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(QCD) Theory Challenges at FCC-ee

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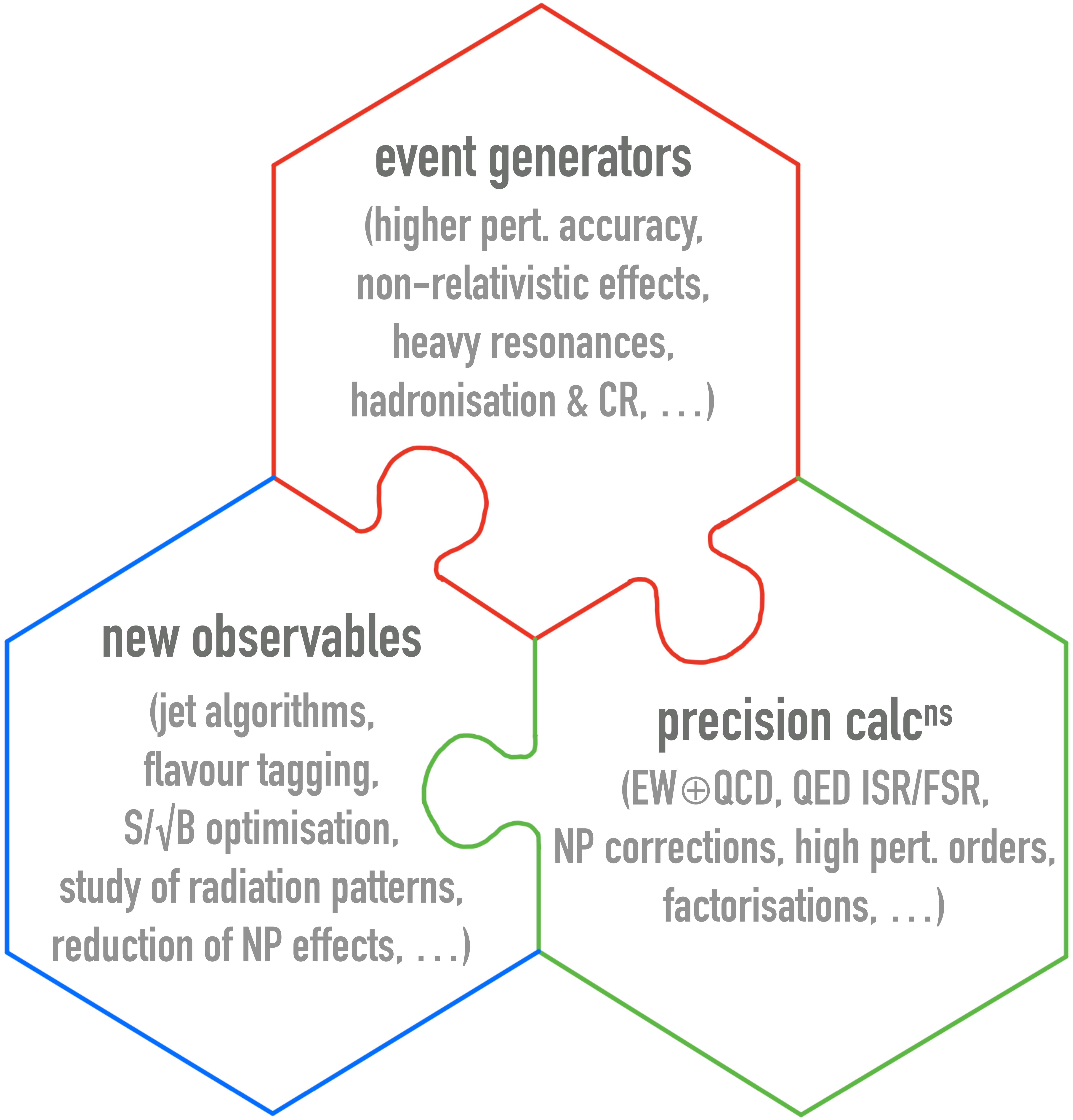
Theory challenges (this talk will focus only on some selected examples)



- Reaching the foreseen performance poses outstanding challenges on TH. Evolution in many areas is demanded‡
  - ➡ NB: cross-pollination across fields essential, global progress is required to match astonishing experimental precision

‡ I will focus on some of the next steps in QCD & some EW aspects (EW & MC generators discussed in depth in other talks at this workshop)

QCD studies in $Z/\gamma^* \to$ jets

Physics at the Z pole (and above)

[P. Janot's talk @ CERN FC workshop 2022]

- Main challenges from EW aspects:
  - EWPO $Z \to qq+X$ @ 3 loops EW (4 loop arguably necessary)
  - Beam calibration ($e^+e^- \to e^+e^-$, $\mu^+\mu^-$, $\gamma\gamma$ @ NNLO EW)
- But **high potential for precision QCD** studies:
  - α_s from R_ℓ (4 loop QCD known, $1/Q^6$ hadronisation corrections)
  - Jets structure: spin correlations, fragmentation, jet observables
  - Study and modelling of non-perturbative effects
  - Heavy quarks (Q) studies (e.g. R_b, asymmetries, fragmentation functions) & flavour tagging (e.g. q/Q vs. g jets)
  - τ decays
  - Calibration/tuning of ML & MC tools (instrumental for higher-energy runs)

Observables	Present value	FCC-ee current syst.
m_Z (keV)	91187500 ± 2100	100
Γ_Z (keV)	2495500 ± 2300 [*]	25
σ^0_{had} (pb)	41480.2 ± 32.5 [*]	4
N_ν ($\times 10^3$) from σ_{had}	2996.3 ± 7.4	1
R_ℓ ($\times 10^3$)	20766.6 ± 24.7	1
$\alpha_s(m_Z)$ ($\times 10^4$) from R_ℓ	1196 ± 30	1.5
R_b ($\times 10^6$)	216290 ± 660	?

An example: A_{FB}

- Heavy quarks challenges: let's consider A_{FB} as an example

[Blondel, Janot 2021]

Observable	present value ± error	FCC-ee **Stat.**	FCC-ee Syst.	Comment and leading exp. error
R_b ($\times 10^6$)	216290 ± 660	**0.3**	< 60	ratio of $b\bar{b}$ to hadrons stat. extrapol. from SLD
$A_{FB}^b, 0$ ($\times 10^4$)	992 ± 16	**0.02**	1-3	b-quark asymmetry at Z pole from jet charge

[Bernreuther et al. 2016]

Of this, the current QCD error is

$$\Delta A_{FB}/A_{FB} \sim \pm 0.003$$

May become a bottleneck at FCC-ee

- N³LO quite hard at the moment (QQg @ 2L, QQ @ 3L): possible workaround with series expansions (e.g. R_b currently known to N³LO up to $O(m_b^4/Q^4)$, "massification" of massless amps, ...) or numerical methods
- Explore fiducial selections to improve perturbative convergence (e.g. cut on acollinearity angle to suppress g → QQ reduces the size of QCD corrections/uncertainties)

