Positivity, BSM, FCC

Joe Davighi (CERN)

Partially based on

JD, Scott Melville, Ken Mimasu, Tevong You, 2308.06226

FCC week, 30th January 2024, Annecy

Two facts:

1. Experimental: strong evidence for a hierarchy $v_{\mathrm{EW}}/\Lambda_{\mathrm{New\ Physics}} < 10^{-1} - 10^{-2}$
2. Theoretical: derivatives of scattering amplitudes satisfy *positivity bounds*

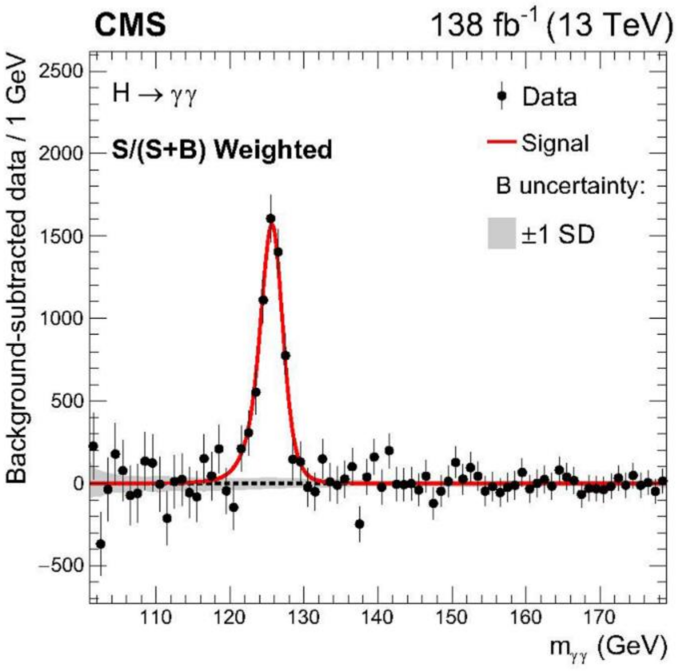
This talk:

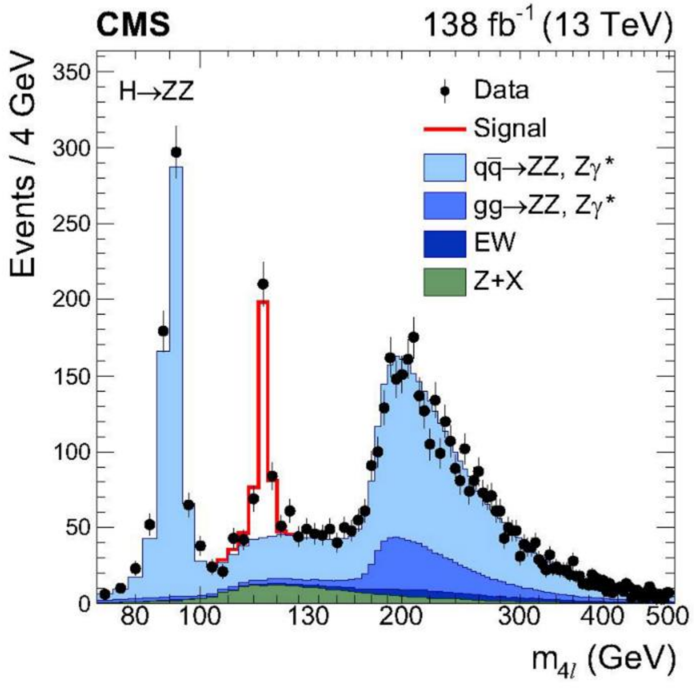
- Positivity allows us to probe fundamental UV properties by measuring SMEFT coefficients
- Positivity can imply correlated tuning between higher-dimension WCs e.g. c_8/c_{10} and v/Λ
    - New EFT perspective on the little hierarchy problem
- Probing these scenarios e^+e^- scattering at **FCC-ee**, which will push back $\Lambda_{\mathrm{New\ Physics}}$ by $\times\ 10$

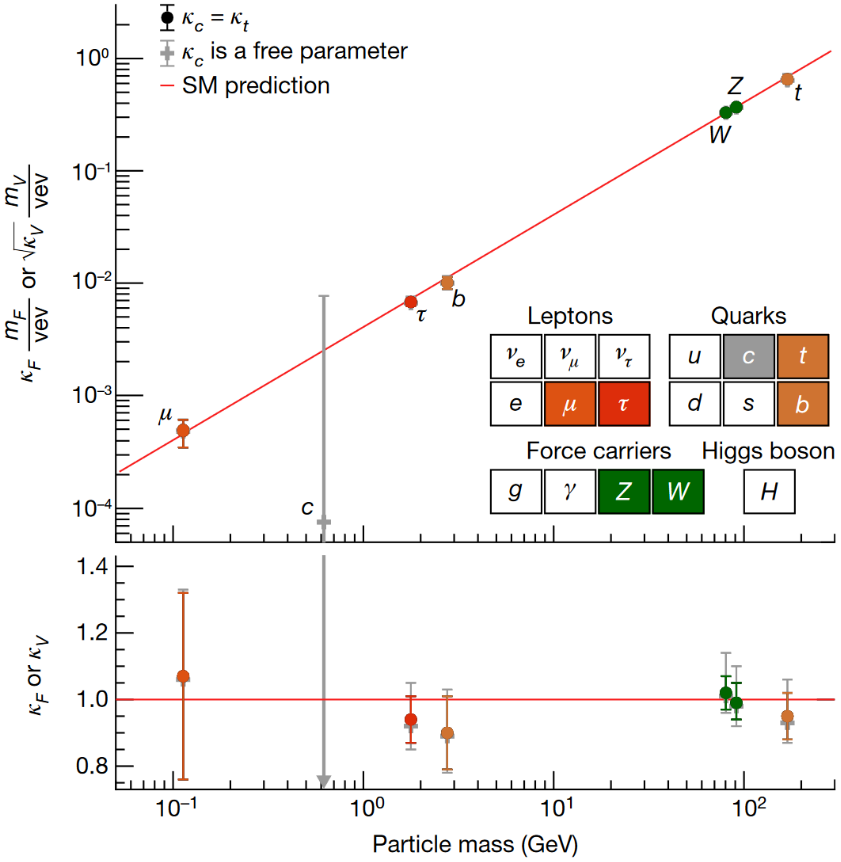
"Positively light" Higgs?

