

# What if cLFV was only manifest in tau decays?

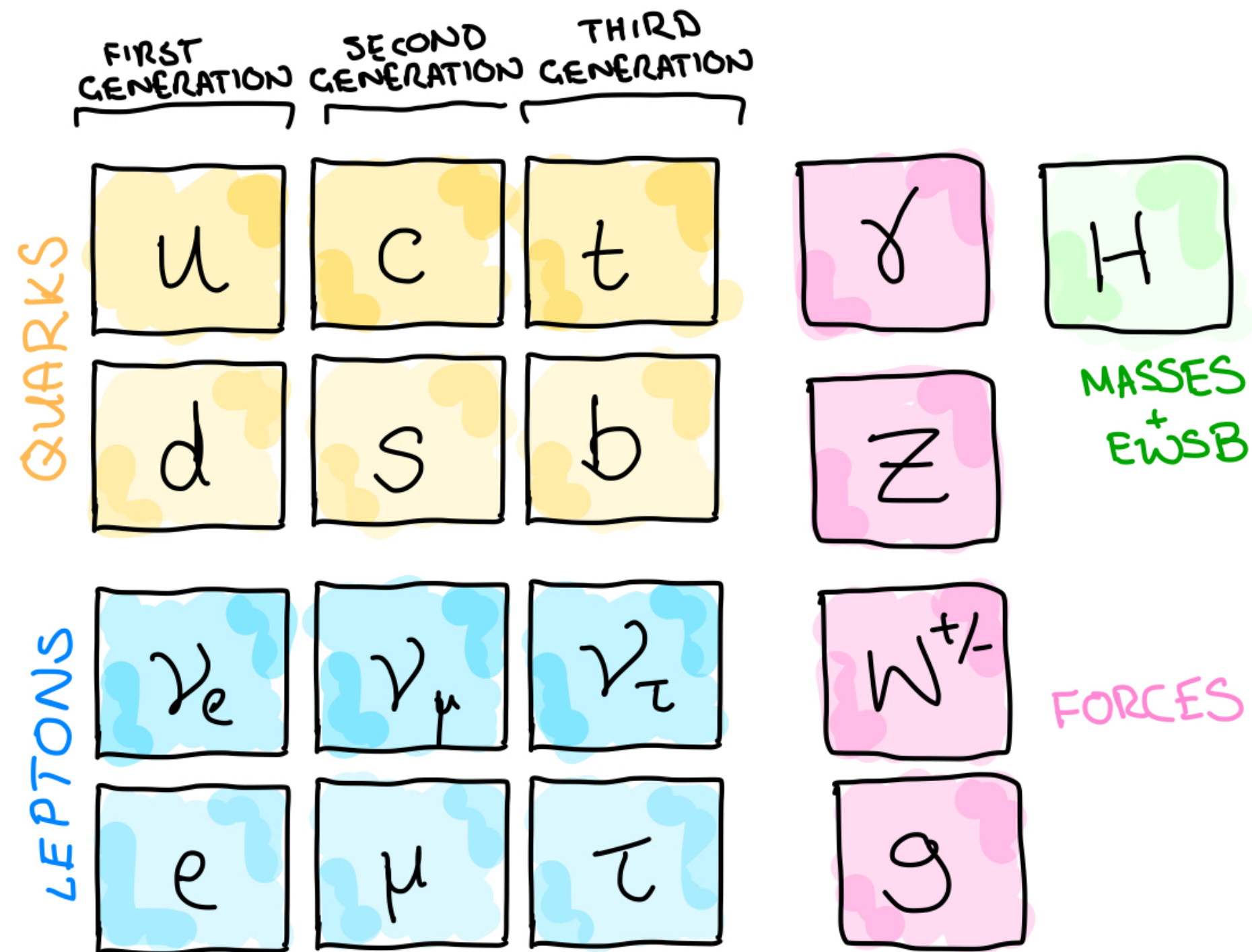
*Dr. Innes Bigaran*

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Based on [IB](#), XG He, M.A. Schmidt, G. Valencia, R. Volkas  
*Phys.Rev.D* 107 (2023) 5, 055001  
arXiv: 2212.09760



# Flavour in the Standard Model



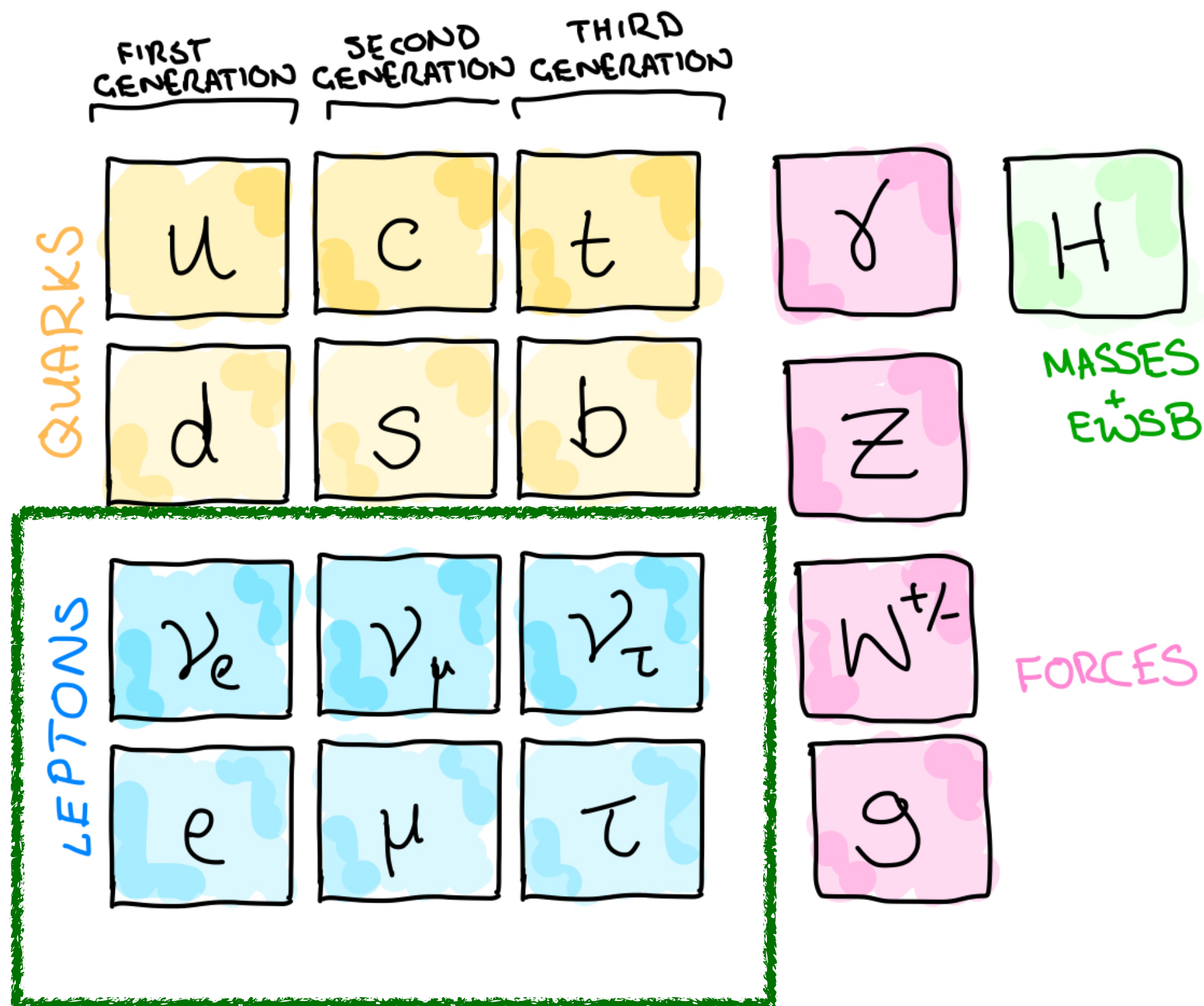
- The SM is a semi-empirical theory. Requires experimental input to fix  $\sim 27$  free parameters to fully prescribe it

Gauge	Force interactions	3 gauge couplings
Higgs	EWSB and W/Z masses	2 Higgs-potential couplings
Flavour	Quark and lepton masses and	$\sim 22$ free parameters

- We need experimentalists to measure these parameters. Can we understand the underlying symmetries that guide them?

“Standard Model Flavour Puzzle”

# Lepton flavour in the Standard Model



- In the SM lepton sector [with no neutrino masses], there is an accidental symmetry “lepton flavour”

$$\mathcal{G}_L = U(1)_e \otimes U(1)_\mu \otimes U(1)_\tau$$

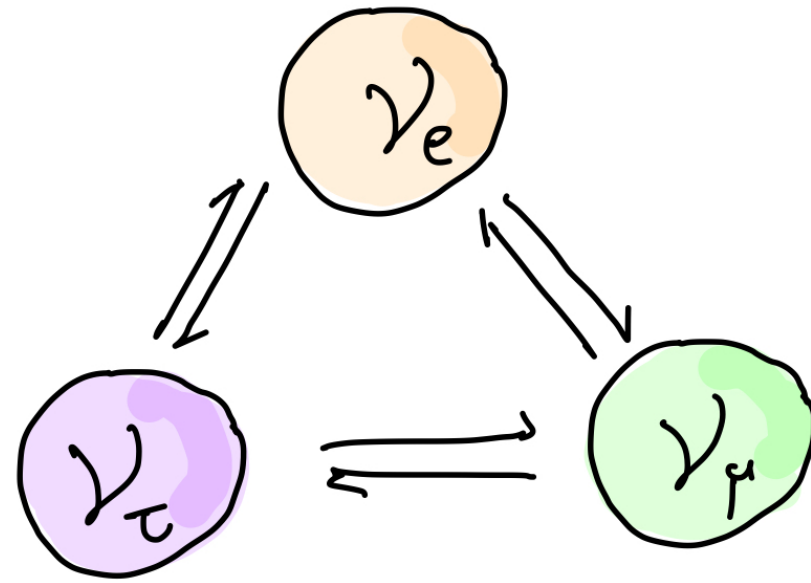
- Flavoured lepton number is conserved in [perturbative] SM interactions, thus also total (sum of flavours) lepton number

$$L = L_\mu + L_e + L_\tau$$

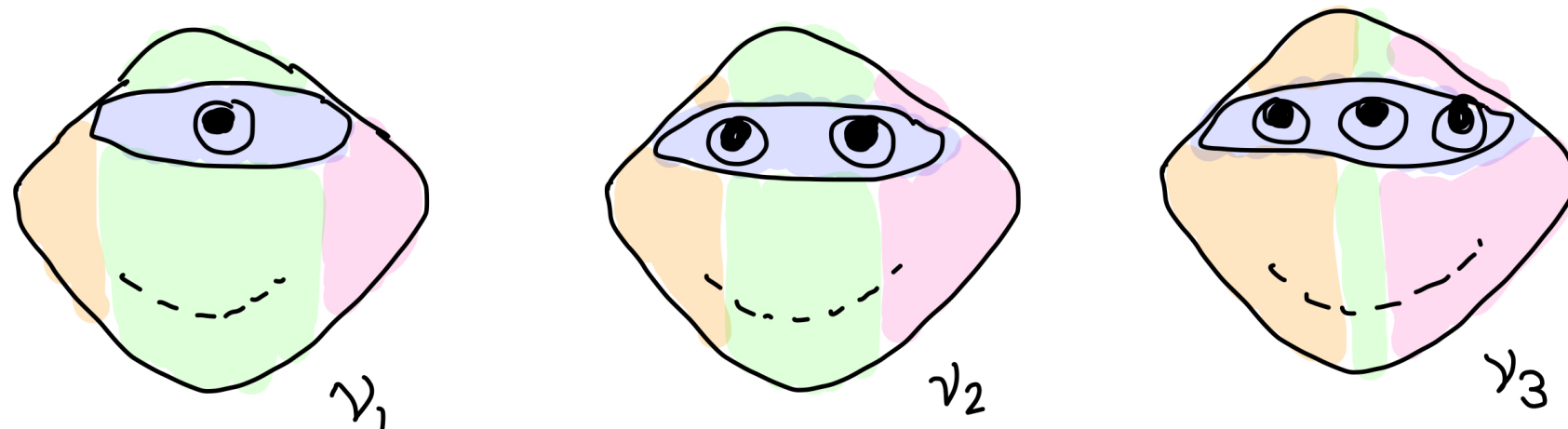
- **Lepton flavour violation (LFV)** is not possible in the SM.



# Leptons in the SM + neutrino masses



Mass eigenstates are linear combinations of flavour eigenstates



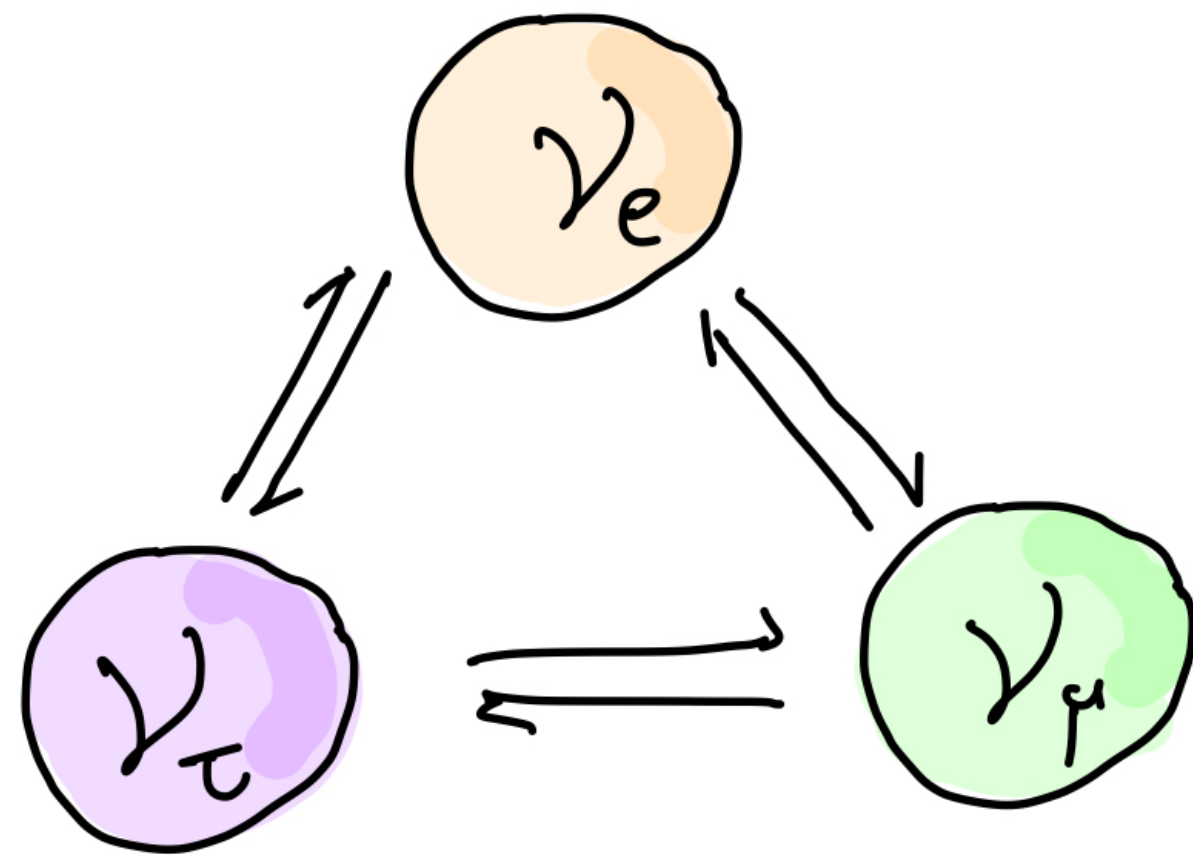
- In the *vanilla* SM, neutrinos are massless.
- Neutrino and neutrino physics provide a probe of lepton flavour symmetries
- Neutrino **flavour oscillations** tell us that beyond the SM lepton flavour *is* violated.

