

ISOSPIN BREAKING IN τ DATA FOR $(g - 2)_\mu$

Mattia Bruno

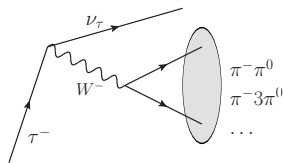
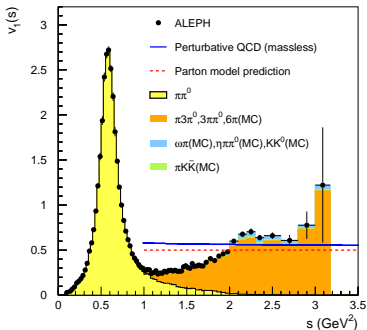
Taku Izubuchi (presenter)

work in collab. with C. Lehner, A. Meyer
for the RBC/UKQCD collaborations



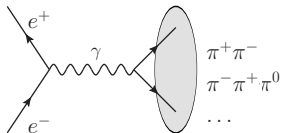
The 17th International Workshops on Tau Lepton Physics,
University of Louisville, December 8th 2023

MOTIVATIONS FOR τ



$V - A$ current

Final states $I = 1$ charged

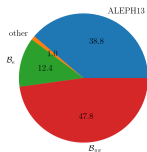
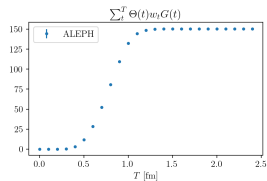
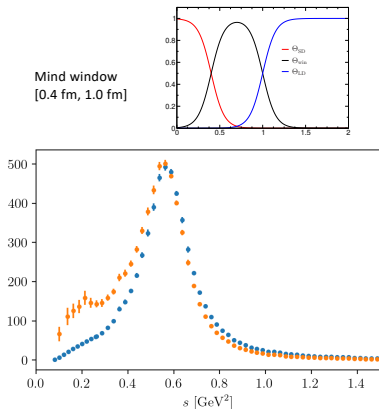


EM current

Final states $I = 0, 1$ neutral

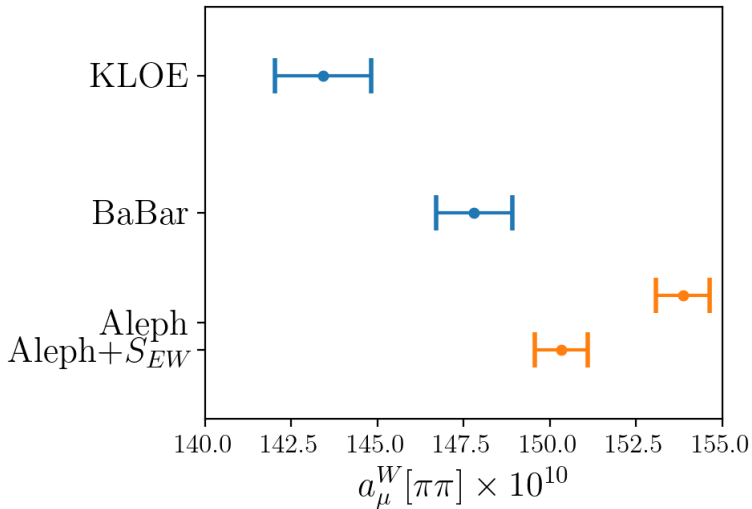
- τ data can improve $a_\mu[\pi\pi]$
- 72% of total Hadronic LO
- competitive precision on a_μ^W

Experimental τ data **very competitive** for intermediate window



UNCORRECTED COMPARISON OF a_μ^W

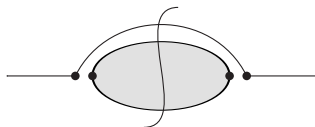
Experimental τ data **very competitive** for intermediate window



(also recent similar analysis by Masjuan et al. 2310.14102)

HADRONIC τ DECAYS

$$\mathcal{M}(P, q, p_1 \cdots p_{n_f} | f) = \frac{G_F V_{ud}}{\sqrt{2}} \bar{u}_\nu(-q) \gamma_\mu^L u_\tau(P) \langle f, p_1 \cdots p_{n_f} | \mathcal{J}_\mu^-(0) | 0 \rangle$$



$$\begin{aligned} d &= \frac{1}{2m} d_q \sum_f d_f \sum_{\text{spin}} |\mathcal{M}(P, q, p_1 \cdots p_{n_f} | f)|^2 \\ &= \frac{1}{2m} d_q \frac{G_F^2 |V_{ud}|^2}{2} \mathcal{L}_{\mu\nu}(P, q) \rho_{\mu\nu}(p) \end{aligned}$$

