
Determination of one angle of the “squashed” (d,s) unitarity triangle at FCC-ee

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Analysis and write-up advanced, plan to submit soon to arXiv

Unitarity triangles

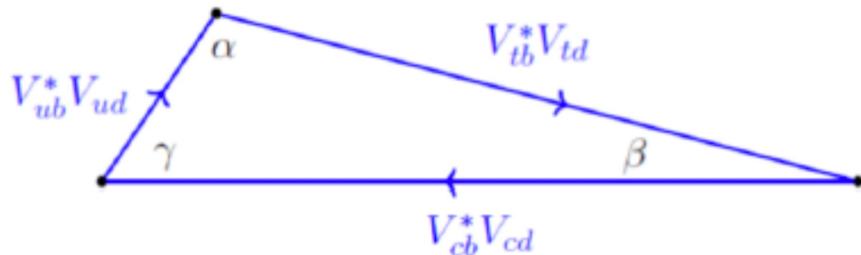
Six triangular relations from unitarity of V_{CKM} .
Among them :

$$V_{CKM} = \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix}$$

The (usual) “(b, d) triangle” :

$$V_{ub}^* V_{ud} + V_{cb}^* V_{cd} + V_{tb}^* V_{td} = 0$$

$$(\lambda^3, \lambda^3, \lambda^3)$$



All angles are (quite) large.

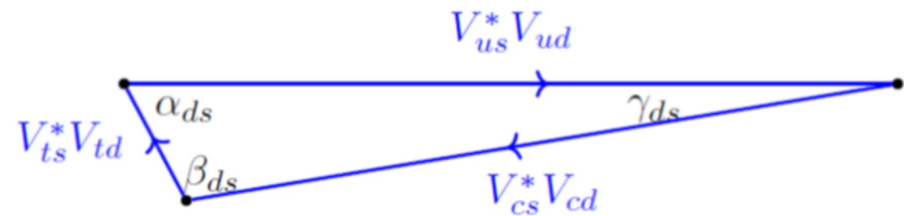
$$(\alpha, \beta, \gamma) \sim (85^\circ, 22^\circ, 66^\circ)$$

Extensively studied experimentally

The “(d, s) triangle” :

$$V_{us}^* V_{ud} + V_{cs}^* V_{cd} + V_{ts}^* V_{td} = 0$$

$$(\lambda, \lambda, \lambda^2)$$



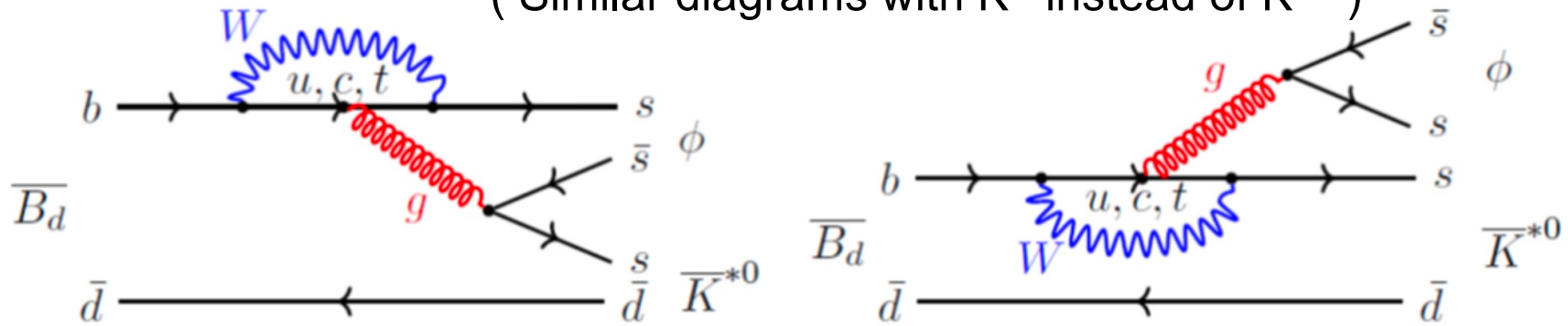
$$(\alpha_{ds}, \beta_{ds}, \gamma_{ds}) \sim (23^\circ, 157^\circ, 0.03^\circ)$$

The “(b, s) triangle”: [2107.02002](#), [2107.05311](#)
(SM: Relations between the angles of these triangles)

FCC-ee : **direct** measurements of the ‘non usual’ triangles. Important consistency checks of the CKM mechanism of the SM. **Here: Study expected precision on α_{ds}**

Measurement of α_{ds}

(Similar diagrams with K^0 instead of K^{*0})



When $K^0 \rightarrow K_S$, or $K^{*0} \rightarrow K_S \pi^0$ (CP eigenstate) :
 interf. between $B \rightarrow \phi K_S$ and $B \rightarrow \bar{B} \rightarrow \phi K_S$ leads to CP violation.

