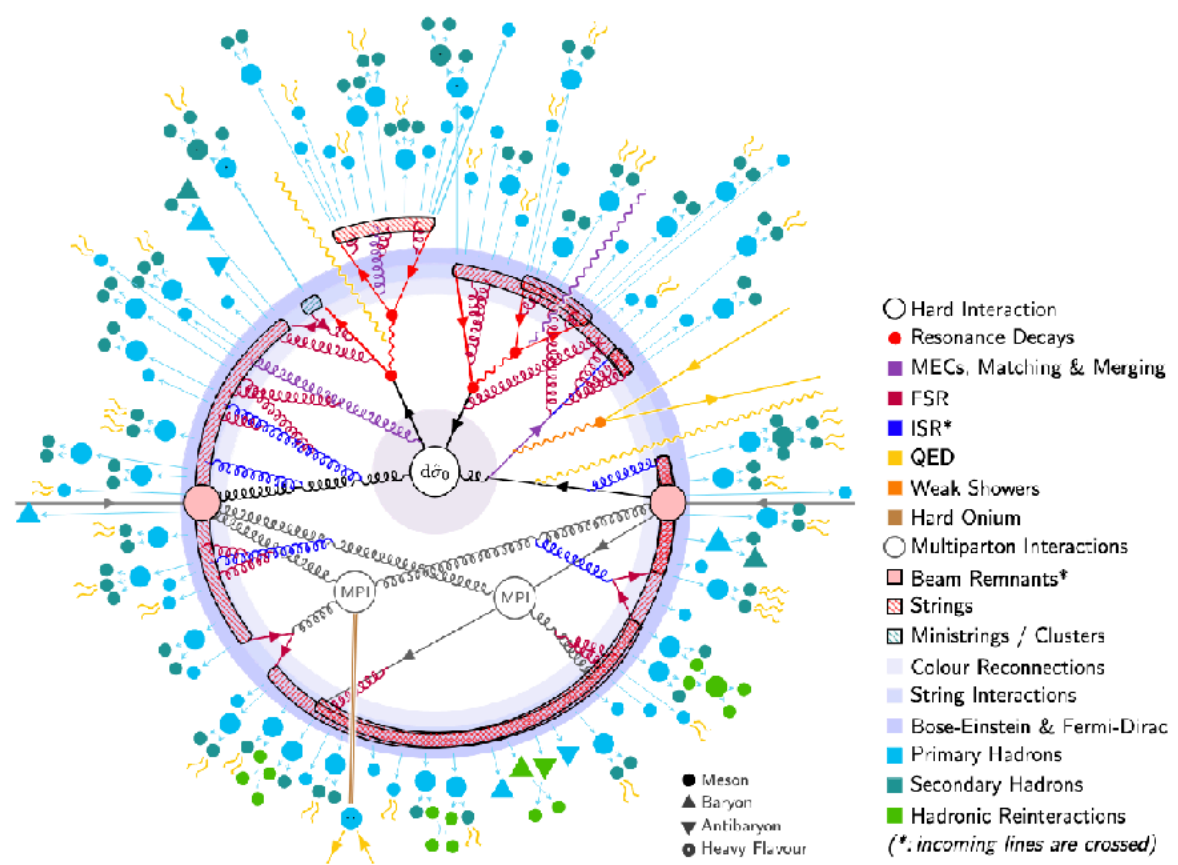
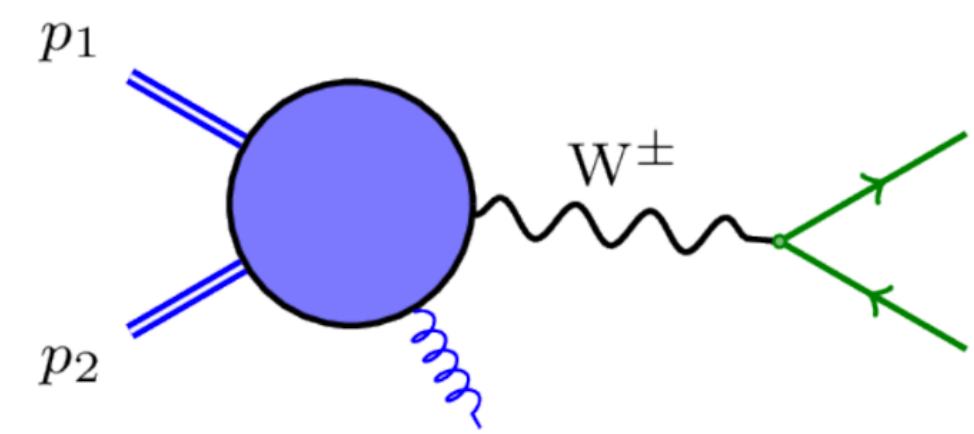


# Monte Carlo generators for FCC-ee



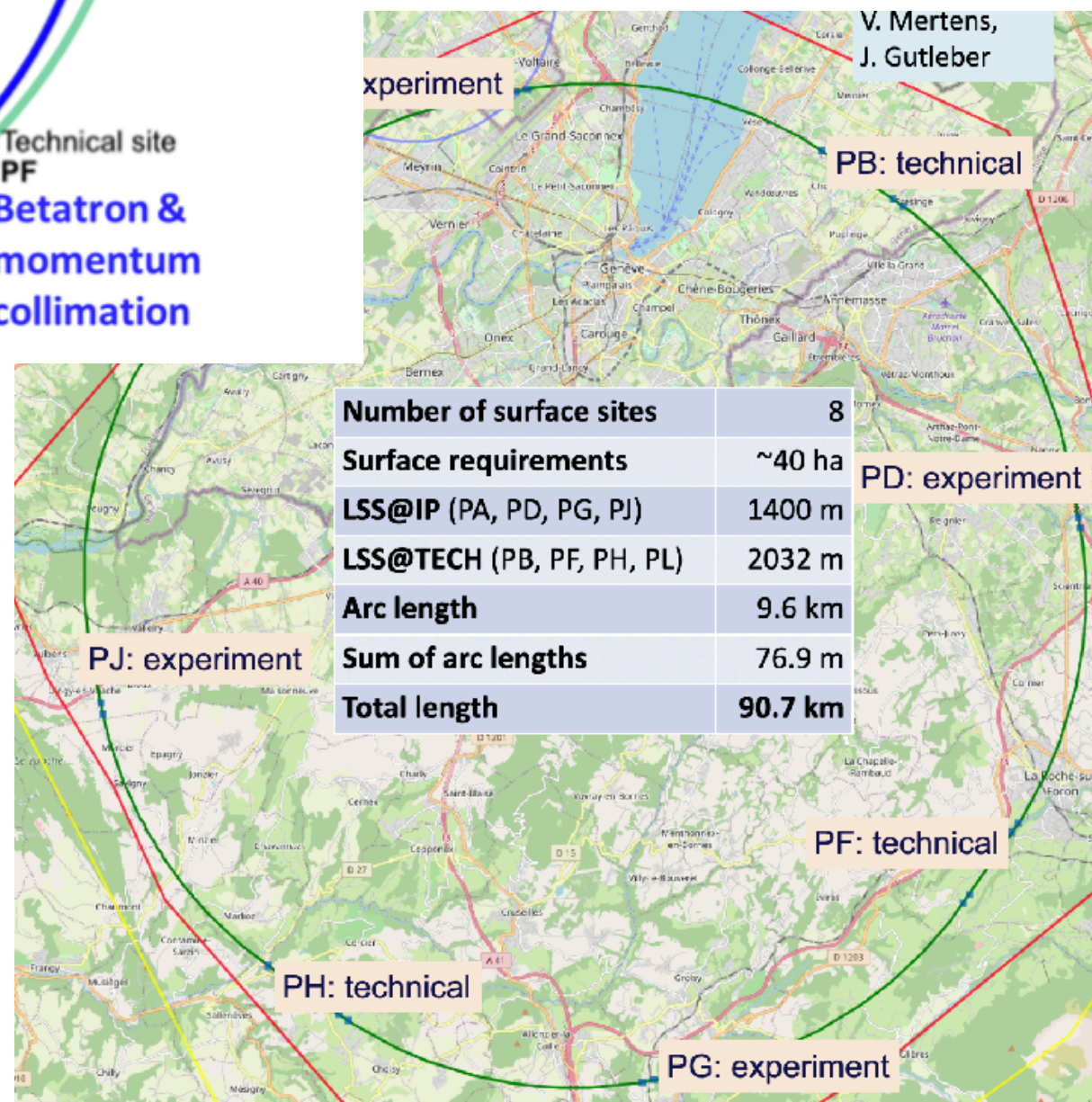
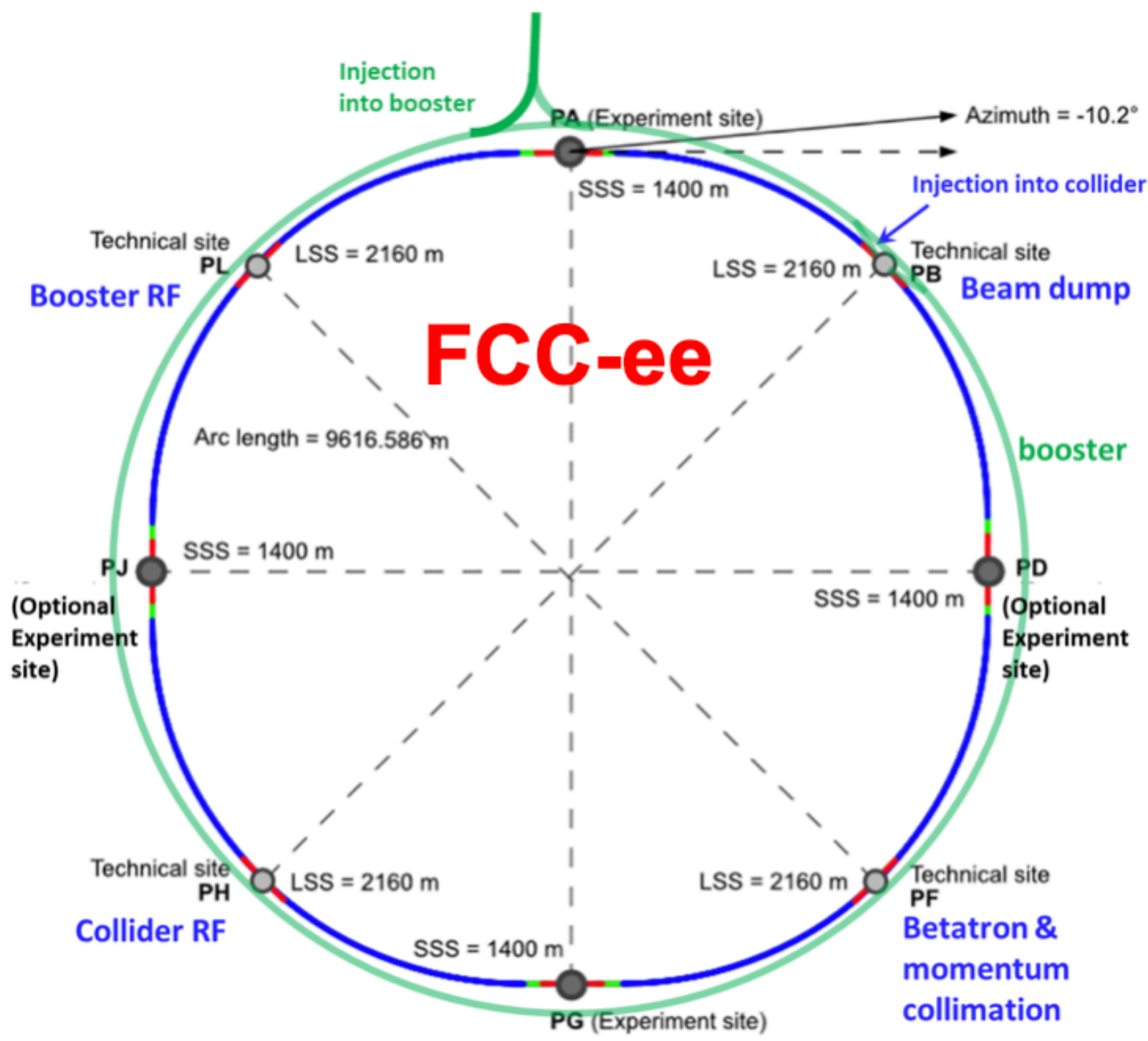
**CLUSTER OF EXCELLENCE**  
QUANTUM UNIVERSE

Jürgen R. Reuter

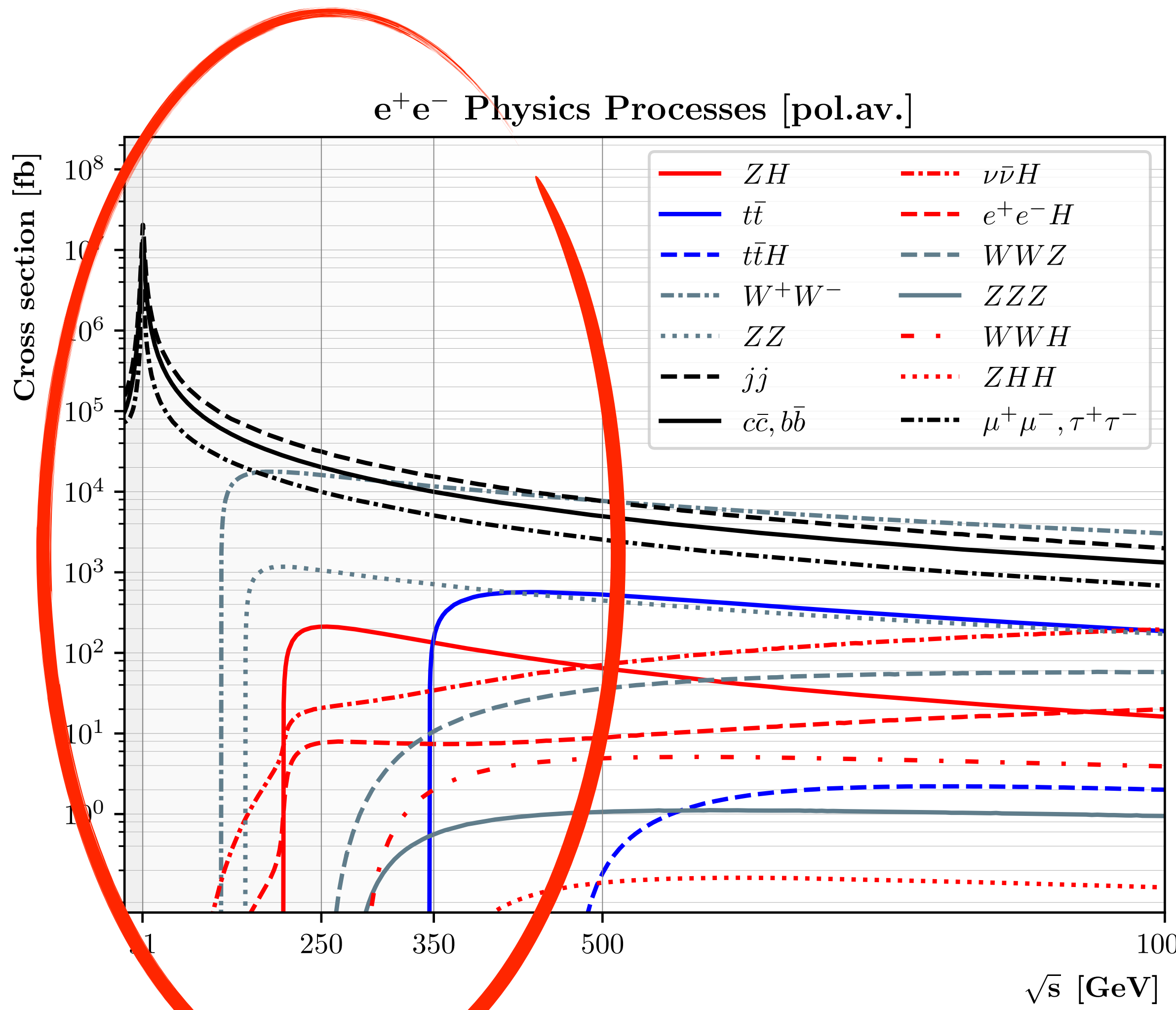




# Physics program to be simulated



- 91 GeV — Z pole running
- 161 GeV — WW threshold
- 240 GeV — ZH threshold
- 365 GeV — tt threshold



# The importance of MC event generators

Why are event generators important?

Because all our forward simulation chain depends on them!

Why are event generators non-trivial?

Because they contain *all* our knowledge of particle physics!





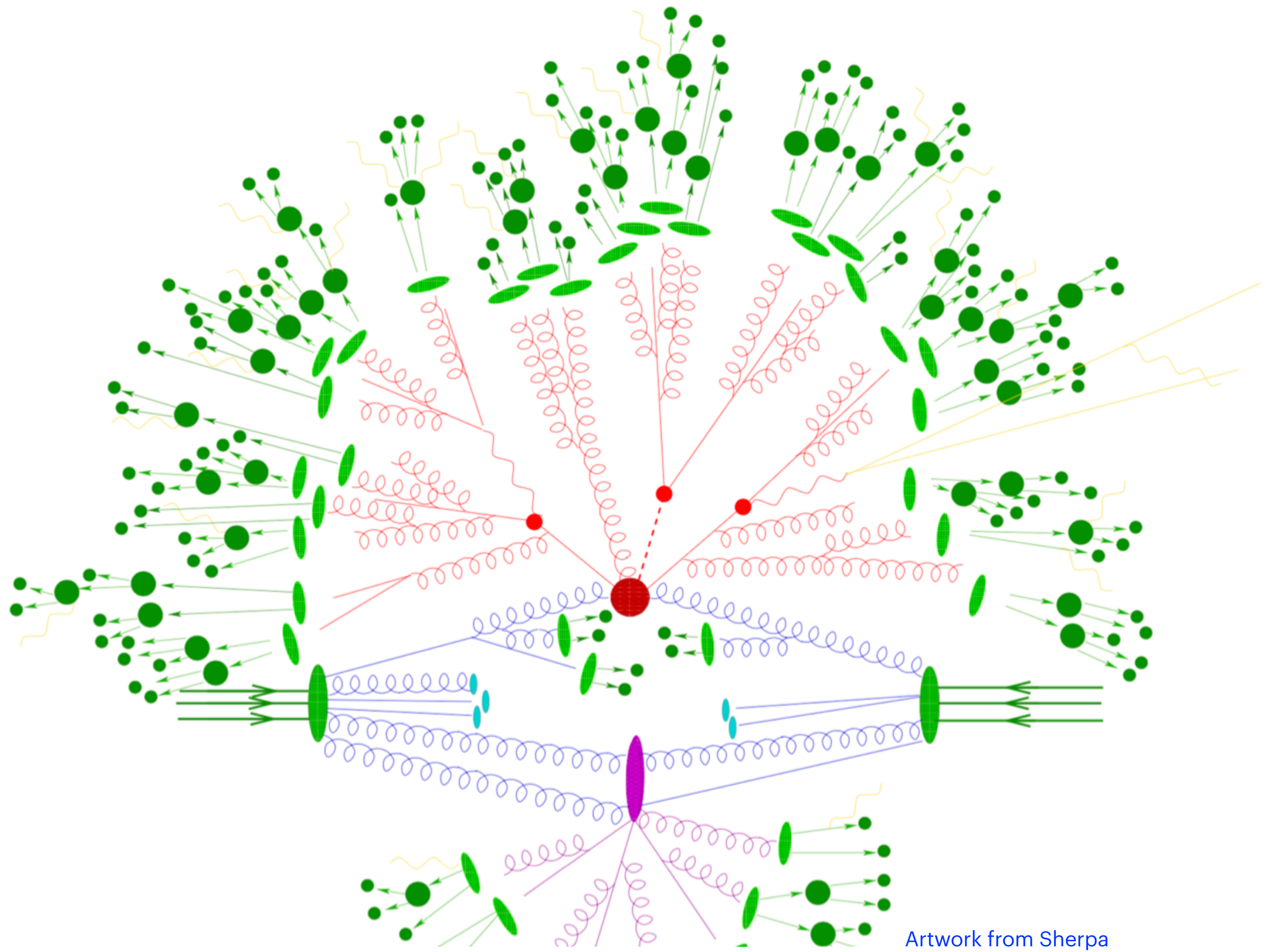
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Artwork from Sherpa





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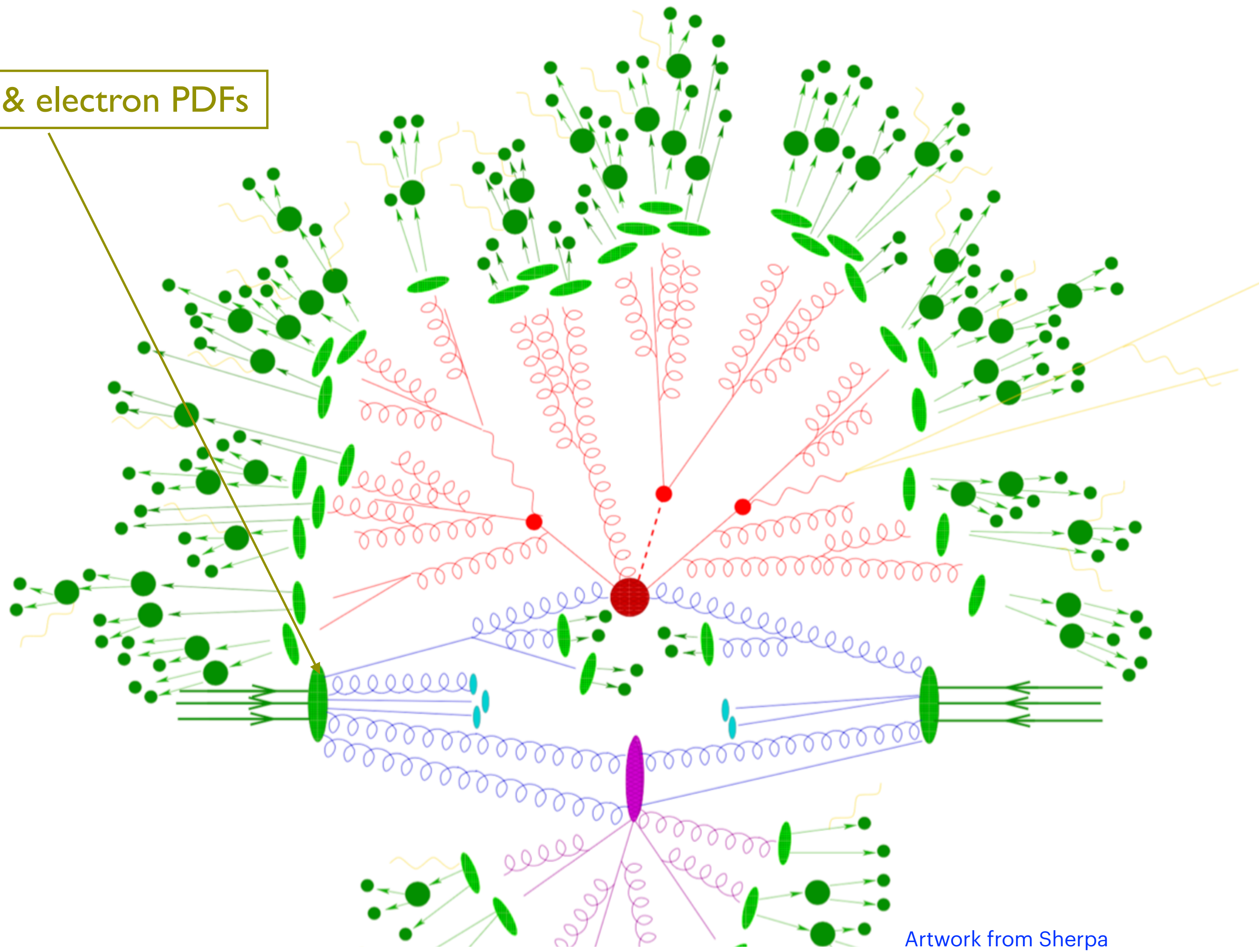
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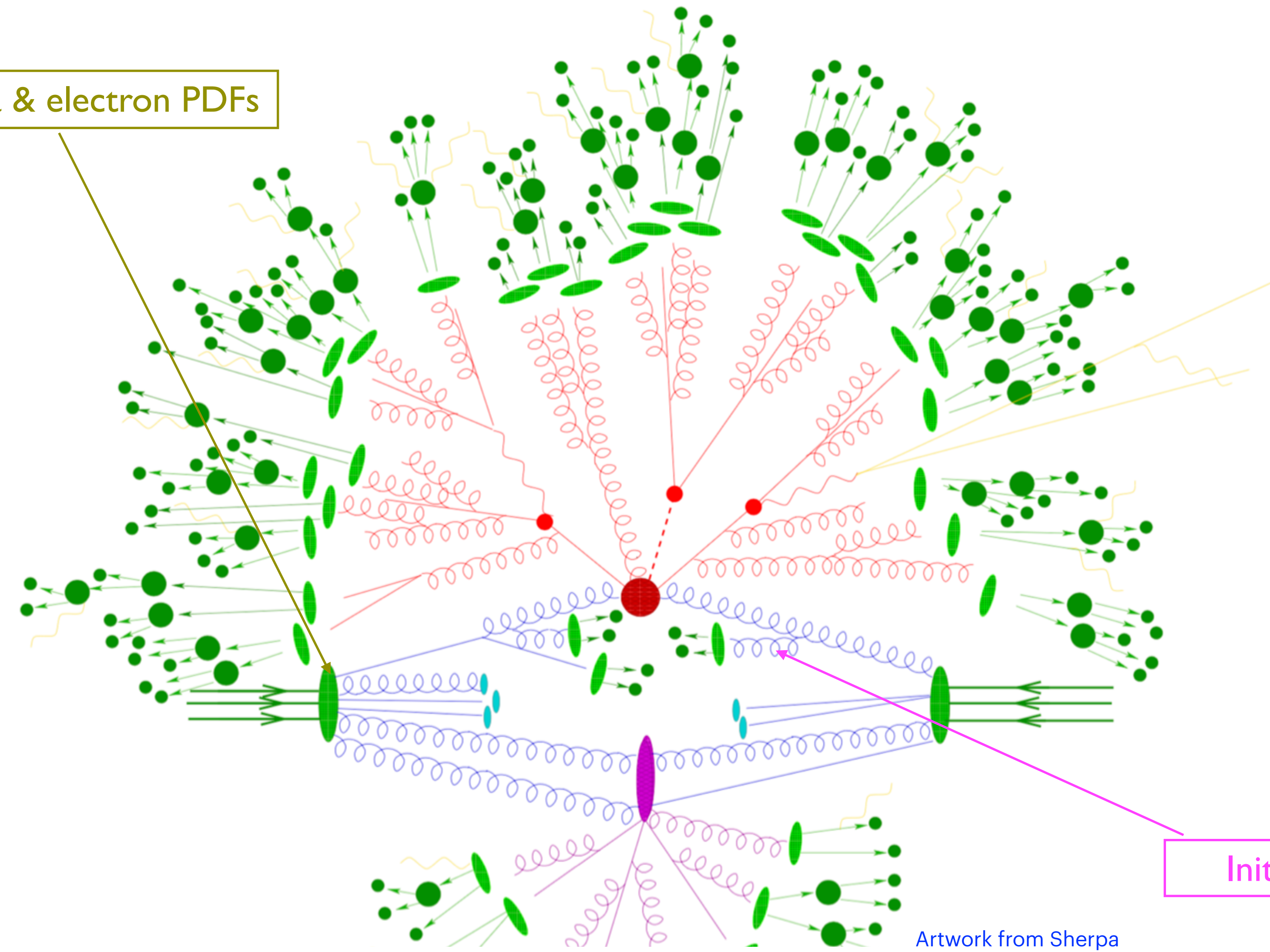
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Initial state QED radiation

