \quad A_i = \mathcal{A}(i, 1) \quad i = 1, 2, 3$$

$$\mathcal{L}_1 = \text{Li}_1 \left(\frac{\sqrt{1 - \frac{t}{t_0}} - 1}{\sqrt{1 - \frac{t}{t_0}} + 1} \right); \quad B_1 = \mathcal{B}(1)$$

Tested $L = 1, N = 3$
Several variants with different
number of free parameters

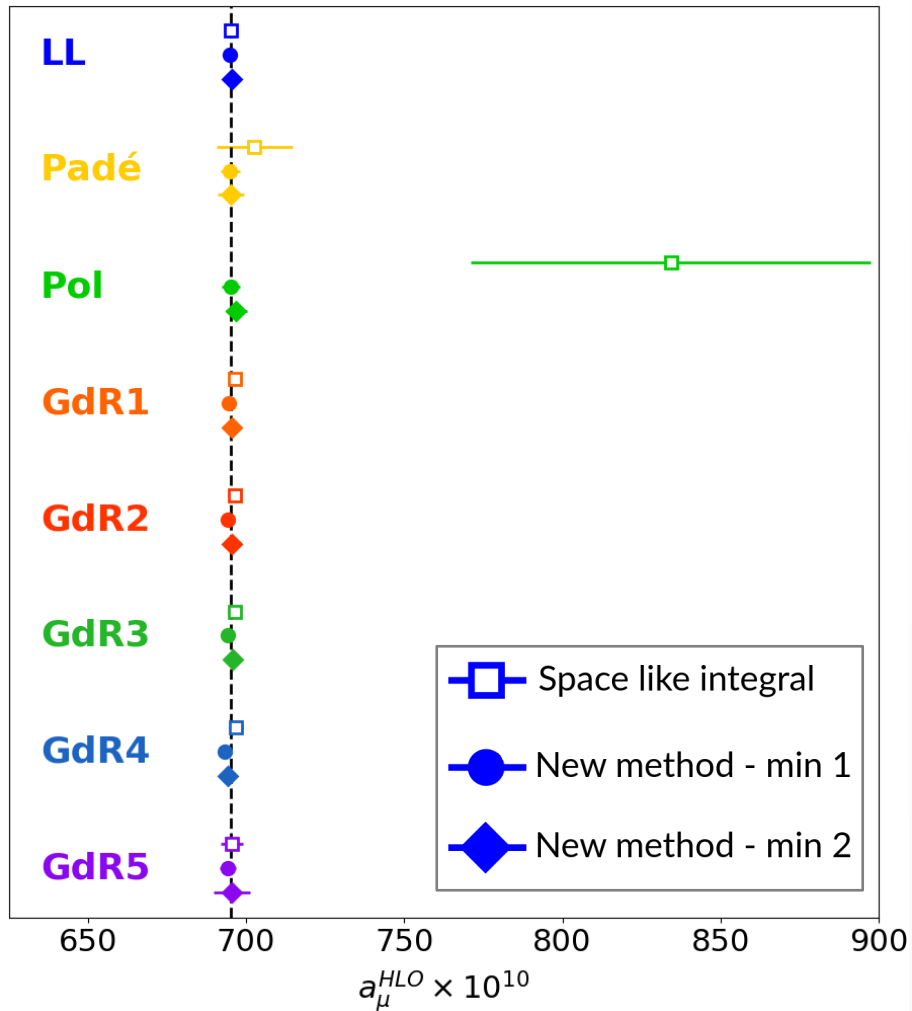
Fit the MUonE data

Simplified fit: simulate the MUonE signal using time-like compilations of $\Delta\alpha_{had}$. Error bars according to the MUonE final statistics.

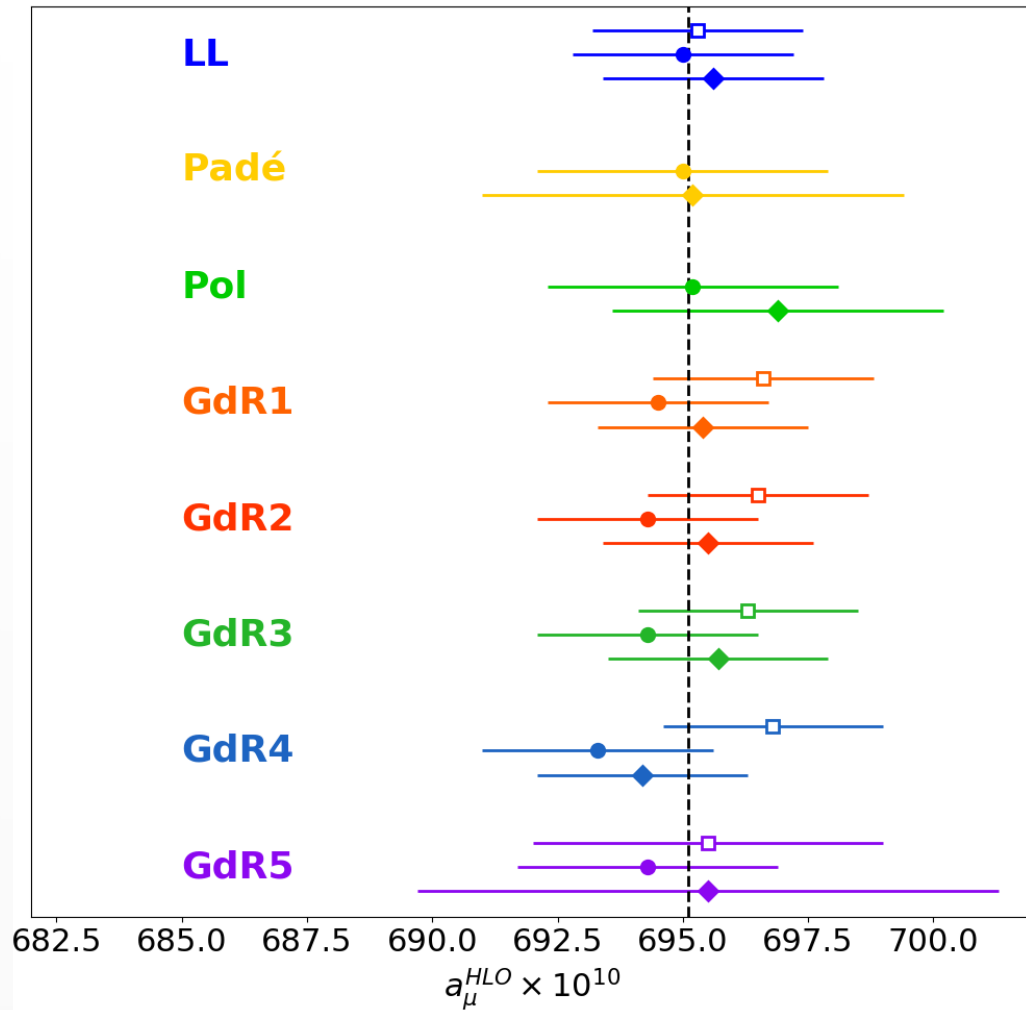
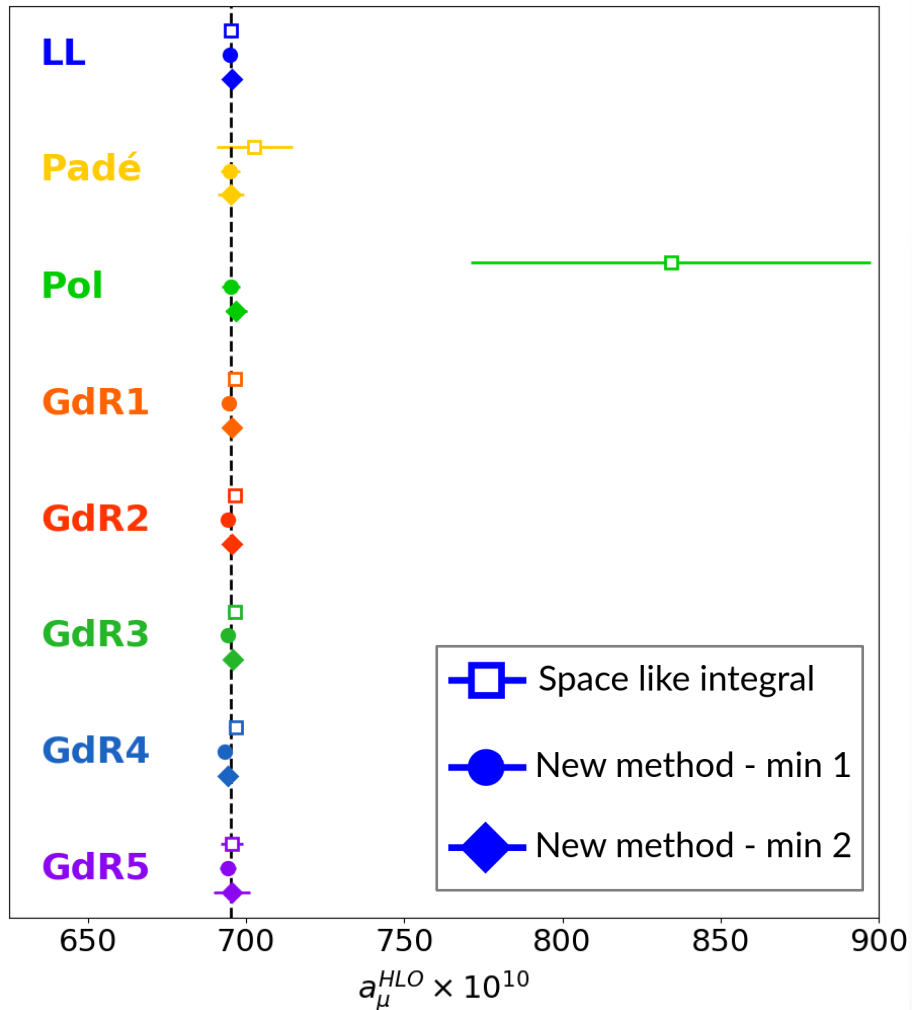


$$\alpha_\mu^{HLO} = \frac{\alpha_0}{\pi} \int_0^1 dx (1-x) \Delta\alpha_{had}[t(x)]$$

Results: a_μ^{HLO}



Results: a_μ^{HLO}



Conclusions



- MUonE will provide an independent calculation of a_{μ}^{HLO} , competitive with the latest evaluations.
- 3 weeks Test Run 2023: proof of concept of the experimental proposal. Data analysis ongoing.
Request for a longer commissioning run in 2025 instrumenting more tracking stations.
- Full apparatus (40 stations) after CERN Long Shutdown 3 (2026-28) to achieve the target precision ($\sim 0.3\%$ stat and similar syst).
- Alternative method to calculate a_{μ}^{HLO} with MUonE data:
less sensitive to the parameterization chosen to model $\Delta\alpha_{\text{had}}(t)$ in the MUonE kinematic range.
Comparable uncertainty to the space-like integral method.

New collaborators are welcome!



INFN +Univ. (Bologna,
Milano-Bicocca, Padova,
Pavia, Perugia, Pisa, Trieste)
Exp-Th



CERN
Exp-Th



Imperial College (London),
Liverpool U. *Exp-Th*
Durham U.



