Isospin breaking in τ data for $(g-2)_\mu$

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for the RBC/UKQCD collaborations

The 17th International Workshops on Tau Lepton Physics,
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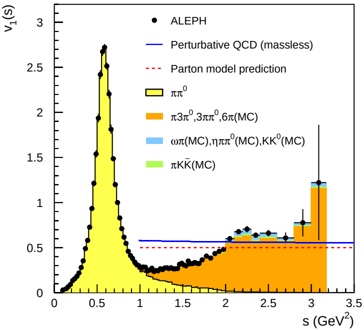
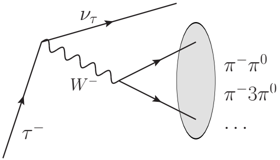
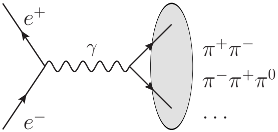
Motivations for τ





$V - A$ current

Final states $I = 1$ charged



EM current

Final states $I = 0, 1$ neutral

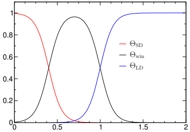
τ data can improve $a_\mu[\pi\pi]$

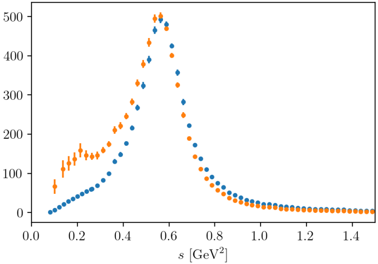
$\rightarrow$ 72% of total Hadronic LO

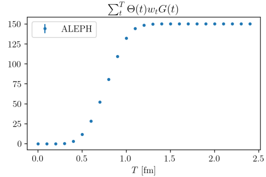
$\rightarrow$ competitive precision on a_μ^W

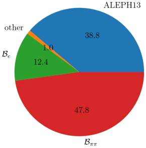
Experimental τ data very competitive for intermediate window

Mind window
[0.4 fm, 1.0 fm]



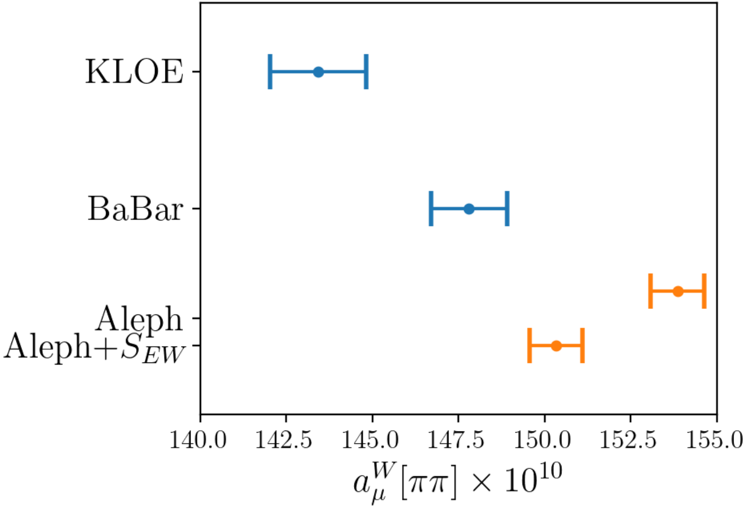






Uncorrected comparison of a_μ^W

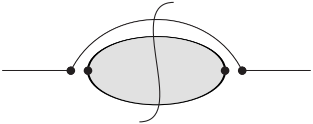
Experimental τ data very competitive for intermediate window



(also recent similar analysis by Masjuan et al. 2310.14102)

Hadronic τ decays

$$\mathcal{M}(P,q,p_1\cdots p_{n_f}|f) = \frac{G_\mathrm{F} V_\mathrm{ud}}{\sqrt{2}}\, \bar{u}_\nu(-q)\gamma^L_\mu u_\tau(P)\ \langle f, p_1\cdots p_{n_f}|\mathcal{J}^-_\mu(0)|0\rangle$$



$$\begin{aligned} d\Gamma &= \frac{1}{2m} d\Phi_q \sum_f d\Phi_f \sum_{\text{spin}} |\mathcal{M}(P,q,p_1\cdots p_{n_f}|f)|^2 \\ &= \frac{1}{2m} d\Phi_q \frac{G_\mathrm{F}^2 |V_\mathrm{ud}|^2}{2} \mathcal{L}_{\mu\nu}(P,q)\, \rho_{\mu\nu}(p) \end{aligned}$$

