Precision EW theory at the Z pole

7th FCC Physics Workshop
(Precision challenges: the Z lineshape)

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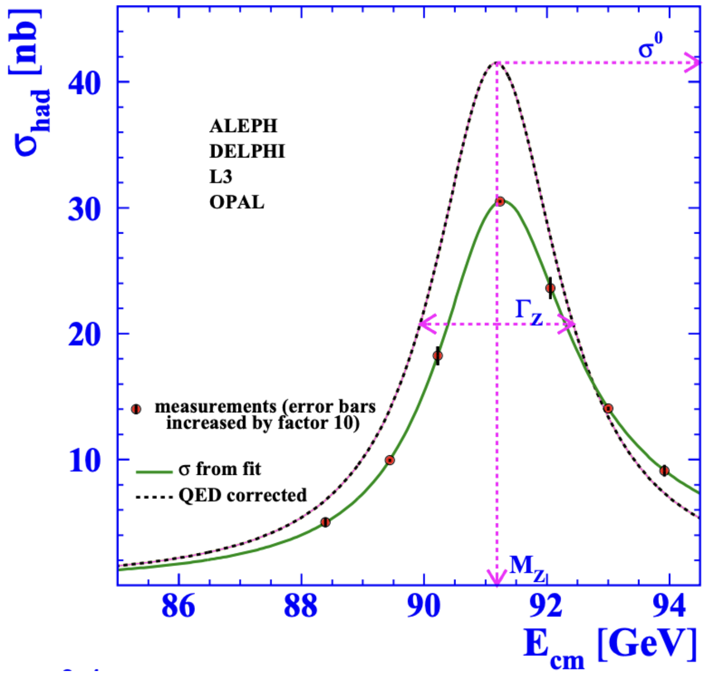
Outline

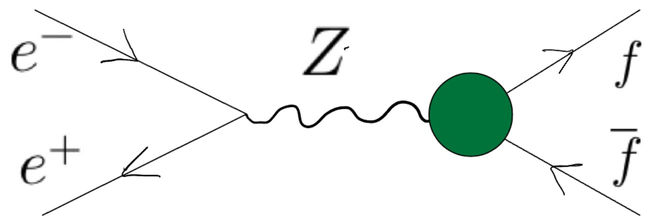
Electroweak (EW) SM predictions at the Z pole

We study the process $e^+e^- \to (Z) \to f\overline{f}$

- Width $\overline{\Gamma}_Z = \sum_{f\backslash t} \overline{\Gamma}_f$,
- Branching ratio $R_f = \overline{\Gamma}_f / \overline{\Gamma}_Z$
- $\sigma_0^f \approx \frac{12\pi}{\overline{M}_Z^2} \frac{\overline{\Gamma}_e \overline{\Gamma}_f}{\overline{\Gamma}_Z^2}$
- $\overline{\Gamma}_f = C\overline{M}_z [R_V^f F_V^f + R_A^f F_A^f]_{s=\overline{M}_Z^2}$
- $F_V^f = |v_f|^2 (1 + \Delta\kappa_v)_{s=\overline{M}_Z^2}$
- $F_A^f = |a_f|^2 (1 + \Delta\kappa_a)_{s=\overline{M}_Z^2}$
- $\overline{M}_Z = M_Z / \sqrt{1 + \frac{\Gamma_Z^2}{M_Z^2}} \approx M_Z - 34\text{MeV}$
- $\overline{\Gamma}_Z = \Gamma_Z / \sqrt{1 + \frac{\Gamma_Z^2}{M_Z^2}} \approx \Gamma_Z - 0.9\text{MeV}$

http://www.cern.ch/LEPEWWG





- R_V^f , R_A^f: capture final-state QED/QCD radiation; known to O(α_s^4), O(α^2), O($\alpha\alpha_s$) Kataev '92 Chetyrkin, Kühn, Kwiatkowski '96 Baikov, Chetyrkin, Kühn, Rittinger '12, v_f, a_f: vector and axial- vector coupling of the $Zf\overline{f}$ vertex
- $\Delta\kappa_v$, $\Delta\kappa_a$, contain higher order EW corrections, which involve loop integral calculations