Krakow IFJ Pan
Exp



**The MUonE
Collaboration**



Cornell U.,
Northwestern U.,
Regis U.,
Virginia U.
Exp



Budker Inst.
(Novosibirsk)
Exp



Demokritos INPP
(Athens) *Exp-Th*



Shanghai
Jiao Tong U.
Exp



PSI (Villigen),
U.Zürich, ETH Zürich
Th

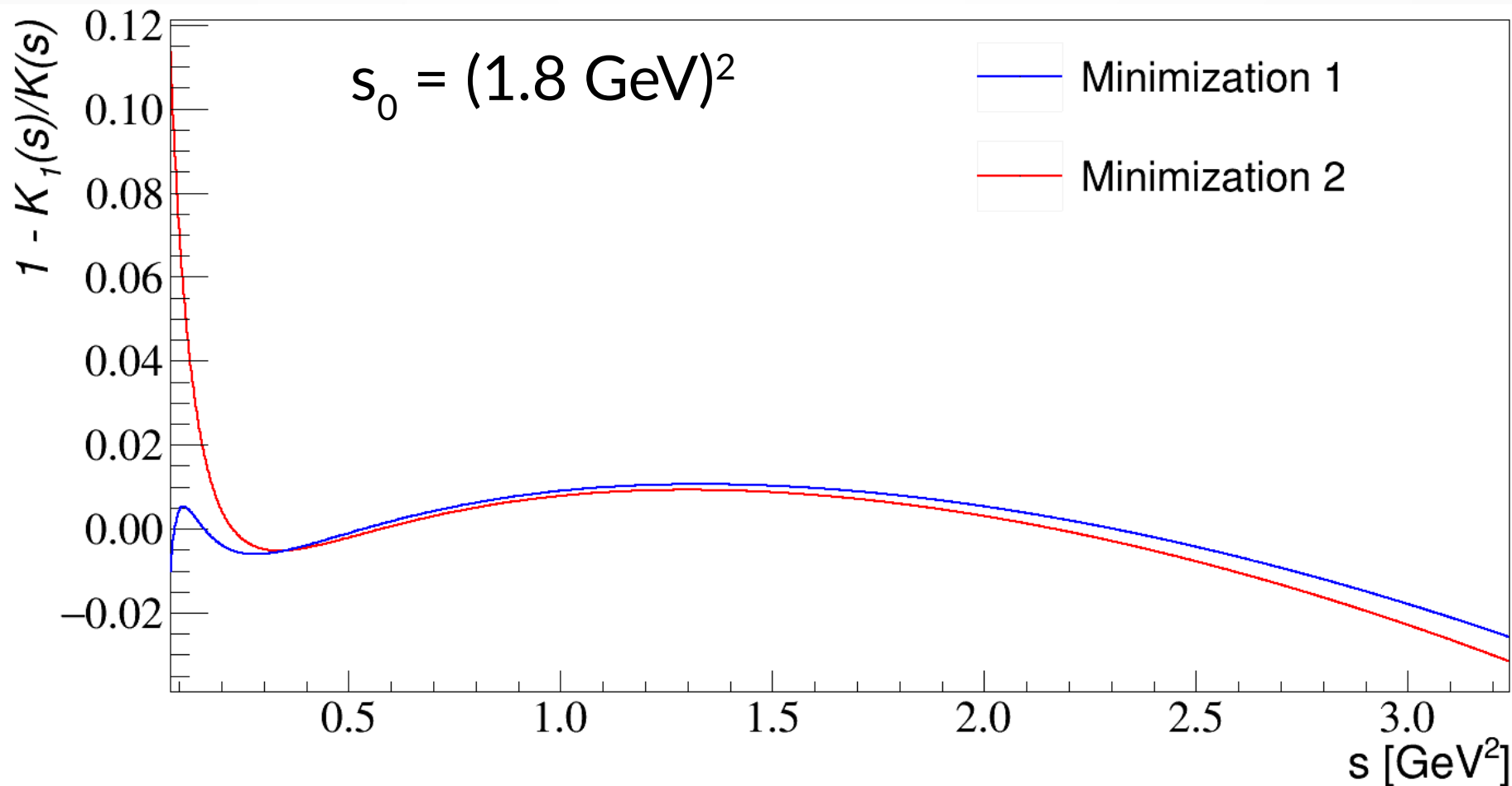


Mainz U.,
Max-Planck Inst.
Exp-Th

+ other involved theorists from: New York City Tech (USA), Vienna U. (A)

BACKUP

Difference $K_1(s) - K(s)$



Results: $a_\mu^{\text{HLO (I)}}$



$$a_\mu^{\text{HLO (I)}} = -\frac{\alpha}{\pi} \sum_{n=1}^3 \frac{c_n}{n!} \frac{d^{(n)}}{dt^n} \Delta\alpha_{had}(t) \Big|_{t=0}$$

Minimization I				$a_\mu^{\text{HLO (I)}} (10^{-10})$				
s_0 values	LL	Padé	Pol	GdR1	GdR2	GdR3	GdR4	GdR5
$(1.8 \text{ GeV})^2$	688.7 ± 2.2	688.7 ± 2.9	688.9 ± 2.9	688.2 ± 2.2	688.0 ± 2.2	688.0 ± 2.2	687.0 ± 2.3	688.0 ± 2.6
$(2.5 \text{ GeV})^2$	691.7 ± 2.2	691.6 ± 3.0	691.8 ± 3.0	691.0 ± 2.2	690.8 ± 2.2	690.8 ± 2.2	689.8 ± 2.3	690.9 ± 2.9
$(12 \text{ GeV})^2$	696.3 ± 2.2	696.3 ± 3.0	696.3 ± 3.2	695.4 ± 2.2	695.3 ± 2.2	695.2 ± 2.2	694.1 ± 2.3	695.3 ± 3.7
Minimization II				$a_\mu^{\text{HLO (I)}} (10^{-10})$				
s_0 values	LL	Padé	Pol	GdR1	GdR2	GdR3	GdR4	GdR5
$(1.8 \text{ GeV})^2$	688.5 ± 2.2	688.1 ± 4.2	689.8 ± 3.3	688.3 ± 2.1	688.4 ± 2.1	688.6 ± 2.2	687.1 ± 2.1	688.4 ± 5.8
$(2.5 \text{ GeV})^2$	689.5 ± 2.2	689.1 ± 4.2	690.8 ± 3.3	689.3 ± 2.1	689.4 ± 2.1	689.6 ± 2.2	688.1 ± 2.1	689.4 ± 5.7
$(12 \text{ GeV})^2$	690.3 ± 2.1	689.9 ± 4.6	691.6 ± 3.6	689.8 ± 2.1	690.1 ± 2.2	690.2 ± 2.2	688.6 ± 2.1	690.0 ± 5.9

$a_\mu^{\text{HLO (I)}} \sim 99\%$ of the total value.

($a_\mu^{\text{HLO}} = 695.1 \times 10^{-10}$ input from time-like data).

Results: a_μ^{HLO} (II, III, IV)



$$a_\mu^{\text{HLO (II)}} = \frac{\alpha}{\pi} \frac{1}{2\pi i} \oint_{|s|=s_0} \frac{ds}{s} c_0 s \Pi_{had}(s) \Big|_{\text{pQCD}} \quad a_\mu^{\text{HLO (III)}} = \frac{\alpha^2}{3\pi^2} \int_{s_{\text{th}}}^{s_0} \frac{ds}{s} [K(s) - K_1(s)] R(s)$$

$$a_\mu^{\text{HLO (IV)}} = \frac{\alpha^2}{3\pi^2} \int_{s_0}^{\infty} \frac{ds}{s} [K(s) - \tilde{K}_1(s)] R(s)$$

Minimization I			
s_0 values	$a_\mu^{\text{HLO (II)}} (10^{-10})$	$a_\mu^{\text{HLO (III)}} (10^{-10})$	$a_\mu^{\text{HLO (IV)}} (10^{-10})$
$(1.8 \text{ GeV})^2$	2.94 ± 0.04	0.43 ± 0.01	2.95 ± 0.05
$(2.5 \text{ GeV})^2$	1.84 ± 0.01	-0.34 ± 0.01	1.79 ± 0.02
$(12 \text{ GeV})^2$	0.208 ± 0.001	-1.695 ± 0.035	0.079 ± 0.001
Minimization II			
s_0 values	$a_\mu^{\text{HLO (II)}} (10^{-10})$	$a_\mu^{\text{HLO (III)}} (10^{-10})$	$a_\mu^{\text{HLO (IV)}} (10^{-10})$
$(1.8 \text{ GeV})^2$	3.23 ± 0.04	0.91 ± 0.02	3.00 ± 0.05
$(2.5 \text{ GeV})^2$	2.54 ± 0.01	1.52 ± 0.02	1.96 ± 0.02
$(12 \text{ GeV})^2$	0.360 ± 0.001	4.85 ± 0.05	0.096 ± 0.001

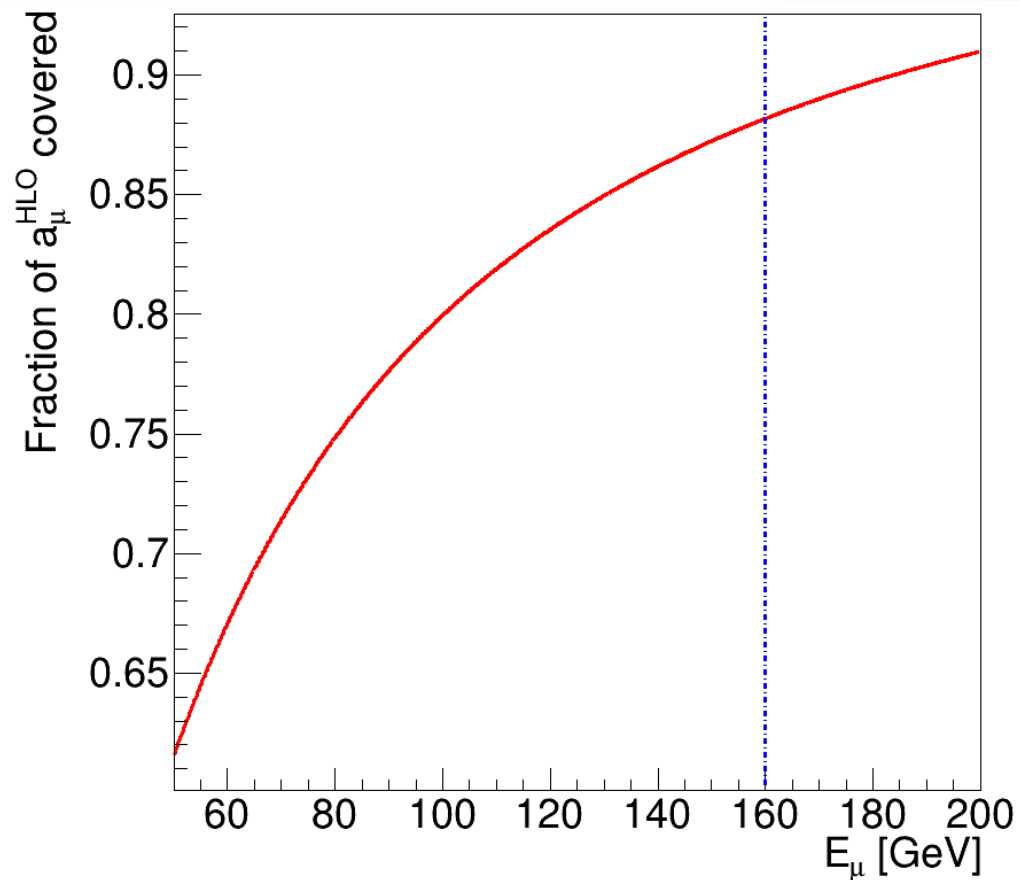
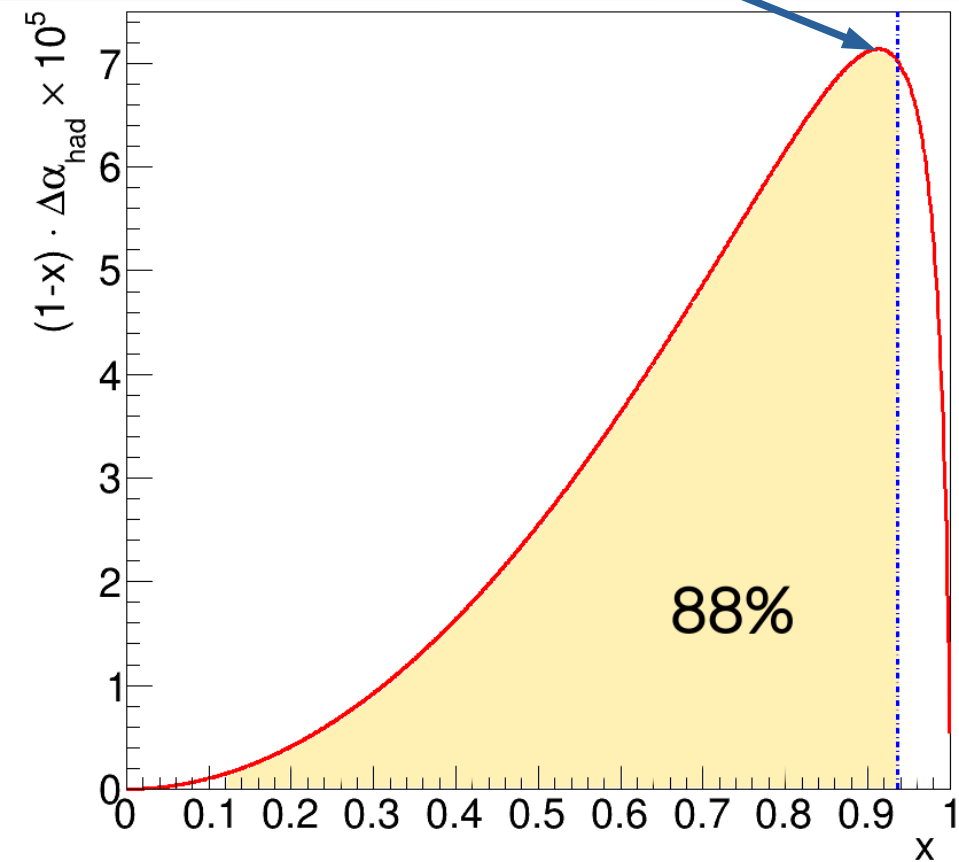
$a_\mu^{\text{HLO (II+III+IV)}} \sim 1\%$ of the total value.

