(Ultimate) precision of theoretical predictions for Bhabha and diphotons

F. Piccinini

Istituto Nazionale di Fisica Nucleare

7th FCC Physics Workshop

LAPP Annecy, January 29 - February 2, 2024

Motivation for precision measurements of the machine luminosity

$$\sigma = \frac{N}{L}$$

Motivation for precision measurements of the machine luminosity

$$\sigma = \frac{N}{L}$$

Several key measurements at an e^+e^- machine depend on L, e.g.

- σ_Z^0, the Z peak cross section
- light neutrino species from radiative return $(e^+e^- \to \nu\bar{\nu}\gamma)$
- Γ_Z from the line-shape of $e^+e^- \to f\bar{f}$
- M_W and Γ_W from line-shape of $e^+e^- \to W^+W^-$ close to threshold
- total cross section for $e^+e^- \to HZ \Longrightarrow HZZ$ coupling and total Γ_H

A recent example: $\mathbf{N}_\nu$ from $\boldsymbol{\Gamma}_Z^{\rm inv}$ at LEP $\mathbf{Z}$ peak measurements

- assuming lepton universality

$$N_\nu \left(\frac{\Gamma_{\nu\bar{\nu}}}{\Gamma_{ll}}\right)_{\rm SM} = \sqrt{\frac{12\pi R_l^0}{\sigma^0_{\rm had} m_Z^2}} - R_l^0 - (3+\delta_\tau)$$

$$\mathbf{N}_\nu = \mathbf{2.9840 \pm 0.0082}$$

$$\delta N_\nu \simeq 10.5 \frac{\delta n_{\rm had}}{n_{\rm had}} \oplus 3.0 \frac{\delta n_{\rm lept}}{n_{\rm lept}} \oplus 7.5 \frac{\delta \mathcal{L}}{\mathcal{L}}$$

$$\frac{\delta \mathcal{L}}{\mathcal{L}} = 0.061\% \Longrightarrow \delta N_\nu = 0.0046$$

ADLO, SLD and LEPEWWG, Phys. Rept. 427 (2006) 257, hep-ex/0509008

A recent example: N_ν from $\Gamma_Z^{\rm inv}$ at LEP Z peak measurements

- assuming lepton universality

$$N_\nu \left(\frac{\Gamma_{\nu\bar\nu}}{\Gamma_{ll}}\right)_{\rm SM} = \sqrt{\frac{12\pi R_l^0}{\sigma^0_{\rm had} m_Z^2}} - R_l^0 - (3+\delta_\tau)$$

$$\mathbf{N_\nu = 2.9840 \pm 0.0082}$$

$$\delta N_\nu \simeq 10.5\frac{\delta n_{\rm had}}{n_{\rm had}} \oplus 3.0\frac{\delta n_{\rm lept}}{n_{\rm lept}} \oplus 7.5\frac{\delta \mathcal{L}}{\mathcal{L}}$$

$$\frac{\delta \mathcal{L}}{\mathcal{L}} = 0.061\% \Longrightarrow \delta N_\nu = 0.0046$$

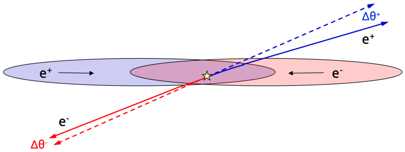
ADLO, SLD and LEPEWWG, Phys. Rept. 427 (2006) 257, hep-ex/0509008

2σ away from SM: hint for BSM? Right handed neutrinos?

Beam-beam effects studied in detail recently

G. Voutsinas, E. Perez, M. Dam, P. Janot, arXiv:1908.01704

- systematics bias on the acceptance due to e.m. beam-beam interactions $\Longrightarrow$ underestimate of luminosity by $\sim 0.1\%$



- together with an update on Bhabha cross sections (see later) $\Longrightarrow$ Luminosity

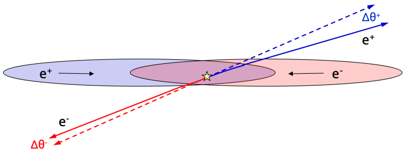
P. Janot, S. Jadach, arXiv:1912.02067

$$\mathbf{N_\nu = 2.9963 \pm 0.0074}$$

Beam-beam effects studied in detail recently

G. Voutsinas, E. Perez, M. Dam, P. Janot, arXiv:1908.01704

- systematics bias on the acceptance due to e.m. beam-beam interactions $\Longrightarrow$ underestimate of luminosity by $\sim 0.1\%$



