# MadGraph5_aMC@NLO for $e^+e^-$ collisions

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# MadGraph5_aMC@NLO

(Alwall et al. 1405.0301; Frederix et al. 1804.10017)

**https://github.com/mg5amcnlo/mg5amcnlo**

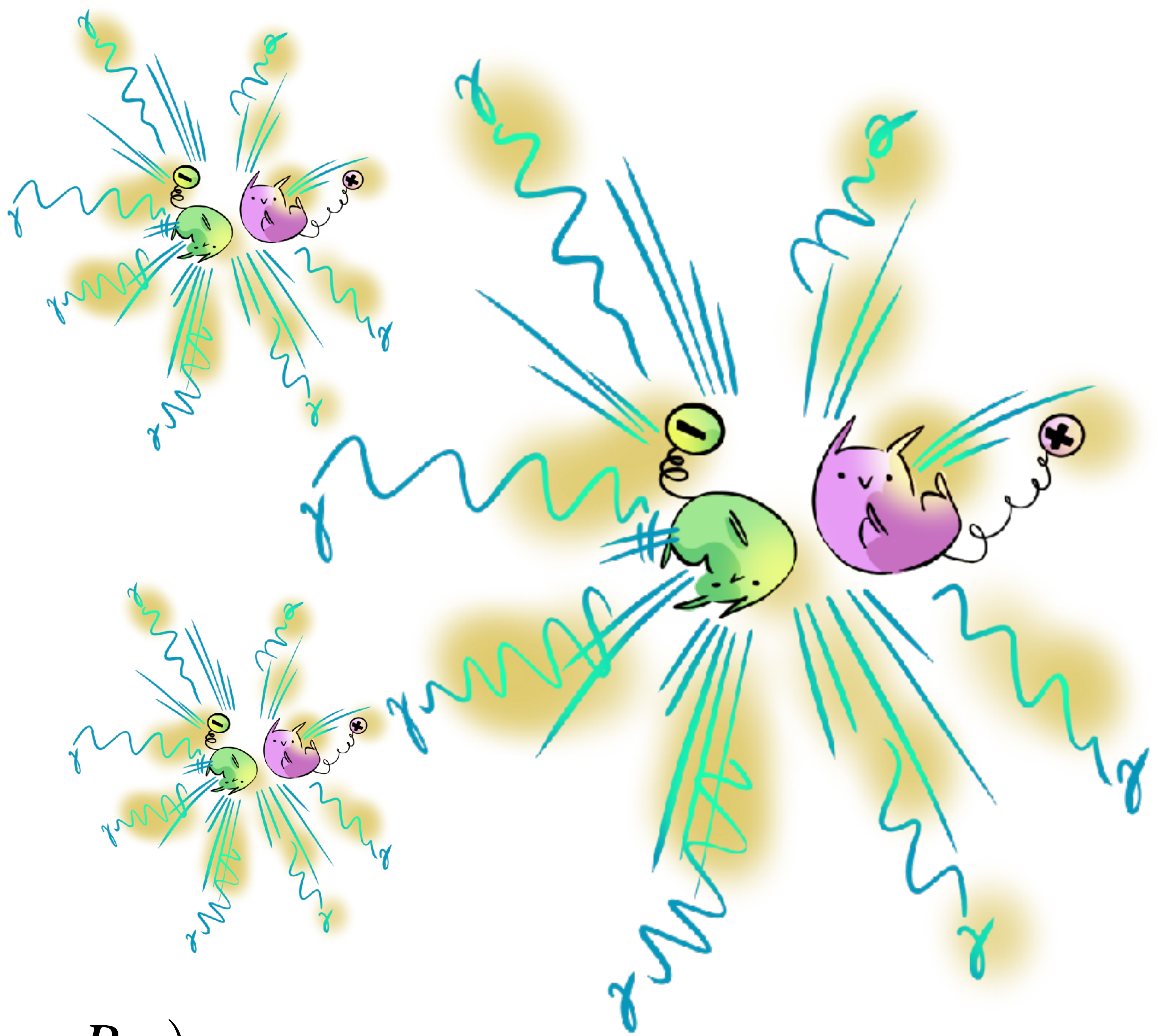
- Automatic computation of LO- and NLO-accurate cross sections (both in the QCD and in the EW coupling, and mixed), working with any Feynrules-generated model
- Several options for QCD matching and merging both LO and NLO.

- Extension to lepton collisions available **from release v3.5.0**: documented in (Frixione, Mattelaer, Zaro, Zhao 2108.10261) [ISR LL+LO EW] and (Bertone, Cacciari, Frixione, Stagnitto, Zaro, Zhao 2207.03265) [ISR NLL+NLO EW]

- In this talk, I will provide some technical details regarding the implementation of ISR NLL and NLO EW, and then present some physical results.

In a collinear factorisation picture, radiation from the initial-state can be factorised in:

- Machine-dependent collective effects: beam-beam interactions, aka ***beamstrahlung***
- Machine-independent universal effects: ***initial-state radiation (ISR)***



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$$d\Sigma_{e^+e^-}(P_{e^+},P_{e^-}) = \sum_{kl}\int dy_+ dy_-\, \underline{\mathcal{B}_{kl}(y_+,y_-)}\, d\sigma_{kl}(y_+P_{e^+},y_-P_{e^-})$$

$$d\sigma_{kl}(p_k,p_l) = \sum_{ij}\int dz_+ dz_+\, \underline{\Gamma_{i/k}(z_+,\mu,m)\,\Gamma_{j/l}(z_-,\mu,m)}\, d\hat{\sigma}_{ij}(z_+p_k,z_-p_l,\mu)$$

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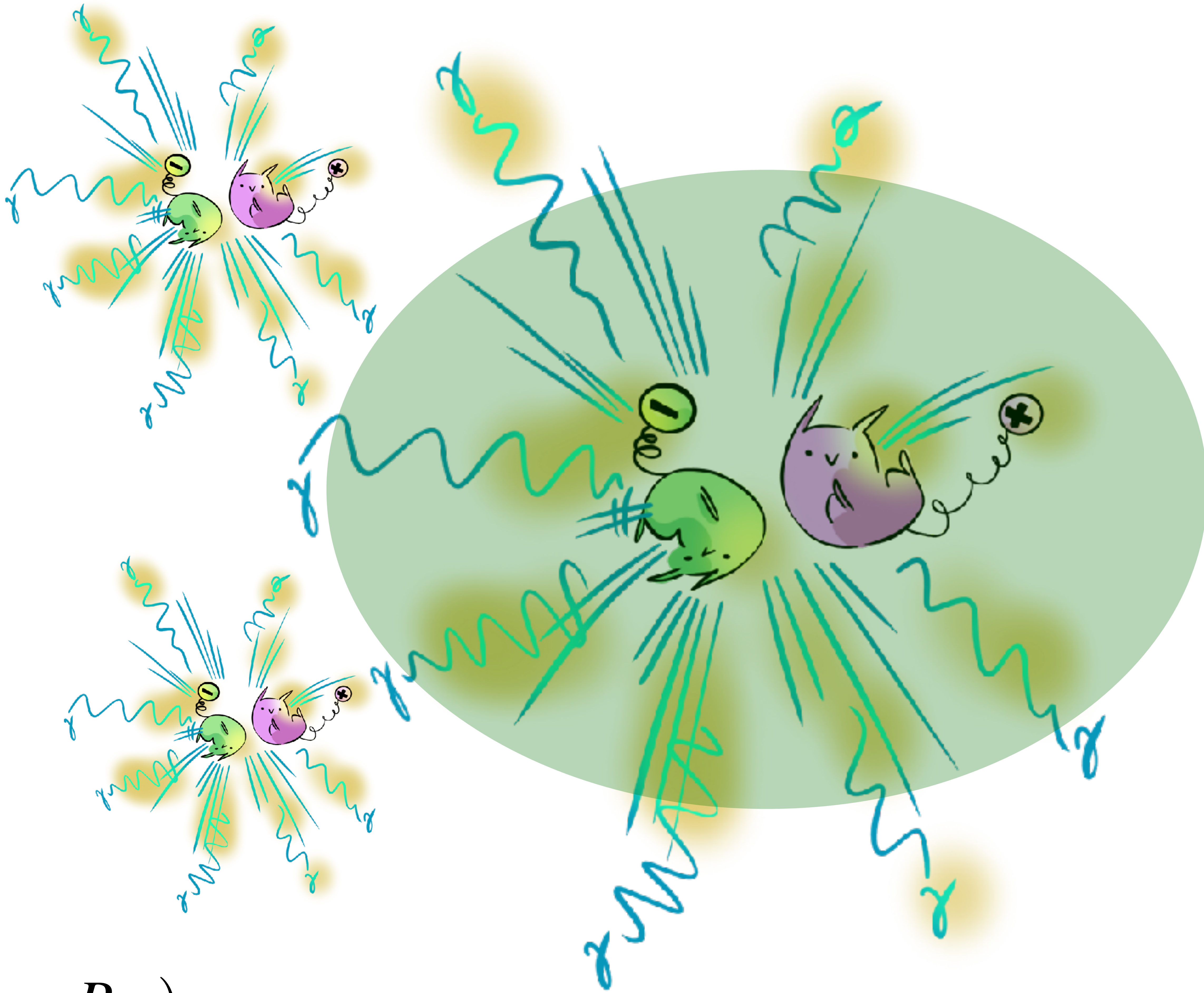



$$d\Sigma_{e^+e^-}(P_{e^+},P_{e^-}) = \sum_{kl}\int dy_+ dy_-\, \mathcal{B}_{kl}(y_+,y_-)\, d\sigma_{kl}(y_+P_{e^+},y_-P_{e^-})$$

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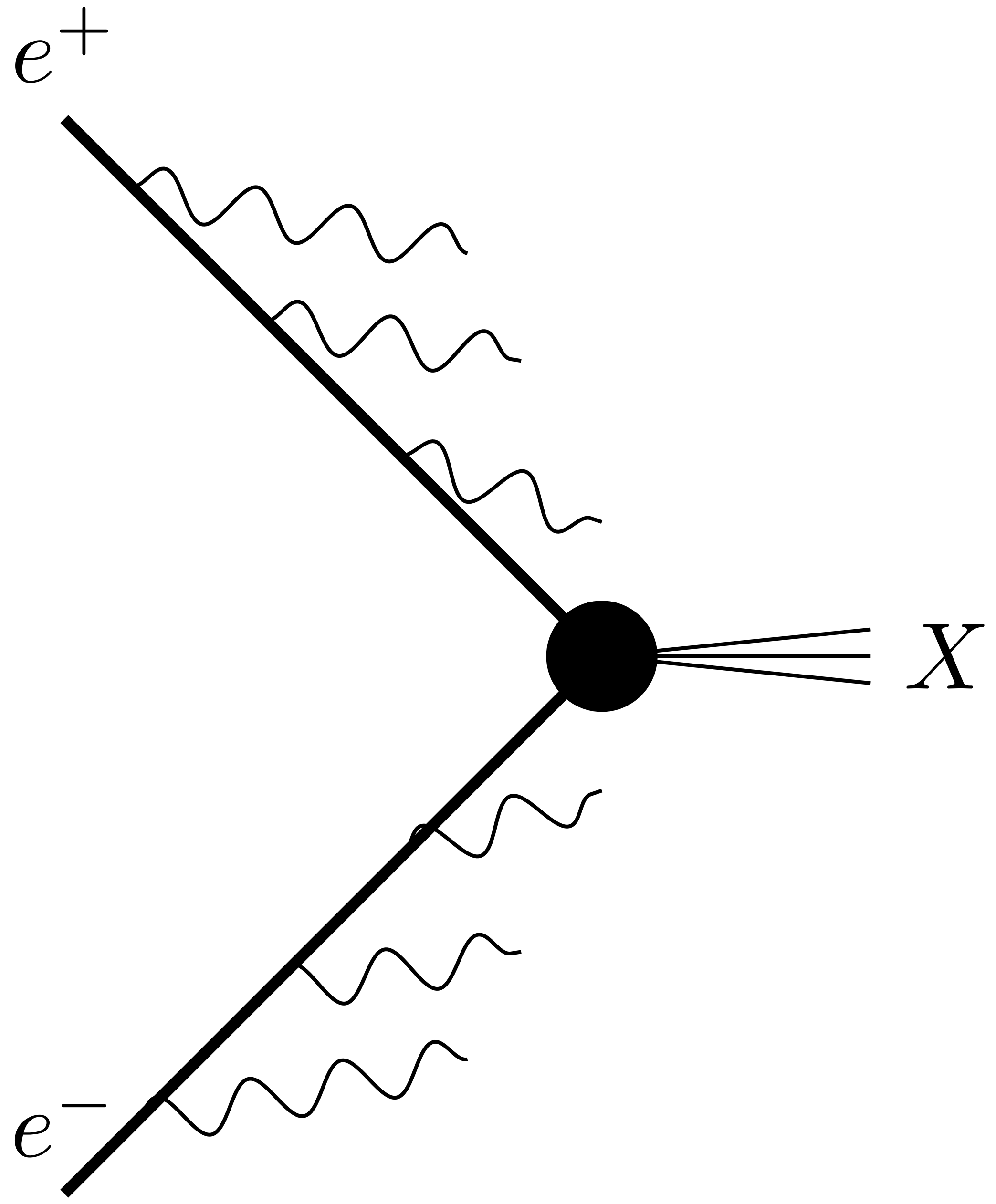
# Initial state radiation (ISR)

Presence in the cross section $d\sigma_{e^+e^-}$ of **potentially large logarithms** due to **collinear photon emissions** in the initial state




$$\simeq \alpha^b \sum_{n=0}^{\infty} \alpha^n \left( c_0^{(n)} + c_1^{(n)} L + \ldots + c_n^{(n)} L^n \right) \quad L = \log\left(\frac{Q^2}{m_e^2}\right)$$

$b$: power of the $\alpha$ in the Born process, $m_e$: electron mass
$Q^2$: typical hard scale of the process e.g. c.o.m. energy squared $s$

Basically all precision observables at $e^+e^-$ colliders affected by ISR!