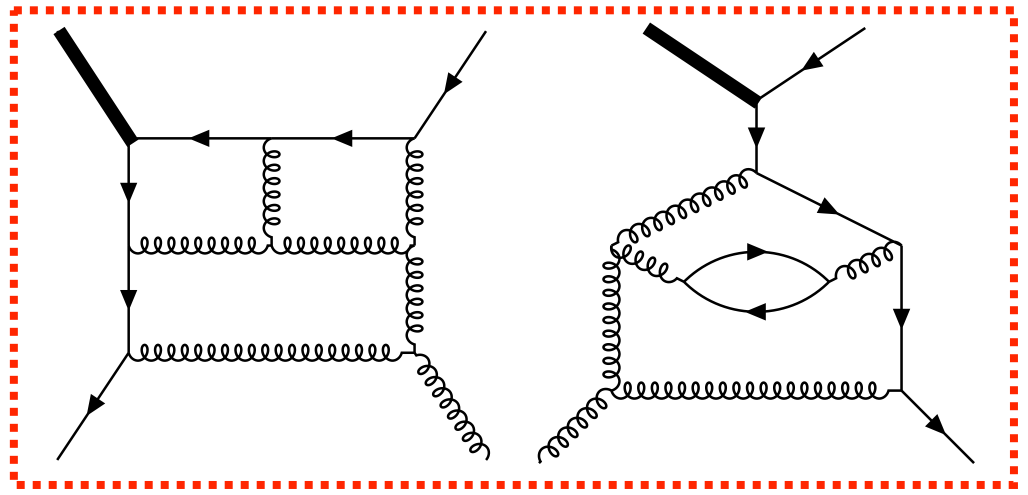
ξ_0 cut	Measured A_{FB}	ΔA_{FB}(stat)	ΔA_{FB}(tune)	ΔA_{FB}(theo. QCD corr)
No cut	0.0998 ± 0.0004	0.00008	0.00014	0.00033
1.50	0.1003 ± 0.0003	0.00011	0.00014	0.00023
1.00	0.1011 ± 0.0002	0.00011	0.00010	0.00016
0.50	0.1023 ± 0.0002	0.00011	0.00010	0.00007
0.30	0.1030 ± 0.0002	0.00011	0.00010	0.00003
0.20	0.1033 ± 0.0001	0.00011	0.00005	0.00002
0.10	0.1035 ± 0.0002	0.00016	0.00005	0.00001

[Alcaraz Mestre 2020]

Moderate cuts seem to reduce the QCD error by an order of magnitude

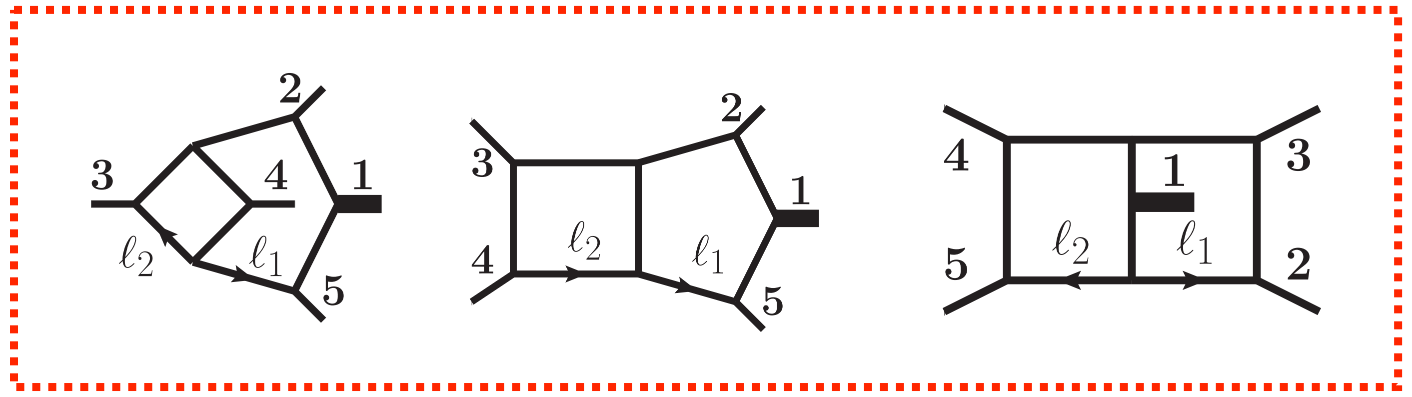
Z/γ* → (light) jets

- Significant improvement needed in FO QCD calculations
  - 3 jets @ N³LO QCD: amplitudes in the making (planar limit), but IR subtraction is an open challenge



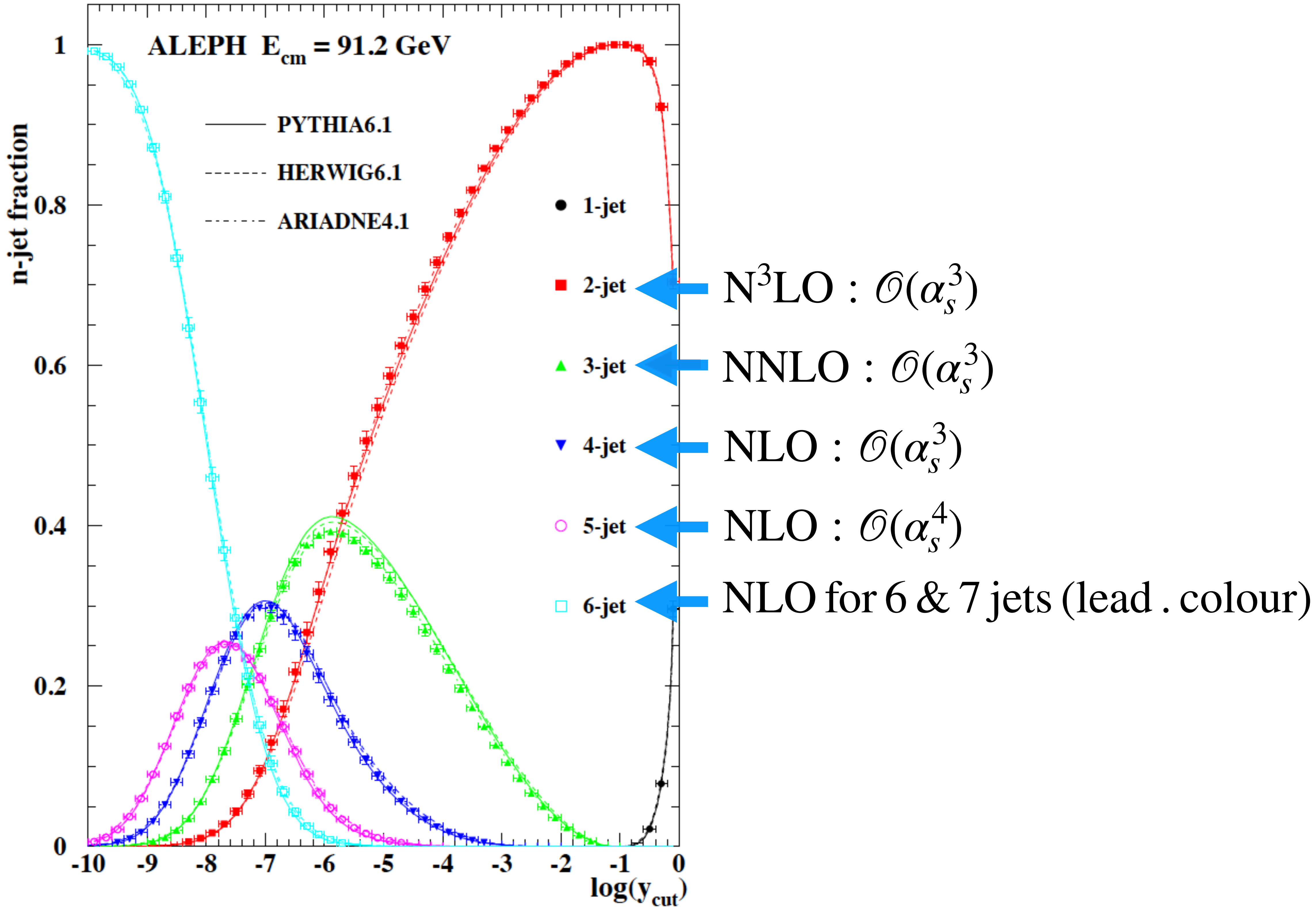
[Gehrmann et al. 2023]