Lorentz + current conservation: single scalar spectral density ρ

$$\begin{aligned} \frac{d}{ds}(s) &= G_F^2 |V_{ud}|^2 \frac{m^3}{32\pi^2} \left(1 + \frac{2s}{m^2}\right) \left(1 - \frac{s}{m^2}\right)^2 \rho(s) \\ &= G_F^2 |V_{ud}|^2 \kappa(s) \rho(s) \end{aligned}$$

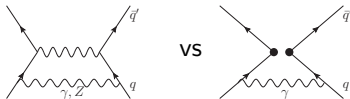
W REGULARIZATION

[Sirlin '82][Marciano, Sirlin '88][Braaten, Li '90]

Effective Hamiltonian $H_W \propto G_F O_{\mu\nu}$

G_F low-energy constant; 4-fermion operator $O_{\mu\nu}$

At $O(\alpha)$ new divergences in EFT \rightarrow need regulator, Z factors



W regularization: $\frac{1}{k^2} \rightarrow -\frac{m_W^2}{k^2(k^2 - m_W^2)}$

[Sirlin '78]

1. universal UV divergences re-absorbed in G_F
2. process-specific corrections in S_{EW} , like a Z factor

Effective Hamiltonian at $O(\alpha)$: $H_W \propto G_F S_{EW}^{1/2} O_{\mu\nu}$
photon at finite $a \neq W$ -regularization

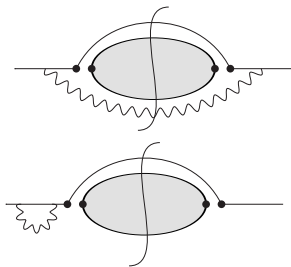
matching required as noted by [Carrasco et al '15][Di Carlo et al '19]

ISOSPIN BREAKING

Initial state

$$\sqrt{S_{EW}} = Z = 1 + \delta Z_\kappa + \delta Z_\rho + \delta Z_{\kappa\rho}$$

$$\begin{aligned} \frac{d}{ds} = & S_{EW} G_F^2 |V_{ud}|^2 \kappa(s) \rho(s) \\ & + G_F^2 |V_{ud}|^2 \delta\kappa(s) \rho(s) \\ & + G_F^2 |V_{ud}|^2 \kappa(s) \delta\rho(s) \\ & + G_F^2 |V_{ud}|^2 \kappa\rho(s) \end{aligned}$$



$\delta\kappa$ known analytically from [Cirigliano et al '00]

$\delta Z_\kappa = -\frac{\alpha}{4\pi} \log(m_W/m)$ [Sirlin '82]

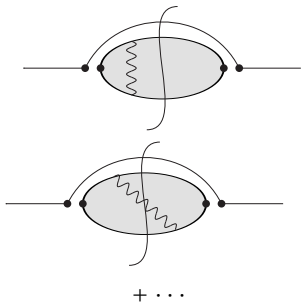
$\delta\kappa$ calculated in fully factorized form [in prep]

ISOSPIN BREAKING

Final state

$$\sqrt{S_{EW}} = Z = 1 + \delta Z_\kappa + \delta Z_\rho + \delta Z_{\kappa\rho}$$

$$\begin{aligned} \frac{d}{ds} = & S_{EW} G_F^2 |V_{ud}|^2 \kappa(s) \rho(s) \\ & + G_F^2 |V_{ud}|^2 \delta\kappa(s) \rho(s) \\ & + G_F^2 |V_{ud}|^2 \kappa(s) \delta\rho(s) \\ & + G_F^2 |V_{ud}|^2 \kappa\rho(s) \end{aligned}$$



$\delta\rho \leftrightarrow$ Lattice QCD+QED calculation

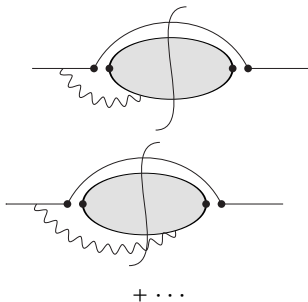
$\delta Z_\rho = \frac{\alpha}{4\pi} c_0$ is like 1-loop $Z_V = 1 + O(g_0^2)$
known from literature/calculable

ISOSPIN BREAKING

Initial-final state

$$\sqrt{S_{EW}} = Z = 1 + \delta Z_\kappa + \delta Z_\rho + \delta Z_{\kappa\rho}$$

$$\begin{aligned} \frac{d}{ds} = & S_{EW} G_F^2 |V_{ud}|^2 \kappa(s) \rho(s) \\ & + G_F^2 |V_{ud}|^2 \delta\kappa(s) \rho(s) \\ & + G_F^2 |V_{ud}|^2 \kappa(s) \delta\rho(s) \\ & + G_F^2 |V_{ud}|^2 \kappa\rho(s) \end{aligned}$$