Lorentz + current conservation: single scalar spectral density ρ

$$\begin{aligned} \frac{d\Gamma(s)}{ds} &= G_\mathrm{F}^2 |V_\mathrm{ud}|^2 \frac{m^3}{32\pi^2} \left(1 + \frac{2s}{m^2}\right)\left(1 - \frac{s}{m^2}\right)^2 \rho(s) \\ &= G_\mathrm{F}^2 |V_\mathrm{ud}|^2\, \kappa(s)\, \rho(s) \end{aligned}$$

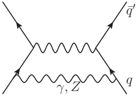
W REGULARIZATION

[Sirlin '82][Marciano, Sirlin '88][Braaten, Li '90]

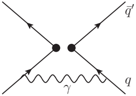
Effective Hamiltonian $H_W \propto G_{\mathrm{F}} O_{\mu\nu}$

G_{F} low-energy constant; 4-fermion operator $O_{\mu\nu}$

At $O(\alpha)$ new divergences in EFT $\to$ need regulator, Z factors



vs



W regularization: $\dfrac{1}{k^2} \to -\dfrac{m_W^2}{k^2(k^2 - m_W^2)}$ [Sirlin '78]

1. universal UV divergences re-absorbed in G_{F}
2. process-specific corrections in S_{EW}, like a Z factor

Effective Hamiltonian at $O(\alpha)$: $H_W \propto G_{\mathrm{F}} S_{EW}^{1/2} O_{\mu\nu}$

photon at finite $a \neq W$-regularization

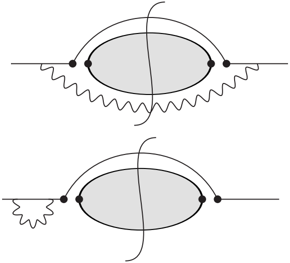
matching required as noted by [Carrasco et al '15][Di Carlo et al '19]

Isospin breaking

Initial state

$$\sqrt{S_{EW}} = Z = 1 + \delta Z_\kappa + \delta Z_\rho + \delta Z_{\kappa\rho}$$

$$\begin{aligned} \frac{d\Gamma}{ds} = & S_{EW} G_F^2 |V_{ud}|^2 \kappa(s)\,\rho(s) \\ & + G_F^2 |V_{ud}|^2\, \delta\kappa(s)\,\rho(s) \\ & + G_F^2 |V_{ud}|^2\, \kappa(s)\,\delta\rho(s) \\ & + G_F^2 |V_{ud}|^2\, \Delta_{\kappa\rho}(s) \end{aligned}$$

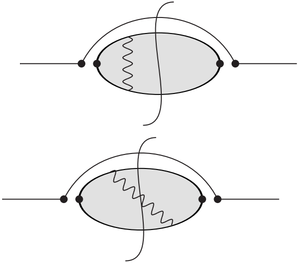


$\delta\kappa$ known analytically from [Cirigliano et al '00]
$\delta Z_\kappa = -\frac{\alpha}{4\pi} \log(m_W/m)$ [Sirlin '82]

$\delta\kappa$ calculated in fully factorized form [in prep]

$$\sqrt{S_{EW}} = Z = 1 + \delta Z_\kappa + \delta Z_\rho + \delta Z_{\kappa\rho}$$

$$\frac{d\Gamma}{ds} = S_{EW} G_F^2 |V_{ud}|^2 \, \kappa(s)\, \rho(s)$$
$$+ G_F^2 |V_{ud}|^2 \, \delta\kappa(s)\, \rho(s)$$
$$+ G_F^2 |V_{ud}|^2 \, \kappa(s)\, \delta\rho(s)$$
$$+ G_F^2 |V_{ud}|^2 \, \Delta_{\kappa\rho}(s)$$



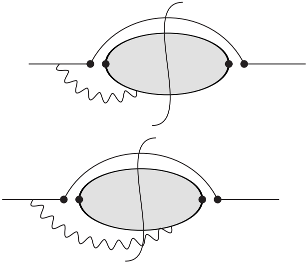
$+ \cdots$

$\delta\rho \leftrightarrow$ Lattice QCD+QED calculation

$\delta Z_\rho = \frac{\alpha}{4\pi} c_0$ is like 1-loop $Z_V = 1 + O(g_0^2)$
known from literature/calculable

$$\sqrt{S_{EW}} = Z = 1 + \delta Z_\kappa + \delta Z_\rho + \delta Z_{\kappa\rho}$$

$$\begin{aligned}\frac{d\Gamma}{ds} = {} & S_{EW} G_F^2 |V_{ud}|^2\, \kappa(s)\, \rho(s) \\ & + G_F^2 |V_{ud}|^2\, \delta\kappa(s)\, \rho(s) \\ & + G_F^2 |V_{ud}|^2\, \kappa(s)\, \delta\rho(s) \\ & + G_F^2 |V_{ud}|^2\, \Delta_{\kappa\rho}(s)\end{aligned}$$