Artwork from Sherpa





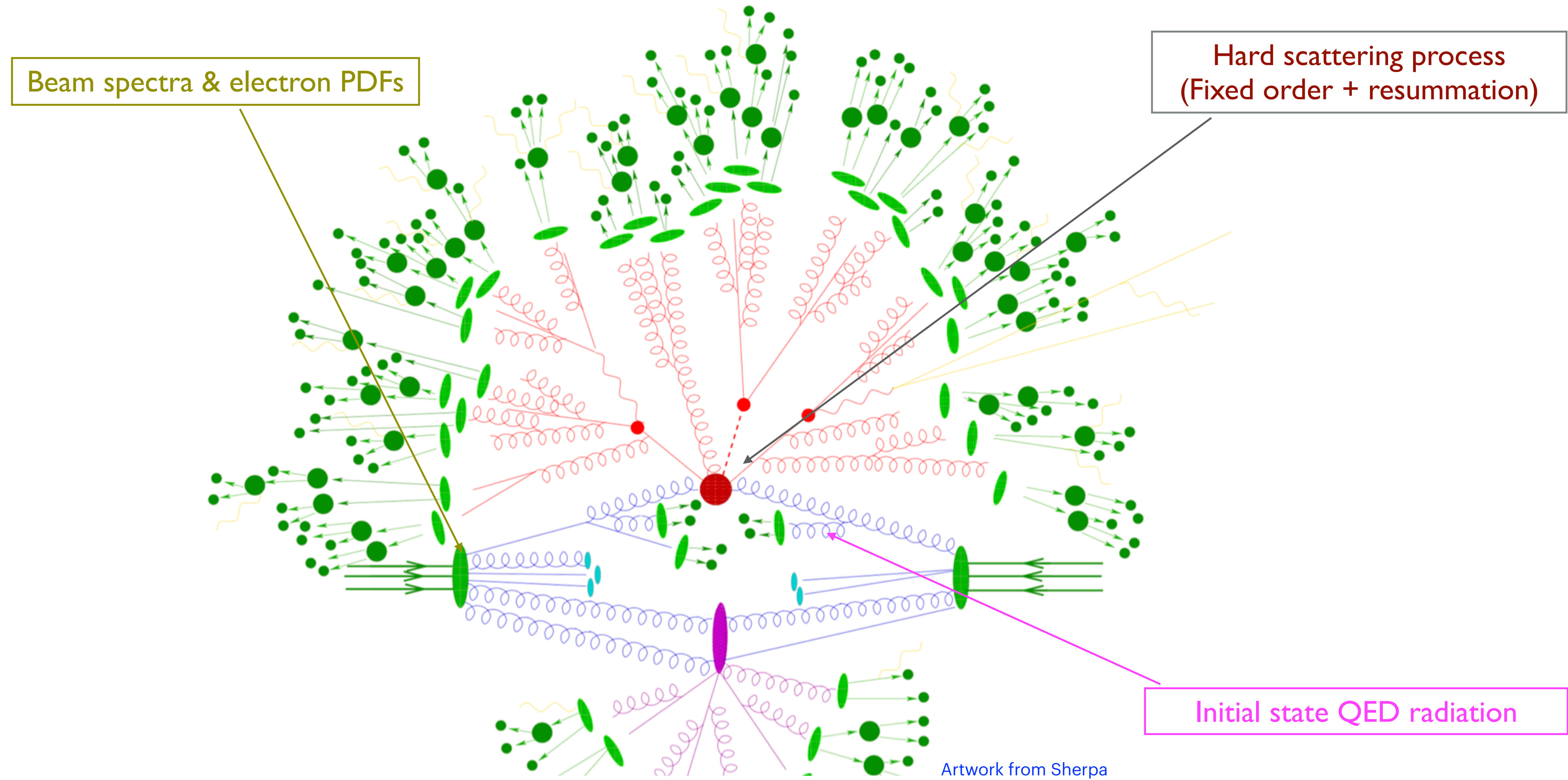
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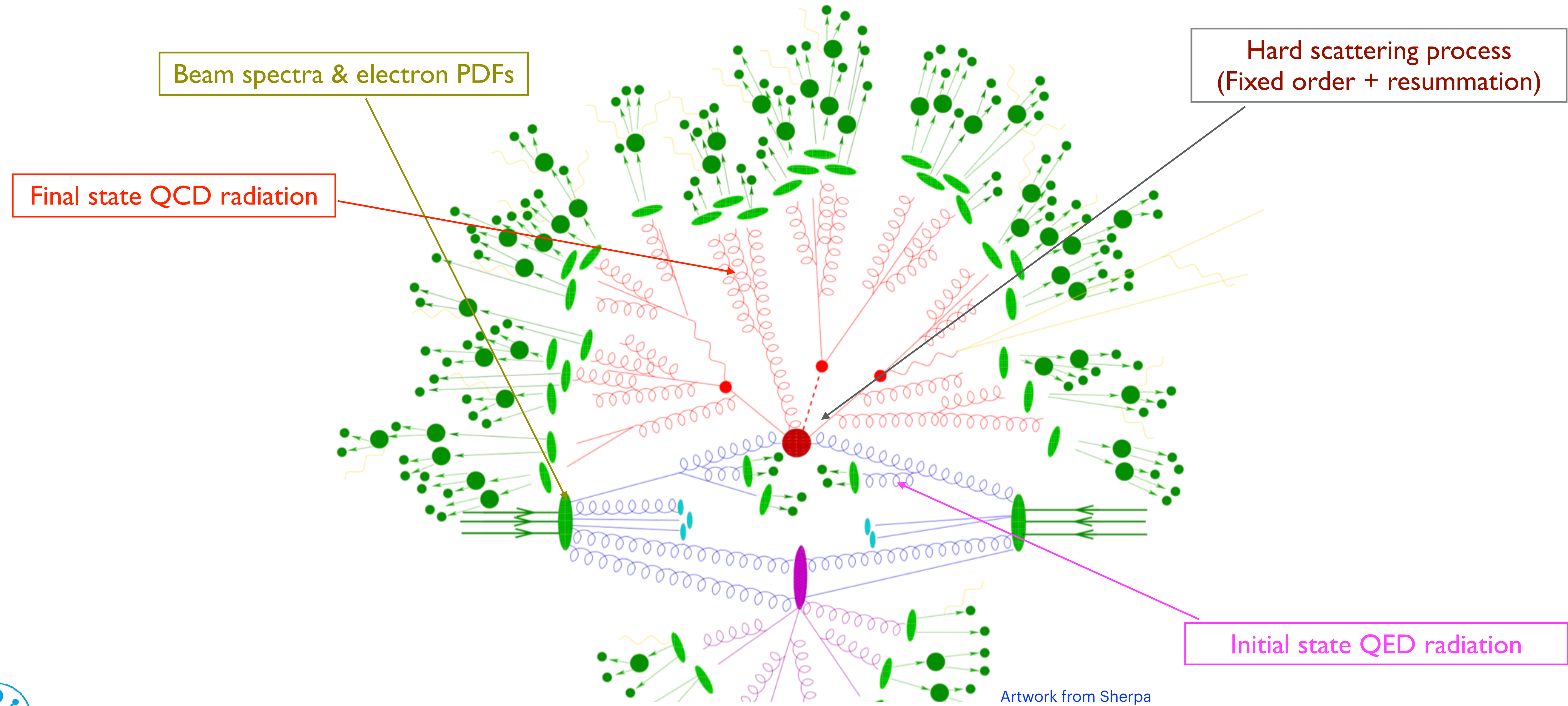
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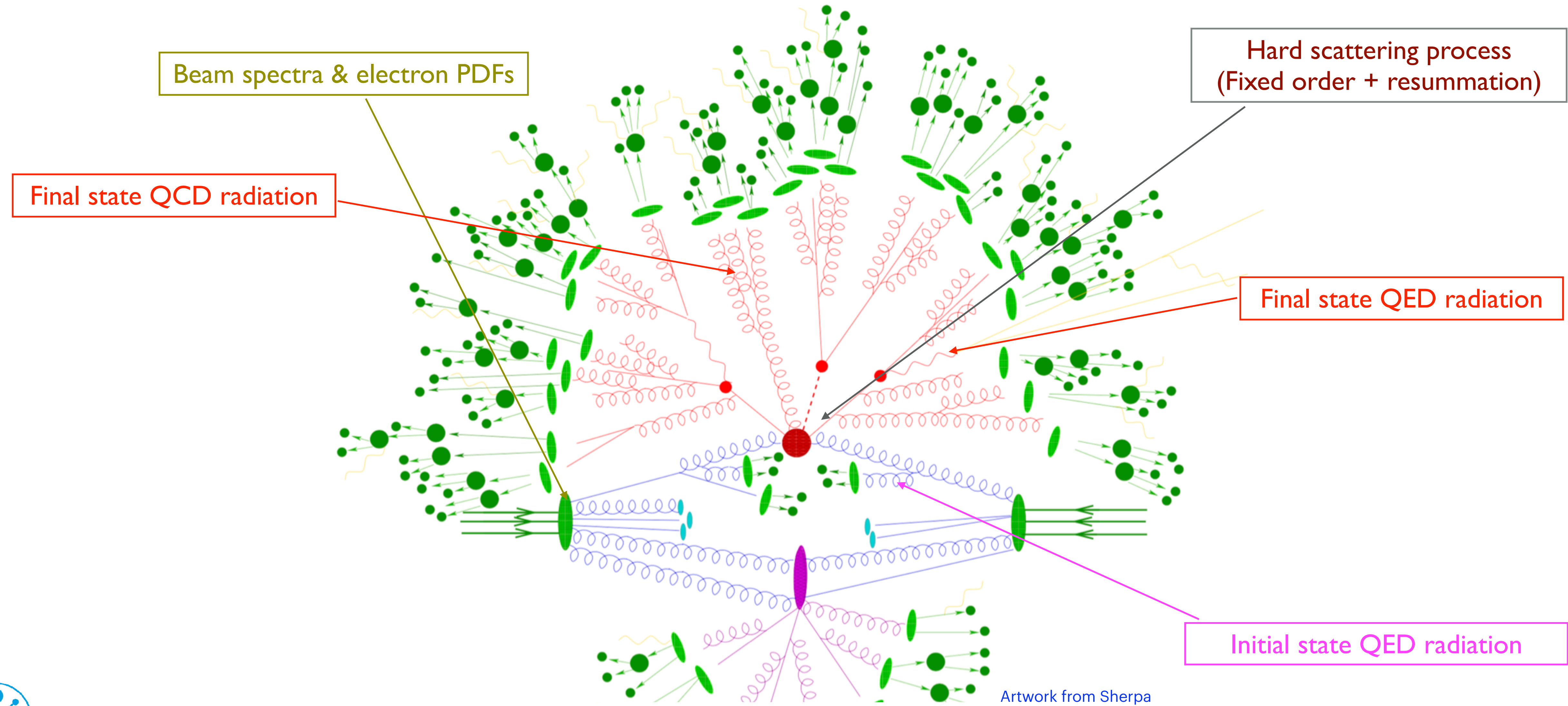
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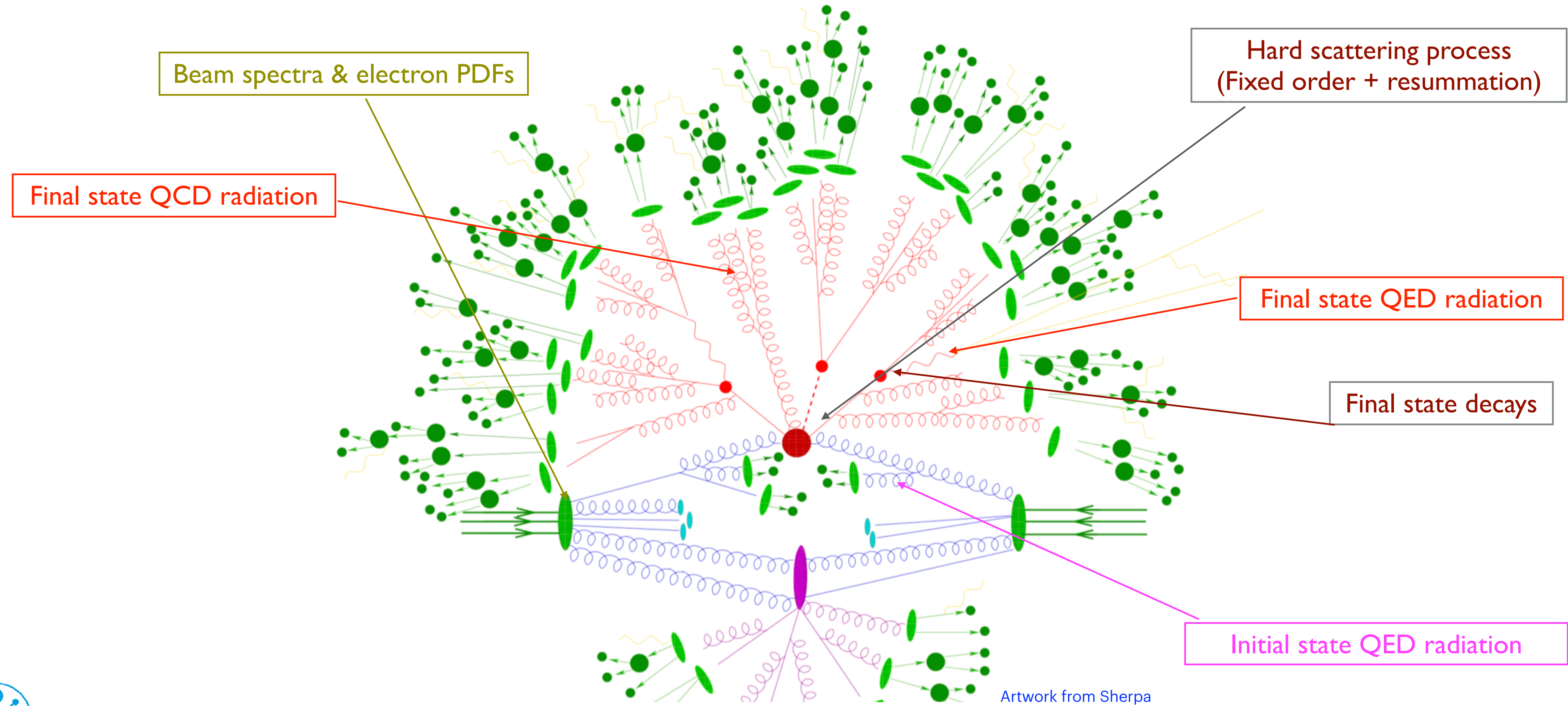
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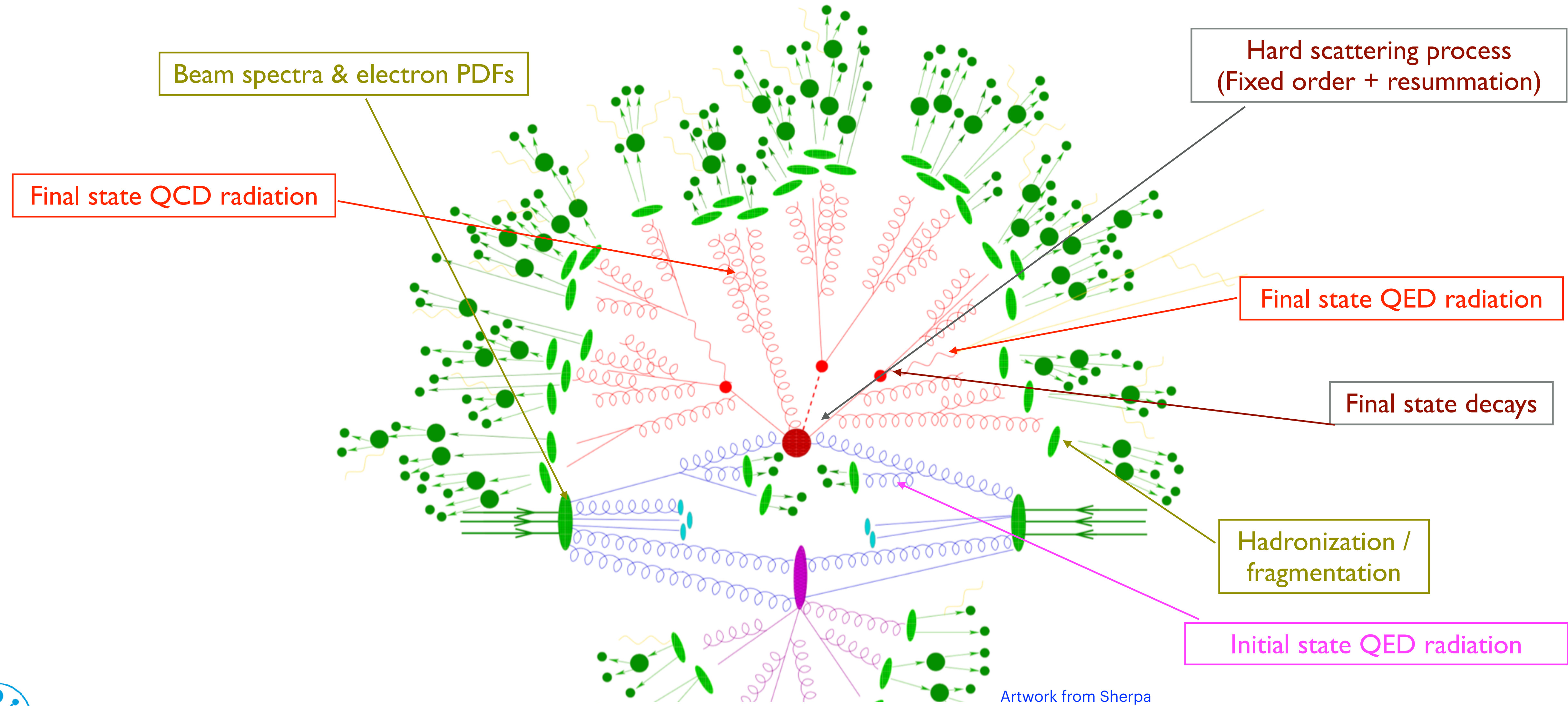
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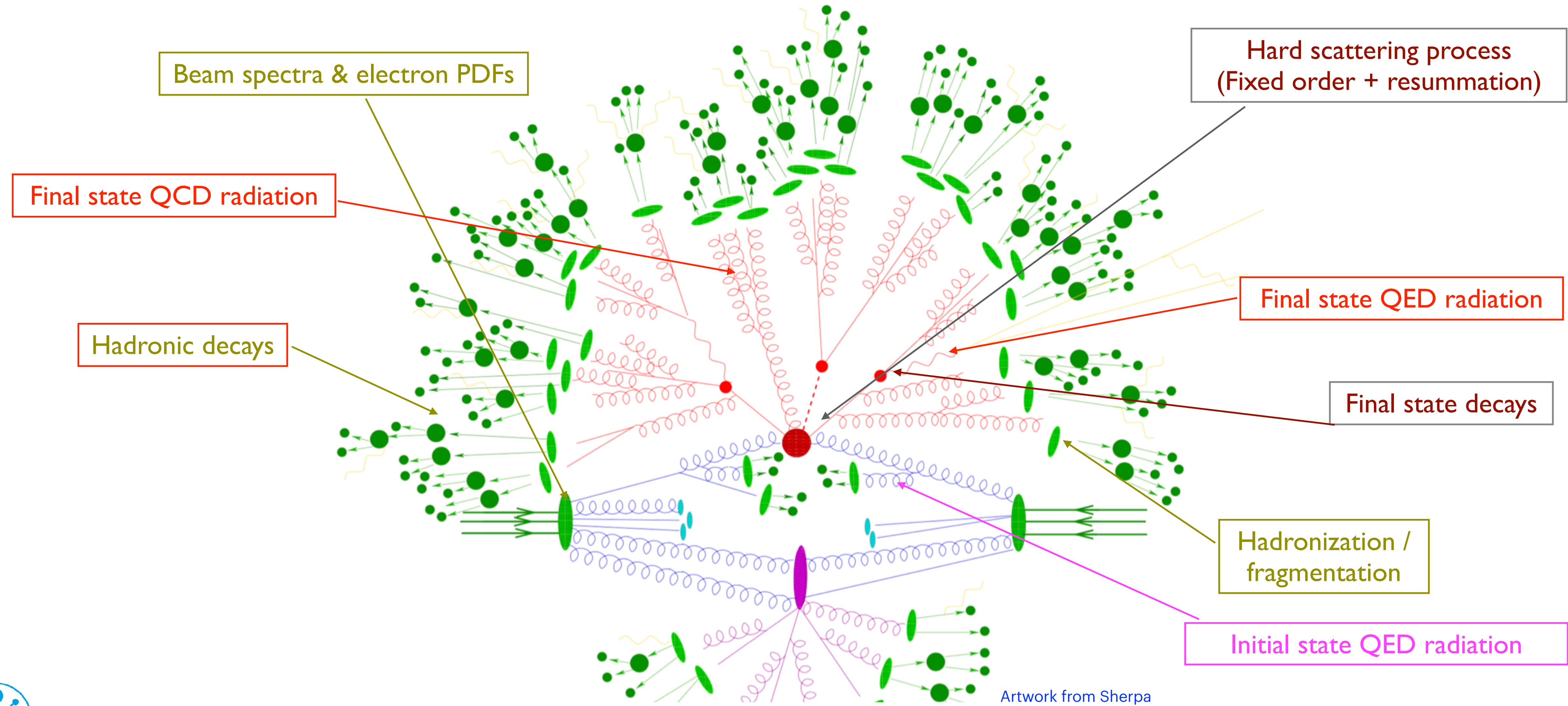
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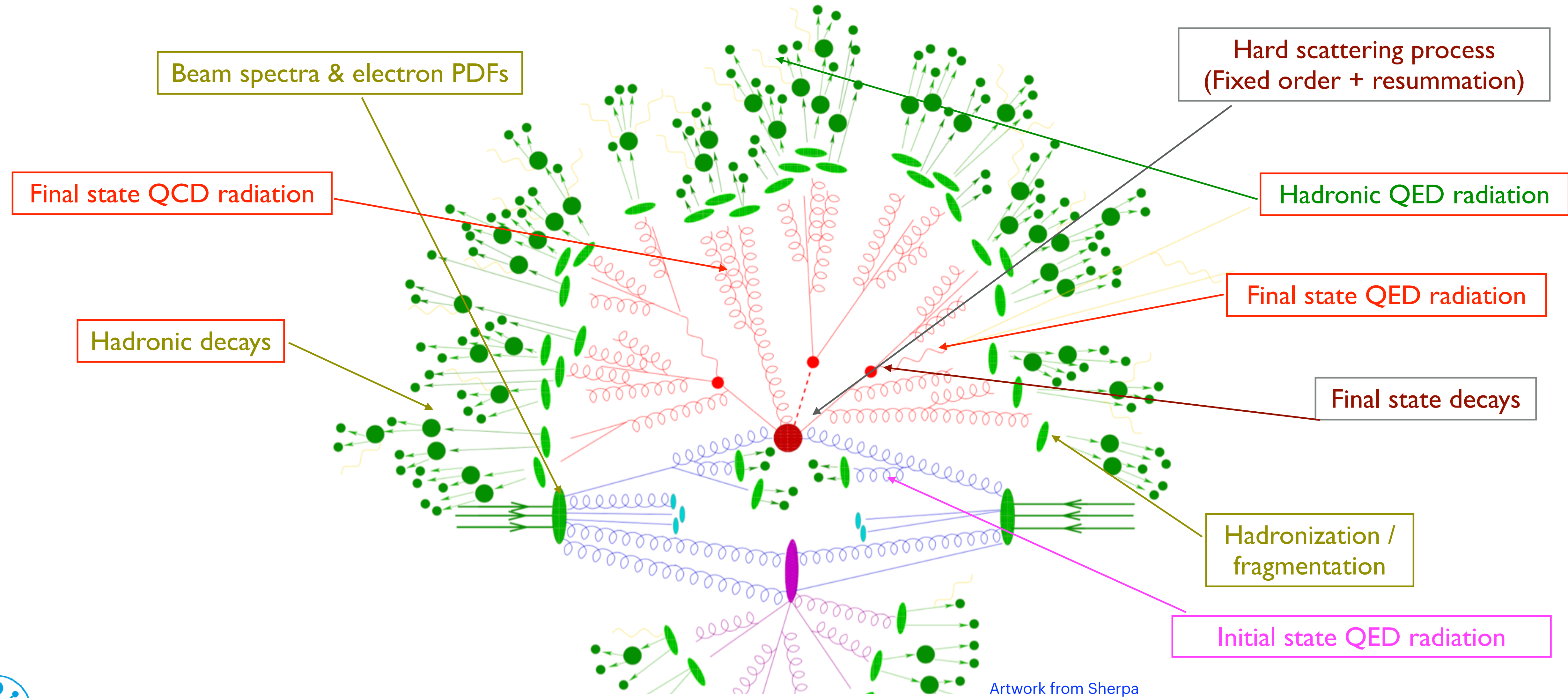
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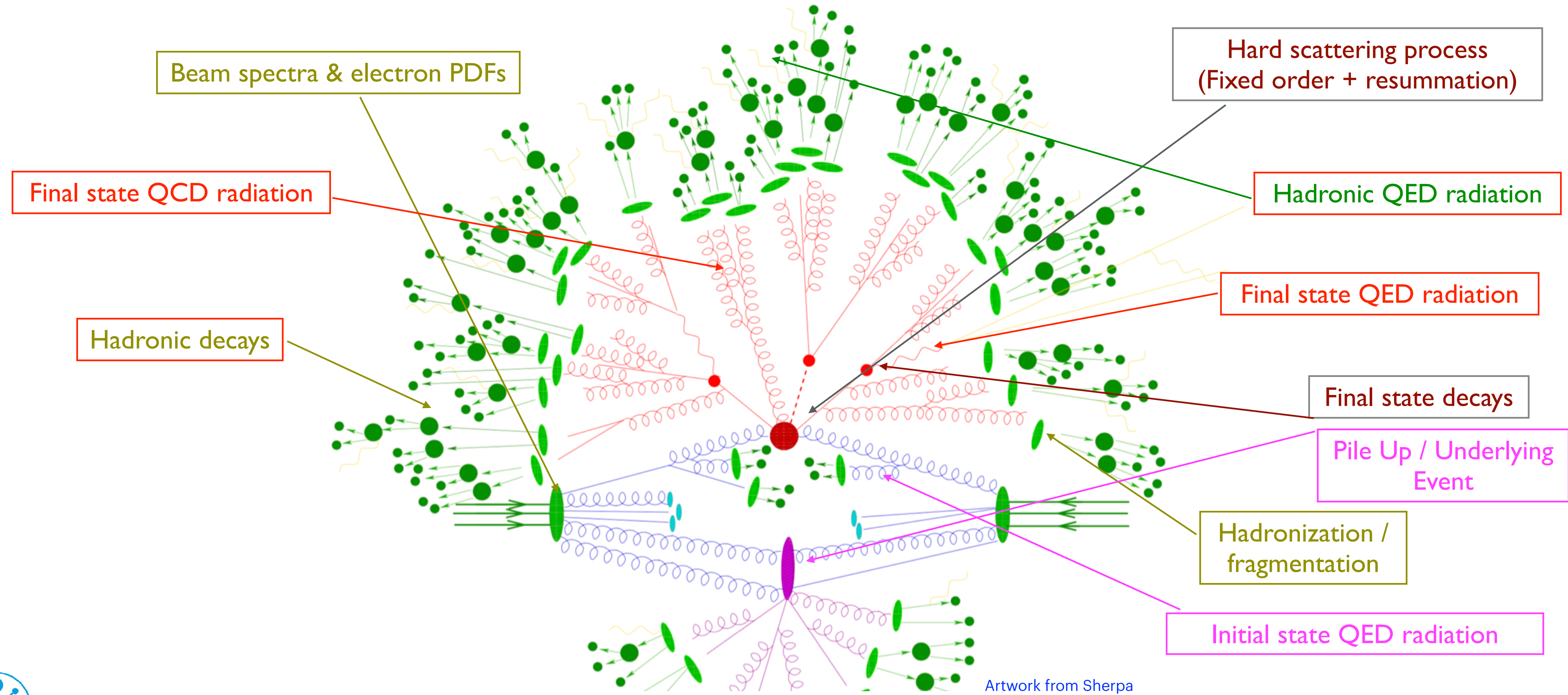
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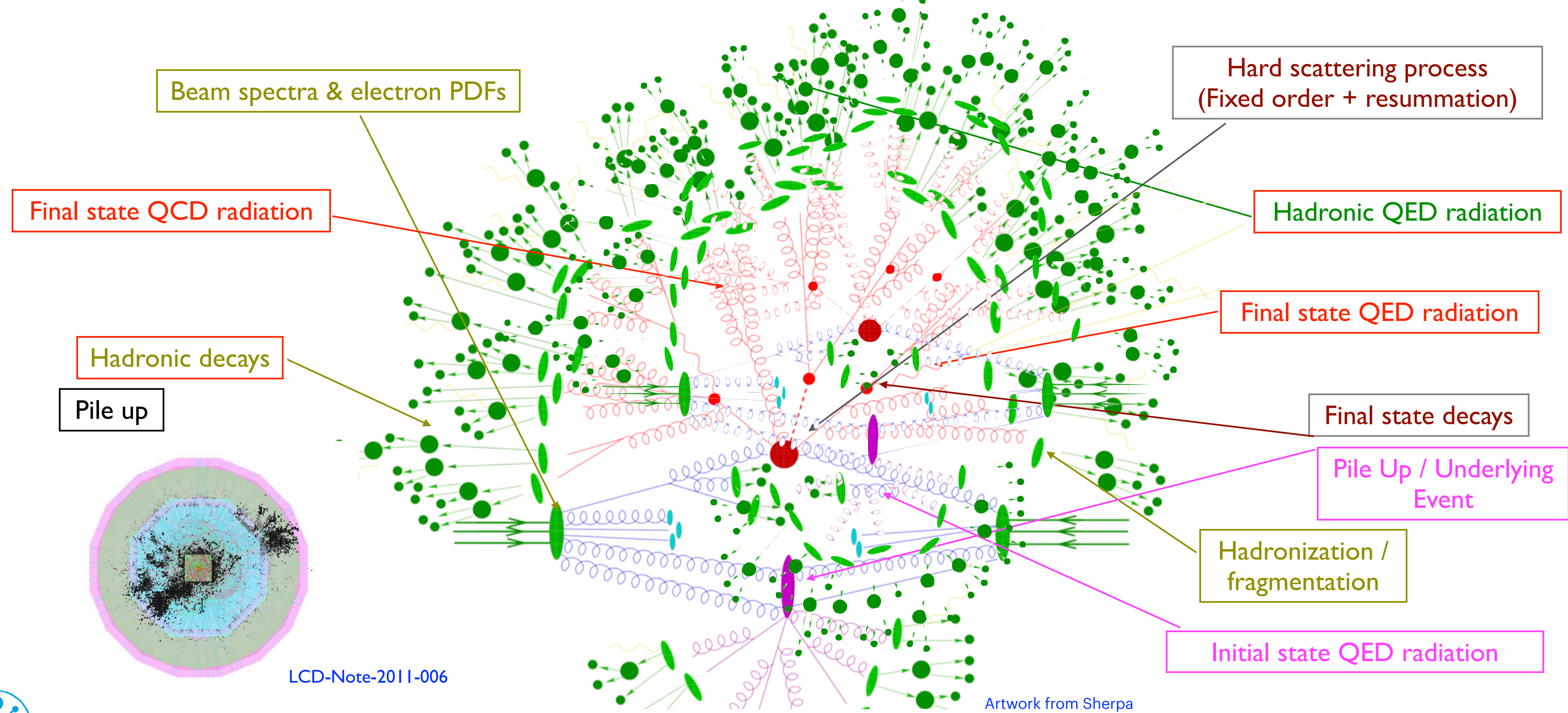
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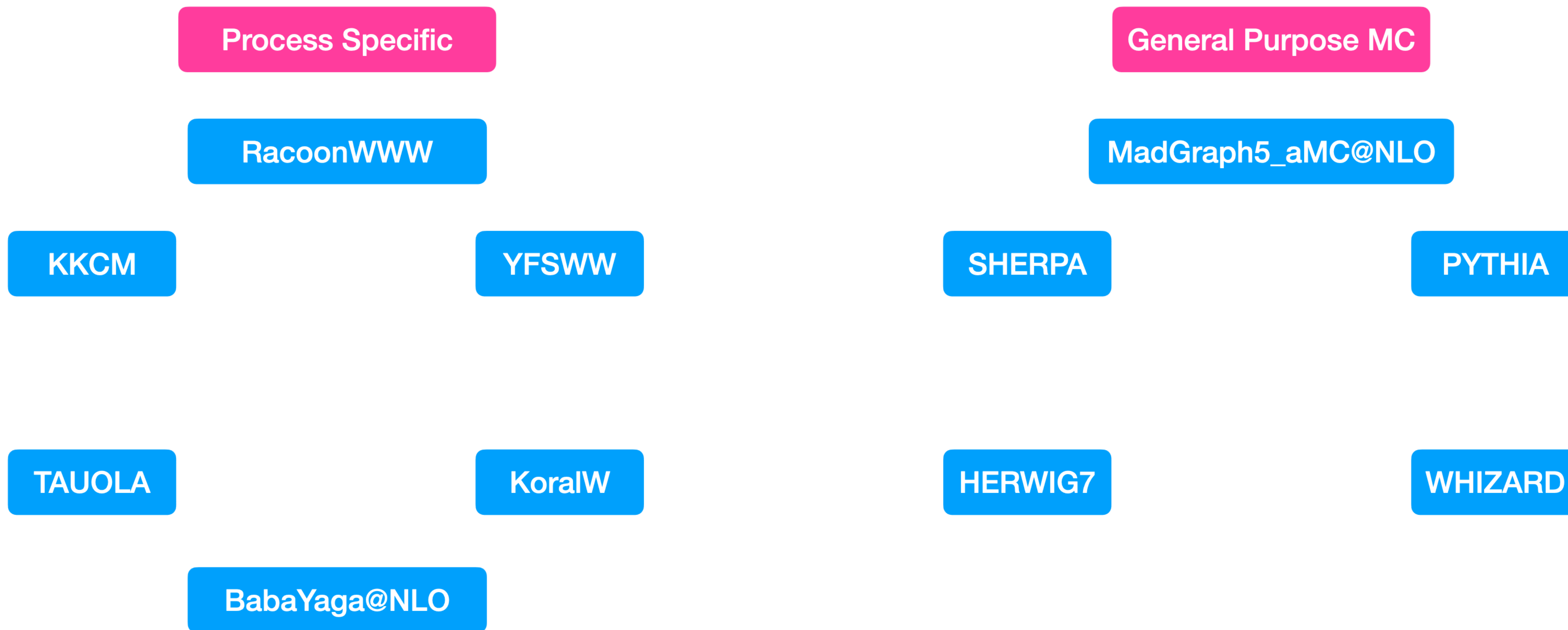
LCD-Note-2011-006

Artwork from Sherpa





# Overview over $e^+e^-$ generators



from Alan Price, 2nd ECFA HF WS, Paestum, 2023



# ECFA H/EW/Top Factory WG3/2 MC Generators

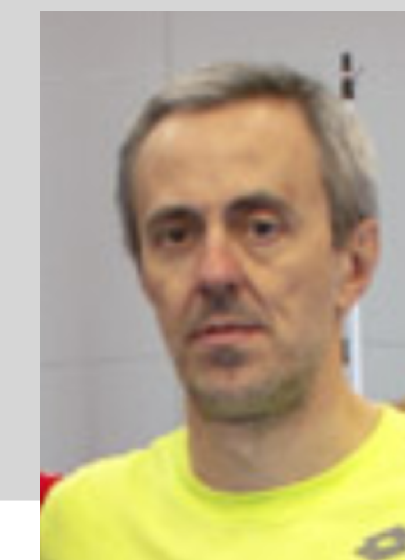
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- $\geq$  100 participants, roughly 30 at CERN
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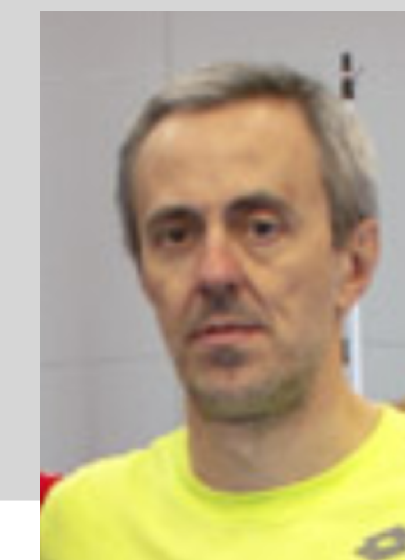


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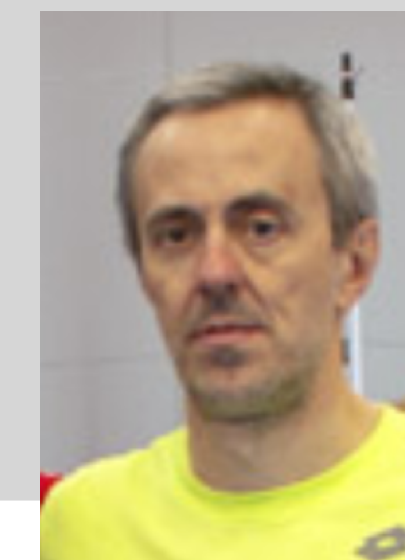
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Apr 24-28, 2023 [https://indico.cern.ch/event/1233329](https://indico.cern.ch/event/1233329/)
- $\geq 120$  participants, roughly 80 at CERN
- Focus: perturbative and non-perturbative QCD



# The scope: lessons learned and where to go

- 📌 LHC a huge success story for Monte Carlos (MCs)
- 📌 Assessment of needs for MCs event for (high-energy)  $e^+e^-$  colliders?
- 📌 Experience from LEP, ILC TDR+250 GeV full simulation, CEPC simulation samples



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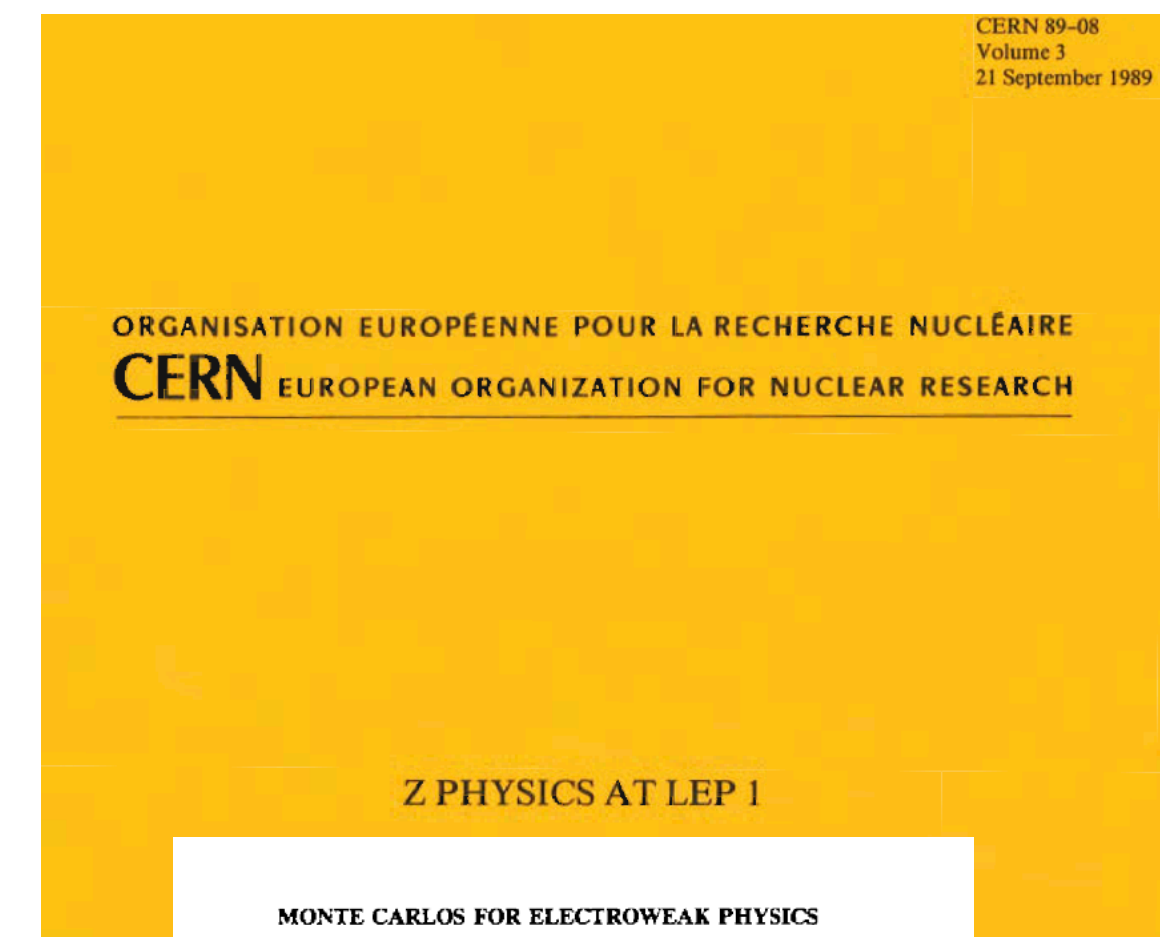
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  - 1.2 Electroweak versus QCD
  - 1.3 Analytic and Monte Carlo formulations
  - 1.4 Monte Carlo techniques
    - 1.4.1 The general recipe
    - 1.4.2 Variance reduction
    - 1.4.3 Multichannel approaches and a-priori weights
    - 1.4.4 Random number sources
- 2 Technical aspects of Monte Carlo and semi-analytical software
  - 2.1 Implementation of weak effects
  - 2.2 Implementation of QED effects
    - 2.2.1 Fixed-order generators and the  $k_0$  problem
    - 2.2.2 Exponentiation - the general structure
    - 2.2.3 The YFS exponentiation scheme
    - 2.2.4 Overview of structure functions in QED
    - 2.2.5 Structure functions for DYNAM2
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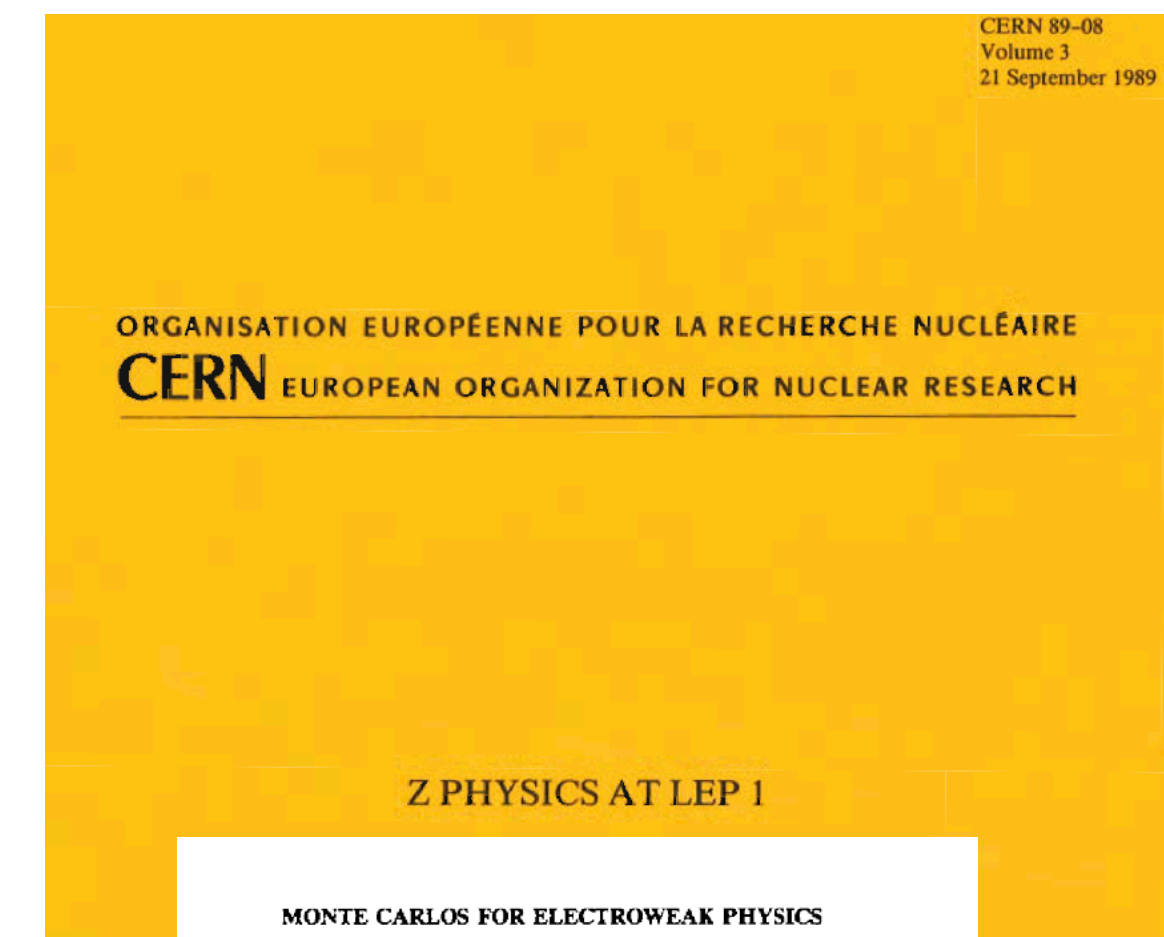




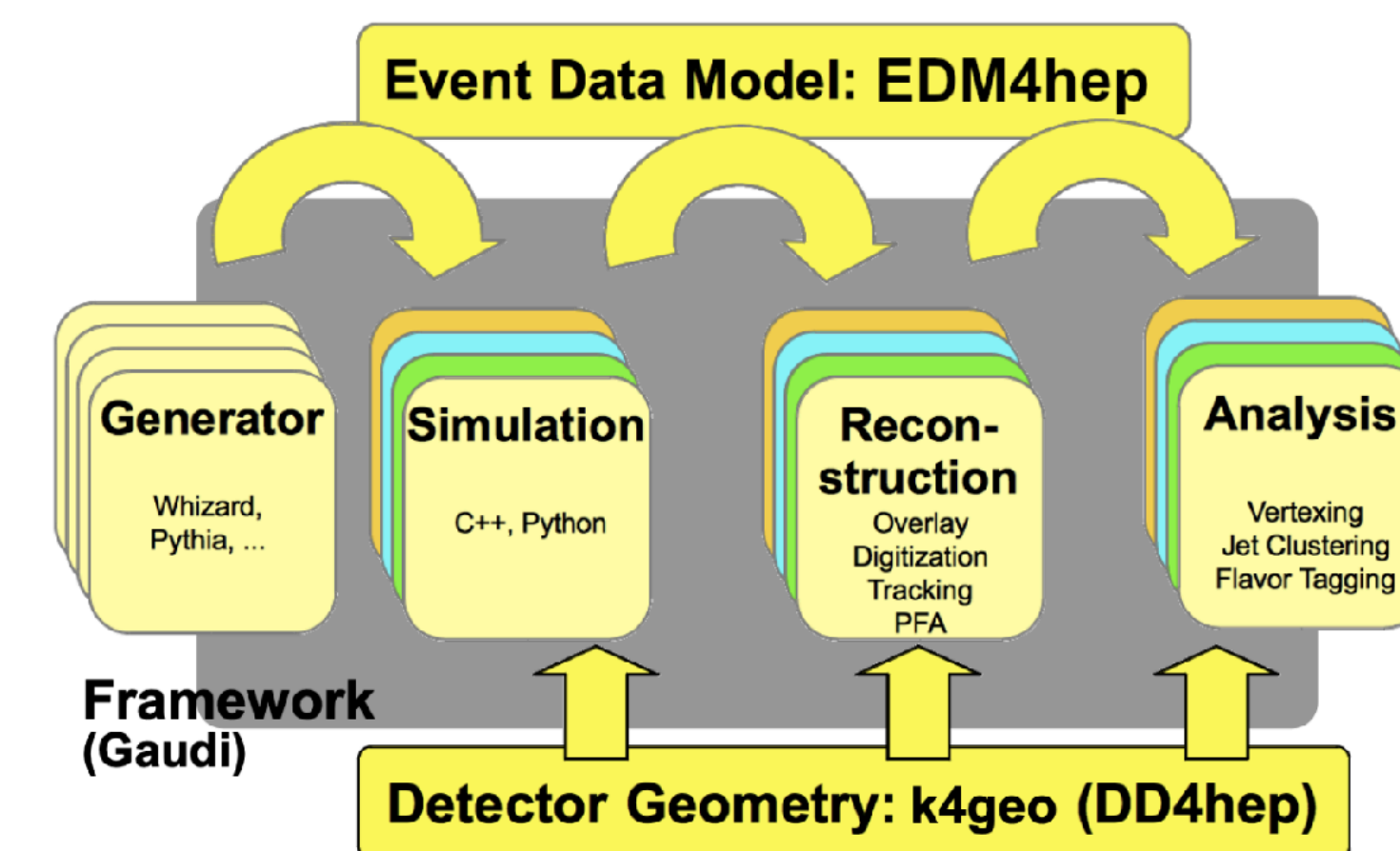
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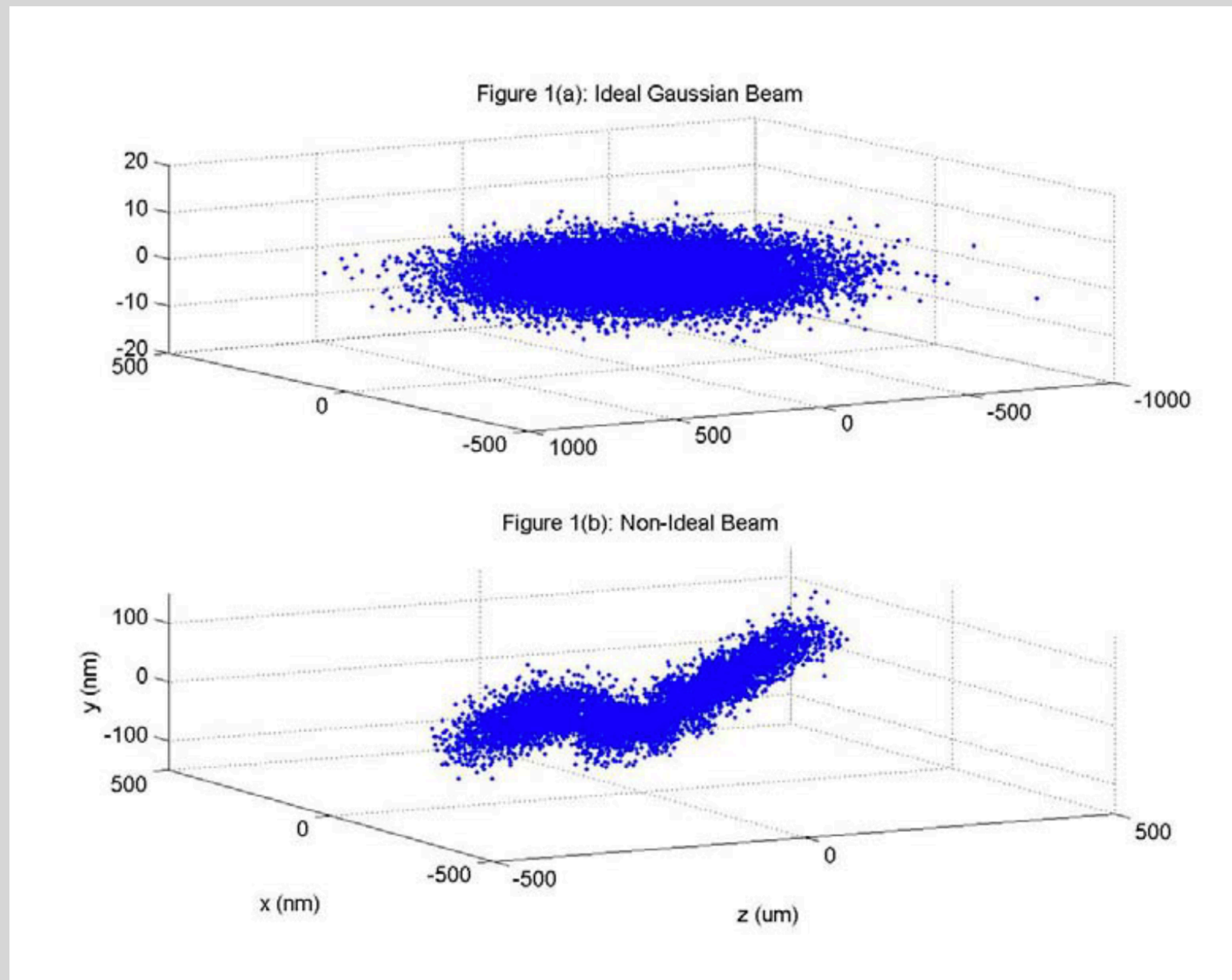
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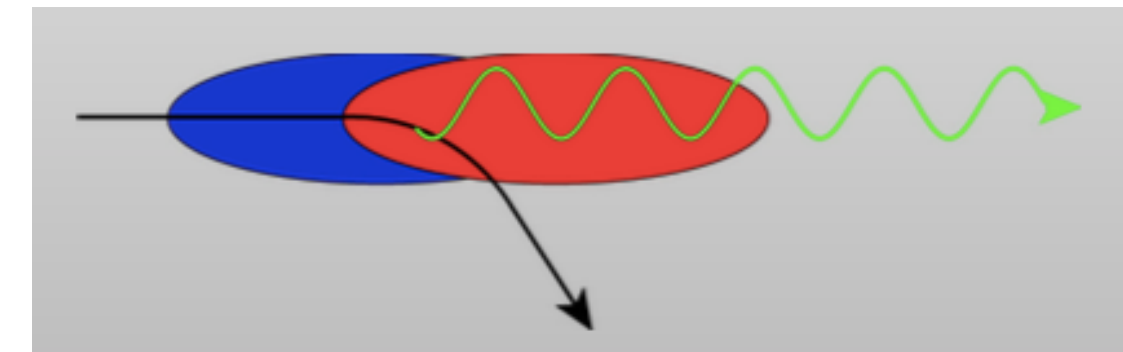
# Beam simulations





# Beam simulations

- Micro-scale bunches create beam structure/-strahlung
- Mostly Gaussian shape for circular machines, but not fully
- Machine simulation with tools like GuineaPig(++), CAIN
- Has to be folded into realistic MC simulations



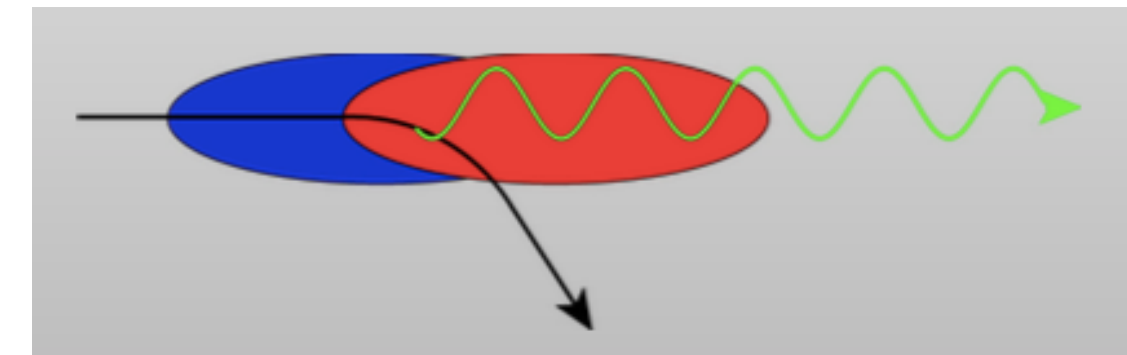
$$L \approx \frac{N}{4\pi\sigma_x\sigma_y} \frac{\eta P_{AC}}{E_{CM}}$$