  - 4 jets @ NNLO QCD: likely within reach in next O(few) years



[Abreu et al. 2023]

Current state of the art



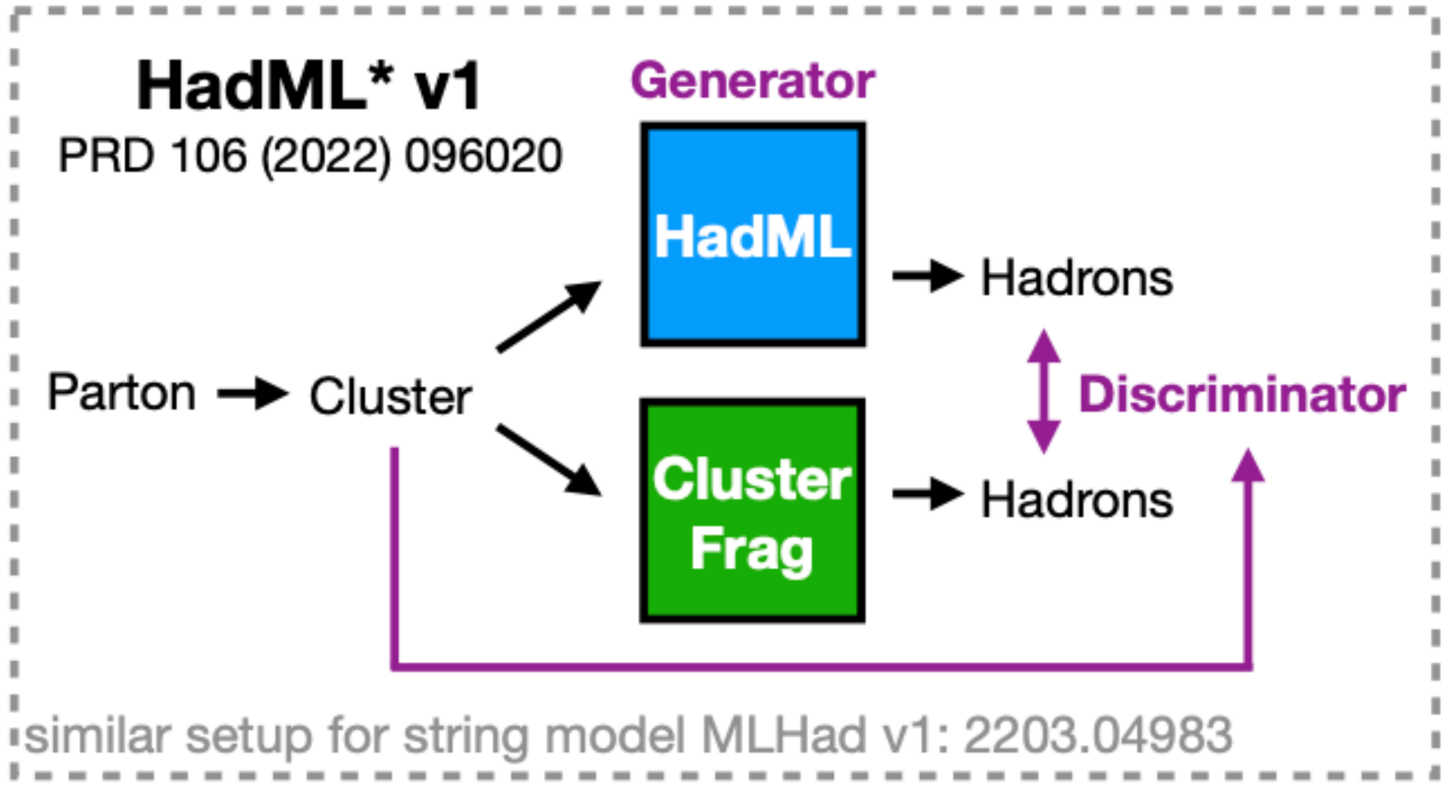
- Advancements in resummed calcⁿˢ in multi-jet observables (global & non-global) will be instrumental: crucial to explore new observable designs to optimise calculability and performance (e.g. small hadronization)

Non-perturbative QCD effects: possible ways forward

- Better MC models/tuning
  - Span of c.o.m. energies crucial for tuning, jointly w/ higher order PSMCs [High-purity samples of g/q/Q jets beneficial]
  - Cross-benefit between stages of FCCee (e.g. Z → jets useful for ZH, CR at WW → jets, …)