*Neutrino masses/oscillation mean*

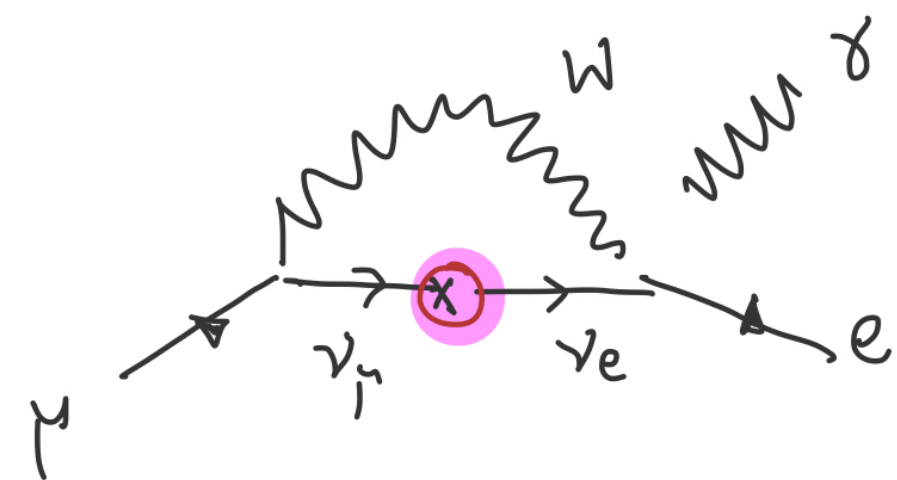
1. *We need BSM physics (an extended theory)*
2. *This new physics violates lepton flavour symmetry*



# So why is a symmetry useful if it's broken?



$$\mathcal{G}_L = U(1)_e \otimes U(1)_\mu \otimes U(1)_\tau$$



$$< O(1e-54)$$

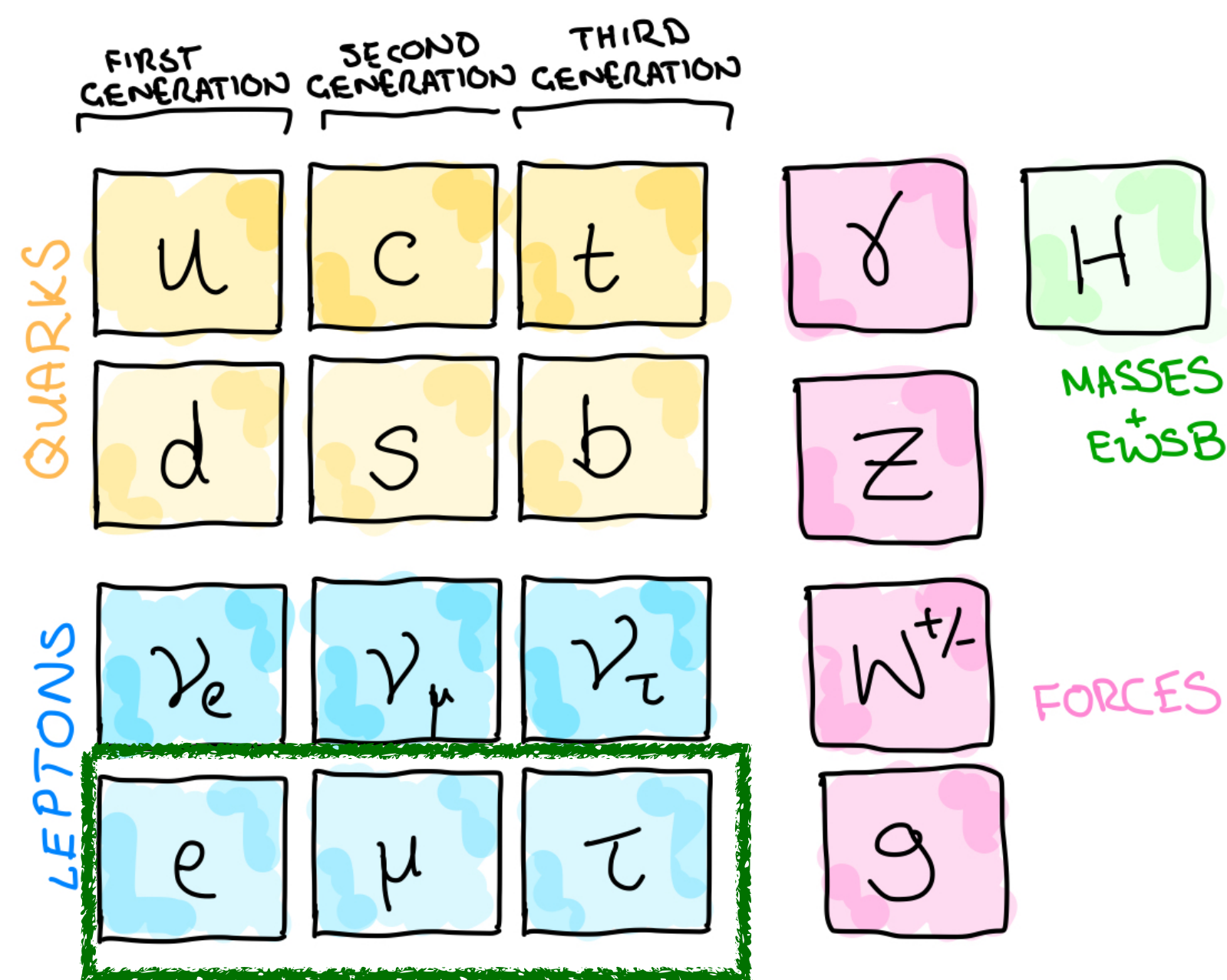
*Petcov, 1977*

- Thus far, our only signal of lepton flavour violation is generated via neutrino masses
- These neutrino masses are *small*.
- It is natural for these effects to be small if they are signals that result from an explicitly broken symmetry of the flavour sector
- We can say the symmetry “**protects**” the size of the observable effects of its breaking
- It remains to be seen if lepton flavour is *only* observably broken in the neutrino sector, though...cLFV?

# Charged LFV (cLFV)

$$\mathcal{G}_L = U(1)_e \otimes U(1)_\mu \otimes U(1)_\tau$$

How is it broken?

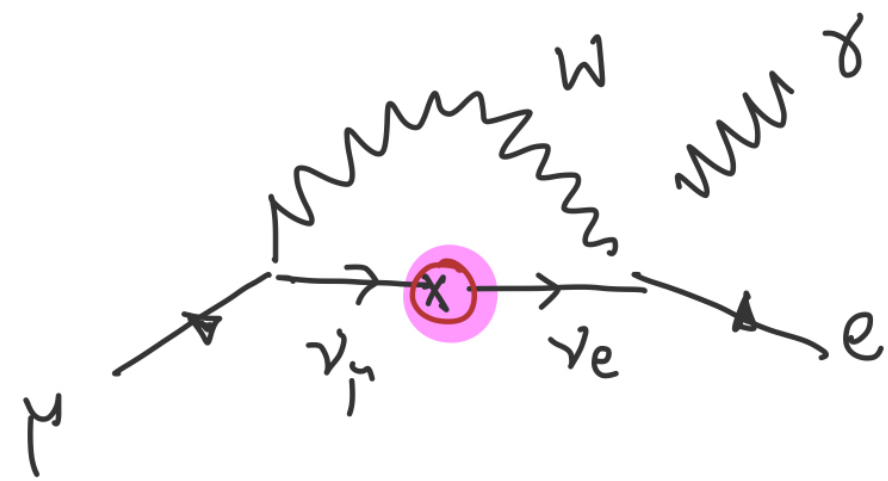


Why am I telling you about cLFV if I am also saying it's all totally self consistent for LFV effects to be small?

- We still haven't explained how neutrinos get their mass. Some new physics needs to generate neutrino masses. We don't know exactly how SM+ that will behave under this narrative.
- Maybe symmetry breaking in the lepton and neutrino sectors is different? Depending how flavour symmetry breaking manifests, e.g. we have right-handed charged leptons.
- In the context of the whole SM flavour puzzle, there is more to unpack here...



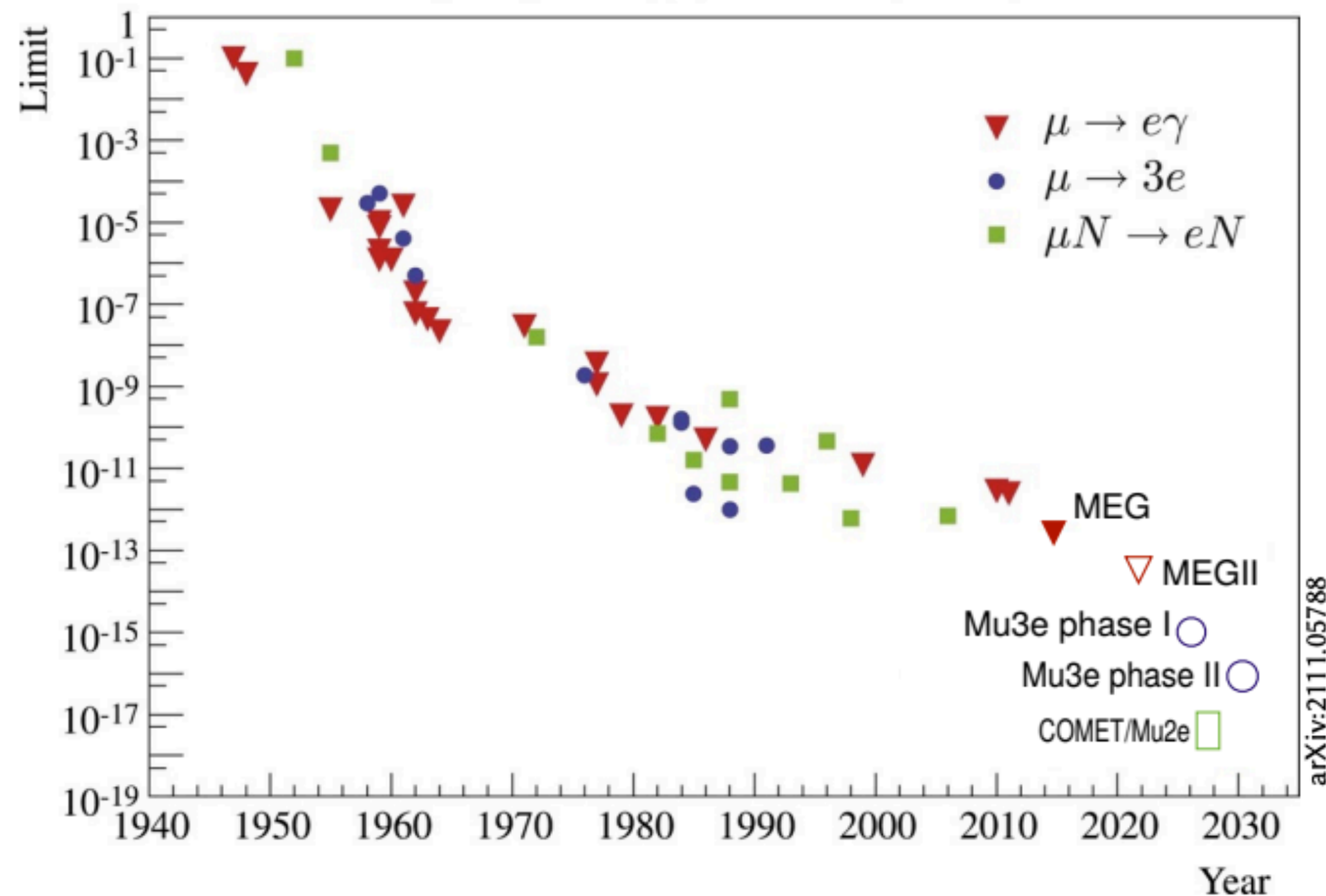
# Charged LFV (cLFV)



$< O(1e-54)$

*Petcov, 1977*

History of  $\mu \rightarrow e\gamma$ ,  $\mu N \rightarrow eN$ , and  $\mu \rightarrow 3e$