$(a_\mu^{\text{HLO}} = 695.1 \times 10^{-10}$ input from time-like data).

$$x < 0.936$$

$$t_{peak} \sim -0.108 \text{ GeV}^2$$

$$x_{peak} \sim 0.92$$

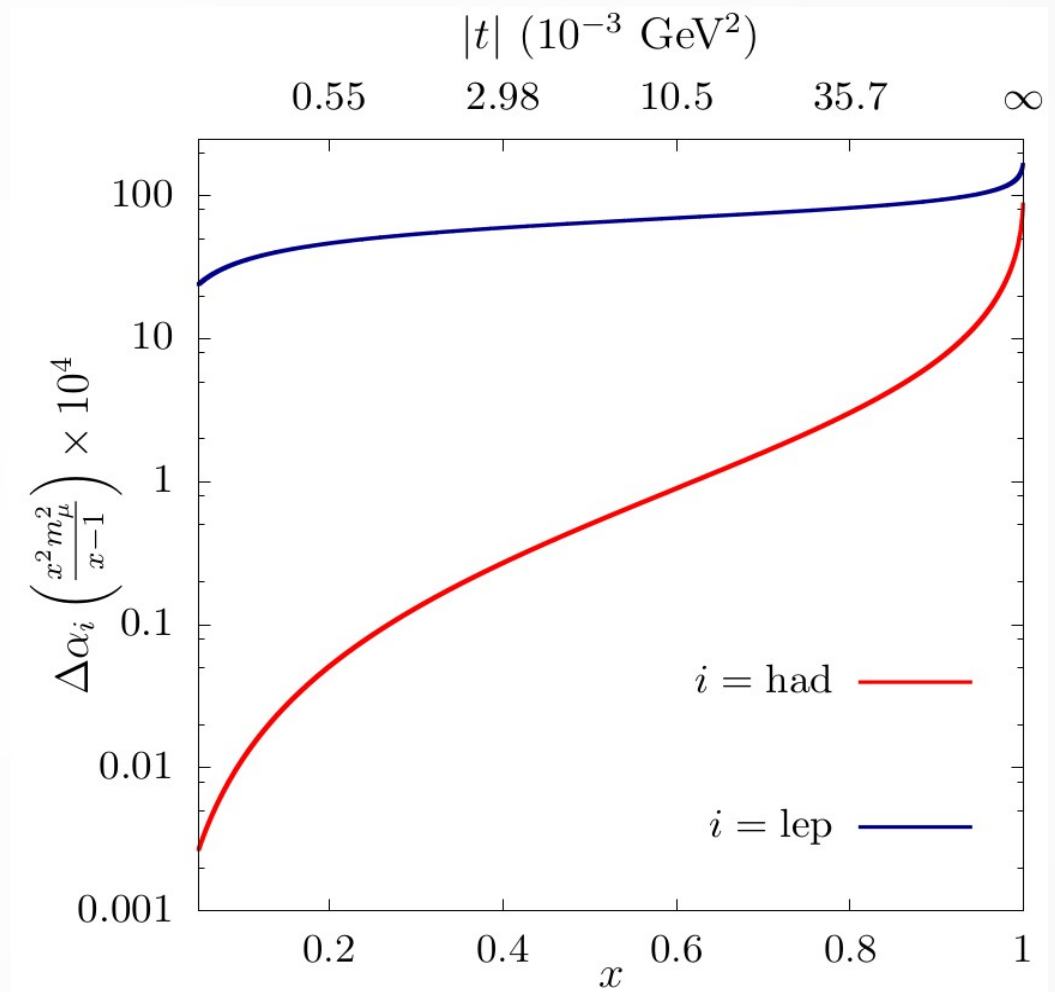


- 160 GeV muon beam on atomic electrons.

$$\sqrt{s} \sim 420 \text{ MeV}$$

$$-0.153 \text{ GeV}^2 < t < 0 \text{ GeV}^2$$

$$\Delta\alpha_{had}(t) \lesssim 10^{-3}$$

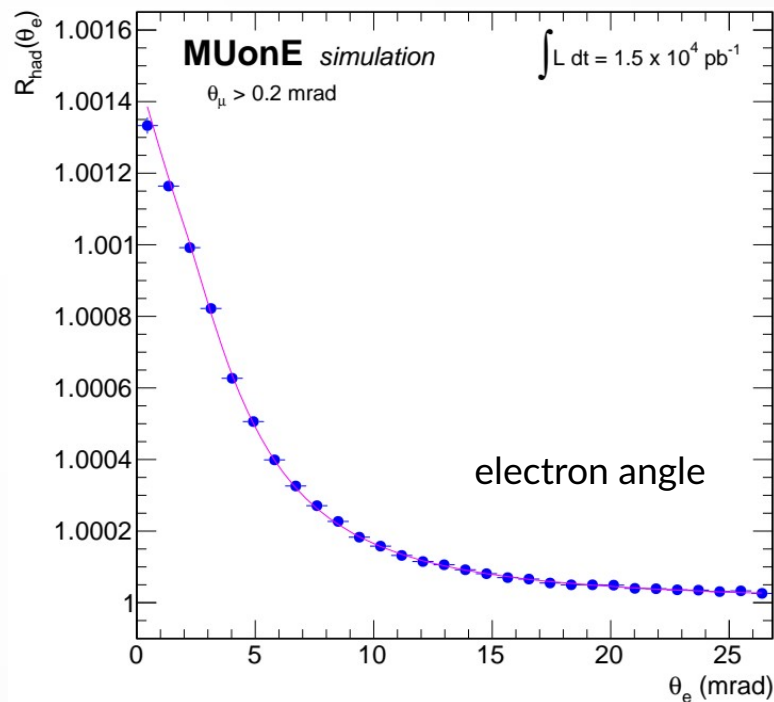
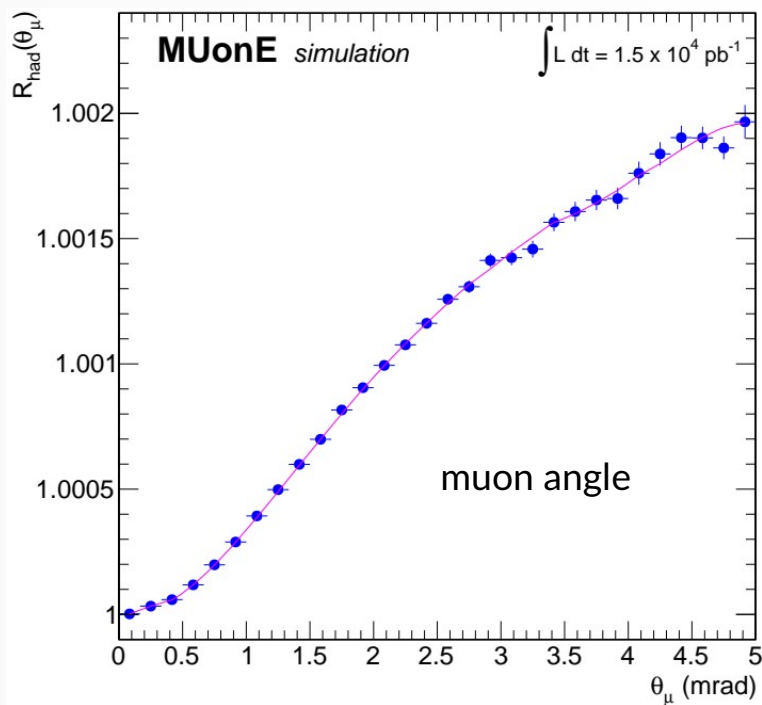


$$R_{\text{had}} = \frac{d\sigma_{\text{data}}(\Delta\alpha_{\text{had}})}{d\sigma_{\text{MC}}(\Delta\alpha_{\text{had}} = 0)} \sim 1 + \frac{2\Delta\alpha_{\text{had}}(t)}{\text{To be measured}}$$

From theoretical calculation

To be measured

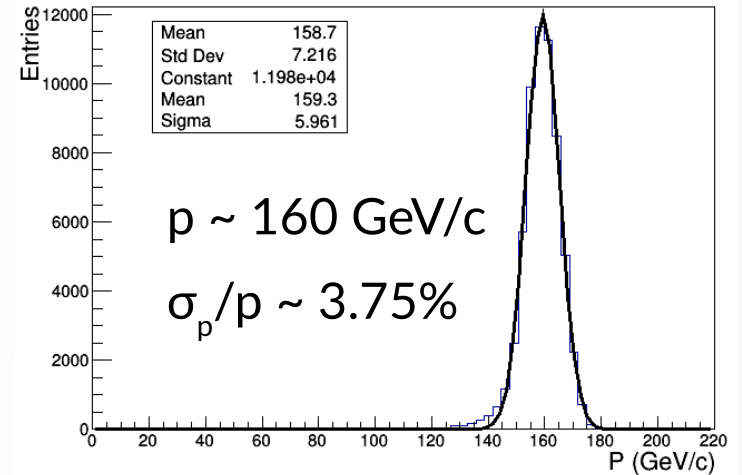
$$\Delta\alpha_{\text{had}}(t) < 10^{-3}$$