Initial and final state radiation

- Extraction of EWPOs (pseudo-observables) from real observables which include QED/QCD initial and final state radiation
- The study of the Z-line shape is possible fully analytically at next-to-leading order (NLO) and some partial higher order results, and including selection cuts, with ZFITTER [D. Y. Bardin, 2001, A. B. Arbuzov, 2006] and TOPAZ0 [G. Montagna, et al., 1999]

ALEPH, DELPHI, L3, OPAL and SLD collaborations analyzed the data taken at the Z-boson resonance

- Measured Z-boson width and mass up to a precision of per-mil level
- Effectively testing 1-loop and 2-loop higher order corrections in the Standard Model, which are at sub per-mil level precision

- Griffin [Lisong Chen, 2023], DIZET v. 6.45 [A. Arbuzov et al., 2023] include SM higher order corrections, e.g.: F_V^f, F_A^f, and can be interfaced with Monte-Carlo programs KoralZ [S. Jadach, et al., 2000] and KKMC [A. Arbuzov, et al., 2021] to account for QED and QCD initial-state and final-state radiation, see also [S. Frixione et al.,2022] and [CERN Yellow Reports: Monographs, arXiv:1809.01830], also see [Juergen Reuter et al., Stefano Frixione, Giovanni Stagnitto in Software Computing: Generators, and the talk by Fulvio Piccinini]

Forward-backward asymmetry at the Z pole

Pseudo-observables (QED effects subtracted), unfolded at the Z peak

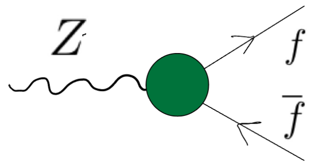
forward-backward asymmetry $A_{\rm FB}^{\rm f\bar{f}} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B} = \frac{3}{4} A_{\rm e} A_{\rm f}$

$$A_{\rm f} = \frac{2\Re e \frac{v_f}{a_f}}{1 + \left(\Re e \frac{v_f}{a_f}\right)^2} = \frac{1 - 4|Q_f|\sin^2\theta_{\rm eff}^{\rm f}}{1 - 4|Q_f|\sin^2\theta_{\rm eff}^{\rm f} + 8Q_f^2(\sin^2\theta_{\rm eff}^{\rm f})^2}$$

Definition of the effective weak mixing angle

$$\sin^2\theta_{\rm eff}^{f} = \frac{1}{4|Q_f|}\left(1 - \Re e \frac{v_f}{a_f}\right) = \left(1 - \frac{\overline{M}_W^2}{\overline{M}_Z^2}\right)\left(1 + \Delta\kappa_Z^f(\overline{M}_Z^2)\right)$$

v_f and a_f are effective vector coupling and axial-vector coupling of the $Zf\bar{f}$ vertex, $\Delta\kappa_Z^f(\overline{M}_Z^2)$ contain the perturbative EW corrections



Perturbative EW corrections

Effective weak mixing angle $\sin^2\theta_{\rm eff}^{\rm b} = \left(1 - \frac{M_W^2}{M_Z^2}\right)\left(1 + \Delta\kappa_Z^{\rm b}(M_Z^2)\right)$

Order	$\Delta\kappa_Z^{\rm b}[10^{-4}]$	Order	$\Delta\kappa_Z^{\rm b}[10^{-4}]$
α	468.945	$\alpha_t^2\alpha_s$	1.362
$\alpha\alpha_s$	-42.655	α_t^3	0.123
$\alpha_t\alpha_s^2$	-7.074	$\alpha_{\rm ferm}^2$	3.866
$\alpha_t\alpha_s^3$	-1.196	$\alpha_{\rm bos}^2$	-0.986

$$\sin^2\theta_{\rm eff}^{\rm b} = s_0 + d_1 L_H + d_2 L_H^2 + d_3\Delta_\alpha + d_4\Delta_t + d_5\Delta_t^2 + d_6\Delta_t L_H + d_7\Delta_{\alpha_s} + d_8\Delta_t\Delta_{\alpha_s} + d_9\Delta_Z$$