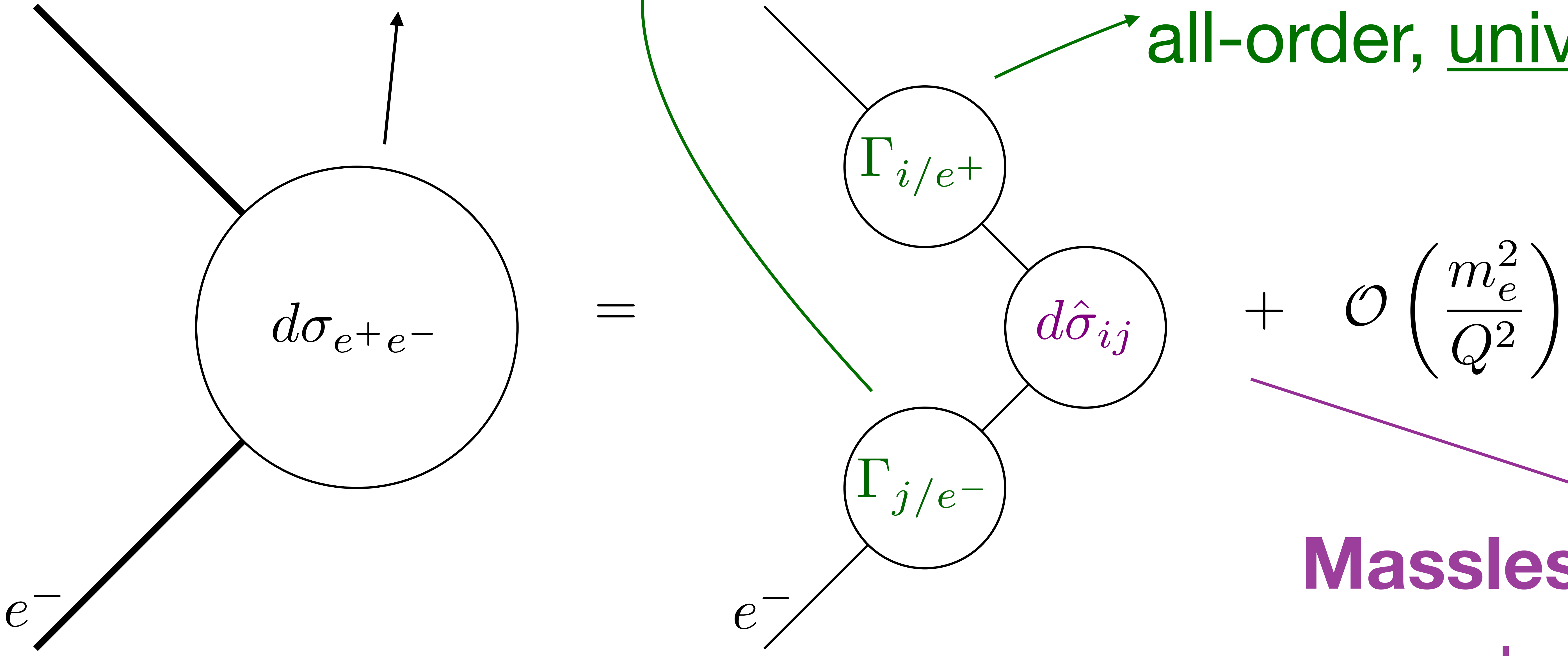
# ISR collinear factorisation

**Massive** particle-level cross section

QED Parton Distribution Functions (PDFs)

$\Gamma_{i/e^+}$ and $\Gamma_{j/e^-}$, *partons* $i, j = e^+, e^-, \gamma, \ldots$

all-order, universal, resumming $\log(Q^2/m_e^2)$




**leading log (LL):** $(\alpha\, L)^k$

**next-to-leading log (NLL):** $\alpha(\alpha\, L)^{k-1}$

...

**Massless** parton-level cross section $d\hat{\sigma}_{ij}$

computed order by order, process-specific

$$d\sigma_{e^+e^-} = \sum_{ij} \int dz_+ dz_- \, \Gamma_{i/e^+}(z_+, \mu^2, m_e^2)\, \Gamma_{j/e^-}(z_-, \mu^2, m_e^2)\, d\hat{\sigma}_{ij}(z_+ p_{e^+}, z_- p_{e^-}, \mu^2) + \mathcal{O}(m_e^2/Q^2)$$

# NLL-accurate QED PDFs

Frixione 1909.03886; Bertone, Cacciari, Frixione, Stagnitto 1911.12040; Frixione 2105.06688;
Bertone, Cacciari, Frixione, Stagnitto, Zaro, Zhao 2207.03265

- PDFs in **three renormalisation schemes** [ $\overline{\mathrm{MS}}$ (where $\alpha$ runs), $\alpha(m_Z)$ and $G_\mu$ (where $\alpha$ is fixed) ] and **two factorisation schemes** [ $\overline{\mathrm{MS}}$ and $\Delta$ (DIS-like, with NLO initial condition maximally simplified ]
- Solution with numerical evolution, plus a **switch to analytical expressions for** $z \to 1$, where the electron PDF $\Gamma_{e^-}$ features an integrable singularity.
- **Photon-initiated partonic contributions** (through the photon PDF $\Gamma_\gamma$) naturally included in the collinear framework at NLL.

Public code eMELA: https://github.com/gstagnit/eMELA

Runtime evaluation too slow → grids in LHAPDF format

Even with grids, eMELA always switches to the analytical solution for $z \to 1$

# Integrating over momentum fractions with ISR

QED PDFs diverge at large-$z$ (like QCD PDFs at small-$z$)

and this region is **never cut-off** (unlike small-$z$ region in QCD)

$$\Gamma_e(z) \to \frac{\hat{\Gamma}_e(z)}{(1-z)^{1-\beta}}$$

$\beta = 0.05$ for $Q \sim 100$ GeV

with $\hat{\Gamma}_e(z)$ at most with a log divergence

A **change of variable under integration** is the solution:

$$t = (1-z)^{1-\gamma} \to \mathrm{d}z\,\Gamma_e(z)\,\hat{\sigma}(z) = \frac{\mathrm{d}t}{1-\gamma}\left[\Gamma_e(z(t))\,(1-z(t))^{\gamma}\right]\hat{\sigma}(z(t))$$

Analytical knowledge around $z = 1$ **crucial** for numerical integration.

In a collinear factorisation picture, radiation from the initial-state can be factorised in:

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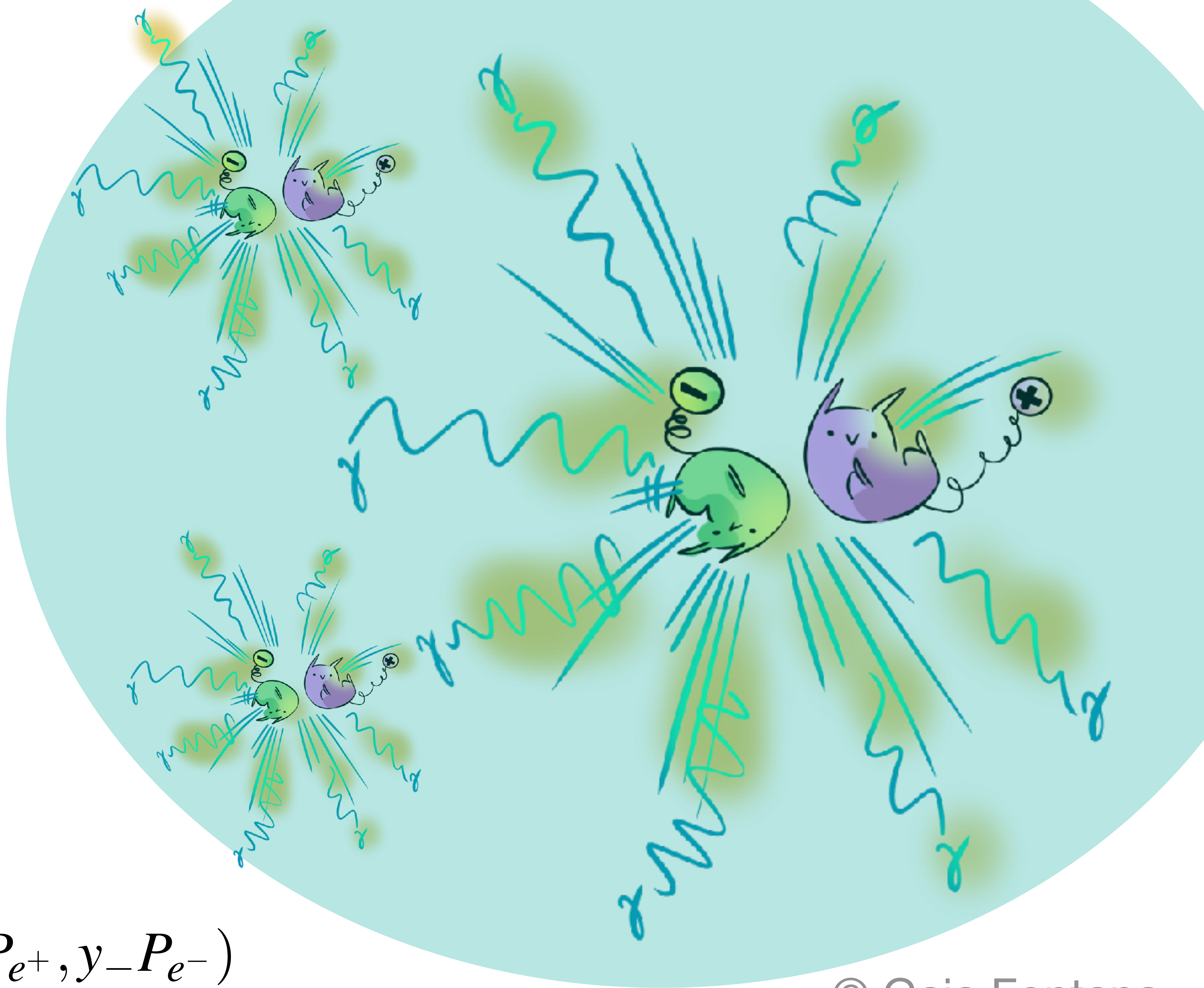

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$$d\Sigma_{e^+e^-}(P_{e^+}, P_{e^-}) = \sum_{kl} \int dy_+ dy_- \, \mathcal{B}_{kl}(y_+, y_-) \, d\sigma_{kl}(y_+ P_{e^+}, y_- P_{e^-})$$

$$d\sigma_{kl}(p_k, p_l) = \sum_{ij} \int dz_+ dz_+ \Gamma_{i/k}(z_+, \mu, m) \Gamma_{j/l}(z_-, \mu, m) \, d\hat{\sigma}_{ij}(z_+ p_k, z_- p_l, \mu)$$

# Beamstrahlung effects

Frixione, Mattelaer, Zaro, Zhao 2108.10261

$$d\Sigma_{e^+e^-}(P_{e^+}, P_{e^-}) = \sum_{kl} \int dy_+ dy_- \, \mathcal{B}_{kl}(y_+, y_-) \, d\sigma_{kl}(y_+ P_{e^+}, y_- P_{e^-})$$

$$\mathcal{B}_{kl}(y_+, y_-) \approx \sum_{n=1}^{N} b_{n,kl}^{(e^+)}(y_+) \, b_{n,kl}^{(e^-)}(y_-)$$

Parameters in $b$ determined through fit to GuineaPig simulations

$$d\Sigma_{e^+e^-}(P_{e^+}, P_{e^-}) = \sum_{n=1}^{N} \sum_{ijkl} \int dx_+ dx_- \, \phi_{i/k,n,kl}^{(e^+)}(x_+, \mu^2, m^2) \, \phi_{j/l,n,kl}^{(e^-)}(x_-, \mu^2, m^2) \times d\hat{\sigma}_{ij}(x_+ P_{e^+}, x_- P_{e^-}, \mu^2, m^2) \,,$$

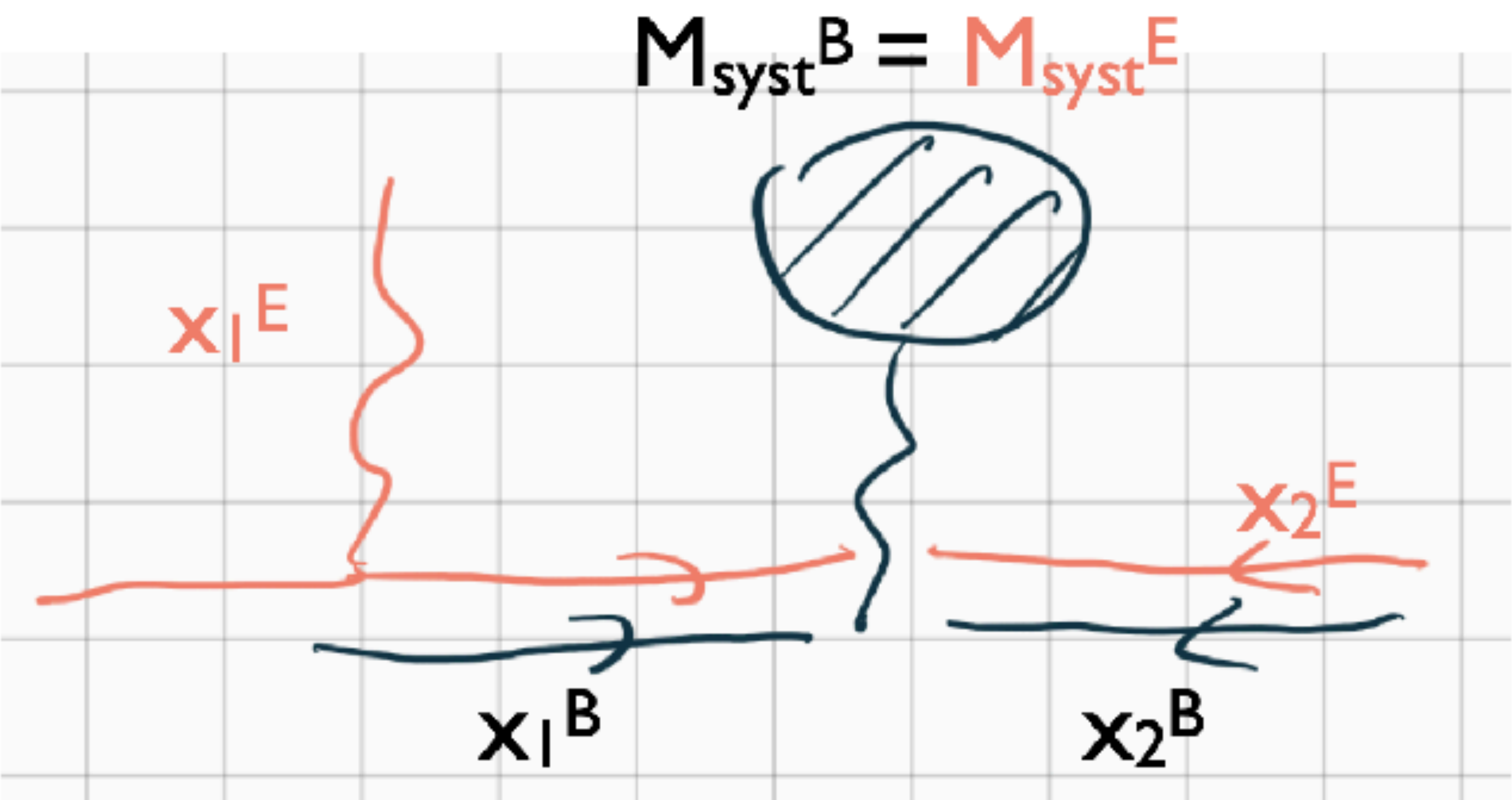
We can store in the grids also beamstrahlung!

$$\phi_{i/k,n,kl}^{(e^\pm)}(x, \mu^2, m^2) = \int dy \, dz \, \delta(x - yz) \, b_{n,kl}^{(e^\pm)}(y) \, \Gamma_{i/k}(z, \mu^2, m^2)$$

# Phase-space mapping in $\mathrm{d}\hat{\sigma}_{ij}$ at NLO EW

- NLO subtraction in MG5_aMC@NLO based on FKS formalism: phase-space partitioned into sectors, each with a different mapping involved.
- Strategy adopted for $pp$ collisions: preserving invariant mass and rapidity of the Born system between event (real emission) and counter-event (Born-like kinematics). These conditions are necessary for the matching with PS.
- For initial-state FKS sectors, this implies that event and counter-event have different Bjorken x's → **highly inefficient with electron ISR**!