Fact 1: experiments establish a hierarchy in Nature

Exhibit A: in ATLAS and CMS we observe a SM-like Higgs $m_h \approx 125\ \mathrm{GeV}$







"Higgs at 10" in Nature: CMS & ATLAS

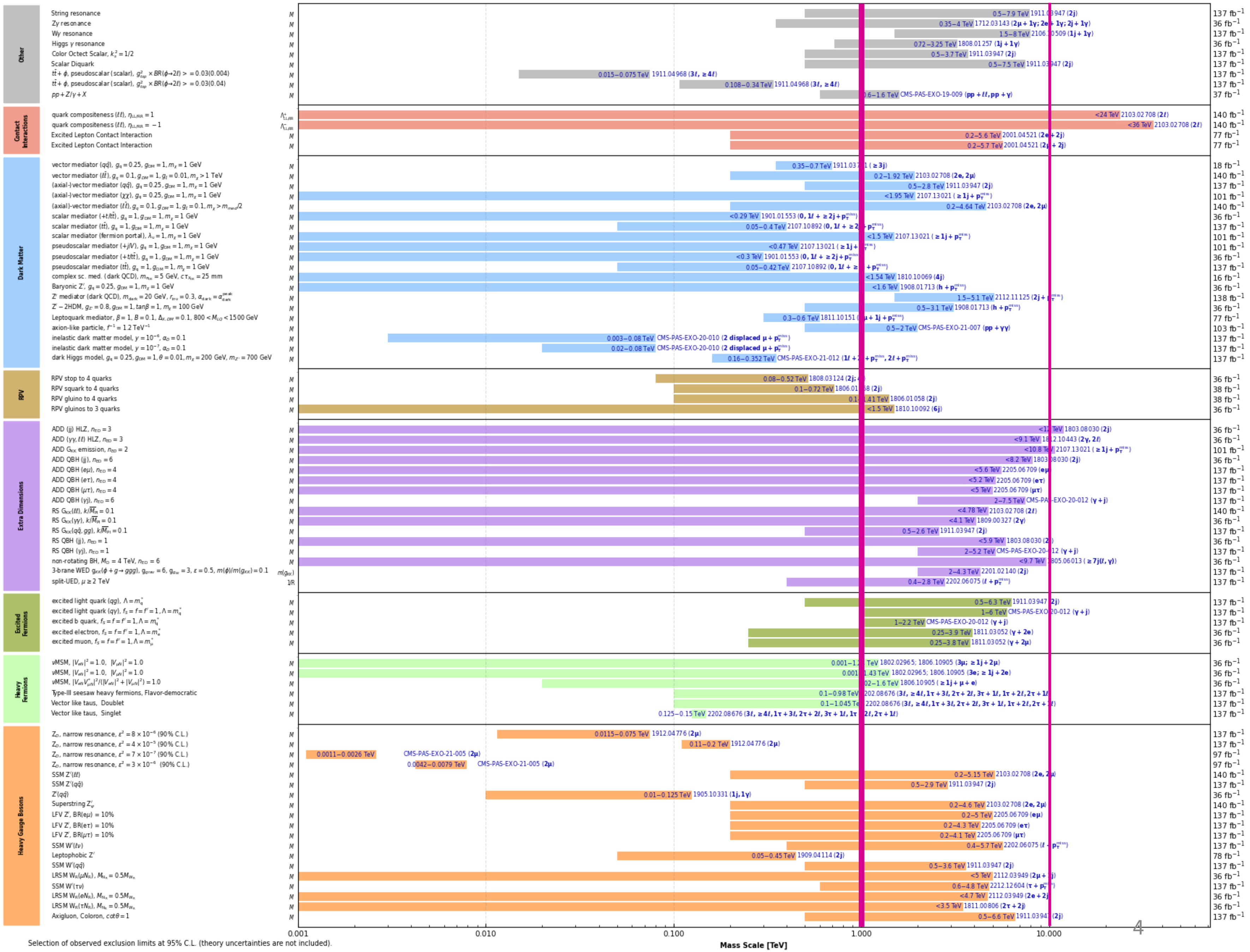
https://twiki.cern.ch/twiki/bin/view/CMSPublic/SummaryPlotsEXO13TeV

Exhibit B:

Direct searches at ATLAS and CMS see no evidence for BSM

$$M_{\mathrm{NP}} \geq \mathrm{TeV}$$

Many ATLAS and CMS direct search limits even approach **10 TeV** exclusion



Selection of observed exclusion limits at 95% C.L. (theory uncertainties are not included).

Exhibit B: It's not just direct searches that point to a hierarchy:

- Precision tests of SM in **flavour** [meson mixing, flavour violating kaon and B-decays] measured at *LHCb, kaon factories, B-factories*, naively probes *very* high effective scales $\Lambda > 10^{5-6}\ \mathrm{TeV}$
  - With a "safe" flavour structure e.g. Minimal Flavour Violation (MFV) or $U(2)^n$, the scale can be brought down to $\mathbf{1-10\ TeV}$

D'Ambrosio, Giudice, Isidori, Strumia, hep-ph/0207036 …

- Precision **electroweak tests of SM** [e.g. Z width, W mass, forward backward asymmetries] probe NP with large couplings to Higgs/leptons/third generation: bounds are $\mathbf{1-10\ TeV}$
  - Bounds still dominated by *LEP II*, but *hadron colliders* also important for some observables

See e.g. Ellis, Madigan, Mimasu, Sanza, You 2012.02779
Bresó-Pla, Falkowski, González-Alonso 2103.12074

All these current measurements complement each other to cover (most) NP scenarios at TeV scale, more-or-less establishing $\boldsymbol{\Lambda_{\text{New Physics}} \gtrsim \text{few TeV} \gg v_{\text{EW}}}$ = the "little hierarchy"

Allwicher, Cornella, Isidori, Stefanek, 2311.00020

See talk by Ben Stefanek

Interestingly, at this scale **explicit BSM models can remain natural**, e.g. if coupled mostly to 3rd generation