- Gaussian shape with specific spreads Avail.: ✓
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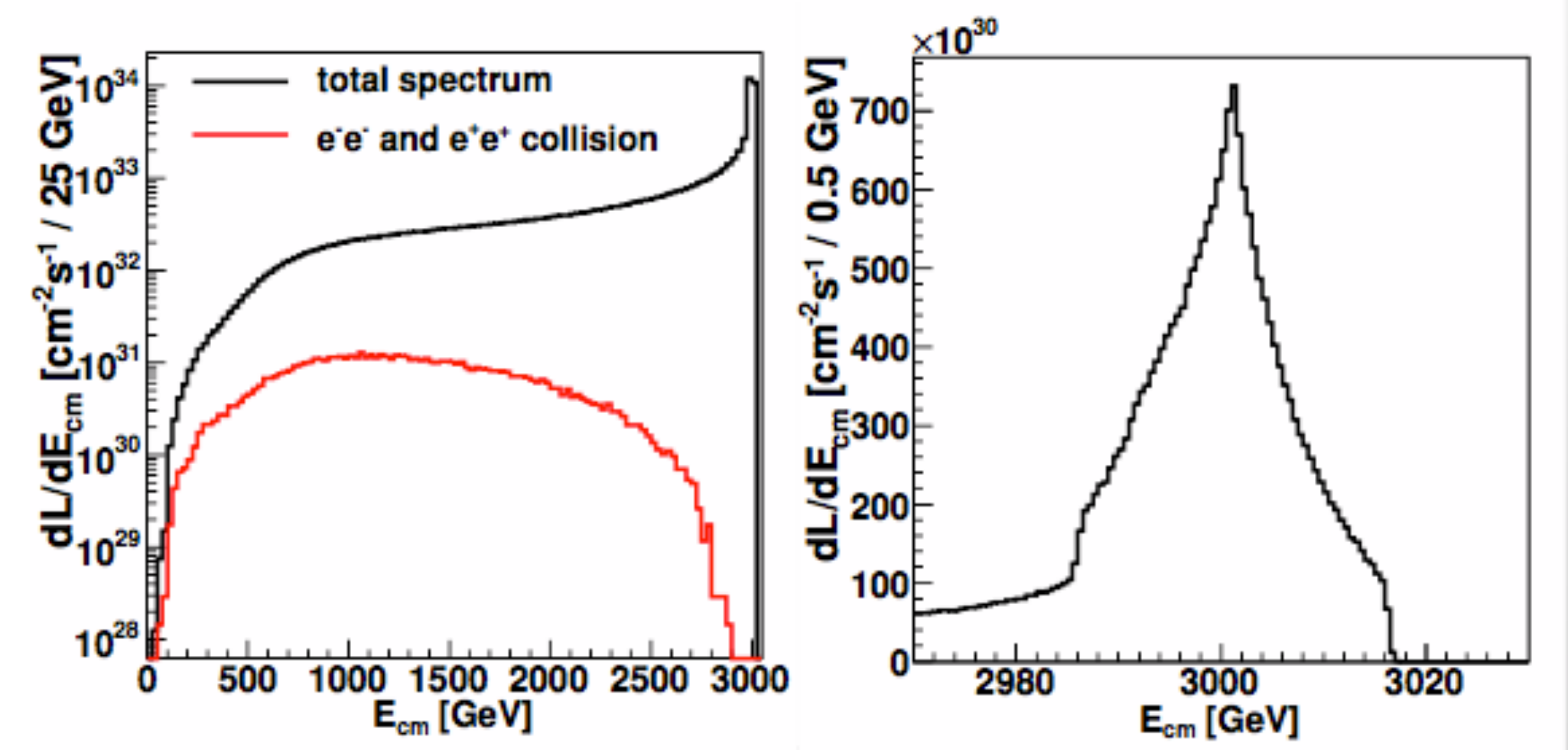
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Dalena/Esbjerg/Schulte [LCWS 2011]

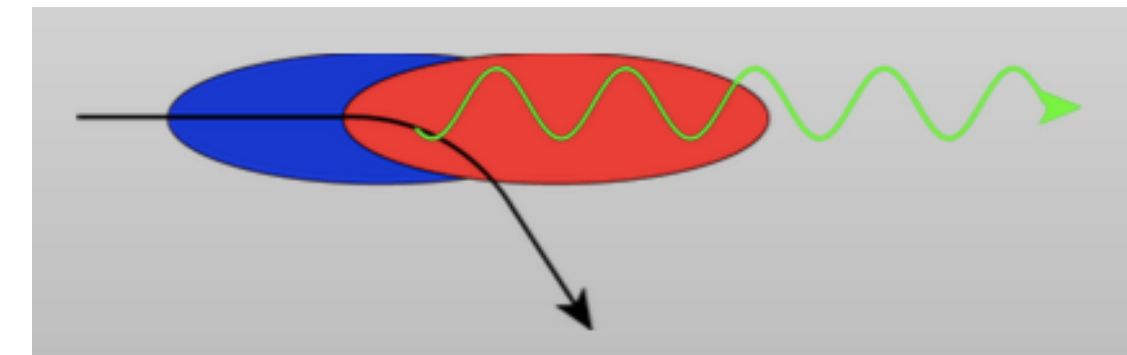




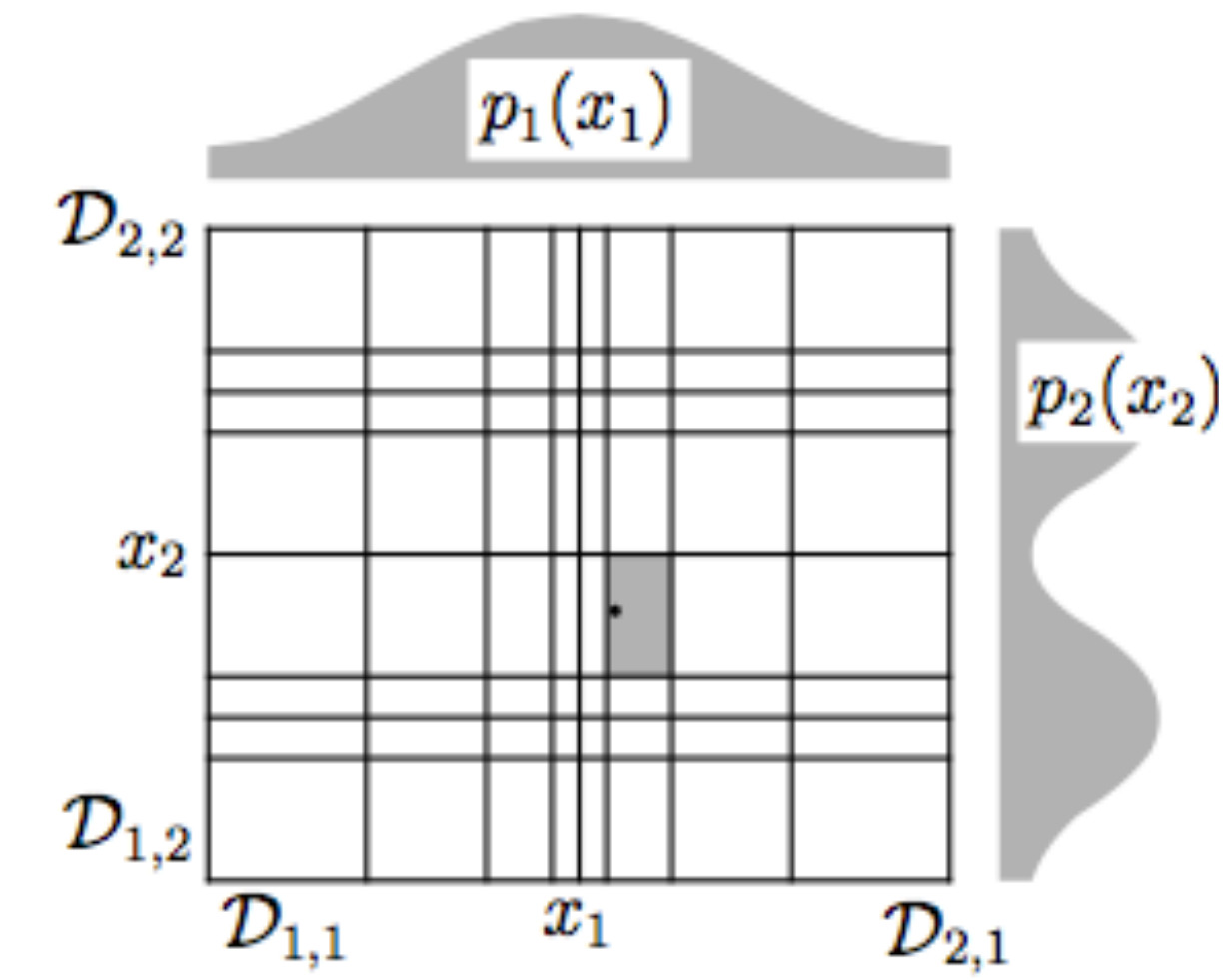
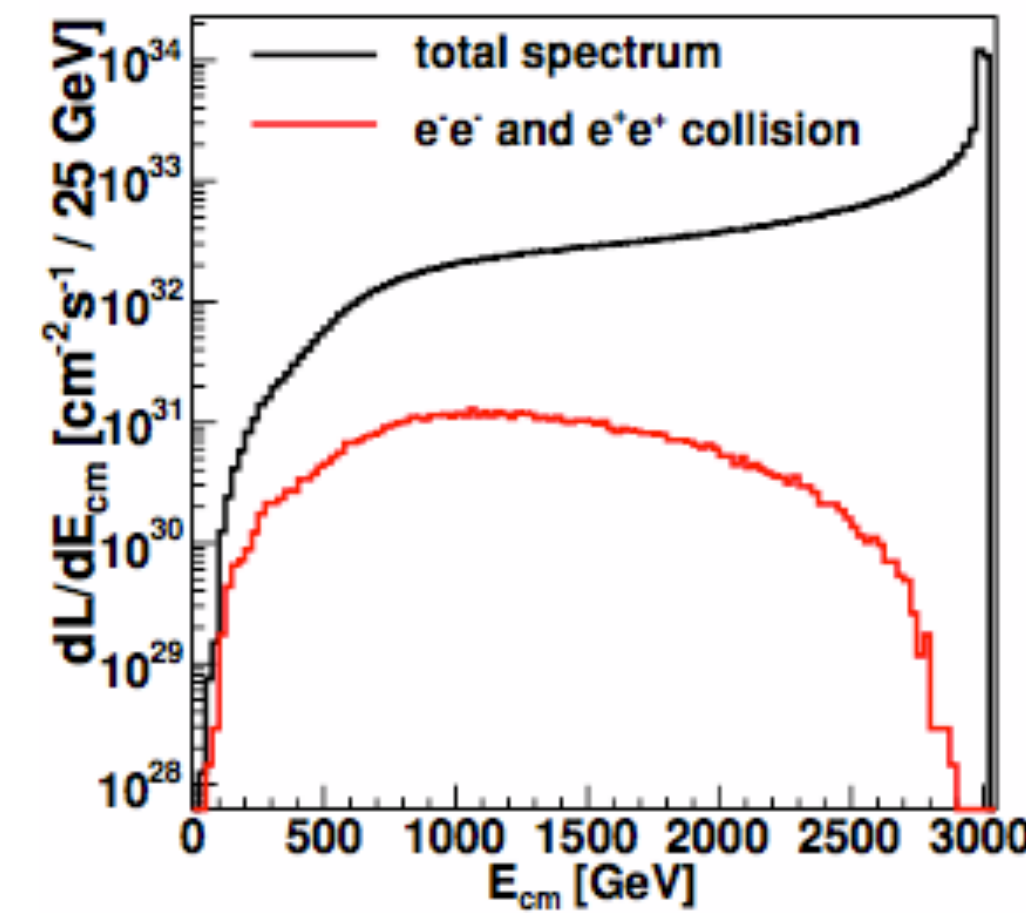
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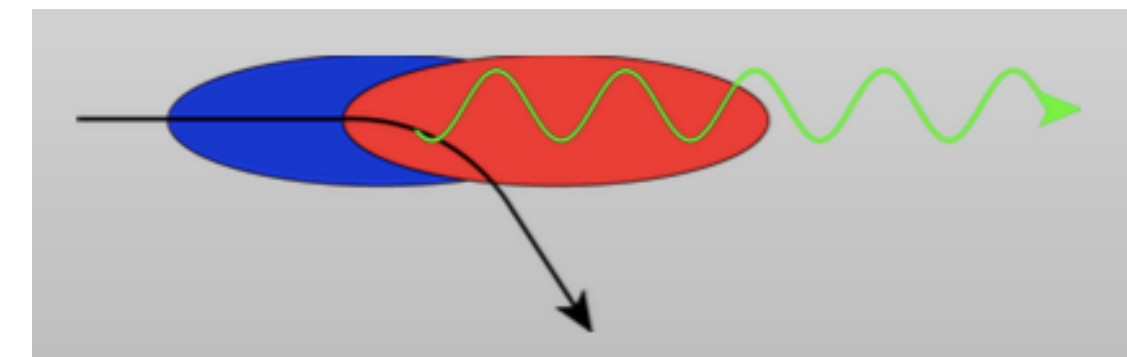


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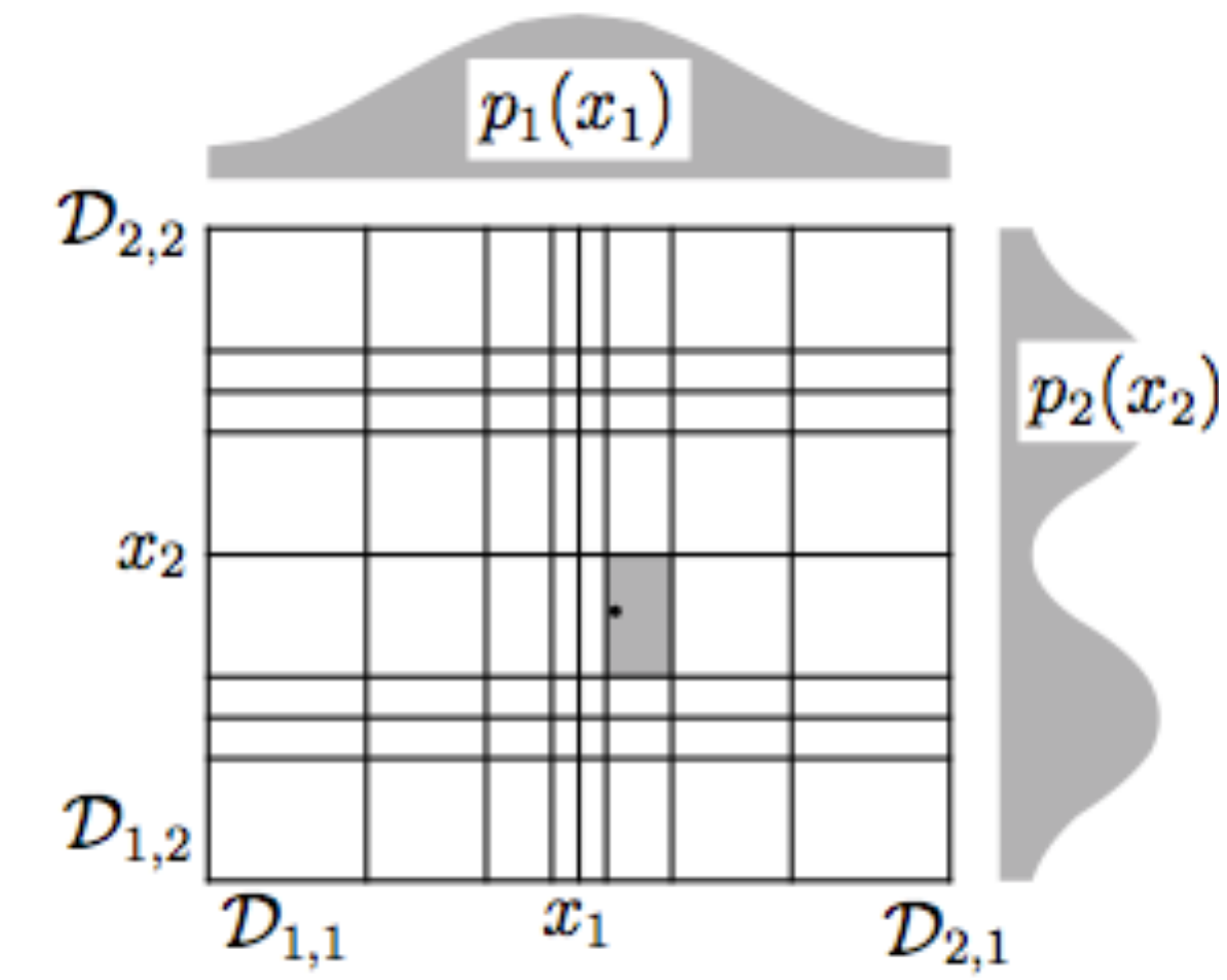
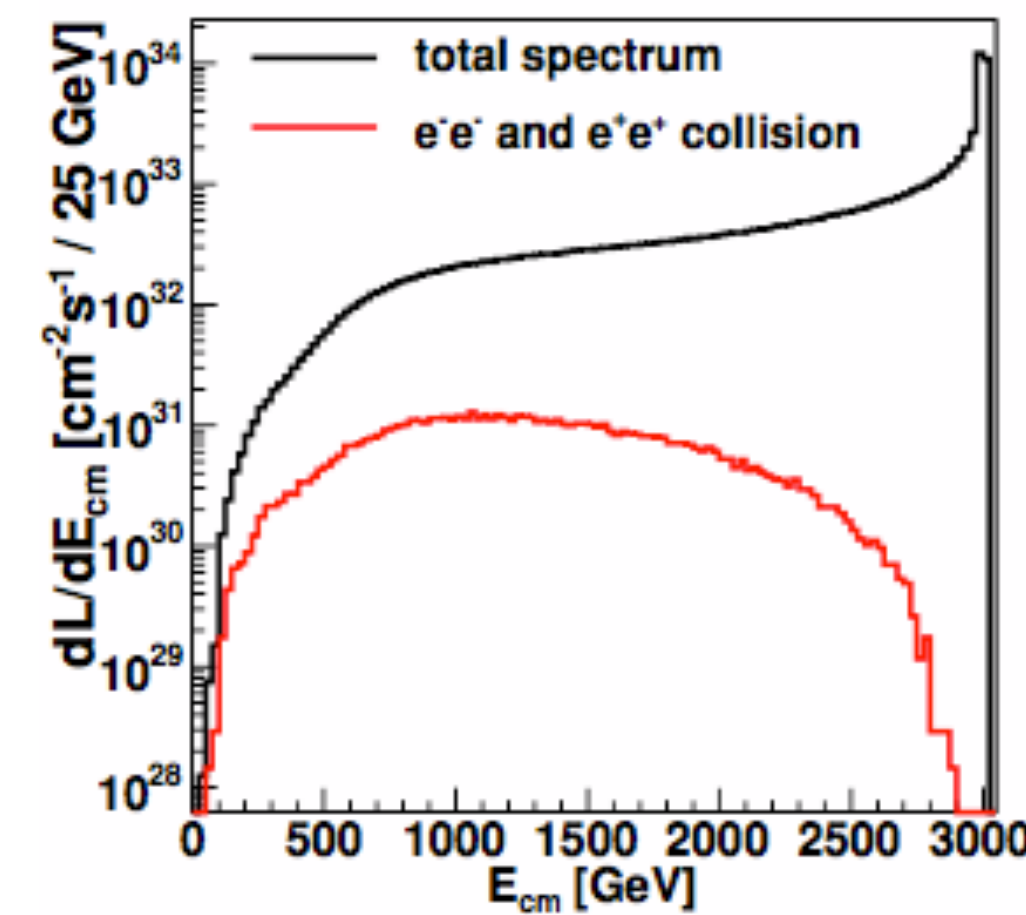
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- Pro (1.): Easy implementation, covers main features
- Con (1.): Gaussian approximative, exceeds nominal collider energy
- Pro (2.): Relatively easy implementation
- Con (2.): Delta peak behaves badly in MC, beams maybe not factorizable/simple power law
- Pro (3.): most exact simulation, generator mode avoids artifacts in tails
- Con (3.): only available (yet) in dedicated tools like LumiLinker and CIRCE2



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Dalena/Esbjerg/Schulte [LCWS 2011]

$$D_{B_1 B_2}(x_1, x_2) \neq D_{B_1}(x_1) \cdot D_{B_2}(x_2)$$

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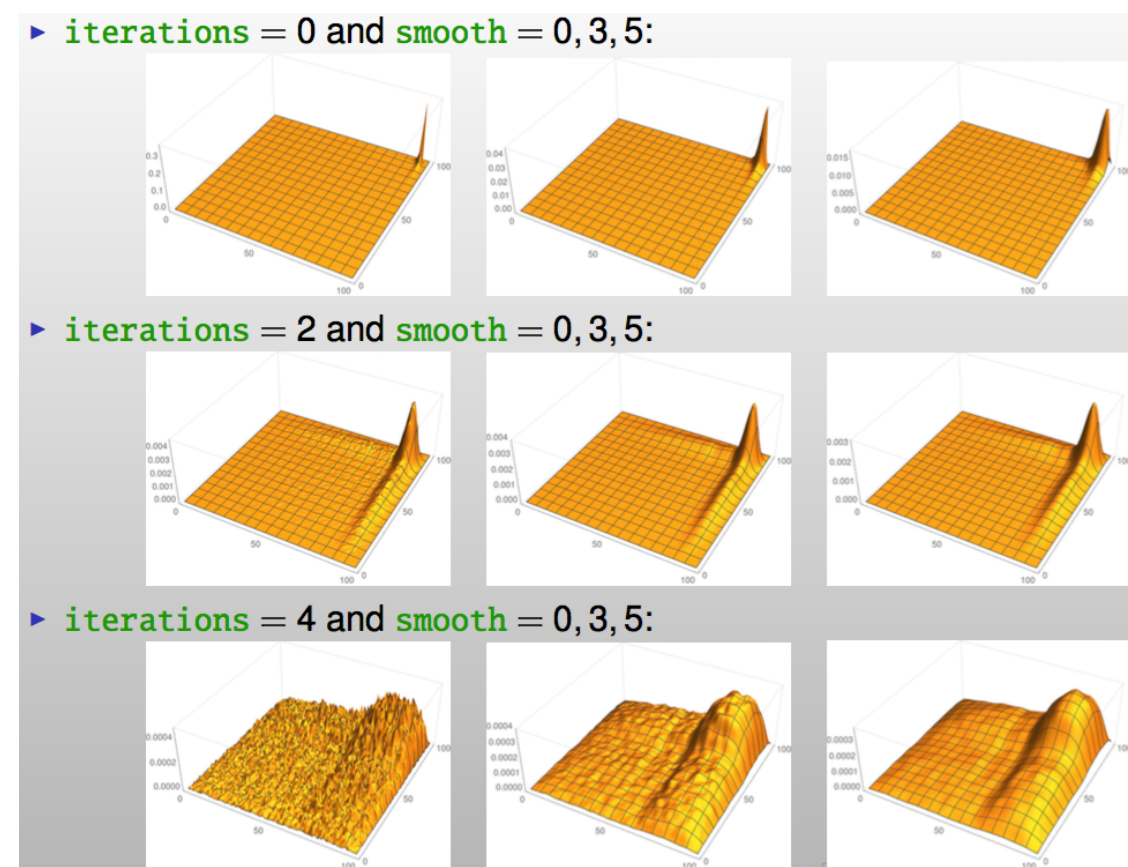
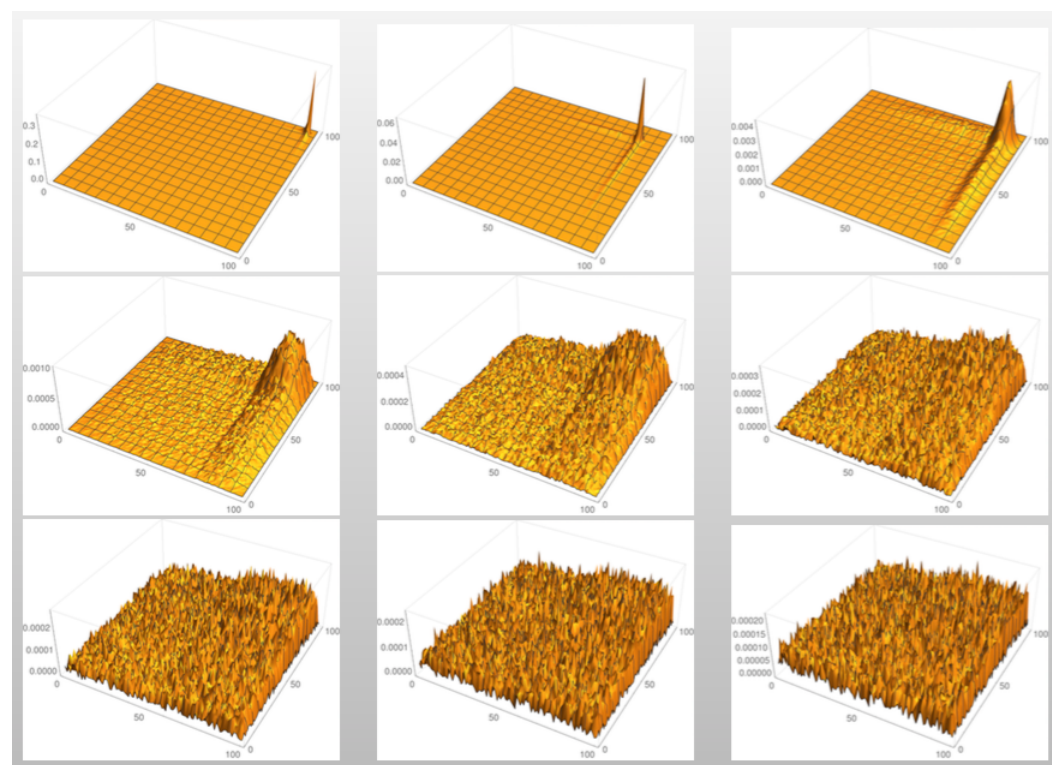




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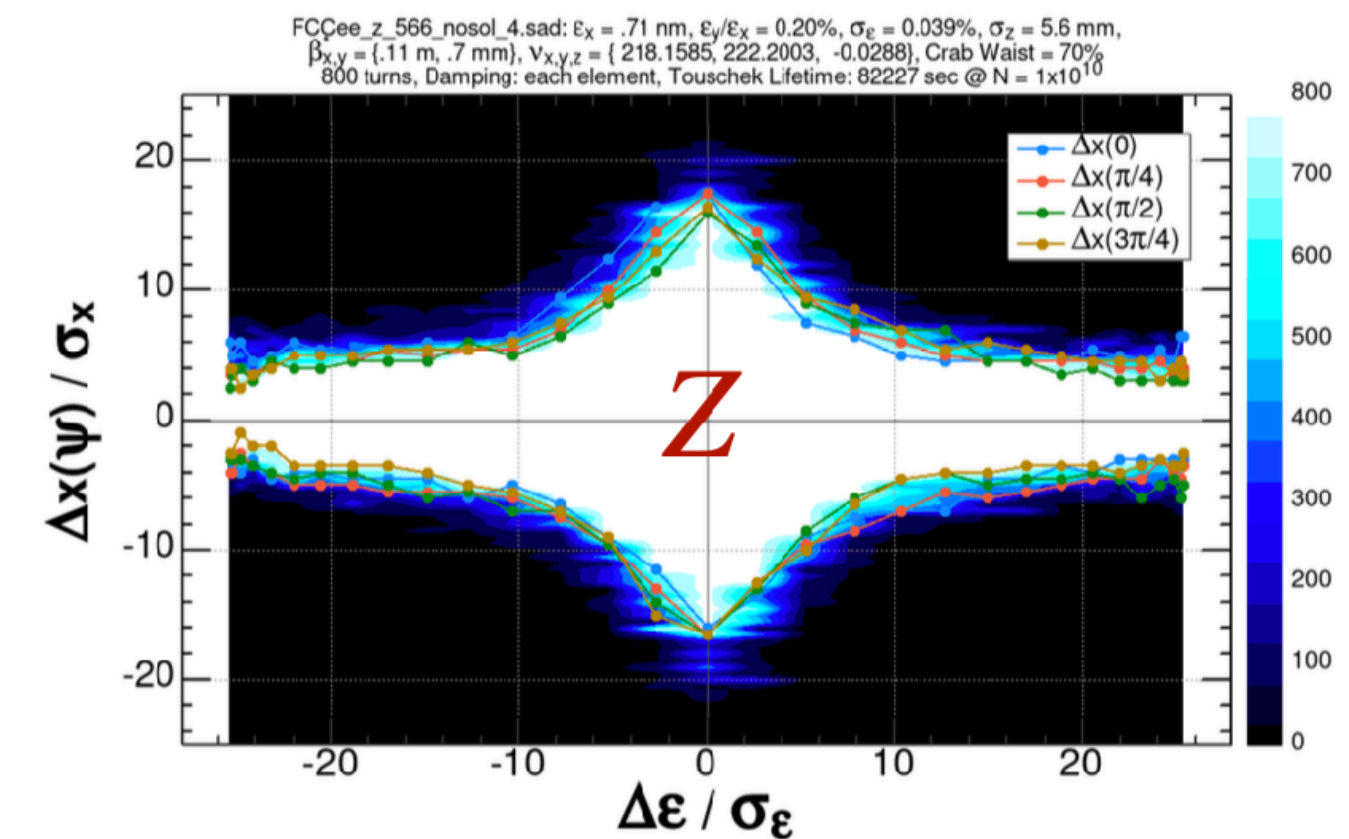
[Thorsten Ohl, 2nd ECFA (MC) WS]

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- FCC+ERLs most likely *not* adequate with parameterized spectra
- Conclusion: CIRCE2-like sampling most versatile/general approach
- Parameterized spectra easier to handle in sampling (esp. NLO simulations)



(171,306 GuineaPig events in  
10,000 bins)

## Dynamic aperture (z-x)



[Katsunobu Oide, FCC week 2023]

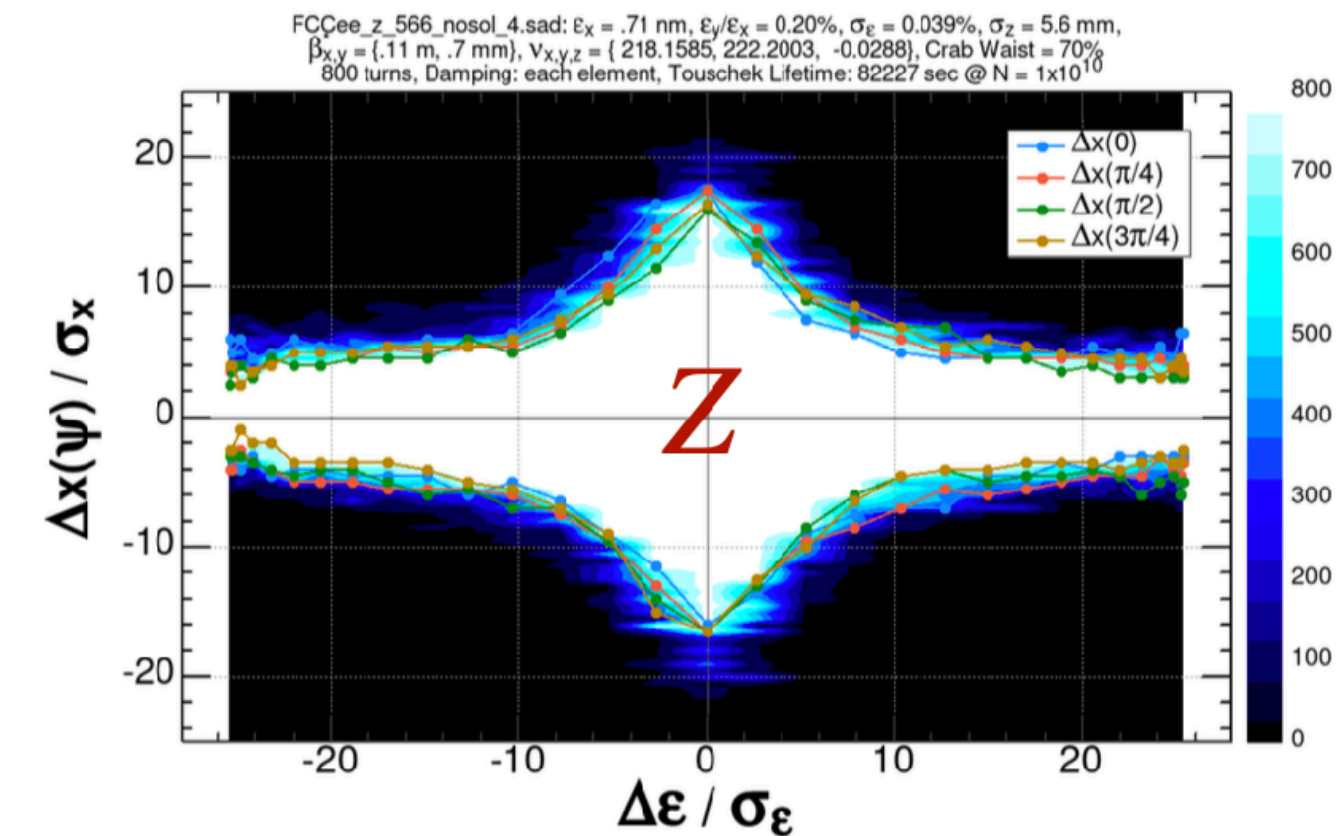


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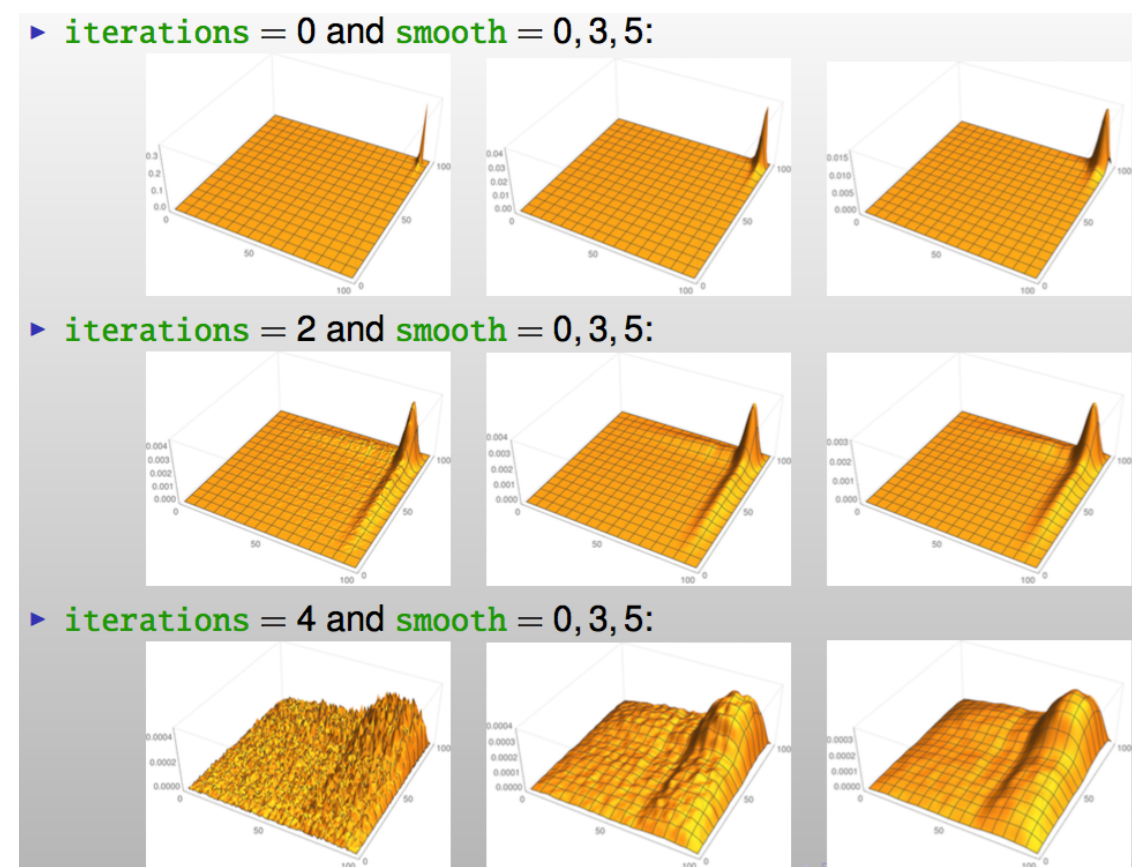
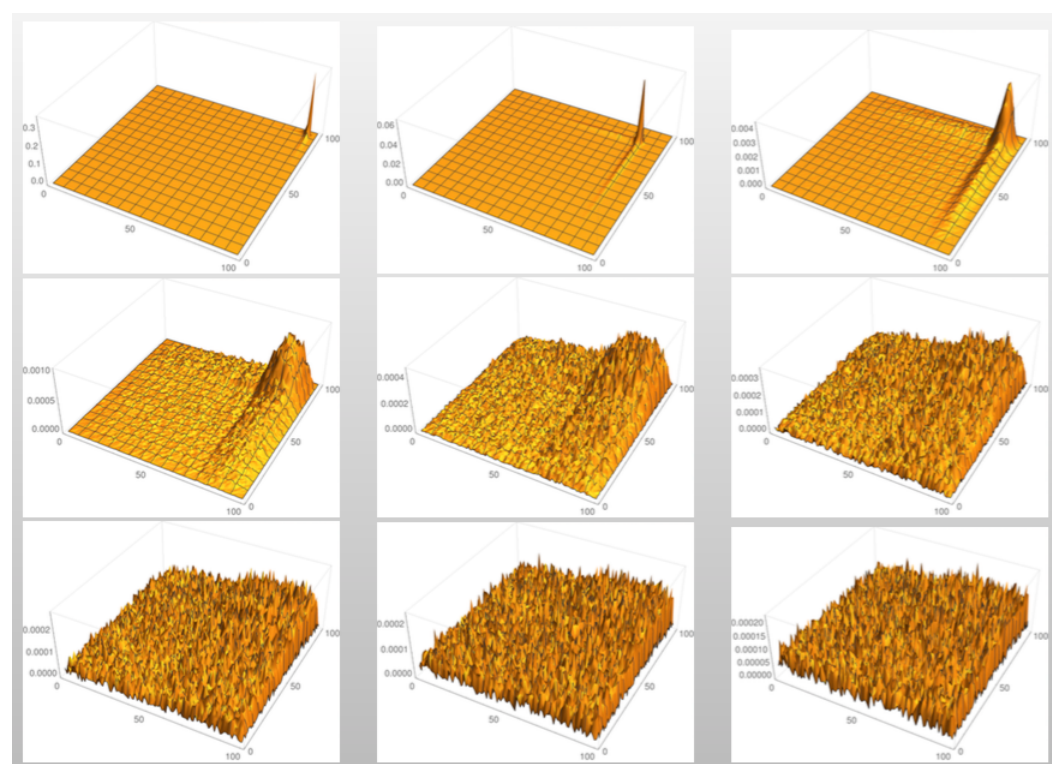
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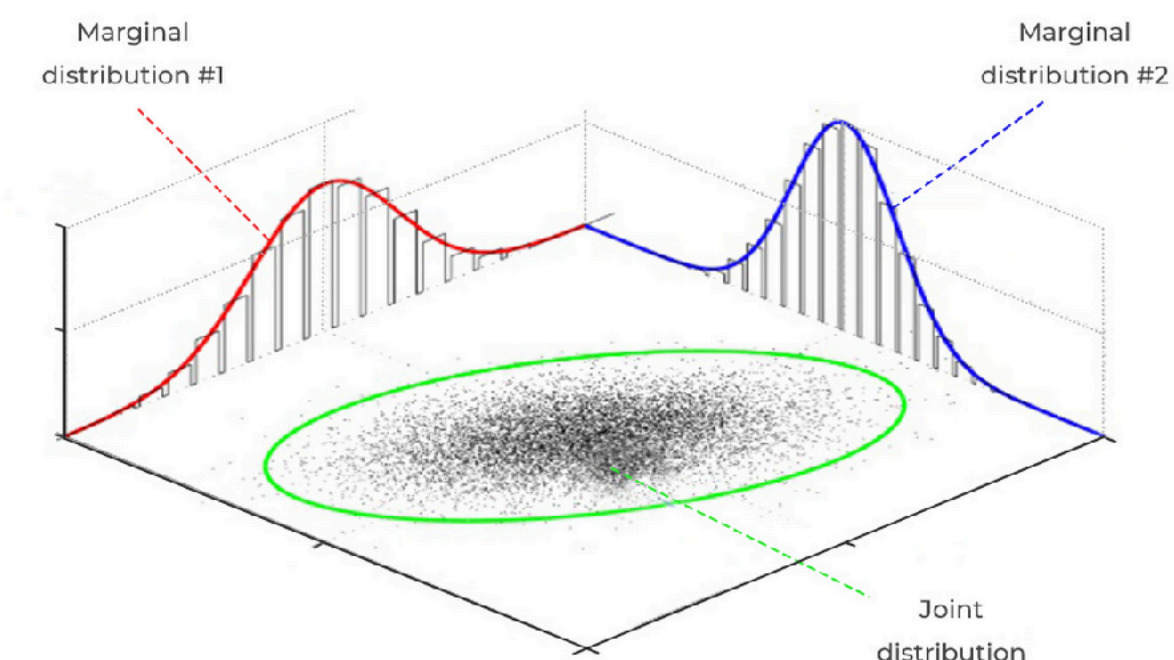
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## Open Issues