[Neglecting direct CP violation in the decay to simplify the formula below :]

$$\Gamma (B_d(t) \rightarrow f) \sim e^{-\Gamma t} [1 \pm (1-2\omega) \sin \Phi_{CKM} \sin(\Delta m_d t)]$$

ω = mistag rate, (well) measured independently

For the B_d :

$$\Phi_{CKM} = \pi - 2 \alpha_{ds}$$

For B_s : $\Phi_{CKM} = \pi + 2 \alpha_{ds}$

Both B and B_s decays give access to α_{ds} via time-dependent CPV asym.
 Must measure both and check that $\Phi_{CKM}(B_s) + \Phi_{CKM}(B_d) = 2\pi$: important consistency check of the SM.

Expected number of signal decays in 150 ab^{-1} at the Z peak

B decay	Br($\times 10^{-6}$)
$\bar{B}^0 \rightarrow \bar{K}^0 \phi$	7.3 ± 0.7
$\bar{B}^0 \rightarrow \bar{K}^{*0} \phi$	10.0 ± 0.5
$\bar{B}_s \rightarrow \phi K^0$	$1.3 \pm 0.6^*$
$\bar{B}_s \rightarrow \phi K^{*0}$	1.14 ± 0.30

B_s signal is small !

(*) $B_s \rightarrow \phi K^0$ not measured yet. This BR corresponds to $B_s \rightarrow K^+ K^- K^0$. Branching used for the numbers below: $1.0 \cdot 10^{-6}$

Expected number of produced B_d and B_s decays:
 (to be x'ed by 2 to account for \bar{B}_d and \bar{B}_s) :

		B_d	B_s
ϕK^0	$K^+ K^- (\pi^+ \pi^-)_{K_s}$	$\sim 4.9 \cdot 10^5$	$\sim 1.7 \cdot 10^4$
ϕK^{*0}	$K^+ K^- (\pi^+ \pi^-)_{K_s} \pi^0$	$\sim 2.1 \cdot 10^5$	$\sim 6.4 \cdot 10^3$

Modelisation of the detector response

- Stand-alone parametrisation of the detector response :

- **Smearing** of the momenta and angles of particles in the decays of interest

- Parametrisation based on typical performance of a light tracker at a future ee detector
- Excellent EM calo resolution

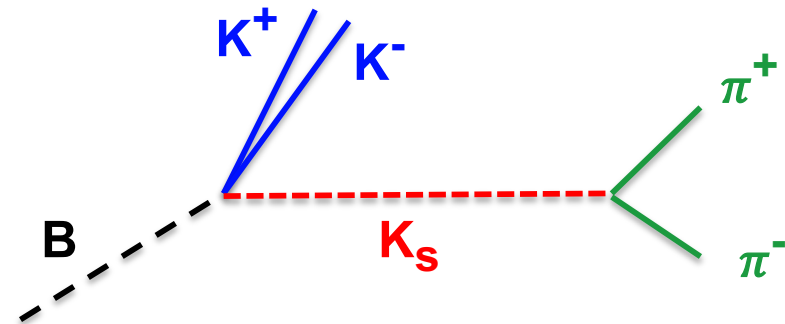
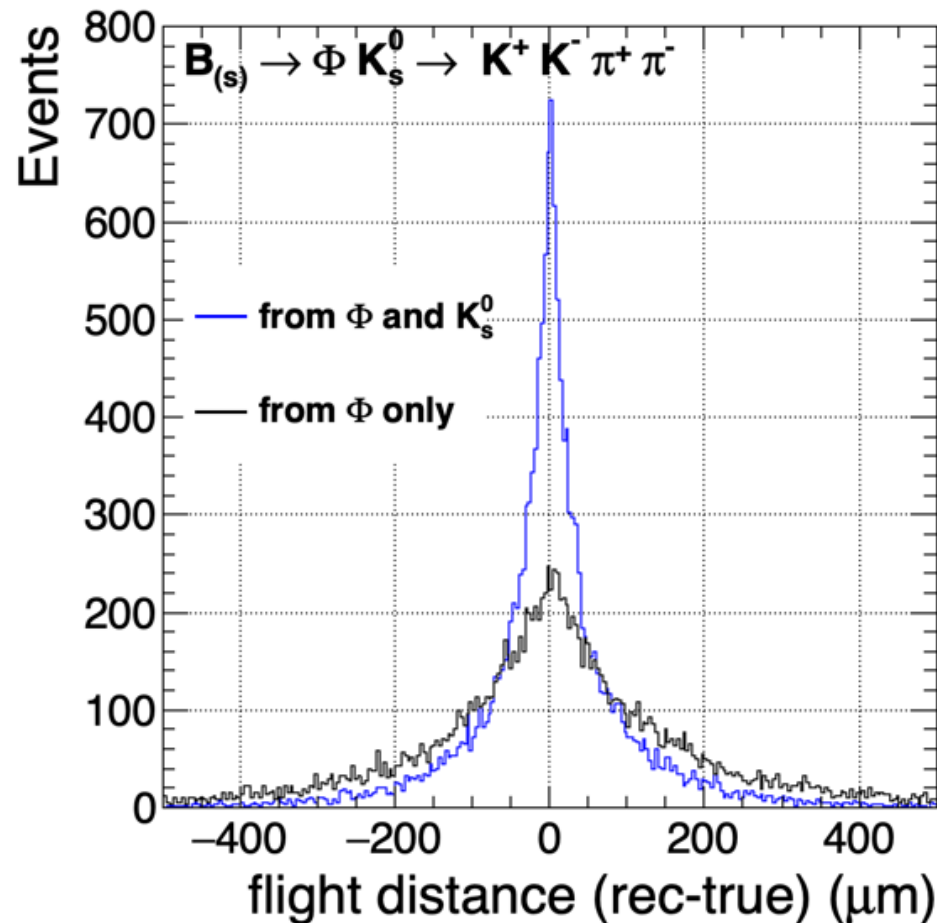
Acceptance :	$ \cos \theta $	< 0.95
Charged particles :		
p_T resolution :	$\frac{\sigma(p_T)}{p_T^2}$	$= 2. \times 10^{-5} \oplus \frac{1.2 \times 10^{-3}}{p_T \sin \theta}$
ϕ, θ resolution :	$\sigma(\phi, \theta) \mu\text{rad}$	$= 18 \oplus \frac{1.5 \times 10^3}{p_T \sqrt[3]{\sin \theta}}$
Vertex resolution :	$\sigma(d_{\text{Im}}) \mu\text{m}$	$= 1.8 \oplus \frac{5.4 \times 10^1}{p_T \sqrt{\sin \theta}}$
e, γ particles :		
Energy resolution :	$\frac{\sigma(E)}{E}$	$= \frac{5 \times 10^{-2}}{\sqrt{E}} \oplus 5 \times 10^{-3}$
EM ϕ, θ resolution :	$\sigma(\phi, \theta) \text{mrad}$	$= \frac{7}{\sqrt{E}}$