e.g. GANs as hadronisation model

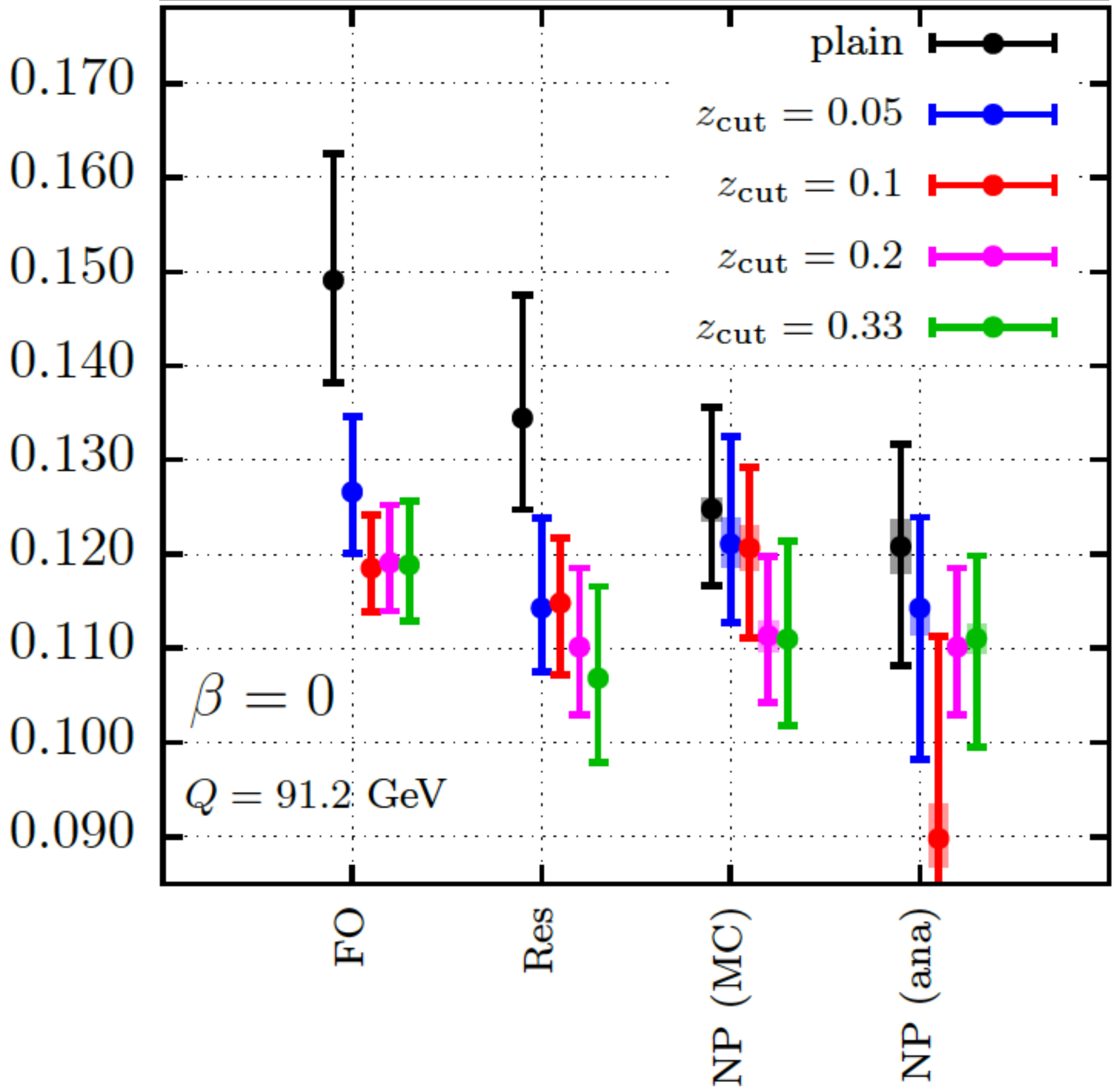
[J. Chan et al. '22–'23]



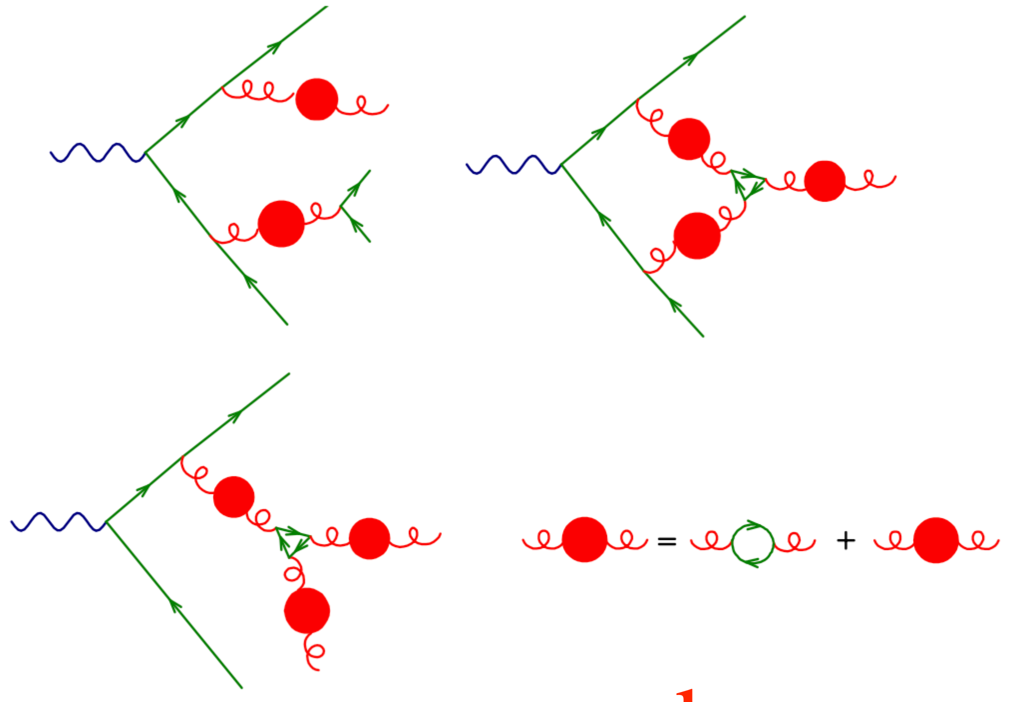
- Observables with smaller sensitivity to soft physics
  - e.g. grooming, albeit unclear whether effective at FCC-ee due to limited phase space

[Marzani et al. 2019]

e.g. α_S from SD thrust



e.g. large-n_F models



- Factorisation theorems and data driven extraction
  - Constrain NP parameters/operators across energies (use of lattice also shows promising prospects)
  - Idea to run below the Z peak might be beneficial
  - Further progress in analytic methods highly desirable

$$\frac{d\sigma}{d\mathcal{O}}(\mathcal{O}) \simeq \frac{d\sigma^{\text{pert.}}}{d\mathcal{O}}\left(\mathcal{O} - \zeta(\mathcal{O})\alpha_0(\Lambda)\frac{\Lambda}{Q}\right)$$