- Machine learning for sampling beam spectra not yet started (expected performance?)
- 2D-/3D-structure of beam spectra (z-dependence, copulas)





# Initial state / beam setup: energy spread + crossing angle

▶ FCC-ee plans for 30 mrad crossing angle  $\Rightarrow$  crossing angle simulations needed

▶ Several MCs offer such simulations:

Asymmetric collisions

beams\_momentum = 500 GeV, 31 GeV



Beams with crossing angle

beams\_momentum = 120 GeV, 120 GeV  
beams\_theta = 15 mrad, -15 mrad

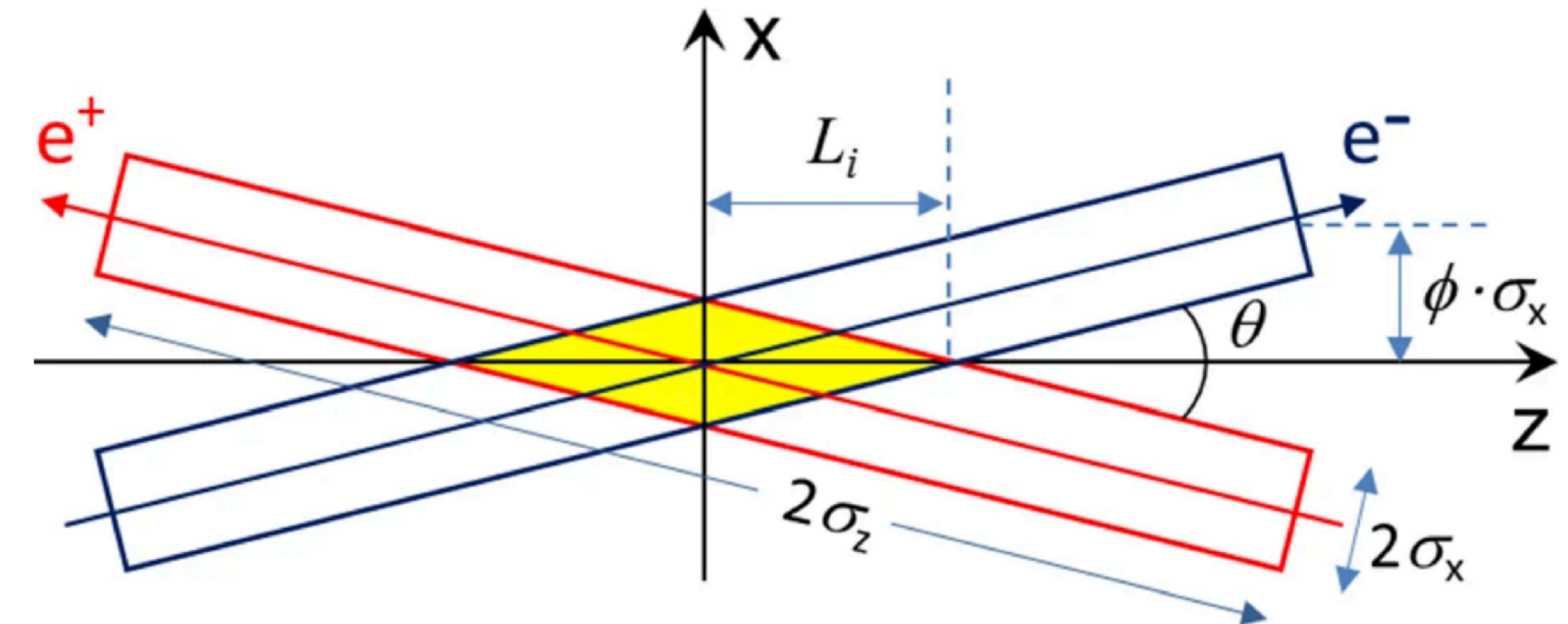


Beams:pzA = 119.987  
Beams:pxA = 1.800  
Beams:pyA = xxx

Beams with rotated crossing angle

beams\_momentum = 120 GeV, 120 GeV  
beams\_theta = 15 mrad, -15 mrad  
beams\_phi = 0, 45 degree

Beams:pzB = -119.987  
Beams:pxB = -1.800  
Beams:pyB = -x.xxx



# Initial state / beam setup: energy spread + crossing angle

▶ FCC-ee plans for 30 mrad crossing angle  $\Rightarrow$  crossing angle simulations needed

▶ Several MCs offer such simulations:

Asymmetric collisions

beams\_momentum = 500 GeV, 31 GeV



Beams with crossing angle

beams\_momentum = 120 GeV, 120 GeV  
beams\_theta = 15 mrad, -15 mrad

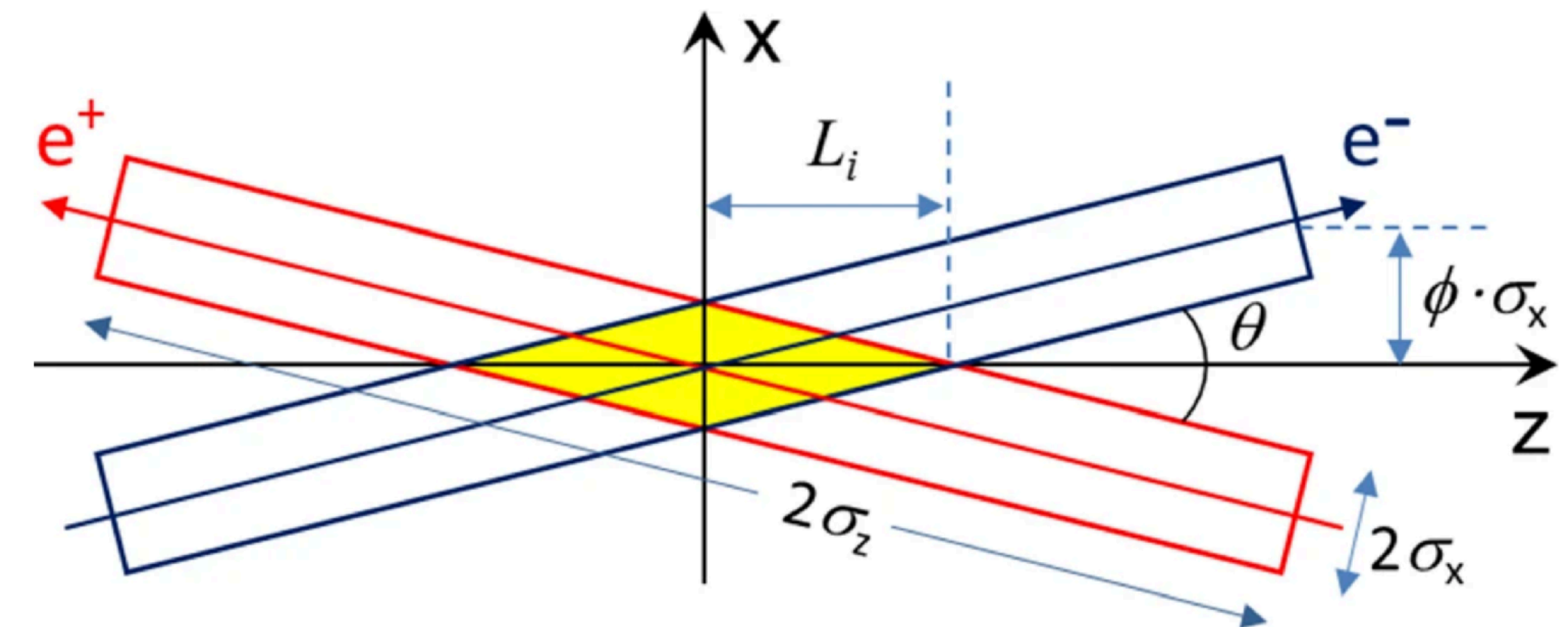


Beams:pzA = 119.987  
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Beams with rotated crossing angle

beams\_momentum = 120 GeV, 120 GeV  
beams\_theta = 15 mrad, -15 mrad  
beams\_phi = 0, 45 degree

Beams:pzB = -119.987  
Beams:pxB = -1.800  
Beams:pyB = -x.xxx



▶ Simulation of beam energy spread: available in many MCs (KKMC, Pythia, Sherpa, Whizard, ...?)

▶ Note: total cross sections do depend on the crossing angle as well as the beam profile

▶ Is there also need for spread in transverse directions?



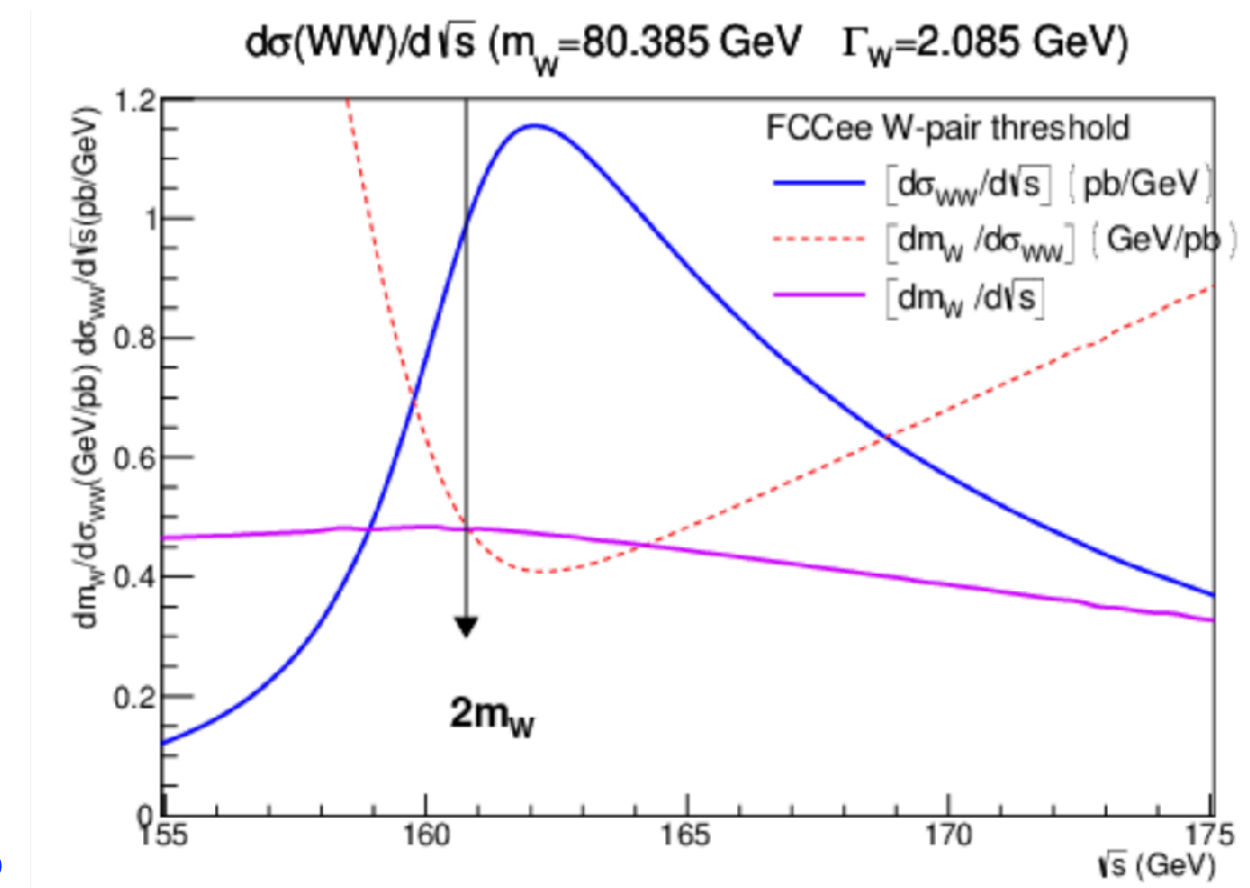
beams = e1, E1  $\Rightarrow$  gaussian  
gaussian\_spread1 = 0.13%  
gaussian\_spread2 = 0.13%



Beams:allowMomentumSpread = on  
Beams:sigmaPzA = 0.156

Beams:sigmaPxA = xxxx

Beams:Shape class

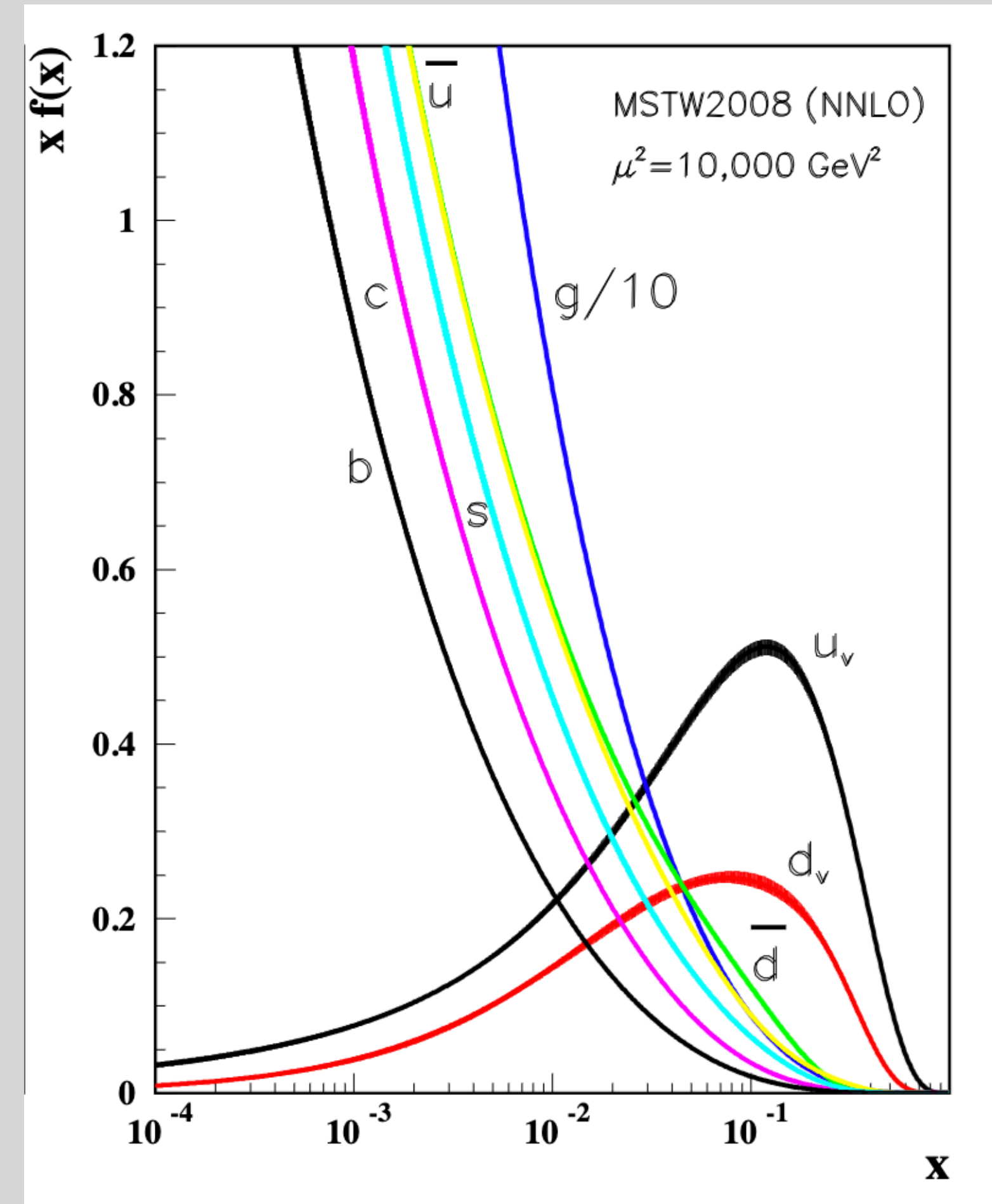
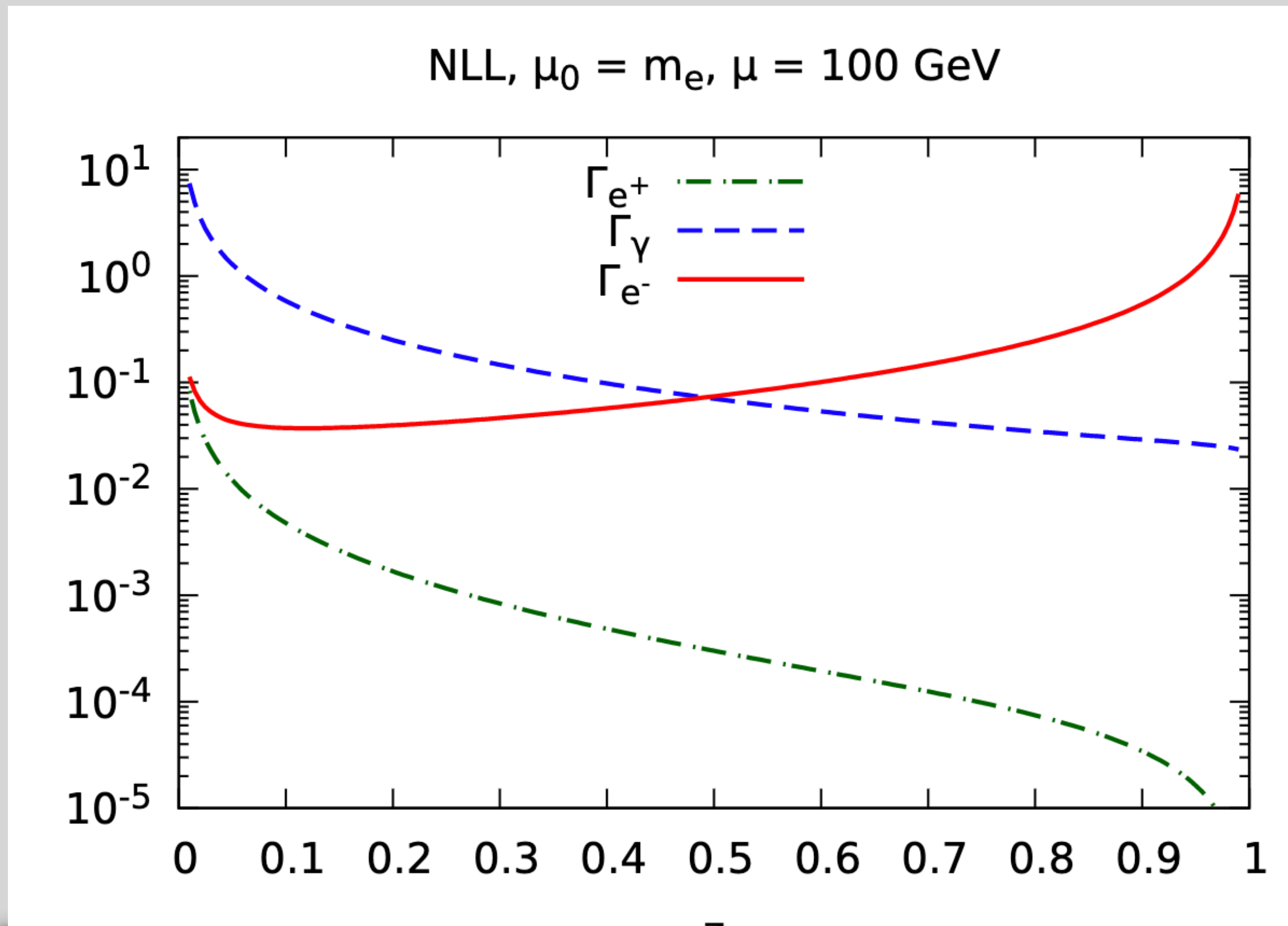


arXiv:1909.12245



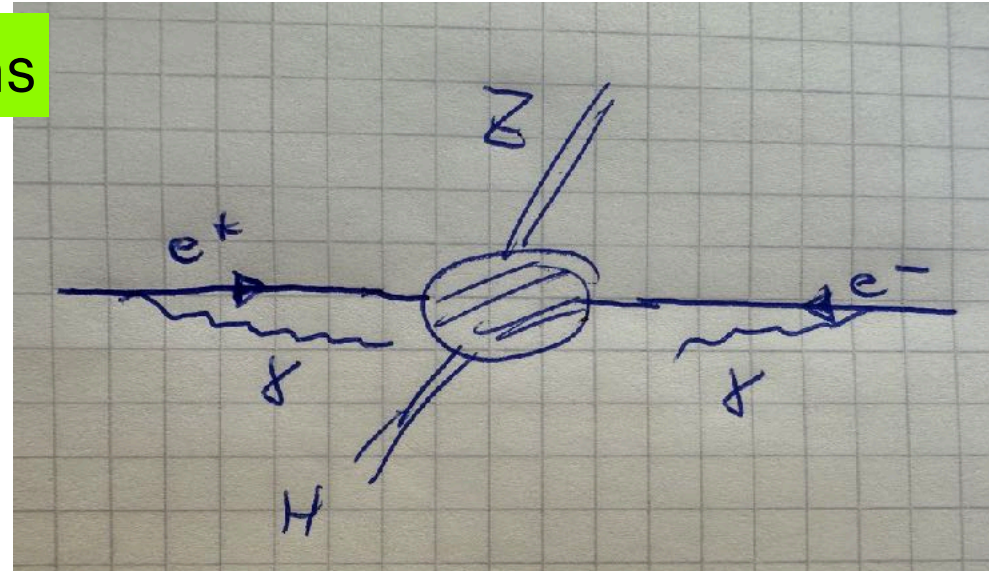


# Initial State Radiation — Lepton PDFs



Collinear logarithms

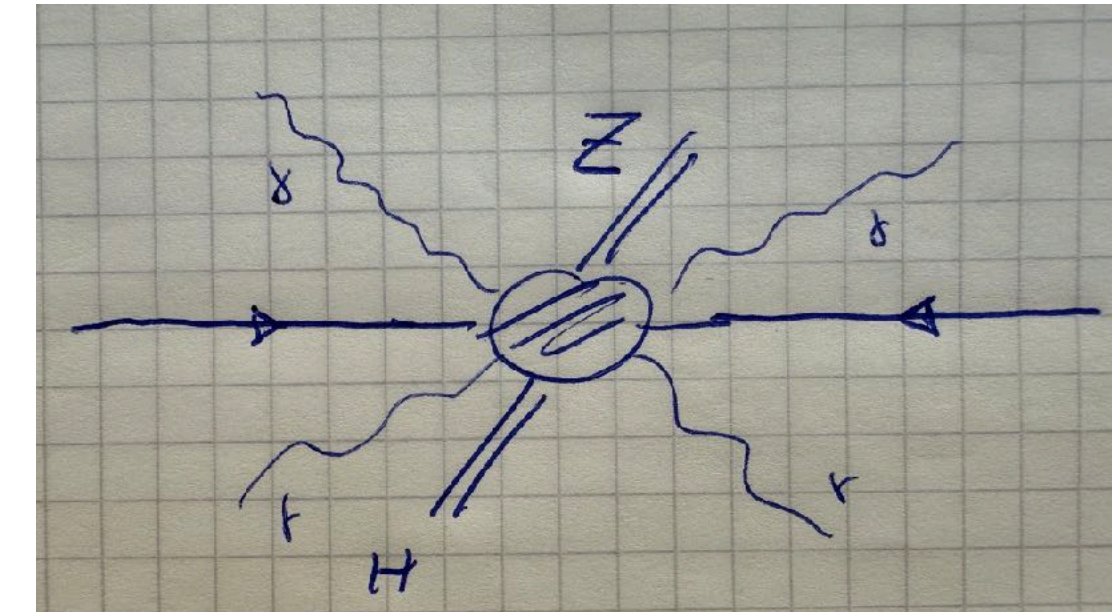
$$L = \log \frac{Q^2}{m^2}$$



$$\sigma = \alpha^b \sum_{n=0}^{\infty} \alpha^n \sum_{i=0}^n \sum_{j=0}^n \varsigma_{n,i,j} L^i \ell^j$$

Soft logarithms

$$\ell = \log \frac{Q^2}{\langle E_\gamma \rangle^2}$$



• Different factorization schemes: focus on collinear logs,  $\log \frac{Q^2}{m_\mu^2}$ , vs. soft logs,  $\log \frac{Q^2}{E_\gamma^2}$ , cf. [2203.12557](#)

• YFS (Yennie-Frautschi-Suura), cf. e.g. [2203.10948](#)

$$d\sigma = \sum_{n_\gamma}^{\infty} \frac{\exp[Y_{res.}]}{n_\gamma!} \prod_{j=1}^{n_\gamma} [dLIPS_j^\gamma S_{res.}(k_j)] [\sigma_0 + \text{corrections}]$$

- Universal soft exponentiation factor, provides  $n_\gamma$  exclusive resolved photons with (almost) exact kinematics
- Exponentiation at amplitude level (CEEX) oder squared ME level (EEX)
- Implemented in LEP legacy MCs (BHLUMI/BHWIDE, KORAL(W/Z), KKMC-ee, YFS(WW/ZZ), also: Sherpa, w.i.p.: Whizard
- Can be systematically improved at fixed-order level by higher-order corrections

• Collinear factorization: universal QED ePDFs,

$$\text{LL: } (\alpha L)^k, \text{ NLL: } \alpha(\alpha L)^{k-1}$$

$$d\sigma_{kl}(p_k, p_l) = \sum_{ij=e^+, e^-, \gamma} \int dz_+ dz_- \Gamma_{i/k}(z_+, \mu^2, m^2) \Gamma_{j/l}(z_-, \mu^2, m^2) \times d\hat{\sigma}_{ij}(z_+ p_k, z_- p_l, \mu^2) + \mathcal{O}\left(\left(\frac{m^2}{s}\right)^p\right)$$



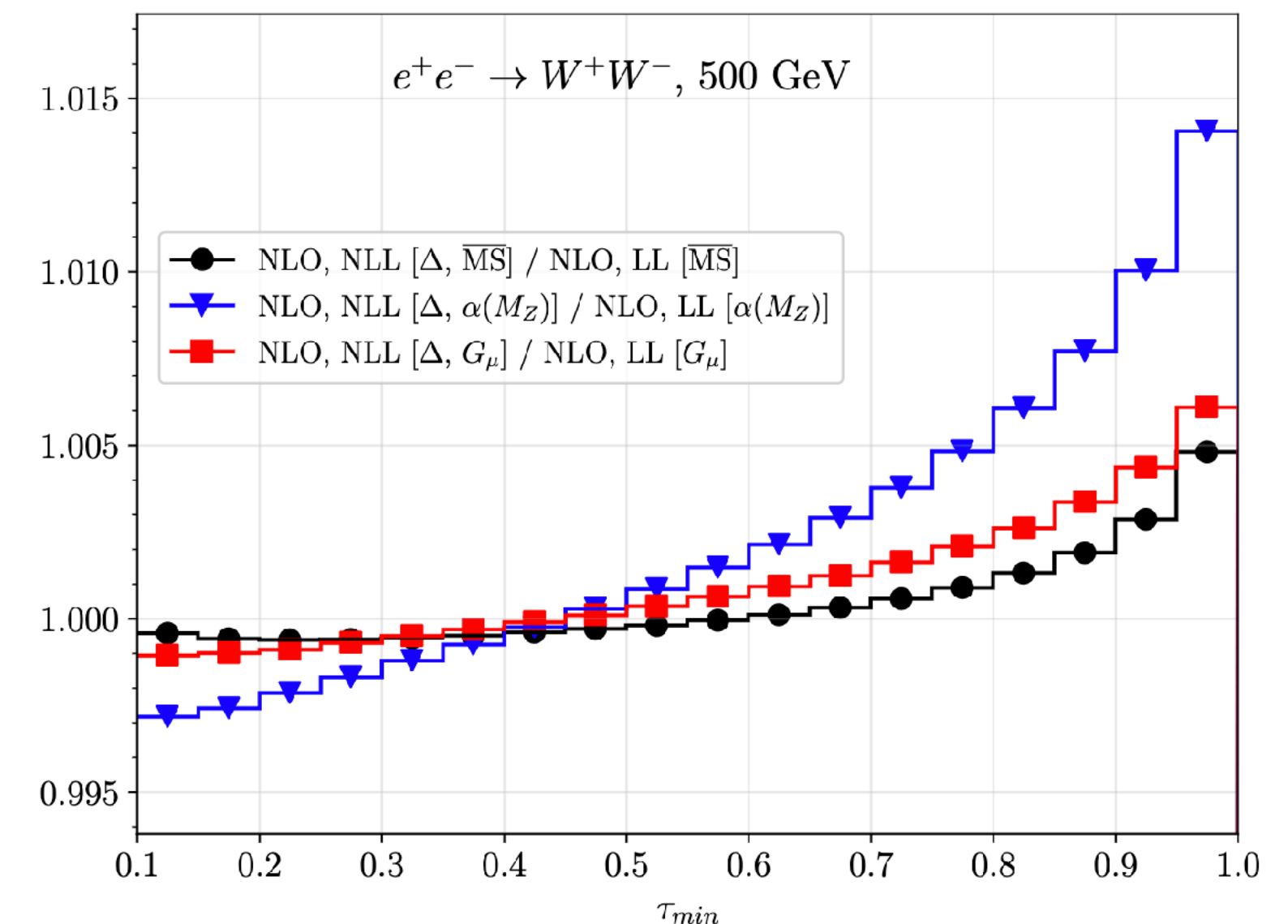
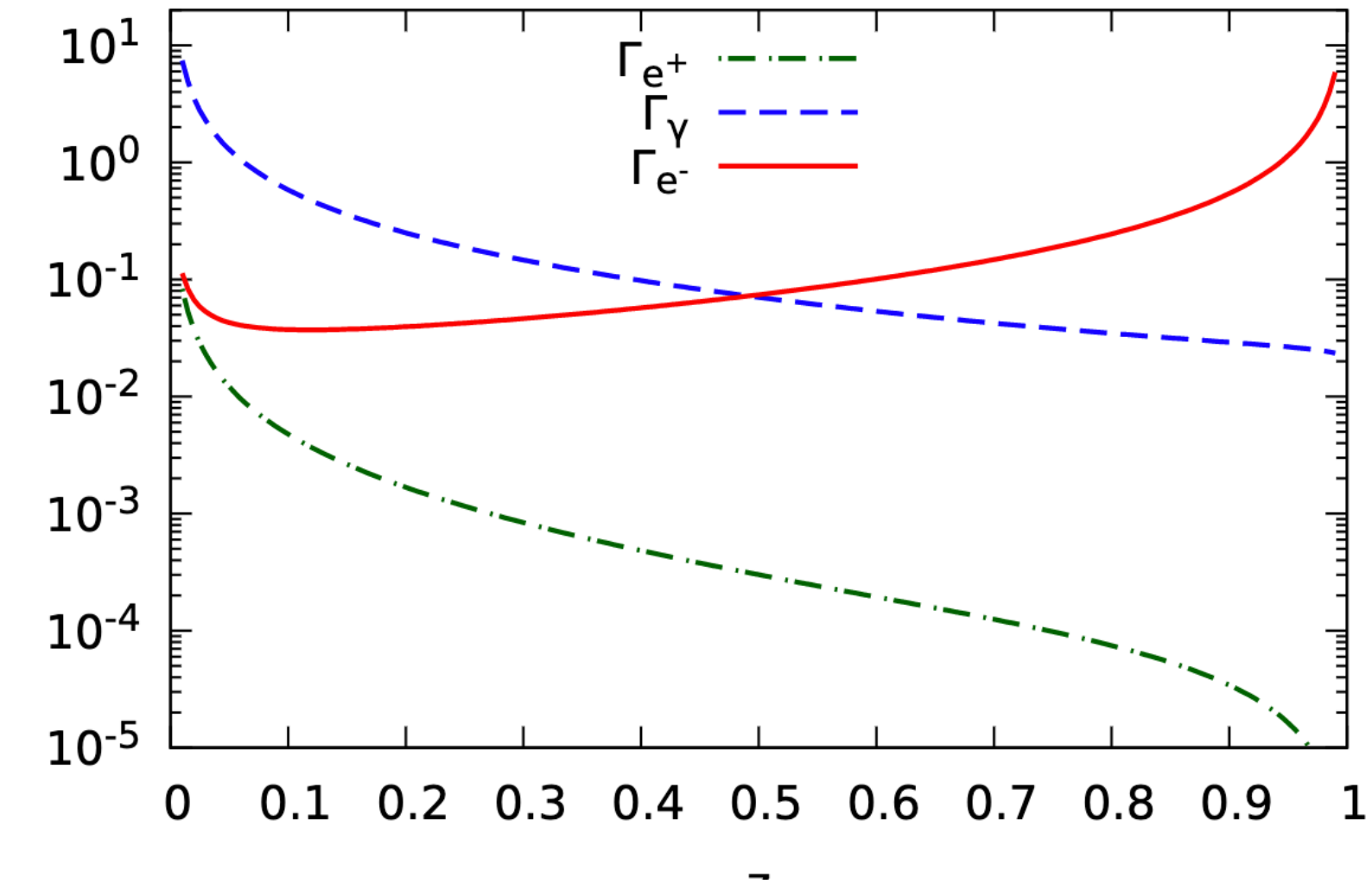
Integrable power-like singularity  $1/(1-z)$  for  $z \rightarrow 1$

- Collinear resummation LO/LL Gribov/Lipatov, 1972; Kuraev/Fadin, 1985;  
Skrzypek/Jadach, 1992; Cacciari/Deandrea/Montagna/Nicrosini, 1992
- NLO QED PDFs, collinear evolution @ NLL  
Frixione, 1909.0388; Bertone/Cacciari/Frixione/Stagnitto, 1911.12040 + 2207.03265
- **Inclusive in all initial-state photons**
- Gives most precise normalization of total cross section: 2-4 per mille
- Numerical stability differs in different QED renormalization schemes, DIS vs.  $\overline{\text{MS}}$
- Also: fast interpolation (CTEQ-like) grids available
- Implementations available in MG5 and Whizard
- Different names in literature: electron structure functions, ISR structure functions
- “Photon PDF” (a.k.a. EPA, Weizsäcker-Williams)  $\Gamma_\gamma$ , peaked at small  $z$
- Very well known from ILC/CLIC simulations: “virtual photon”-induced processes

↪ Talks by Stefano Frixione & Giovanni Stagnitto

ePDFs for polarized leptons !?

NLL,  $\mu_0 = m_e, \mu = 100 \text{ GeV}$



Beam polarization (transversal, longitudinal, arbitrary)

```
beams_pol_density = @([<spin entries>]), @([<spin entries>])  
beams_pol_fraction = <degree beam 1>, <degree beam 2>
```

$ m  = 2$	massless
$ m  = 2j + 1$	massive

Initial-state spin-density matrix:  $\rho$



Beam polarization (transversal, longitudinal, arbitrary)

```
beams_pol_density = @(<spin entries>), @(<spin entries>)  
beams_pol_fraction = <degree beam 1>, <degree beam 2>
```

Unpolarized beams

$$\rho = \frac{1}{|m|} \mathbb{I}$$

$$\begin{array}{ll} |m| = 2 & \text{massless} \\ |m| = 2j + 1 & \text{massive} \end{array}$$

Initial-state spin-density matrix:  $\rho$

```
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```

Beam polarization (transversal, longitudinal, arbitrary)

```
beams_pol_density = @(<spin entries>), @(<spin entries>)  
beams_pol_fraction = <degree beam 1>, <degree beam 2>
```

Unpolarized beams

$$\rho = \frac{1}{|m|} \mathbb{I}$$

Circular polarization

$$\rho = \text{diag} \left( \frac{1 \pm f}{2}, 0, \dots, 0, \frac{1 \mp f}{2} \right)$$

$$\begin{array}{ll} |m| = 2 & \text{massless} \\ |m| = 2j + 1 & \text{massive} \end{array}$$

Initial-state spin-density matrix:  $\rho$