- together with an update on Bhabha cross sections (see later) $\Longrightarrow$ Luminosity

P. Janot, S. Jadach, arXiv:1912.02067

$$\mathbf{N_\nu = 2.9963 \pm 0.0074}$$

Luminosity is a key quantity for a precision e^+e^- collider

Reference processes for luminosity

Instead of getting the luminosity from machine parameters, it's more effective to exploit the relation

$$\sigma = \frac{N}{L} \quad \rightarrow \quad L = \frac{N_{\rm ref}}{\sigma_{\rm theory}} \qquad \frac{\delta L}{L} = \frac{\delta N_{\rm ref}}{N_{\rm ref}} \oplus \frac{\delta \sigma_{\rm theory}}{\sigma_{\rm theory}}$$

Reference processes for luminosity

Instead of getting the luminosity from machine parameters, it's more effective to exploit the relation

$$\sigma = \frac{N}{L} \quad \to \quad L = \frac{N_{\rm ref}}{\sigma_{\rm theory}} \qquad \frac{\delta L}{L} = \frac{\delta N_{\rm ref}}{N_{\rm ref}} \oplus \frac{\delta \sigma_{\rm theory}}{\sigma_{\rm theory}}$$

Reference processes required to have

- **large rates** (so as not to be statistics limited)
- **low backgrounds**
- **good control of systematics**
  - particle ID, acceptance, . . . (see following talks)
  - **theory**
    - differential cross sections calculable with high theoretical precision
    - fully exclusive Monte Carlo generators required
    - negligible room for possible NP contributions

Luminosity precision level

- In the past (LEP)
  - ⋆ Small-angle Bhabha scattering at LEP: ∼0.05%
- In the past/at present (flavour factories)
  - ⋆ Large-angle QED processes as $e^+e^- \to e^+e^-$ (Bhabha), $e^+e^- \to \gamma\gamma$, $e^+e^- \to \mu^+\mu^-$, to achieve a typical precision at the level of $1 \div 0.1\%$

Luminosity precision level

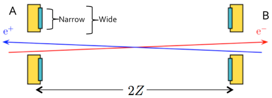
- In the past (LEP)
  - ⋆ Small-angle Bhabha scattering at LEP: ∼0.05%
- In the past/at present (flavour factories)
  - ⋆ Large-angle QED processes as $e^+e^- \to e^+e^-$ (Bhabha), $e^+e^- \to \gamma\gamma$, $e^+e^- \to \mu^+\mu^-$, to achieve a typical precision at the level of $1 \div 0.1\%$
- **Realistic uncertainty target for future e^+e^- colliders?**

Luminosity precision level

- In the past (LEP)
  - ⋆ Small-angle Bhabha scattering at LEP: ∼0.05%
- In the past/at present (flavour factories)
  - ⋆ Large-angle QED processes as $e^+e^- \to e^+e^-$ (Bhabha), $e^+e^- \to \gamma\gamma$, $e^+e^- \to \mu^+\mu^-$, to achieve a typical precision at the level of $1 \div 0.1\%$
- **Realistic uncertainty target for future e^+e^- colliders?**
  - at Z pole 10^{-4} or better for the overall luminosity calibration
  - $\mathcal{O}(10^{-4})$ at $\sqrt{s} \sim 2M_W$ to get $\Delta M_W \simeq 1$ MeV
  - 10^{-5} or better for point-to-point luminosity control

SABS general features

- **Bhabha scattering strongly peaked in the forward region** $d\sigma/d\theta \sim 1/\theta^3$
  $\Longrightarrow$ special lumi detector (LumiCal) covering the region $\theta < 100$ mrad centered around the outgoing beams

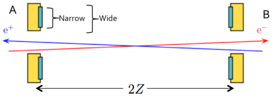


M. Damm, talk at ECFA MiniWorkshop: Luminosity, 16/12/2022

- where it is QED dominated $\sqrt{|t|} \sim \mathcal{O}(1 - 2\,\mathrm{GeV})$ at $\sqrt{s} \sim M_Z$

SABS general features

- **Bhabha scattering strongly peaked in the forward region** $d\sigma/d\theta \sim 1/\theta^3$ $\Longrightarrow$ special lumi detector (LumiCal) covering the region $\theta < 100$ mrad centered around the outgoing beams