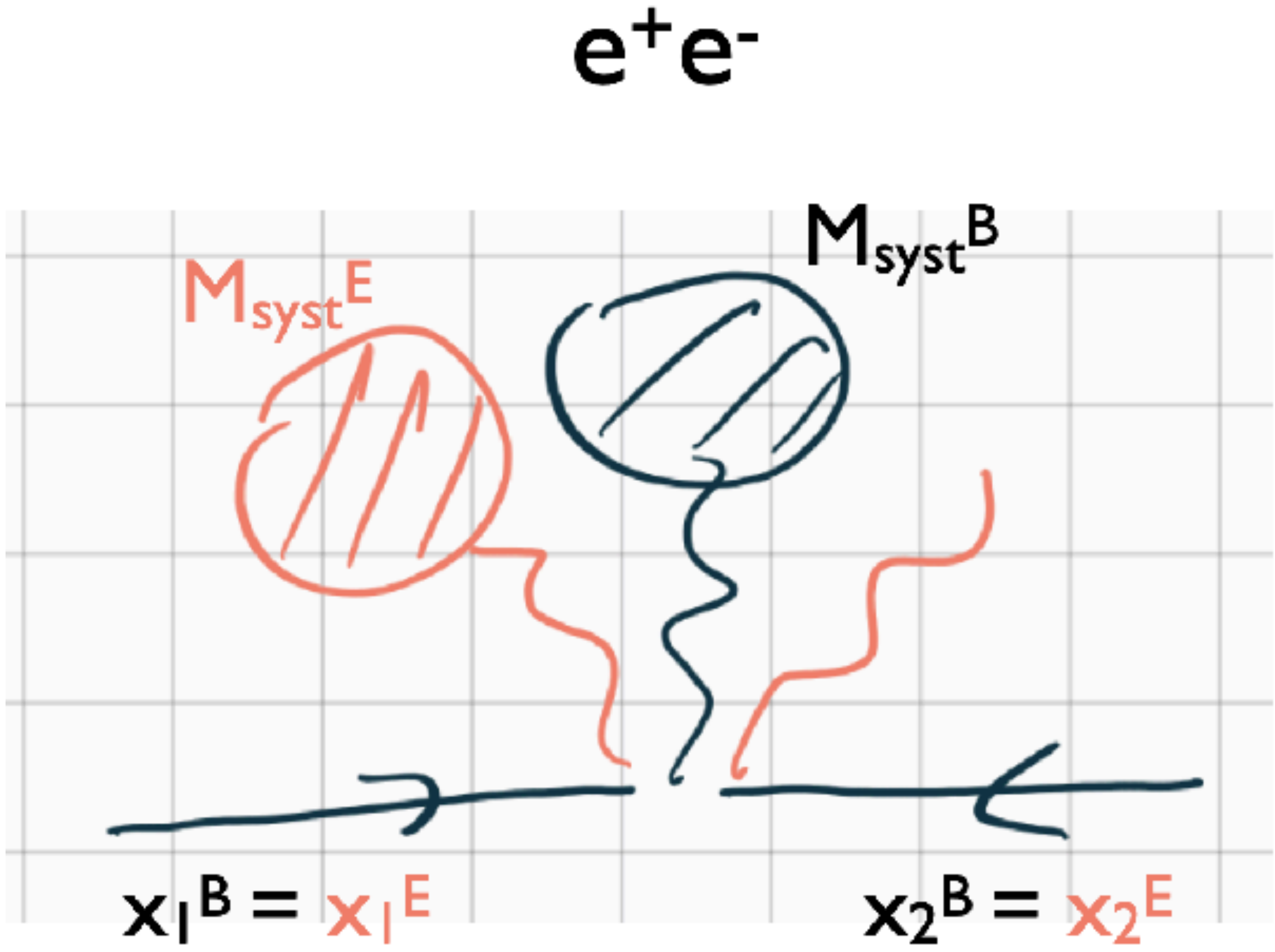
pp (event projection)




by Marco Zaro

# Phase-space mapping in $\mathrm{d}\hat{\sigma}_{ij}$ at NLO EW

- A new mapping, which preserves the Bjorken x's, has been designed: generate $k_i$ of real emission parton with energy fraction $\xi_i$; generate other momenta in c.m. frame with invariant mass $(1 - \xi_i)\,\hat{s}$; boost the momenta to recover total momentum conservation.
- The new mapping has made it possible to do NLO EW at e+e- colliders *almost* for any process.
- Exception e.g. processes proceeding via a resonating, pure s-channel (e.g. $e^+e^- \to \mu^+\mu^-$, with energy close to $m_Z$), where you want to preserve the mass of the Born system.




by Marco Zaro

# How to run $e^+e^-$ @ NLL+NLO EW (v3.5.0)

- Install eMELA: https://github.com/gstagnit/eMELA (provider for QED PDFs)
- Specify in the runcard the relevant PDF (different name implies LL/NLL, fact. scheme, ren. scheme, beamstrahlung eg. `NLL_DELTA_ALGMU` for PDF at NLL in Delta factorisation scheme and $G_\mu$ renormalisation scheme):

```
pdlabel = emela
lhaid = [LHAID of the chosen PDF set, as in pdfsets.index]
```

(Internally, MG5_aMC will take care of setting the finite part of the IR and UV counter-terms accordingly)

- Generate the process: `generate e+ e- > w+ w- [QED]`

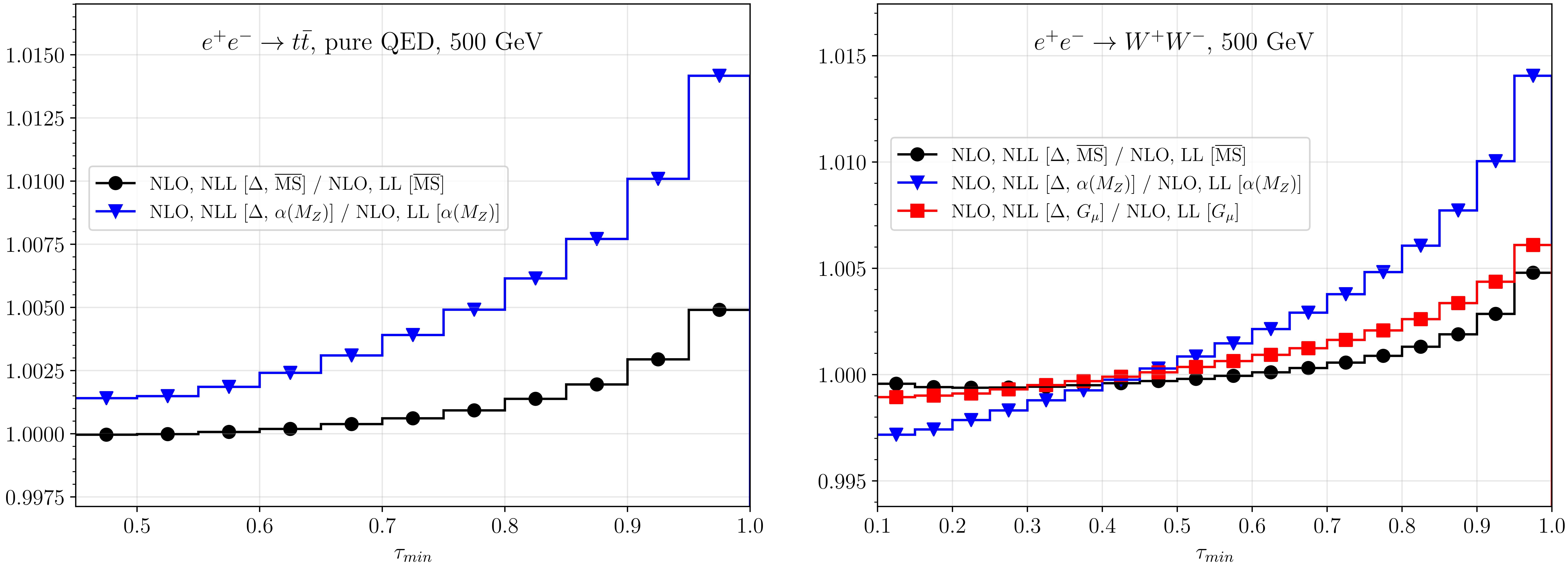
**More details and caveats here:**
https://answers.launchpad.net/mg5amcnlo/+faq/3324

# Some studies on physical cross sections

- From: [Bertone, Cacciari, Frixione, Stagnitto, Zaro, Zhao 2207.03265]
- Processes:
  - $e^+e^- \to q\bar{q}(\gamma)$ [pure QED, with real and virtual radiation limited to initial state]
  - $e^+e^- \to W^+W^-(X)$ [full EW]
  - $e^+e^- \to t\bar{t}(X)$ [full EW] and $e^+e^- \to t\bar{t}(X)$ [pure QED]
- $\mu = \sqrt{s} = 500$ GeV (qualitatively similar results in the range 50-500 GeV)
- We focus on the cumulative cross section:

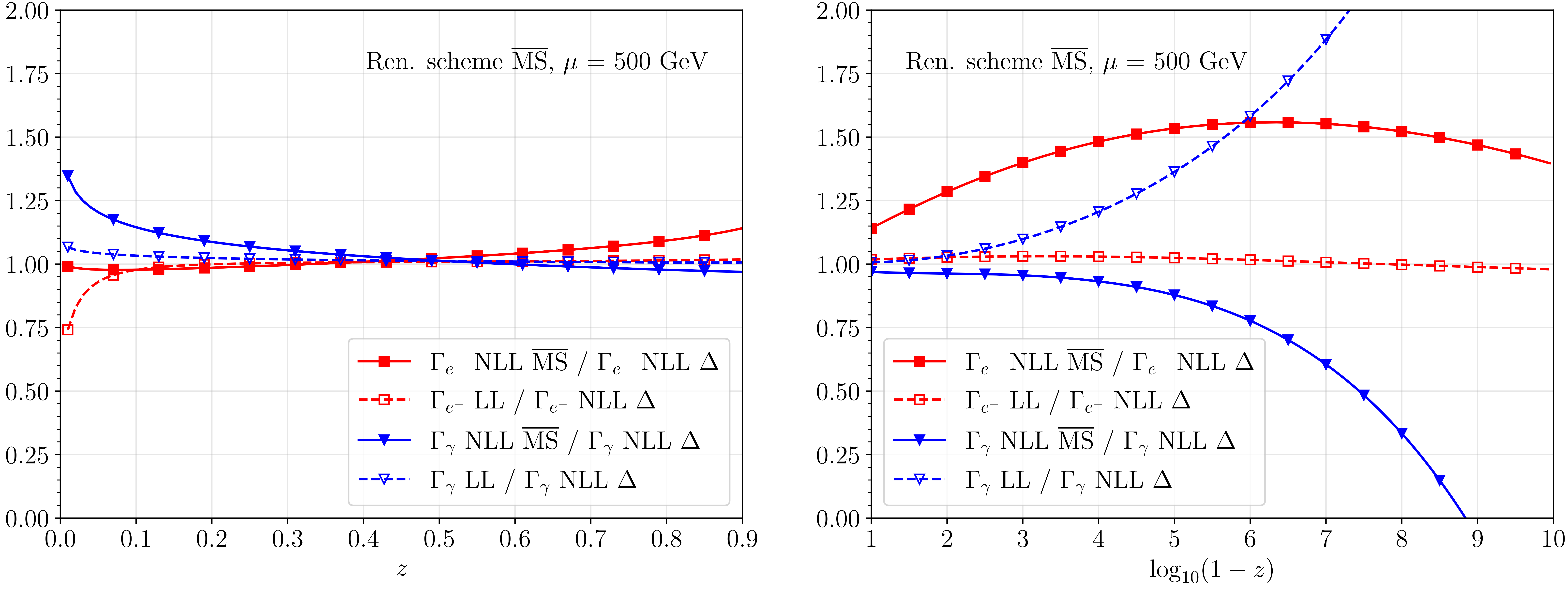
$$\sigma(\tau_{min}) = \int d\sigma \, \Theta(\tau_{min} \le M_{p\bar{p}}^2/s) \,, \quad p = q, t, W^+$$

# Impact of NLL




**Non trivial pattern**, impossible to account in some universal manner.
NLL-accurate PDFs are phenomenologically important for precision studies.