```
beams_pol_density = @()
```

```
beams_pol_density = @(\pm j)  
beams_pol_fraction = f
```



Beam polarization (transversal, longitudinal, arbitrary)

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 beams\_pol\_fraction = <degree beam 1>, <degree beam 2>

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beams\_pol\_density = @( $\pm j$ )  
 beams\_pol\_fraction = f

Transversal polarization (along axis)

$$\rho = \begin{pmatrix} 1 & 0 & \dots & \dots & \frac{f}{2} e^{-i\phi} \\ 0 & 0 & \ddots & & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & & \ddots & 0 & 0 \\ \frac{f}{2} e^{i\phi} & \dots & \dots & 0 & 1 \end{pmatrix}$$

beams\_pol\_density = @( $j, -j, j:-j:\exp(-I*\phi)$ )  
 beams\_pol\_fraction = f



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beams\_pol\_density = @( $j, -j, j:-j:\exp(-I*\phi)$ )  
 beams\_pol\_fraction = f

Polarization along arbitrary axis ( $\theta, \Phi$ )

$$\rho = \frac{1}{2} \cdot \begin{pmatrix} 1 - f \cos \theta & 0 & \dots & \dots & f \sin \theta e^{-i\phi} \\ 0 & 0 & \ddots & & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & & \ddots & 0 & 0 \\ f \sin \theta e^{i\phi} & \dots & \dots & 0 & 1 + f \cos \theta \end{pmatrix}$$

beams\_pol\_density = @( $j:j:1-\cos(\theta),$   
 $j:-j:\sin(\theta)*\exp(-I*\phi), -j:j:1+\cos(\theta)$ )  
 beams\_pol\_fraction = f





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Polarization along arbitrary axis ( $\theta, \Phi$ )

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 $j:-j:\sin(\theta)*\exp(-I*\phi), -j:j:1+\cos(\theta)$ )  
 beams\_pol\_fraction = f

Diagonal / arbitrary density matrices

$$\rho = (x_{m,m'})$$

beams\_pol\_density = @({ $m:m':x_{m,m'}$ })







Getty Villa, Pacific Palisades, Etruscan, 525 BC



📌 Trivial things: cut selections (angular cuts etc.), clustering, particle containers

unstable "W+" { decay\_helicity = 0 }

📌 Factorization into production and decay (w/ full spin correlation, **intermediate polarization**)

📌 Final state polarizations (w/ spin density matrices) — not directly usable in event formats

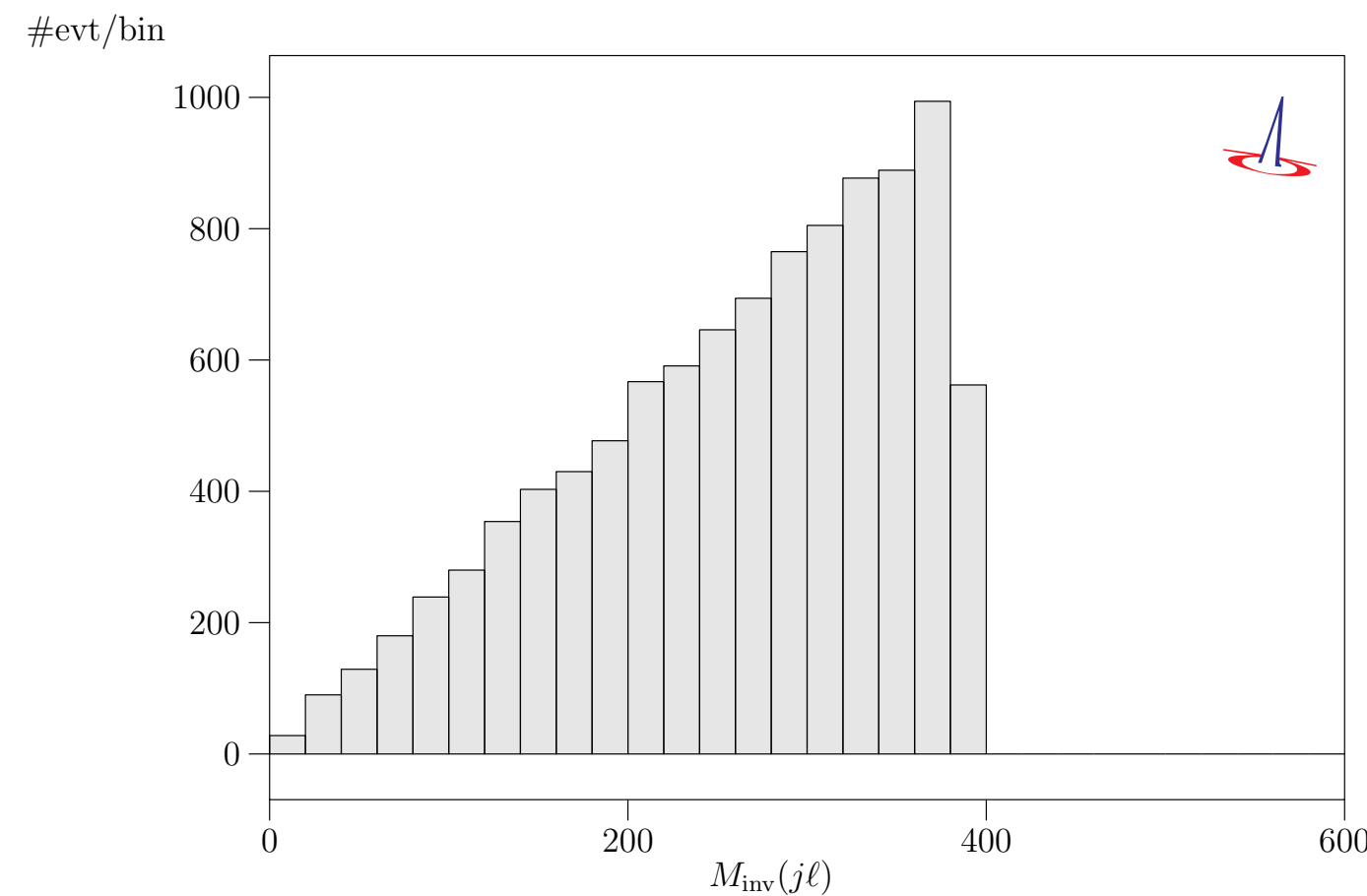
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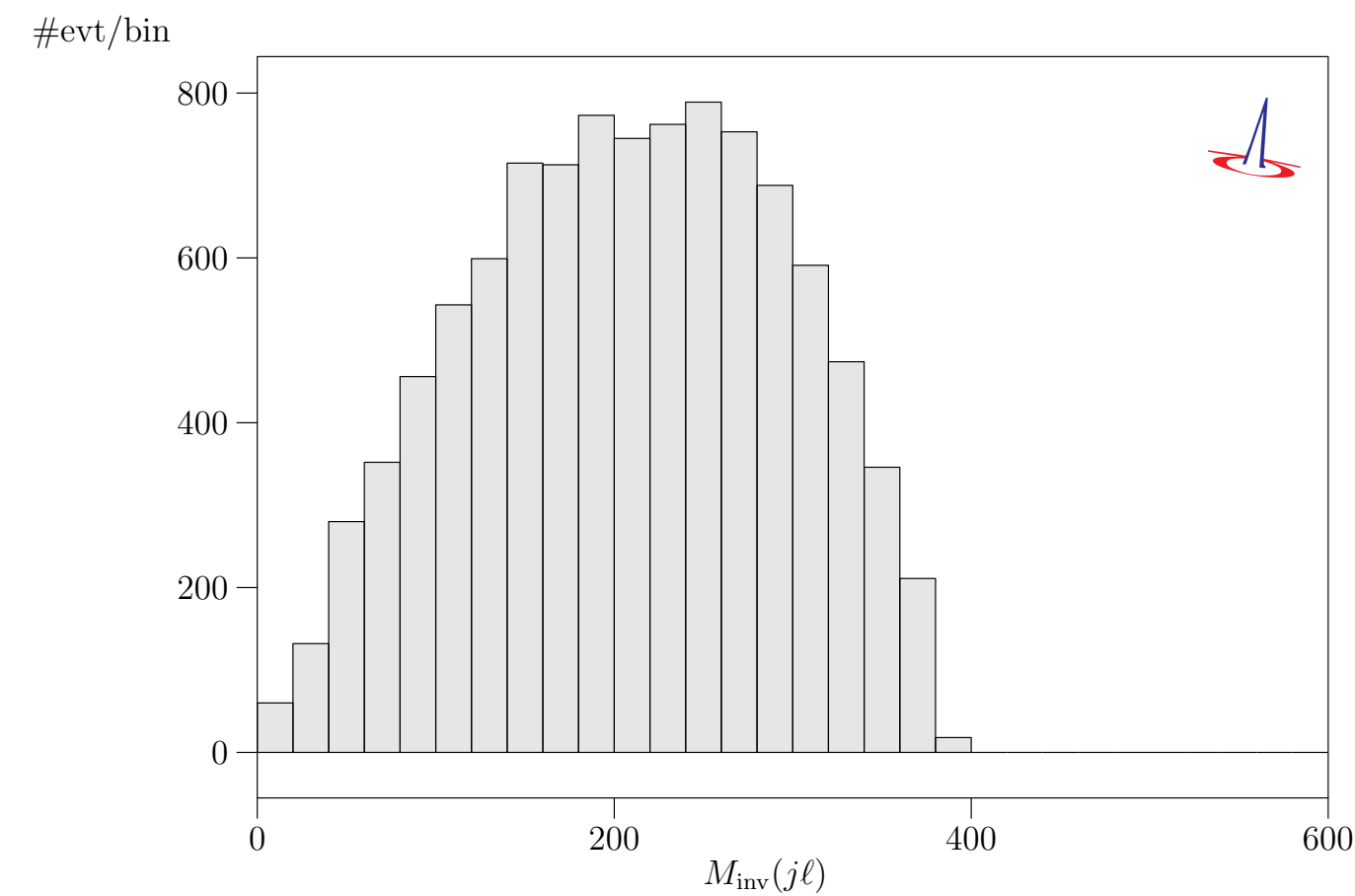
📌 Final state polarizations (w/ spin density matrices) — not directly usable in event formats

unstable “W+” { decay\_helicity = 0 }

?isotropic\_decay = true



simulate (casc)



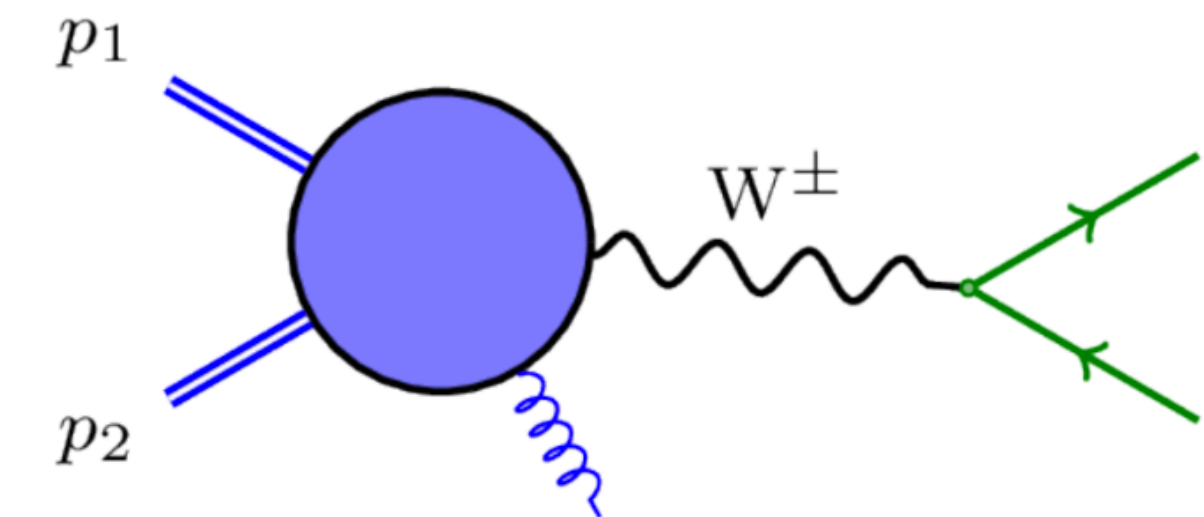
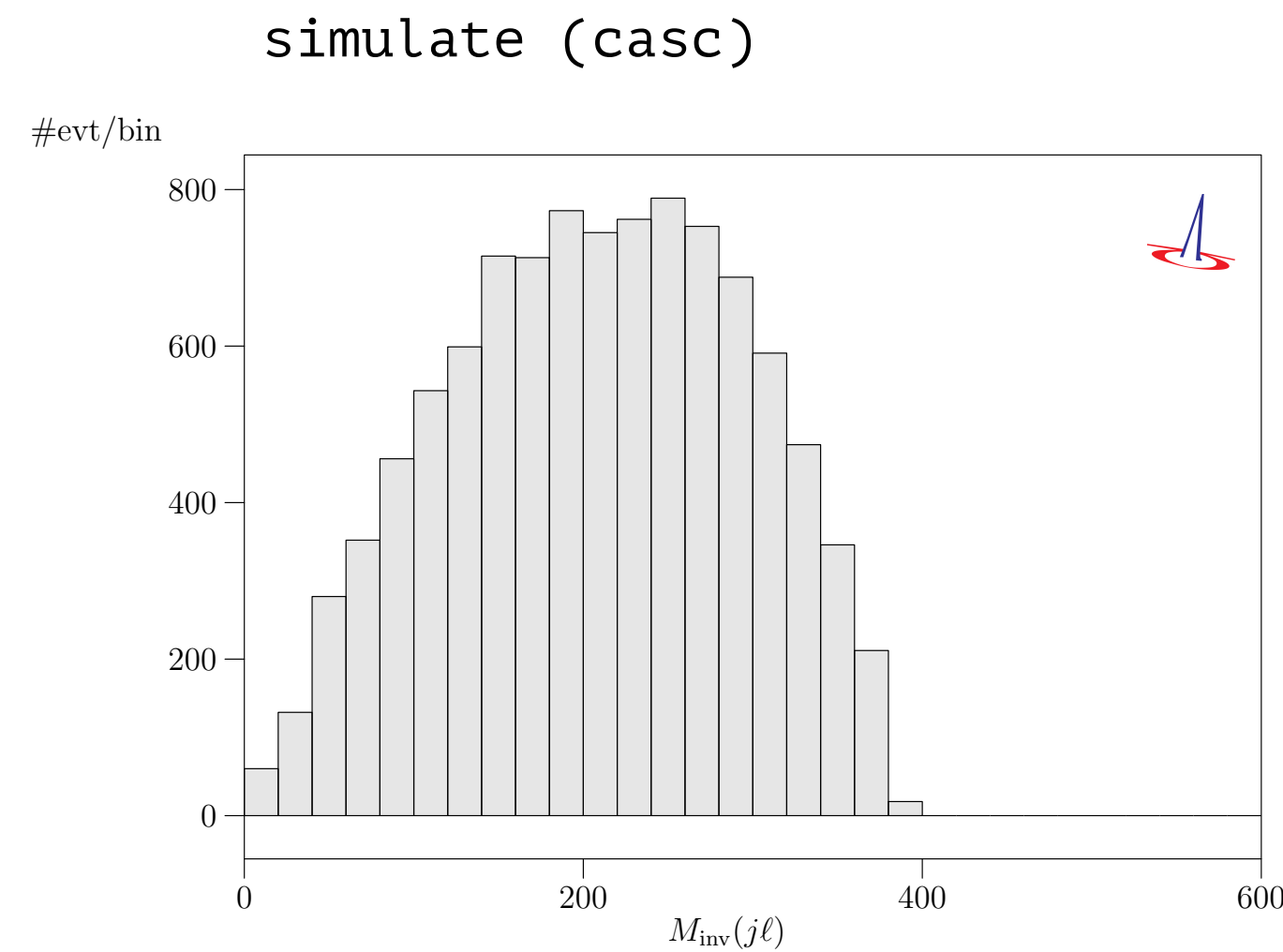
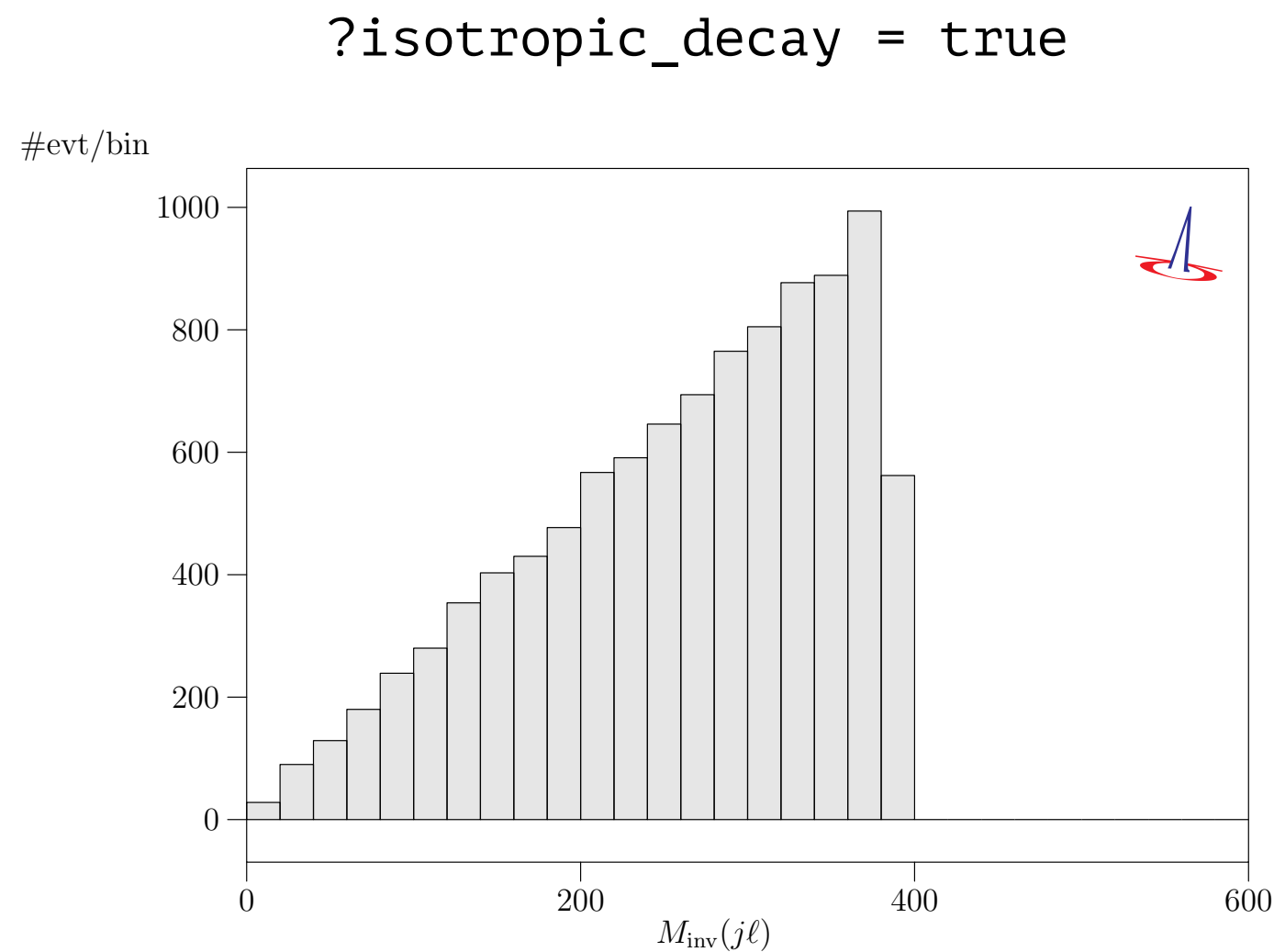


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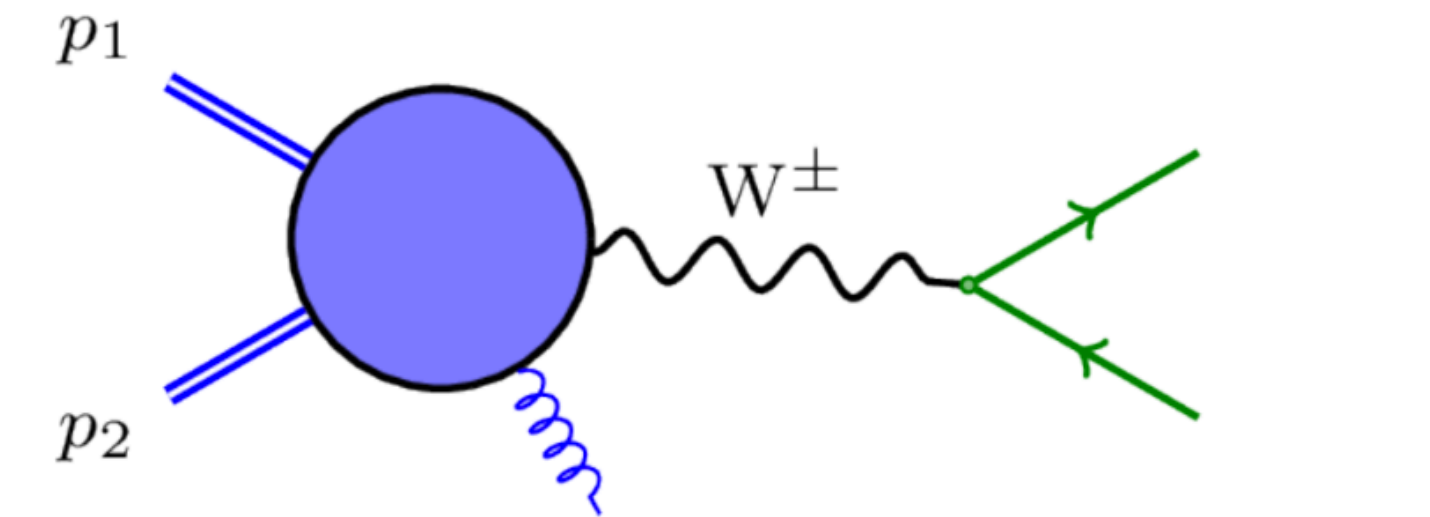
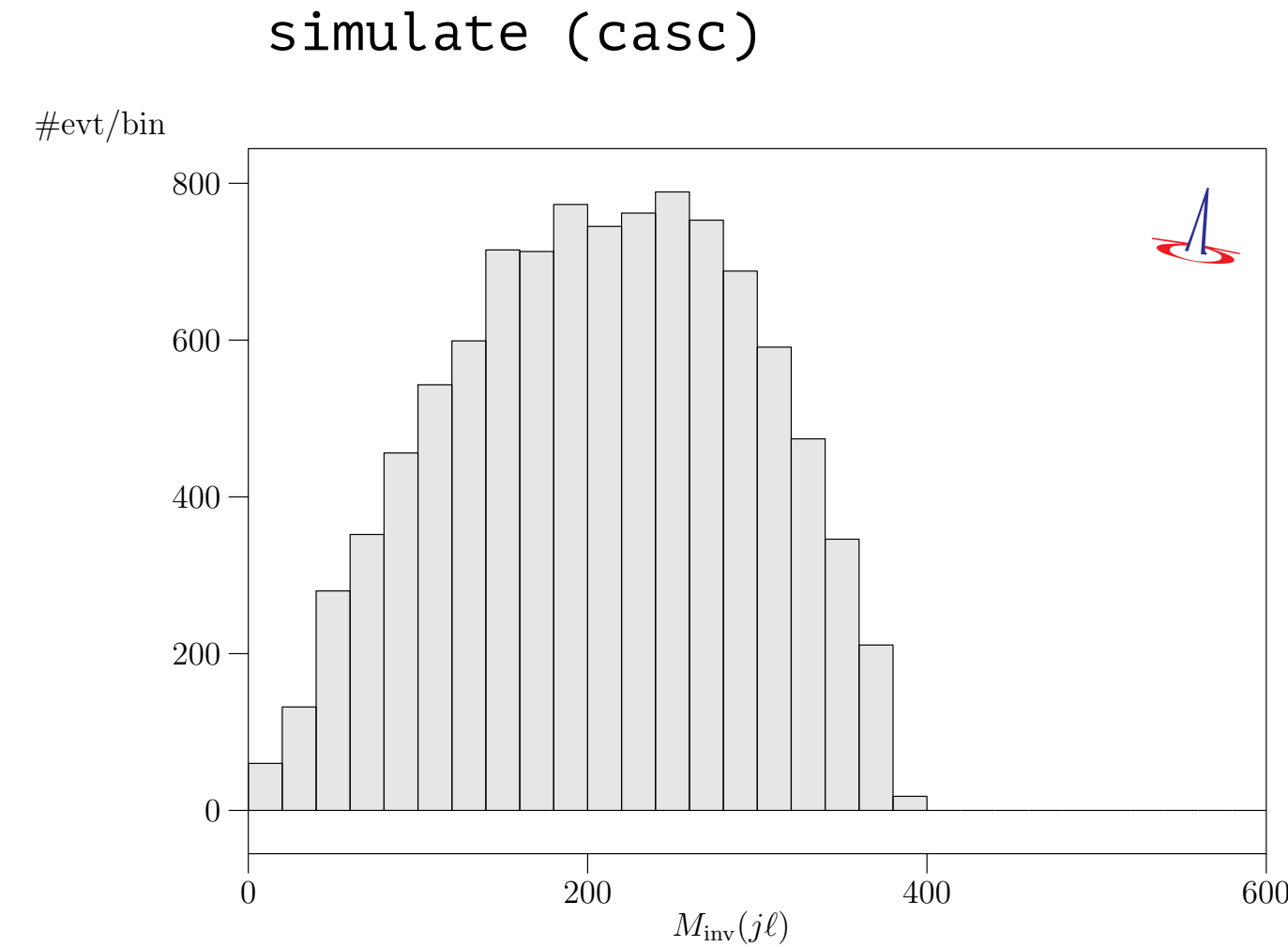
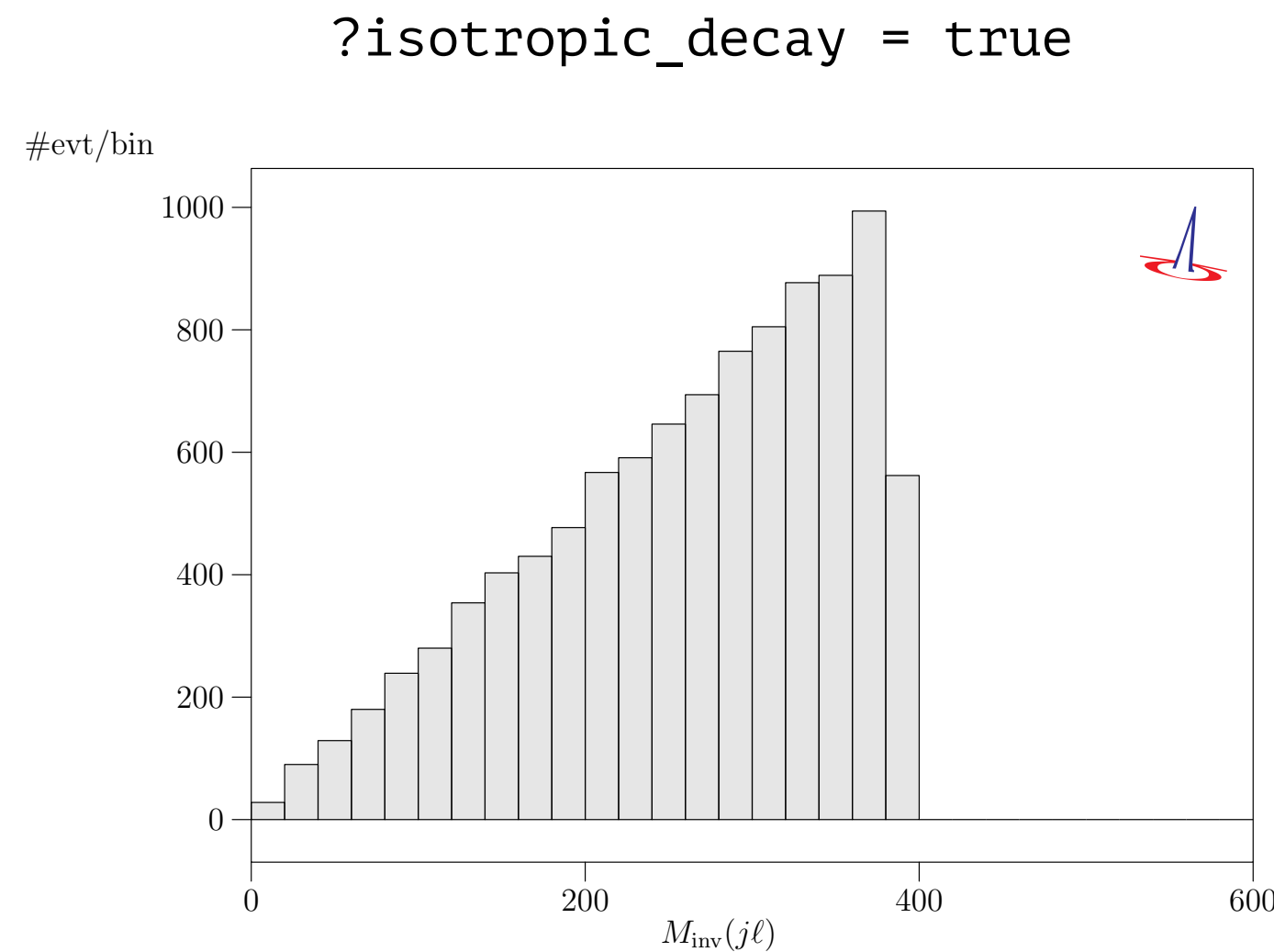
$$M_\lambda = \mathbf{P}_\mu \cdot \frac{-g_{\mu\nu} + \frac{k^\mu k^\nu}{k^2}}{k^2 - M_V^2 + iM_V\Gamma_V} \cdot \mathbf{D}_\nu$$

$$-g^{\mu\nu} + \frac{k^\mu k^\nu}{k^2} \longrightarrow \sum_\lambda \epsilon_\lambda^{\mu*} \epsilon_\lambda^\nu$$

On-shell vector bosons (NWA or DPA)

unstable “W+” { decay\_helicity = 0 }

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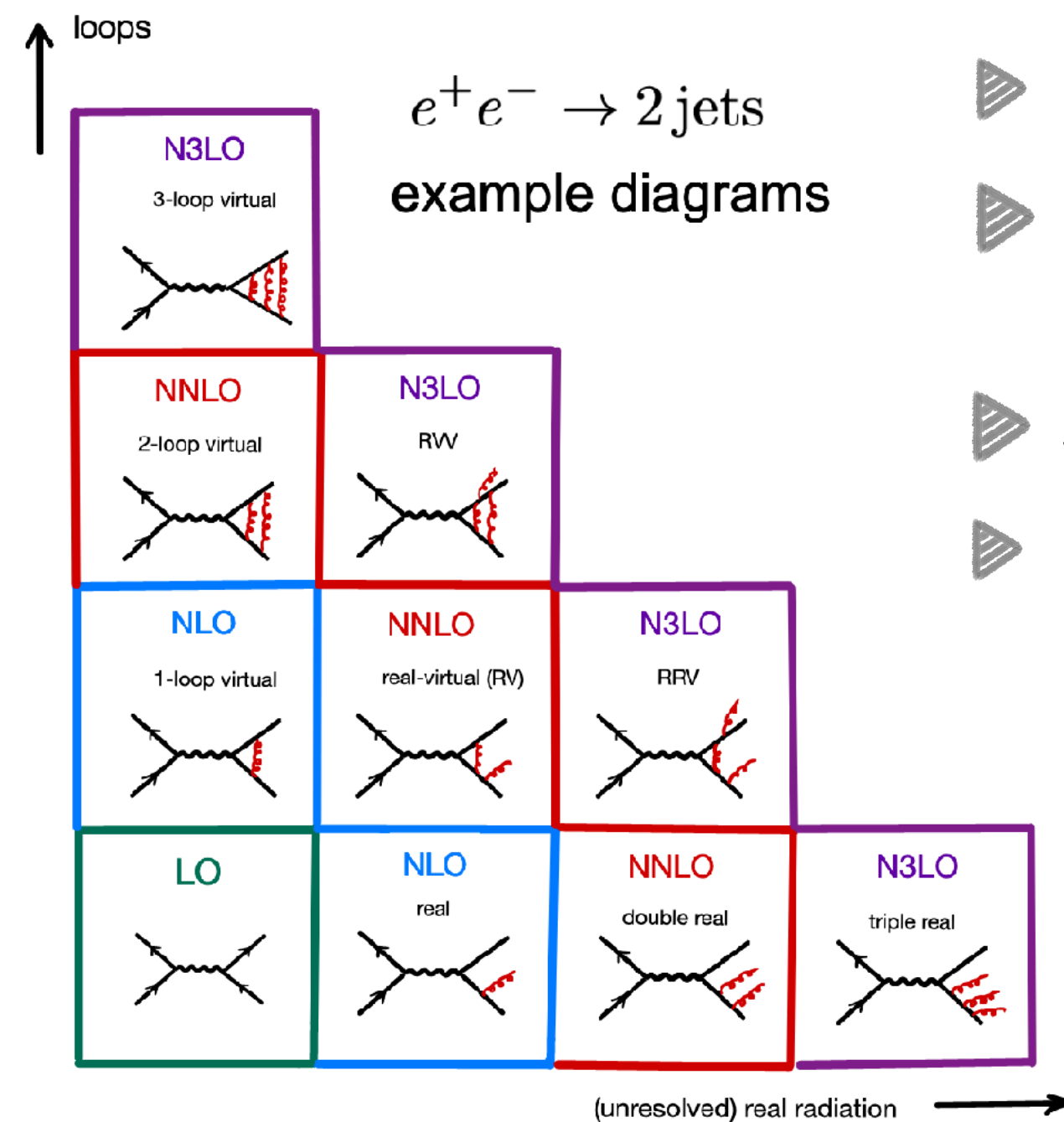
$$-g^{\mu\nu} + \frac{k^\mu k^\nu}{k^2} \longrightarrow \sum_\lambda \epsilon_\lambda^{\mu*} \epsilon_\lambda^\nu$$

On-shell vector bosons (NWA or DPA)

- Intermediate polarization of resonances (e.g. W / Z / [t]):** projection to on-shell state
- Necessary to have this in machinery of established MC generator to calibrate simulation
- Automated for LO — work in progress for NLO

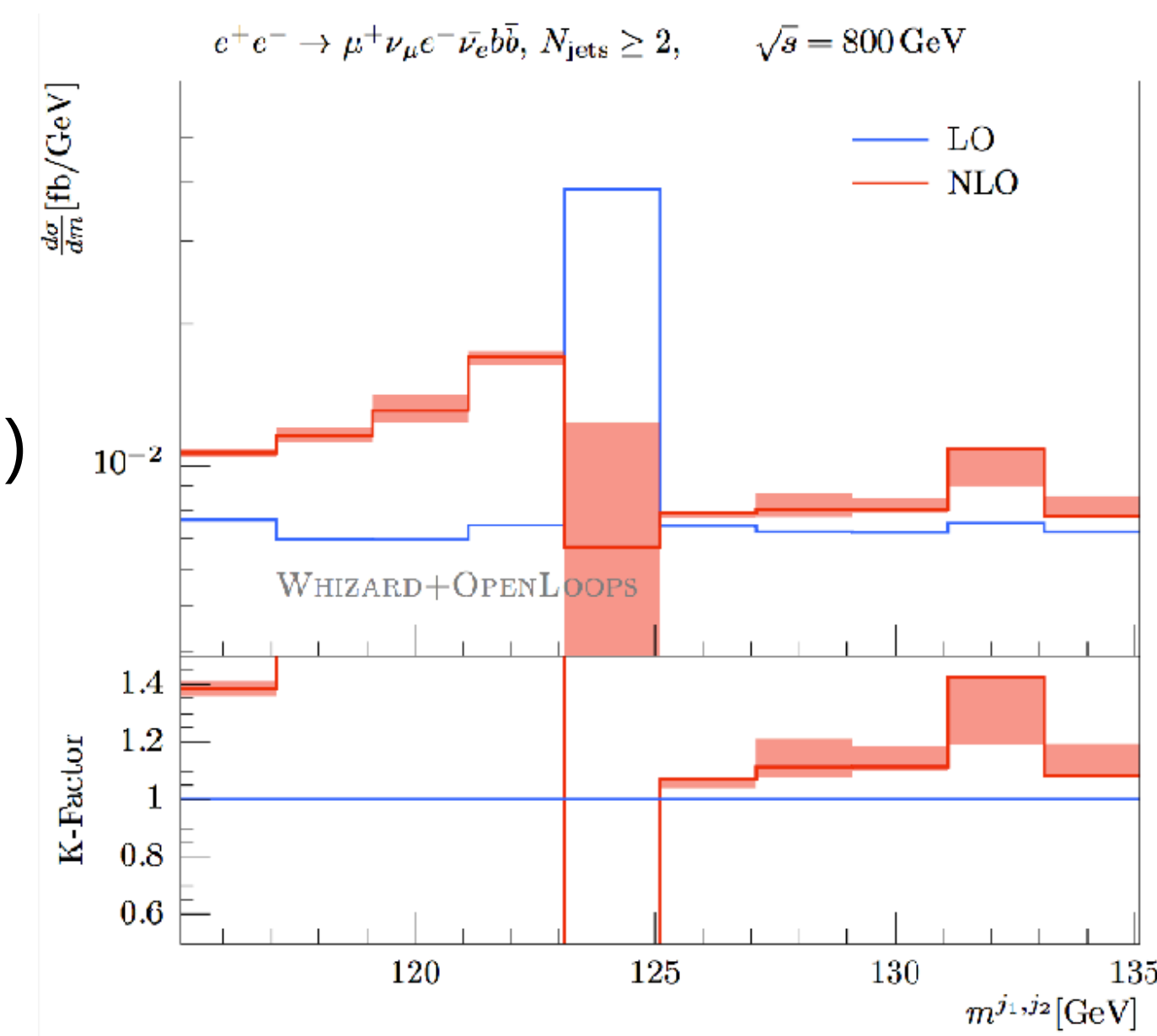


# The “Exclusive” Frontier — fN(N)LO, Automation in MCs



$e^+e^- \rightarrow 2\text{jets}$   
example diagrams

- ▶ LO + NLO QCD ⊕ EW automated: Sherpa, MG5, Whizard
- ▶ Note the fine-prints
- ▶ Signal + background samples (full SM QFT interference level)
- ▶ Need  $e^+e^- \rightarrow 2f, 3f, 4f, 5f, 6f, [7-10f]$  @ NLO QCD ⊕ EW (arbitrary cuts, fully differential)



## NLO EW

Pia Bredt, Phd thesis, DESY, 2022, arXiv:2212.04393

$\sqrt{s}$ [GeV]	MCSANcEe[37]		WHIZARD+RECOLA			$\sigma^{\text{sig}}$ (LO/NLO)
	$\sigma_{\text{LO}}^{\text{tot}}$ [fb]	$\sigma_{\text{NLO}}^{\text{tot}}$ [fb]	$\sigma_{\text{LO}}^{\text{tot}}$ [fb]	$\sigma_{\text{NLO}}^{\text{tot}}$ [fb]	$\delta_{\text{EW}}$ [%]	
250	225.59(1)	206.77(1)	225.60(1)	207.0(1)	-8.25	0.4/2.1
500	53.74(1)	62.42(1)	53.74(3)	62.41(2)	+16.14	0.2/0.3
1000	12.05(1)	14.56(1)	12.0549(6)	14.57(1)	+20.84	0.5/0.5

## NLO QCD

	$\sigma_{\text{LO}}$ [fb]	$\sigma_{\text{NLO}}$ [fb]	$K$
$e^+e^- \rightarrow jj$	622.737(8)	639.39(5)	1.027
$e^+e^- \rightarrow jjj$	340.6(5)	317.8(5)	0.933
$e^+e^- \rightarrow jjjj$	105.0(3)	104.2(4)	0.992
$e^+e^- \rightarrow jjjjj$	22.33(5)	24.57(7)	1.100
$e^+e^- \rightarrow jjjjjj$	3.583(17)	4.46(4)	1.245
$e^+e^- \rightarrow t\bar{t}$	166.37(12)	174.55(20)	1.049
$e^+e^- \rightarrow t\bar{t}j$	48.12(5)	53.41(7)	1.110
$e^+e^- \rightarrow t\bar{t}jj$	8.592(19)	10.526(21)	1.225
$e^+e^- \rightarrow t\bar{t}jjj$	1.035(4)	1.405(5)	1.357



# The "Exclusive" Frontier — fN(N)LO, Automation in MCs

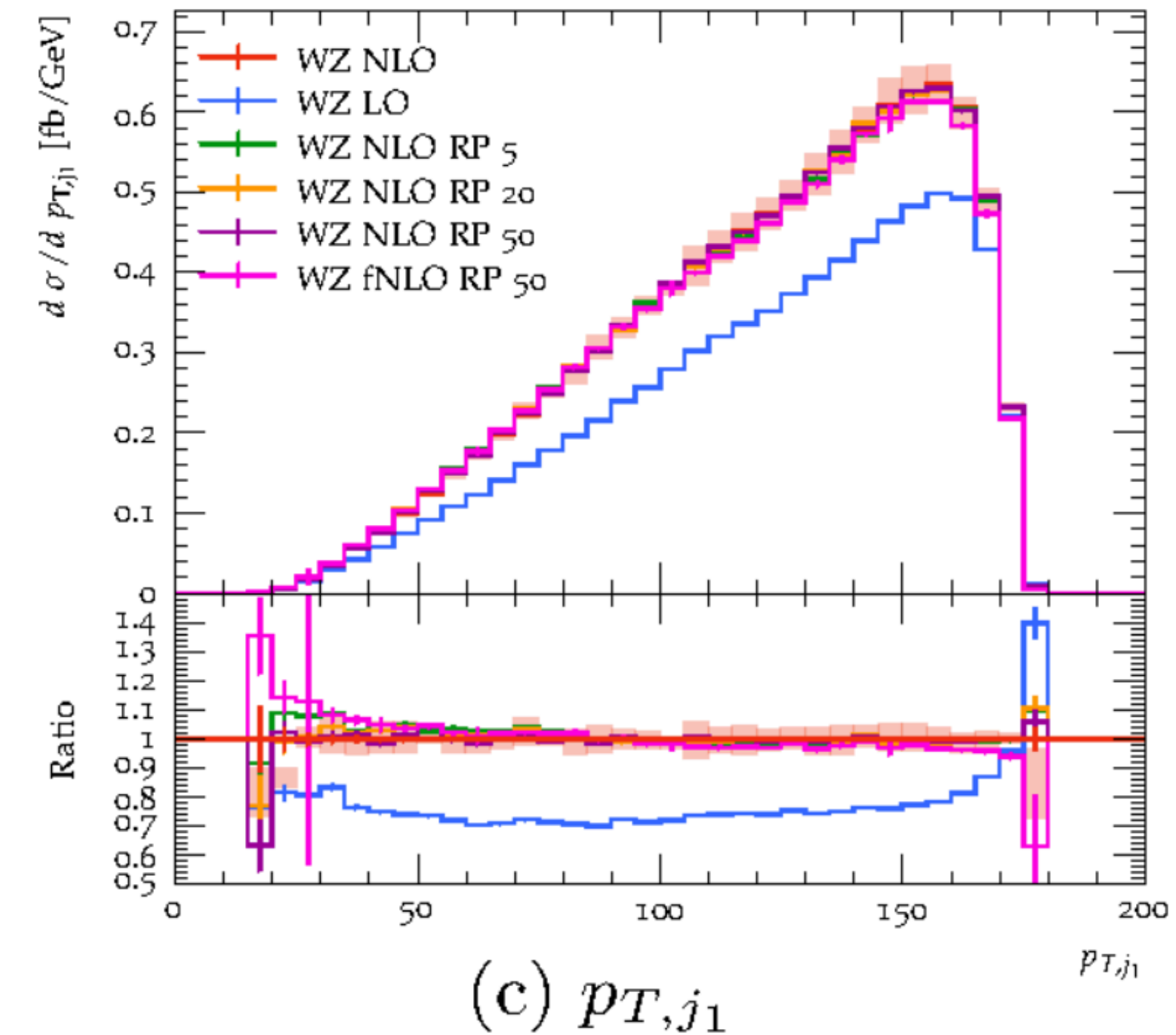
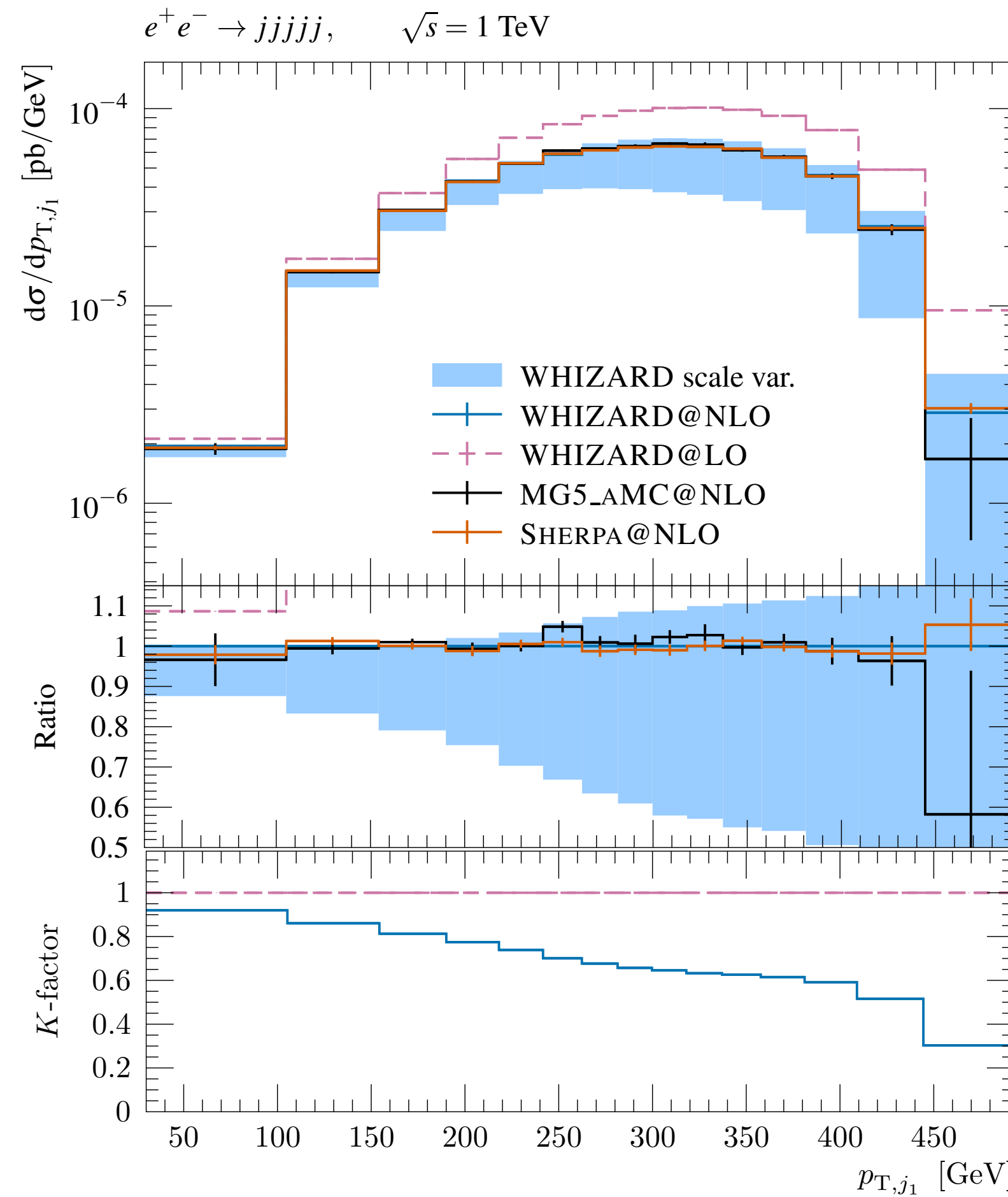
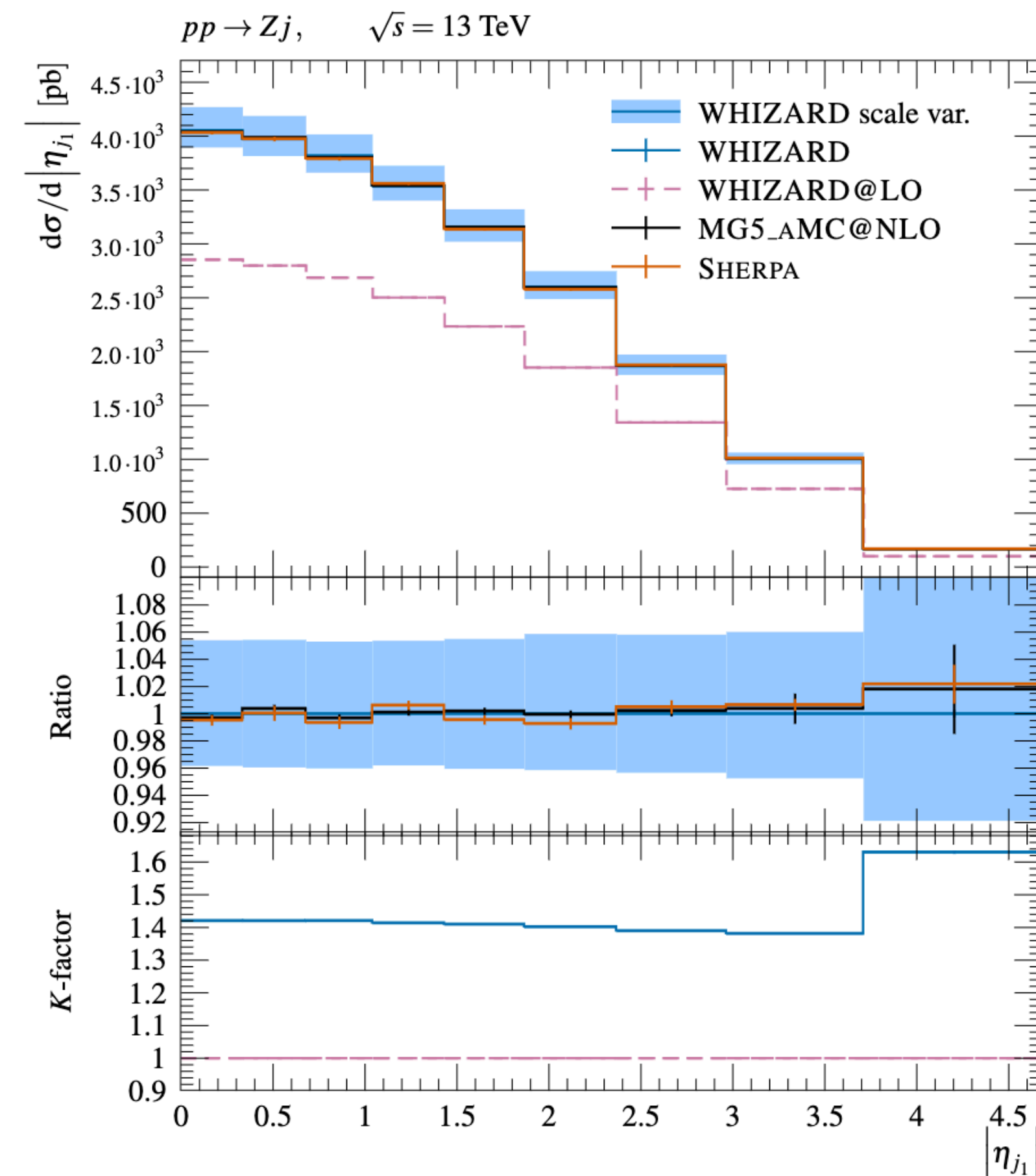
pp @ 13 TeV, NLO QCD

ee @ 1 TeV, NLO QCD

ILC 500:  $e^+e^- \rightarrow t\bar{t}j$

$$\mu_R = H_T/2 \quad \text{with} \quad H_T := \sum_i \sqrt{p_{T,i}^2 + m_i^2}$$

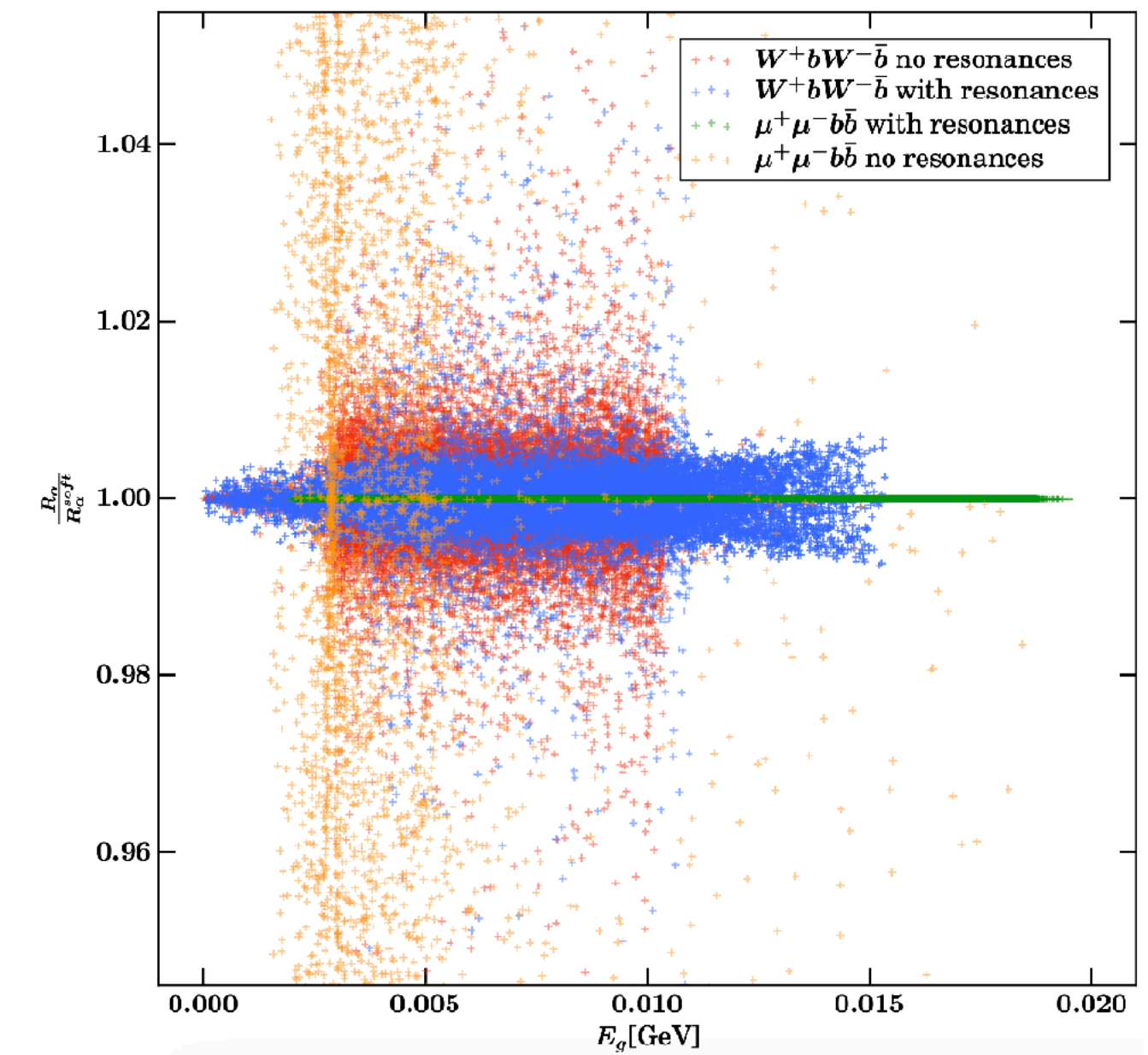
(a)





# N(N)LO Automation in MC — Going beyond

- MC NLO implementation relies on 2 building blocks: Subtraction (Catani-Seymour or Frixione/Kunszt/Signer)
- also: resonance-aware FKS subtraction cf. Ježo/Nason, 1509.09071; Chokoufé, 2017
- Photon isolation, photon recombination, light-, b-, c-jet selection
- Covers also loop-induced processes (“LO”, virtual-squared)



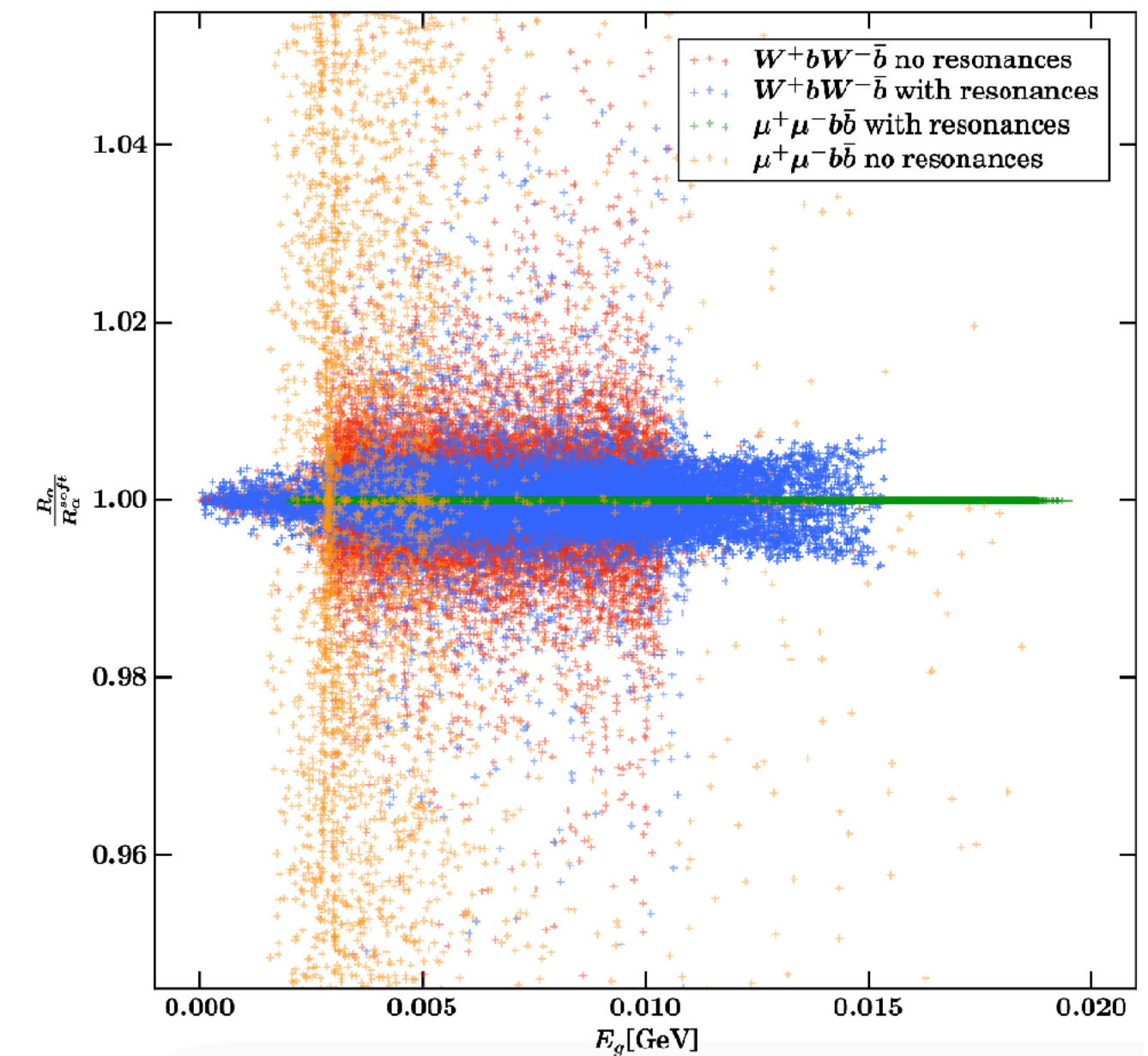
↪ Talk by Qian Song (Thu)

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## Two major bottlenecks to NNLO

- Virtual integrals with many mass scales / off-shell legs  