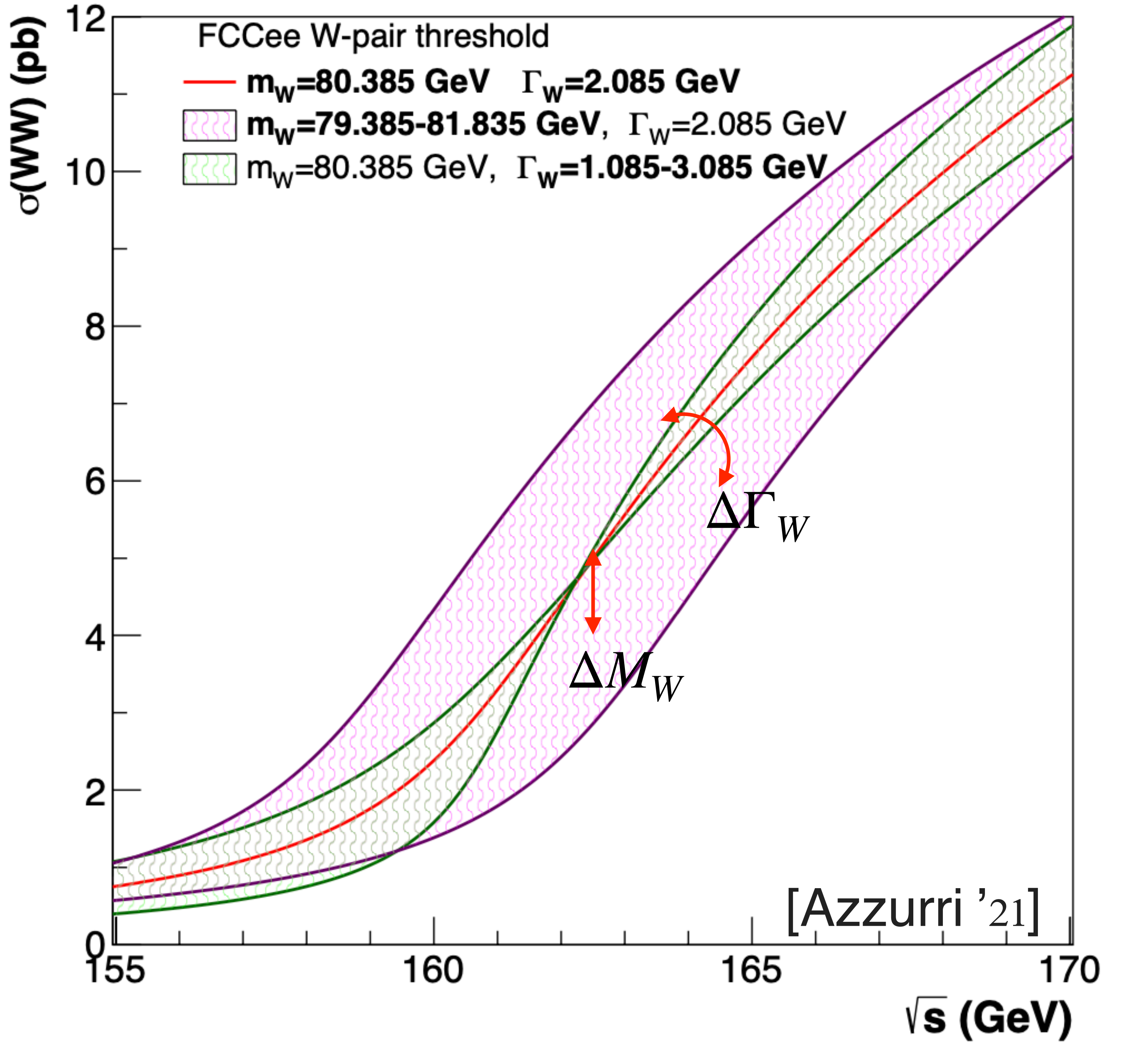
WW threshold

WW threshold scan and W mass and width

- XS known at NLO (EW) + NNLO (unstable particles EFT). OK for $\delta m_W \sim$ 5-6 MeV. Effect of selection cuts in EFT to be understood

[Denner, Dittmaier, Roth, Wieders '05; Actis, Beneke, Falgari, Schwinn '08]

	$\sigma(e^-e^+ \to \mu^-\bar{\nu}_\mu u\bar{d}\,X)$(fb)				
$\sqrt{s}$ [GeV]	Born	Born (ISR)	NLO	$\hat{\sigma}^{(3/2)}$	$\sigma^{(3/2)}_{\mathrm{ISR}}$
158	61.67(2)	45.64(2) [−26.0%]	49.19(2) [−20.2%]	−0.001 [−0.0‰]	0.000 [+0.0‰]
161	154.19(6)	108.60(4) [−29.6%]	117.81(5) [−23.6%]	0.147 [+1.0‰]	0.087 [+0.6‰]
164	303.0(1)	219.7(1) [−27.5%]	234.9(1) [−22.5%]	0.811 [+2.7‰]	0.544 [+1.8‰]
167	408.8(2)	310.2(1) [−24.1%]	328.2(1) [−19.7%]	1.287 [+3.1‰]	0.936 [+2.3‰]
170	481.7(2)	378.4(2) [−21.4%]	398.0(2) [−17.4%]	1.577 [+3.3‰]	1.207 [+2.5‰]



Reaching precision of 0.3-0.5 MeV (lep. channel) requires NNLO EW

$$\Delta\sigma_{\mathrm{WW}}(\mathrm{T}) < 0.8\ \mathrm{fb}$$

NB: no W BRs: ~0.04% in table units at √s = 161 GeV

- Recent calculation of $\mathcal{O}(\alpha_s\alpha)$ terms ~ O(0.034%) [Li et al. 2024]
- Can be further improved using higher-orders ISR (NLL and beyond) [Frixione's talk]

FCC-ee as a Higgs factory

Theory challenges at the ZH threshold

- Example: total cross section will be measured with 0.2%-0.5% accuracy. Necessary TH for (EW) production:
  - $e^+e^- \to Z\,H$ (available), $H\,\nu\,\nu$ (e^+e^-) @ 2 loops EW (beyond reach at the moment) [Chen, Guan, He, Liu, Ma '22; Freitas, Song '21–'22]
  - Mixed QCD⊗EW @ 2 loops under control [Gong et al. '17]
- Wealth of data in hadronic decays relies on QCD input

Current and future uncertainties in total Higgs decay rates

Decay	current unc. $\delta\Gamma$ [%]				future unc. $\delta\Gamma$ [%]			
	$\mathrm{Th}_{\mathrm{Intr}}$	$\mathrm{Th}^{m_q}_{\mathrm{Par}}$	$\mathrm{Th}^{\alpha_s}_{\mathrm{Par}}$	$\mathrm{Th}^{m_H}_{\mathrm{Par}}$	$\mathrm{Th}_{\mathrm{Intr}}$	$\mathrm{Th}^{m_q}_{\mathrm{Par}}$	$\mathrm{Th}^{\alpha_s}_{\mathrm{Par}}$	$\mathrm{Th}^{m_H}_{\mathrm{Par}}$
$H \to b\bar{b}$	< 0.4	1.4	0.4	–	0.2	0.6	< 0.1	–
$H \to \tau^+\tau^-$	< 0.3	–	–	–	< 0.1	–	–	–
$H \to c\bar{c}$	< 0.4	4.0	0.4	–	0.2	1.0	< 0.1	–
$H \to \mu^+\mu^-$	< 0.3	–	–	–	< 0.1	–	–	–
$H \to W^+W^-$	0.5	–	–	2.6	0.3	–	–	0.1
$H \to gg$	3.2	< 0.2	3.7	–	1.0	–	0.5	–
$H \to ZZ$	0.5	–	–	3.0	0.3	–	–	0.1
$H \to \gamma\gamma$	< 1.0	< 0.2	–	–	< 1.0	–	–	–
$H \to Z\gamma$	5.0	–	–	2.1	1.0	–	–	0.1