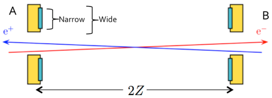


M. Damm, talk at ECFA MiniWorkshop: Luminosity, 16/12/2022

- where it is QED dominated $\sqrt{|t|} \sim \mathcal{O}(1 - 2\,\mathrm{GeV})$ at $\sqrt{s} \sim M_Z$
- **Large statistics $\Longrightarrow$ ideal process for the point-to-point lumi control**

SABS general features

- **Bhabha scattering strongly peaked in the forward region** $d\sigma/d\theta \sim 1/\theta^3$
  $\Longrightarrow$ special lumi detector (LumiCal) covering the region $\theta < 100$ mrad centered around the outgoing beams



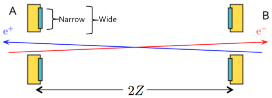
M. Damm, talk at ECFA MiniWorkshop: Luminosity, 16/12/2022

- where it is QED dominated $\sqrt{|t|} \sim \mathcal{O}(1 - 2\,\mathrm{GeV})$ at $\sqrt{s} \sim M_Z$
- **Large statistics $\Longrightarrow$ ideal process for the point-to-point lumi control**

- **Systematics (theory)**
  - **QED corrections**
  - **hadronic contribution to photon vacuum polarization**

SABS general features

- **Bhabha scattering strongly peaked in the forward region** $d\sigma/d\theta \sim 1/\theta^3$ $\Longrightarrow$ special lumi detector (LumiCal) covering the region $\theta < 100$ mrad centered around the outgoing beams



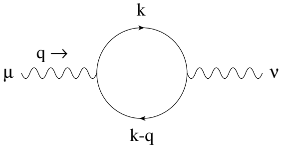
M. Damm, talk at ECFA MiniWorkshop: Luminosity, 16/12/2022

- where it is QED dominated $\sqrt{|t|} \sim \mathcal{O}(1 - 2\,\mathrm{GeV})$ at $\sqrt{s} \sim M_Z$
- **Large statistics $\Longrightarrow$ ideal process for the point-to-point lumi control**

- **Systematics (theory)**
  - **QED corrections**
  - **hadronic contribution to photon vacuum polarization**

- **Systematics (exp)** (see following talks)

Vacuum Polarization



- $\alpha \to \alpha(q^2) \equiv \frac{\alpha}{1-\Delta\alpha(q^2)}$ $\quad\quad \Delta\alpha(q^2) = \Delta\alpha_{e,\mu,\tau,\mathrm{top}}(q^2) + \Delta\alpha^{(5)}_{\mathrm{had}}(q^2)$

- $\Delta\alpha^{(5)}_{\mathrm{had}}$ is an intrinsically non-perturbative contribution. It can be calculated from $e^+e^- \to hadrons$ data using dispersion relations

$$\Delta\alpha^{(5)}_{\mathrm{had}}(q^2) = -\frac{q^2\alpha}{3\pi}\left[P\!\!\int_{4m_\pi^2}^{E^2_{cut}} \frac{R^{data}_{had}(s)}{s(s-q^2)} ds + P\!\!\int_{E^2_{cut}}^{\infty} \frac{R^{pQCD}_{had}(s)}{s(s-q^2)} ds \right]$$

- it is affected by an uncertainty, due to low energy data on $\sigma_{had}(s)$ which is improving with time
  - low energy physics of muon $g-2$ is triggering new data and efforts in $\Delta\alpha_{\mathrm{had}}$

QED corrections, order of magnitudes

- after LEP, several progresses in perturbative (two-loop) (three-loop calculations to μe scattering ongoing) contributions to QED Bhabha scattering and different matching schemes between fixed order and multiphoton emission (e.g. YFS and parton shower for exclusive event generation)

QED corrections, order of magnitudes

- after LEP, several progresses in perturbative (two-loop) (three-loop calculations to μe scattering ongoing) contributions to QED Bhabha scattering and different matching schemes between fixed order and multiphoton emission (e.g. YFS and parton shower for exclusive event generation)

Loosely and schematically, the corrections to the LO cross section can be arranged as (collinear log $L \equiv \log \frac{Q^2}{m_e^2}$)