Location: M2 beamline at CERN

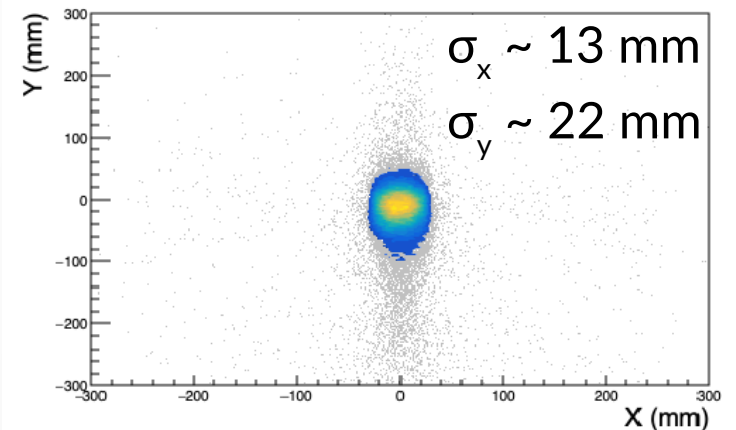


Beam momentum

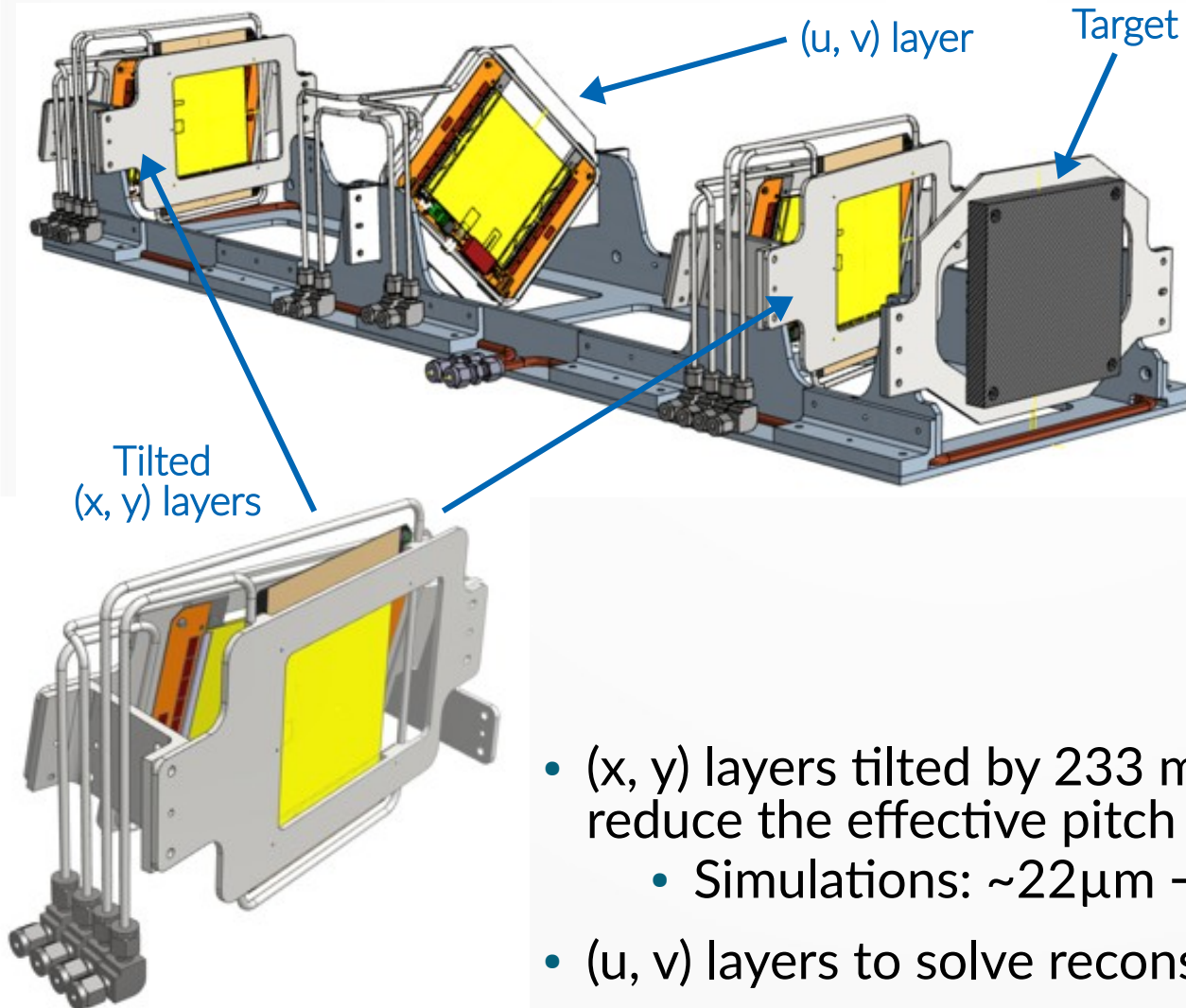


- Location: upstream the COMPASS detector (CERN North Area).
- Low divergence muon beam: $\sigma_{x'} \sim \sigma_{y'} \sim 0.2 \text{ mrad}$.
- Spill duration $\sim 5 \text{ s}$. Duty cycle $\sim 25\%$.
- Maximum rate: 50 MHz ($\sim 2\text{-}3 \times 10^8 \mu^+/\text{spill}$).

Beam spot



Tracking station



Stringent request:
relative position within a station
must be stable at $10\ \mu\text{m}$.



Low CTE material:
INVAR (CTE $\sim 1.2 \times 10^{-6}\ \text{K}^{-1}$)

Laser holographic system
to monitor stability.

- (x, y) layers tilted by 233 mrad:
reduce the effective pitch and improve spatial resolution.
 - Simulations: $\sim 22\ \mu\text{m} \rightarrow \sim 10\ \mu\text{m}$.
- (u, v) layers to solve reconstruction ambiguities.

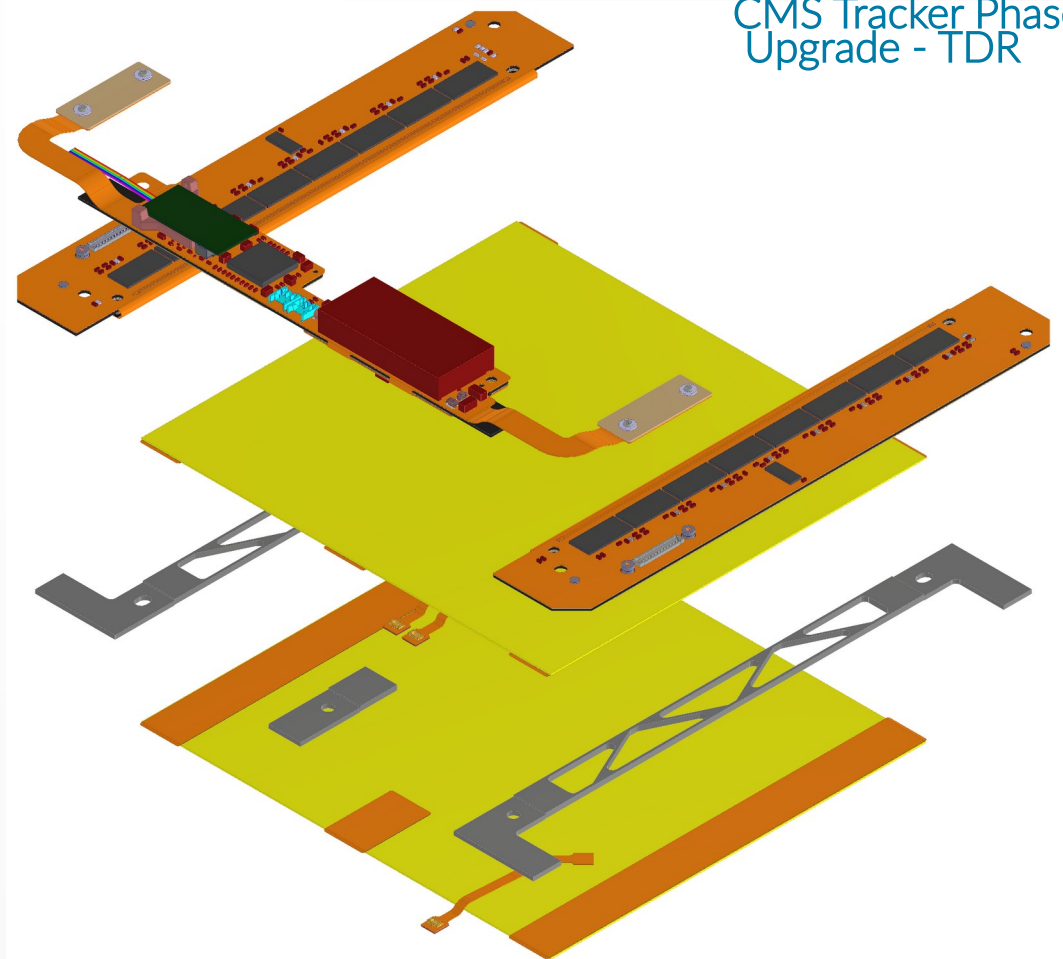
Tracker: CMS 2S modules



Silicon strip sensors currently in production for the CMS-Phase2 upgrade.

Two close-by strip sensors reading the same coordinate:

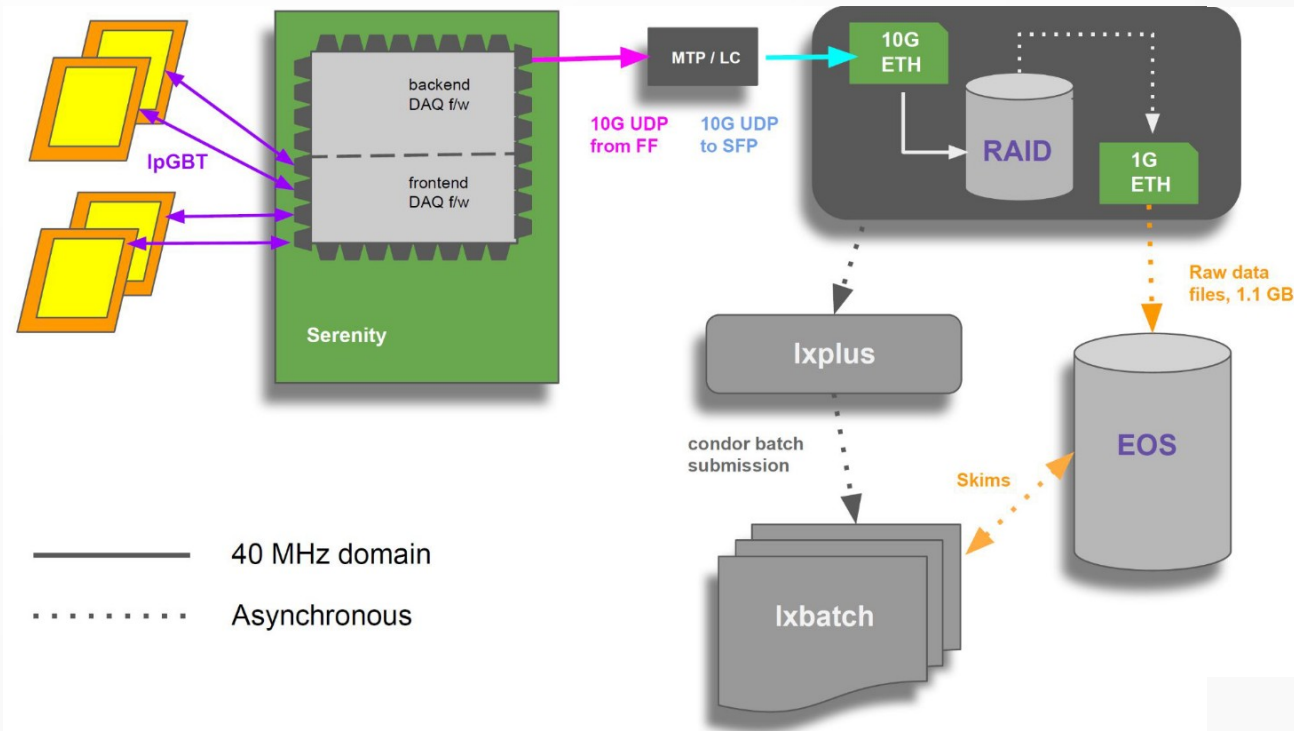
- Suppress background of single sensor hits.
- Reject large angle tracks.
- Pitch: $90\ \mu\text{m}$
- Digital readout
- Readout rate: 40 MHz
- Area: $10 \times 10\ \text{cm}^2$ ($\sim 90\ \text{cm}^2$ active)
- Thickness: $2 \times 320\ \mu\text{m}$



DAQ system

Frontend control and readout via **Serenity** board
(to be used in the CMS-Phase2 upgrade).