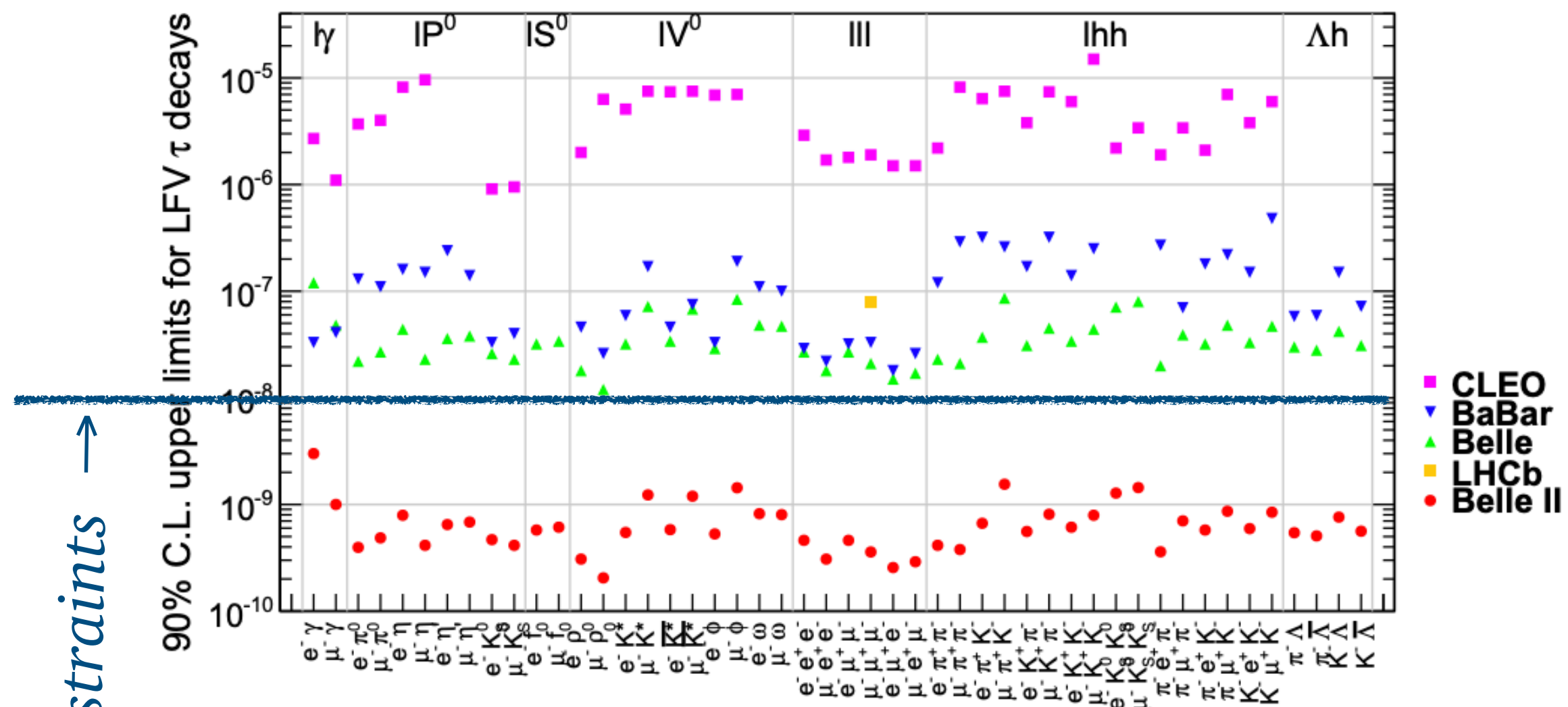


- SU(2) symmetry of the SM links neutrinos and (left handed) charged leptons : can we see LFV in the charged-lepton sector?
- Many searches for cLFV have focussed on muon decays:
  - Muon mass  $\sim 105$  MeV and lifetime  $\sim 2.2 \mu s$
  - ‘Goldilocks mass’: no hadronic decays, but does decay!
- Of course, if we only extended the SM by neutrino masses and don’t allow for other LFV, we expect only small cLFV effects (*symmetry!*). Other LFV BSM models yield larger signals.

So an observable detection of cLFV is genuine sign of new physics!



# New prospects for tau physics



Belle II Physics Book, 1808.10567

- The tau is more challenging than the muon: smaller lifetime, decays also into mesons...
- But: it can decay into both electrons and muons, so probe all types of lepton flavour transitions
- Belle II plans to improve many of the tau decay sensitivities by up to two orders of magnitude in the BRs.
- Even current bounds from tau decays are very strong constraints on many new physics models coupling to third generation leptons.

# Today's focus

Muon physics is still very exciting. But what if there was a well-motivated physics reason for why we haven't seen  $\mu \rightarrow e$  cLFV? Could cLFV with  $\tau$ 's be right around the corner?

# Lepton flavour *triality*

The idea: each charged lepton is *charged* under this  $Z_3$  (flavour triality)

$$\Psi_T \rightarrow \left( e^{2\pi i/3} \right)^T \Psi$$

$T$  is a triality charge.

electron	:	$T=1$	} In line with generation.
muon	:	$T=2$	
tau	:	$T=3$	

- Ernest Ma, 2010 “Quark and lepton flavour triality” 1006.3524
- Motivated by the success of non-Abelian discrete flavour symmetry  $A_4$  to explain neutrino tribimaximal mixing. (Altarelli + Feriglio 0512103, He, Kuem + Volkas 0601001)
- In the charged lepton sector,  $A_4$  breaks to an approximate  $Z_3$  “lepton triality”: a discrete subgroup of lepton flavour.
- Each charged lepton is assigned a triality charge.
- This same residual symmetry can be a feature of many other (discrete) flavour models for the lepton sector.



# What does this say about cLFV?

## Charge assignments:

$L_i$  has triality  $T=i$

$e_{R,i}$  has triality  $T=i$

‡ Triality sums modulo 3

## Implications:

$\mu \rightarrow e \gamma$   
 $T=2$        $T=1$   
 $\Delta T \neq 0$

$\tau \rightarrow \nu e e$   
 $T=3$        $T = -2 + 1 + 1 = 0$   
 $0 \pmod 3 = 3$   
 $\Rightarrow \Delta T = 0$  !

- If triality is a good symmetry, then it should be conserved.
- This permits certain cLFV processes, and forbids others. If triality is ultimately broken by some small parameter, then this approximate symmetry suppresses the size of any triality breaking processes [ $\nu$  mass requirement]
- So if the lepton flavour symmetry has triality as a feature, then we are MUCH more likely to see cLFV in tau decays.

# Where should we look for cLFV?

Triality-preserving charged-lepton decays:

Observable	Present constraint	Projected sensitivity
$\text{BR}(\tau^- \rightarrow \mu^- \mu^- e^+)$	$< 1.7 \times 10^{-8}$ [1]	$2.6 \times 10^{-10}$ [2]
$\text{BR}(\tau^- \rightarrow \mu^+ e^- e^-)$	$< 1.5 \times 10^{-8}$ [1]	$2.3 \times 10^{-10}$ [2]

## Lepton-flavour-violating tau decays from triality

Innes Bigaran,<sup>1,2,3,\*</sup> Xiao-Gang He,<sup>4,5,†</sup> Michael A. Schmidt,<sup>6,‡</sup>

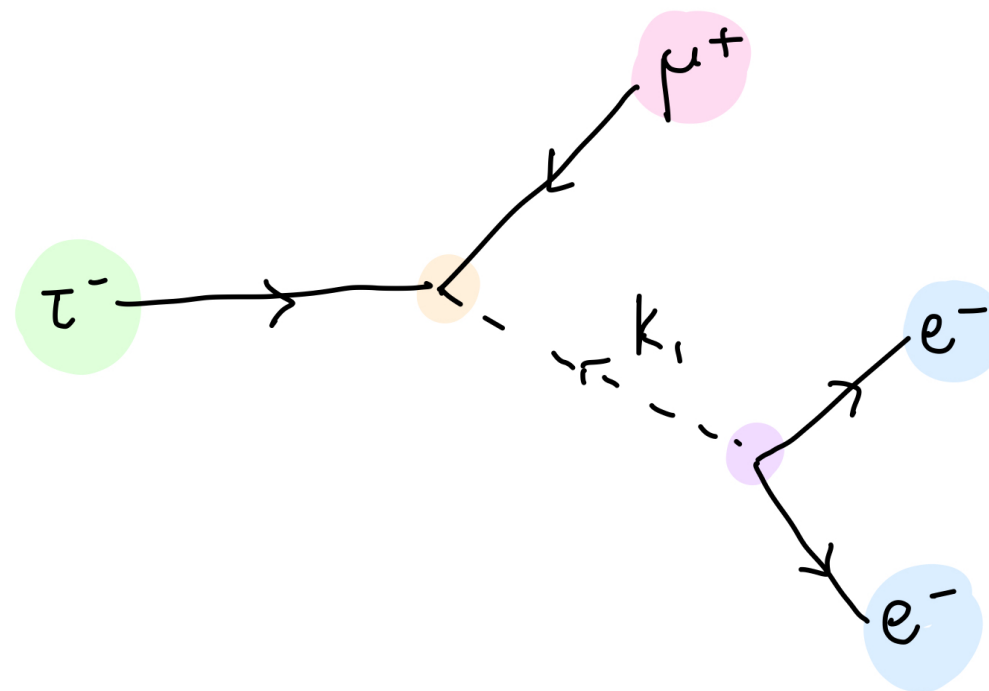
German Valencia,<sup>7,§</sup> and Raymond Volkas<sup>3,¶</sup>

*Phys.Rev.D* 107 (2023) 5, 055001  
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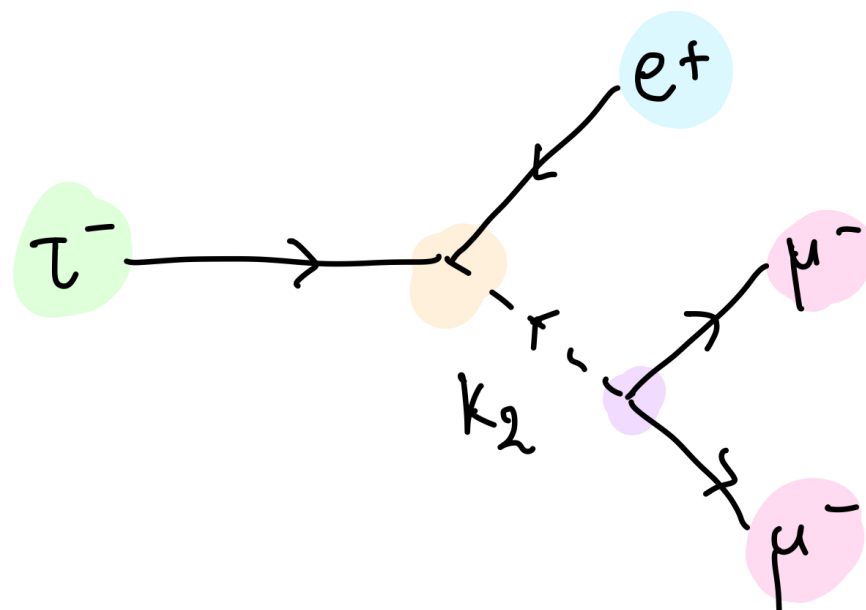
- There are two triality-preserving cLFV decays
- The sensitivity to these decays at experiment is expected to be improved significantly at Belle II
- If triality is a good lepton flavour symmetry, these are our **best bet** for finding signs of the underlying new physics violating lepton flavour beyond the SM
- What we did in 2212.09760: with triality as a starting point, established simple extensions featuring doubly charged scalar bileptons and studied the phenomenology of these models and decays

Observable
$\text{BR}(\tau^- \rightarrow \mu^- \mu^- e^+)$
$\text{BR}(\tau^- \rightarrow \mu^+ e^- e^-)$

T=1 scalar



T=2 scalar



# Electroweak Singlet Scalars

- Most simple extensions are electroweak singlet scalar *bileptons* : they couple directly to two leptons. Triality forbids other SM interactions, making their phenomenology straightforward.
- Mediating this interaction at tree-level, we can either assign the scalar a triality of T=2 or T=1
- The T=1 scalar  $k_1$  mediates  $\tau^- \rightarrow \mu^+ e^- e^-$