$$L_H = \log\left(\frac{M_{\rm H}}{125.7}\right), \qquad \Delta_t = \left(\frac{m_{\rm t}}{173.2}\right)^2 - 1, \qquad \Delta_Z = \frac{M_{\rm Z}}{91.1876} - 1,$$

$$\Delta_\alpha = \frac{\Delta\alpha}{0.0059} - 1, \qquad \Delta_{\alpha_s} = \frac{\alpha_{\rm s}}{0.1184} - 1$$

$$s_0 = 0.232704, \quad d_1 = 4.723\times10^{-4}, \quad d_2 = 1.97\times10^{-4}, \quad d_3 = 2.07\times10^{-2},$$
$$d_4 = -9.733\times10^{-4}, \quad d_5 = 3.93\times10^{-4}, \quad d_6 = -1.38\times10^{-4},$$
$$d_7 = 2.42\times10^{-4}, \quad d_8 = -8.10\times10^{-4}, \quad d_9 = -0.664$$

Input Parameters: M_Z, Γ_Z, M_W, Γ_W, M_H, m_t, α_s, and $\Delta\alpha$, which is the shift of the electromagnetic fine structure constant due to light fermion loops between the scales $q^2 = 0$ and M_Z^2, $\alpha_t = y_t^2/(4\pi)$, y_t is top Yukawa coupling

Z-boson form factors at two-loop accuracy

[I. Dubovyk, A. Freitas, J. Gluza, T. Riemann, J. Usovitsch, Electroweak pseudo-observables and Z-boson form factors at two-loop accuracy, JHEP 08 (2019) 113.]

Form fact.	Born	$\mathcal{O}(\alpha)$	$\mathcal{O}(\alpha\alpha_s)$	$\mathcal{O}(\alpha\alpha_s)$ non-fact.	$\mathcal{O}(\alpha_t\alpha_s^2,\ \alpha_t\alpha_s^3,\ \alpha_t^2\alpha_s,\ \alpha_t^3)$	$\mathcal{O}(N_f^2\alpha^2)$	$\mathcal{O}(N_f\alpha^2)$	$\mathcal{O}(\alpha_{bos}^2)$
F_V^ℓ $[10^{-5}]$	39.07	−24.86	2.41	–	0.35	1.47	2.37	0.27
F_A^ℓ $[10^{-5}]$	3309.44	118.59	9.46	–	1.22	8.60	2.60	0.45
$F_{V,A}^\nu$ $[10^{-5}]$	3309.44	127.56	9.46	–	1.22	8.60	3.83	0.39
$F_V^{u,c}$ $[10^{-5}]$	544.88	−44.80	7.29	−0.39	1.02	−1.67	3.25	0.33
$F_A^{u,c}$ $[10^{-5}]$	3309.44	120.79	9.46	−0.98	1.22	8.60	3.27	0.44
$F_V^{d,s}$ $[10^{-5}]$	1635.01	5.84	9.64	−0.80	1.32	0.71	3.45	0.37
$F_A^{d,s}$ $[10^{-5}]$	3309.44	123.78	9.46	−1.14	1.22	8.60	3.11	0.42
F_V^b $[10^{-5}]$	1635.01	−26.16	9.64	3.13	1.32	0.71	1.77	1.05
F_A^b $[10^{-5}]$	3309.44	78.26	9.46	4.45	1.22	8.60	0.13	1.18

Table: *Contributions of different perturbative orders to the Z vertex form factors. A fixed value of M_W has been used as input, instead of G_μ. N_f^n refers to corrections with n closed fermions loops, whereas α_{bos}^2 denotes corrections without closed fermions loops. Furthermore, $\alpha_t = y_t/(4\pi)$ where y_t is the top Yukawa coupling.*

- Some progress towards three-loop Electroweak with fermionic three-loop corrections at $\mathcal{O}(\alpha^2\alpha_s)$ [Lisong Chen and Ayres Freitas, JHEP 03 (2021) 215]

Theory input from different origins

- We compute EW higher order corrections to put new limits on new physics $\rightarrow$ multi-loop corrections in full SM
- Extraction of EWPOs (pseudo-observables) from real observables $\rightarrow$ QED/QCD, additional SM effects
- Other Electroweak parameters (input parameters) $\rightarrow$ m_t, α_s, extracted from other processes

Theory uncertainties

- Theory error estimate is not well defined, ideally $\delta_{th} \ll \delta_{exp}$
- Common methods [Ayres Freitas, arXiv:1401.2447v4]:
  - Count prefactors (α, N_c, N_f , ...)
  - Extrapolation of perturbative series
  - Renormalization scale dependence
  - Renormalization scheme dependence

Theory status

- 1-loop and leading 2-loop corrections
  Veltman, Passarino, Sirlin, Marciano, Bardin, Hollik, Riemann, Degrassi, Kniehl, ...