[Abreu ea., Badger ea., Baglio ea., Brønnum-Hansen ea.](#)
- IR pole treatment / subtraction [CS, FKS, NS, Stripper, qT/sub-jettiness etc.](#)



↪ Talk by Qian Song (Thu)

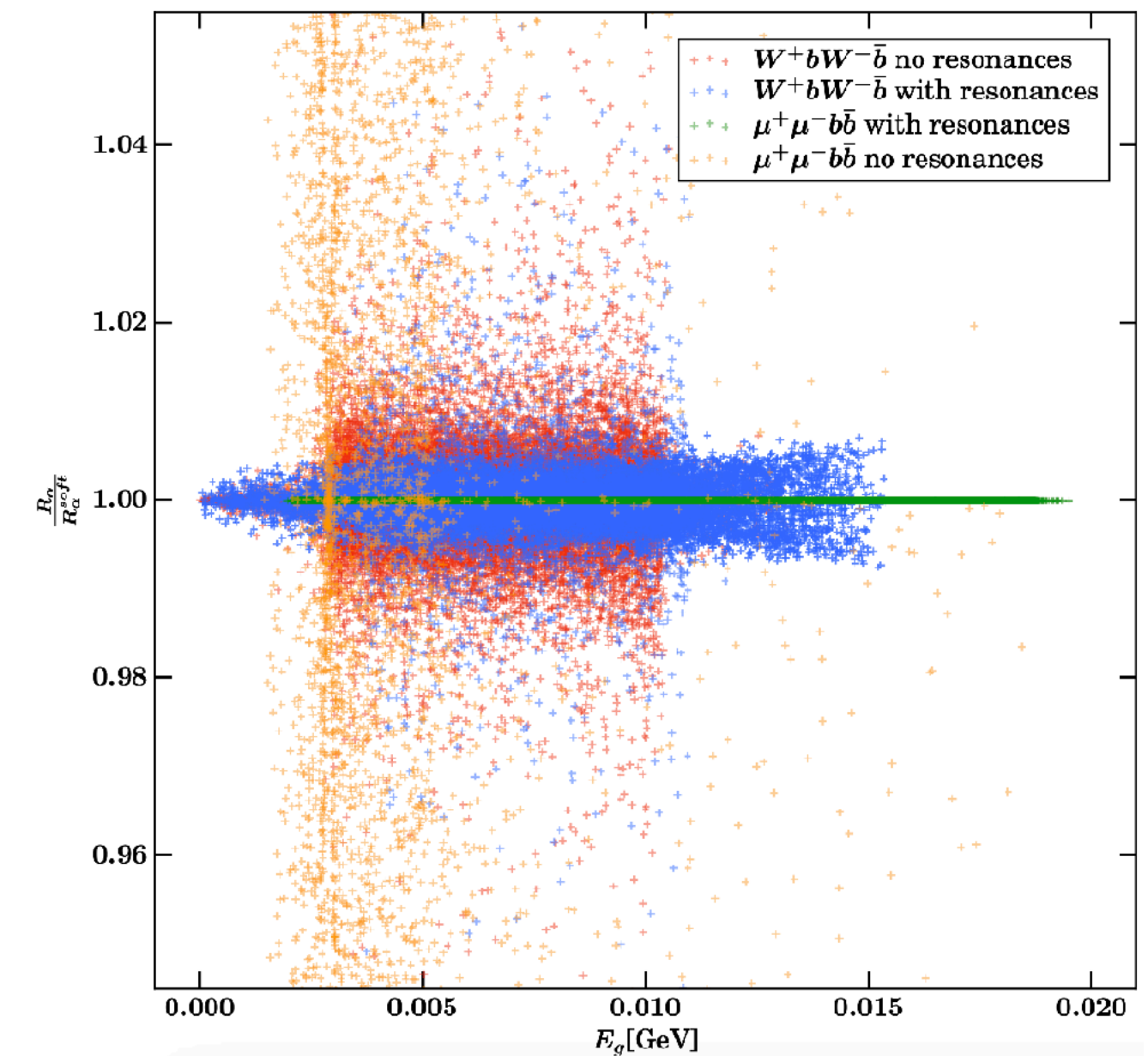


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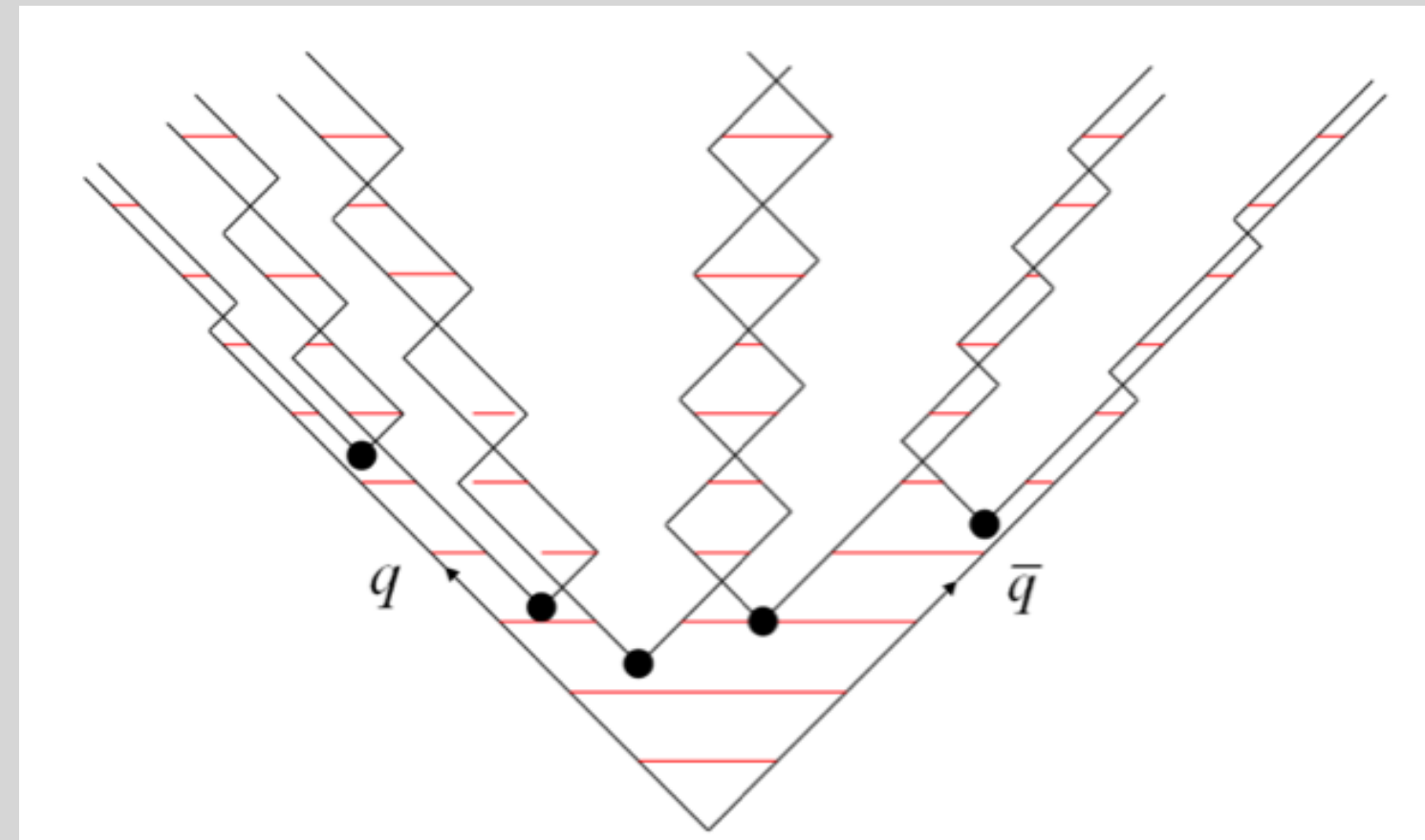
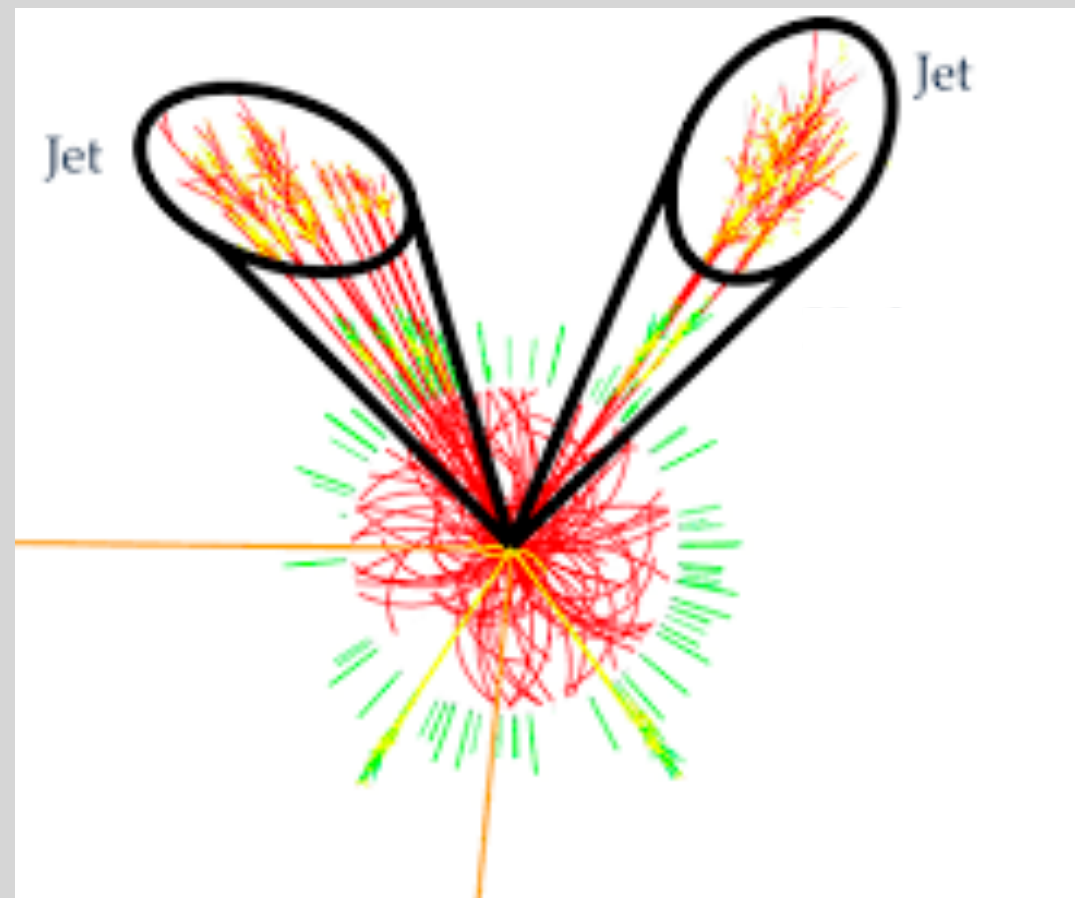
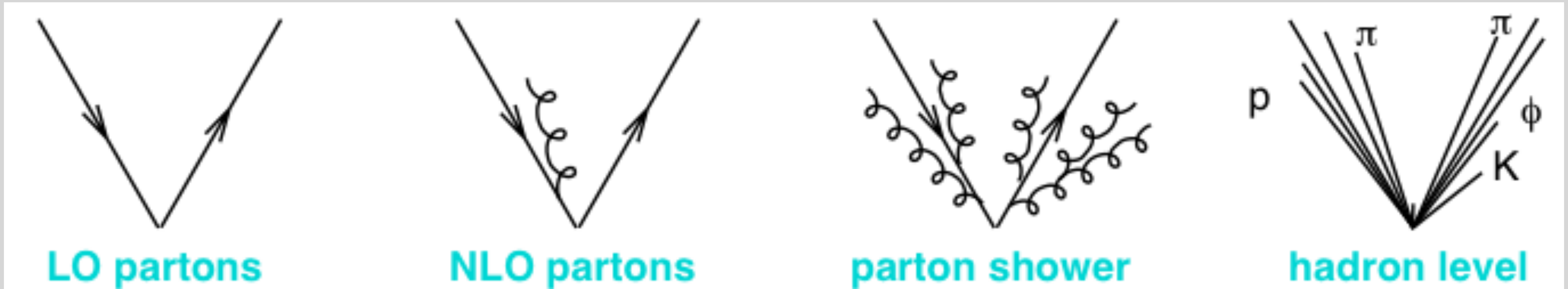
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- FKS soft/eikonal subtraction sufficient for low-energy machines
- NNLO QED (massive, virtuals pending): McMule [Signer ea.](#) [Whizard]
- Baby steps to NNLO automation: [Griffin](#) [Chen/Freitas, 2023](#) [↪ Talk by Qian Song \(Thu\)](#)
- for NNLO EW need for full-fledged soft+collinear NNLO subtraction [Stripper]

# Parton Showers, Matching, Hadronization

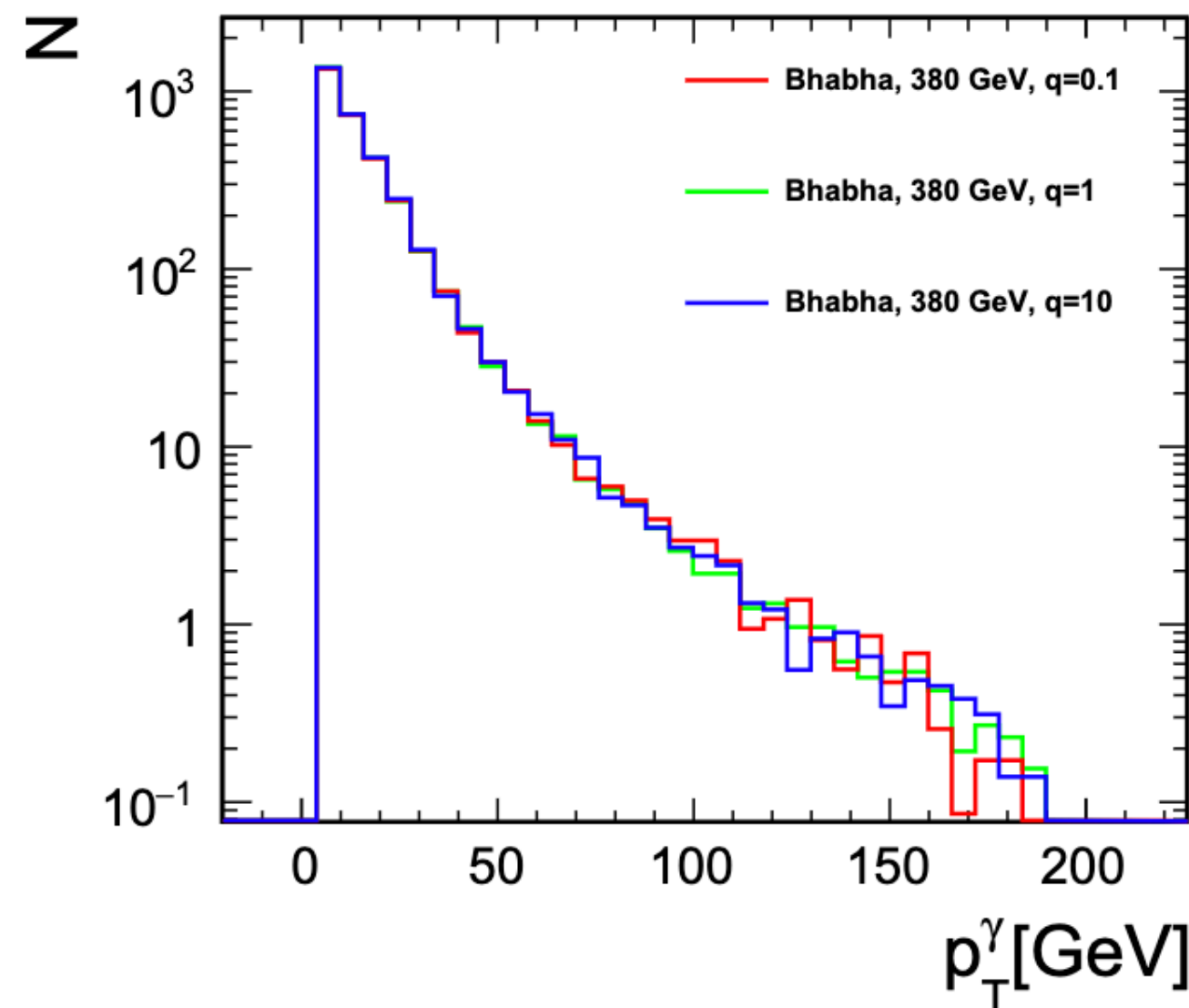
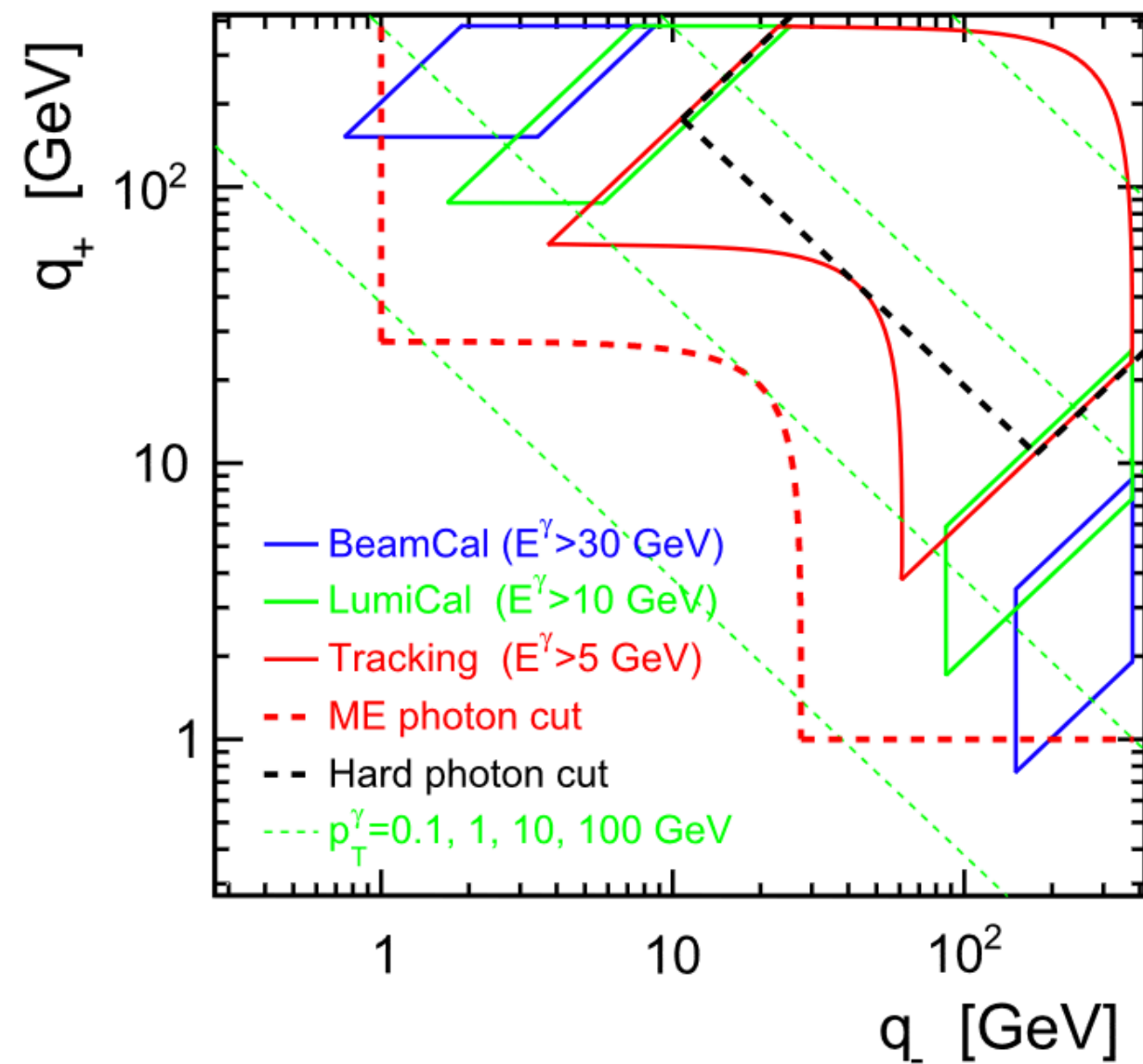




# Exclusive photons

## QED ISR [+FSR], matching

- Explicit photon from fix-order (LO/NLO/NNLO) matrix element (best description)
- “Shower-recoil approach”: generate  $p_{\perp}$  according to  $\frac{\alpha}{\pi} \cdot \log \frac{p_{\perp}^2}{m_e^2}$
- Boost according to the generated  $p_{\perp}$  (avail. for for ISR, EPA or ISR+EPA)
- Algorithm applied recursively (similar to massive NLO EW ISR PS construction)
- Recursive algorithm resembles a photon shower with  $n$  exclusive photons



J. Kalinowski/W. Kotlarski/P. Sopicki/A.F. Zarnecki, 2020



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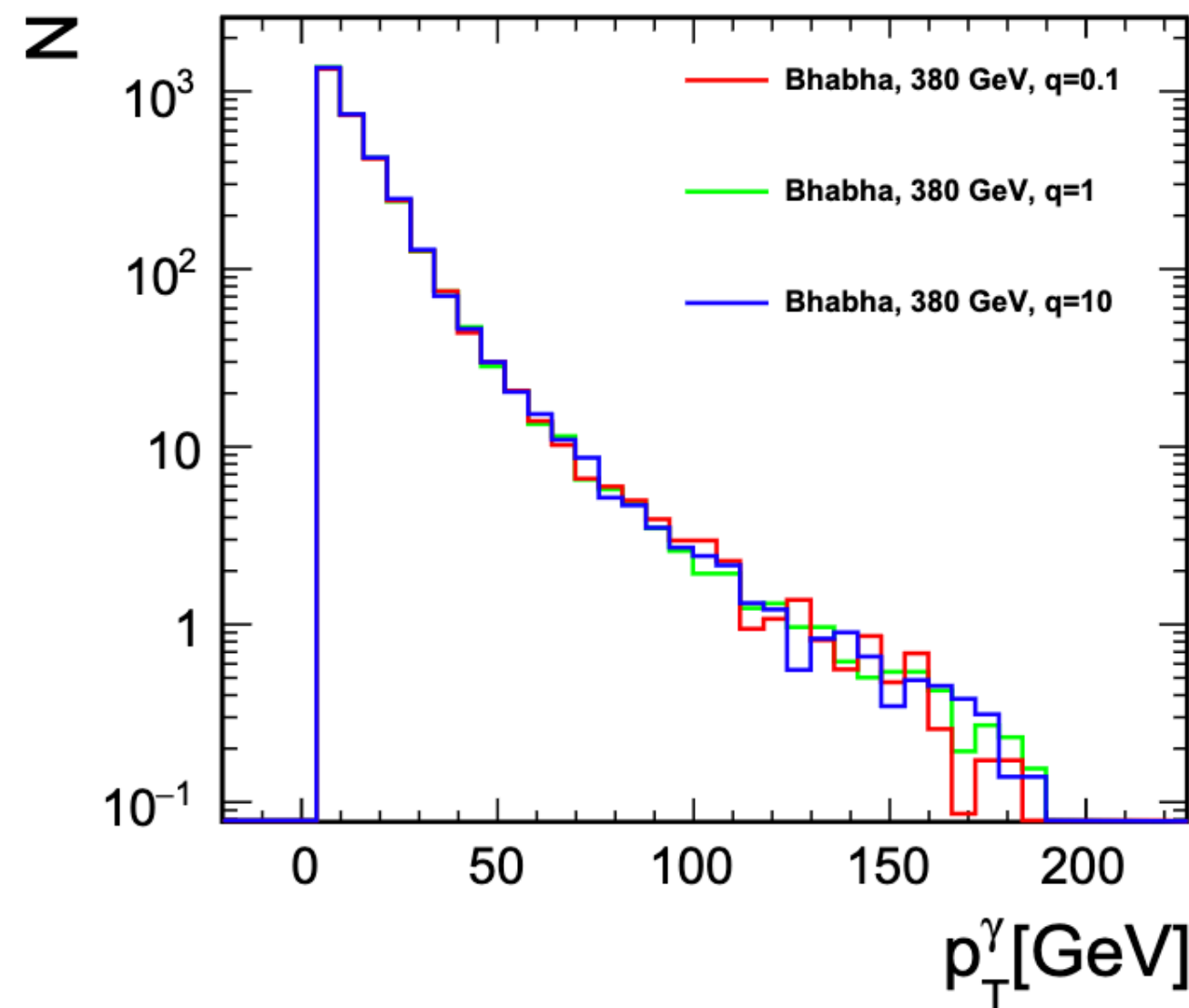
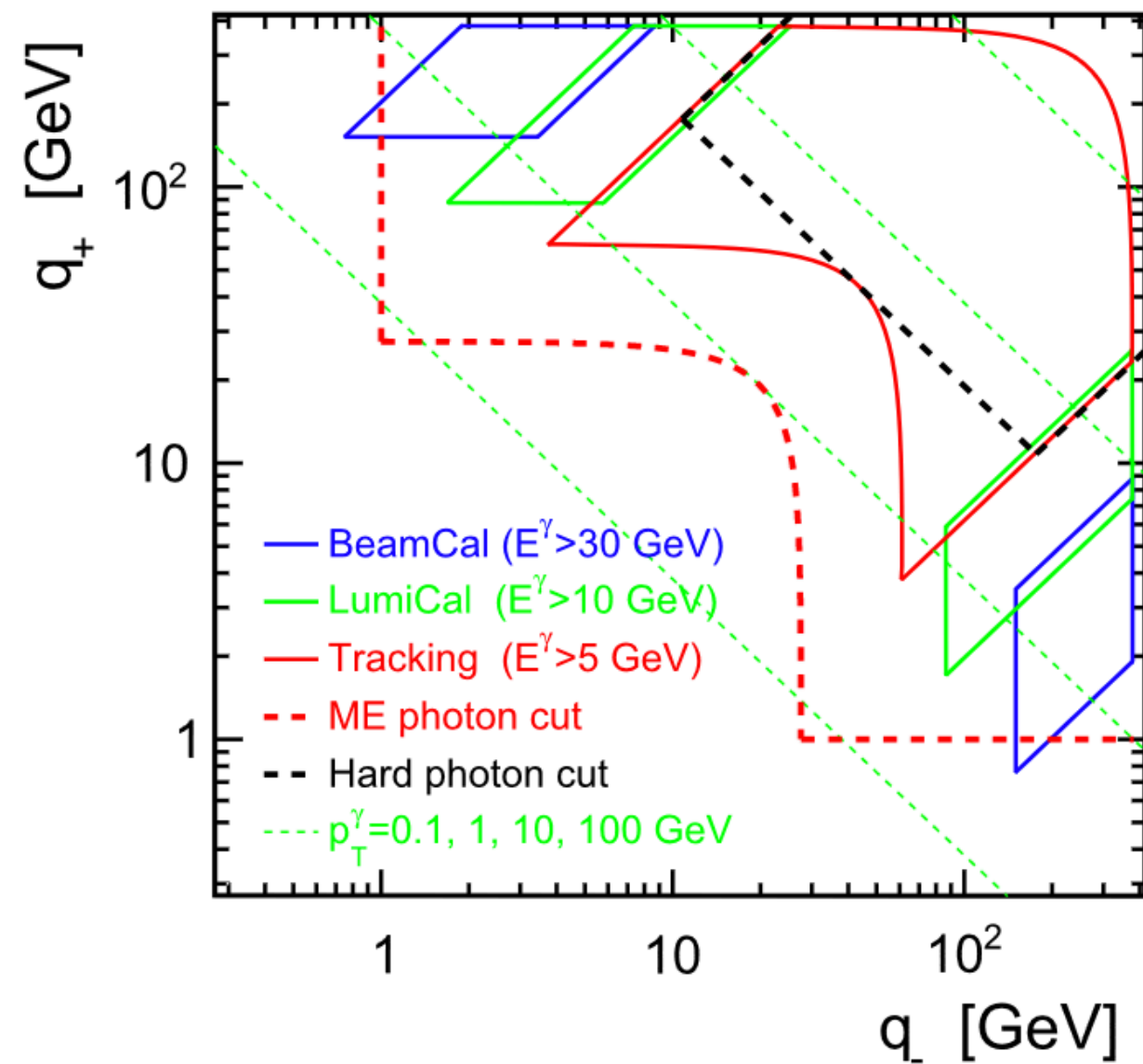
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## Full QED shower

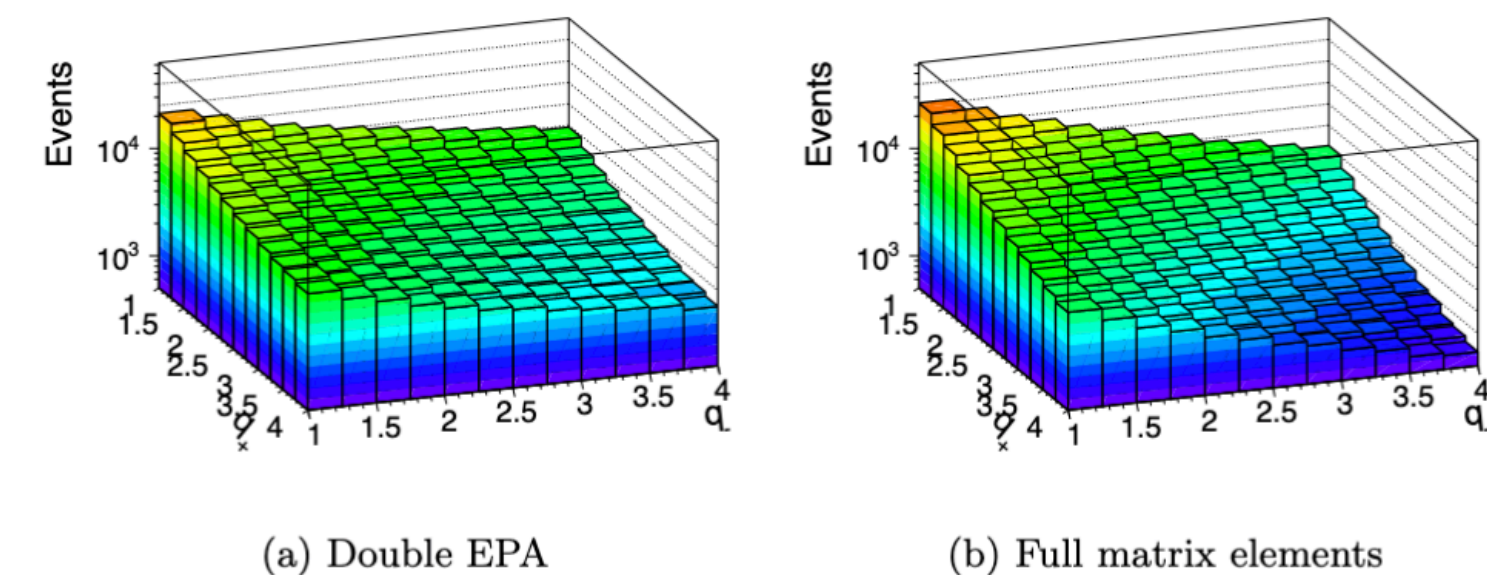
- Based either on dipoles or antennae, for ISR separate, for FSR interleaved [?]
- Can then be combined with POWHEG/MC@NLO/XXX-type matching
- Can be combined with resummation in (semi-)automated ways ... w.i.p.

## Matching between EPA/ $\gamma$ PDF + beam $\gamma$

M. Berggren/W. Kilian/K. Mękała/JRR



J. Kalinowski/W. Kotlarski/P. Sopicki/A.F. Zarnecki, 2020



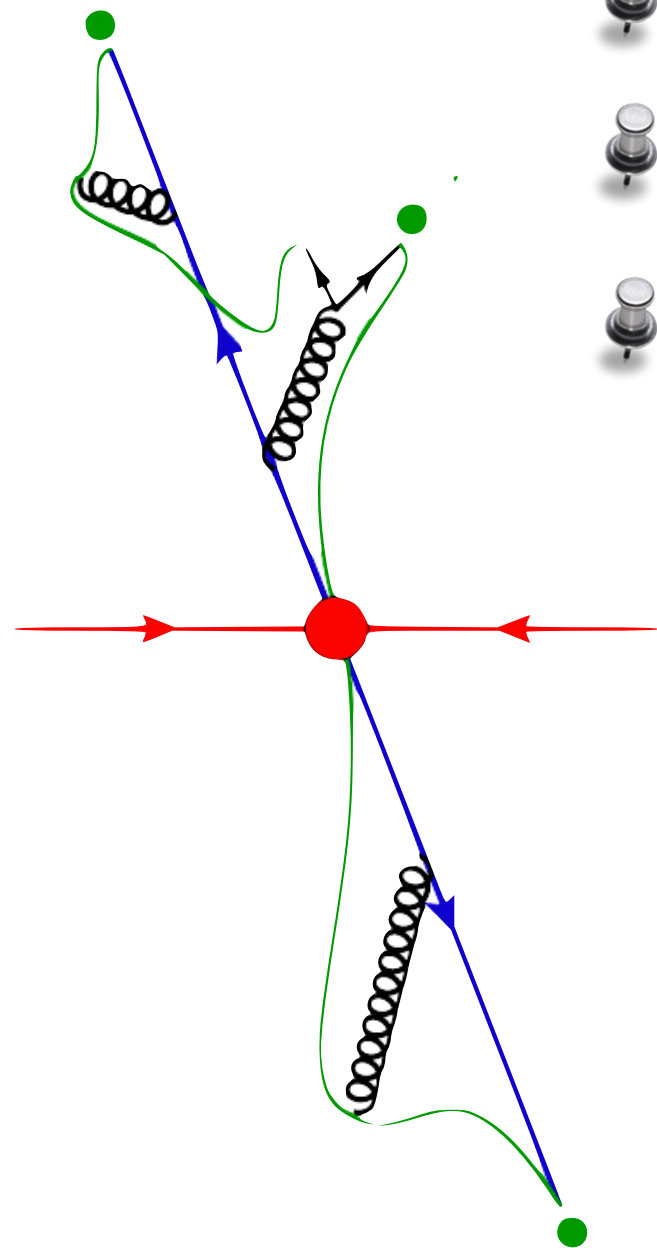
(a) Double EPA

(b) Full matrix elements

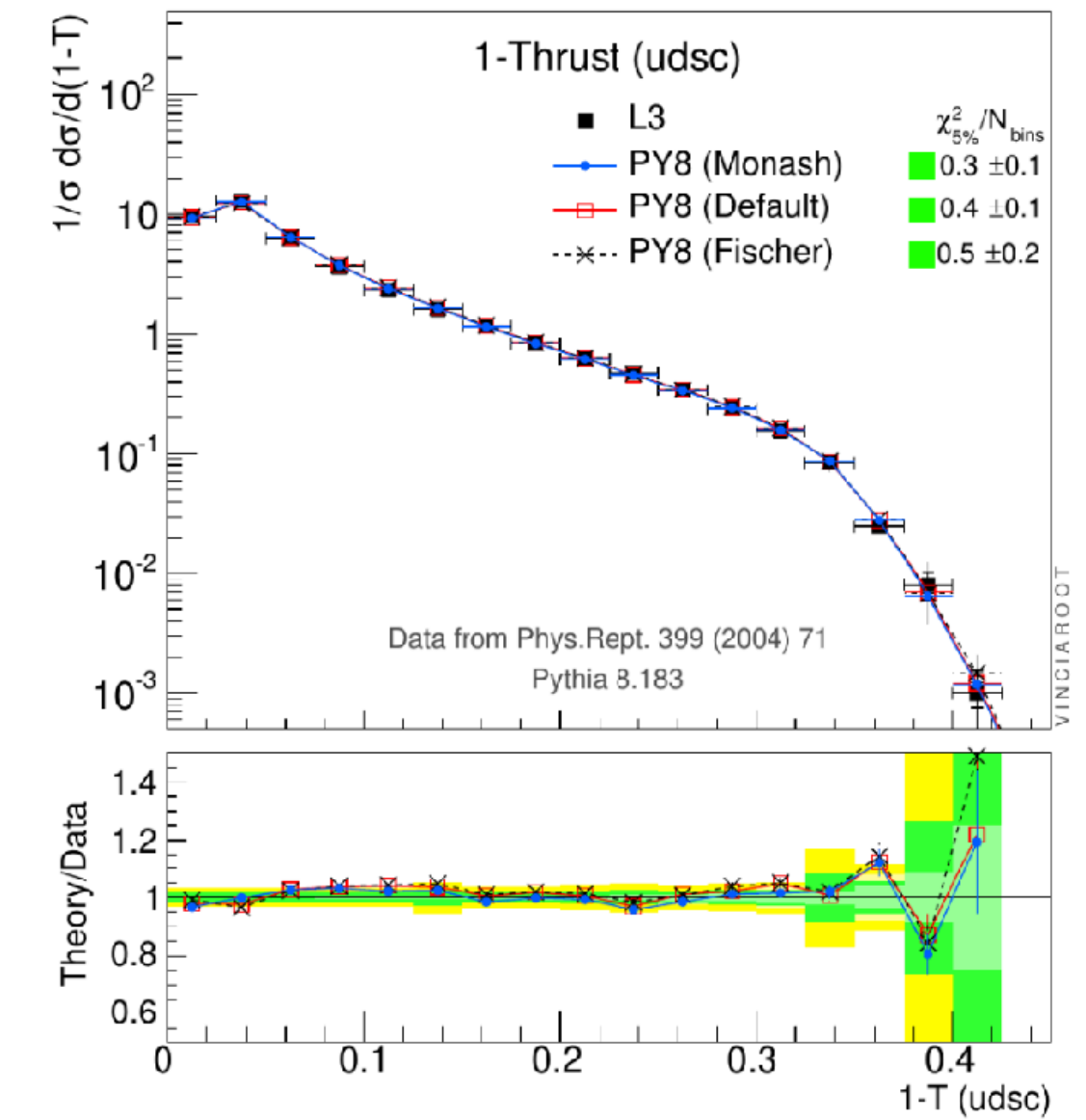


# Parton shower / hadronization

- Machinery of parton showers well advanced, recap of CERN workshop 04/2023
- Tuning: automated tools w/ built-in correlations (Professor, AutoTunes, Apprentice, ...)
- Global event shapes,  $\alpha_s$ , charge multiplicity, hadron multiplicity
- Possible NLL parton showers (final state only!) for  $e^+e^-$ :



Shower	Ordering	NLL Validation
PanScales [2002.11114]	$10 \leq \beta < 1$	Fixed and all order numerical tests for a range of observables
Alaric [2208.06057]	$k_t$ ( $\beta = 0$ )	Analytical, numerical tests for global event shapes
Deductor [2011.04777]	$k_t, \Lambda$ ( $\beta = 0, 1$ )	Analytical and numerical tests for thrust
Manchester-Vienna [2003.06400]	$k_t$ ( $\beta = 0$ )	Analytical for thrust and multiplicity



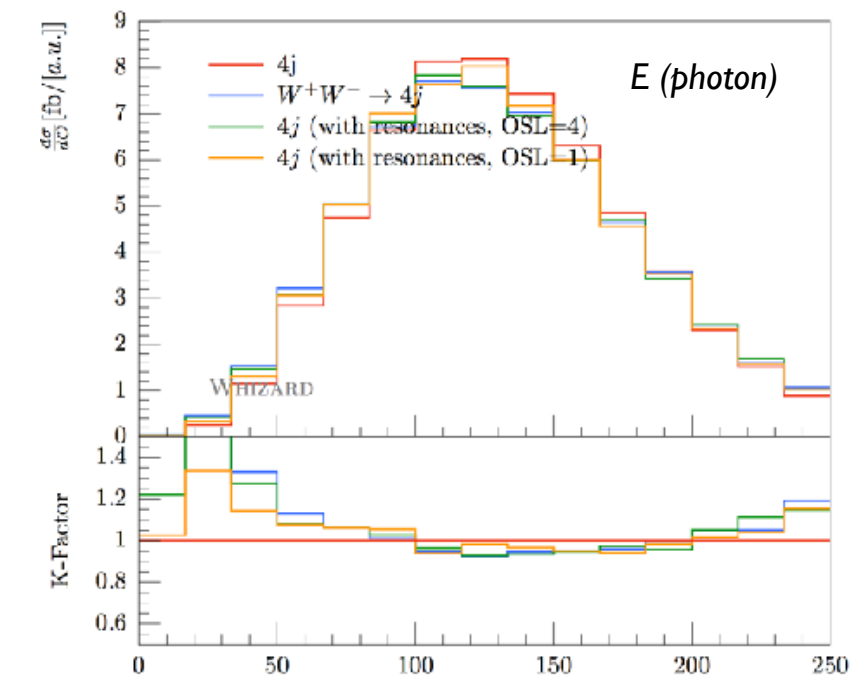
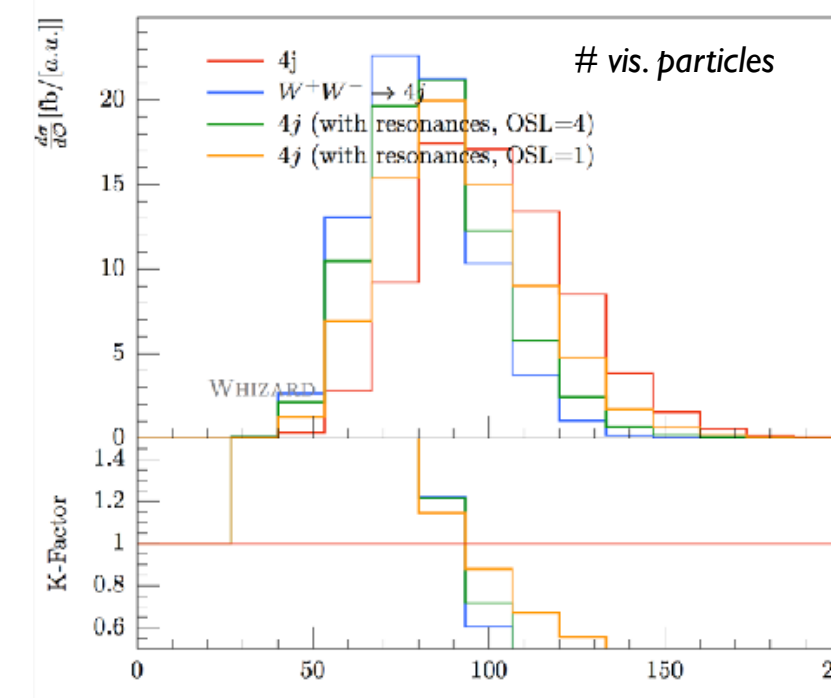
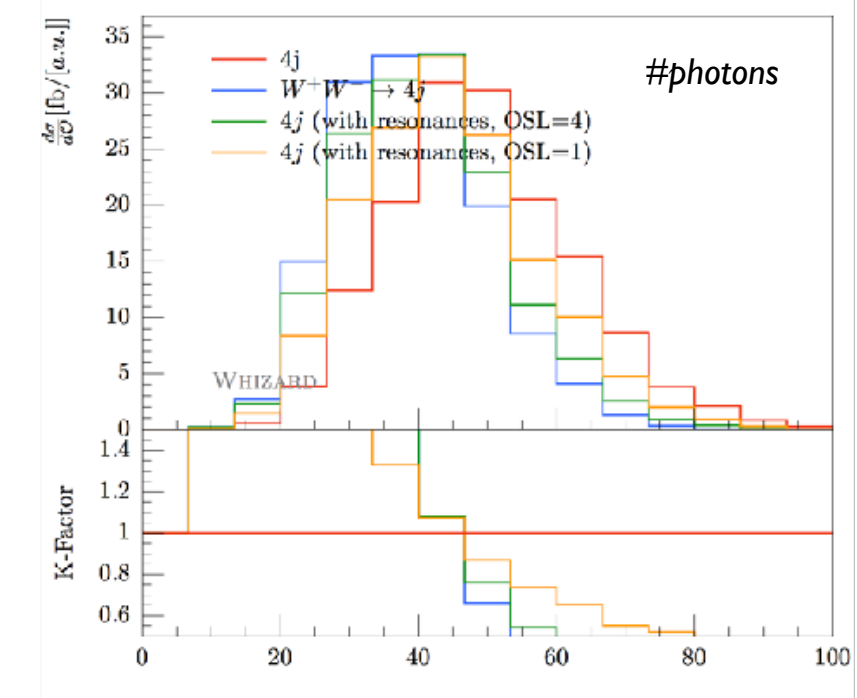
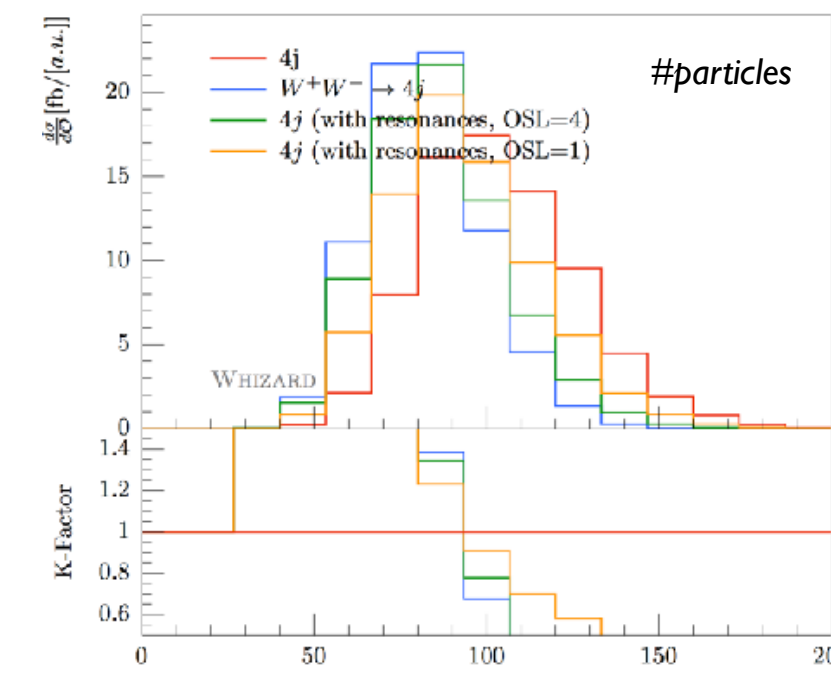
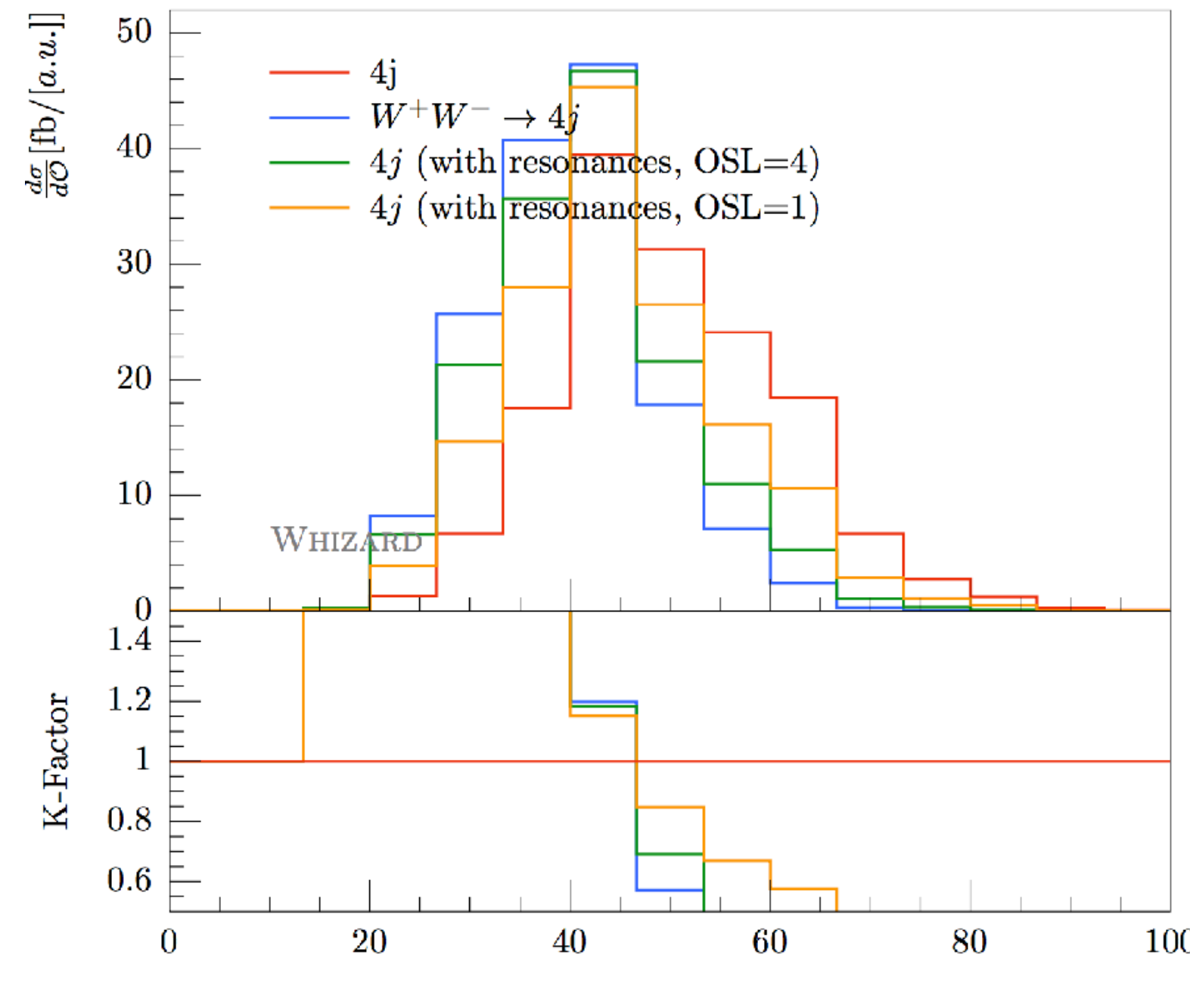
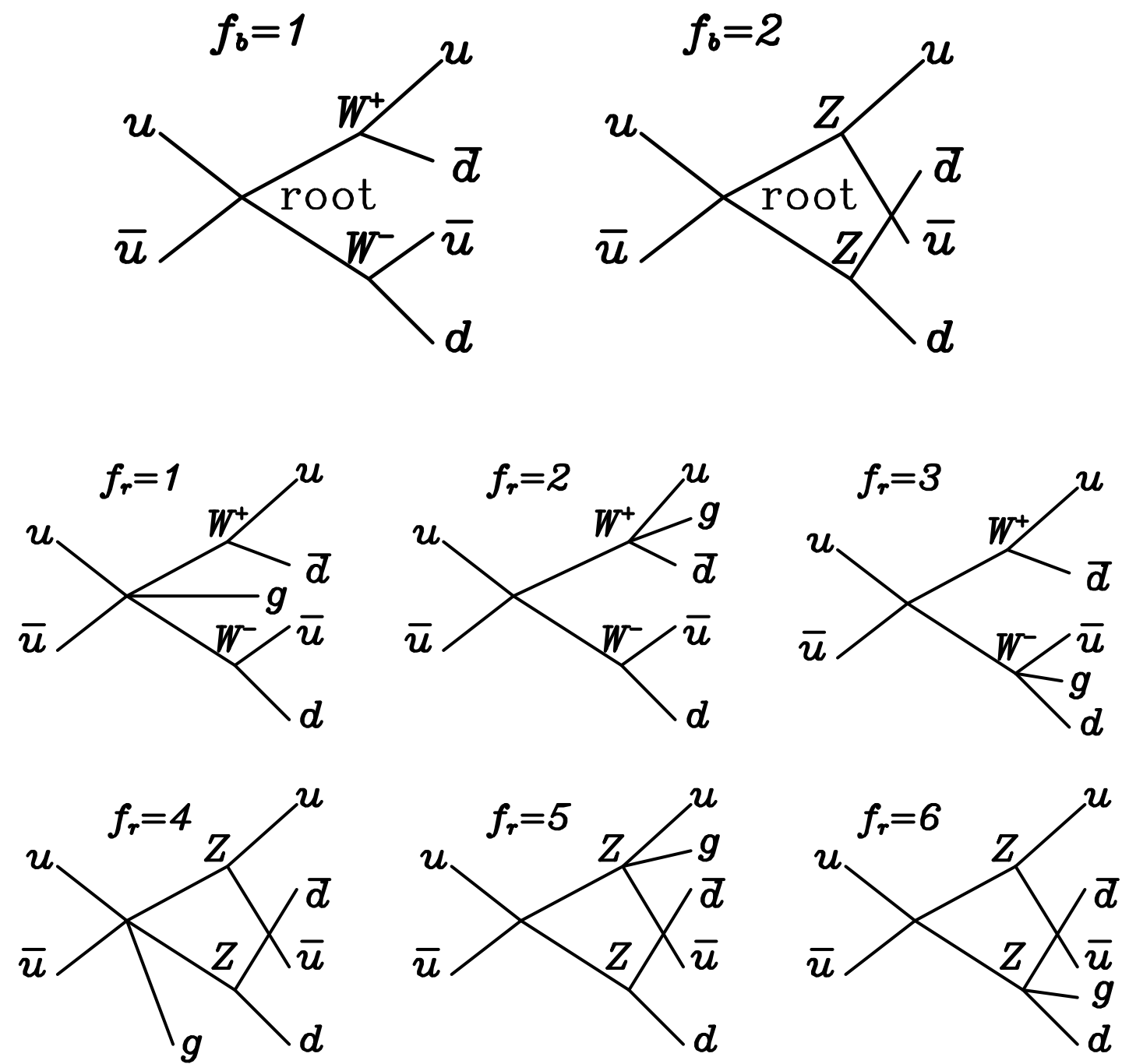
↳ Talk by Melissa van Beekveld / Pier Monni

- Ongoing work towards NNLL showers, sub-leading color (FCC = full color correlations)
- NLO matching automated, different approaches, different error estimates;
- NNLO matching still process-dependent; also does not yet preserve NNLL accuracy
- Elephant in the room: fragmentation  $\Rightarrow$  no paradigm shift/quantum leap in last 30 years

Gigantic clean data sets from Z pole and above will necessitate new models / theory

# (Resonance) Matching to shower / hadronization

- ❖ **Problem:**  $e^+e^- \rightarrow jjjj$  not dominated by highest  $\alpha_s$  power, but by resonances
- ❖ **Solution:** proper merging w/ resonant subprocesses by resonance histories
- ❖ **MC generators allow to pass resonance history to Shower MC**





# Dedicated tools for special processes

**PACKED WITH PRECISION-MADE,  
MISSION-SPECIFIC TOOLS.**



**GRIP. PUNCH. ADJUST. DRIVE. WRENCH. PICK.  
SCRAPE. HAMMER. OH YEAH...AND CUT.**

# In memoriam: Staszek Jadach



Stanisław ("Staszek") Jadach, 1943 — 2023

**RAPIDITY GENERATOR FOR MONTE-CARLO CALCULATIONS  
OF CYLINDRICAL PHASE SPACE**

**S. JADACH**

*Institute of Physics, Jagellonian University, Cracow, Poland*

Received 1 November 1974

Potentially a severe impact on the development of LEP legacy Monte Carlos, YFS-style tools (the whole KKMC, YFS-WW/ZZ, Photos, Tauola, BHLumi/BHWide !

Important rôle of Belle 2 program: active usage of many of these programs!



Bhabha cross sect. depends on detector acceptance angles

$$\sigma_{Bh} \simeq 4\pi\alpha^2 \left( \frac{1}{t_{\min}} - \frac{1}{t_{\max}} \right) = 4\pi\alpha^2 \left( \frac{t_{\max} - t_{\min}}{\bar{t}^2} \right), \quad \bar{t} = \sqrt{t_{\min} t_{\max}}$$

Machine	$\theta_{\min} \div \theta_{\max}$ [mrad]	$\sqrt{s}$ [GeV]	$\bar{t}/s \simeq \bar{\theta}^2/4$	$\sqrt{\bar{t}}$ [GeV]
LEP	28 ÷ 50	$M_Z$	$3.5 \times 10^{-4}$	1.70
FCCee	64 ÷ 86	$M_Z$	$13.7 \times 10^{-4}$	3.37
FCCee	64 ÷ 86	240	$13.7 \times 10^{-4}$	8.9
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ILC	31 ÷ 77	500	$6.0 \times 10^{-4}$	12.2
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[Maciej Skrzypek; Brussels Topical Workshop]



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Current BHLUMI precision forecast for FCCee			
Type of correction / Error	$M_Z$ (2019) [1]	240 GeV	350 GeV [2]
(a) Photonic $\mathcal{O}(L_e\alpha^2)$	0.027%	0.032%	0.033%
(b) Photonic $\mathcal{O}(L_e^3\alpha^3)$	0.015%	0.026%	0.028%
(c) Vacuum polariz.	0.009%	0.020%	0.022%
(d) Light pairs	0.010%	0.015%	0.015%
(e) Z and s-channel $\gamma$ exchange	0.09%	0.25% (0.034%)	0.5% (0.07%)
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(g) Technical Precision	[0.027%]		