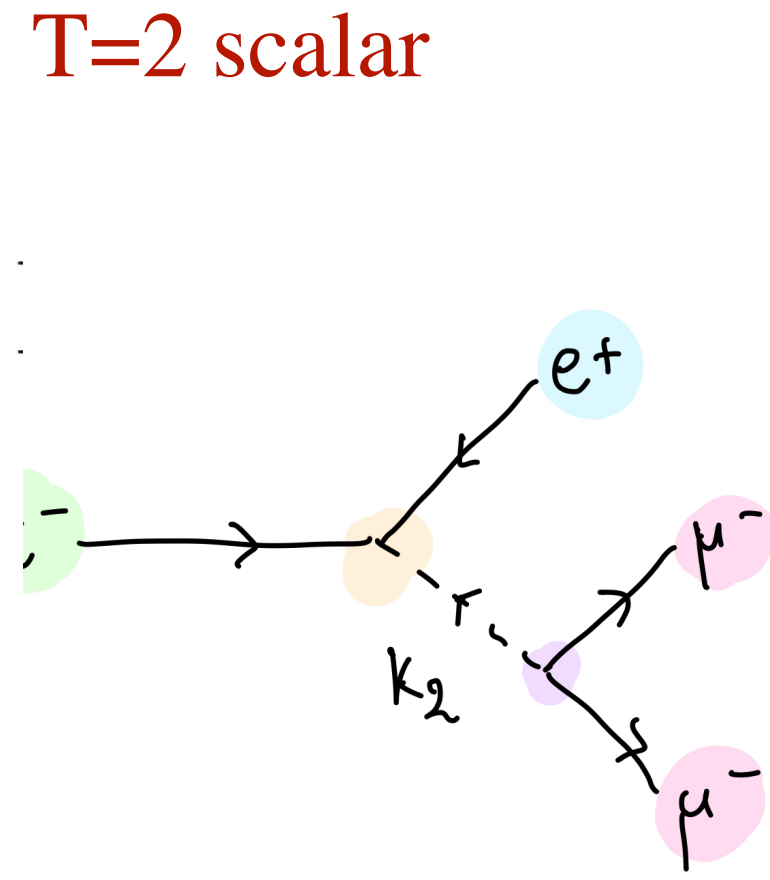
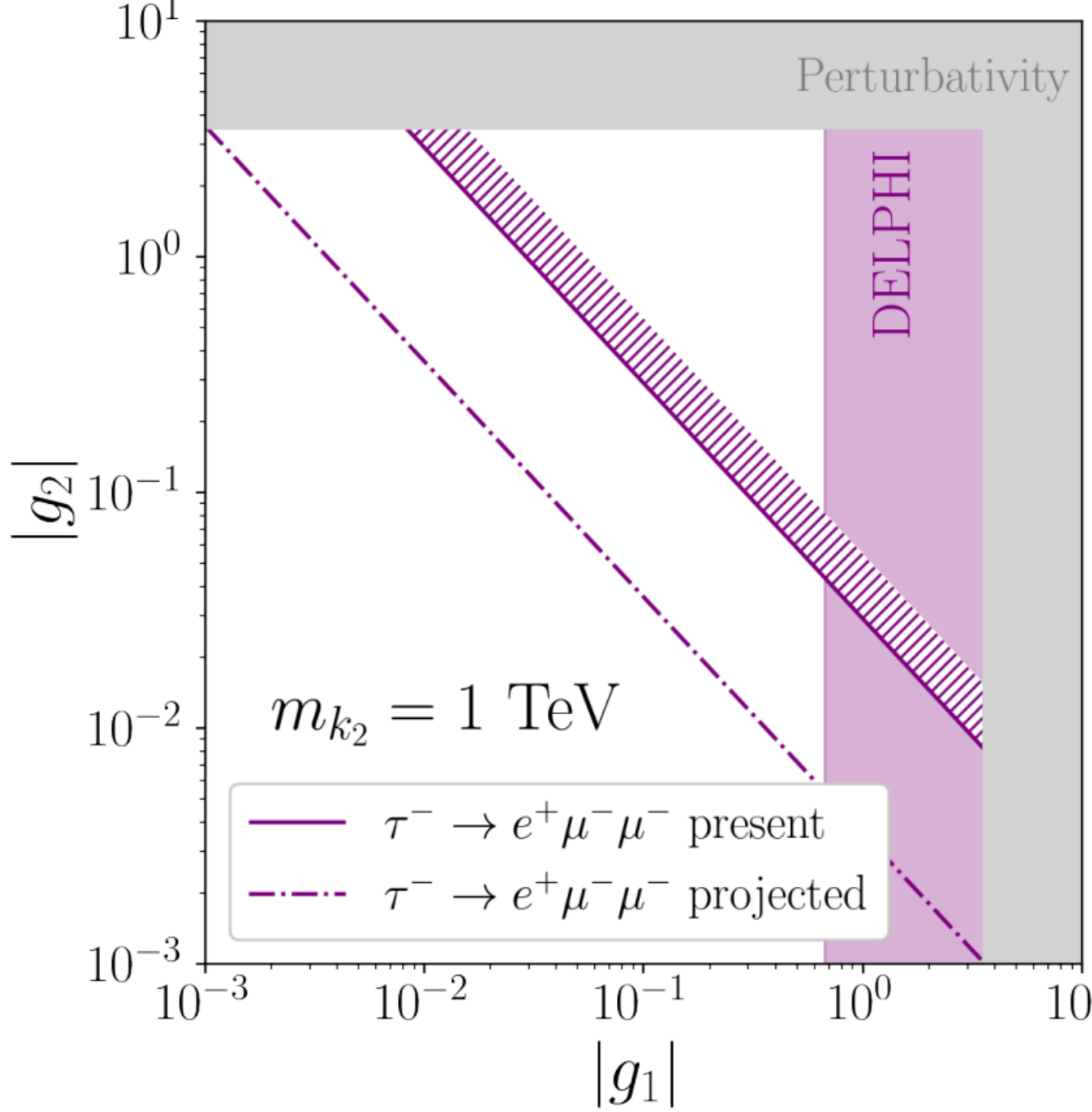
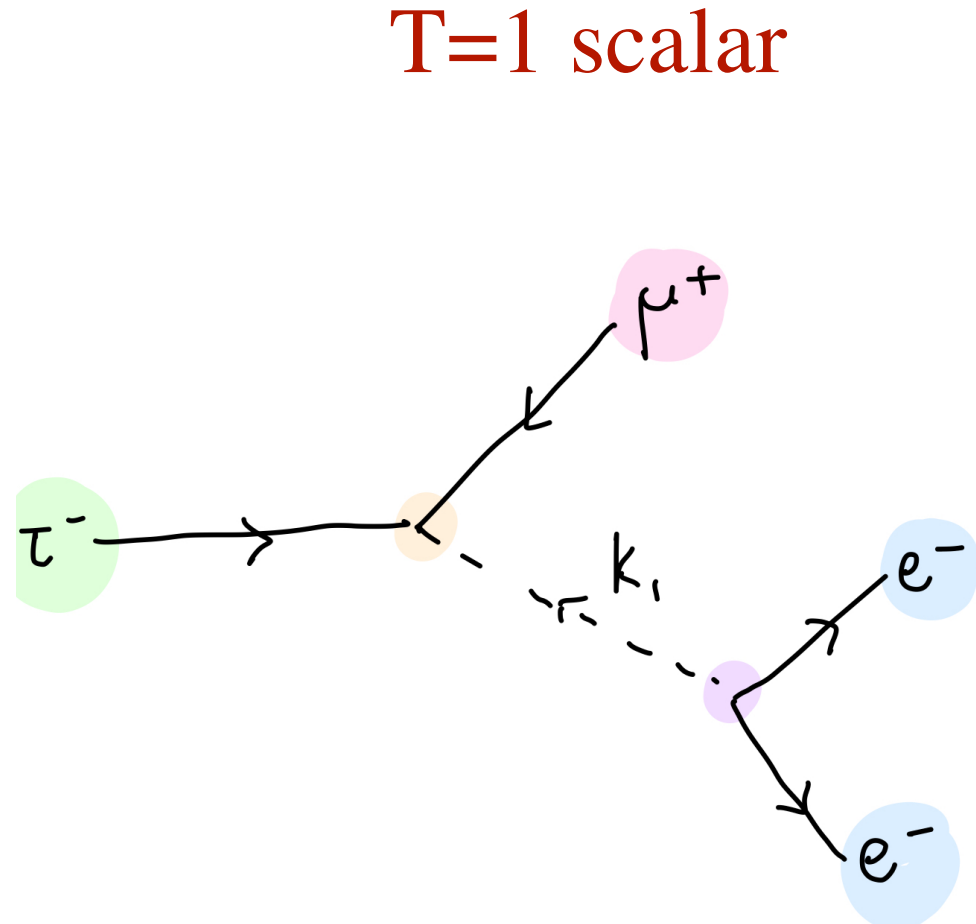
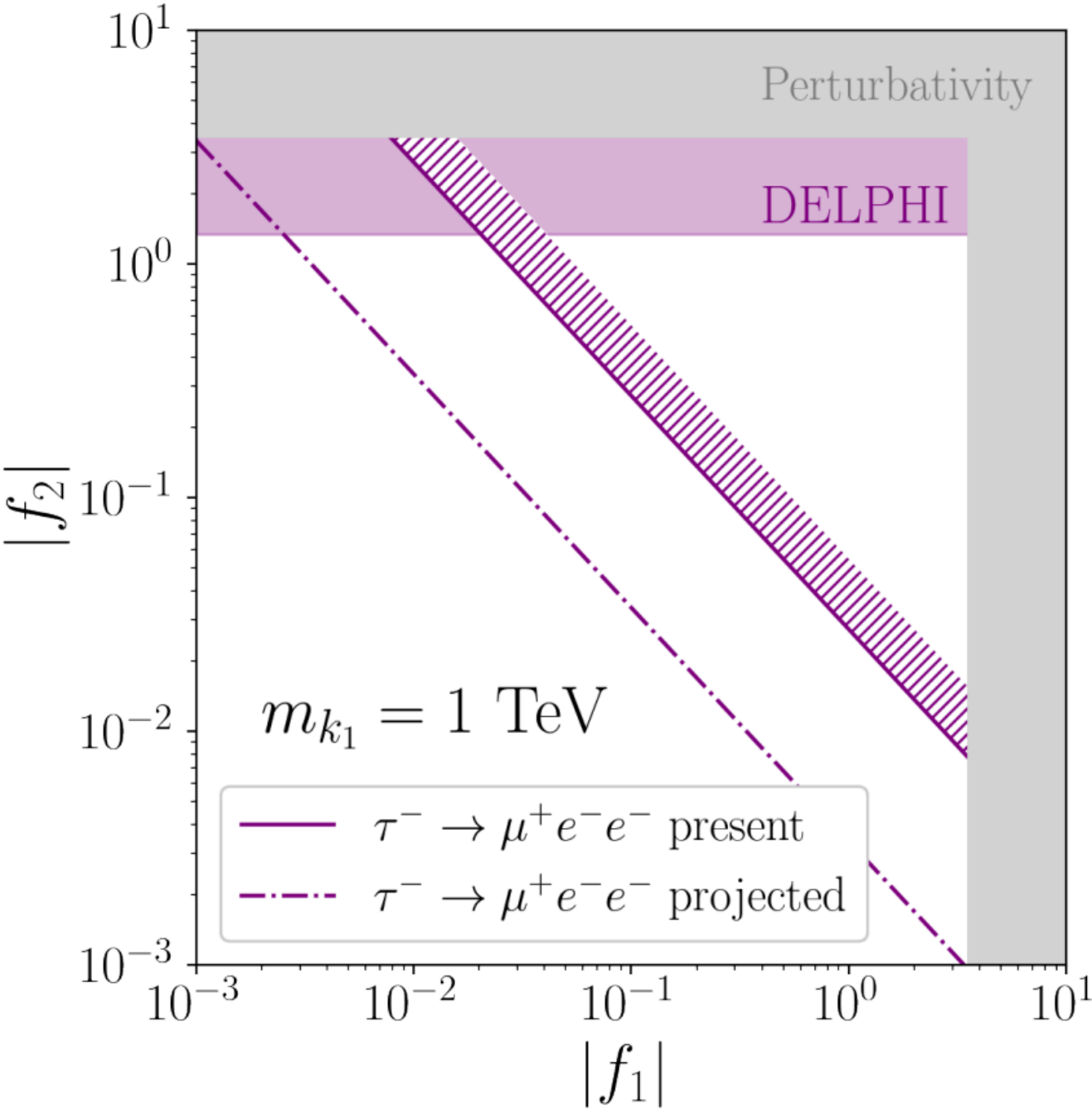
$$\mathcal{L}_{k_1} = \frac{1}{2} (2f_1 \overline{\tau}_R^c \mu_R + f_2 \overline{e}_R^c e_R) k_1 + h.c.$$

- The T=2 scalar  $k_2$  mediates  $\tau^- \rightarrow e^+ \mu^- \mu^-$

$$\mathcal{L}_{k_2} = \frac{1}{2} (2g_1 \overline{\tau}_R^c e_R + g_2 \overline{\mu}_R^c \mu_R) k_2 + h.c.$$



# Electroweak Singlet Scalars

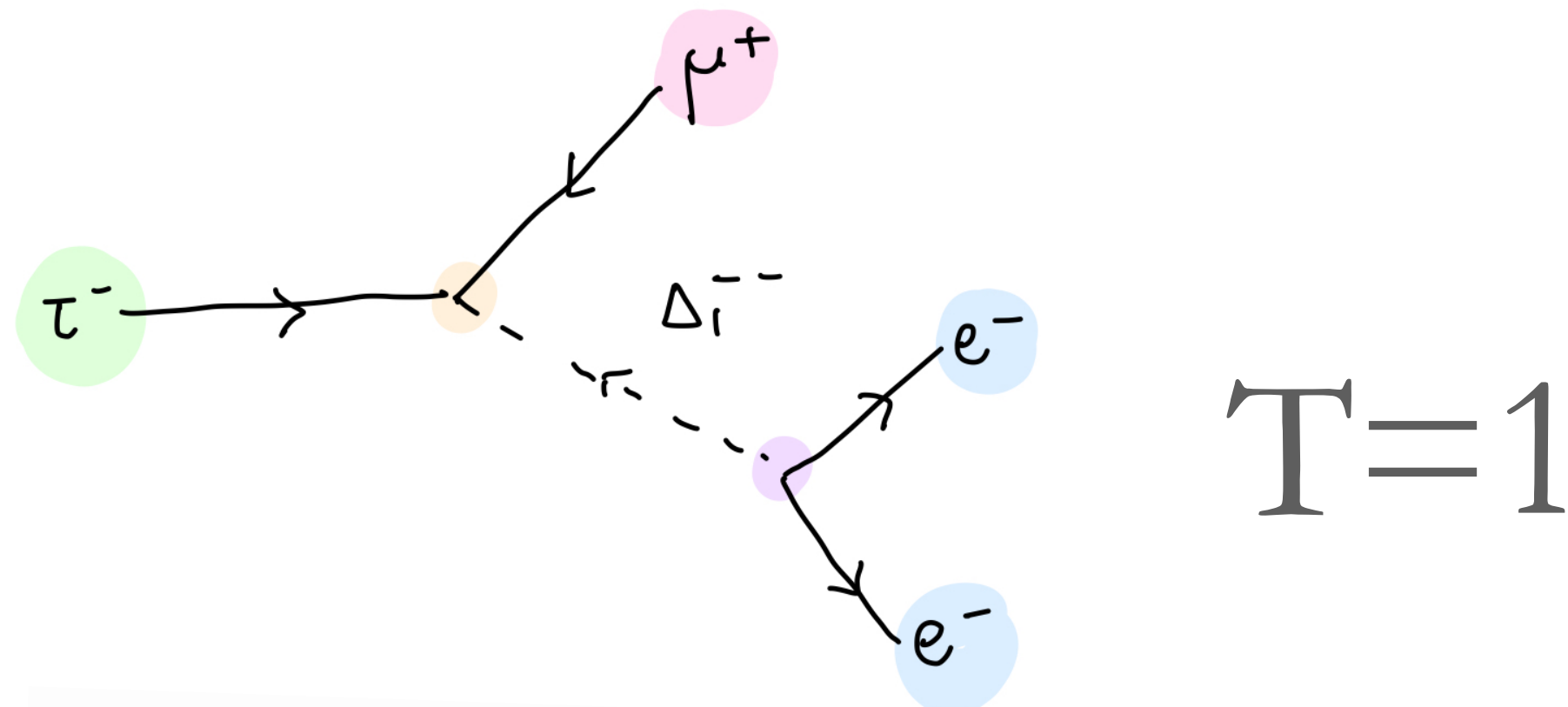


$$\mathcal{L}_{k_1} = \frac{1}{2} (2f_1 \bar{\tau}_R^c \mu_R + f_2 \bar{e}_R^c e_R) k_1 + h.c.$$