$\kappa\rho$ from EFT and two-pion channel

[Cirigliano et al' 00]

$Z_{\kappa\rho} = \frac{\alpha}{\pi} (1 + \frac{3}{2}\bar{Q}) \log(m_W/m)$ from W -regularization [Sirlin '82]

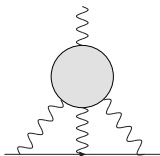
Working on inclusive strategy from Lattice [in prep]

ISOSPIN BREAKING

Strategy

1. take experimental d/ds (e.g. Aleph13, Belle08)
2. $\delta\kappa$ initial state corrections: analytic, under control
3. $\kappa\rho$ initial-finite mixed rad. corr:
analytically known for intermediate two-pion channel
effective field theory [Cirigliano et al '01, '02]
meson dominance models [Flores-Talpa et al. '06, '07]
new results from phenomenological models [Roig et al '23]
4. define $\delta_{EM} \equiv \kappa(s) + \delta\kappa(s) + \kappa\rho(s)$ and calculate:
$$\frac{1}{S_{EW}G_F^2|V_{ud}|^2} \frac{1}{\delta_{EM}(s)} \frac{d^{\text{exp}}}{ds} = \rho(s) + \delta\rho(s)$$
5. Laplace transform to Euclidean time
6. add difference $e\ell - \tau$ evaluated from LQCD+QED

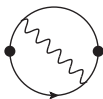
SYNERGY



from QCD we need a **4-point function** $f(x, y, z, t)$:
known kernel with details of photons and muon line
1 pair of point sources (x, y) , sum over z, t exact at sink
stochastic sampling over (x, y) (based on $|x - y|$)

Successful strategy: x10 error reduction

[RBC '16]



from QCD we need a **4-point function** $f(x, y, z, t)$:
 $(g - 2)_\mu$ kernel + photon propagator

Similar problem → re-use HLbL point sources!

The RBC & UKQCD collaborations

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CONTRIBUTION TO a_μ

Time-momentum representation

[Bernecker, Meyer, '11]

$$G^\gamma(t) = \frac{1}{3} \sum_k \int d\vec{x} \langle j_k^\gamma(x) j_k^\gamma(0) \rangle \quad \rightarrow \quad a_\mu = 4\alpha^2 \sum_t w_t G^\gamma(t)$$

Isospin decomposition of u, d current

$$j_\mu^\gamma = \frac{i}{6} (\bar{u}\gamma_\mu u + \bar{d}\gamma_\mu d) + \frac{i}{2} (\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d) = j_\mu^{(0)} + j_\mu^{(1)}$$

$$\frac{i}{2} (\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d), \left[\begin{array}{c} I = 1 \\ I_3 = 0 \end{array} \right] \quad \rightarrow \quad j_\mu^{(1,-)} = \frac{i}{2} (\bar{u}\gamma_\mu d), \left[\begin{array}{c} I = 1 \\ I_3 = -1 \end{array} \right]$$

Isospin 1 charged correlator $G_{11}^W = \frac{1}{3} \sum_k \int d\vec{x} \langle j_k^{(1,+)}(x) j_k^{(1,-)}(0) \rangle$

$$G_{II'}^\gamma \equiv \frac{1}{3} \sum_k \int d\vec{x} \langle j_k^{(I)}(x) j_k^{(I')}(0) \rangle, \quad \delta G^{11} \equiv G_{11}^\gamma - G_{11}^W$$

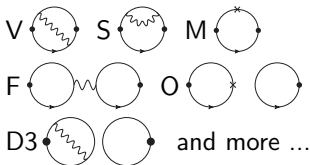
FINAL STATE

IB corrections for charged (τ) and neutral (ee)
difference of τ and ee spectral densities in Euclidean time

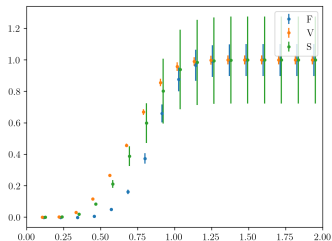
[MB et al PoS'18]

[MB Edinburgh'22]

many (quark) diagrams involved:



plot = window \times μ -kernel \times ($V - F$)
48l phys.pion mass ensemble



first calculation of all diagrams [BMWc '20]

ongoing RBC/UKQCD effort

significant stat. improvement for leading-diagrams

first results for sub-leading diagrams

INCLUSIVITY PROBLEM

Take $\kappa\rho$ from EFT \rightarrow restrict to two-pion channel
discard G_{00}^γ , keep G_{01}^γ