- Common FCCSW : Full MC events + response of the **IDEA detector with DELPHES**
 - Detailed description of tracks, accounting for multiple scattering
 - Genuine vertex fitting

Vertex reconstruction

- Resolution on K_S , ϕ and B decay vertices : crucial to suppress the backgrounds
- Resolution on B_S decay vertex: crucial for time-dependent measurements !

Vertexing code (F. Bedeschi) extended recently to also handle neutrals. See FB's report at the [October Physics Perf. meeting](#)



Using only the tracks from the ϕ :
resolution $\sim 168 \mu\text{m}$ only !
(very small angular separation of the two kaon tracks)

Adding the K_S “neutral tracks” : $70 \mu\text{m}$

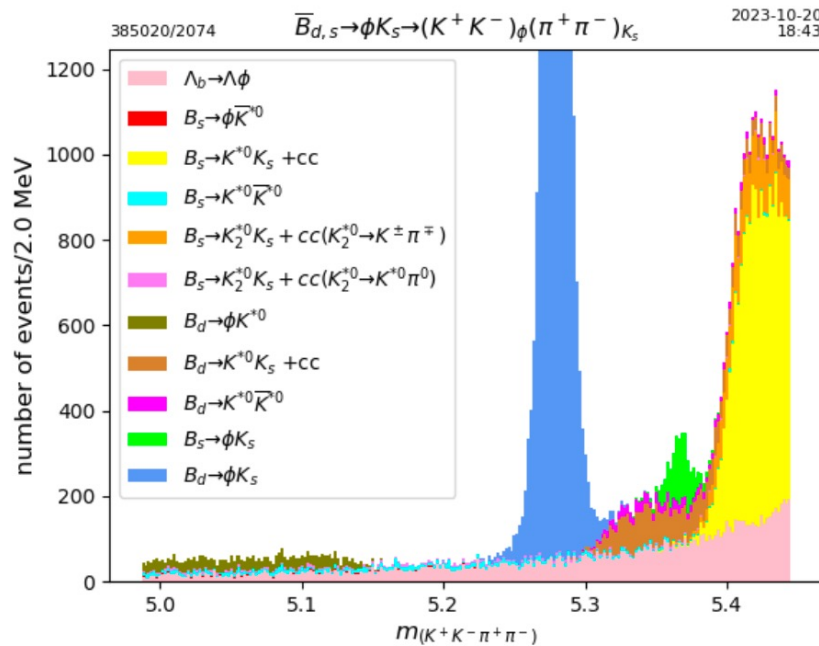
(for comparison: in $B_S \rightarrow D_S (KK\pi)$ K ,
resolution was $\sim 20 \mu\text{m}$)

Very good compared to the average flight distance of the B ($\sim 3 \text{ mm}$) and no significant dilution of the B_S time-dependent CP asymmetries.