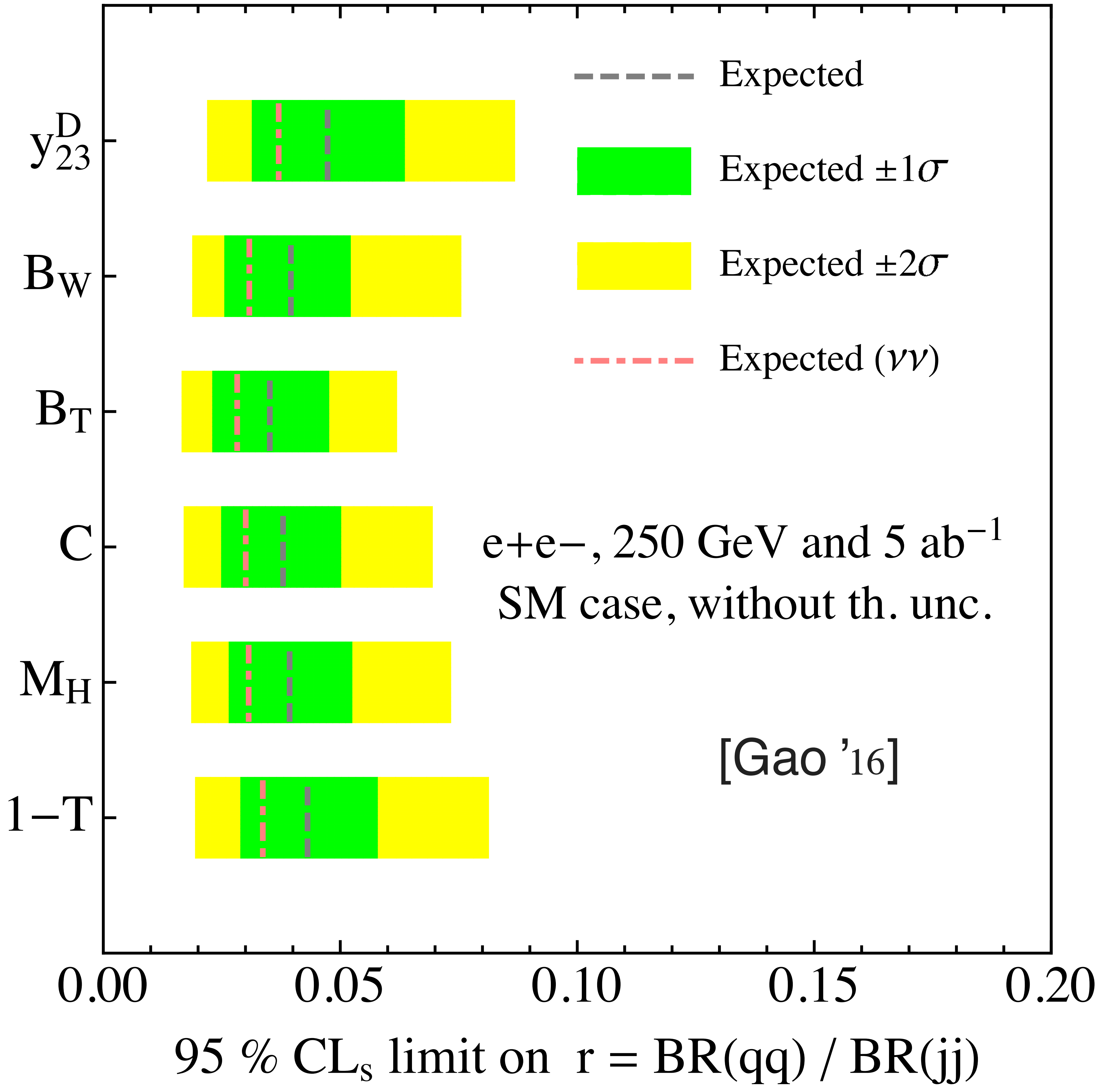
[Table from J. de Blas' talk at FCC week 2023]

Projected reduction of intrinsic TH uncertainties for total rates in line with what can be achieved with future calculations; improvement needed in parametric uncertainties

Hadronic Higgs decays

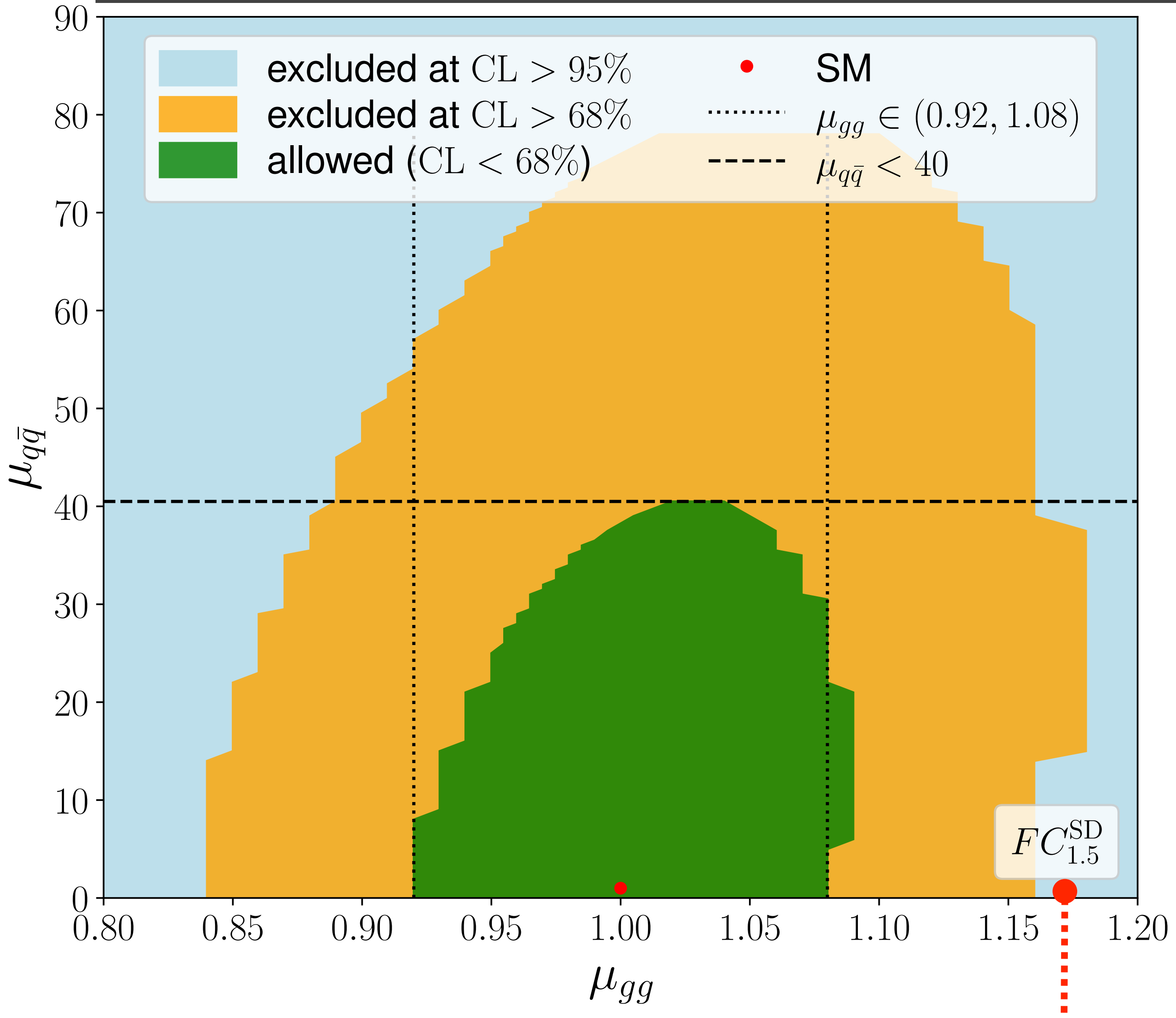
- New opportunities in differential distributions: e.g. strange Yukawa/Higgs BRs from shapes
  - Progress in perturbative calc^ns ongoing (many subtleties); major obstacle is the precise assessment of hadronization