- Completed 2-loop results for M_W, Z-pole observables
  Freitas, Hollik, Walter, Weiglein '00 Awramik, Czakon, Freitas '06 Awramik, Czakon '02 Hollik, Meier, Uccirati '05,07 Onishchenko, Veretin '02 Awramik, Czakon, Freitas, Kniehl '08 Awramik, Czakon, Freitas, Weiglein '04 Freitas '13,14 Dubovyk, Freitas, Gluza, Riemann, Usovitsch '16,18

- Leading 3- and 4-loop results (enhanced by y_t and/or N_f)
  Chetyrkin, Kühn, Steinhauser '95 Schröder, Steinhauser '05 Faisst, Kühn, Seidensticker, Veretin '03 Chetyrkin et al. '06 Boughezal, Tausk, v. d. Bij '05 Boughezal, Czakon '06 Chen, Freitas '20

Historical time stamps for Electroweak $\sin^2\theta^{\rm b}_{\rm eff}$

- One-loop corrections to the $\sin^2\theta^{\rm b}_{\rm eff}$ [A. Akhundov, D. Bardin, T. Riemann, Electroweak one loop corrections to the decay of the neutral vector boson, Nucl. Phys. B276 (1986) 1.] [W. Beenakker, W. Hollik, The width of the Z boson, Z. Phys. C40 (1988) 141.]

- Two-loop electroweak corrections to the $\sin^2\theta^{\rm b}_{\rm eff}$ [Awramik, M. Czakon, A. Freitas, B. Kniehl, Two-loop electroweak fermionic corrections to $\sin^2\theta^{\rm b}_{\rm eff}$, Nucl. Phys. B813 (2009) 174-187.] [I. Dubovyk, A. Freitas, J. Gluza, T. Riemann, J. Usovitsch, The two-loop electroweak bosonic corrections to $\sin^2\theta^{\rm b}_{\rm eff}$, Phys. Lett. B762 (2016) 184-189.]

- The leap in complexity going to three loop may be judged by how long it took to go from one to two loops calculation

Electroweak precision physics

	Experiment	Theory uncertainty	Main source
$M_W\,[\mathrm{MeV}]$	80385 ± 15	4	$N_f^2\alpha^3$, $N_f\alpha^2\alpha_\mathrm{s}$
$\sin^2\theta^\mathrm{l}_\mathrm{eff}\,[10^{-5}]$	23153 ± 16	4.5	$N_f^2\alpha^3$, $N_f\alpha^2\alpha_\mathrm{s}$
$\Gamma_Z\,[\mathrm{MeV}]$	2495.2 ± 2.3	0.4	$N_f^2\alpha^3$, $N_f\alpha^2\alpha_\mathrm{s}$, $\alpha\alpha_\mathrm{s}^2$
$\sigma^0_\mathrm{had}\,[\mathrm{pb}]$	41540 ± 37	6	$N_f^2\alpha^3$, $N_f\alpha^2\alpha_\mathrm{s}$
$R_f = \Gamma_Z^f/\Gamma_Z^\mathrm{had}\,[10^{-5}]$	21629 ± 66	15	$N_f^2\alpha^3$, $N_f\alpha^2\alpha_\mathrm{s}$

- The number of Z-bosons collected at LEP is 1.7×10^7
- New results fermionic $\mathcal{O}(\alpha^2\alpha_\mathrm{s})$ [Lisong Chen and Ayres Freitas, JHEP 03 (2021) 215] are not included, yet
- N_f counts the number of closed fermion loops