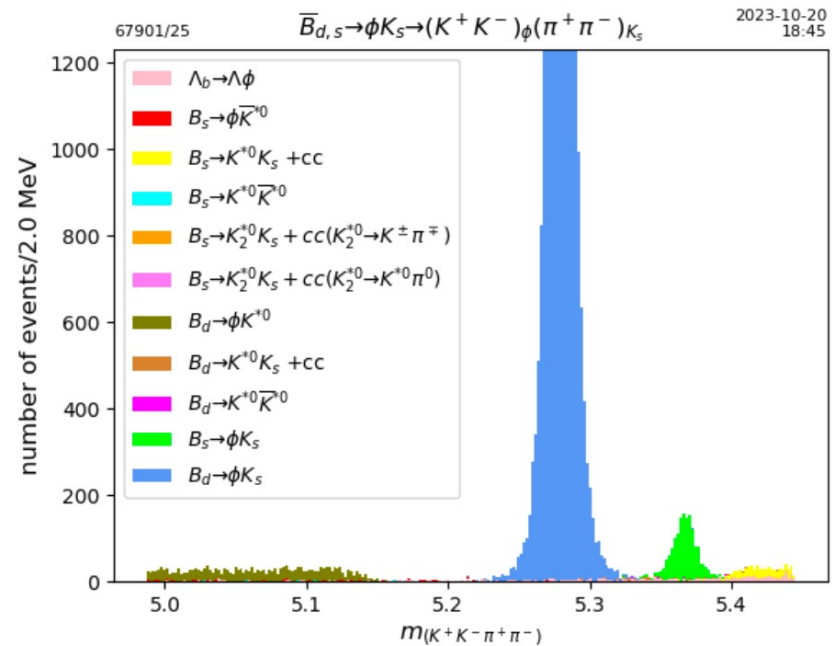
Backgrounds: exclusive final states

Potential background contributions :

- Final states w/o long-lived particles, e.g. $\phi(KK)$ $\rho(\pi\pi)$ or $\phi(KK)$ $f_0(\pi\pi)$
 - Demand $M(\pi\pi) \sim K_S$ mass and $\pi\pi$ -vertex detached from ϕ
- Final states with K_S (or Λ), e.g. $B \rightarrow K^{*0} K_S$ with $K^{*0} \rightarrow K^\pm \pi^\mp$
 - They include a π instead of a K , or a ρ instead of a π
 - Can be reduced even w/o PID thanks to the mass resolution



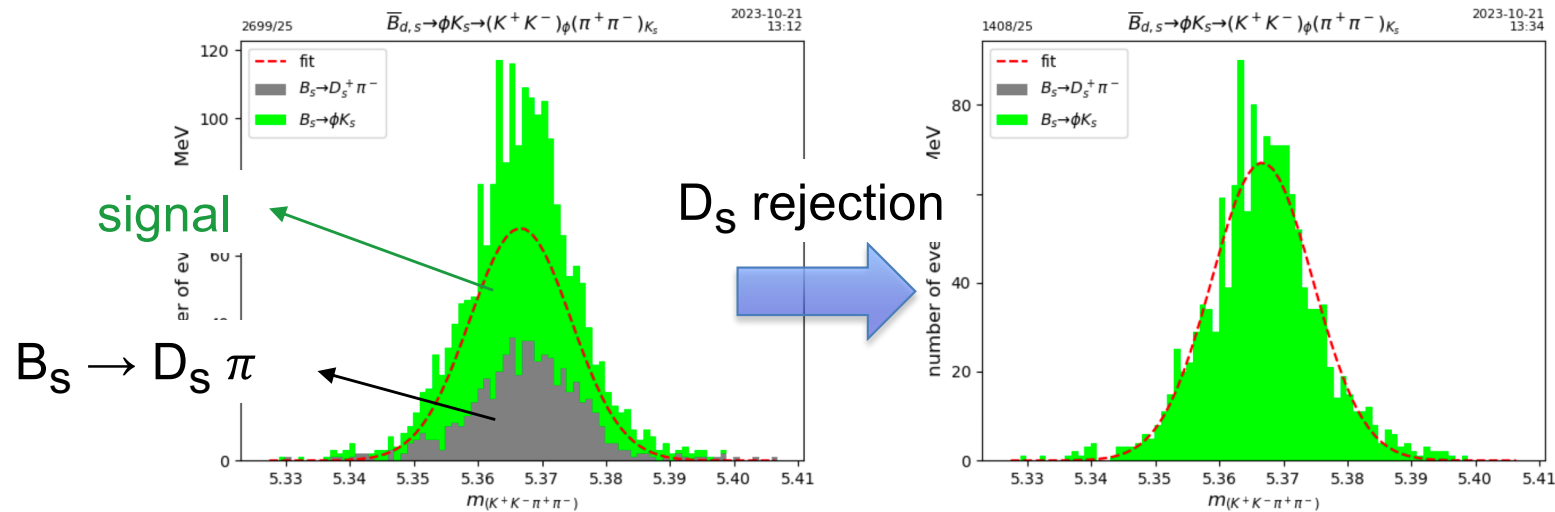
w/o mass cuts



With $m(K^+K^-)$ and $m(\pi^+\pi^-)$ cuts

Backgrounds: (semi-) exclusive final states

- Decays of $B_S \rightarrow D_S (\rightarrow \phi \pi) \pi \rightarrow K K \pi \pi$
 - Branching is 400x larger than $B_S \rightarrow \phi K_S \rightarrow K K \pi \pi$
 - Can be suppressed by an explicit reconstruction of $D_S \rightarrow \phi \pi$
 - (take the tracks from the ϕ , fit them with a 3rd track, if the chi2 is good and the vertex mass is consistent with the D_S mass, reject)