See e.g. JD, Stefanek 2305.16280 ; JD, Gosnay, Miller, Renner 2312.13346 …

If FCC-ee Z pole run measures SM-like EWPOs, will push back $\Lambda_{\text{New Physics}}$ by ~an order of magnitude: the **most effective way to explore BSM at 10 TeV** & probe naturalness

Either we will see clear deviations from SM on the Z-pole (reasonable to expect if NP is natural!), or the little hierarchy v/Λ is made even bigger…

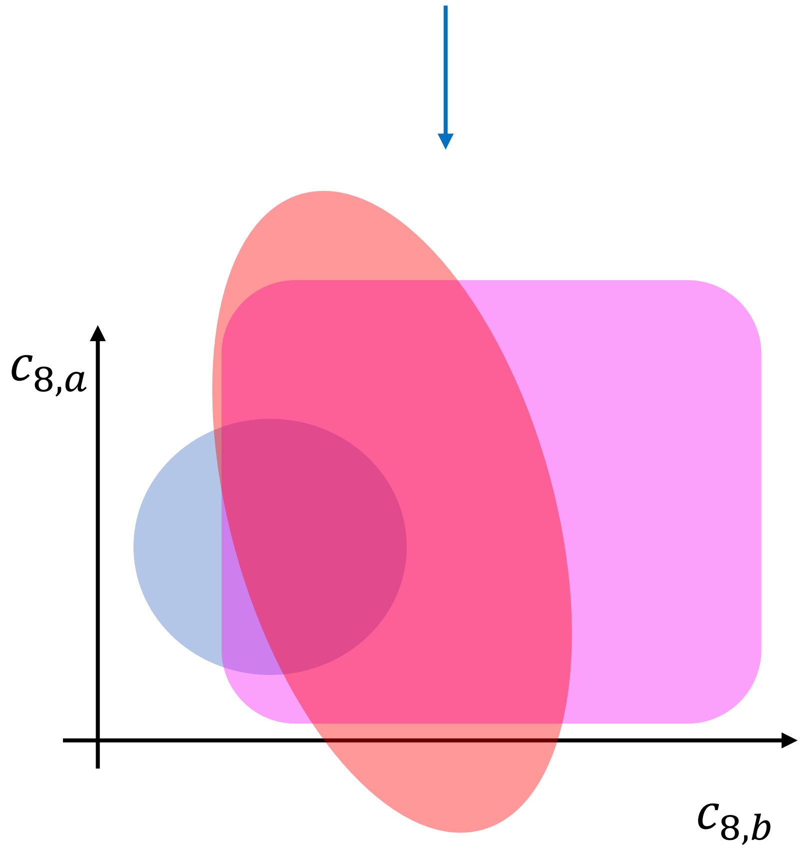
Fact 2: Positivity bounds connect UV to EFT

"Positivity" is about mapping collections of UV models to regions in (SM) EFT space they populate

$\Longrightarrow$ complementary middle ground to (a) studying one BSM model at a time, and (b) doing pure bottom-up EFT

$\mathcal{L}_{\text{UV}} = ?$

Causality, Unitarity, Locality



<u>Positivity for collider BSM</u>

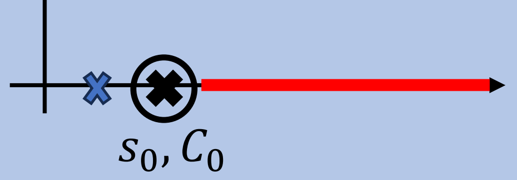
- What are these positivity constraints on SMEFT?
- Sensitivity to their violation at future colliders?
- Can positivity shed light on model building / the little hierarchy problem / discovering new particles?

Positivity ab ovo

Causality

Amplitude $\mathcal{A}_{AB\to AB}(s,t=0)$ is an analytic function of s up to known singularities

$$\mathcal{A}_{\text{EFT}}(s,0) = \frac{c_6}{\Lambda^2}s + \frac{c_8}{\Lambda^4}s^2 + \cdots$$



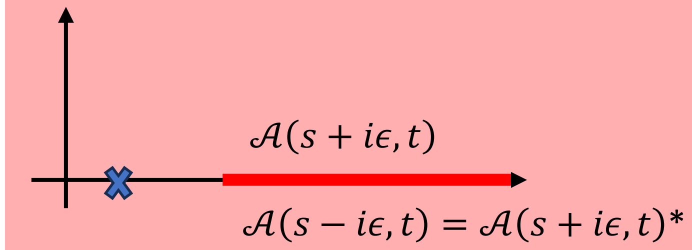
IR Observable:

$$\partial_s^n \mathcal{A}_{\text{EFT}}(s,0)\Big|_{s_0} \sim \int_{C_0} \frac{ds}{s^{n+1}} \mathcal{A}_{\text{EFT}}(s,0)$$

Unitarity

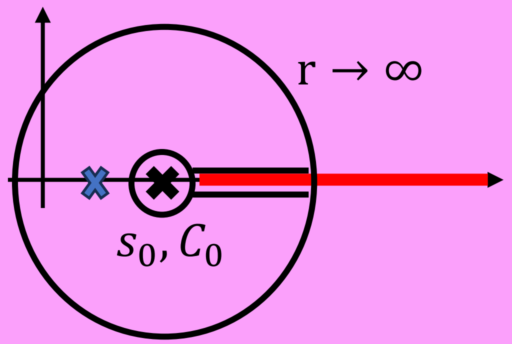
$$S^\dagger S = 1 \implies i\left(\mathcal{A}^\dagger - \mathcal{A}\right) = \mathcal{A}^\dagger \mathcal{A}$$

The discontinuity of $\mathcal{A}_{AB\to AB}$ across the branch cut is positive



Locality

Cauchy: $\partial_s^n \mathcal{A}_{\text{EFT}}|_{s_0}$ = Positive + $\int_{\text{infinity}} \frac{\mathcal{A}_{\text{UV}} ds}{s^{n+1}}$.