e.g. H → qq BRs from event shapes

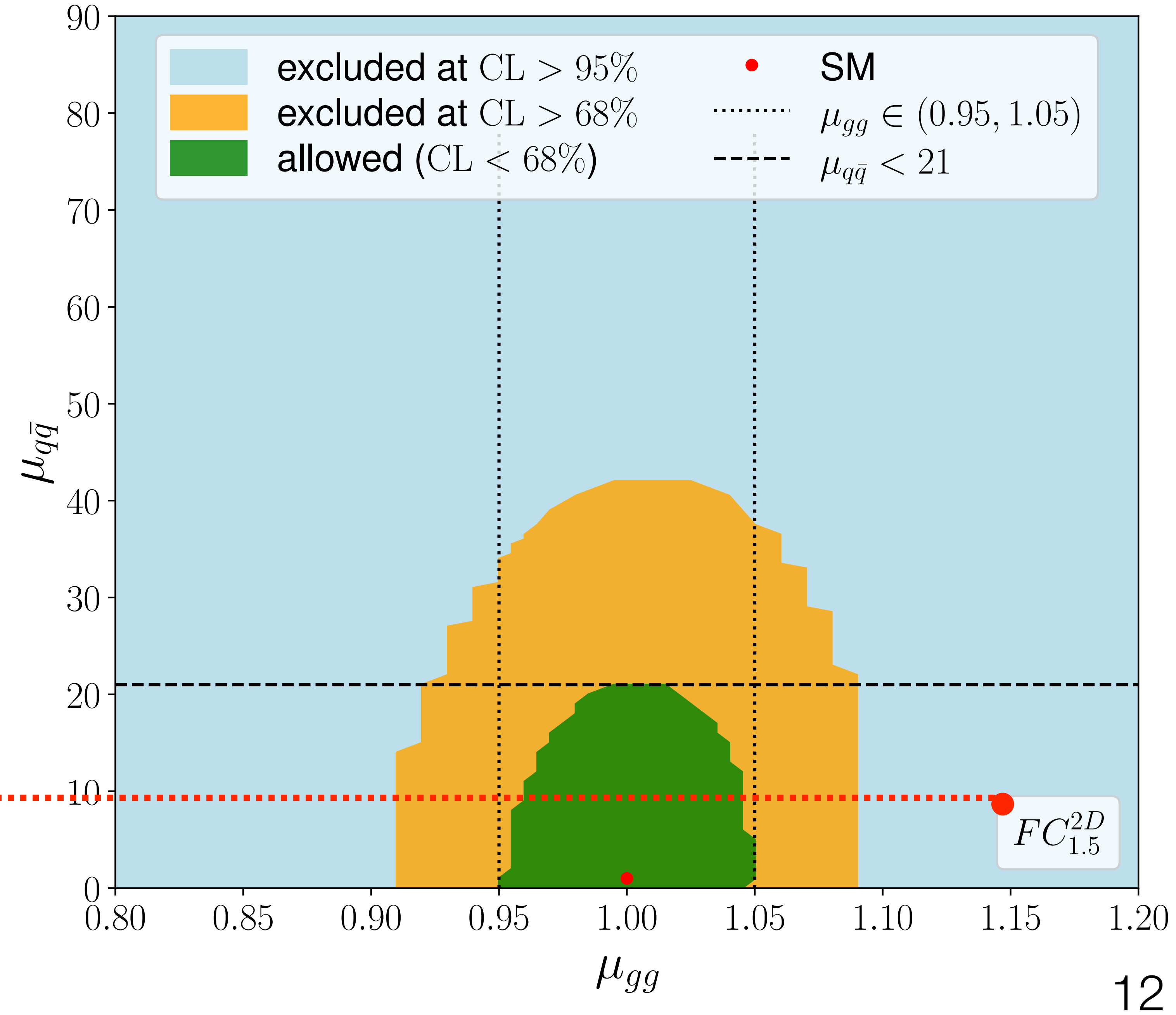


e.g. H → gg & H → qq BRs from fractional moments of EEC

[Knobbe, Krauss, Reichelt, Schumann '23]



with Soft Drop



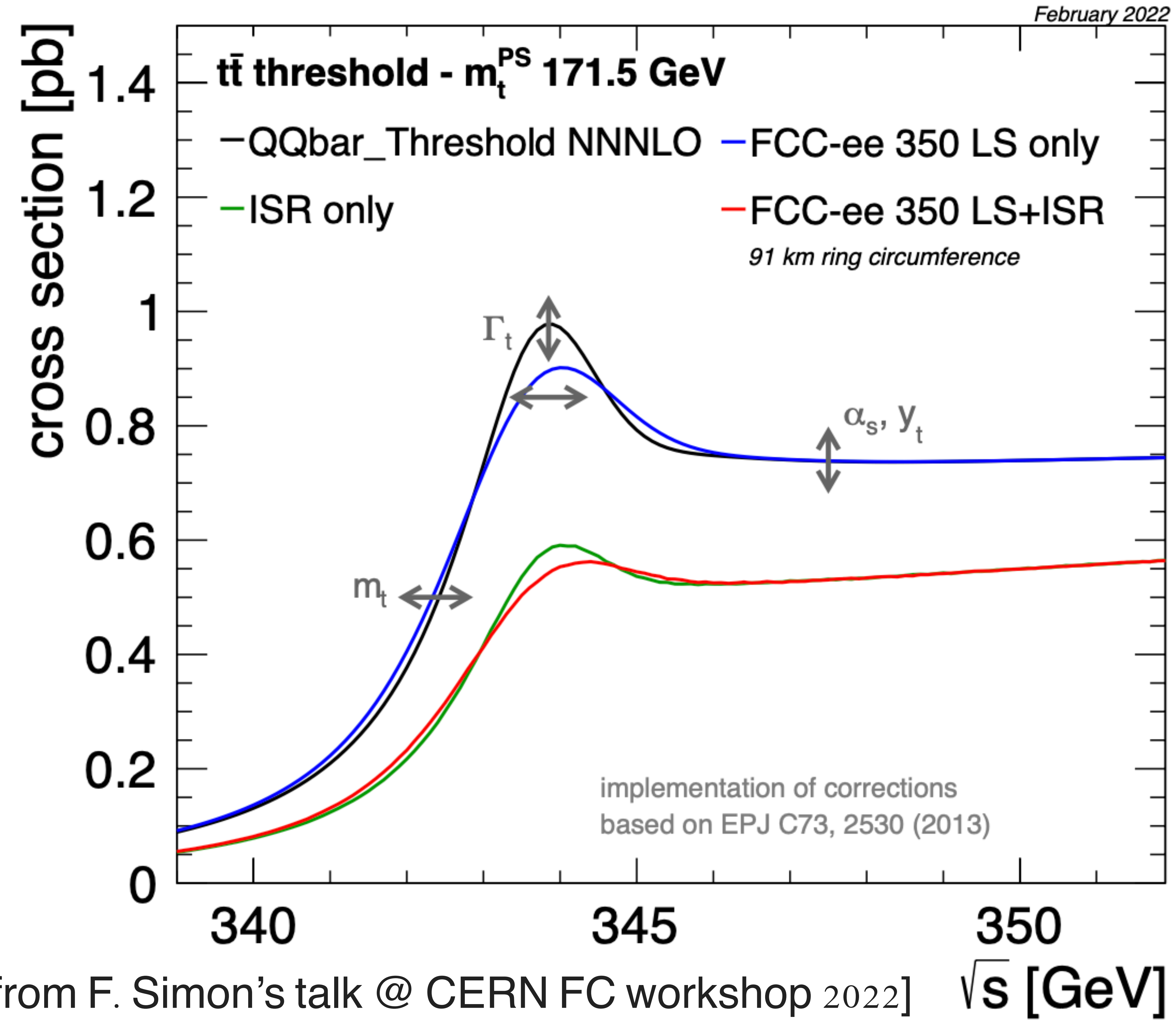
constraints from two independent hemispheres

tt threshold scan

Top physics

- Huge potential from threshold scan: up to per-mille accuracy on cross section & asymmetries
- Access to top mass and width, as well as strong coupling and top Yukawa coupling
- e.g. projected exp. target for top mass $\delta m_t \sim 20$ MeV