- More generally: background candidates come from a random combination of a K_S (e.g. from fragmentation) and a ϕ that comes from a D_S
 - E.g. $D_S \rightarrow \phi l \nu$, or $D_S \rightarrow \phi \pi + X$
 - Suppressed by a partial reconstruction of D_S decays: if ϕ + track can be fit to a common vertex, with mass $< D_S$ mass + $O(2x \text{ resolution})$, reject

Backgrounds: inclusive bb and cc events

Inclusive PYTHIA sample (signal events removed), passed through DELPHES

- 1 billion of bb events, 500 millions of cc events

Reconstruction:

- Identify “primary tracks”, and consequently, “secondary tracks”
- Build ϕ and K_S candidates (opposite-charge tracks, same hemisphere, fit to a common vertex, mass consistent with ϕ or K_S)
 - K_S candidates: restricted to $L < 1.5$ m from the IP
- Build $B_{(S)}$ candidates from pairs of ϕ and K_S (same hemisphere, vertex fit of ϕ tracks + K_S)

In a first step: assume perfect PID – i.e. use MC-truth information to demand:

ϕ legs = Kaons

K_S legs = Pions

Signal efficiency: $\sim 58\%$

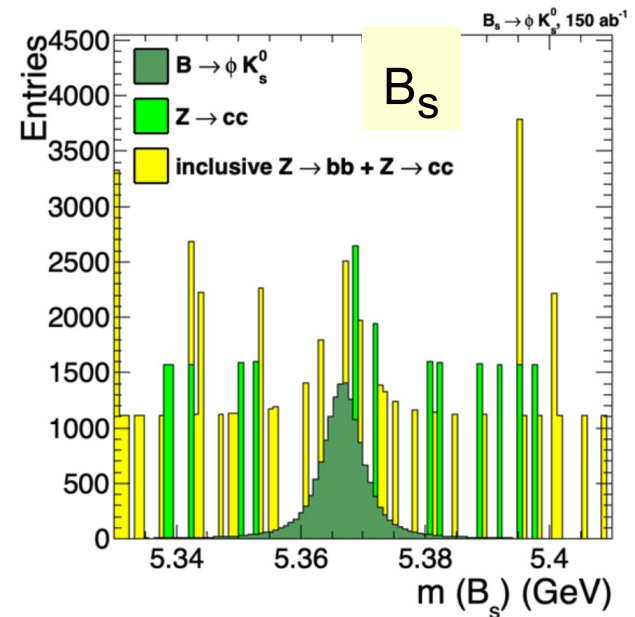
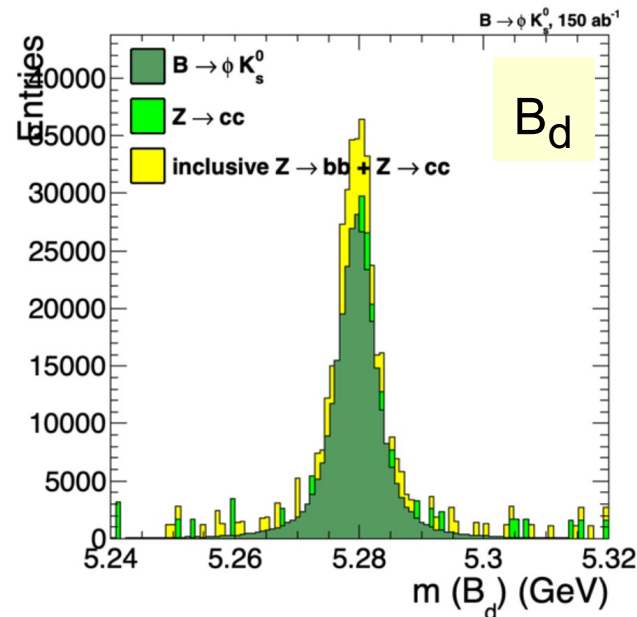
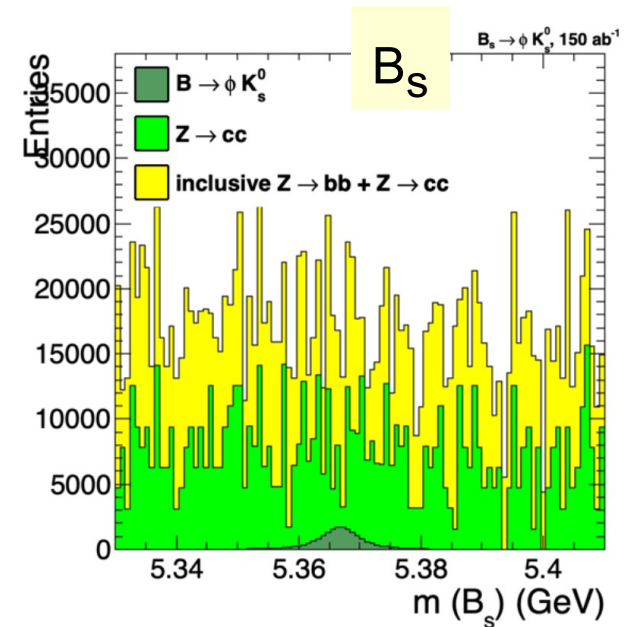
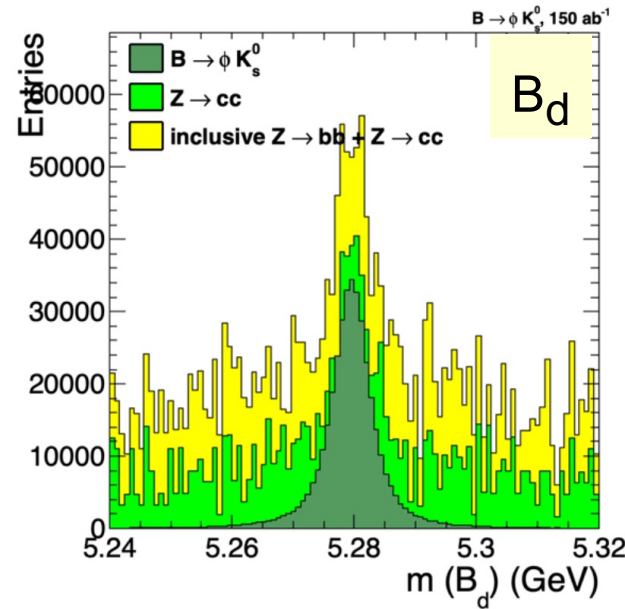
Main loss = acceptance for the K_S