For a local theory,
$\lim_{s\to\infty} \mathcal{A} \leq \#\, s \log^2 s < \#\, s^2$

$$\partial_s^{\geq 2} \mathcal{A}_{\text{EFT}}(s,0)|_{s_0} \geq 0 \qquad \textbf{Positivity Bound}$$

Adams, Arkani-Hamed, Dubovsky, Nicolis, Rattazzi, hep-th/0602178

SMEFT $e^+e^- \to e^+e^-$

Fuks, Liu, Zhang, Zhou, 2009.02212

To put positivity to work in pheno, we need EFT contributions to an *elastic scattering* process $AB \to AB$

Lepton colliders provide a perfect laboratory. Independent contributions from four dim-8 SMEFT operators:

$$\mathcal{O}_8^{(1)} = \partial^\nu \left(\bar{e}_i \gamma^\mu e_i\right) \partial_\nu \left(\bar{e}_i \gamma_\mu e_i\right) \,,$$
$$\mathcal{O}_8^{(2)} = \partial^\nu \left(\bar{e}_i \gamma^\mu e_i\right) \partial_\nu \left(\bar{L}_i \gamma_\mu L_i\right) \,,$$
$$\mathcal{O}_8^{(3)} = D^\nu \left(\bar{e}_i L_i\right) D_\nu \left(\bar{L}_i e_i\right) \,,$$
$$\mathcal{O}_8^{(4)} = \partial^\nu \left(\bar{L}_i \gamma^\mu L_i\right) \partial_\nu \left(\bar{L}_i \gamma_\mu L_i\right) \,,$$

↓

$\partial_s^2 \mathcal{A}_{\text{SMEFT}}(e^+e^- \to e^+e^-) \geq 0$ Positivity Bound

↓

$c_8^{(1)} \leq 0$ $c_8^{(3)} \geq 0$ $c_8^{(4)} \leq 0$ $c_8^{(2)} \leq 2\sqrt{c_8^{(1)} c_8^{(4)}}$ $-\left(c_8^{(2)} + c_8^{(3)}\right) \leq 2\sqrt{c_8^{(1)} c_8^{(4)}}$

If one can establish experimentally that one of these inequalities is violated

$\Longrightarrow$ unitarity / causality / locality must be violated at higher energies!

SMEFT $e^+e^- \to e^+e^-$: dimension-8 is enough!

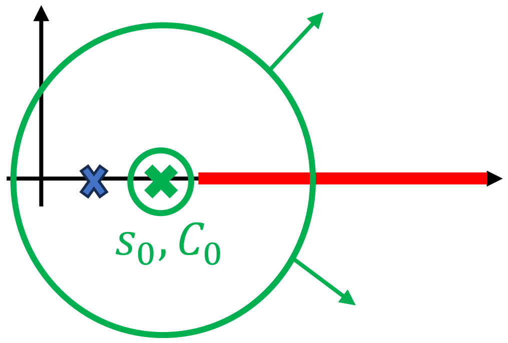
Another fundamental implication of positivity:

Saturating the dimension-8 positivity bounds $[c_8^{(i)} = 0]$ is only possible for a **free theory**!

$[\partial_s^2 \mathcal{A}_{\text{SMEFT}}$ is a *positive integral* of $\mathcal{A}_{\text{SMEFT}}$. It can only equal zero if the entire SMEFT contribution to $\mathcal{A}_{\text{EFT}}$, resumming *all* higher-dimension operators, is exactly zero]

$\Longrightarrow$ **If you experimentally exclude NP in e^+e^- scattering at dimension-8, you have "proven" there is SM only** in this process. There is no such statement for dimension-6.

$$\partial_s^n \mathcal{A}_{\text{EFT}}\Big|_{s_0} = \textbf{Positive} + 0$$



SMEFT $e^+e^- \to e^+e^-$

Fuks, Liu, Zhang, Zhou, 2009.02212

Experimental prospects in future lepton colliders (a somewhat unfair comparison for FCC-ee…)

$$\mathcal{O}_8^{(1)} = \partial^\nu \left(\bar{e}_i \gamma^\mu e_i\right) \partial_\nu \left(\bar{e}_i \gamma_\mu e_i\right) ,$$
$$\mathcal{O}_8^{(2)} = \partial^\nu \left(\bar{e}_i \gamma^\mu e_i\right) \partial_\nu \left(\bar{L}_i \gamma_\mu L_i\right) ,$$
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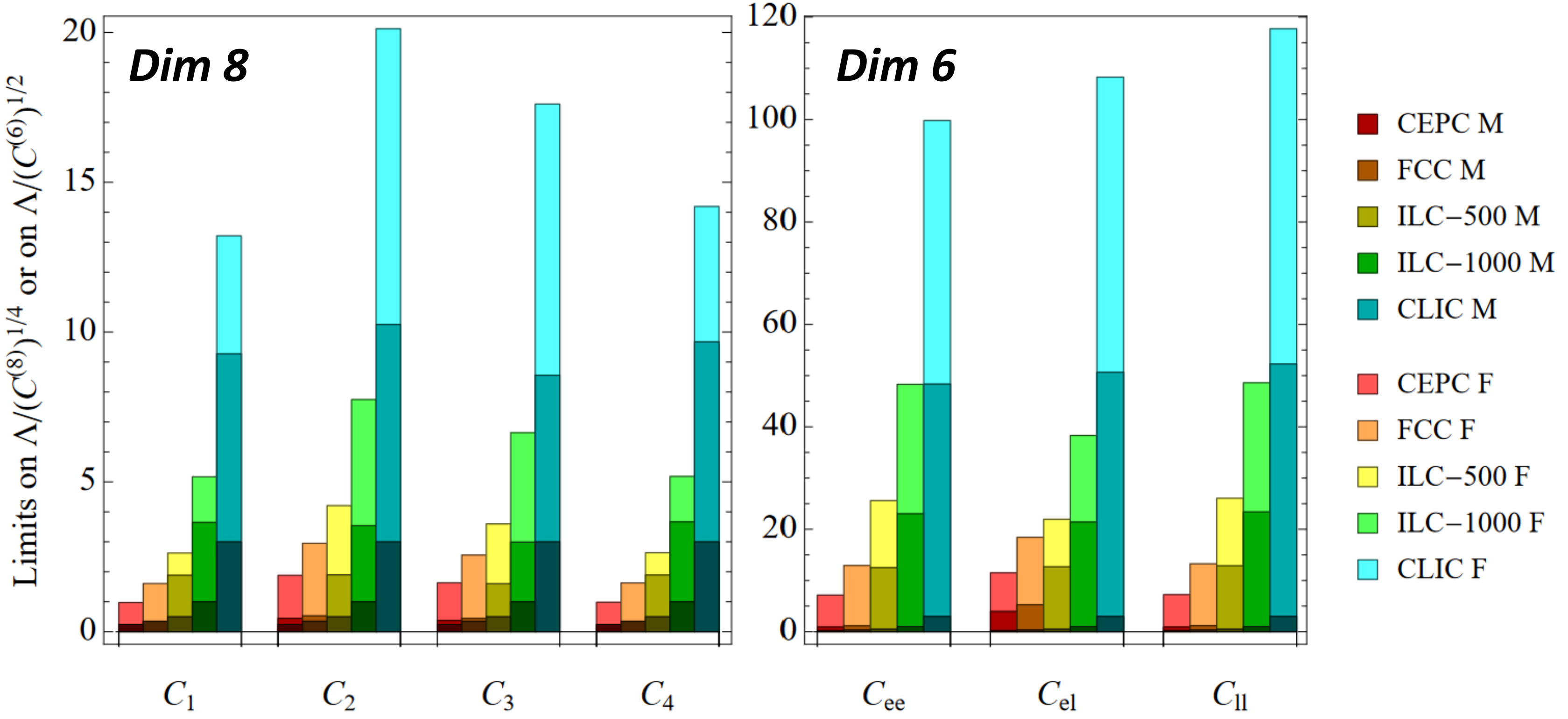
Collider	Runs: Energy [GeV] (Luminosity [ab^{-1}])			
FCC-ee	161 (10),	240 (5),	350 (0.2),	365 (1.5)
CLIC	380 (0.5),	1500 (2),	3000 (4)	
μC	10000 (10)			