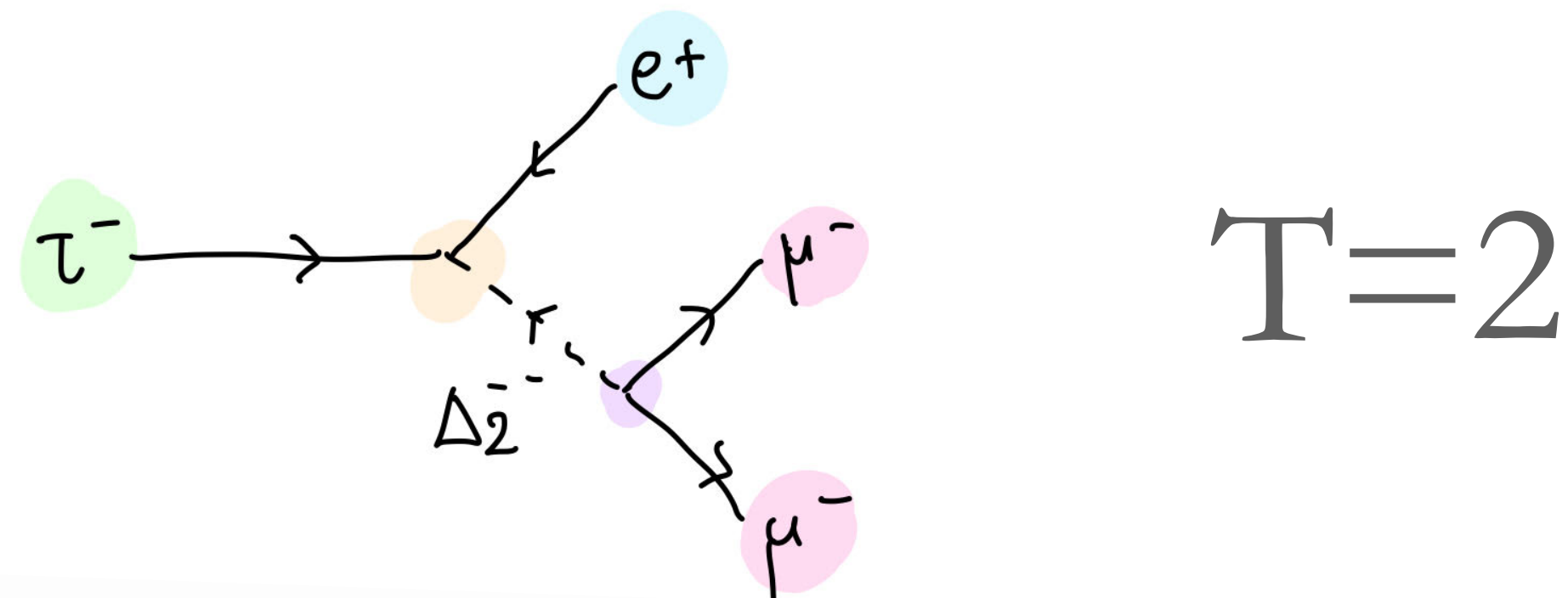
$$\mathcal{L}_{k_2} = \frac{1}{2} (2g_1 \bar{\tau}_R^c e_R + g_2 \bar{\mu}_R^c \mu_R) k_2 + h.c.$$

- Doubly-charged scalar bilepton is constrained to be TeV scale by direct searches
- Contribution from (T=1) [T=2] to  $e^+ e^- \rightarrow (e^+ e^-) [\tau^+ \tau^-]$  constrained by DELPHI

# Electroweak Triplet Scalars

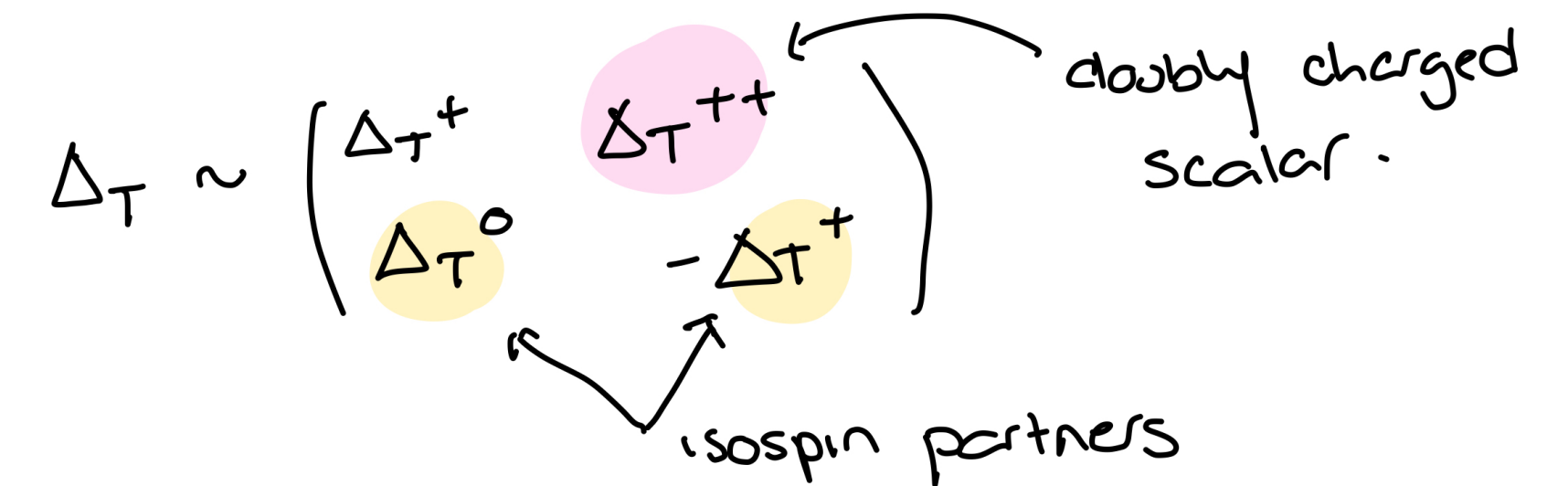


$$\mathcal{L}_{\Delta_1} = \frac{1}{2} (2f_1 \bar{L}_3^c \Delta_1 L_2 + f_2 \bar{L}_1^c \Delta_1 L_1) + h.c.$$



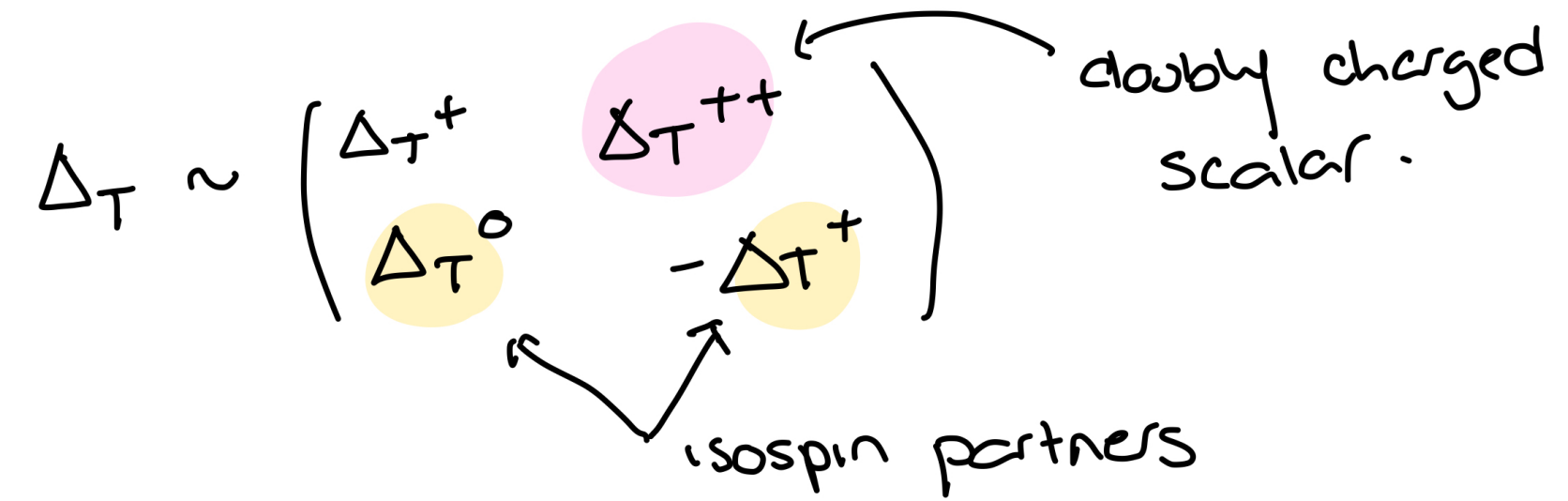
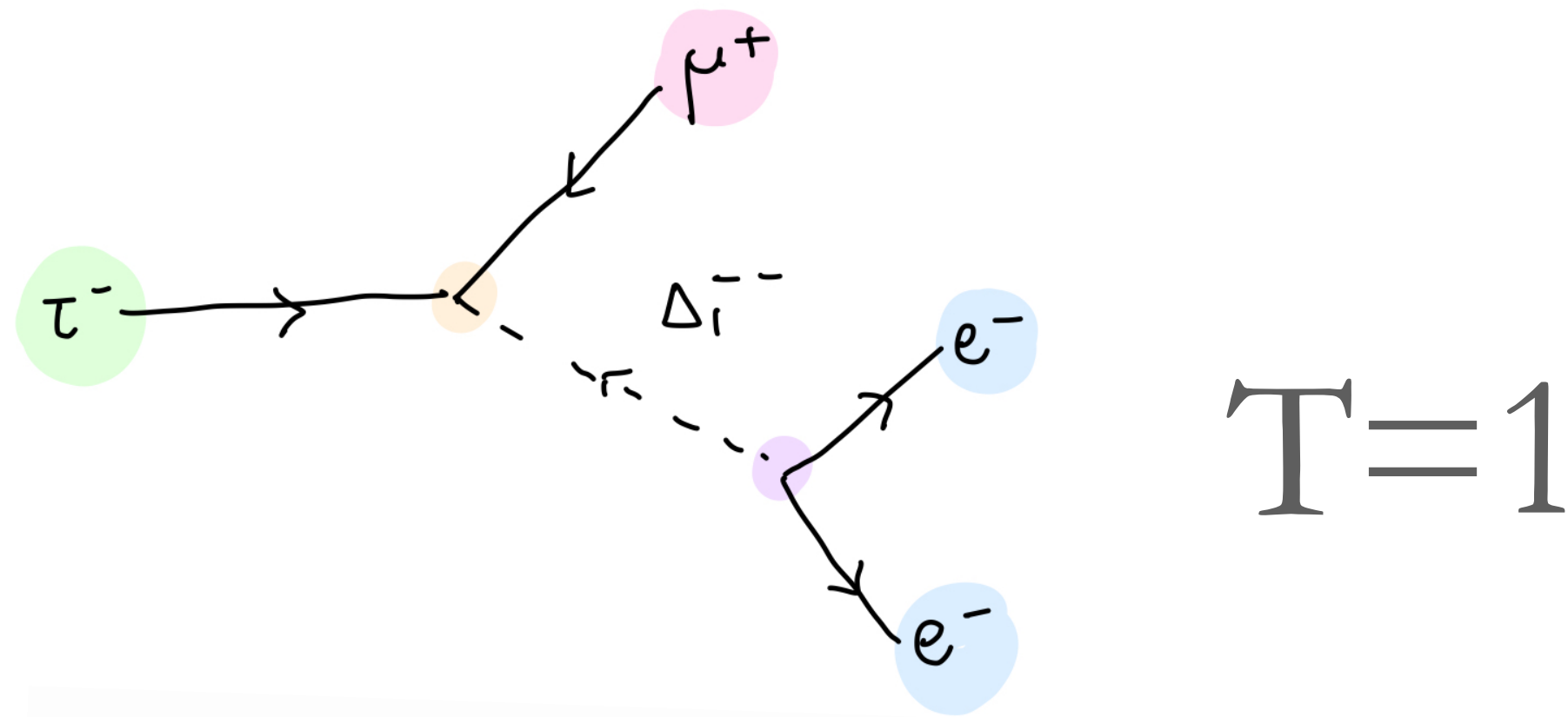
$$\mathcal{L}_{\Delta_2} = \frac{1}{2} (2g_1 \bar{L}_3^c \Delta_2 L_1 + g_2 \bar{L}_2^c \Delta_2 L_2) + h.c.$$

- Much richer phenomenology. Can be thought of as an extension of the singlet scalar study.
- Electroweak triplet scalar bileptons



- Similarly constrained to be TeV scale by bilepton searches
- Couple to lepton doublet: constraints also from neutrino interactions

# Electroweak Triplet Scalars



$$\mathcal{L}_{\Delta_1} = \frac{1}{2} (2f_1 \bar{L}_3^c \Delta_1 L_2 + f_2 \bar{L}_1^c \Delta_1 L_1) + h.c.$$

Example of expanded interaction terms:

$$\begin{aligned} \bar{L}_3^c i\sigma_2 \Delta_1 L_2 &= -(\tau_L)^c \mu_L \Delta_1^+ - \frac{1}{\sqrt{2}} \left[ (\tau_L)^c \nu_{\mu L} + (\nu_{\tau L})^c \mu_L \right] \Delta_1^+ + (\nu_{\tau L})^c \nu_{\mu L} \Delta_1^0, \\ \bar{L}_1^c i\sigma_2 \Delta_1 L_1 &= -(e_L)^c e_L \Delta_1^{++} - \sqrt{2} (e_L)^c \nu_{eL} \Delta_1^+ + (\nu_{eL})^c \nu_{eL} \Delta_1^0. \end{aligned}$$