Lattice calculation fully inclusive in energy (cut at m_τ) and channels

G_{01} **mostly** dominated by $\pi\pi$. Is it correct?

simple estimate $a^W[3\pi] \leq 20\%$ of $a^W[2\pi]$ [MB Edinburgh '22]

Isospin-breaking in 2π and 3π from [Colangelo et al 22][Hoferichter et al '23]

IB correction of $a^W[3\pi] \approx -1 \cdot 10^{-10}$

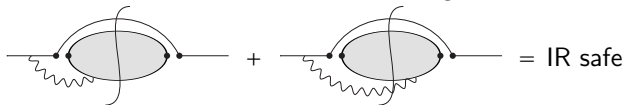
IB correction of $a^W[2\pi] \approx +1 \cdot 10^{-10}$

warning if precision from Lattice $\ll 2 \cdot 10^{-10}$

LONG-DISTANCE

Final dream

Infrared divergences cancel out in total rate
cancellations occur in subset of diagrams



Intermediate two-pion channel
effective field theory
meson dominance models

[Cirigliano et al '01, '02]

[Flores-Talpa et al. '06, '07]

Full estimate in LQCD+QED

[MB et al in prep]

integral of inf.vol. kernel w/ 3-point QCD correlators

working on proof for analytic continuation, ie no inverse problem

CONCLUSIONS

Windows very powerful quantities: **intermediate window** a_μ^W
hadronic τ -decays can shed light on tension lattice vs e^+e^-

τ data **very competitive** on intermediate window
historic tension w/ ee data and in IB τ effects
preliminary analysis Aleph $< 0.5\%$ accuracy on a_μ^W
(old) LQCD IB effects precision $O(1.5) \cdot 10^{-10}$ [MB Edinburgh '22]
new EuroHPC allocation, blinding

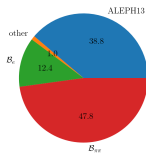
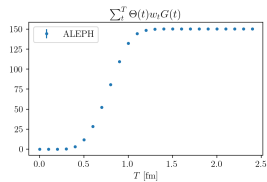
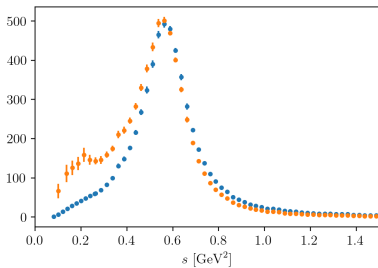
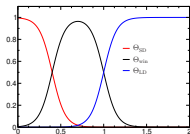
→ Goal for ee - τ correction is with error $\sim O(1) \times 10^{-10}$

Work in progress to finalize full formalism [MB et al, in prep]
W-regularization and short-distance corrections
(re-)calculation of initial state rad.cor.
initial-final rad.cor: proof for analytic continuation
numerical calculation of final state IB corrections
relevant also for QED correction to HVP

Thanks for your attention

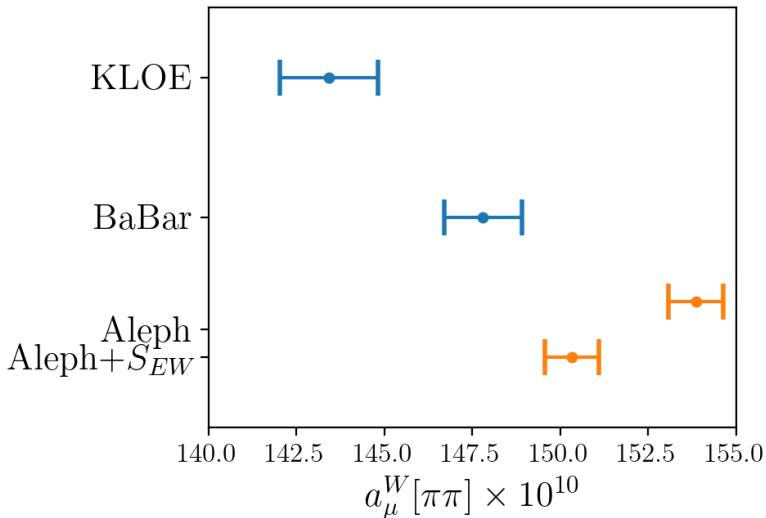
Experimental τ data **very competitive** for intermediate window

Mind window
[0.4 fm, 1.0 fm]



UNCORRECTED COMPARISON OF a_μ^W

Experimental τ data **very competitive** for intermediate window



(also recent similar analysis by Masiuan et al. 2310.14102)