LO	α^0		
NLO	αL	α	
NNLO	$\frac{1}{2}\alpha^2 L^2$	$\frac{1}{2}\alpha^2 L$	$\frac{1}{2}\alpha^2$
h.o.	$\sum_{n=3}^{\infty} \frac{\alpha^n}{n!} L^n$	$\sum_{n=3}^{\infty} \frac{\alpha^n}{n!} L^{n-1}$	$\cdots$

Red: matched PS, YFS, SF + NLO

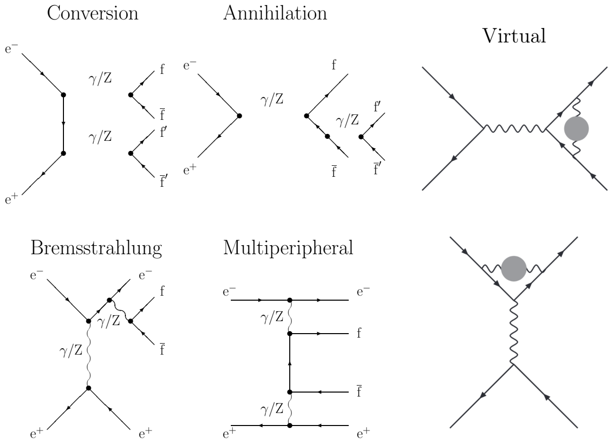
QED corrections, order of magnitudes

- after LEP, several progresses in perturbative (two-loop) (three-loop calculations to μe scattering ongoing) contributions to QED Bhabha scattering and different matching schemes between fixed order and multiphoton emission (e.g. YFS and parton shower for exclusive event generation)

Loosely and schematically, the corrections to the LO cross section can be arranged as (collinear log $L \equiv \log \frac{Q^2}{m_e^2}$)

LO	90%		
NLO	10%	0.5%	
NNLO	0.5%	0.05%	0.01%
h.o.	0.01%	$\cdots$	$\cdots$

- virtual and real pair corrections part of the NNLO calcualtion



P. Janot, S. Jadach, arXiv:1912.02067

Past and recent updates

- **theoretical error in SABS at LEP1 by the end of operation**

Type of correction/error	(%)	(%)	updated (%)
missing photonic $O(\alpha^2 L)$	0.100	0.027	0.027
missing photonic $O(\alpha^3 L^3)$	0.015	0.015	0.015
vacuum polarization	0.040	0.040	0.040
light pairs	0.030	0.030	0.010
Z-exchange	0.015	0.015	0.015
total	0.110	0.061	0.054

I column: S. Jadach, O. Nicrosini et al. Physics at LEP2 YR 96-01, Vol. 2
A. Arbuzov et al., Phys. Lett. B389 (1996) 129
II column: B.F.L. Ward, S. Jadach, M. Melles, S.A. Yost, hep-ph/9811245
III column: G. Montagna et al., Nucl. Phys. B547 (1999) 39

- **experimental systematics: 0.034%**

G. Abbiendi et al., (OPAL), Eur. Phys. J. C14 (2000) 373

Past and recent updates

- **theoretical error in SABS at LEP1 by the end of operation**

Type of correction/error	(%)	(%)	updated (%)
missing photonic $O(\alpha^2 L)$	0.100	0.027	0.027
missing photonic $O(\alpha^3 L^3)$	0.015	0.015	0.015
vacuum polarization	0.040	0.040	0.040
light pairs	0.030	0.030	0.010
Z-exchange	0.015	0.015	0.015
total	0.110	0.061	0.054

I column: S. Jadach, O. Nicrosini et al. Physics at LEP2 YR 96-01, Vol. 2
A. Arbuzov et al., Phys. Lett. B389 (1996) 129
II column: B.F.L. Ward, S. Jadach, M. Melles, S.A. Yost, hep-ph/9811245
III column: G. Montagna et al., Nucl. Phys. B547 (1999) 39

- **experimental systematics: 0.034%**

G. Abbiendi et al., (OPAL), Eur. Phys. J. C14 (2000) 373

- **recent reanalysis**
  - "The path to 0.01% theoretical luminosity precision for the FCC-ee"

    S. Jadach, W. Placzek, M. Skrzypek, B.F.L. Ward and S.A. Yost, Phys Lett B790 (2019) 314

  - "Improved Bhabha cross section at LEP and the number of light neutrino species"

    P. Janot and S. Jadach, Phys. Lett. B803 (2020) 135319

  - "Study of theoretical luminosity precision for electron colliders at higher energies"