← n.b. excluding run on Z-pole!



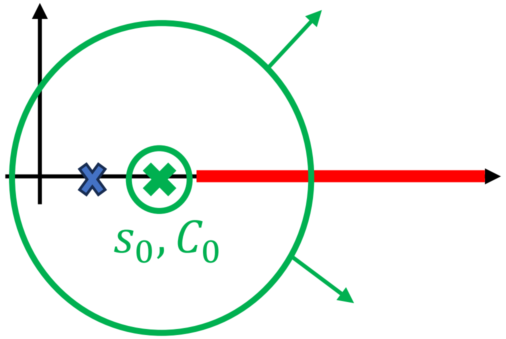
FCC:
For order-1 couplings, FCC-ee probes scales $\mathbf{\Lambda = 1 - 3}$ **TeV** for bounds on individual dimension-8 operators

SMEFT $e^+e^- \to e^+e^-$: positivity at dimension-6?

We do not have positivity bounds at dimension-6. *Why not?*

$$c_6 \sim \frac{\partial \mathcal{A}_{\text{EFT}}}{\partial s}\Big|_0 \text{, but causality + unitarity + locality only guarantees } \frac{\partial \mathcal{A}_{\text{UV}}}{\partial s} < \log^2 s$$

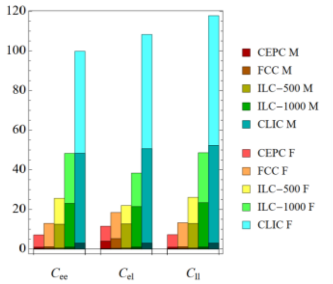
$\Longrightarrow$ we cannot 'cross out' the contour at infinity



BUT we can consider a restricted (but still large) class of UV models, for which "$c_6 > 0$":

In addition to UV being causal + unitary + local, assume UV amplitudes is bounded as $\mathcal{A}_{\text{UV}} < s$.

Roughly, this is true if **there is no s-channel resonance mediating $AB \to AB$ scattering**



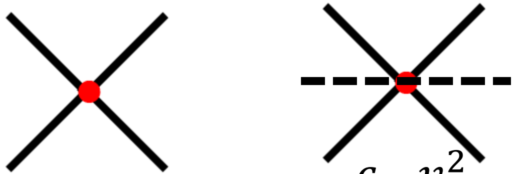
Sensitivity to dimension-6 positivity at FCC-ee up to scales $\mathbf{\Lambda \sim 10 - 20\ TeV}$

If measure dim-6 positivity violation, you expect new resonance in the *s*-channel!

Positivity and the Little Hierarchy

Consider $AB \to AB$ scattering on the background whereon a scalar field gets a VEV $\langle \boldsymbol{H} \rangle \neq \boldsymbol{0}$

$$\mathcal{L} = \frac{c_8}{\Lambda^4}\mathcal{O}_8 + \frac{c_{10,v}}{\Lambda^6}|H|^2\mathcal{O}_8 + \cdots$$



The positivity bound $\partial_s^2 \mathcal{A}_{\text{EFT}} > 0$ is NOT just $c_8 > 0$, but $\bar{c_8} \coloneqq c_8 + \frac{c_{10,v} v^2}{\Lambda^2} > 0$

If the UV is such that $c_8 > 0$, $c_{10,v} < 0$ and $|c_8|/|c_{10,v}| \ll 1$, positivity can only be satisfied if

$$\frac{v_{\text{EW}}^2}{\Lambda_{\text{NP}}^2} < \frac{|c_8|}{|c_{10,v}|} \ll 1 \qquad \textit{Positively Light Higgs?}$$

Positivity **correlates tuning in scales** to a seemingly unrelated **tuning in other higher-dimension WCs**

JD, Melville, Mimasu, You, 2308.06226

Positivity and the Little Hierarchy

Can disentangle the c_{10} contribution to the bound by producing $h(h)$ in the final state.