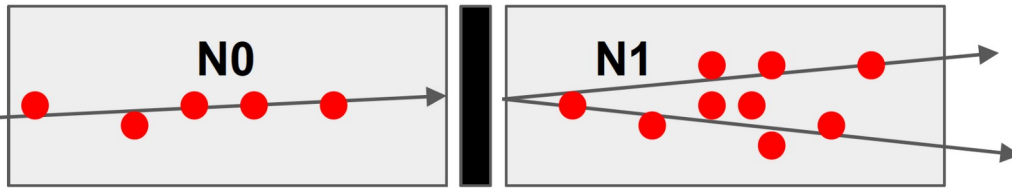
- **Asynchronous beam:** triggerless readout of the 2S modules @40MHz.
- Event aggregator on FPGA.
- Further data aggregation on the PC.
- Transmission to EOS into ~1GB files.



Test Run Analysis



Online event selection



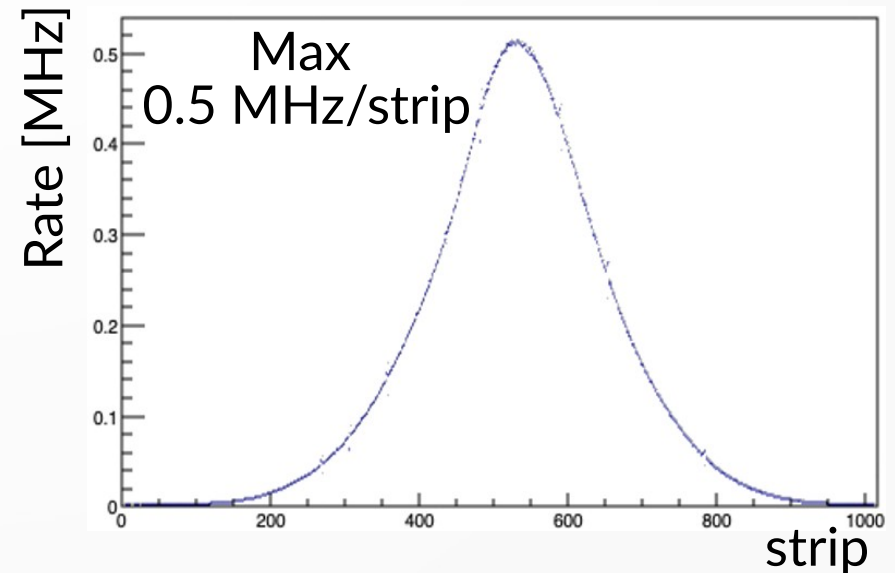
Select potential elastic events by looking at the number of hits in two consecutive stations:

- $N_{\text{hits}}^0 \geq 5 \ \&\&$
- $N_{\text{hits}}^1 \geq 5 \ \&\&$
- $N_{\text{hits}}^1 - N_{\text{hits}}^0 \geq 3-5$

Reduce the data flow to 1%-2%
Can be easily implemented on FPGA.

Beam rate

$1-2 \times 10^8 \ \mu/\text{spill}$
(1 spill = 5s)



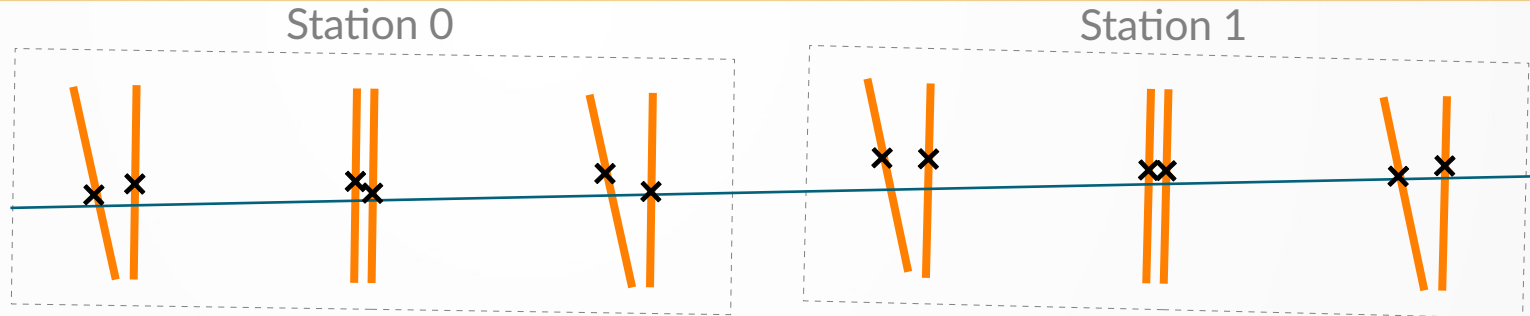
Goal:
count the total number of muons per run (input for expected luminosity)

Test Run Analysis



Alignment

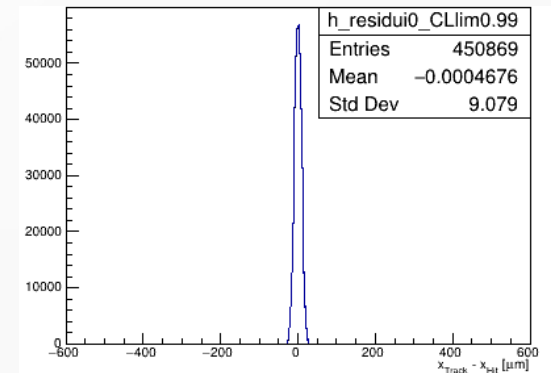
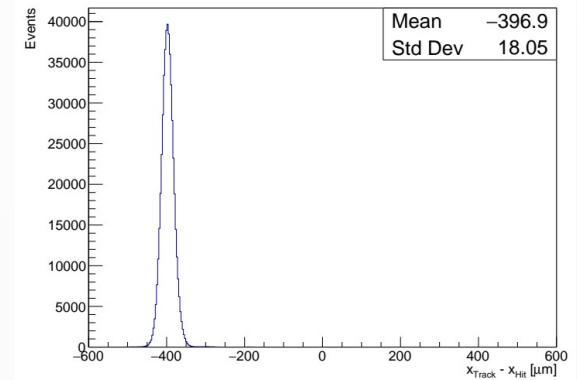
Currently:



- Track based iterative procedure:
2 alignment parameters per module
(offset in the measured direction
and rotation angle around the beam axis).
- Align the coordinate orthogonal to the
measurement direction by measuring
the image of module's middle line.

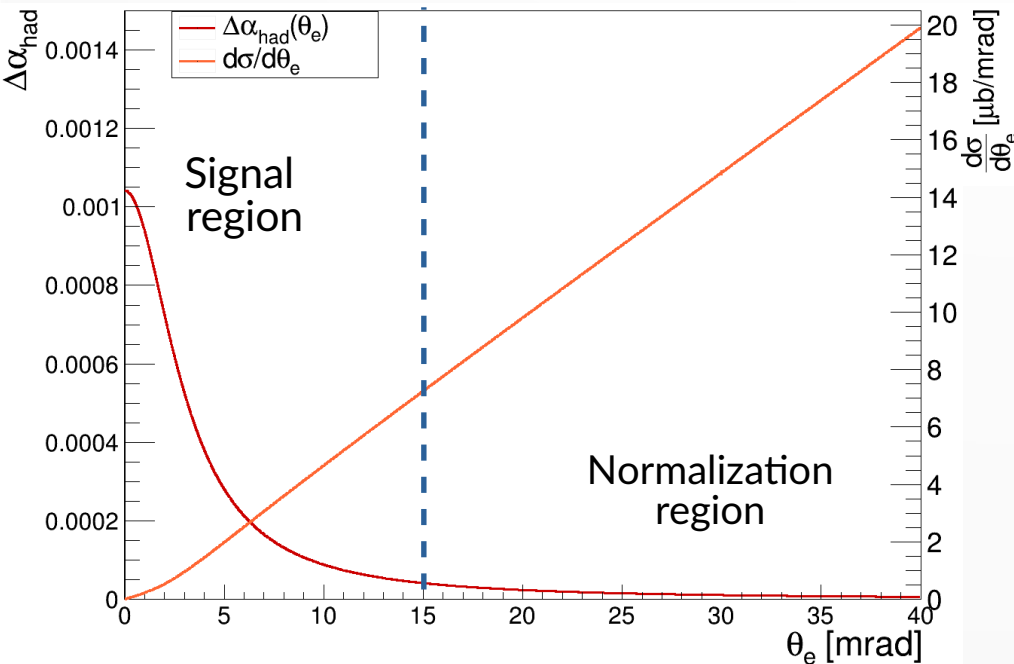
Ongoing work:

- Include the hardware metrology measurements
as starting point of the track based alignment.
- Global alignment.



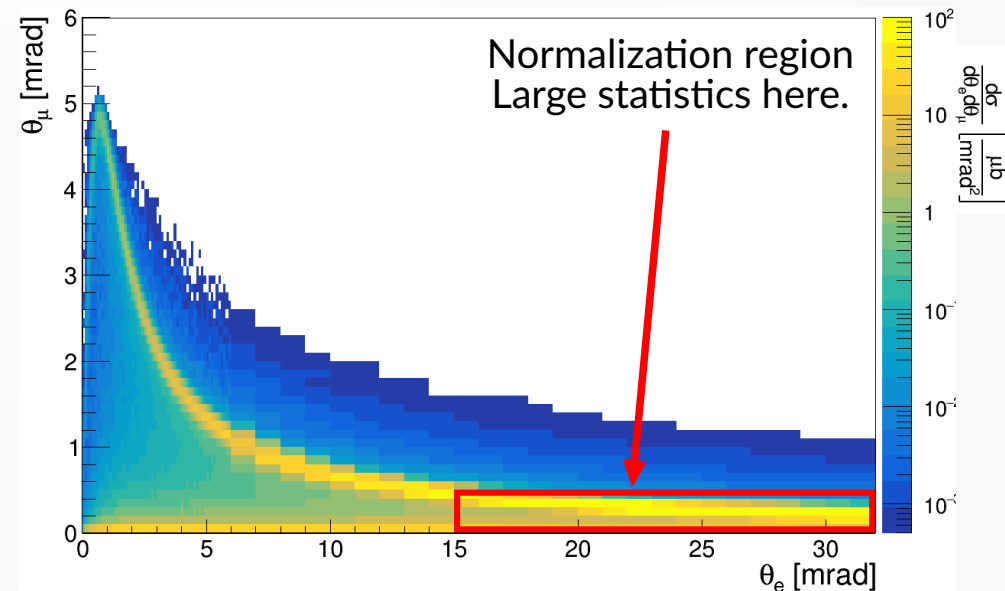
Strategy for the systematic effects

Main systematics have large effects in the normalization region.
(no sensitivity to $\Delta\alpha_{\text{had}}$ here)



Promising strategy:

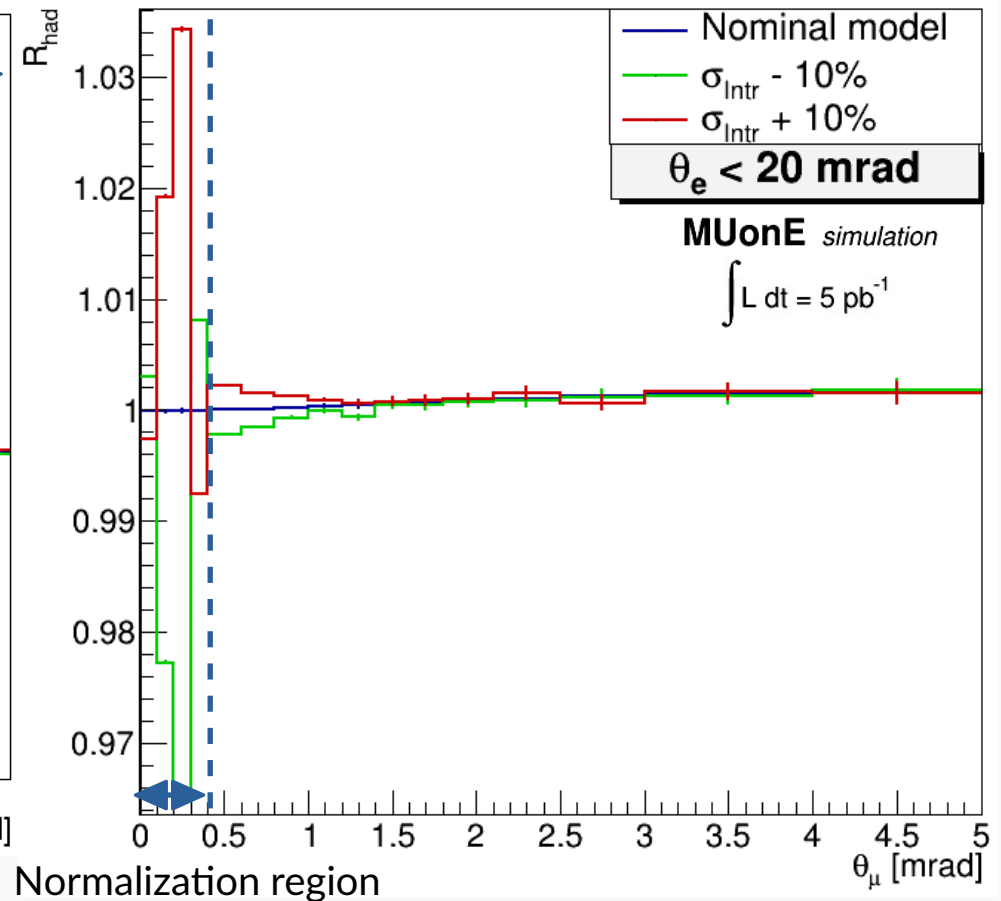
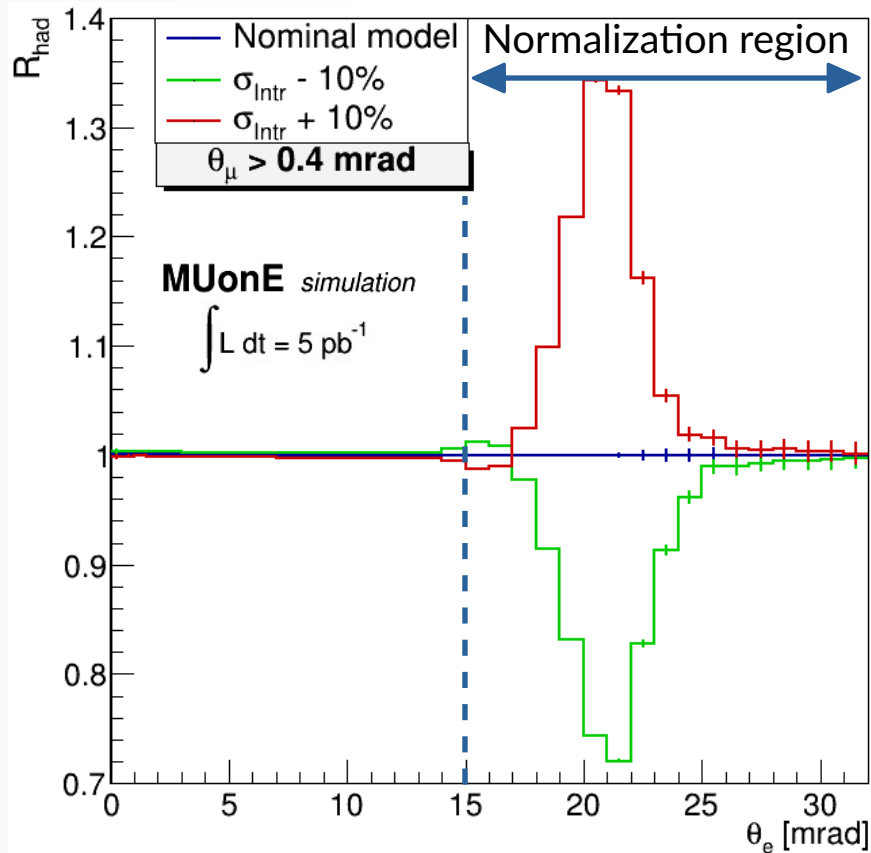
- Study the main systematics in the normalization region.
- Include residual systematics as nuisance parameters in a combined fit with signal.

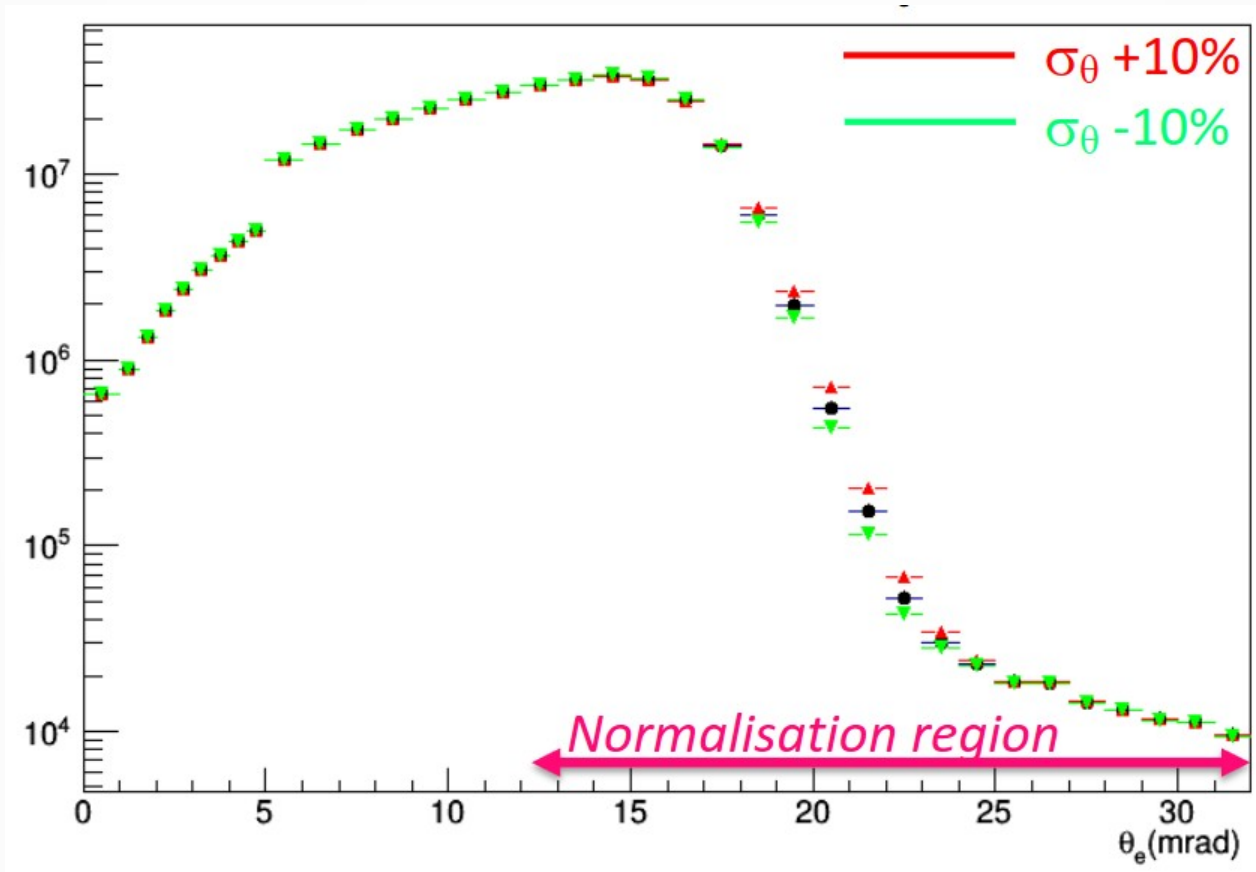


Systematic error on the angular intrinsic resolution



$\pm 10\%$ error on the angular intrinsic resolution.





The need of including systematic effects in the analysis



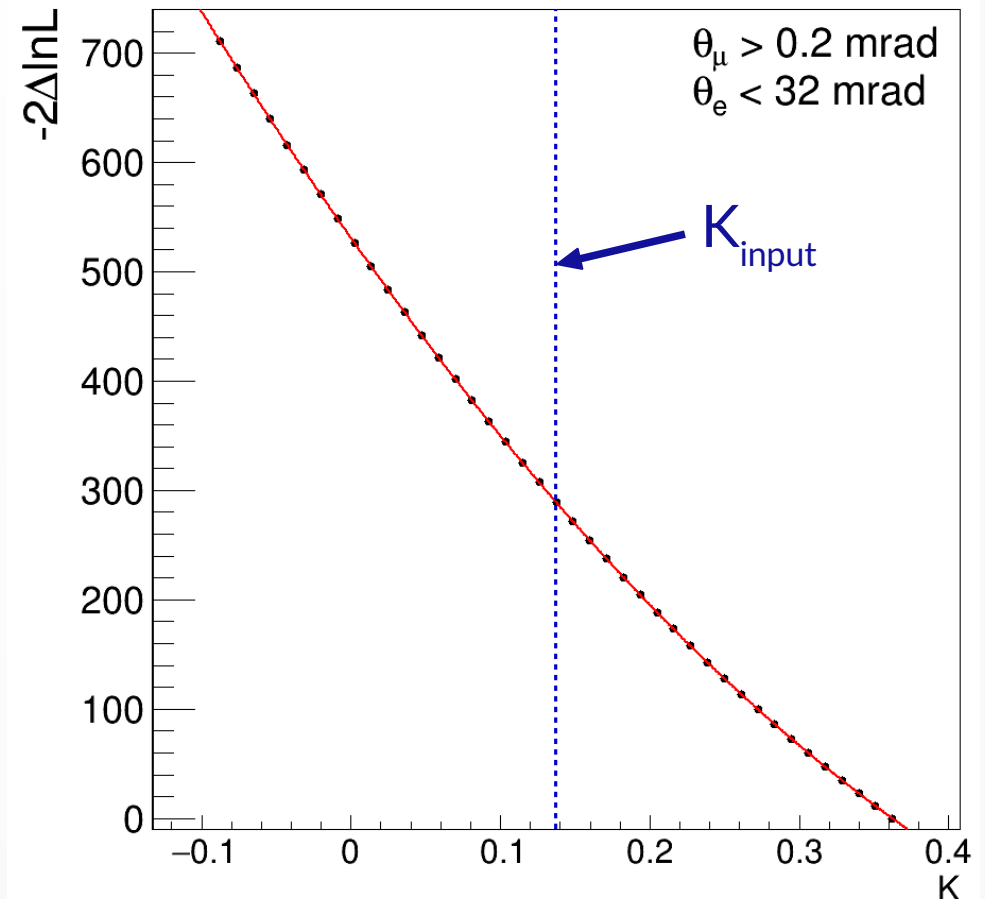
What if systematic effects are not included in the template fit?