Great challenge for theory to match this precision: intrinsic (e.g. higher order) & parametric (e.g. strong coupling from Z pole) uncertainties



[Plot from F. Simon's talk @ CERN FC workshop 2022]

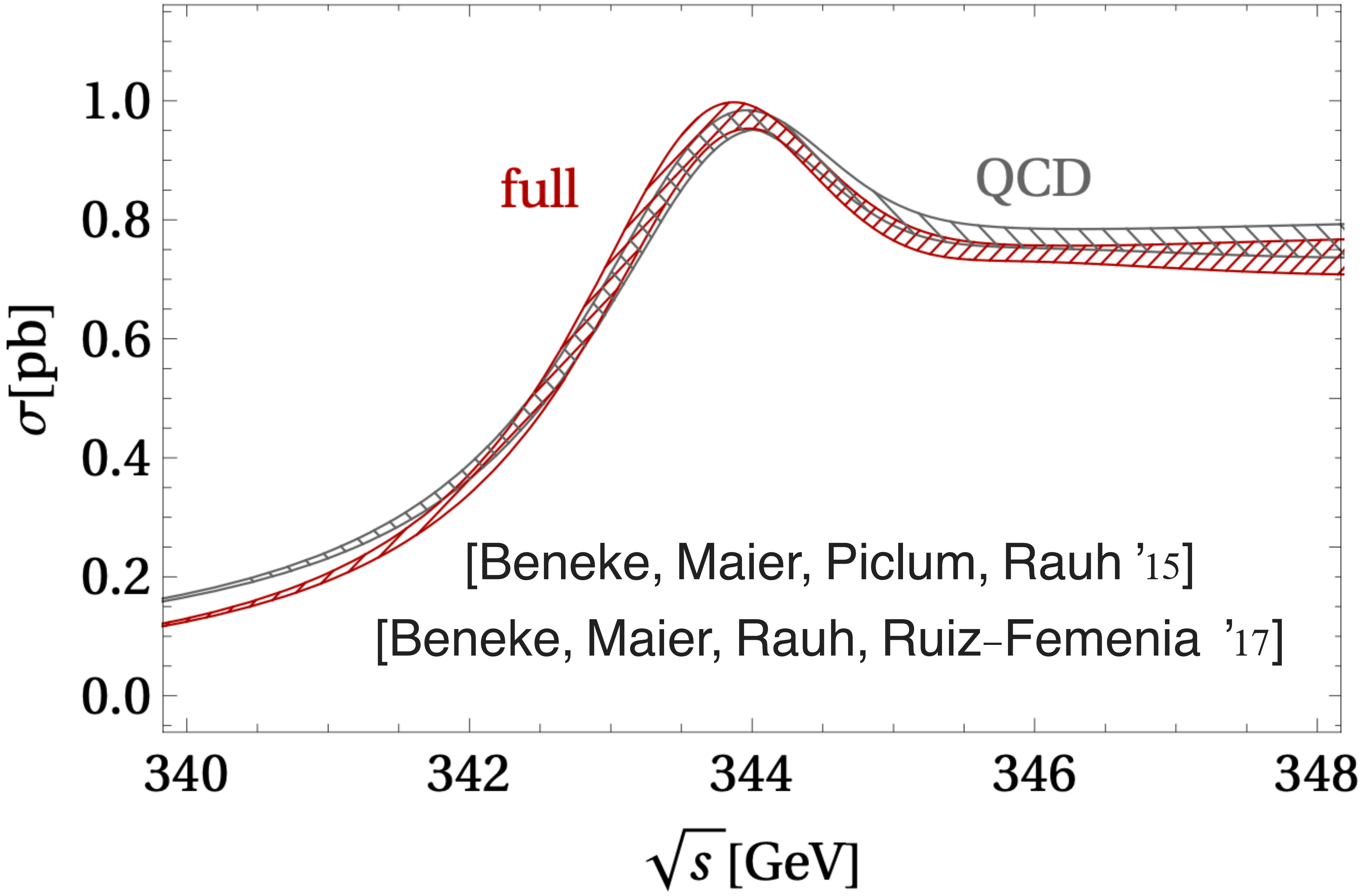
Top physics: theory for threshold scan

- PNRQCD predictions known to N³LO (also including EW+non-resonant effects @ NNLO)

$$R \sim v \sum_k \left(\frac{\alpha_s}{v}\right)^k \cdot \{ \underbrace{1\,(\text{LO})}\ ; \underbrace{\alpha_s, v\,(\text{NLO})}; \underbrace{\alpha_s^2, \alpha_s v, v^2\,(\text{NNLO})}; \underbrace{\alpha_s^3, \alpha_s^2 v, \alpha_s v^2, v^3\,(\text{N3LO})}; \dots \}$$

[Beneke, Kiyo, Marquard, Penin, Piclum, Steinhauser '15]

- Uncertainty in top mass (potential subtracted) **$\delta m_t \sim$ 40 MeV**. Towards exp. target (20 MeV):
  - Some improvements will come from **matching of N³LO+NNLL** (ingredients available)
  - Needs **NLL ISR (possibly including soft modes)**
  - Ultimately might require **N⁴LO in PNRQCD needed** (currently out of reach)



- Continuum regime: recent calculation of full N³LO QCD (reaching ~0.1-0.5% QCD errors)

[Chen, Guan, He, Liu, Ma '22]

Concluding remarks

- Astounding physics programme at FCCee, drastic reduction of experimental uncertainties: theory precision likely to be among the main bottlenecks

- Many (if not all) areas of theory calculations need to be involved (fixed order QCD + EW, resummations in QCD & QED, effective field theories, non-perturbative QCD, event generators, new observables,…)

  - ➡ Many challenges are technical in nature: hard calculations, currently beyond reach but likely to become achievable with the evolution of the field in the coming decades, and substantial work

  - ➡ Also deep conceptual questions, which need significant breakthroughs to improve their understanding: e.g. non-perturbative QCD (hadronisation, CR), EFT calculations, high-order QCD+EW MCs currently a bottleneck in several studies

  - ➡ New opportunities from data-driven approaches, crucial to think of how to exploit it for modelling aspects and theory uncertainties (e.g. heavy flavour & gluon fragmentation, hadronization modelling, …)

  - ➡ Huge step forward demanded for MCs (QCD/EW, ISR, HO for jet processes, NR QCD, resonances)