Prospects in future lepton colliders (still an unfair comparison…)

- $AB \to AB$: $l^+l^- \to l^+l^-, l = \{e, \mu\}$ $\Longrightarrow$ c_8/Λ^4 sensitivity
- $AB \to ABh$: $l^+l^- \to l^+l^-h$ $\Longrightarrow$ $c_{10,v}/\Lambda^6$ sensitivity (dominant SM bkg = $e^+e^- \to Zh$)

$$\mathcal{O}_8^{(1)} = \partial^\nu \left(\bar{e}_i\gamma^\mu e_i\right) \partial_\nu \left(\bar{e}_i\gamma_\mu e_i\right) \,,$$
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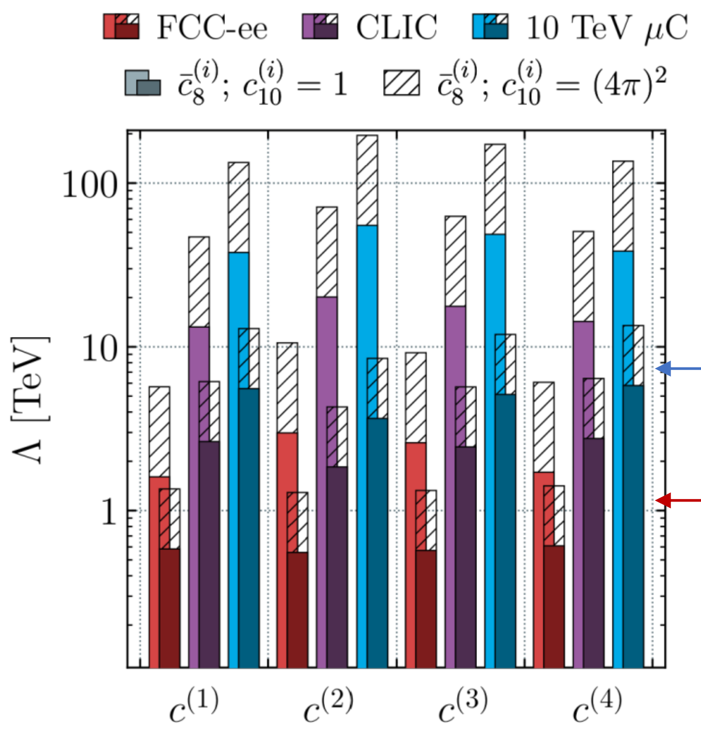

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CLIC	380 (0.5),	1500 (2),	3000 (4)	
μC	10000 (10)			

Can reach 10 (5) TeV sensitivity at μC for $|H|^2\mathcal{O}_8^{(i)}$, in the limit of 4π (1) couplings

FCC-ee

Reach 1 TeV sensitivity for $|H|^2\mathcal{O}_8^{(i)}$ if large coupings

Summary

Positivity bounds relate classes of UV models to patterns in low-energy EFT coefficients; offers new ways of thinking about BSM at future colliders.

At FCC-ee minus the Z pole run (an example of "diversity" program):

- Dim-8 positivity for 4-lepton processes: sensitivity to scales **1-3 TeV** for order-1 couplings
- Dim-6 positivity and inference of new states; sensitivity to scales up to **15 TeV** or so…
- Dim-8 vs Dim-10 measurements from $e^+e^- \to e^+e^-(h)$, can potentially measure a pattern of SMEFT coefficients that are only consistent with restricted range of allowed Higgs vevs; sensitivity up to **1 TeV** if strongly coupled

What about FCC-hh?

FCC-hh would probe counterpart operators involving quarks, naively reaching much higher scales…

Backup Slides

Example

Real scalar field ϕ

$$\mathcal{L} = \frac{c_8}{\Lambda^4}(\partial\phi)^4$$

Contribution to the amplitude

$$\mathcal{A}_{\phi\phi\to\phi\phi}(s,t) = 8c_8\,\frac{s^2+t^2+u^2}{\Lambda^4}$$

Positivity bound

$$\partial_s^2\mathcal{A}_{\text{EFT}}(s,0)|_{s_0} \geq 0 \qquad \Longrightarrow \qquad c_8 > 0$$

Example (continued beyond the forward limit)

Real scalar field ϕ

$$\mathcal{L} = \frac{c_8}{\Lambda^4}(\partial\phi)^4 + \frac{c_{10,\partial}}{\Lambda^6}(\partial\phi)^2(\partial\partial\phi)^2$$

We still have $c_8 > 0$ from $\partial_s^2 \mathcal{A}_{\text{EFT}}(s,0)$

There are also bounds *beyond the forward limit* i.e. scattering angle $\theta \neq 0$ i.e. Mandelstam $t \neq 0$

These bounds here take the form

De Rham, Melville, Tolley, Shou 1702.06134
Bellazzini, Riva, Serra, Sgarlata, 1710.02539

$$|c_{10,\partial}| < \mathcal{O}(1)\ c_8$$

So we *cannot tune* c_8 to be much smaller than c_{10} without violating positivity

But there are no such “sandwich bounds” on the $c_{10,v}$ that we consider in “positively light Higgs”

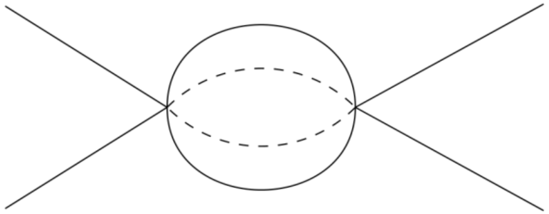
IR loops?

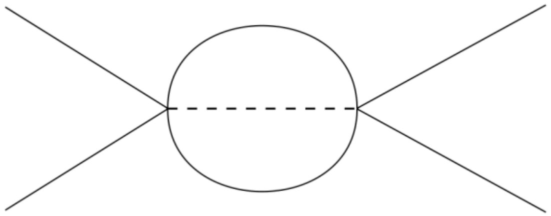
Davighi, Melville, Mimasu, You, 2308.06226

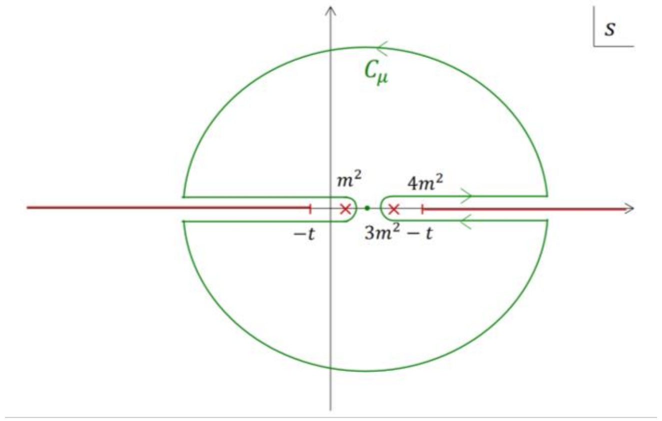
Subleading effect that *strengthens* the positivity bound by contributing positively to $\int_{\text{branch cut}} \text{Disc}\,\mathcal{A}$