    S. Jadach, W. Placzek, M. Skrzypek and B.F.L. Ward, Eur. Phys. J. C81 (2021) 1047

Updates and FCC-ee projections

LEP Publication in:	1994		2000		2019	
LumiCal generation	1^{st}	2^{nd}	1^{st}	2^{nd}	1^{st}	2^{nd}
Photonic $\mathcal{O}(\alpha^2 L_e)$	0.15%	0.15%	0.027%	0.027%	0.027%	0.027%
Photonic $\mathcal{O}(\alpha^3 L_e^3)$	0.09%	0.09%	0.015%	0.015%	0.015%	0.015%
Z exchange	0.11%	0.03%	0.09%	0.015%	0.090%	0.015%
Vacuum polarization	0.10%	0.05%	0.08%	0.040%	0.015%	0.009%
Fermion pairs	0.05%	0.04%	0.05%	0.040%	0.010%	0.010%
Total	0.25%	0.16%	0.13%	0.061%	0.100%	0.037%

P. Janot and S. Jadach, Phys. Lett. B803 (2020) 135319

Type of correction / Error	Update 2018	FCC-ee forecast
(a) Photonic $[O(L_e\alpha^2)]$ $O(L_e^2\alpha^3)$	0.027%	0.1×10^{-4}
(b) Photonic $[O(L_e^3\alpha^3)]$ $O(L_e^4\alpha^4)$	0.015%	0.6×10^{-5}
(c) Vacuum polariz.	0.014% [26]	0.6×10^{-4}
(d) Light pairs	0.010% [18, 19]	0.5×10^{-4}
(e) Z and s-channel γ exchange	0.090% [11]	0.1×10^{-4}
(f) Up-down interference	0.009% [28]	0.1×10^{-4}
(f) Technical Precision	(0.027)%	0.1×10^{-4}
Total	0.097%	1.0×10^{-4}

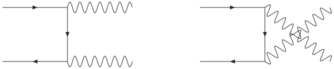
S. Jadach, W. Placzek, M. Skrzypek, B.F.L. Ward and S.A. Yost, Phys Lett B790 (2019) 314

Interesting alternative: $e^+e^- \to \gamma\gamma$

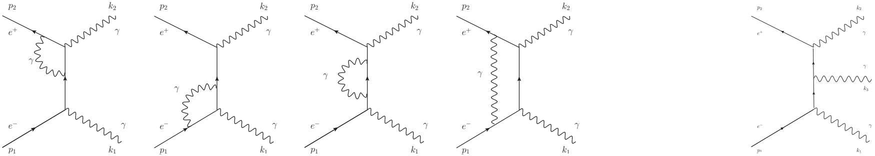
- LO diagrams (pure QED)

Interesting alternative: $e^+e^- \to \gamma\gamma$

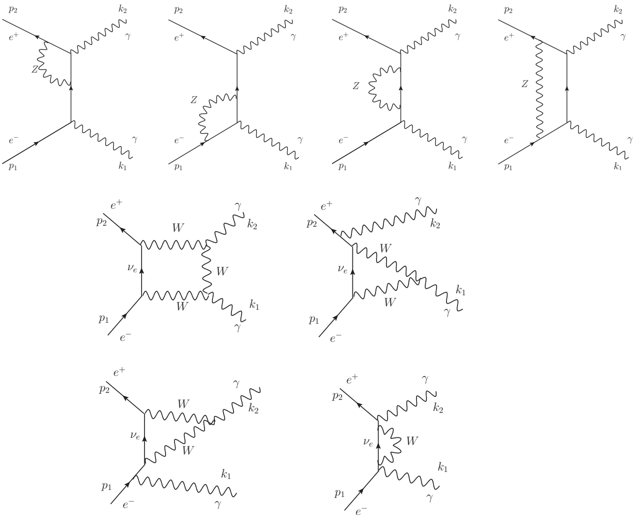
- LO diagrams (pure QED)