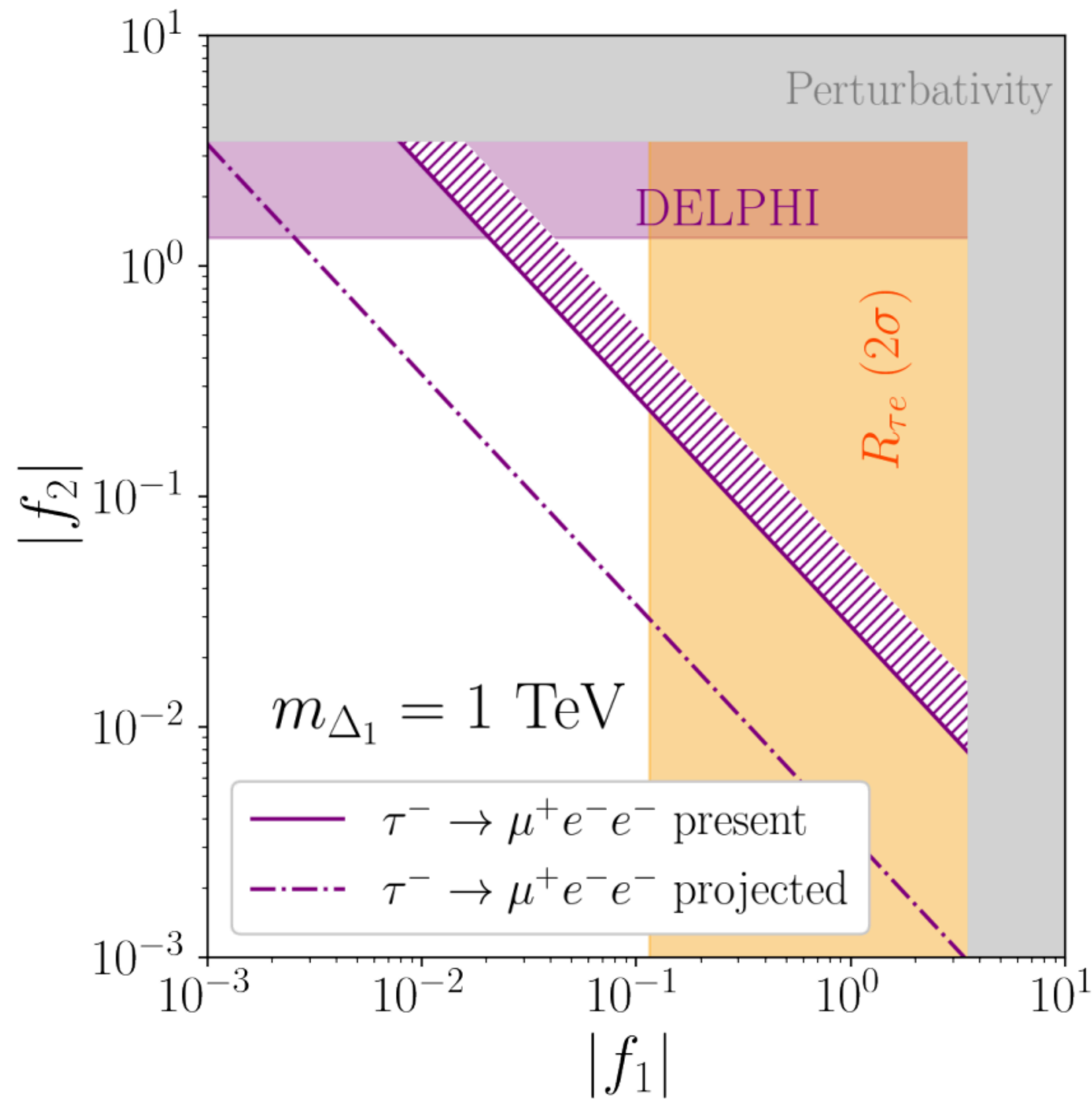
- Isospin partners: single and doubly charged components
- LFU ratios of tau decays also constrain parameter space, e.g.

$$R_{\tau e} = \frac{\Gamma(\tau \rightarrow \mu + \nu) \Gamma_{SM}(\mu \rightarrow e + \nu)}{\Gamma_{SM}(\tau \rightarrow \mu + \nu) \Gamma(\mu \rightarrow e + \nu)}$$

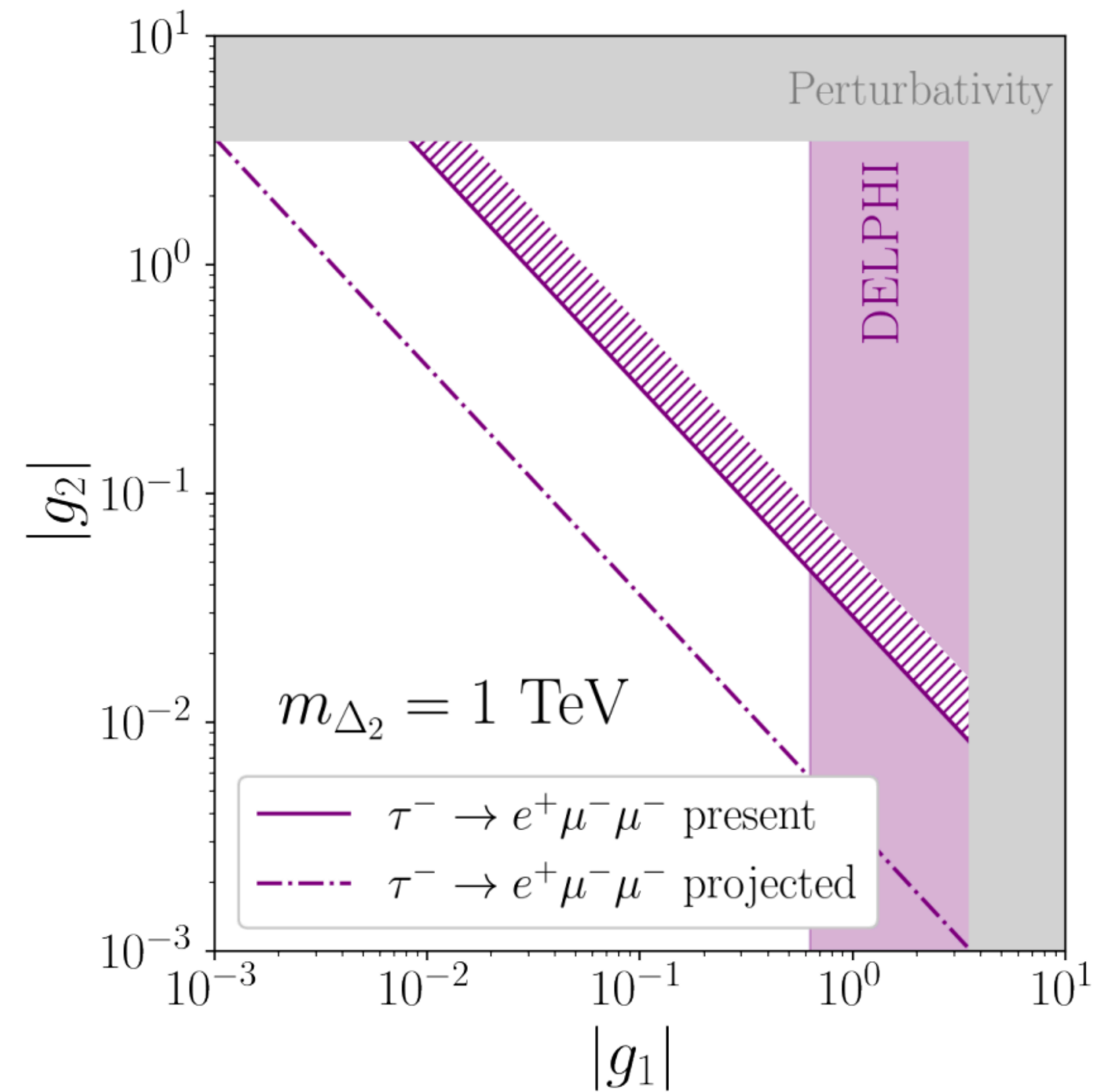
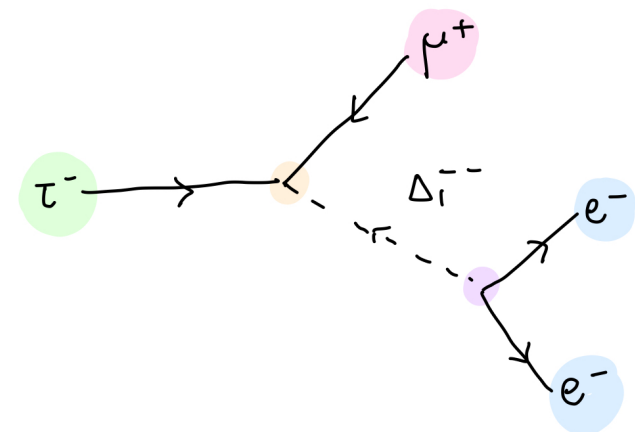
- Majorana-mass type interaction induced by neutral component. Constrained in structure by triality... [we will return to this later]



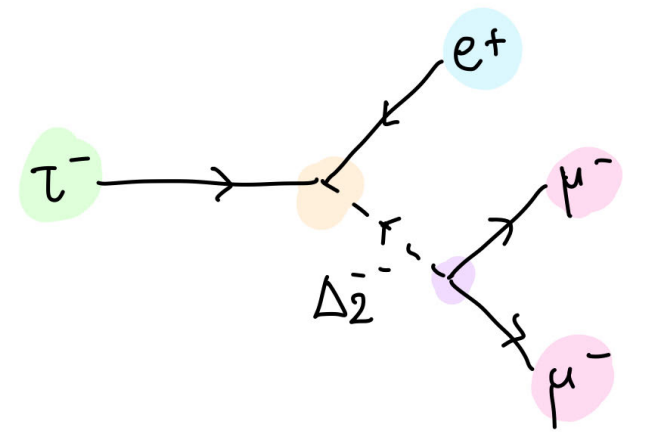
# Electroweak Triplet Scalars



**T=1 scalar**



**T=2 scalar**



$$\mathcal{L}_{\Delta_1} = \frac{1}{2} (2f_1 \bar{L}_3^c \Delta_1 L_2 + f_2 \bar{L}_1^c \Delta_1 L_1) + h.c.$$

$$\mathcal{L}_{\Delta_2} = \frac{1}{2} (2g_1 \bar{L}_3^c \Delta_2 L_1 + g_2 \bar{L}_2^c \Delta_2 L_2) + h.c.$$

# Brief recap of the story so far.

- Lepton flavour triality could be a residual symmetry in the flavour sector which constrains observable cLFV decays
- Moreover, motivated by flavour triality we could expect cLFV tau decays as outlined to be the first signal of LFV new physics
- Simple UV completions: triality-preserving extensions to the SM contain scalar bileptons EW triplet and singlet phenomenology study shows there's regions of parameter space that would predict a signal within reach of Belle II.

# Tau decay kinematic study in light of triality

Observable	Present constraint	Projected sensitivity
$\text{BR}(\tau^- \rightarrow \mu^- \mu^- e^+)$	$< 1.7 \times 10^{-8}$ [1]	$2.6 \times 10^{-10}$ [2]
$\text{BR}(\tau^- \rightarrow \mu^+ e^- e^-)$	$< 1.5 \times 10^{-8}$ [1]	$2.3 \times 10^{-10}$ [2]

$$\frac{d^2 \Gamma (\tau^- \rightarrow l_i^- l_i^- l_j^+)}{dm_{--}^2 dm_{+-}^2} = \frac{1}{256\pi^3 m_\tau^3} |\overline{\mathcal{M}}|^2$$

$\downarrow$  invariant mass of  $l_i^- l_i^-$   
 $\downarrow$  invariant mass of  $l_i^- l_j^+$   
 $\downarrow$  Wilson coefficient  $\sim$  couplings.

Expt. limits assume  $\sim 1$

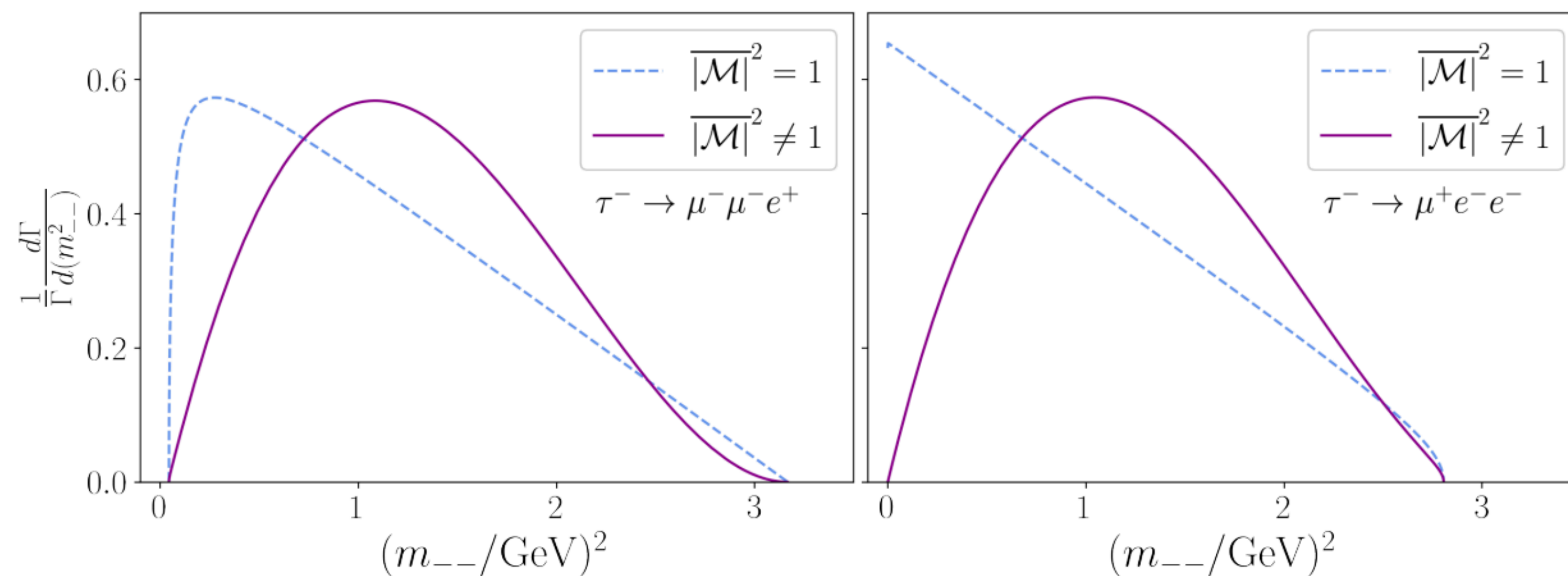
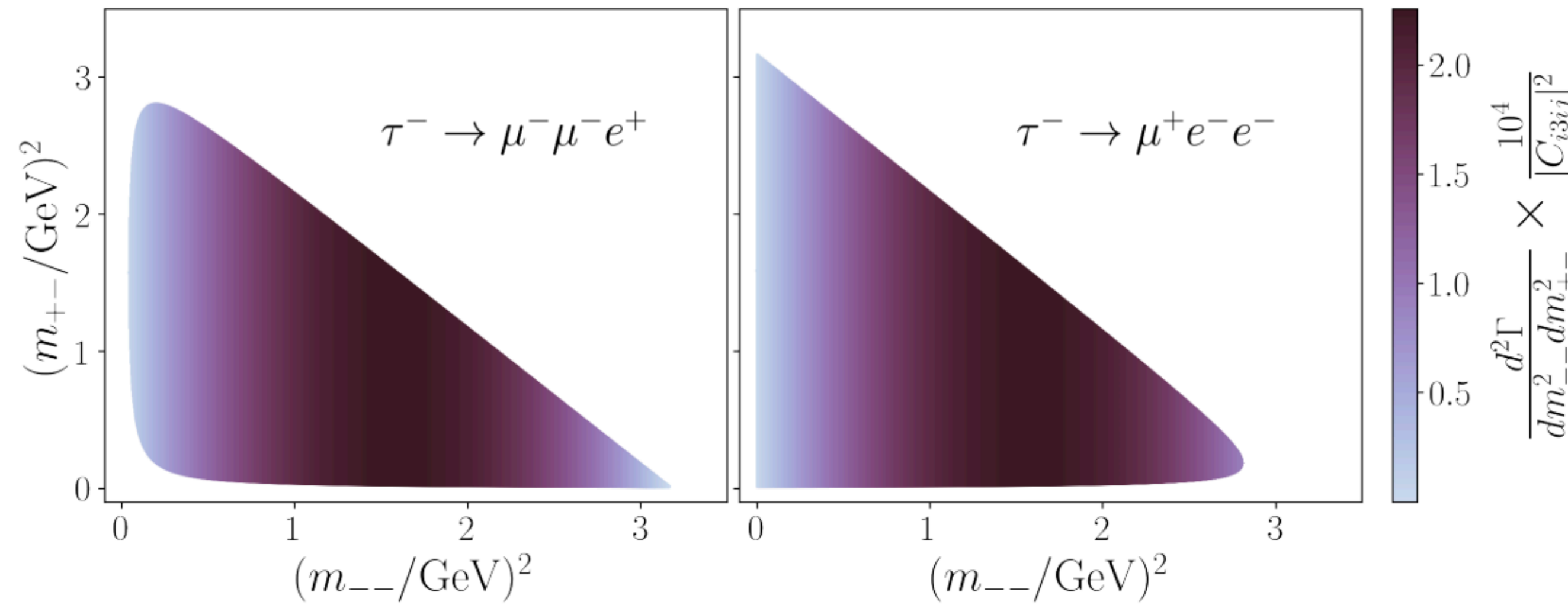
(BSM here is too heavy to go on-shell)