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(c) Vacuum polariz.	$0.6 \times 10^{-4}$	$1.0 \times 10^{-4}$	$1.1 \times 10^{-4}$
(d) Light pairs	$0.5 \times 10^{-4}$	$0.4 \times 10^{-4}$	$0.4 \times 10^{-4}$
(e) Z and s-channel $\gamma$ exch.	$0.1 \times 10^{-4}$	$1.0 \times 10^{-4(*)}$	$1.0 \times 10^{-4(*)}$
(f) Up-down interference	$0.1 \times 10^{-4}$	$0.09 \times 10^{-4}$	$0.1 \times 10^{-4}$
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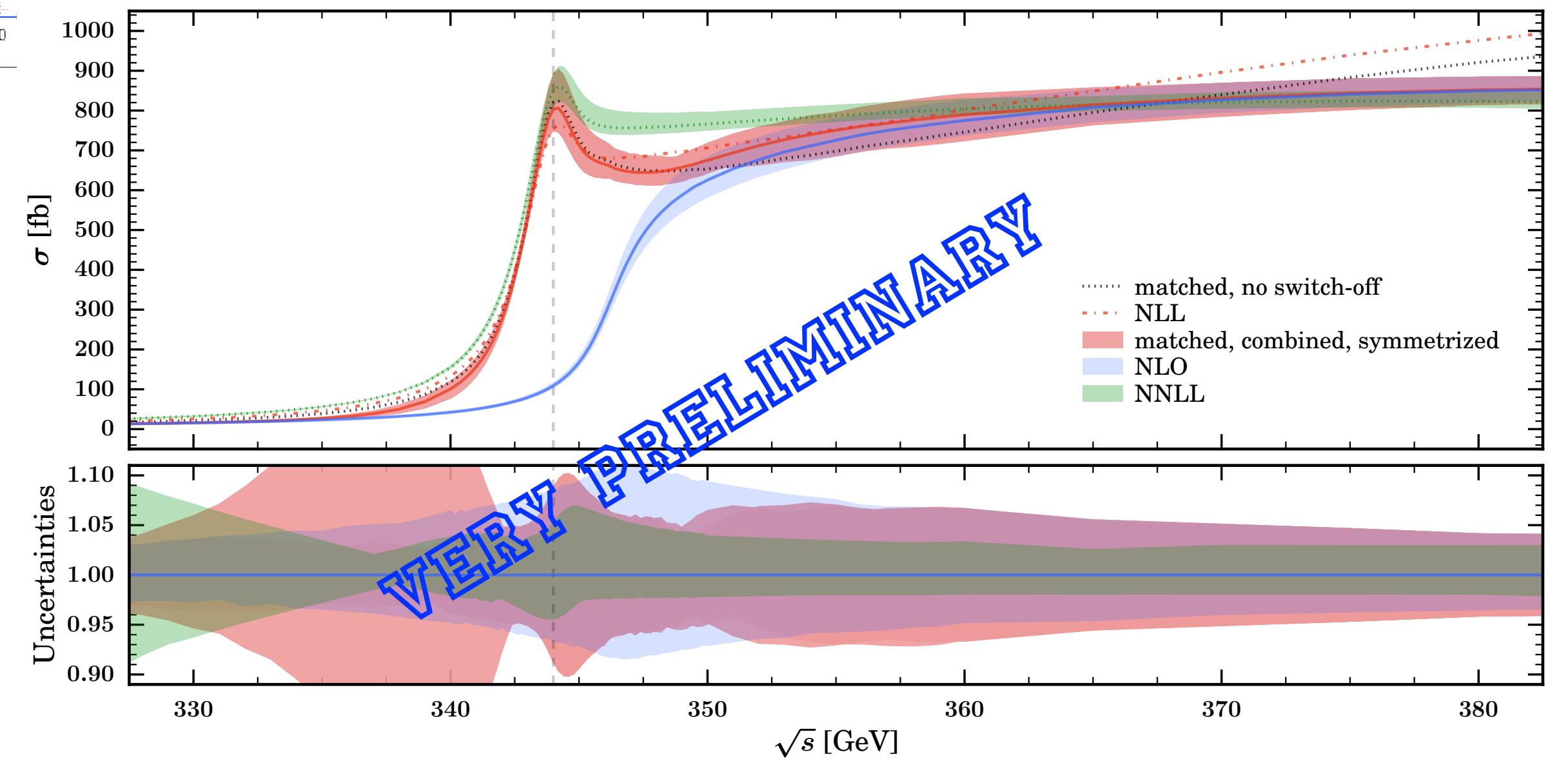
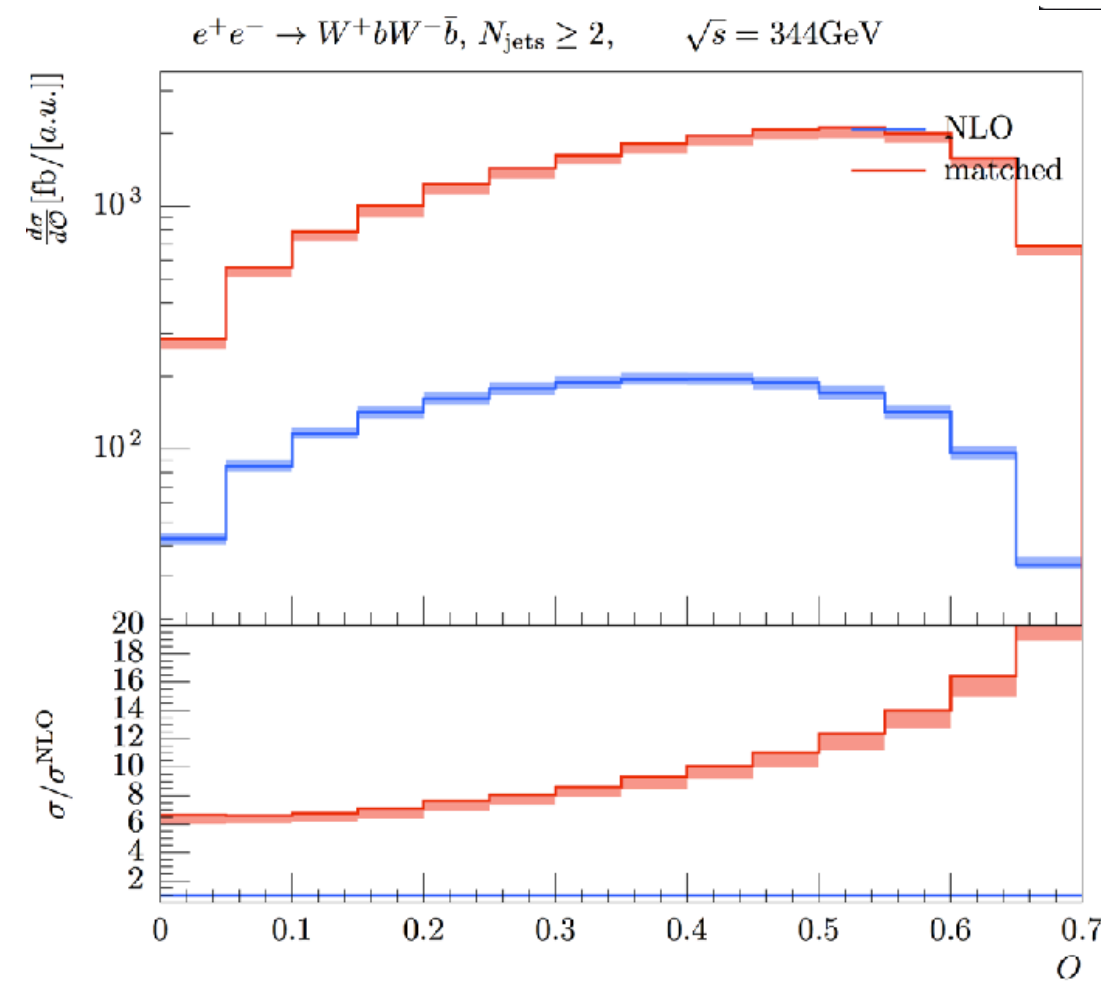
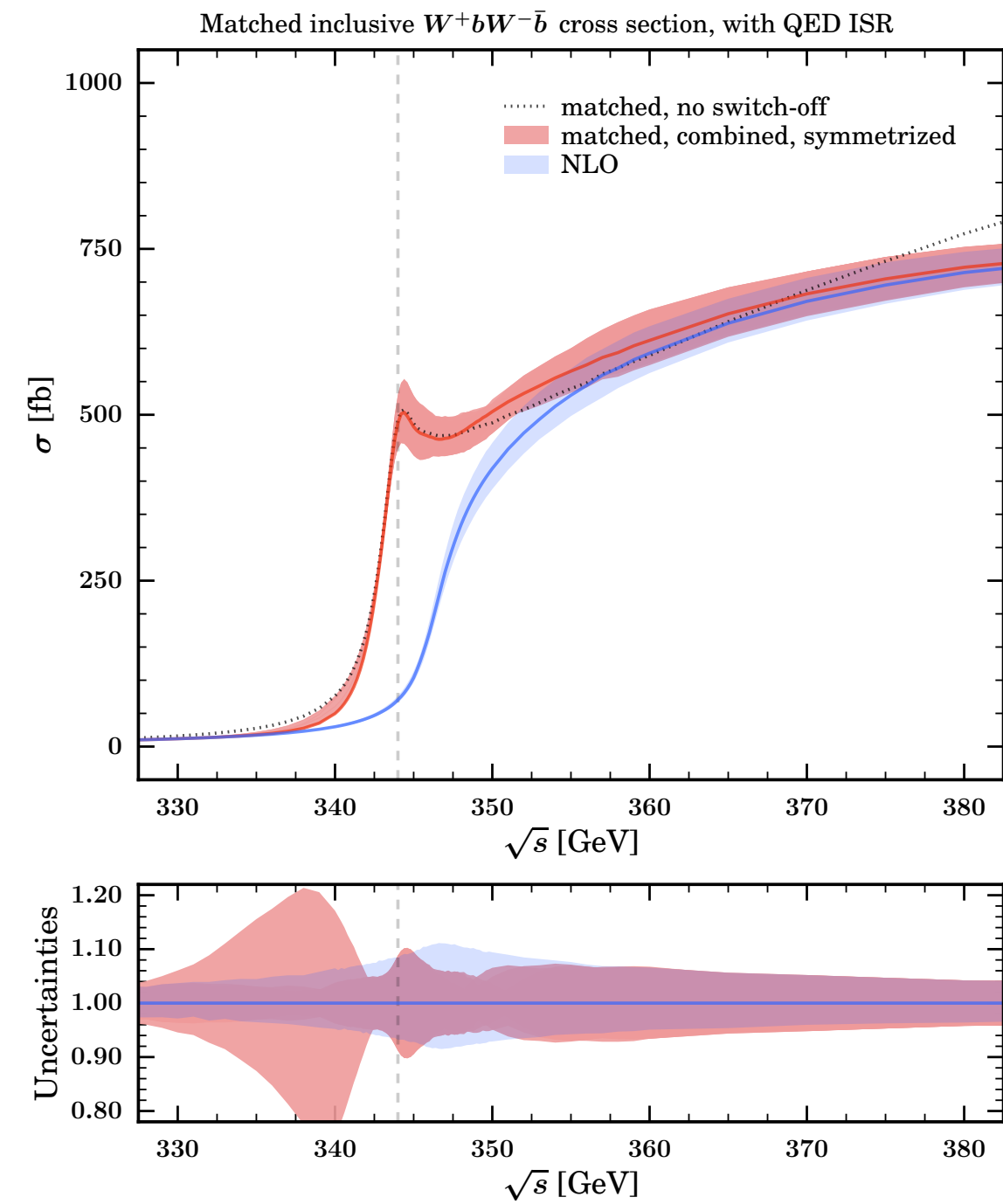
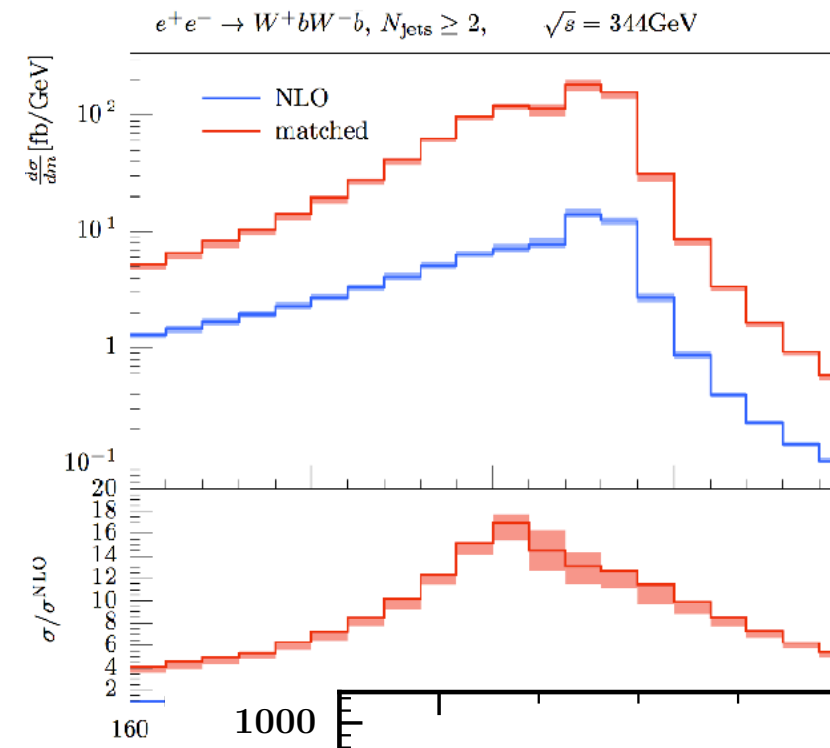
- Technical precision needs 2nd code: BHLumi vs. BabaYaga (NNLO in hard process possible)
- Major ingredients: hadronic vacuum polarization, EW corrections, light fermion pairs
- Inclusion of 4f, 4f +  $\gamma$ , 5f, 6f backgrounds necessary at matrix element level





- Special implementations for  $WW / tt$  threshold (resummation)
- Differential distributions at top threshold, systematics
- Exclusive Top threshold NLL-NLO QCD matched available
- Recent improvement in axial form factor matching
- Technical issues (person power)
- Improvement needed (e.g. shower matching)

no cuts $\sigma_{\text{tot}}$ [fb]	$\sqrt{s} = 200$ GeV		$\sqrt{s} = 500$ GeV	
	Born	best	Born	best
YFSWW3	659.64(07)	622.71(19)	261.377(34)	279.086(97)
RACOONWW	659.51(12)	621.06(14)	261.400(70)	280.149(86)
(Y-R)/Y	0.02(2)%	0.27(4)%	-0.01(3)%	-0.38(5)%
bare cuts $\sigma_{\text{tot}}$ [fb]	$\sqrt{s} = 200$ GeV		$\sqrt{s} = 500$ GeV	
	Born	best	Born	best
YFSWW3	627.18(07)	592.68(19)	181.507(33)	197.933(84)
RACOONWW	627.22(12)	590.94(14)	181.507(63)	198.696(76)
(Y-R)/Y	-0.01(2)%	0.29(4)%	0.00(4)%	-0.39(6)%



# BSM Modelling in Simulation

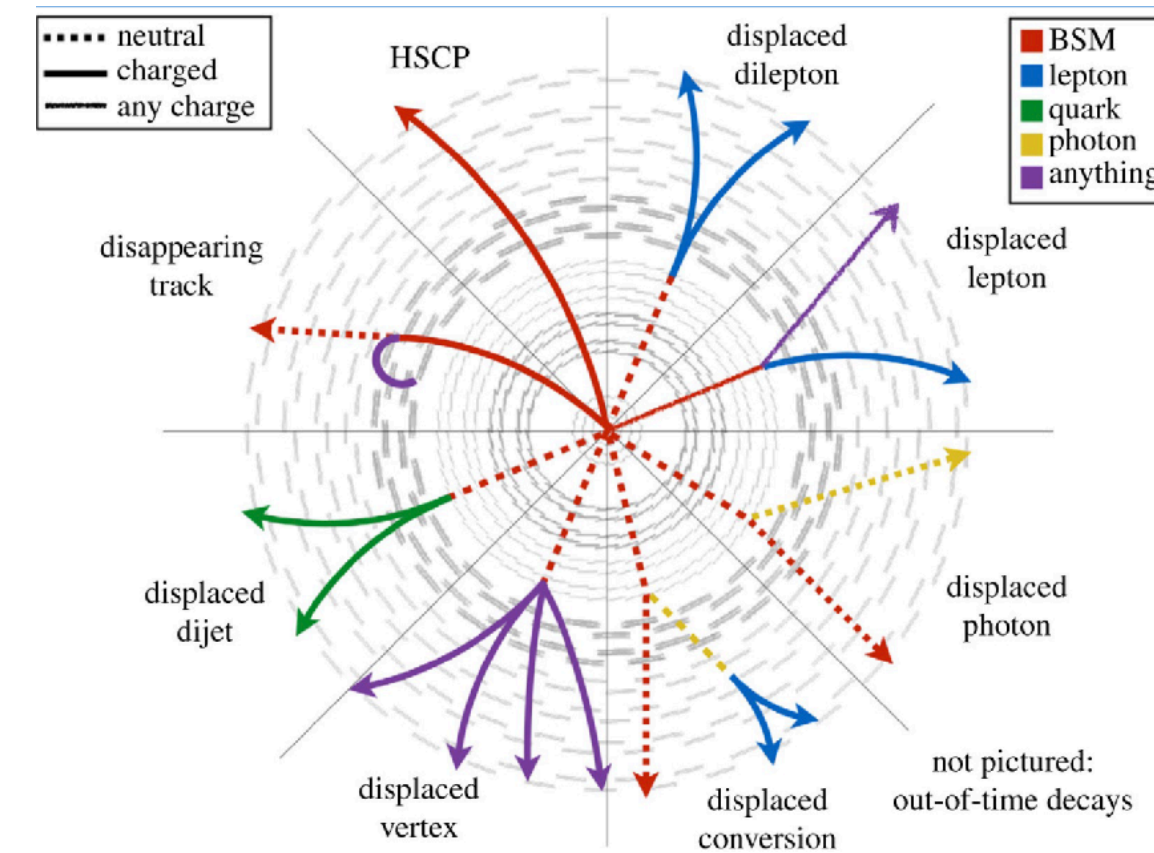




# BSM Models: UFO magic

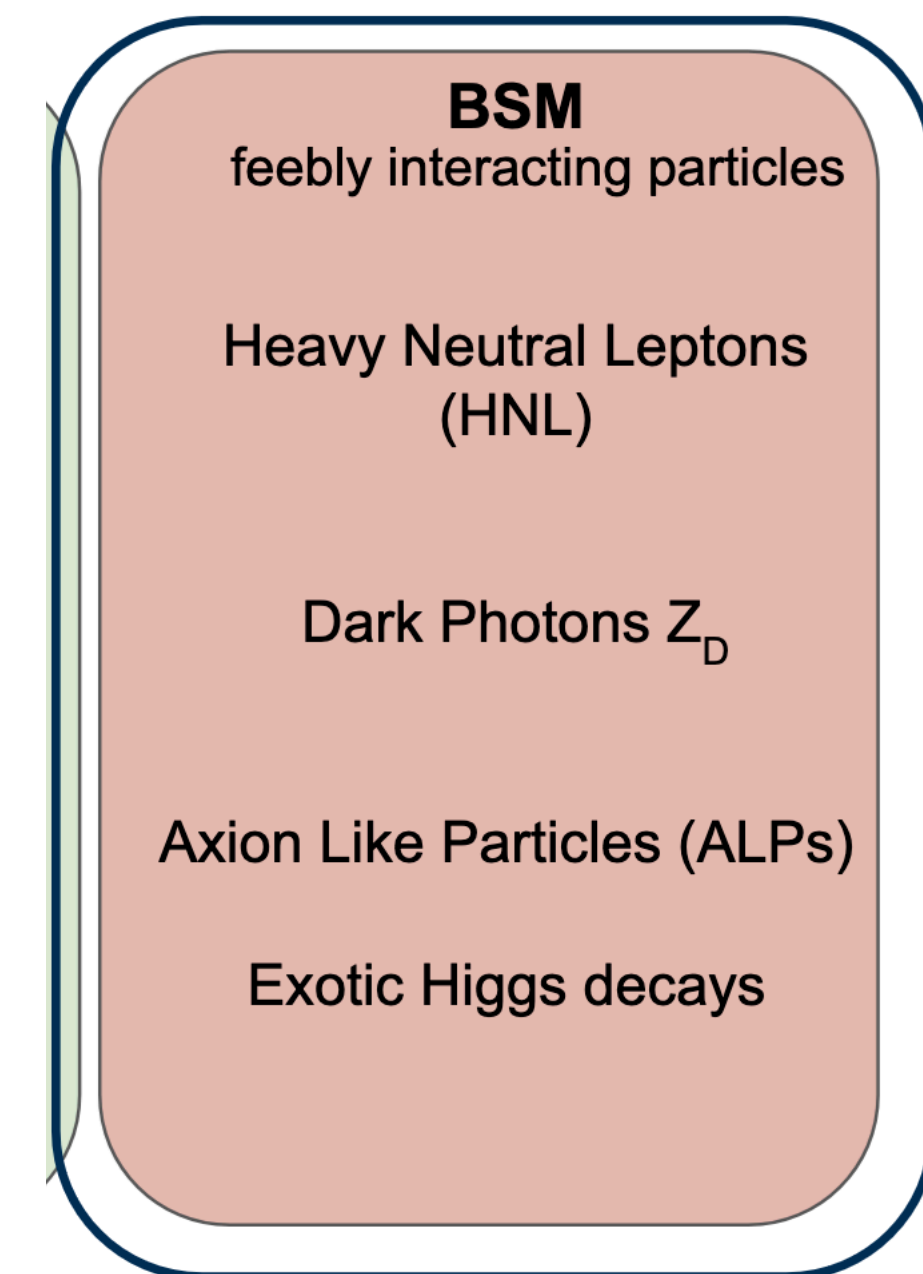
- BSM models from Lagrangian level tools (LanHEP, SARAH, FeynRules)
- Transferred to MC generator via UFO format: v1 [1108.2040](#) v2: [2304.09883](#)
- Allows for all Lagrangian-based BSM models

- Spin 0, 1/2, 1, 3/2, 2 supported (some 3/2, 2 features missing in some MC)
- Majorana fermions and fermion-number violating vertices
- 5-, 6-, 7-, 8-, ... point vertices (optimization for code generation pending)
- Arbitrary Lorentz structures in vertices (especially "full" SMEFT / HEFT)
- Keeping track of the order of insertions
- Customized propagators
- Exotic colored objects (sextets, decuplets, epsilon structures)
- (S)LHA-style input files from spectrum generators to MC generators (scans!)
- Automated calculations of widths (UFO side vs. MC generator side)
- Long-lived particles, displaced vertices, oscillations in decays (not all MCs yet)
- Lots of bug reports and constructive feedback from many different users
- LO fully supported, NLO (QCD) available on UFO side, but not all MCs



LLPs that are semi-stable or decay in the sub-detectors are predicted in a variety of BSM models:

- Heavy Neutral Leptons (HNLs)
- RPV SUSY
- Dark photons
- ALPs
- Dark sector models



# Conclusions & Outlook



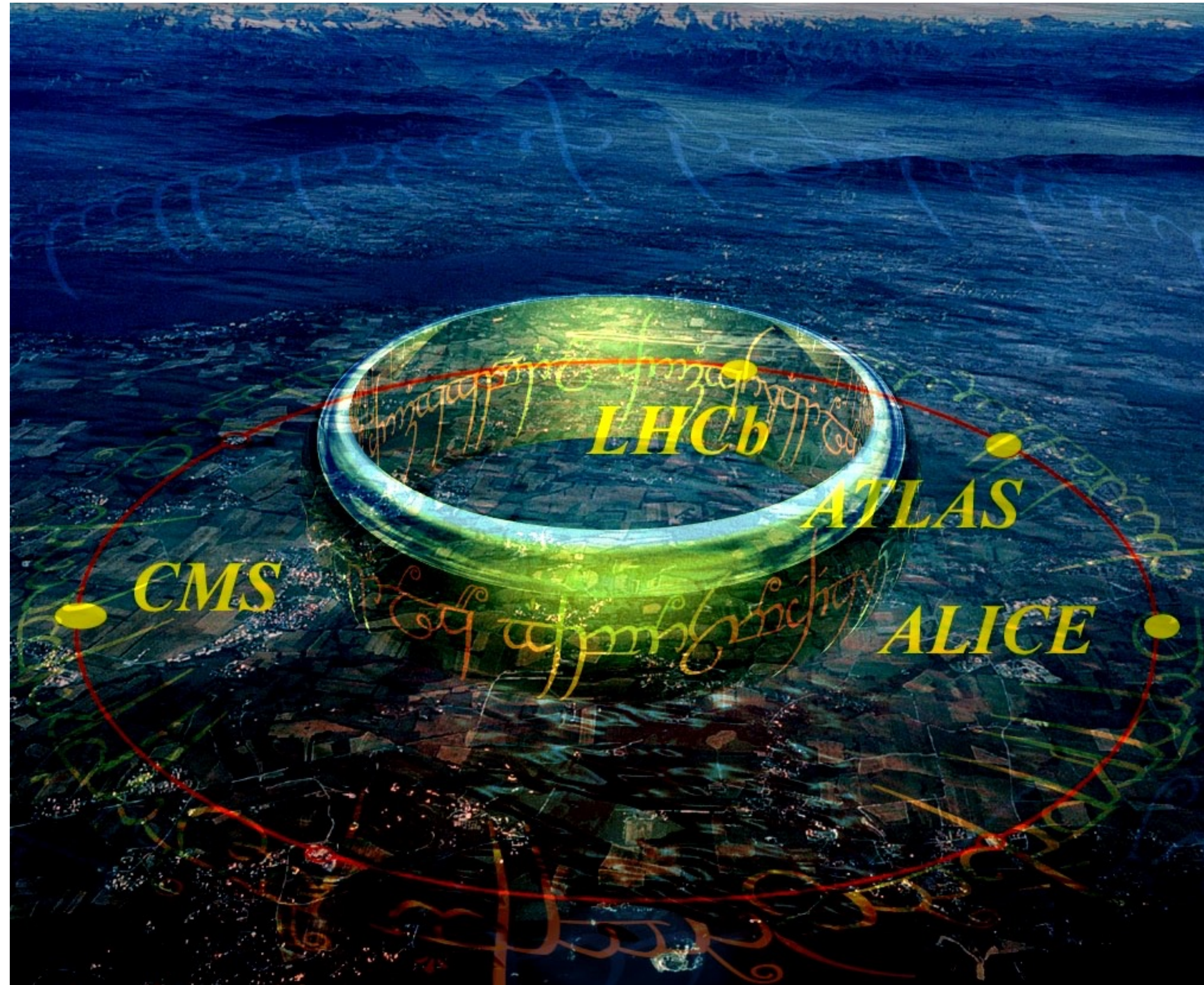
- Monte-Carlo event generators implement *all* necessary SM and BSM physics
- Modularity and redundancy of codes very important
- Fixed-order NLO QCD+EW for SM and NLO QCD BSM under control (mostly)
- First attempts to go to NNLO for QED (with certain caveats)
- LL/NLL ePDF in collinear factorization vs. YFS soft/eikonal factorization
- Matching prescriptions for exclusive photon radiation
- Different focus in different generators: no *a priori* best strategy for QED (and EW) corrections
- More studies, test cases and benchmarks needed: also 2nd and 3rd implementations important!
- Technical aspects: crossing angle, polarization (density matrices), beam spectra, event formats ....
- Will depend a lot on support on young researchers/theorists working
- Also need for dedicated MCs, e.g. for luminosity measurement ( $e^+e^- \rightarrow e^+e^-, \gamma\gamma, W^+W^-, t\bar{t}$ )
- Not to forget: QCD showers + fragmentation [FCC-ee Z-pole will boost to a new precision!]







# One Ring To Find Them,





# One Ring To Rule Them Out





**B A C K U P**

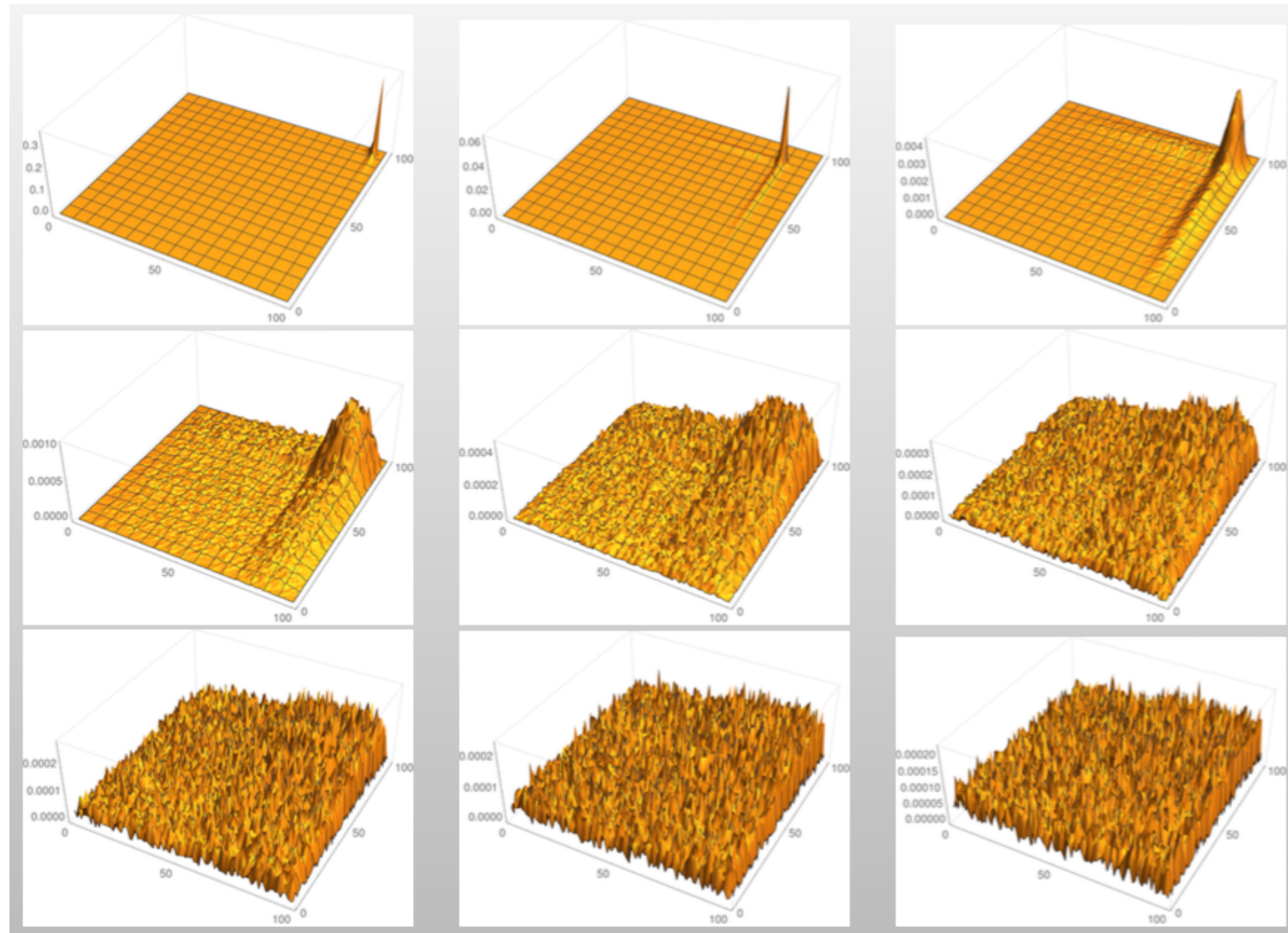
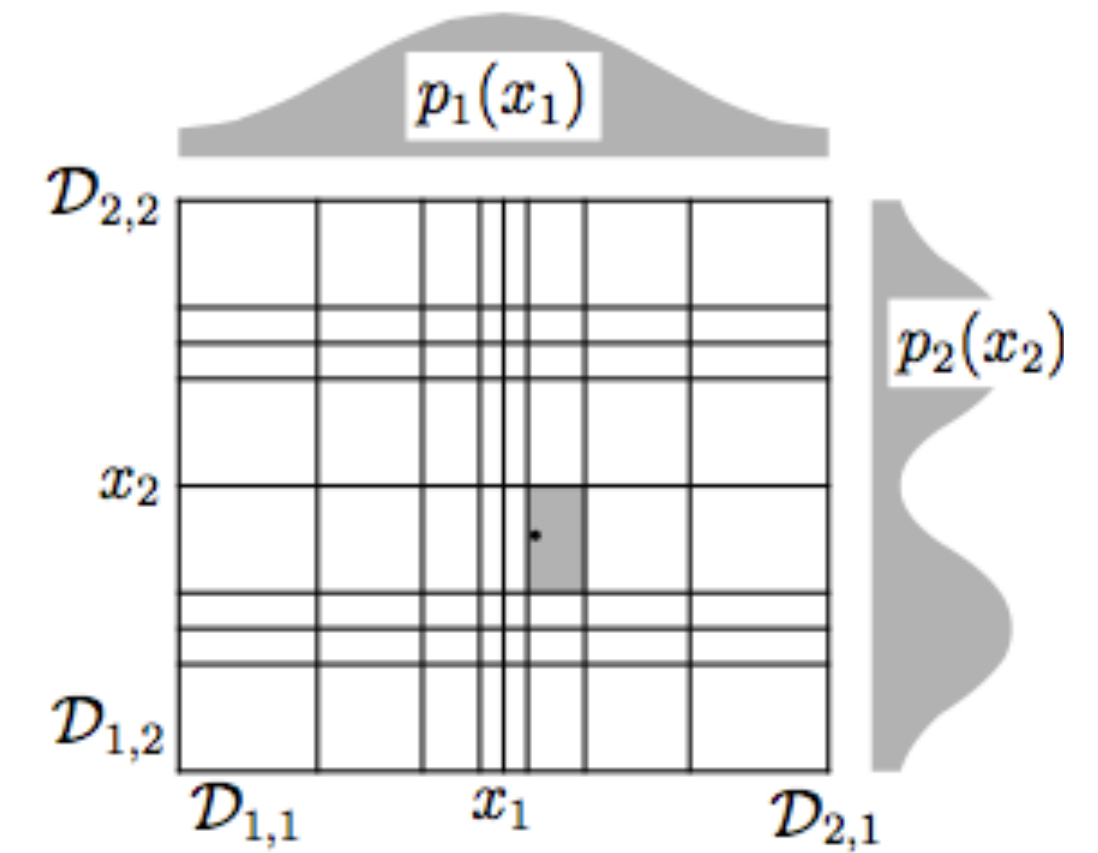


# Beam simulations (technical details)

CIRCE2 algorithm T. Ohl, 1996, 2005

↳ Talk by Thorsten Ohl 06/2023: <https://indico.cern.ch/event/1266492/>

- Adapt **2D factorized variable width histogram** to steep part of distribution
- Smooth correlated fluctuations with moderate **Gaussian filter** [suppresses artifacts from limited GuineaPig statistics]
- Smooth **continuum/boundary bins separately** [avoid artificial beam energy spread]



(171,306 GuineaPig events in 10,000 bins)

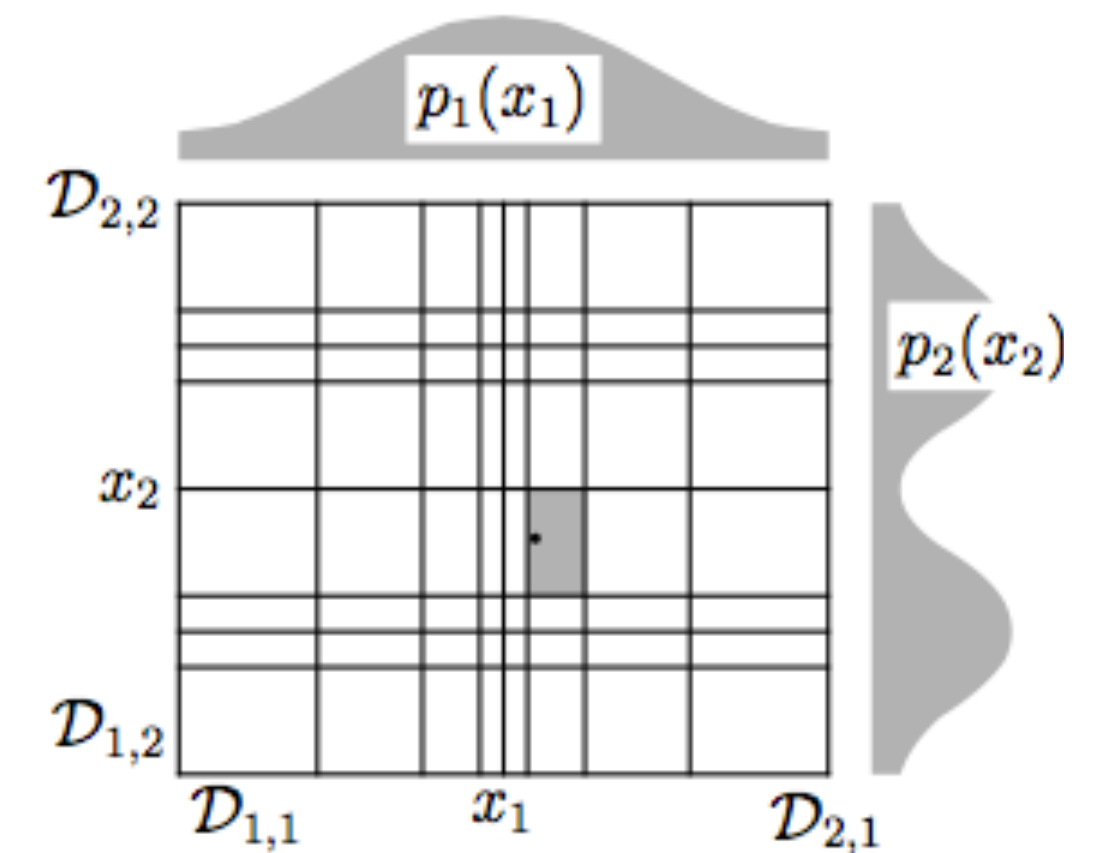


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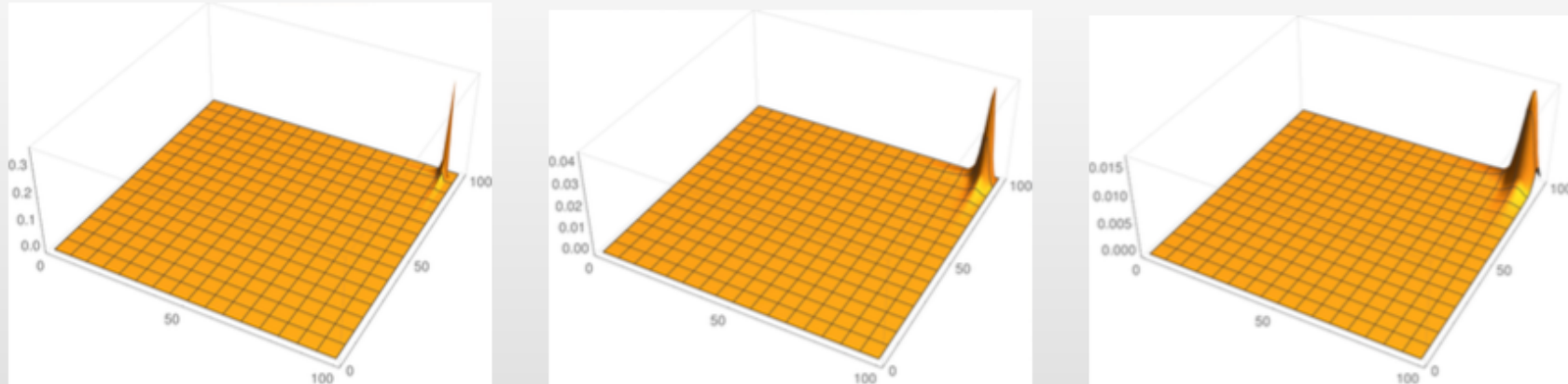
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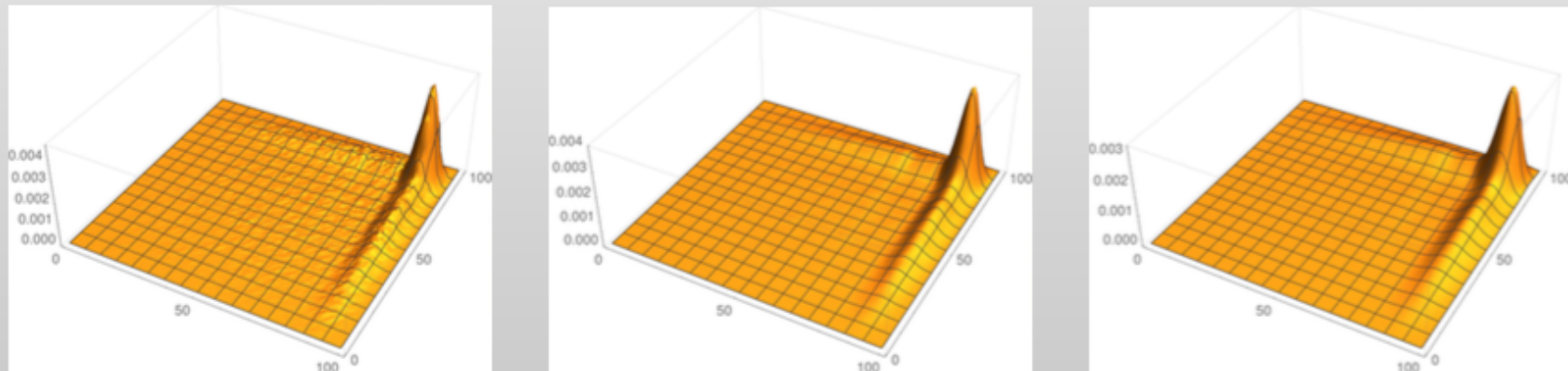
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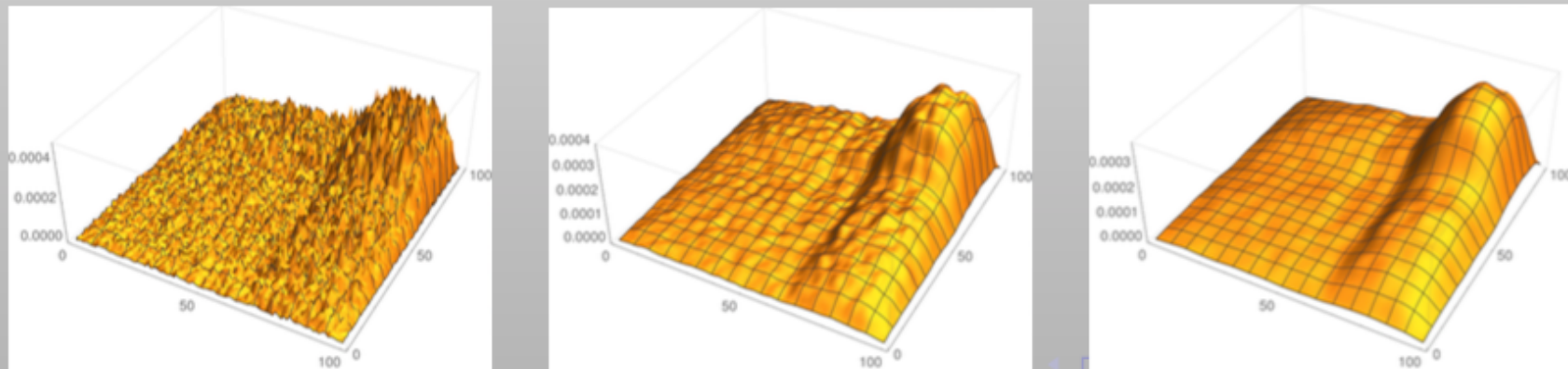
▶ **iterations = 0** and **smooth = 0, 3, 5:**



▶ **iterations = 2** and **smooth = 0, 3, 5:**



▶ **iterations = 4** and **smooth = 0, 3, 5:**



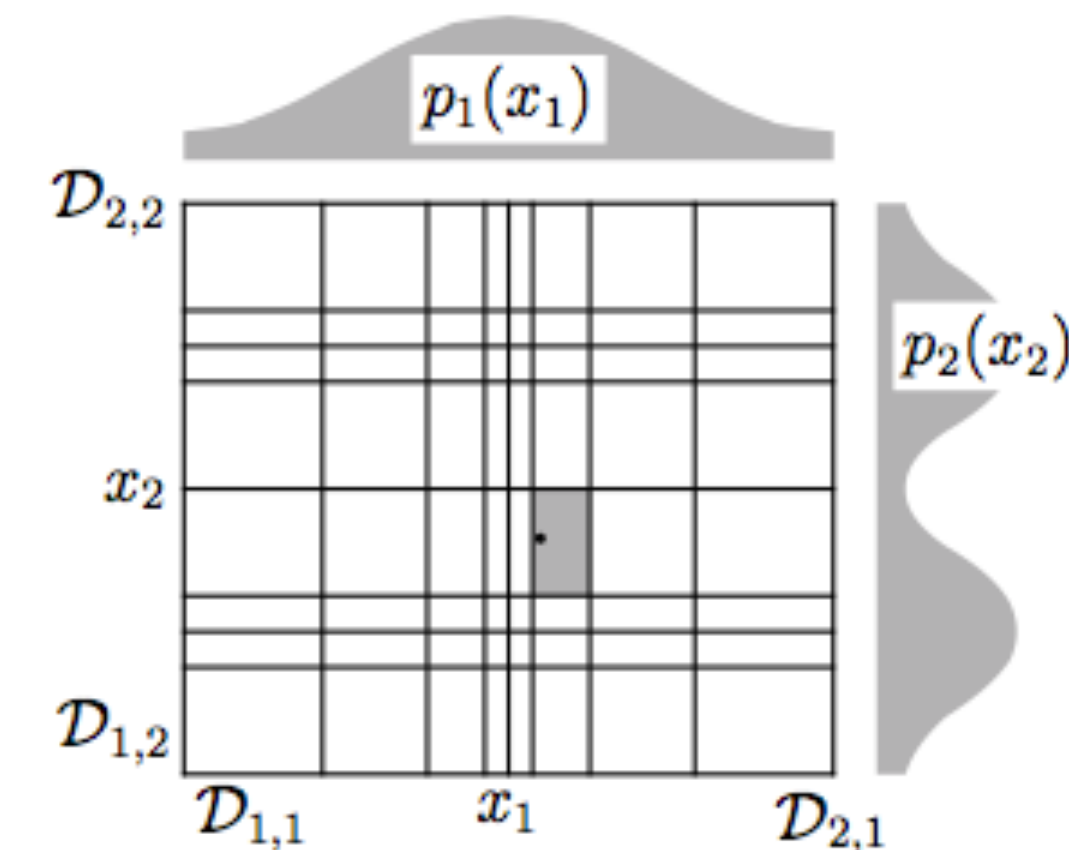


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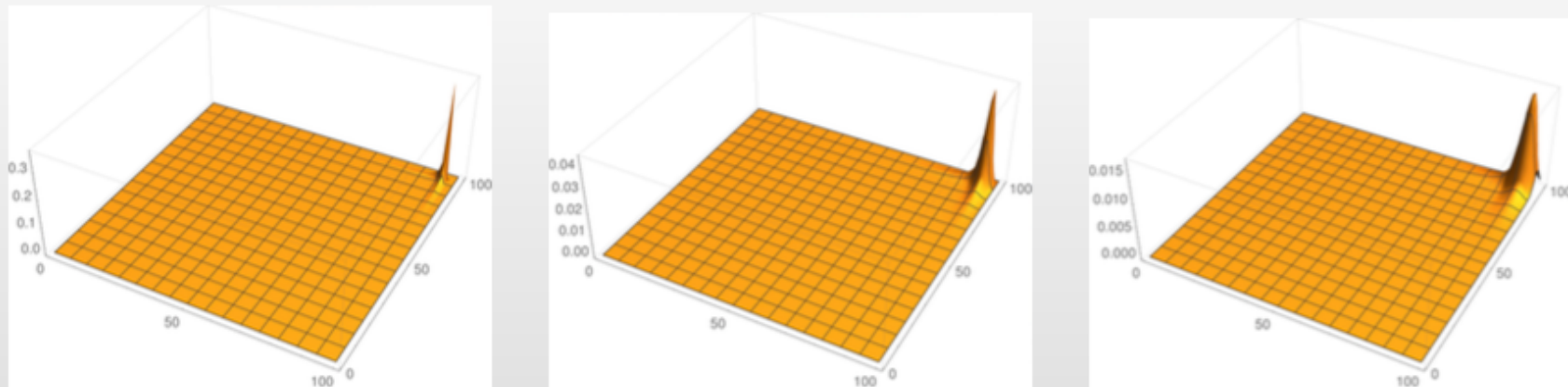
CIRCE2 algorithm T. Ohl, 1996, 2005

↳ Talk by Thorsten Ohl 06/2023: <https://indico.cern.ch/event/1266492/>

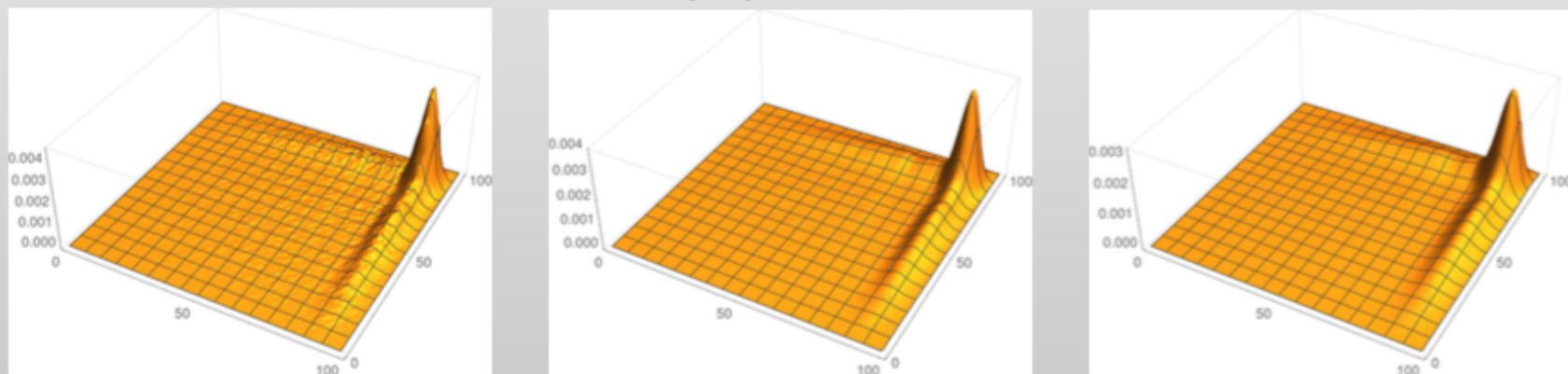
- Adapt **2D factorized variable width histogram** to steep part of distribution
- Smooth correlated fluctuations with moderate **Gaussian filter** [suppresses artifacts from limited GuineaPig statistics]
- Smooth **continuum/boundary bins separately** [avoid artificial beam energy spread]



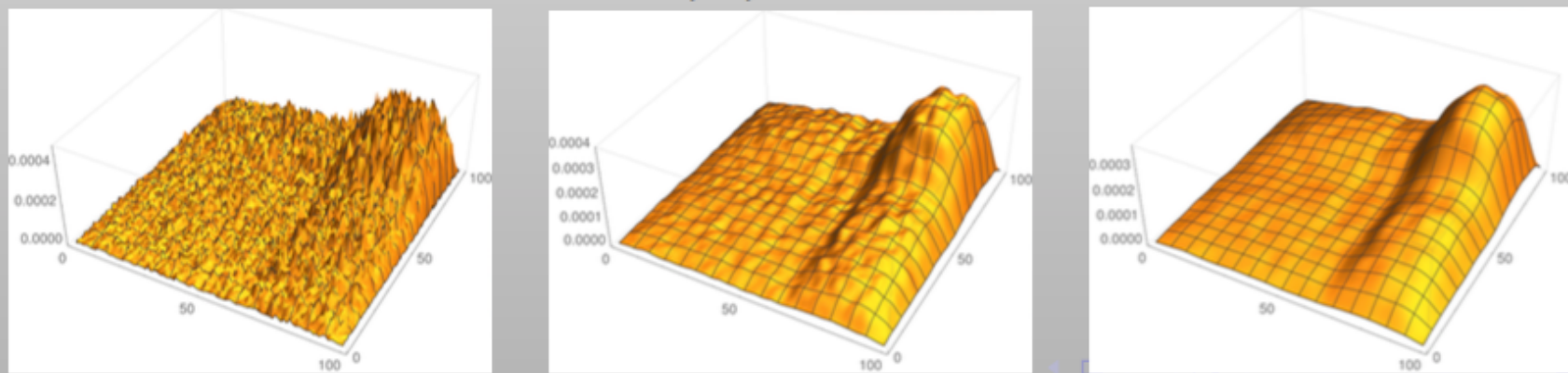
► **iterations = 0** and **smooth = 0, 3, 5:**



► **iterations = 2** and **smooth = 0, 3, 5:**



► **iterations = 4** and **smooth = 0, 3, 5:**



## 1. Run Guinea-Pig++ with