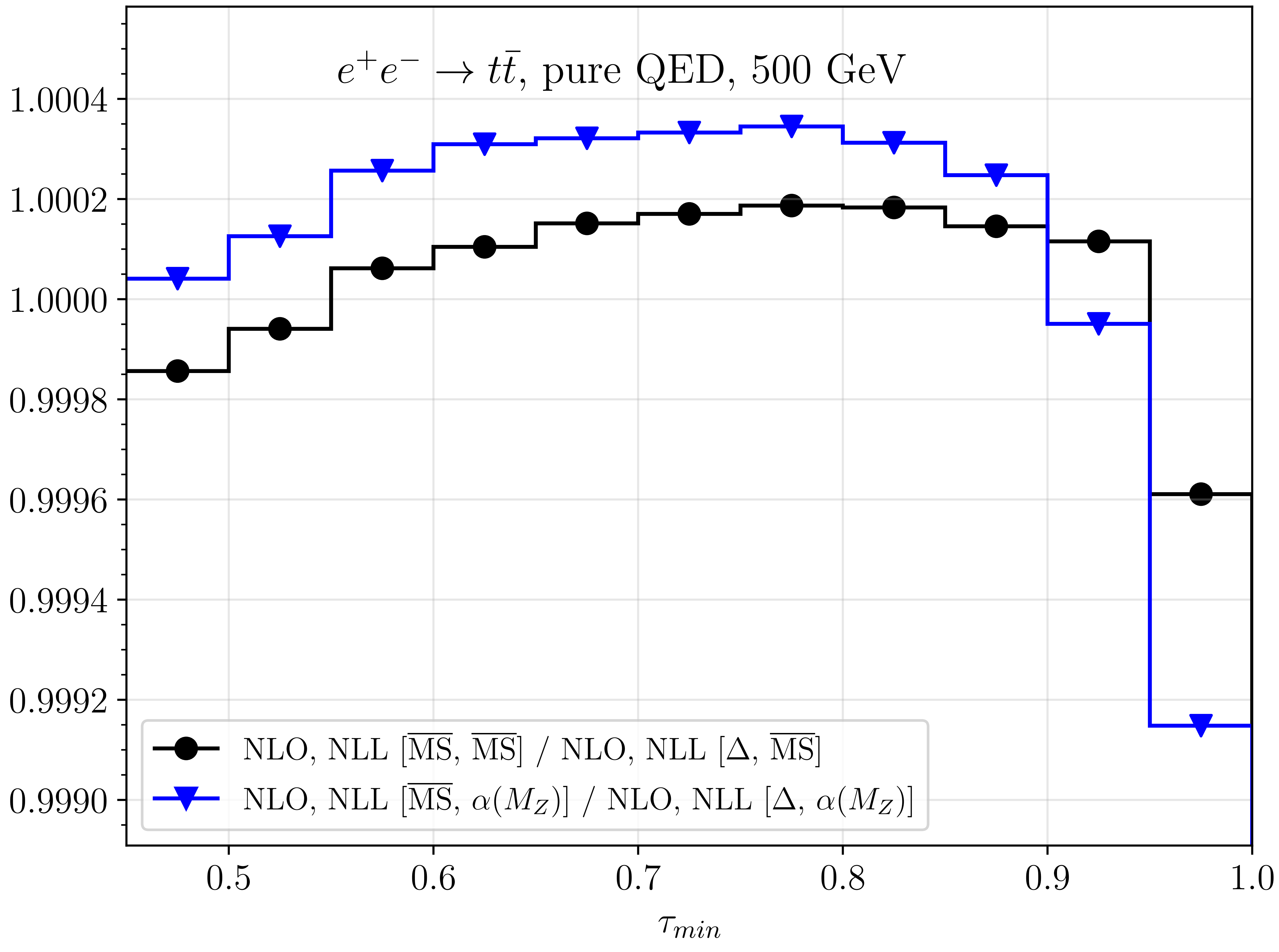
# Dependence on factorisation scheme

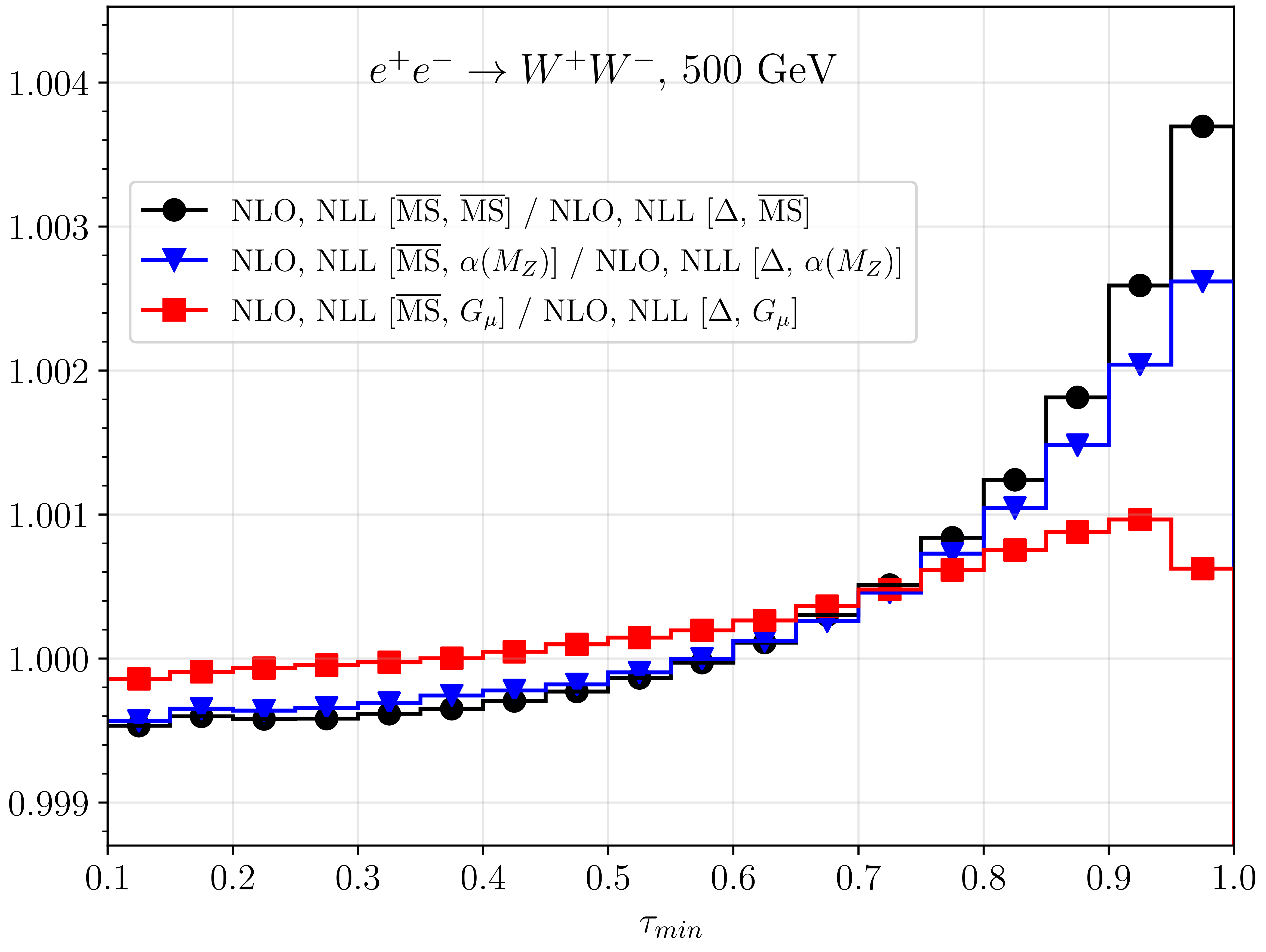



<u>At the PDF level</u>, $\mathcal{O}(1)$ difference between $\overline{\text{MS}}$ and $\Delta$ scheme.

Electron at NLL in the $\Delta$ scheme closer to the LL value.

# Dependence on factorisation scheme

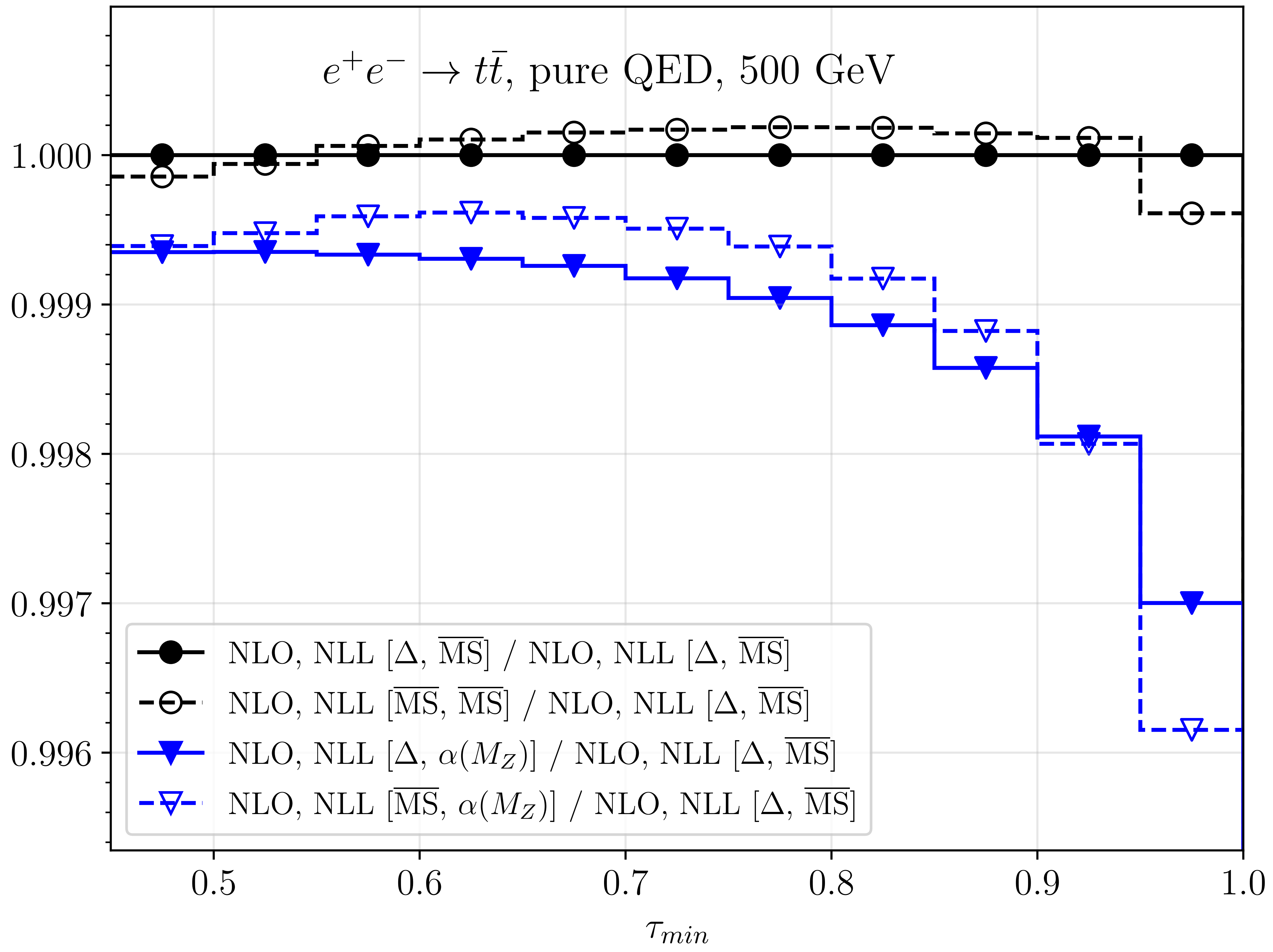


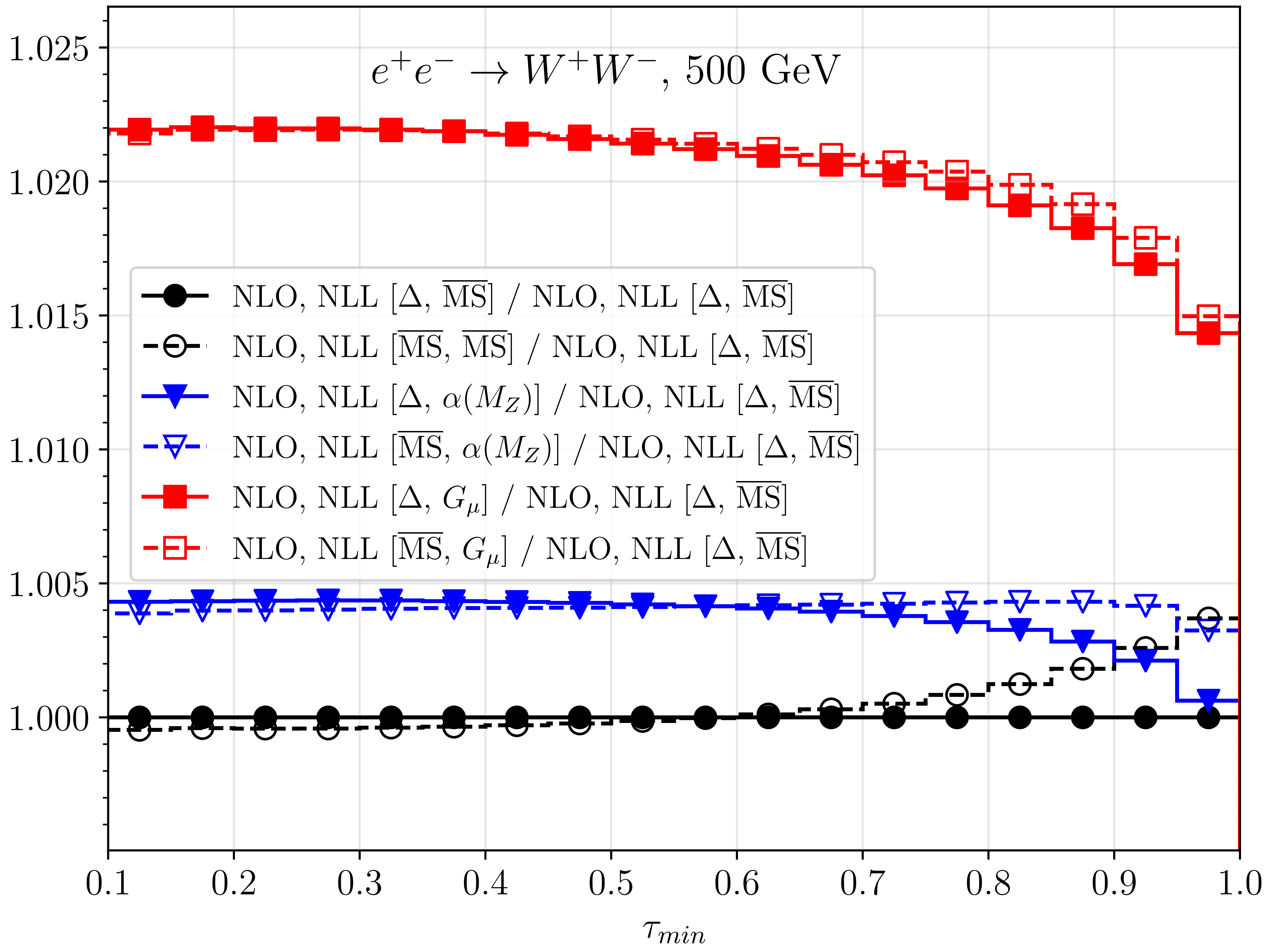




At the cross section level, $\mathcal{O}(10^{-4} - 10^{-3})$ difference between fact. schemes.