- Bounds on these decays have phase-space dependence that isn't taken into account
- In particular, phase space of three body decays depends on the nature of the effective interaction and mediators present (R. H. Dalitz (1953))
- Limits assume that there's no kinematic dependence in the matrix element, which is not necessarily true
- Discriminating power of three-body phase space of taus studied in e.g. Dassinger et al 0707.0988, Celis et al 1403.5781 ++, though not in these decay channels above.





# Phase space distributions



- Both scalar triplet/singlet models contribute to both processes via a RH or LH vector interaction. Differential decay rates are of the same form
- Different structure of the Dalitz plots arise from different decay kinematics. These differential decay rates are *not* the same as those from the “phase space” approximation ( $M \sim 1$ ).
- A bound is derived from looking at what we *expect* to see, and thus constraining the size of the effect. We need internal experimental information to be able to recast constraints, though... (e.g. detection efficiency as a function of invariant masses)

# Neutrino masses in the triality-based models

Dirac masses

$$\mathcal{L} \supset -y_{ij} \bar{L}_i \nu_{Rj} \tilde{H} + h.c.$$

$$y = \begin{pmatrix} y_{11} & 0 & 0 \\ 0 & y_{22} & 0 \\ 0 & 0 & y_{33} \end{pmatrix}$$

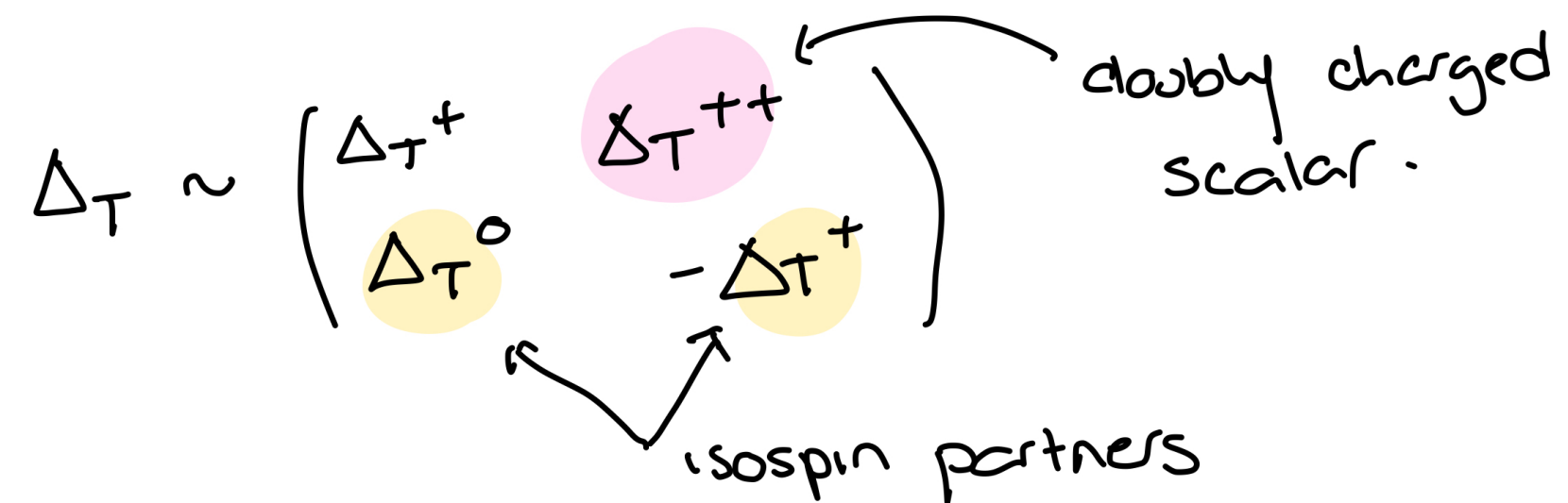
Majorana masses

$$\mathcal{L} \supset k_{ij} (L_i H) (L_j H)$$

$$k = \begin{pmatrix} 0 & k_{12} & 0 \\ k_{12} & 0 & 0 \\ 0 & 0 & k_{33} \end{pmatrix}$$

- In my abstract, I promised a connection to neutrino masses.
- If we add **RH neutrinos** with three flavours, generationally-assigned triality, we can also obtain a triality-preserving neutrino mass matrix, but with a restricted texture.
- **Lepton triality needs to be broken to achieve observed neutrino mass texture.**

# Neutrino masses in the triality-based models



$$\mathcal{L}_{\Delta_1} = \frac{1}{2} (2f_1 \overline{L}_3^c \Delta_1 L_2 + f_2 \overline{L}_1^c \Delta_1 L_1) + h.c.$$

$$\begin{aligned} \overline{L}_3^c i\sigma_2 \Delta_1 L_2 &= -(\overline{\tau_L})^c \mu_L \Delta_1^{++} - \frac{1}{\sqrt{2}} \left[ (\overline{\tau_L})^c \nu_{\mu L} + (\overline{\nu_{\tau L}})^c \mu_L \right] \Delta_1^+ + (\overline{\nu_{\tau L}})^c \nu_{\mu L} \Delta_1^0 \\ \overline{L}_1^c i\sigma_2 \Delta_1 L_1 &= -(\overline{e_L})^c e_L \Delta_1^{++} - \sqrt{2} (\overline{e_L})^c \nu_{eL} \Delta_1^+ + (\overline{\nu_{eL}})^c \nu_{eL} \Delta_1^0 \end{aligned}$$

- Lepton triality needs to be broken to achieve observed neutrino mass texture.

*Breaking triality to generate neutrino masses isn't bad, neutrino masses are small so triality protection helps to keep triality breaking effect small.*

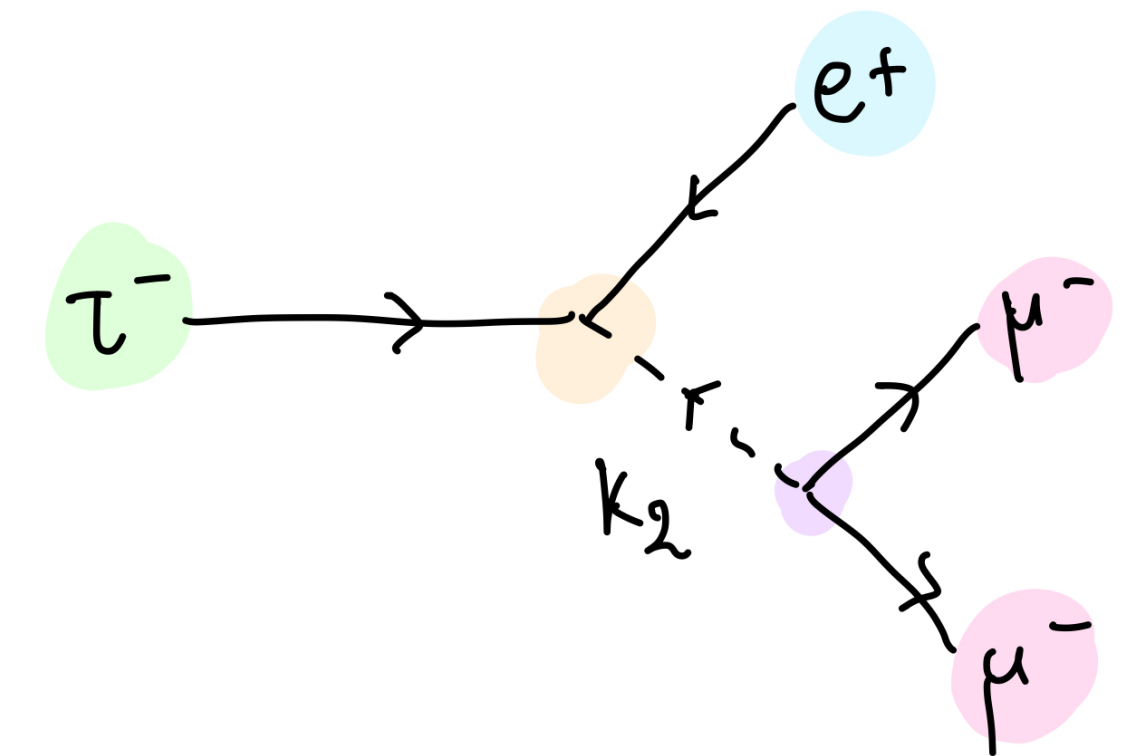
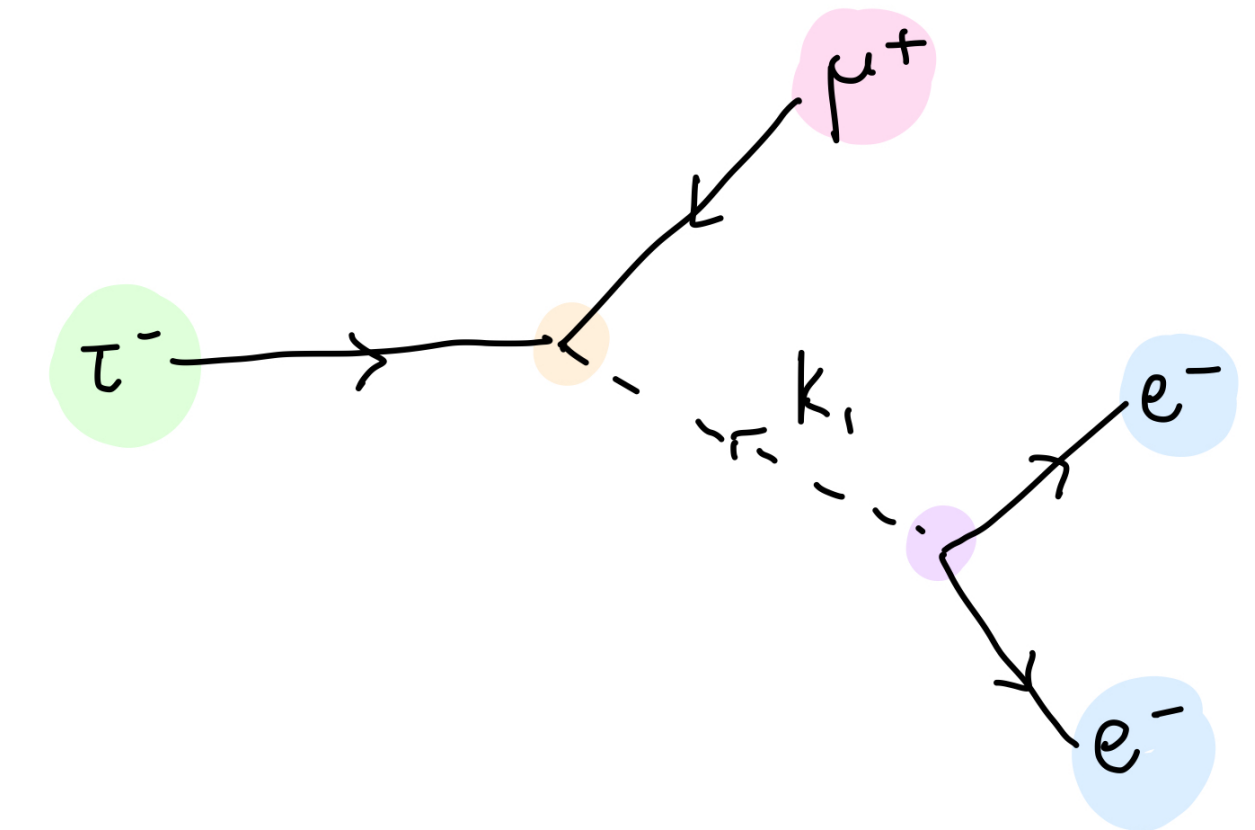
- e.g. in the T=1 triplet model, we can see from the Lagrangian that if the neutral component gets a vev then we obtain a nonzero Majorana neutrino mass, though with a restricted flavour structure (Type II seesaw). Naturally small vev due to soft breaking by a cubic Higgs coupling.