Backgrounds: inclusive bb and cc events

Basic reconstruction :

- Cuts against(semi-) exclusive processes:
- $L(K_S) > 1 \text{ mm}$
 - No reco'ed D_S

Remaining bckgd:
mostly
combinatorial

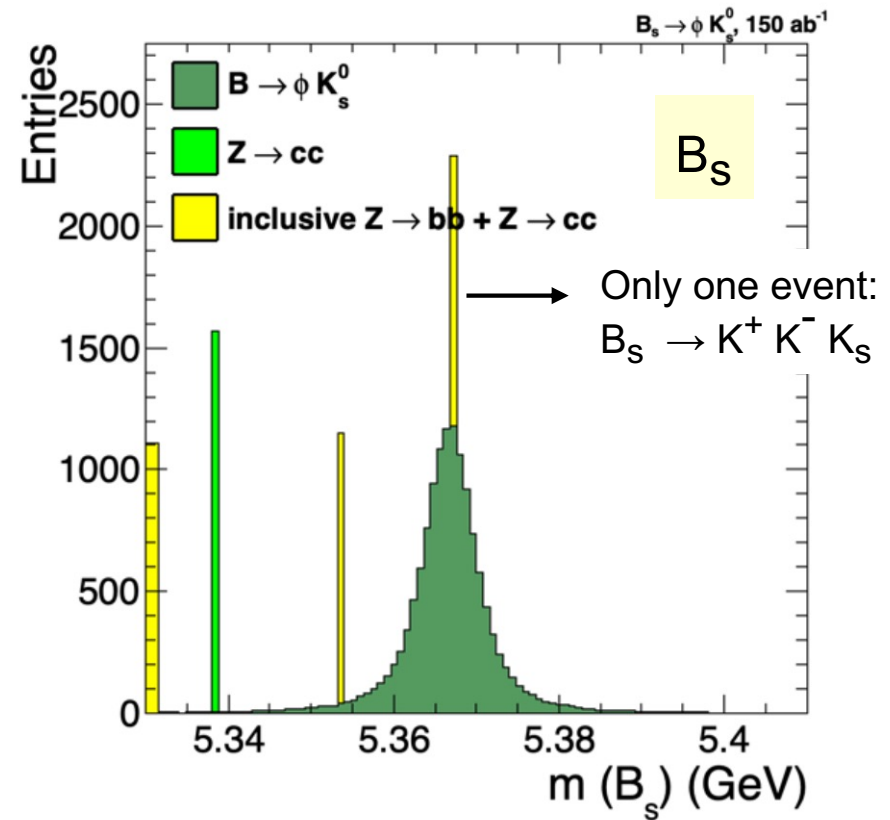
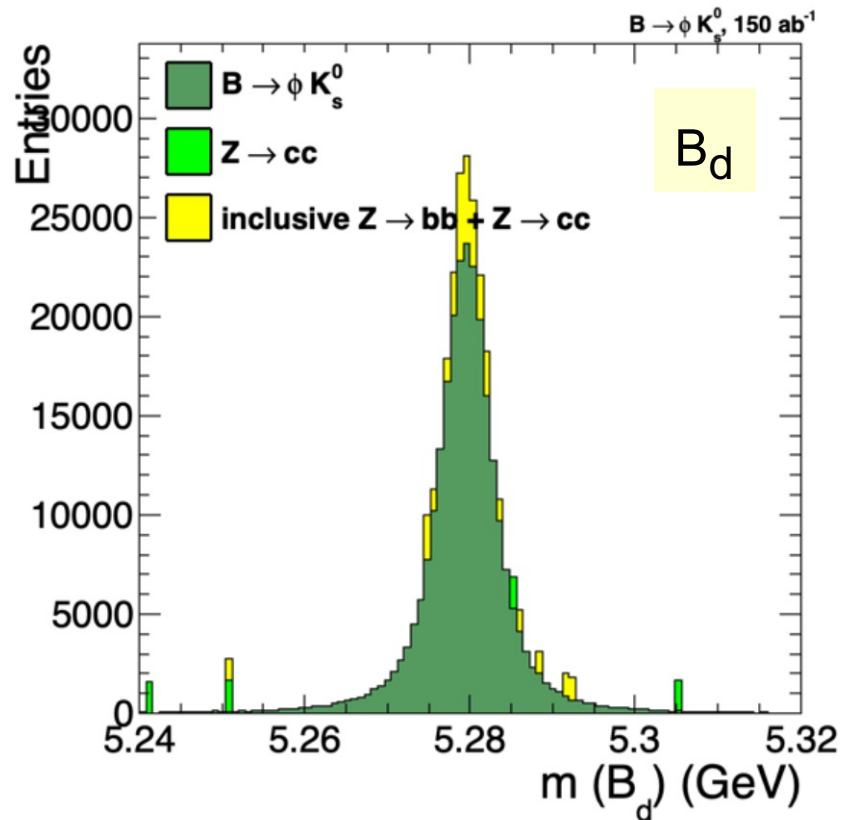