See also Bellazzini, Riembau, Riva, 2112.12561

Assuming $|c_{10}| \gg |c_8|$, consider corrections to $\partial_s^2 \mathcal{A}_{\text{EFT}}$:







$$\frac{\text{Disc}_s \mathcal{A}(s,0)}{2\pi i} = \frac{|c_{10}|^2 s^6}{\Lambda^{12}} \left(\frac{N_3}{(16\pi^2)^3} + \frac{N_2}{(16\pi^2)^2} \frac{v^2}{s} \right) \quad \Longrightarrow \quad c_8 + c_{10} \frac{v^2}{\Lambda^2} > \frac{|c_{10}|^2 s_{\text{max}}^4}{\Lambda^8} \left(\frac{2N_3}{3(16\pi^2)^3} + \frac{N_2}{(16\pi^2)^2} \frac{v^2}{s_{\text{max}}} \right)$$

UV? First Attempt

Davighi, Melville, Mimasu, You, 2308.06226

Light fields: Dirac Ψ, complex H

Heavy field: real scalar ϕ

UV interaction: $L_{\text{UV}} \supset (y\,\phi\bar{\Psi}\Psi + \text{h.c.}) + \mu g_3 \phi^3 - \mu g_1 \phi |H|^2$

Integrating out ϕ generates

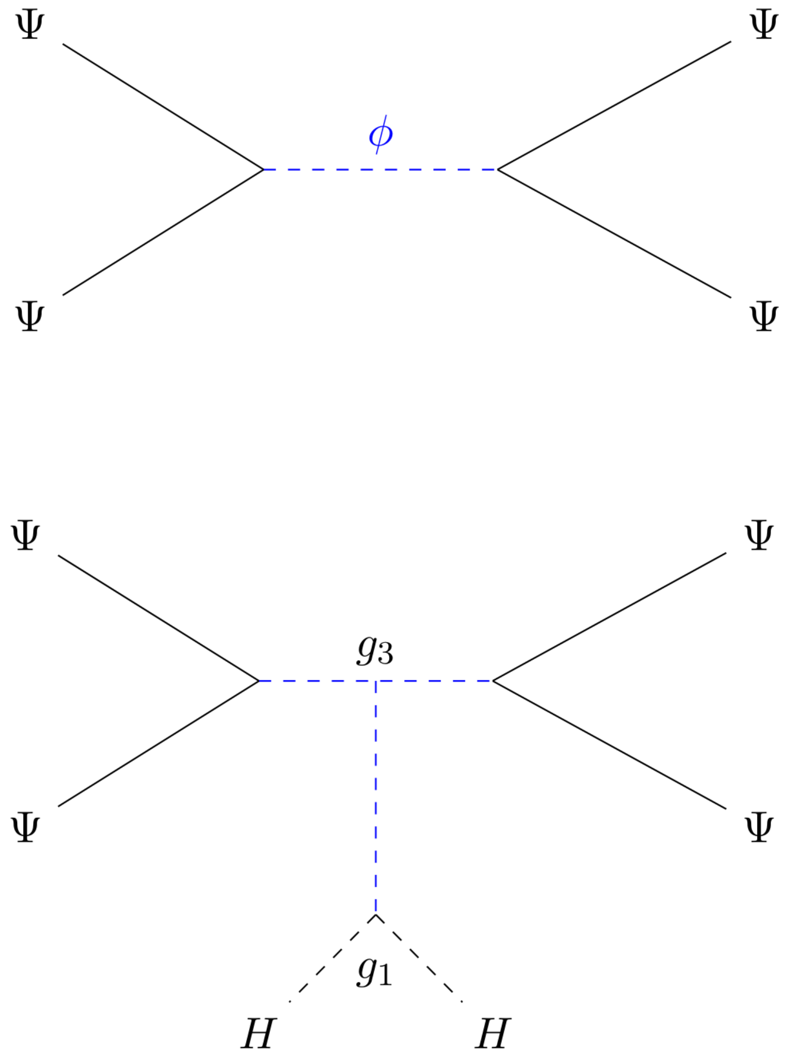
$$L_{\text{EFT}} \supset \left(c_8 + \frac{c_{10}|H|^2}{v^2}\right)\mathcal{O}_8, \qquad \mathcal{O}_8 = -\bar{\Psi}\Psi\partial_\mu\bar{\Psi}\partial^\mu\Psi$$

WCs: $c_8 = (y+\bar{y})^2, \quad c_{10} = (y+\bar{y})^2 \frac{4g_1(3g_3-g_1)\mu^2}{M^2}$

c_8 positive; c_{10} sign arbitrary

Positivity $\partial_s^2 \mathcal{A}_{\Psi\Psi\to\Psi\Psi} > 0 \implies \frac{v^2}{M^2} < \frac{1}{4g_1(3g_3-g_1)}\frac{M^2}{\mu^2}$

BUT **perturbativity breaks down** for $(g_i\mu)^2 \geq 4\pi M^2$! Cannot "stretch" positivity-implied hierarchy by more than **1-loop factor**



Second Attempt: SM extension

WIP with Maria Ramos and Guilherme Guedes

Light fields: Ψ (SM lepton), complex H (SM Higgs)

Two heavy fields: **fermion N** (RH ν), scalar S (in rep $(\mathbf{1},\mathbf{1})_{1(2)}$)

UV interactions: $L_{\text{UV}} \supset y\, H\bar{\Psi}N + \kappa S\bar{\Psi}^c\Psi - \lambda S\bar{N}N + \text{h.c}$

Integrating out S, N generates (assume same mass M)