Overview of future experiments as of 2021

	Experiment uncertainty			Theory uncertainty
	ILC	**CEPC**	**FCC-ee**	**Current**
$M_W\,[\mathrm{MeV}]$	3-4	3	1	4
$\sin^2\theta^{\mathrm{l}}_{\mathrm{eff}}\,[10^{-5}]$	1	2.3	0.6	4.5
$\Gamma_Z\,[\mathrm{MeV}]$	0.8	0.5	0.1	0.4
$R_f\,[10^{-5}]$	14	17	6	15

- FCC-ee Tera-Z operating at 88-95 GeV producing 5×10^{12} visible Z decays, 5 orders of magnitude more events than at LEP
- FCC-ee Tera-Z reproduces the LEP data in 23 hours and is planned to operate for 5 years
- To match the precision of the experiment we compute 3-loop and 4-loop Standard Model predictions

Overview of future experiments as of 2022

	Experiment uncertainty			Theory uncertainty
	ILC	**CEPC**	**FCC-ee**	**Current**
$M_W\,[\text{MeV}]$	3-4	3	~~1~~0.3	4
$\sin^2\theta^{\text{l}}_{\text{eff}}\,[10^{-5}]$	1	2.3	?0.6	4.5
$\Gamma_Z\,[\text{MeV}]$	0.8	0.5	~~0.1~~0.025	0.4
$R_f\,[10^{-5}]$	14	17	~~6~~1	15

- Recent update from [Alain Blondel, Patrick Janot, Eur.Phys.J.Plus 137 (2022) 1]
- To match the precision of the experiment we compute 3-loop and 4-loop Standard Model predictions

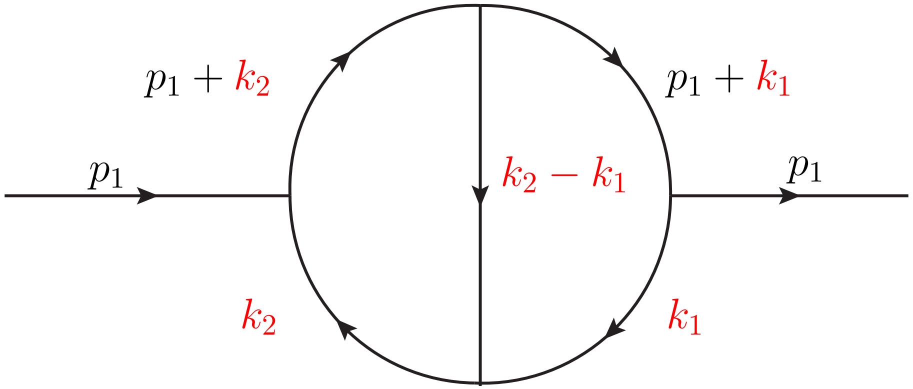
Actual calculation

- We generate systematically Feynman diagrams at 3-loop order and 4-loop order
- Public codes are FeynArts [T. Hahn, 2001] and QGRAF [P. Nogueira, 1993]
- Number of Feynman diagrams grows factorially
- 3-loop full Electroweak order $\sim 400\,000$ Feynman diagrams
- 4-loop involves more than 10 million Feynman diagrams

Summary

- To match the precision of FCC-ee Tera-Z, 1-2 orders improvement in SM theory calculations is required
- At the Z-pole 3-loop and leading 4-loop EW higher order corrections are needed
- Tools like Griffin and DIZET have to be flexible with multi-loop/leg merging for QED MC
- Development of new calculational techniques (numerical/semi-numerical) are crucial
- Results can only be shared as numerical grid or fit formula