Final selection

Kinematic cuts to suppress combinatorial background:

- loose cuts on $p(\phi)$ and $p(K_S)$ (> 1 GeV)
- $p(B) > 10$ GeV
- reco'ed energy in signal hemisphere (w/o B candidate) < 28 GeV

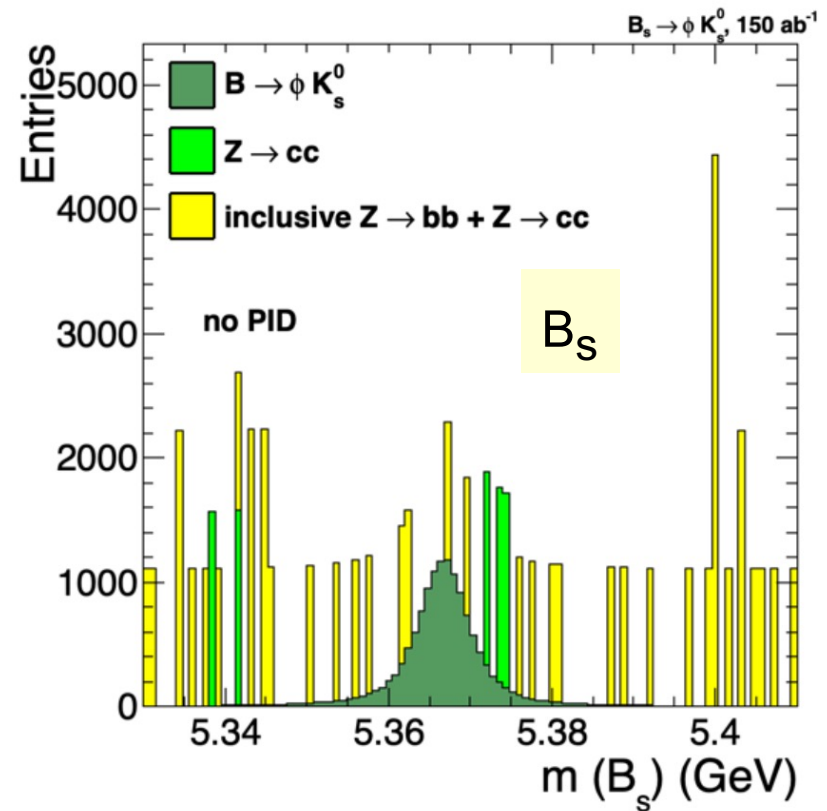
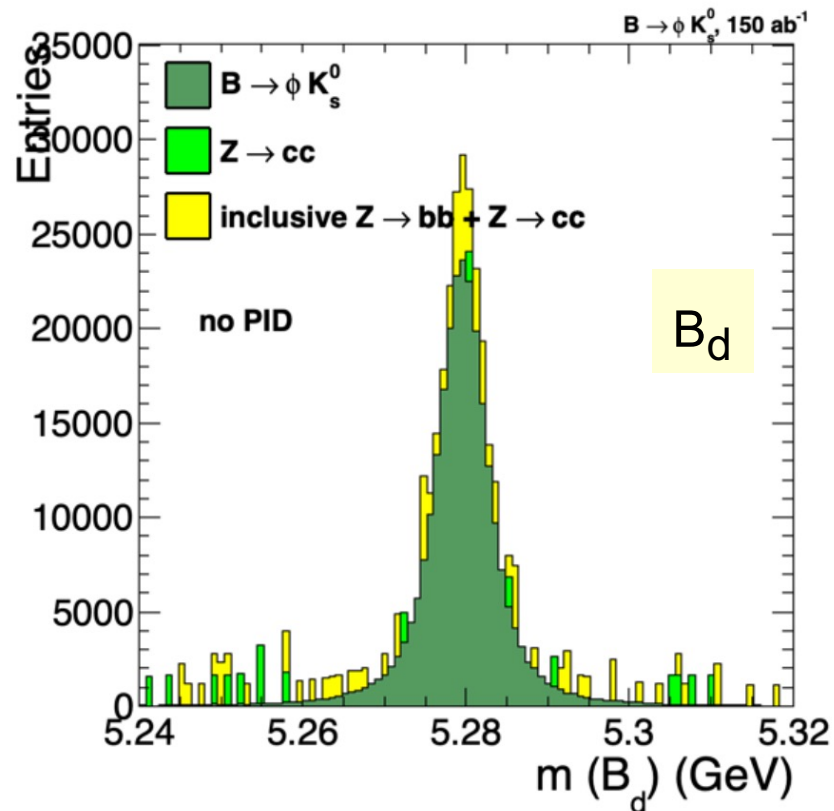
Final signal efficiency:
~ 40%