$+\cdots$

$\Delta_{\kappa\rho}$ from EFT and two-pion channel [Cirigliano et al' 00]

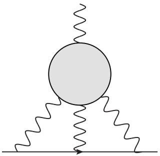
$Z_{\kappa\rho} = \frac{\alpha}{\pi}(1 + \frac{3}{2}\bar{Q}) \log(m_W/m)$ from W-regularization [Sirlin '82]

Working on inclusive strategy from Lattice [in prep]

Strategy

1. take experimental $d\Gamma/ds$ (e.g. Aleph13, Belle08)
2. $\delta\kappa$ initial state corrections: analytic, under control
3. $\Delta_{\kappa\rho}$ initial-finite mixed rad. corr:
   analytically known for intermediate two-pion channel
   effective field theory [Cirigliano et al '01, '02]
   meson dominance models [Flores-Talpa et al. '06, '07]
   new results from phenomenological models [Roig et al '23]
4. define $\delta\Gamma_{EM} \equiv \kappa(s) + \delta\kappa(s) + \Delta_{\kappa\rho}(s)$ and calculate:
$$\frac{1}{S_{EW} G_{\mathrm{F}}^2 |V_{\mathrm{ud}}|^2} \frac{1}{\delta\Gamma_{EM}(s)} \frac{d\Gamma^{\mathrm{exp}}}{ds} = \rho(s) + \delta\rho(s)$$
5. Laplace transfrom to Euclidean time
6. add difference $ee - \tau$ evaluated from LQCD+QED

Synergy



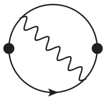
from QCD we need a 4-point function $f(x, y, z, t)$:

known kernel with details of photons and muon line

1 pair of point sources (x, y), sum over z, t exact at sink

stochastic sampling over (x, y) (based on $|x - y|$)

Successfull strategy: x10 error reduction [RBC '16]



from QCD we need a 4-point function $f(x, y, z, t)$:

$(g-2)_\mu$ kernel + photon propagator

Similar problem $\rightarrow$ re-use HLbL point sources!

The RBC & UKQCD collaborations

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BNL and BNL/RBRC
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Taku Izubuchi
Yong-Chull Jang
Chulwoo Jung
Christopher Kelly
Meifeng Lin
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Shigemi Ohta (KEK)
Amarjit Soni
Raza Sufian
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Callum Radley-Scott
Chris Sachrajda

Stony Brook University
Fangcheng He
Sergey Syritsyn (RBRC)

Contribution to a_μ

Time-momentum representation [Bernecker, Meyer, '11]

$$G^\gamma(t) = \tfrac{1}{3}\sum_k \int d\vec{x}\ \langle j_k^\gamma(x) j_k^\gamma(0)\rangle \quad \to \quad a_\mu = 4\alpha^2 \sum_t w_t G^\gamma(t)$$

Isospin decomposition of u, d current

$$j_\mu^\gamma = \tfrac{i}{6}\left(\bar{u}\gamma_\mu u + \bar{d}\gamma_\mu d\right) + \tfrac{i}{2}\left(\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d\right) = j_\mu^{(0)} + j_\mu^{(1)}$$

$$\tfrac{i}{2}\left(\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d\right), \begin{bmatrix} I = 1 \\ I_3 = 0 \end{bmatrix} \quad \to \quad j_\mu^{(1,-)} = \tfrac{i}{\sqrt{2}}\left(\bar{u}\gamma_\mu d\right), \begin{bmatrix} I = 1 \\ I_3 = -1 \end{bmatrix}$$

Isospin 1 charged correlator $G_{11}^W = \tfrac{1}{3}\sum_k \int d\vec{x}\ \langle j_k^{(1,+)}(x) j_k^{(1,-)}(0)\rangle$

$$G_{II'}^\gamma \equiv \frac{1}{3}\sum_k \int d\vec{x} \langle j_k^{(I)}(x)\, j_k^{(I')}(0)\rangle\,, \quad \delta G^{11} \equiv G_{11}^\gamma - G_{11}^W$$

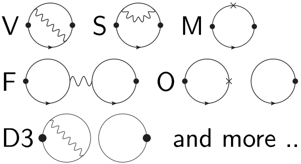
Final state

IB corrections for charged (τ) and neutral (ee) [MB et al PoS'18]

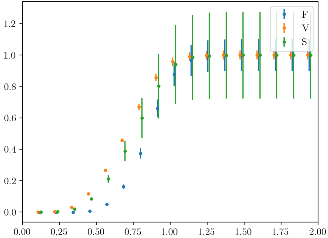
difference of τ and ee spectral densities in Euclidean time

[MB Edinburgh'22]

many (quark) diagrams involved:



and more ...