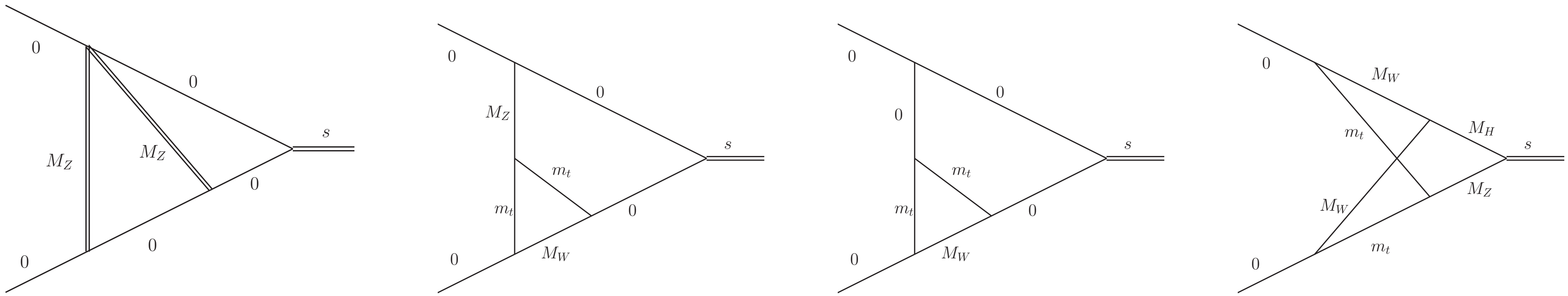
Feynman integral



$$I(a_1, \ldots, a_5) = \int \frac{d^D k_1 d^D k_2}{[k_1{}^2]^{a_1} [(p_1 + k_1)^2]^{a_2} [k_2{}^2]^{a_3} [(p_1 + k_2)^2]^{a_4} [(k_2 - k_1)^2]^{a_5}}$$

- To make Standard Model predictions we compute several thousand different integrals with different values for $\{a_f\}$
- Calculating each Feynman integral individually for every new choice of $\{a_f\}$ is inefficient

Samples of Feynman integrals for the $Z\bar{b}b$ vertex

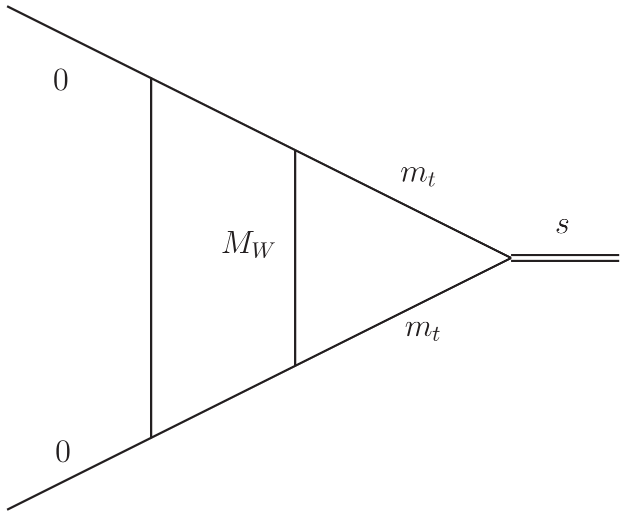


- Number of closed loops grows with the perturbative order
- From Feynman diagrams we can project to scalar integrals
- Feynman integrals are UV and infrared divergent
- Regularized in dimensional regularization with $\epsilon = (4 - D)/2$, D the space time dimension

Numerical evaluation

- Integrals are divergent like $1/\epsilon^{2L}$, L the loop-number
- A cancellation of all divergences is required
- Large cancellations between the terms; require high numerical precision
- General methods for Feynman integral computation: sector decomposition [T. Binoth, G. Heinrich, 2000, G. Heinrich, 2008], Mellin-Barnes approach [V. A. Smirnov:1999, B. Tausk,1999], system of differential equations [Kotikov, 1991, Remiddi, 1997, Gehrmann, Remiddi, 2000]
- General tools to compute Feynman integrals numerically for precision physics: pySecDec [G. Heinrich, S. Jahn,S.P. Jones,M. Kerner,F. Langer,2022], FIESTA5 [A.V. Smirnov, , N. D. Shapurov, L. I. Vysotsky, 2021], DiffExp [M. Hidding, 2006.05510] and AMFlow [Liu, Xiao and Ma, Yan-Qing,2201.11669], SeaSyde [T. Armadillo, R. Bonciani, S. Devoto, N. Rana, A. Vicini]