**Large cancellations in the $\overline{\text{MS}}$ fact. scheme.**

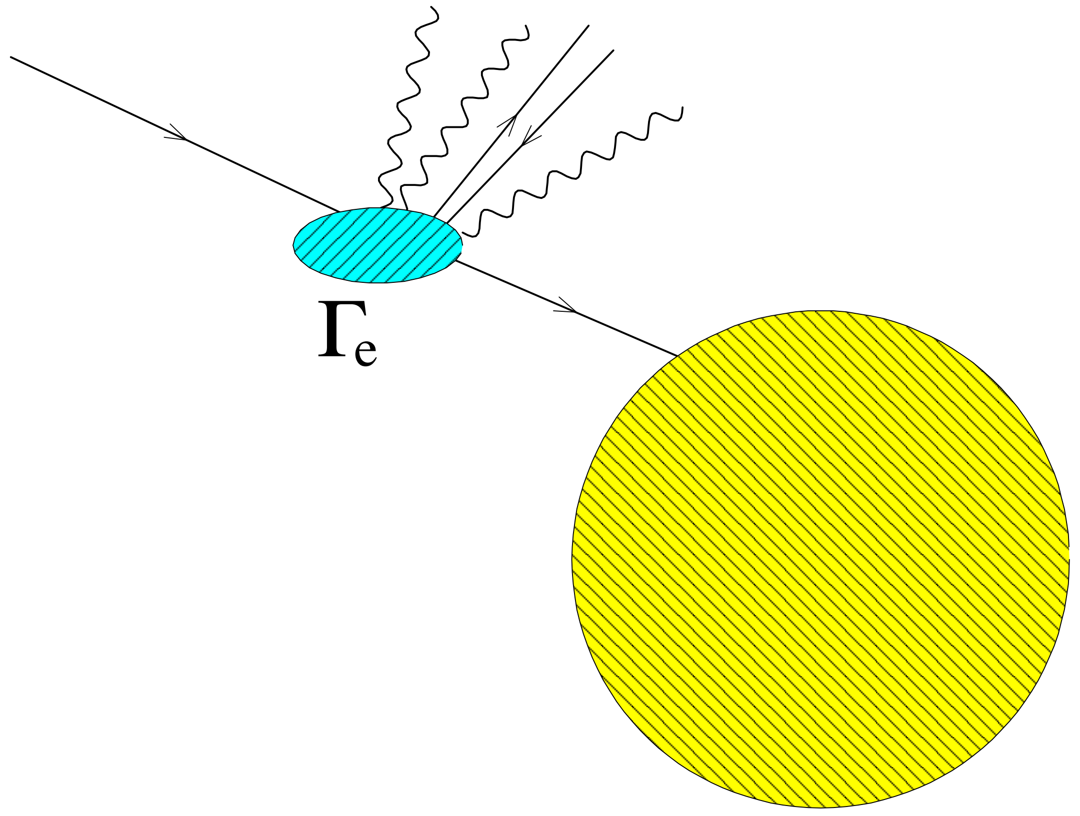
# Dependence on renormalisation scheme

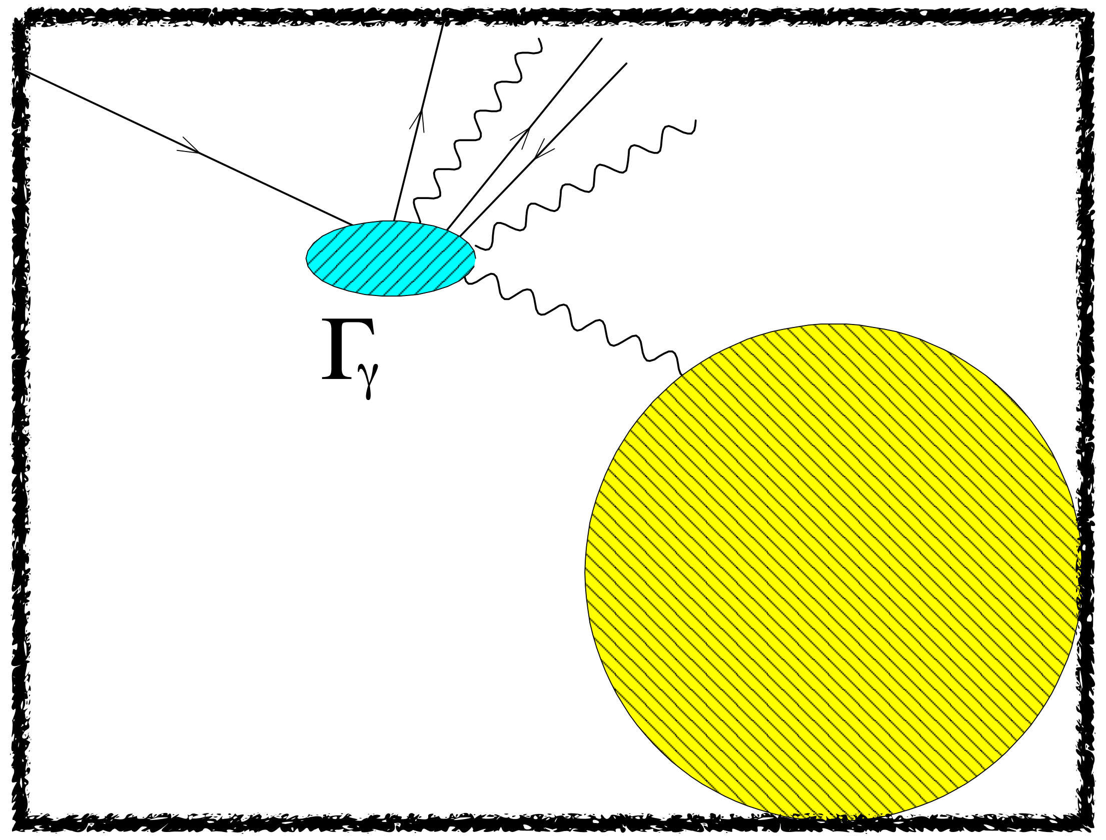



Ren. scheme dependence significantly **larger** than the fact. scheme one.
Mostly a normalisation effect.

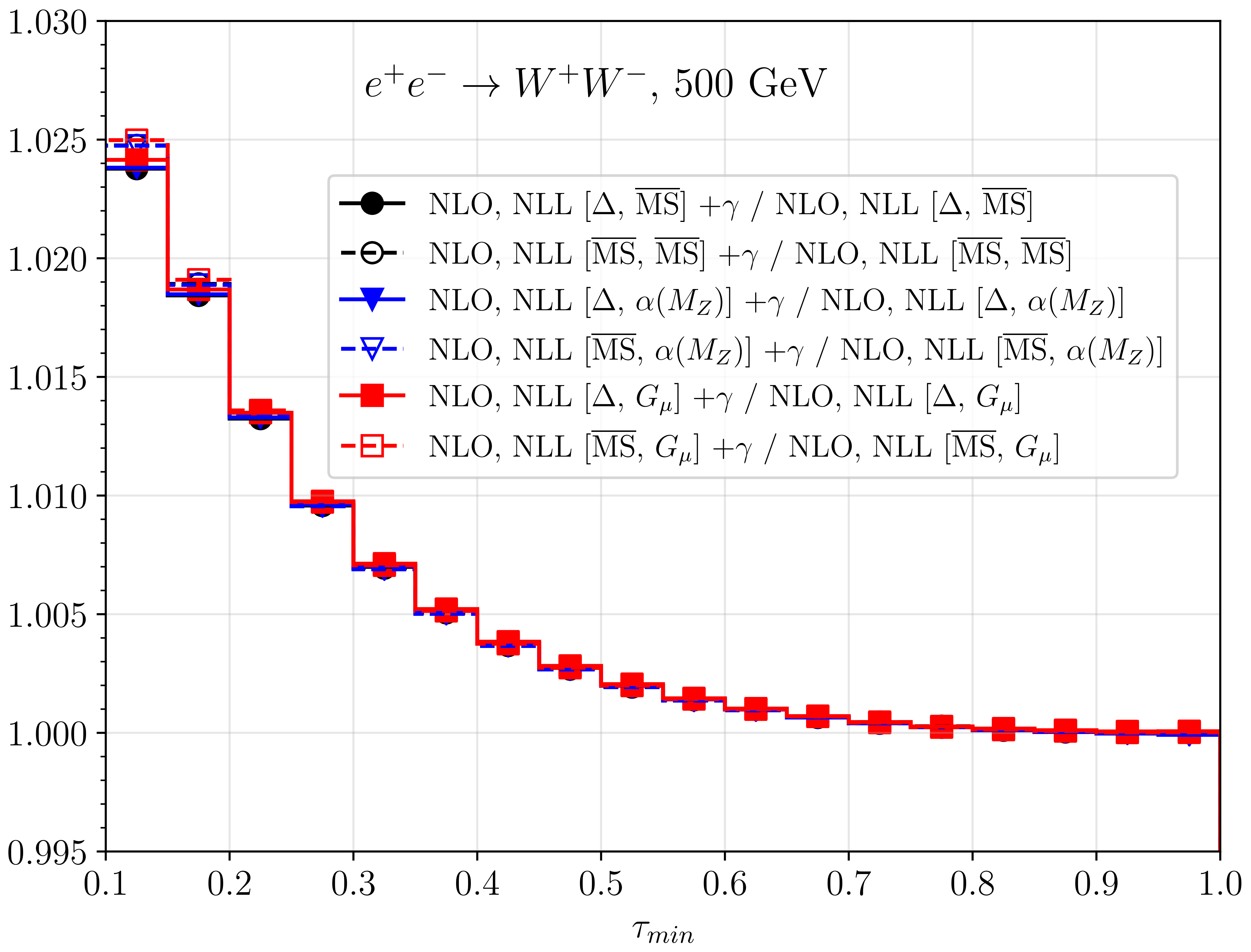
# Impact of photon-induced contributions



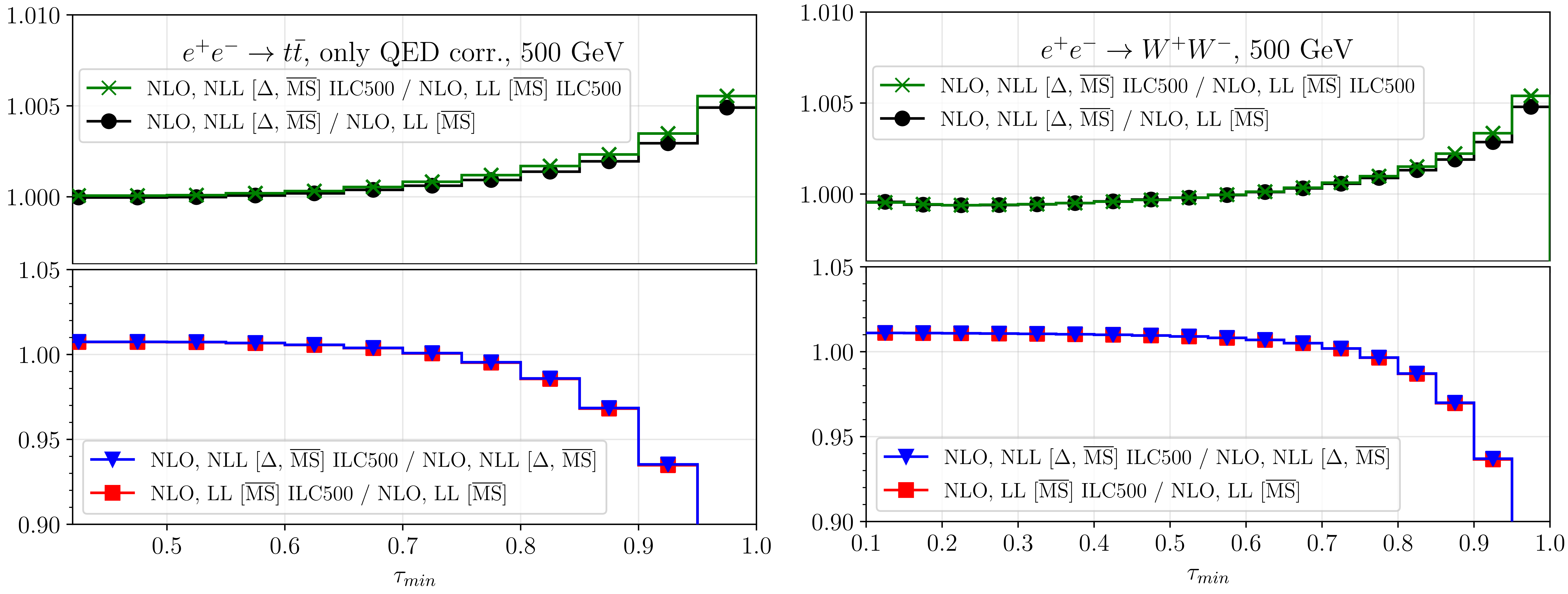




- At LO, i.e. $\mathcal{O}(\alpha^2)$, both $W^+W^-$ and $t\bar{t}$ feature a $\gamma\gamma$ channel.
- Photon PDF $\Gamma_\gamma$ only suppressed by a power of $\alpha$ w.r.t. $\Gamma_{e^-}$, and peaked at small-$z$ values.

Both effects can lead to **physical effects** e.g. $W^+W^-$ at small $\tau_{min}$.