plot = window $\times$ μ-kernel $\times(V-F)$

48I phys.pion mass ensemble

first calculation of all diagrams [BMWc '20]

ongoing RBC/UKQCD effort

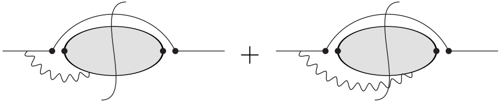
- significant stat. improvement for leading-diagrams
- first results for sub-leading diagrams

Take $\Delta_{\kappa\rho}$ from EFT $\rightarrow$ restrict to two-pion channel
discard G_{00}^{γ}, keep G_{01}^{γ}

Lattice calculation fully inclusive in energy (cut at m_τ) and channels
G_{01} mostly dominated by $\pi\pi$. Is it correct?
simple estimate $a^W[3\pi] \leq 20\%$ of $a^W[2\pi]$ [MB Edinburgh '22]

Isospin-breaking in 2π and 3π from [Colangelo et al 22][Hofericther et al '23]
IB correction of $a^W[3\pi] \approx -1 \cdot 10^{-10}$
IB correction of $a^W[2\pi] \approx +1 \cdot 10^{-10}$
warning if precision from Lattice $\ll 2 \cdot 10^{-10}$

Infrared divergences cancel out in total rate
cancellations occur in subset of diagrams

 $= \text{IR safe}$

Intermediate two-pion channel
effective field theory [Cirigliano et al '01, '02]
meson dominance models [Flores-Talpa et al. '06, '07]

Full estimate in LQCD+QED [MB et al in prep]
integral of inf.vol. kernel w/ 3-point QCD correlators
working on proof for analytic continuation, ie no inverse problem

Conclusions

Windows very powerful quantities: intermediate window a_μ^W
hadronic τ-decays can shed light on tension lattice vs e^+e^-

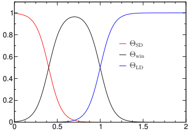
τ data very competitive on intermediate window
historic tension w/ ee data and in IB τ effects
preliminary analysis Aleph $< 0.5\%$ accuracy on a_μ^W
(old) LQCD IB effects precision $O(1.5) \cdot 10^{-10}$ [MB Edinburgh '22]
new EuroHPC allocation, blinding
$\longrightarrow$ Goal for ee-τ correction is with error $\sim O(1) \times 10^{-10}$

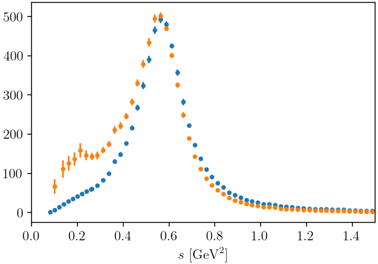
Work in progress to finalize full formalism [MB et al, in prep]
W-regularization and short-distance corrections
(re-)calculation of initial state rad.cor.
initial-final rad.cor: proof for analytic continuation
numerical calculation of final state IB corrections
relevant also for QED correction to HVP

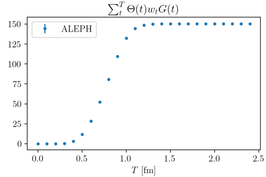
Thanks for your attention

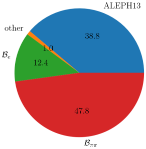
Experimental τ data very competitive for intermediate window

Mind window
[0.4 fm, 1.0 fm]



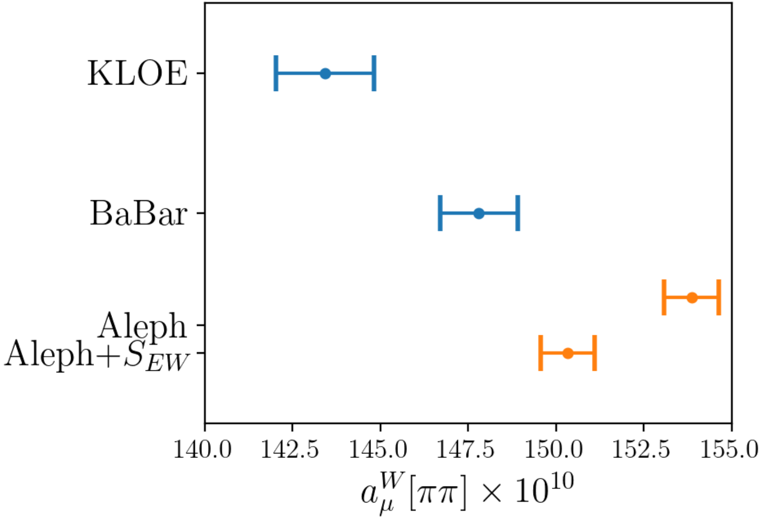






Uncorrected comparison of a_μ^W

Experimental τ data very competitive for intermediate window



(also recent similar analysis by Masjuan et al. 2310.14102)