# Summary

IB, XG He, M.A. Schmidt, G. Valencia, R. Volkas  
arXiv: 2212.09760

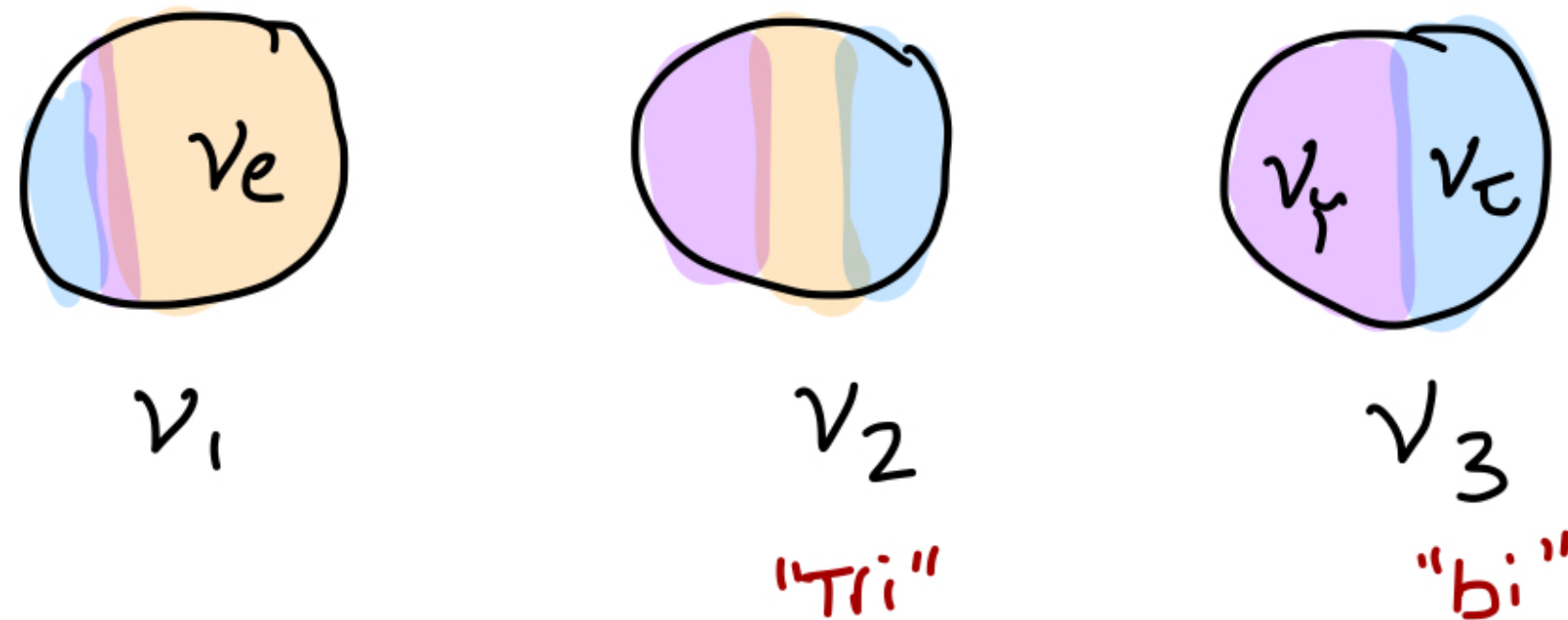
- Lepton triality: assign lepton flavours different “charges” under a  $Z_3$   
Motivates the search for cLFV signals in tau decays, and explains non-observation of cLFV in  $\mu$  to  $e$  transitions
- Motivated by a residual  $Z_3$  flavour symmetry in the lepton sector: can guide flavour model-building
- Minimal models furnished by EW singlet and triplet bileptons
- Dominant signals of cLFV in models with lepton flavour triality are in tau three-lepton decays. Discovery could be around the corner...



Thank you!

# Backup

# Tribimaximal mixing



$\theta_{12} \sim 35.3^\circ$	$\rightarrow 33.4^\circ$
$\theta_{23} \sim 45^\circ$	$\rightarrow 49.1^\circ$
$\theta_{13} = 0$	$\rightarrow 8.54^\circ$
$\delta$ undetermined.	$\rightarrow 197^\circ$

- In 2011, Daya Bay and Reno measured a nonzero  $\theta_{13}$ , inconsistent with tribimaximal mixing
- Green data shows the results from 2022 NuFit collaboration fit to neutrino oscillation data.



# Breaking triality to generate neutrino masses

$$\mathcal{L}_5 = -\frac{k_{ij}}{4} (L_i H)^T C (L_j H) + h.c.$$

Triality  $\rightarrow$  
$$K = \begin{pmatrix} 0 & k_{12} & 0 \\ k_{12} & 0 & 0 \\ 0 & 0 & k_{33} \end{pmatrix}$$

Singlet complex scalar  $S$ ,  $T=1$

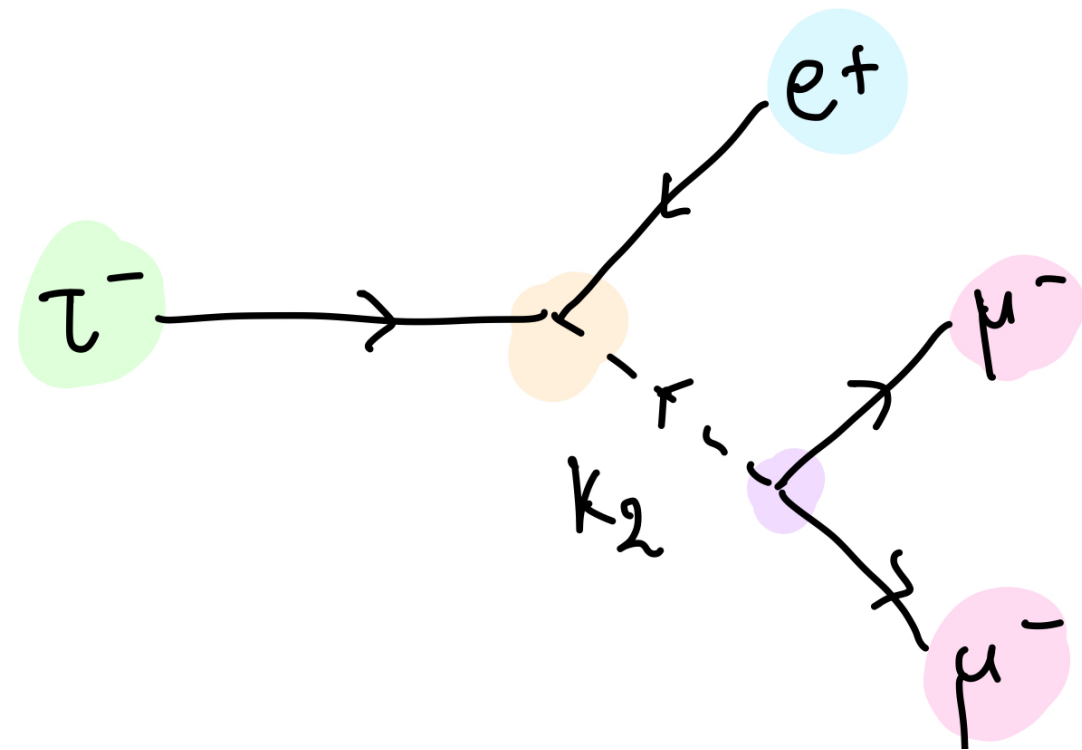
$S^{(4)} (L_i H)^T C (L_j H)$  operator ( $D=6$ )

$\Rightarrow$  leads to additional triality preserving couplings and if  $S$  gets a vev breaking triality we can have all zero entries populated.

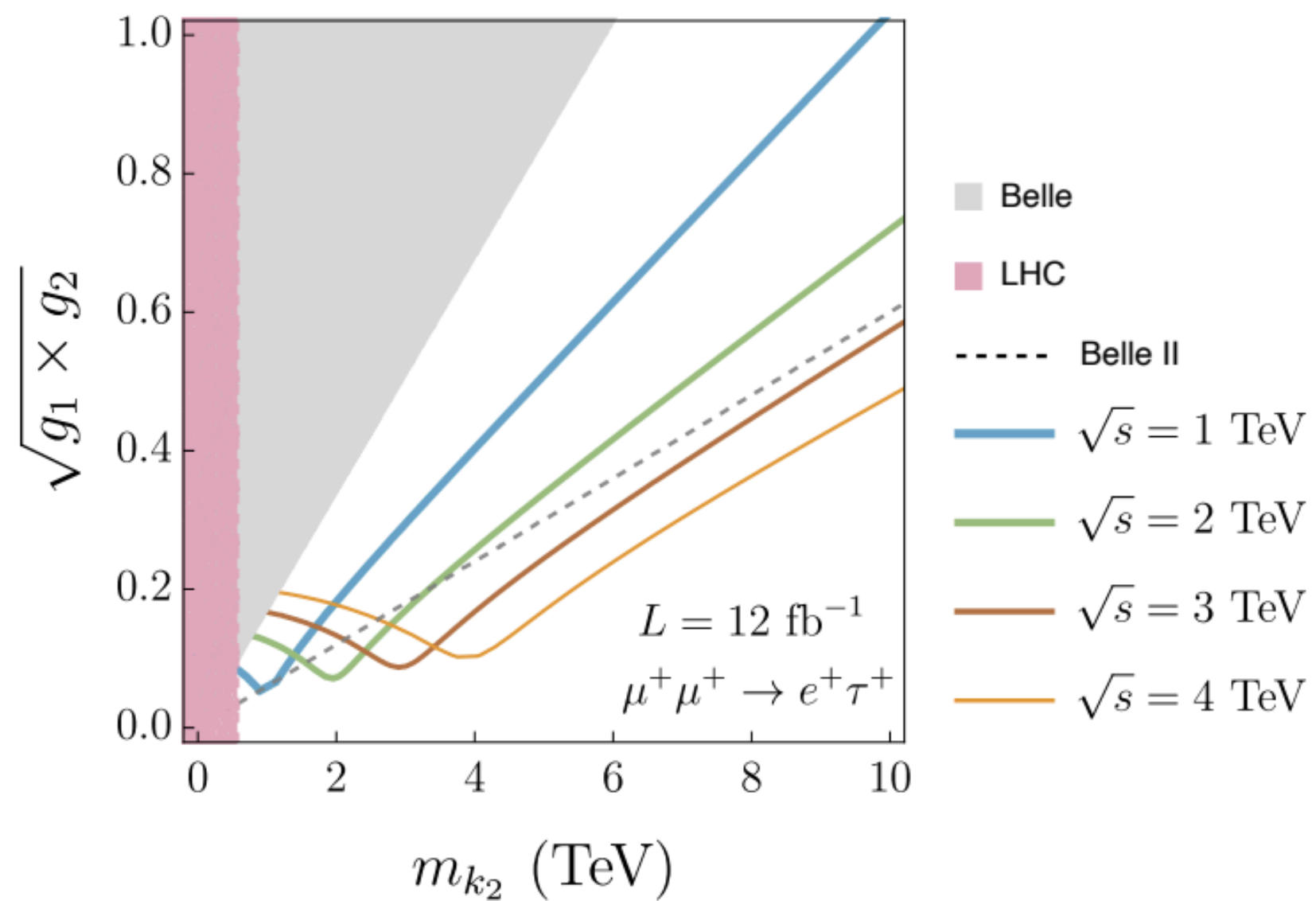
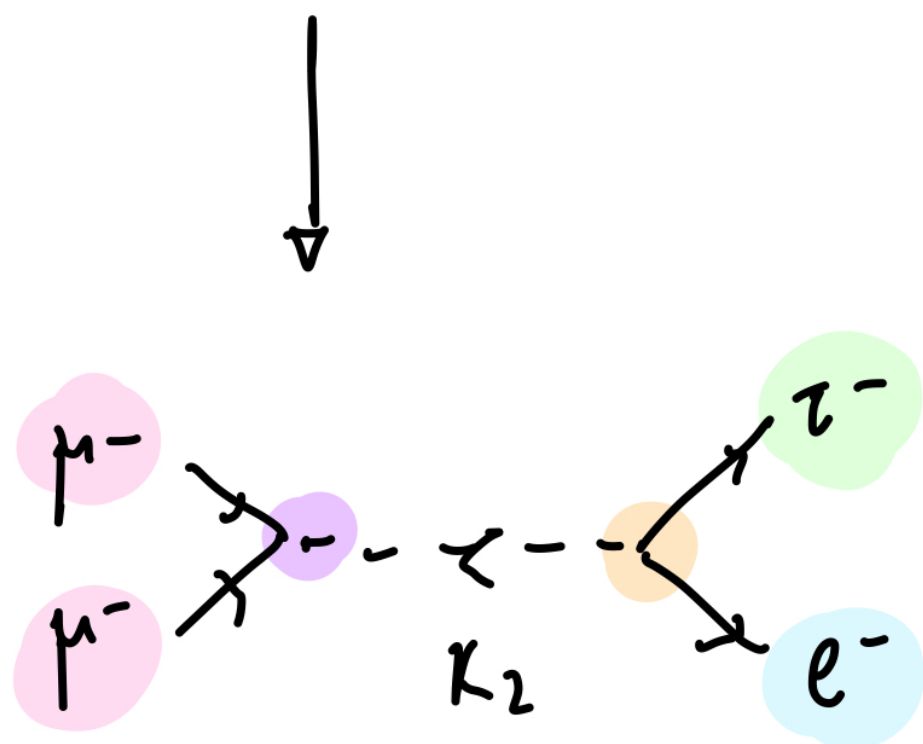
- A diagonal Dirac neutrino matrix together with a general RH neutrino Majorana mass matrix can generate the required neutrino mass and mixing parameters.

# Same-sign muon collider signals

G. Lichtenstein M.A. Schmidt, G. Valencia, R.Volkas , 2307.11369



- Related by crossing-symmetry to processes that could be probed at a same-sign muon collider ( $\mu$ TRISTAN, 2201.06664)



- Demonstrated reach of this experiment after a year of data taking for these models