```
do_lumi=7; num_lumi=100000000; num_lumi_eg=100000000; num_lumi_gg=100000000;
```

to produce lumi. [eg] [eg].out with  $(E_1, E_2)$  pairs.

[Large event numbers, as Guinea-Pig++ will produce only a small fraction!]

## 2. Run circe2\_tool.opt with steering file

```
{ file="ilc500/beams.circe" # to be loaded by WHIZARD
  { design="ILC" roots=500 bins=100 scale=250 # E in [0,1]
    { pid/1=electron pid/2=positron pol=0 # unpolarized e-/e+
      events="ilc500/lumi.ee.out" columns=2 # <= Guinea-Pig
      lumi = 1564.763360 # <= Guinea-Pig
      iterations = 10 # adapting bins
      smooth = 5 [0,1) [0,1) # Gaussian filter 5 bins
      smooth = 5 [1] [0,1) smooth = 5 [0,1) [1] } } }
```

to produce correlated beam description

## 3. Run WHIZARD with SINDARIN input:

3 simulation options

```
beams = e1, E1 => circe2
$circe2_file = "ilc500.circe"
$circe2_design = "ILC"
?circe_polarized = false
```

1. Unpolarized simulation with unpol. spectra
2. Pol. simulation: unpol. spectra + pol. beams
3. Polarized spectrum with helicity luminosities