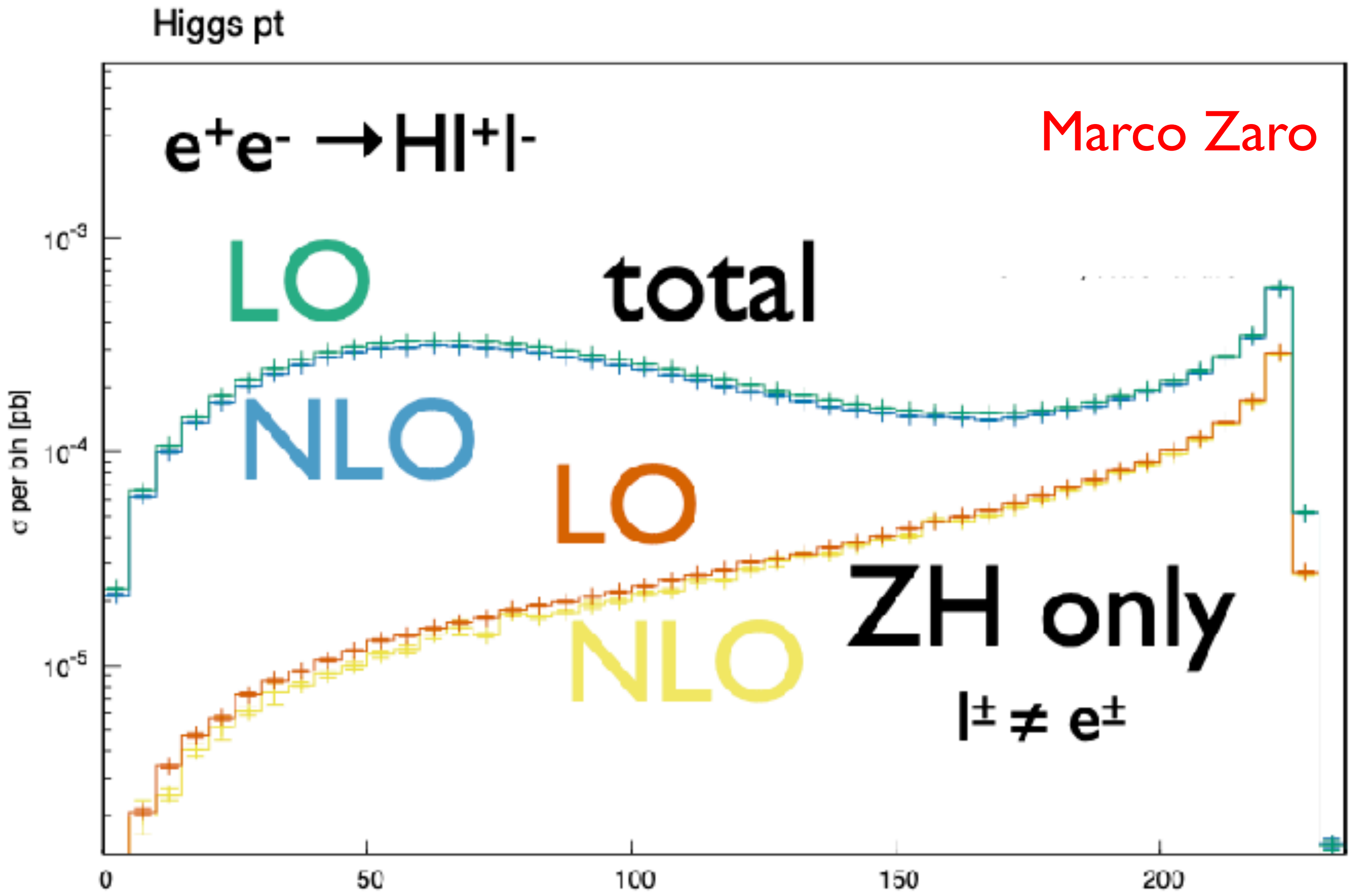
# Effect of beamstrahlung (ILC500*)

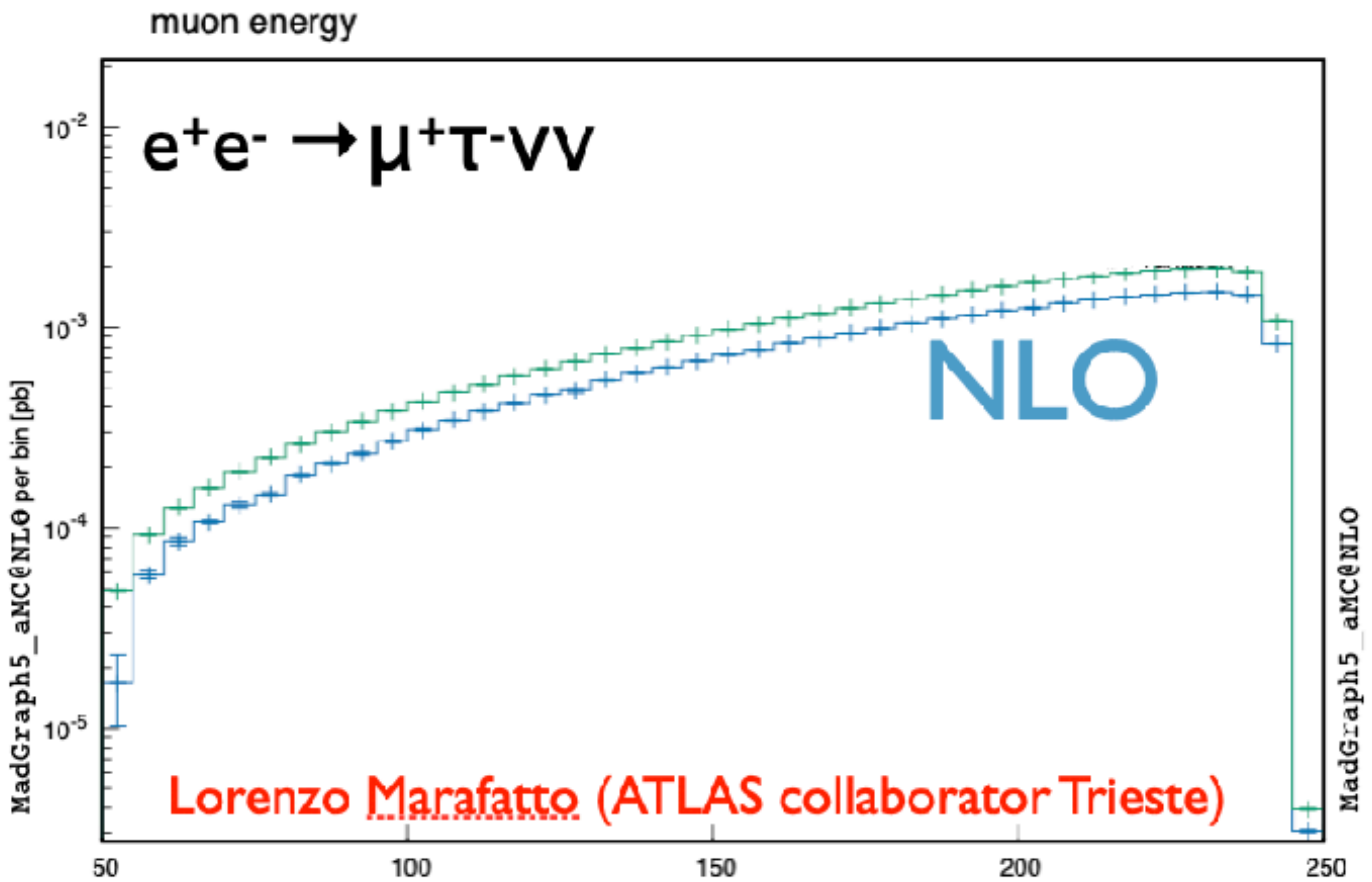



Beamstrahlung effects have a clearly visible impact (independent of LL/NLL)

*Beamstrahlung for FCC-ee can be included as well, but it is not shipped with eMELA
Required parameters can be found in 2108.10261

# Unstable particles with complex-mass scheme: $e^+e^- \to Hl^+l^-$ and $e^+e^- \to \mu^+\tau^-\nu\nu$




- Qualitatively similar results to Denner, Dittmaier, Roth, Weber, hep-ph/0302198
- Results obtained in 15mins (on a cluster) @ 0.1%

Inclusive timing profile :

| | |
|---|---|
| Overall slowest channel | 0:06:15 |
| Average channel running time | 0:03:42 |
| Aggregated total running time | 8:05:57 |

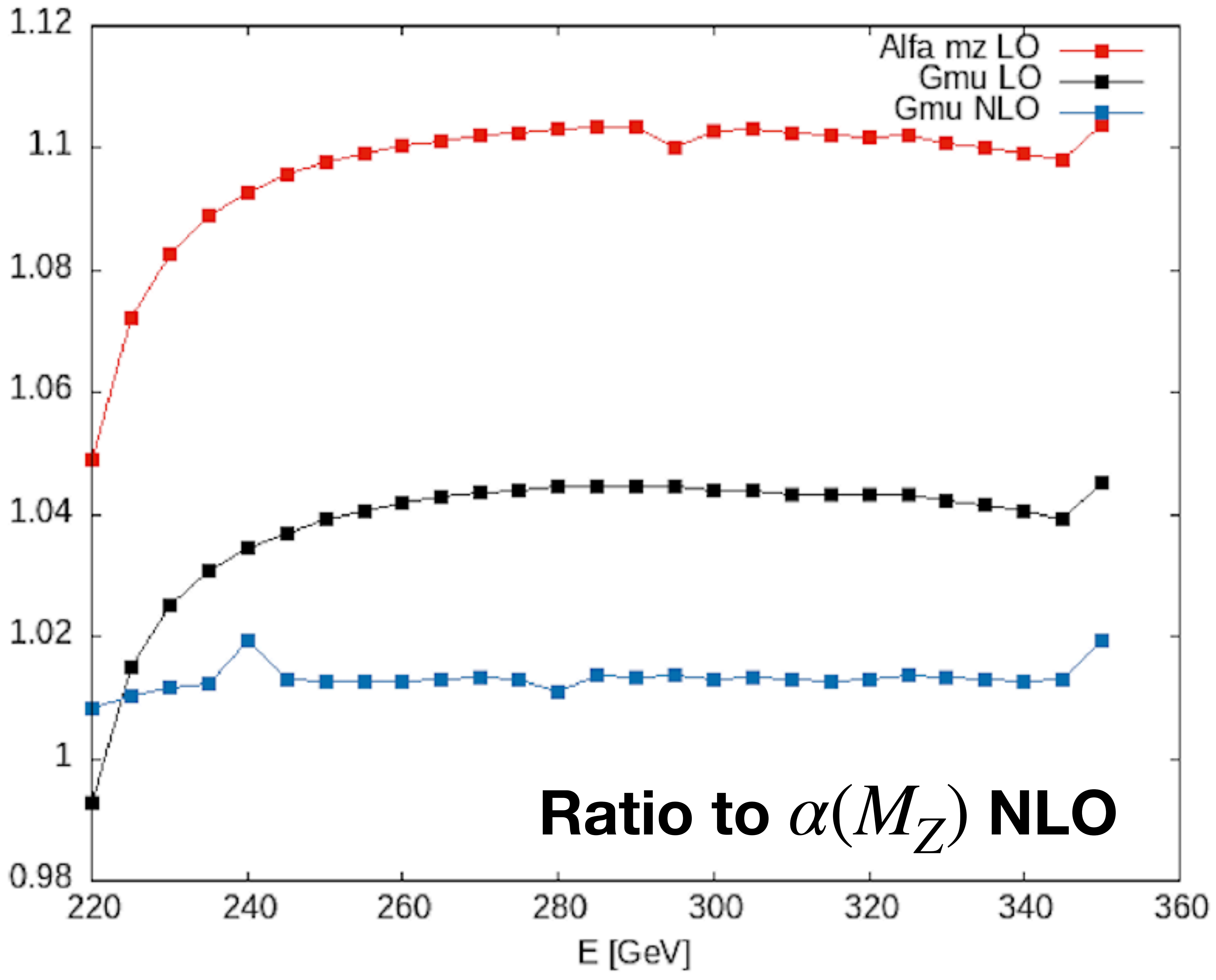
- Very preliminary results
- Running time seems not to be an issue: for a 0.1%-accurate run:

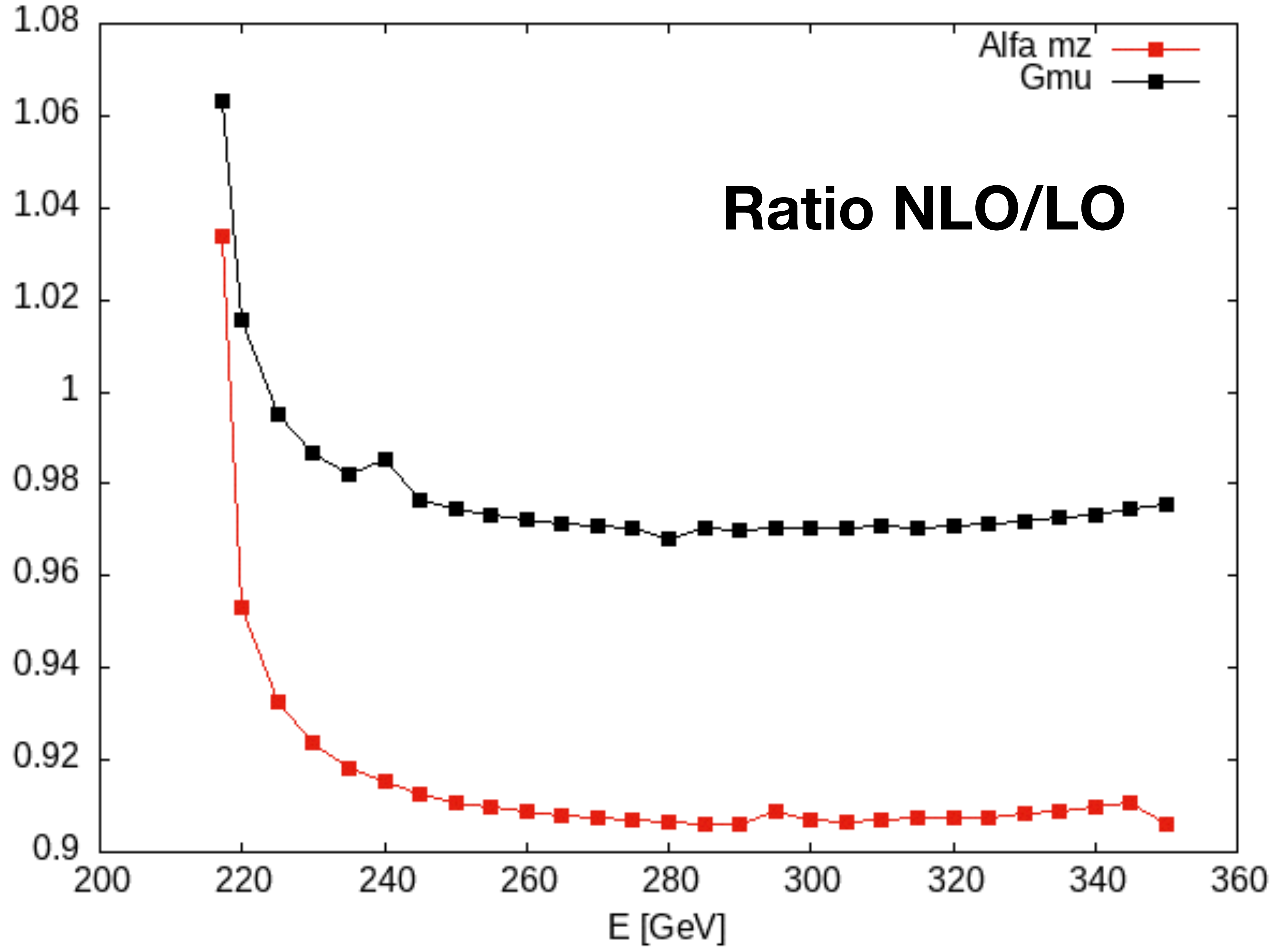
Inclusive timing profile :

| | |
|---|---|
| Overall slowest channel | 0:20:06 |
| Average channel running time | 0:13:09 |
| Aggregated total running time | 1 day, 14:34:39 |