$c_8 \sim |\kappa|^2, \quad c_{10} \sim |y|^2\kappa\lambda$

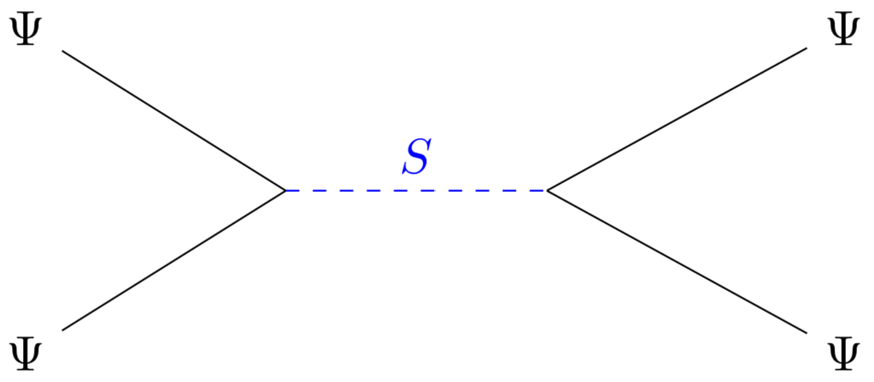
Again c_8 positive, c_{10} sign arbitrary

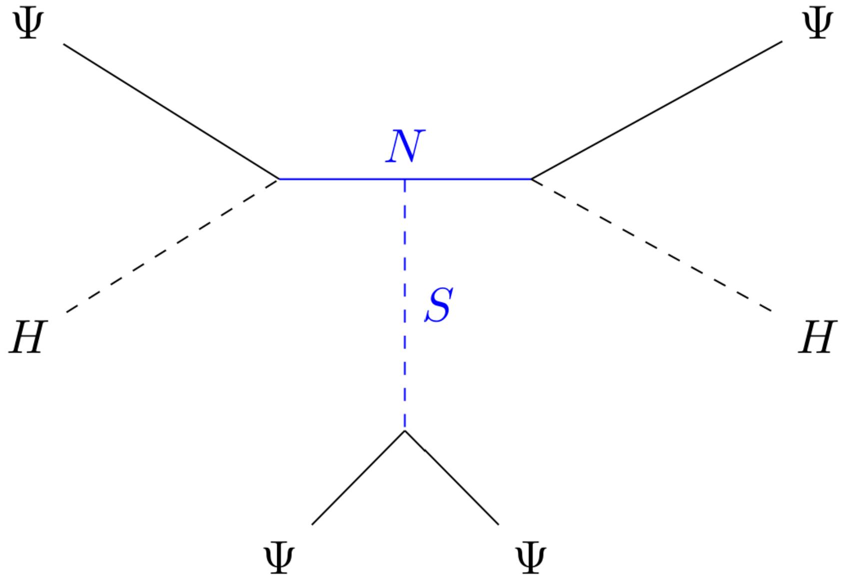
But this time, we have a coupling in the numerator of c_8/c_{10}:

$$\frac{c_8}{c_{10}} \sim \frac{\kappa}{\lambda|y|^2}$$

Positivity: $\partial_s^2 \mathcal{A}_{\Psi\Psi\to\Psi\Psi} > 0 \;\Rightarrow \frac{v^2}{M^2} < \frac{\kappa}{\lambda|y|^2}$ for $\kappa < 0$.

We can seemingly go to *small couplings* $\kappa \ll 1$ without violating perturbativity.

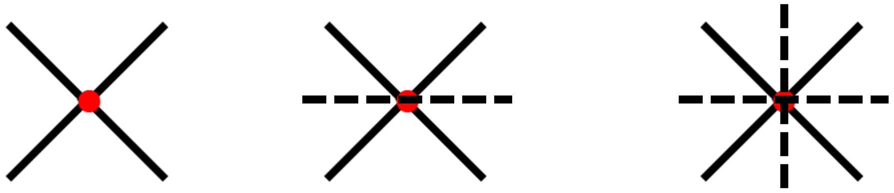




Summing the tower

Vacuum: $\langle \boldsymbol{H} \rangle \neq \mathbf{0}$**.** In general $AB \to AB$ receives contributions from whole tower of operators:

$$\mathcal{L}_{\mathrm{EFT}} \supset \frac{c_8}{\Lambda^4}\mathcal{O}_8 + \frac{c_{10,v}}{\Lambda^6}|H|^2\mathcal{O}_8 + \frac{c_{12,v}}{\Lambda^6}|H|^4\mathcal{O}_8 + \cdots = f_8\left(\frac{|H|^2}{\Lambda^2}\right)\mathcal{O}_8$$



$\sim c_8 s^2$ $\qquad \sim \frac{c_{10} v^2}{\Lambda^2} s^2$ $\qquad \sim \frac{c_{12} v^4}{\Lambda^4} s^2$

"Re-summing", positivity condition is really

$$f_8\left(\frac{|H|^2}{\Lambda^2}\right) > 0$$

For this to restrict v^2/Λ^2 requires a "violation" of the natural EFT power counting $\frac{1}{\mathrm{n!}}\partial^n f|_{\mathrm{vac}} \sim \mathcal{O}(1)$

Disentangling c_{10}

- $AB \to AB \Longrightarrow \bar{c}_8/\Lambda^4$
- $AB \to ABh, ABhh \Longrightarrow vEc_{10}/\Lambda^6, E^2c_{10}/\Lambda^6$

Enough to verify positivity is satisfied

Can we disentangle the c_{10} contribution?

$$\text{Positivity: } \bar{c}_8 \coloneqq c_8 + \frac{c_{10,v}v^2}{\Lambda^2} \geq 0$$

Technically, we **cannot** extract an unambiguous positivity bound on vev v from this without either:

- A direct determination of "bare" $c_8 = f_8(0)$; unmeasurable in scattering! or
- An independent determination/inference of the scale Λ

BUT

If we measure $c_{10,v} < 0$ and infer $|c_{10}| \gg \bar{c}_8 > 0$ even under conservative hypothesis $\Lambda = 1\text{ TeV}$* as per observed little hierarchy, this is indirect evidence for being in the "positively light" region of EFTs

* if $\Lambda \uparrow$, inferred $|c_{10}|/c_8 \uparrow$ also