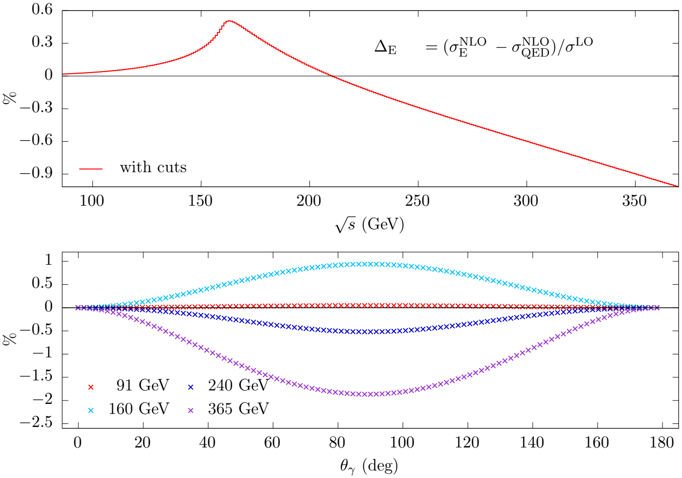
- QED NLO diagrams



NLO virtual weak diagrams (no fermionic blobs)

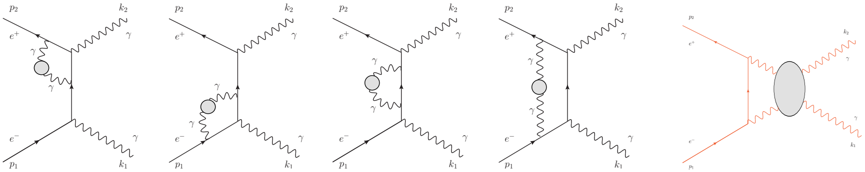


Pure weak corrections



C.M. Carloni Calame, M. Chiesa, G. Montagna, O. Nicrosini, FP, Phys. Lett. B798 (2019) 134976

- “small” at FCC-ee energies, larger for higher energies



$$\sigma^{\rm NNLO}_{\Delta\alpha_{had}} \pm \delta\sigma \overset{\text{very naive!}}{\approx} \left(\sigma^{\rm NLO}_{\rm QED} - \sigma^{\rm LO}\right) \times \left[\Delta\alpha_{had}(s) \pm \delta\Delta\alpha_{had}\right]$$

$\sqrt{s}$ (GeV)	$\Delta\alpha_{had}(s)^*$	$\delta\sigma/\sigma_{LO}$ [1]	$\delta\sigma/\sigma_{LO}$ [2]
91	$(276.7 \pm 1.2) \cdot 10^{-4}$	$2.8 \cdot 10^{-5}$	$3.7 \cdot 10^{-6}$
160	$(309.1 \pm 1.2) \cdot 10^{-4}$	$3.0 \cdot 10^{-5}$	$3.8 \cdot 10^{-6}$
240	$(333.2 \pm 1.2) \cdot 10^{-4}$	$3.1 \cdot 10^{-5}$	$3.9 \cdot 10^{-6}$
365	$(358.5 \pm 1.2) \cdot 10^{-4}$	$3.4 \cdot 10^{-5}$	$4.0 \cdot 10^{-6}$

- LbL contribution, with its uncertainty, should be quantified

*from F. Jegerlehner's recent `hadr5n16.f`

$e^+e^- \to \gamma\gamma$: Pros and cons

✓ at LO, purely QED process, ***at any energy***

✓ at NLO, weak corrections (loops with Z & $W^\pm$), but not fermionic loops yet (in particular, ***no hadronic loops***)

✓ hadronic vacuum polarization (**and its uncertainty**) enters only at NNLO (2-loops, order α^2)

✓ $d\sigma/d\cos\theta \sim 1/\sin^2\theta$) $\Longrightarrow$ lowest angle acceptance less critical than for Bhabha

✗ Large Bhabha background, in particular at Z pole

✗ At NNLO also Ligh-by-Light contribution present, (**with its uncertainty**)

✗ Statistics lower than Bhabha for respective typical event selections

✗ Lack of independent MC codes for cross-checks/validation

Available dedicated tools/generators

- Bhabha scattering
  - BHLUMI 4.04
  - BHWIDE
  - BabaYaga
  - MCGPJ
- Di-photon
  - BabaYaga
  - BKQED

Summary

- Further detailed investigating of di-photon production for precision determination of the integrated luminosity would be important
- Bhabha scattering preferred for the point-to-point luminosity control (leading systematics tend to cancel in the point-to-point comparison)
  Needed further studies on the correlations between lumi measurements at different c.o.m. energies
- Radiation of additional fermion pairs currently not implemented in the dedicated MC codes
- Detailed quantitative analysis of beamstrahlung on lumi determination necessary
- the path towards 10^{-4} (or even better) precision in luminosity seems viable