# Studying renormalisation schemes:

$$e^+e^- \to HZ$$



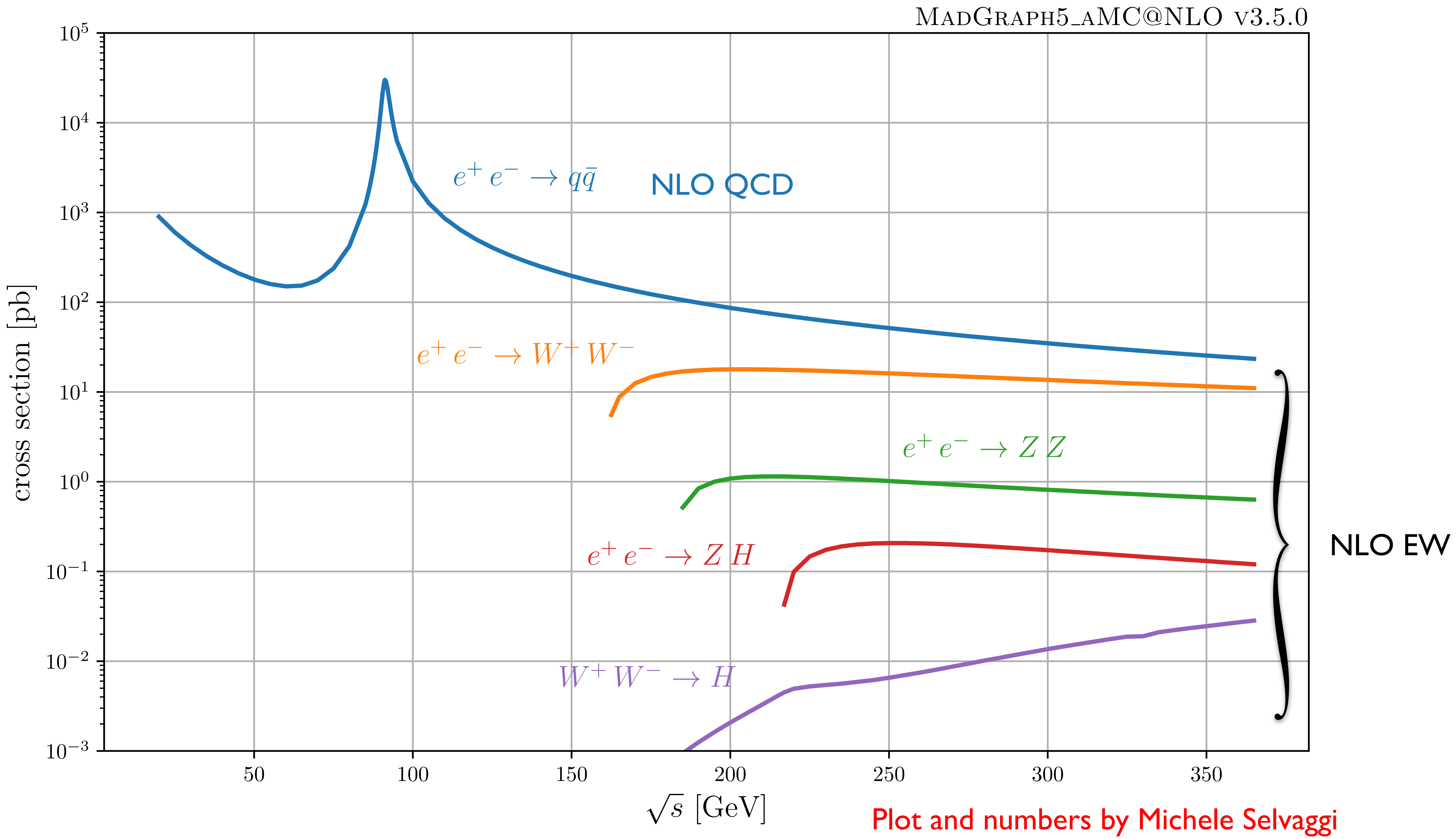

Plot and numbers by Riccardo Lubello (bachelor student)

Scheme dependence reduced when moving from LO to NLO EW

# Conclusions

- New functionalities for lepton colliders in MG5_aMC from v3.5.0: complete automation of (most of the) relevant processes at NLL+NLO EW
- The code has still some limitations, due to the underlying phase-space mapping. Work in progress.
- Exclusive event generation and matching to fixed-order are the next big steps (see Stefano's talk)
- **Try the code, do pheno, and please report bugs/issues!**

# A final propaganda plot




Plot and numbers by Michele Selvaggi

# BACKUP

# QED Parton Shower

see for instance review in 0912.0749

Introduction of a cutoff $x_+ = 1 - \epsilon$, with $\epsilon \ll 1$, to regularise splitting kernels:

$$P_+(z) = \theta(x_+ - z)P(z) - \delta(1 - z)\int_0^{x_+} \mathrm{d}x\, P(x)$$

By introducing a Sukadov form factor: $\Pi(s_1, s_2) = \exp\left(-\frac{\alpha}{2\pi}\int_{s_2}^{s_1}\frac{\mathrm{d}s'}{s'}\int_0^{x_+}\mathrm{d}z\, P(z)\right)$

one can recast the evolution equation in an iterative integral form:

$$D(x, s) = \sum_{n=0}^{\infty}\prod_{i=1}^{n}\left\{\int_{m_e^2}^{s_{i-1}}\frac{\mathrm{d}s_i}{s_i}\Pi(s_{i-1}, s_i)\frac{\alpha}{2\pi}\int_{x/(z_1\cdots z_{i-1})}^{x_+}\frac{\mathrm{d}z_i}{z_i}P(z_i)\right\}\Pi(s_n, m_e^2)D\left(\frac{x}{z_1\cdots z_n}, m_e^2\right)$$

which can be solved by means of a MC algorithm

# QED Parton Shower

see for instance review in 0912.0749

It allows for exclusive photon emission in the context of collinear factorisation.

Photon energies dictated by distribution in $z$, whereas angles are generated independently according to the YFS formula, valid in the soft limit:

$$\cos\theta_l \propto -\sum_{i,j=1}^{N} \eta_i \eta_j \frac{1-\beta_i\beta_j\cos\theta_{ij}}{(1-\beta_i\cos\theta_{il})(1-\beta_j\cos\theta_{jl})}$$

with $\eta_i$ a charge factor and $\beta_i$ the speed of the emitting particle.

Algorithm adopted in BabaYaga $[e^+e^- \to e^+e^-, e^+e^- \to \mu^+\mu^-, e^+e^- \to \gamma\gamma]$

hep-ph/0003268, hep-ph/0103117, hep-ph/0312014, hep-ph/0801.3360, hep-ph/0607181

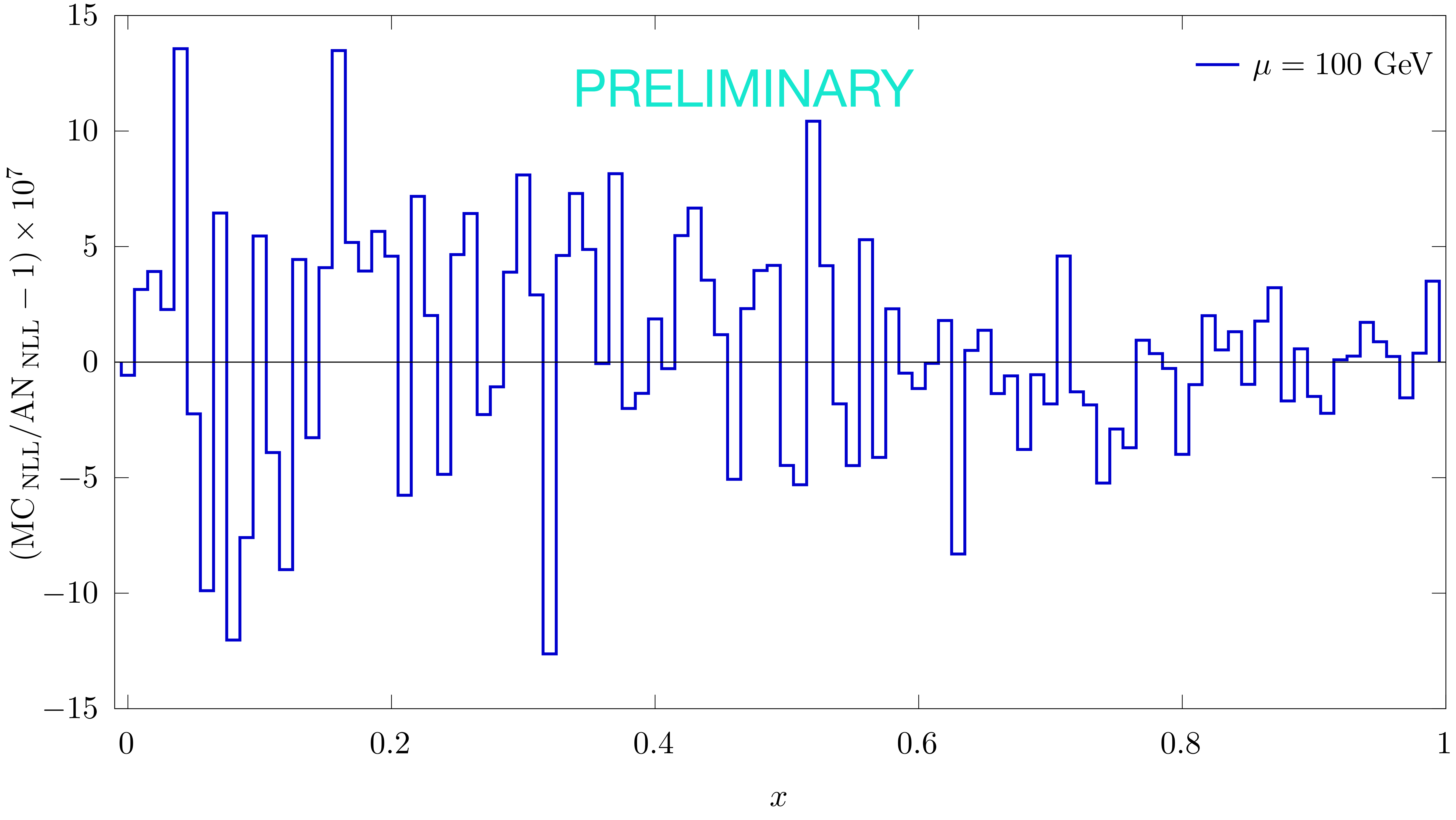
Balossini, Bignamini, Carloni Calame, Lunardini, Montagna, Nicrosini, Piccinini

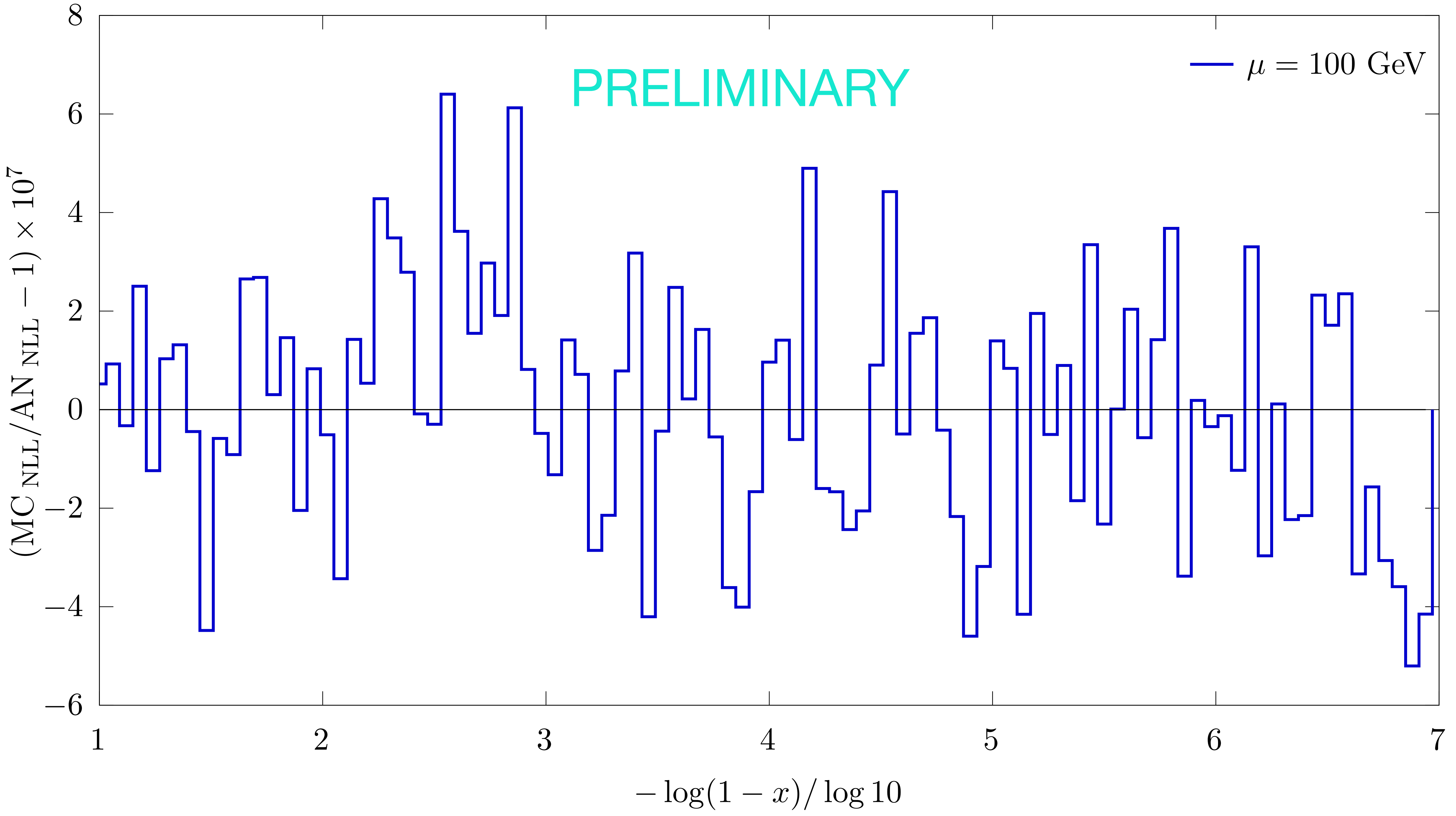
BabaYaga also includes a matching to NLO QED in the short distance cross section