Simplified situation:

- 1 fit parameter (K).

$$\Delta\alpha_{had}(t) \simeq -\frac{1}{15}Kt$$

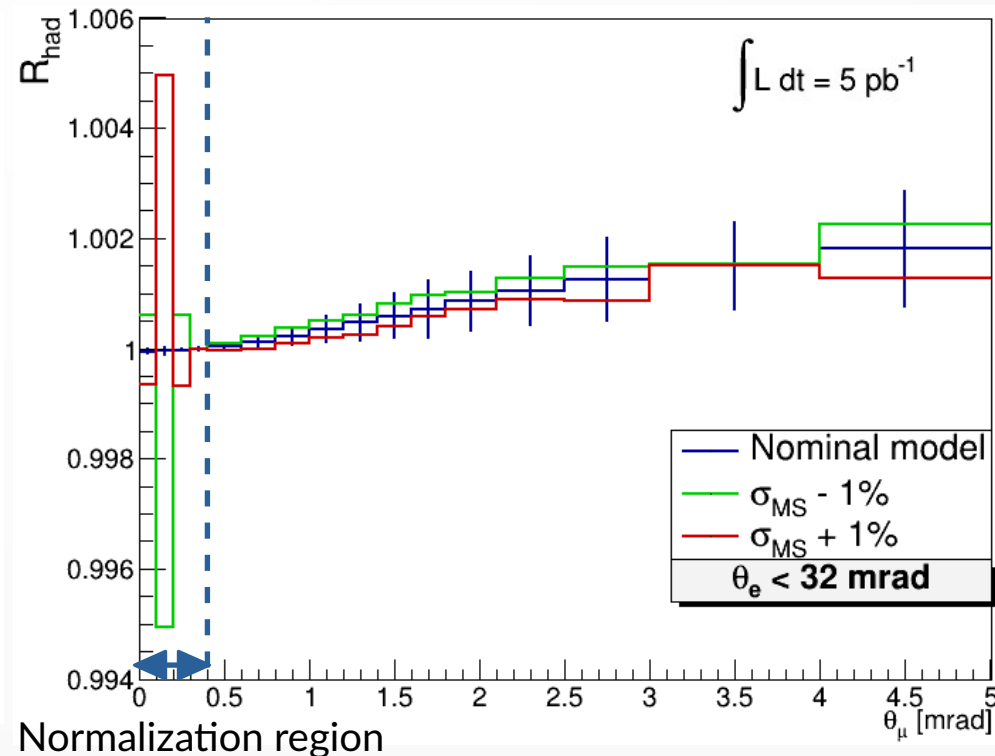
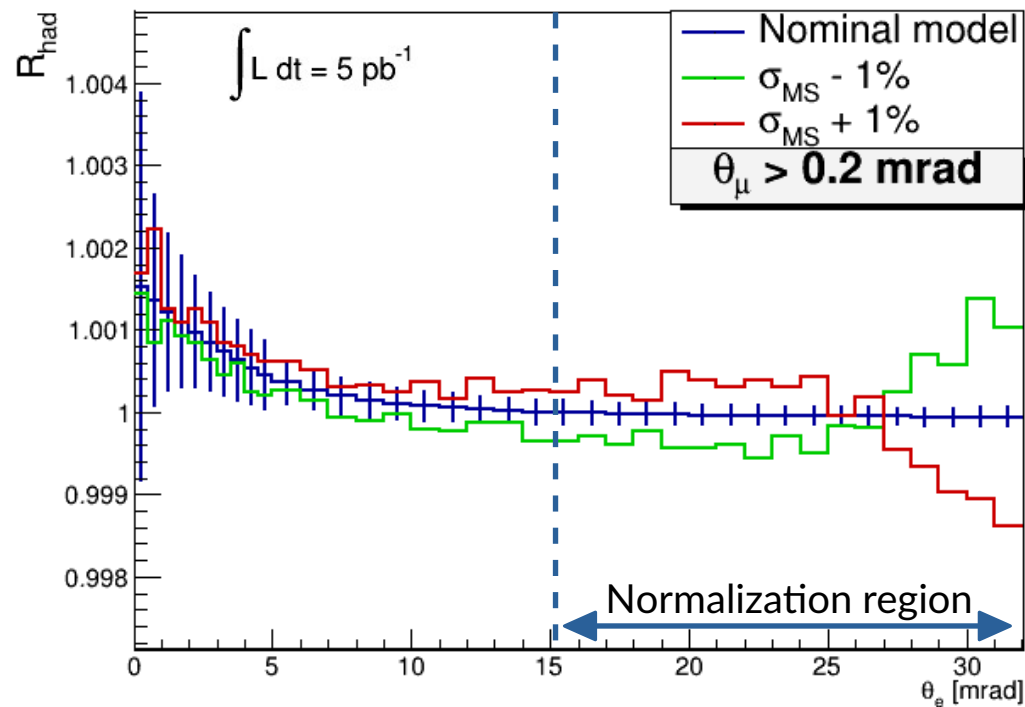
- $L = 5 \text{ pb}^{-1}$.
 $\sim 10^9$ elastic events
(~ 4000 times less than the final statistics)
- Shift in the pseudo-data sample:
 $\sigma_{\text{Intr}} \rightarrow \sigma_{\text{Intr}} + 5\%$.



Systematic error on the multiple scattering

Expected precision on the multiple scattering model: $\pm 1\%$

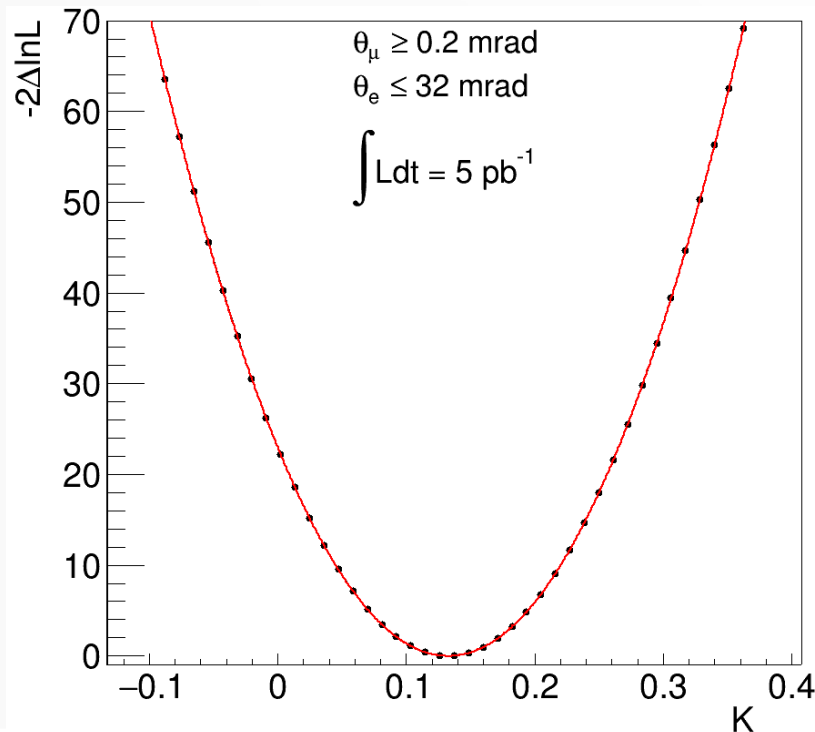
G. Abbiendi et al JINST (2020) 15 P01017



Combined fit signal + systematics



- Include residual systematics as nuisance parameters in the fit.
- Simultaneous likelihood fit to K and systematics using the Combine tool.



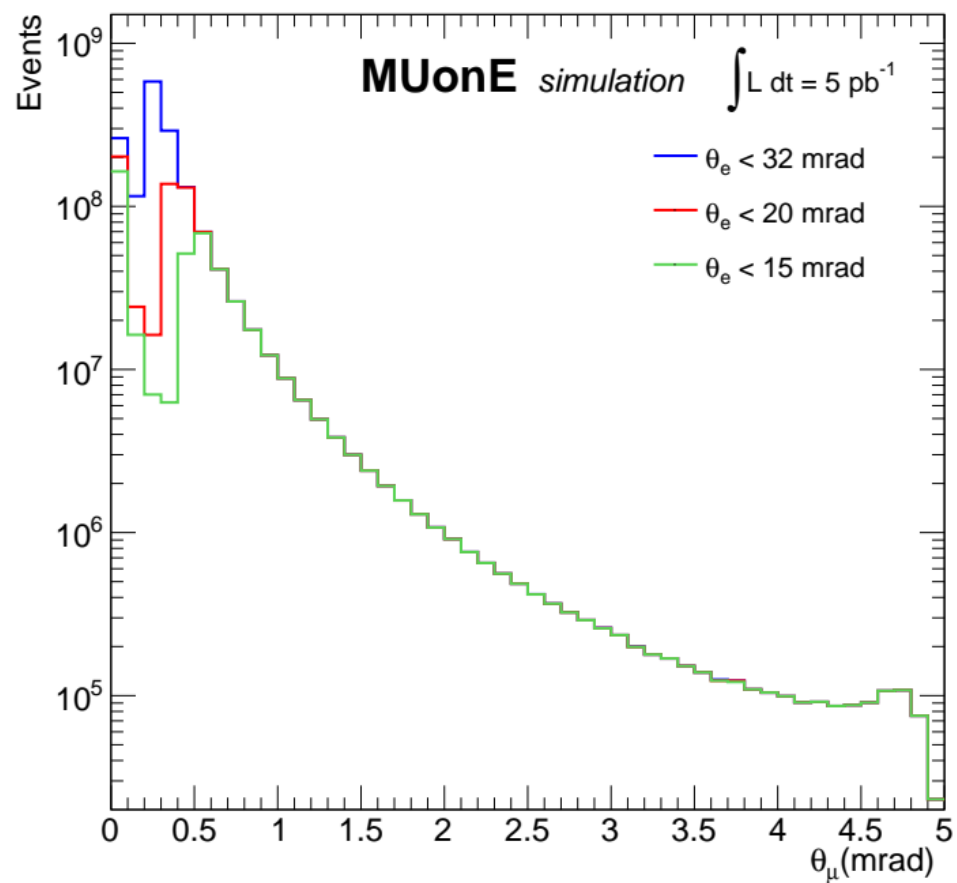
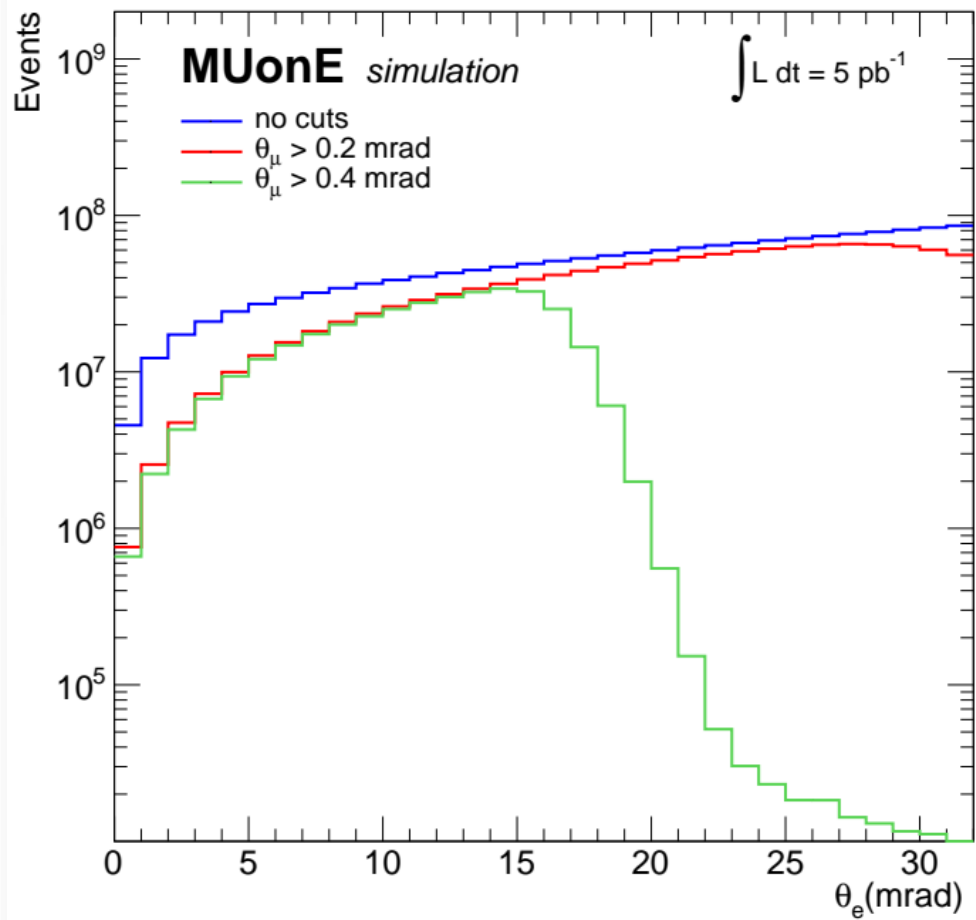
- $K_{\text{ref}} = 0.137$
- shift MS: +0.5%
- shift intr. res: +5%
- shift E_{beam} : +6 MeV

Selection cuts	Fit results
	$K = 0.133 \pm 0.028$
$\theta_e \leq 32 \text{ mrad}$	$\mu_{\text{MS}} = (0.47 \pm 0.03)\%$
$\theta_\mu \geq 0.2 \text{ mrad}$	$\mu_{\text{Intr}} = (5.02 \pm 0.02)\%$
	$\mu_{E_{\text{Beam}}} = (6.5 \pm 0.5) \text{ MeV}$
	$\nu = -0.001 \pm 0.003$

Similar results also for different selection cuts.