State of the art 6 years ago

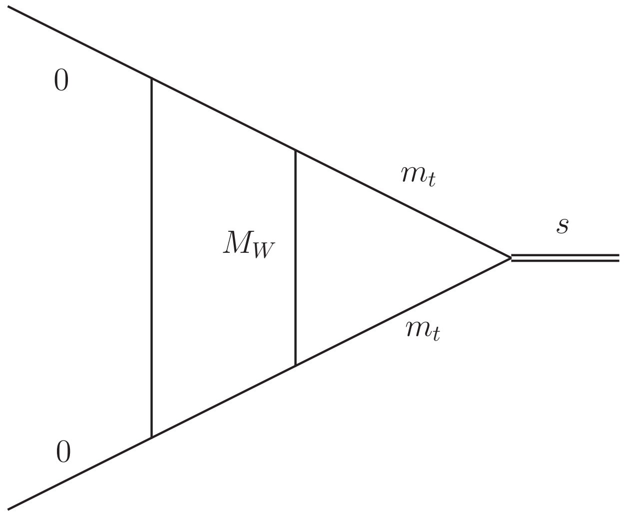


- In physical regions $\left(\frac{s}{M_Z^2}, \frac{M_W^2}{M_Z^2}, \frac{m_t^2}{M_Z^2}\right) = \left(1, \left(\frac{401925}{4559382}\right)^2, \left(\frac{433000}{227969}\right)^2\right)$
- Arbitrary kinematic point, but with restricted accuracy
- A complementary mixture of Mellin-Barnes integral and sector decomposition methods

$$\text{soft13}^{d=4-2\epsilon}[1,1,1,1,1,1,0] = 0.93453624 + 0.54089756\, i$$
$$+(0.1901137256 - 0.6583157563\, i)/\epsilon - 0.209548413480837 0/\epsilon^2$$

- One kinematic point in 1 day

The two-loop example

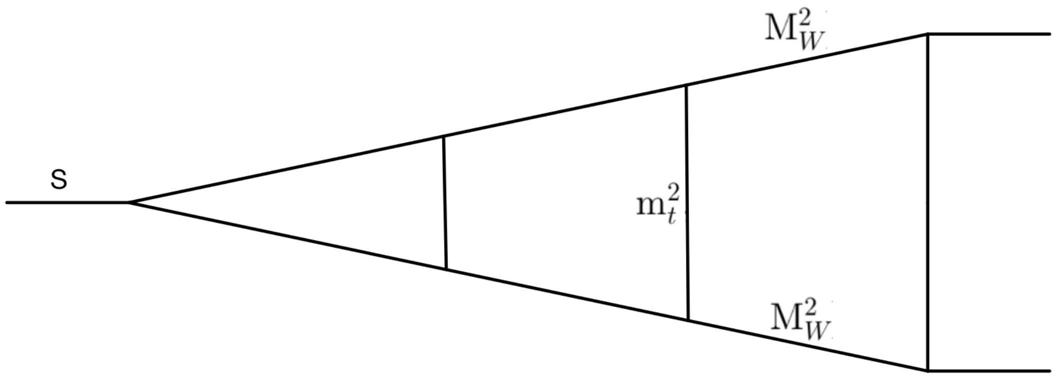


- With the program AMFlow

$$
\begin{aligned}
&\text{soft13}^{d=4-2\epsilon}[1,1,1,1,1,1,0] \\
&= (0.934536247523241 + 0.540897568924577\, i) \\
&+ (0.190113725674667 - 0.658315756362794\, i) 1/\epsilon \\
&- 0.2095484134808370/\epsilon^2
\end{aligned}
$$

- Arbitrary kinematic point in 5 minutes

The state of the art 2021 - automatic computations



- In physical regions $(s, M_W^2, m_t^2) = (1, (\frac{401925}{4559382})^2, (\frac{433000}{227969})^2)$

-> v3t181$^{d=4-2\epsilon}$[1, 1, 1, 1, 1, 1, 1, 1, 1, -3, 0, 0] $=$

$$\frac{2.00000000000}{\epsilon^3} + \frac{9.8700393436 + 18.8495559213\, i}{\epsilon^2} - \frac{26.507336797 - 41.196707081\, i}{\epsilon} + (2.29574523 + 201.06880207\, i) + O(\epsilon)$$

- Fully automated with DiffExp[pySecDec]