# Towards a “NLL” QED Parton Shower

C. M. Carloni Calame, S. Frixione, G. Montagna, F. Piccinini, GS

With a NLL iterative solution, we recover the known (non-singlet) NLL PDFs




WIP towards exclusive kinematics of final-state photons and singlet components

# NLL-accurate QED PDFs

Frixione 1909.03886; Bertone, Cacciari, Frixione, Stagnitto 1911.12040; Frixione 2105.06688; Bertone, Cacciari, Frixione, Stagnitto, Zaro, Zhao 2207.03265

- PDFs in **three different renormalisation schemes:** $\overline{\text{MS}}$ (where $\alpha$ runs), $\alpha(m_Z)$ and $G_\mu$(where $\alpha$ is fixed); **two different factorisation schemes:** $\overline{\text{MS}}$ and $\Delta$ (DIS-like, with NLO initial condition maximally simplified).

$$\Gamma_{e^-}^{[0],\overline{\text{MS}}}(z,\mu_0^2) = \Gamma_{e^-}^{[0],\Delta}(z,\mu_0^2) = \delta(1-z)$$

$$\Gamma_{e^-}^{[1],\overline{\text{MS}}}(z,\mu_0^2) = \left[\frac{1+z^2}{1-z}\left(\log\frac{\mu_0^2}{m^2} - 2\log(1-z) - 1\right)\right]_+ \quad , \quad \Gamma_{e^-}^{[1],\Delta}(z,\mu_0^2) = \log\frac{\mu_0^2}{m^2}\left[\frac{1+z^2}{1-z}\right]_+$$

Evolution operator and short-distance cross section modified, such that $\hat{\sigma}_N(\mu^2)\, E_N(\mu^2,\mu_0^2)\, \Gamma_N(\mu_0^2)$ independent on the fact. scheme (up to NLO)

# Large-$z$ analytical expressions for $\Gamma_{e^-}$

$$\Gamma_{e^-}^{\mathrm{NLL}}(z,\mu^2) = \frac{e^{-\gamma_{\mathrm{E}}\xi_1} e^{\hat{\xi}_1}}{\Gamma(1+\xi_1)} \xi_1 (1-z)^{-1+\xi_1} h(z,\mu^2)$$

$$\xi_1 = 2t + \mathcal{O}(\alpha^2)$$

$$\hat{\xi}_1 = \frac{3}{2} t + \mathcal{O}(\alpha^2)$$

$$t = \frac{1}{2\pi b_0} \log \frac{\alpha(\mu)}{\alpha(\mu_0)}$$

$$h^{\overline{\mathrm{MS}}}(z,\mu^2) = 1 + \frac{\alpha(\mu_0)}{\pi}\left[\left(\log\frac{\mu_0^2}{m^2} - 1\right)\left(A(\xi_1) + \frac{3}{4}\right) - 2B(\xi_1) + \frac{7}{4} + \left(\log\frac{\mu_0^2}{m^2} - 1 - 2A(\xi_1)\right)\underline{\log(1-z)} - \underline{\log^2(1-z)}\right]$$

$$h^{\Delta}(z,\mu^2) = \frac{\alpha(\mu)}{\alpha(\mu_0)} + \frac{\alpha(\mu)}{\pi}\log\frac{\mu_0^2}{m^2}\left(A(\xi_1) + \log(1-z) + \frac{3}{4}\right)$$

$$A(\xi_1) = \frac{1}{\xi_1} + \mathcal{O}(\xi_1)$$

$$B(\xi_1) = -\frac{\pi^2}{6} + 2\zeta_3\xi_1 + \mathcal{O}(\xi_1^2)$$

Logarithmic terms artefacts of the $\overline{\mathrm{MS}}$ fac. scheme, **absent in the $\Delta$ scheme.**

Here shown in the $\overline{\mathrm{MS}}$ ren. scheme and with a single-fermion family; evolution with multiple fermion families with their mass thresholds and different ren. schemes (e.g. $\alpha(m_Z)$, $G_\mu$) amount to a redefinition of $\xi_1$ and $\hat{\xi}_1$.

# NLL-accurate QED PDFs

Frixione 1909.03886; Bertone, Cacciari, Frixione, Stagnitto 1911.12040; Frixione 2105.06688; Bertone, Cacciari, Frixione, Stagnitto, Zaro, Zhao 2207.03265

- PDFs in **three different renormalisation schemes**: $\overline{\mathrm{MS}}$ (where $\alpha$ runs), $\alpha(m_Z)$ and $G_\mu$(where $\alpha$ is fixed); **two different factorisation schemes**: $\overline{\mathrm{MS}}$ and $\Delta$ (DIS-like, with NLO initial condition maximally simplified).

$$\alpha_R = \alpha_{\overline{\mathrm{MS}}}(m_Z) - \Delta_{\overline{\mathrm{MS}}\to R}\,\alpha^2_{\overline{\mathrm{MS}}}(m_Z) + \mathcal{O}(\alpha^3)$$

Modified evolution to reabsorb the running of alpha, leading to $\mathcal{O}(\alpha^3)$ w.r.t. $\overline{\mathrm{MS}}$ results

→ naively neglecting the running of $\alpha$ leads to $\mathcal{O}(\alpha^2)$ differences w.r.t. $\overline{\mathrm{MS}}$

$$\mathbb{P}_R^{[0,k]} = \mathbb{P}_{\overline{\mathrm{MS}}}^{[0,k]}$$

$$\mathbb{P}_R^{[1,k]} = \mathbb{P}_{\overline{\mathrm{MS}}}^{[1,k]} + \left(2\pi b_0^{(k)} \log\frac{\mu^2}{m_{k+1}^2} + D^{(k)}\right)\mathbb{P}_{\overline{\mathrm{MS}}}^{[0,k]}$$

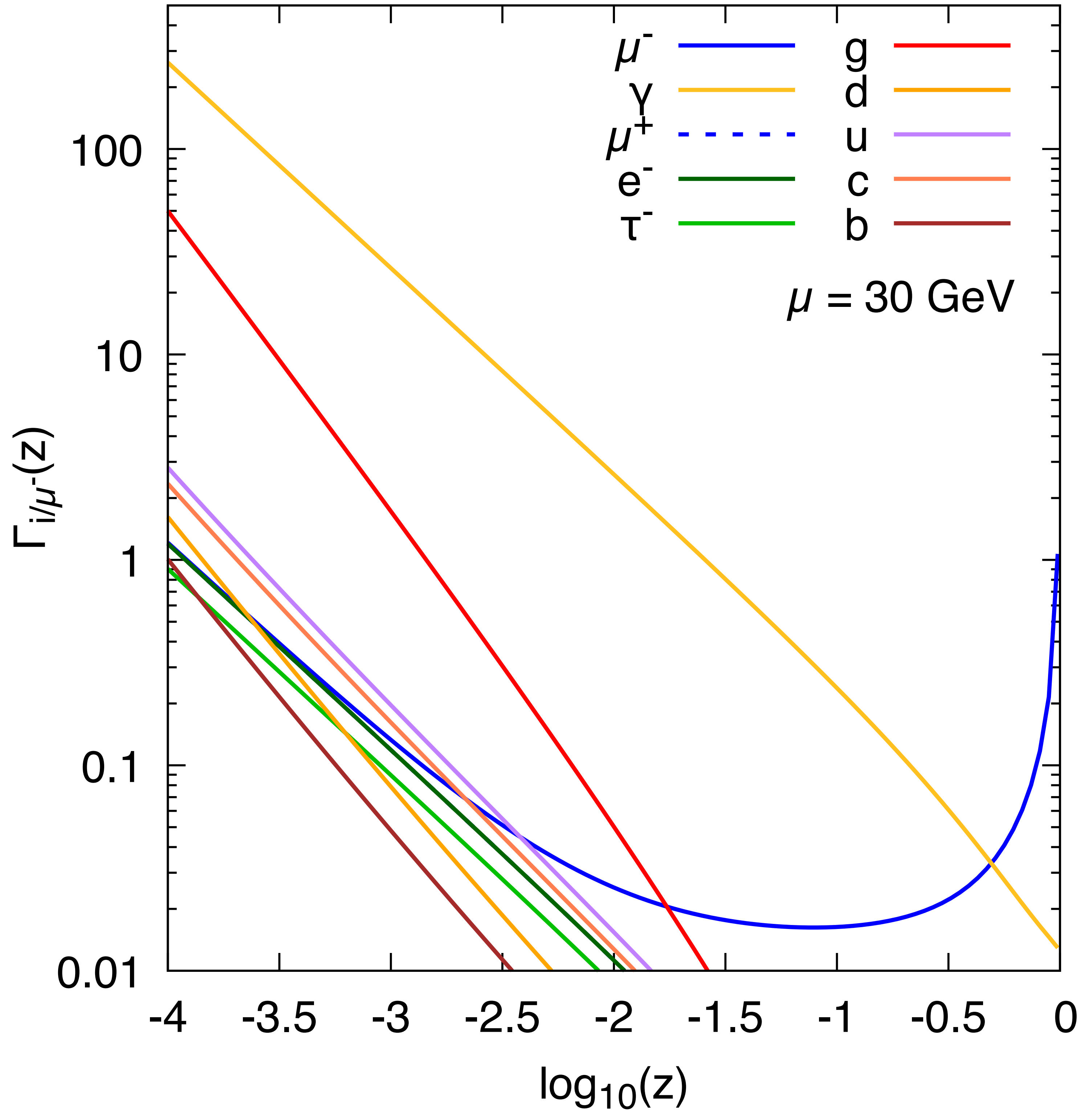
$$D^{(k)} = 2\pi \sum_{i=k+1}^{M} b_0^{(i)} \log\frac{m_i^2}{m_{i+1}^2} + 2\pi\Delta_{\overline{\mathrm{MS}}\to R}$$

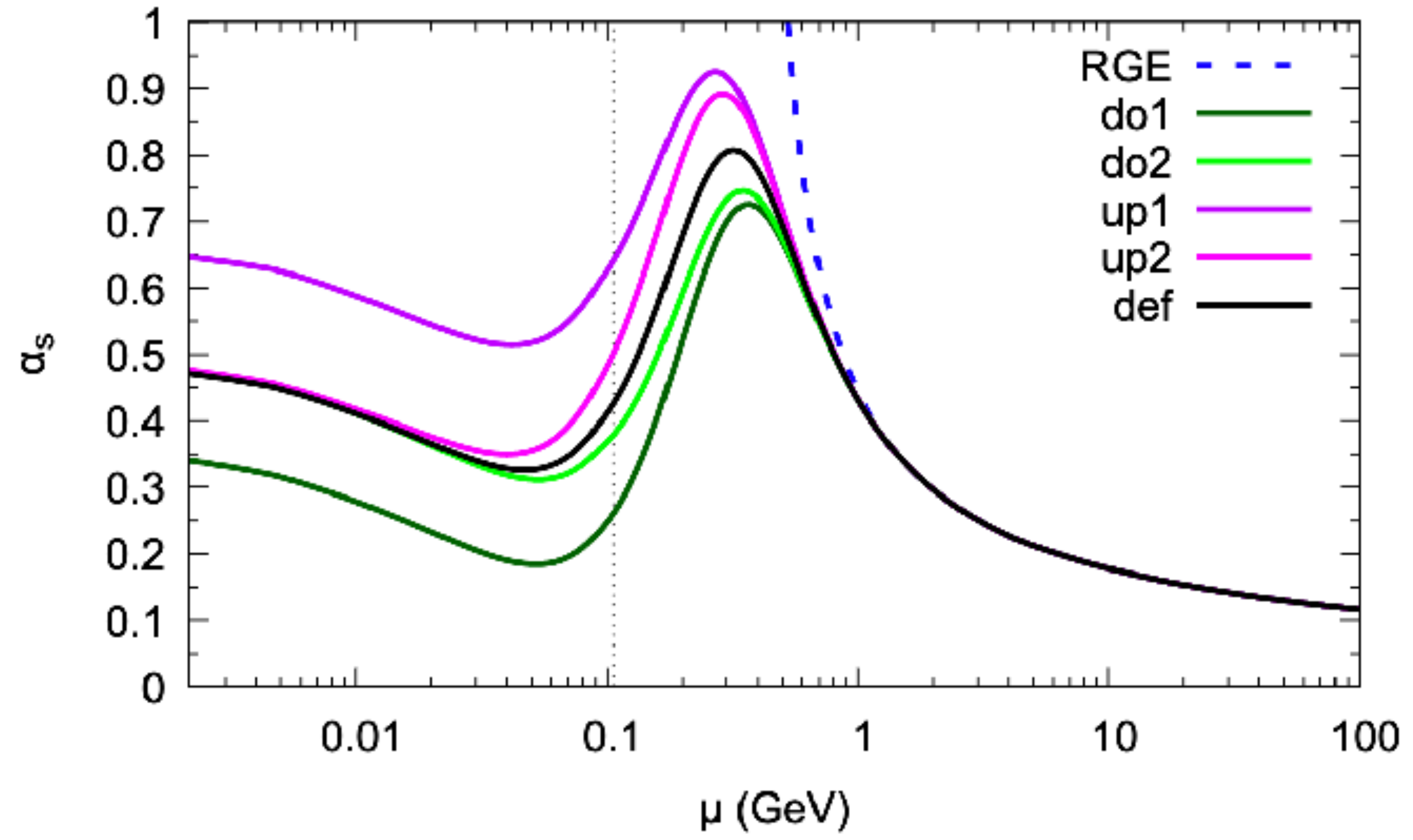
# ISR for muon colliders

(Frixione, GS 2309.07516)

Higher energy ($\sqrt{s} \sim$ 1-10 TeV) → smaller $z$ probed, $z \sim \sqrt{M^2/s}$

Copious emission of QCD particles at small-$z$, relevant when $M^2 \ll s$




Coupled QED+QCD LL muon PDFs, with a novel treatment of $\alpha_s$ at small scales