Next steps:

- Test the procedure for the MuonE design statistics.
- Improve the modelization of systematic effects.



GEANT4 simulations



TB2017 (resolution $\sim 7\mu\text{m}$)

TB2018 (resolution $\sim 40\mu\text{m}$)

Tracker only

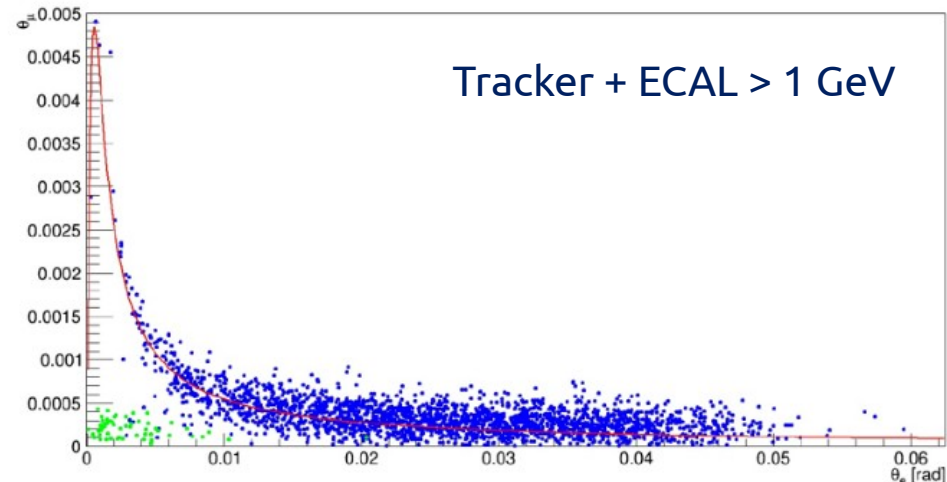
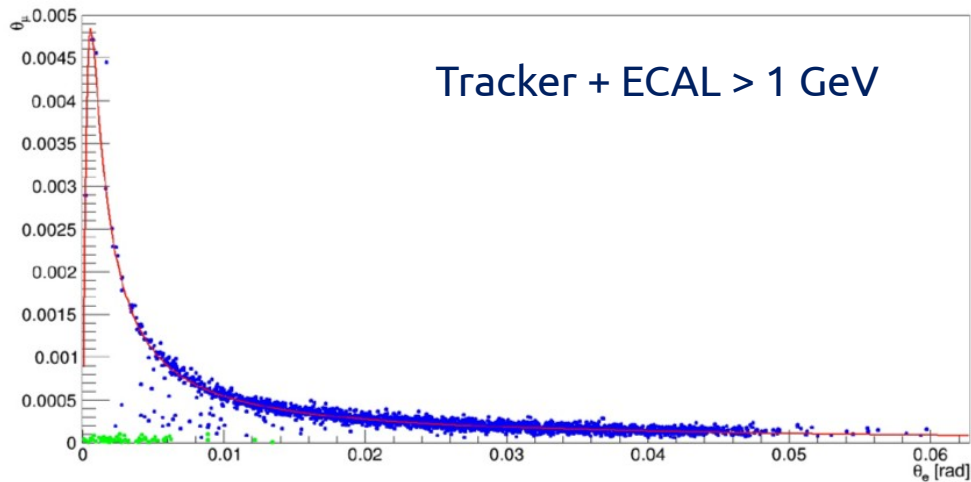
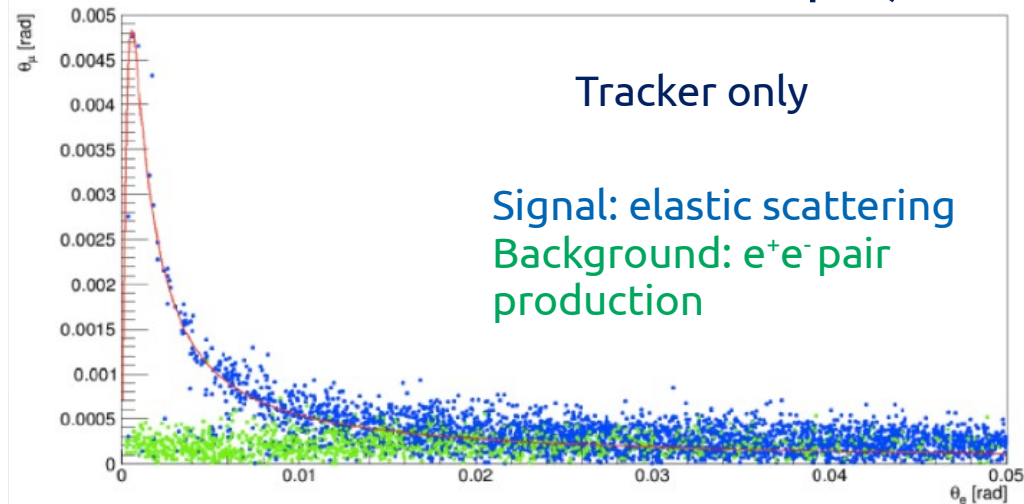
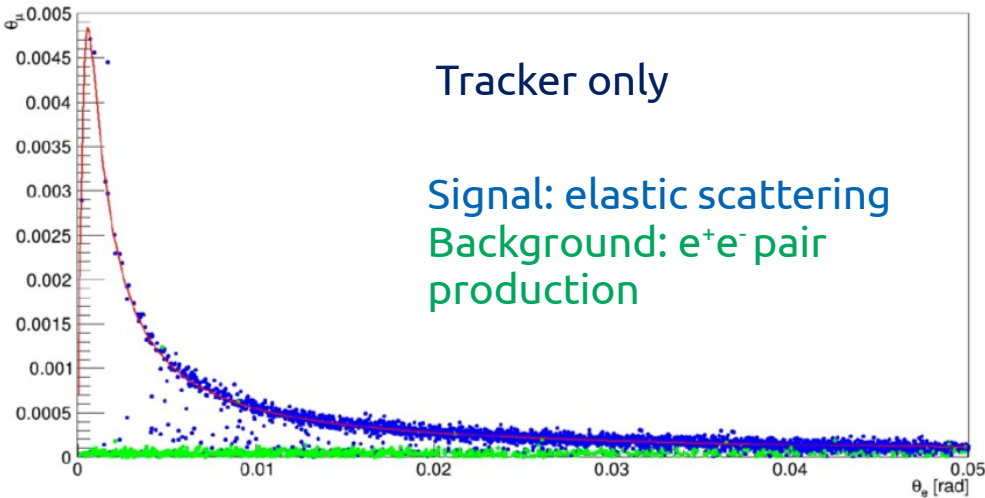
Signal: elastic scattering
Background: e^+e^- pair
production

Tracker only

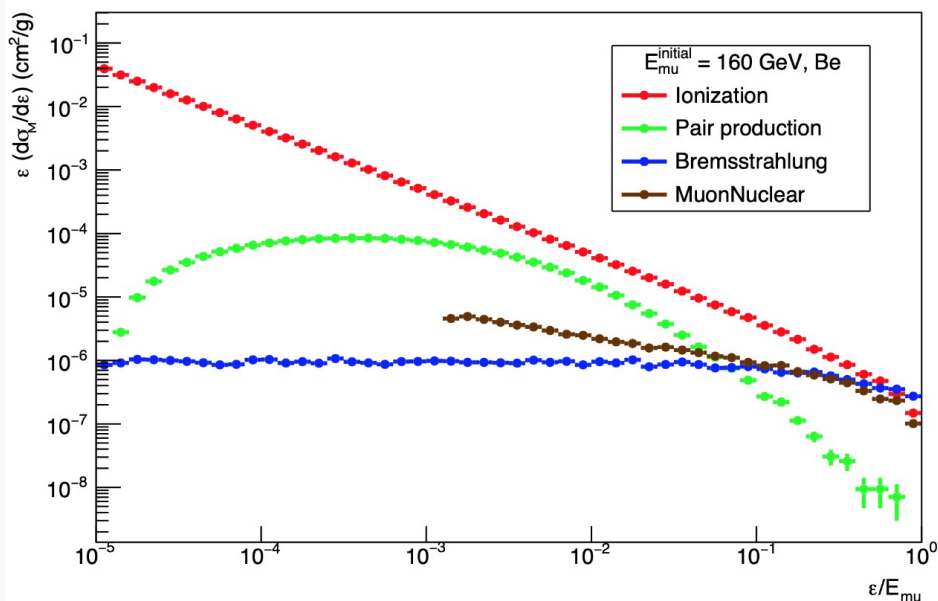
Signal: elastic scattering
Background: e^+e^- pair
production

Tracker + ECAL > 1 GeV

Tracker + ECAL > 1 GeV

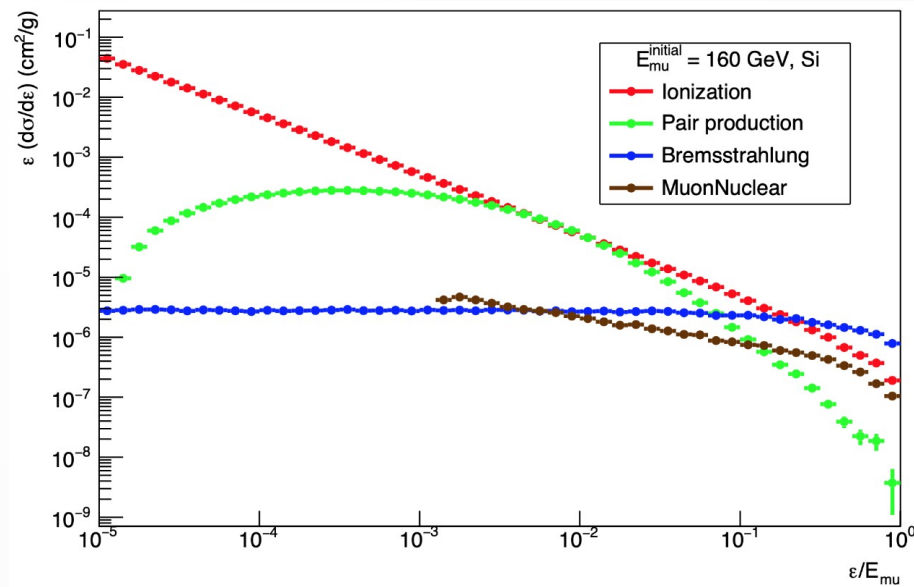


Backgrounds



MESMER

- $\mu e^- \rightarrow \mu e^- \gamma$
- $\mu e^- \rightarrow \mu e^- e^+ e^-$
- $\mu N \rightarrow \mu N e^+ e^-$



GEANT4

- $\mu N \rightarrow \mu N \gamma$
- $\mu N \rightarrow \mu X$