Low MC statistics for B_s ! Still, confidence that the background can be kept small.
Lower limit: $S / B > 5 - 6$ under the B_s peak.

Final selection (without any PID)

Perfect PID was assumed for the previous plots.
Assuming no PID requirement at all :

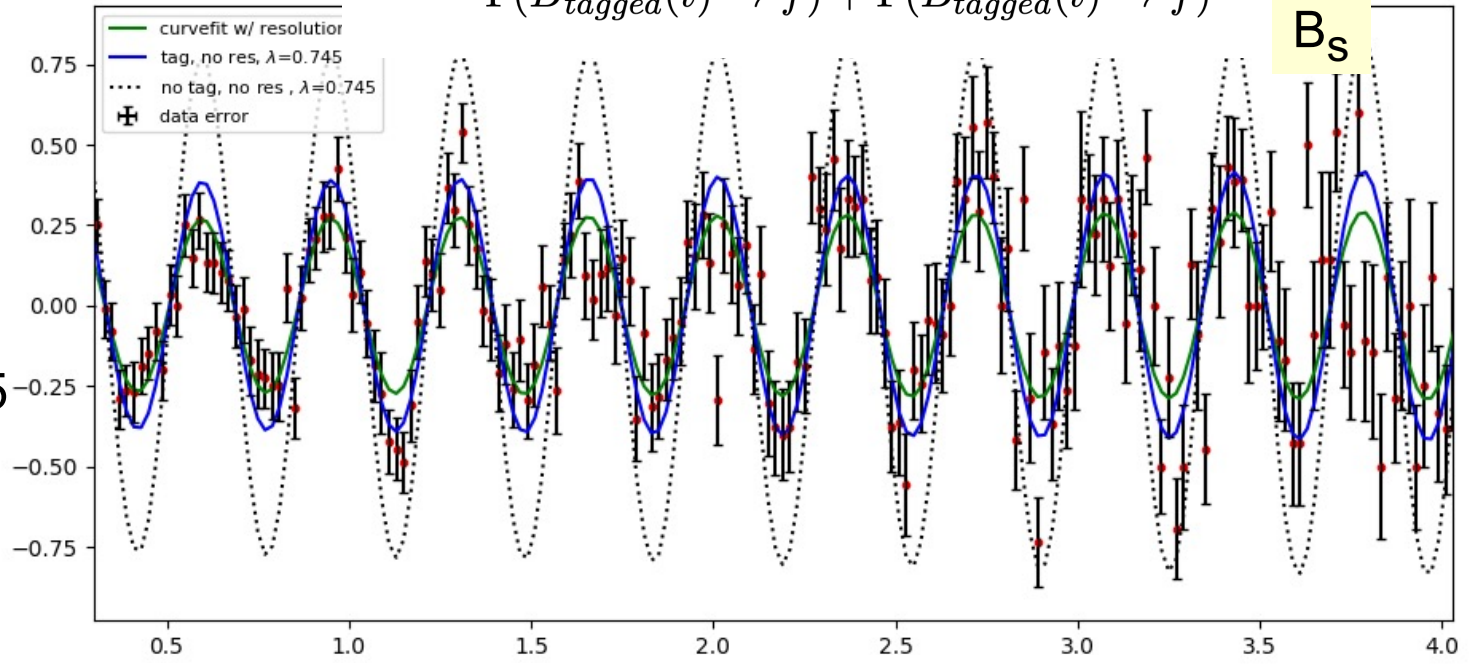


Not much effect for B_d . For B_s , PID is crucial – background would be as large as the signal without any PID.

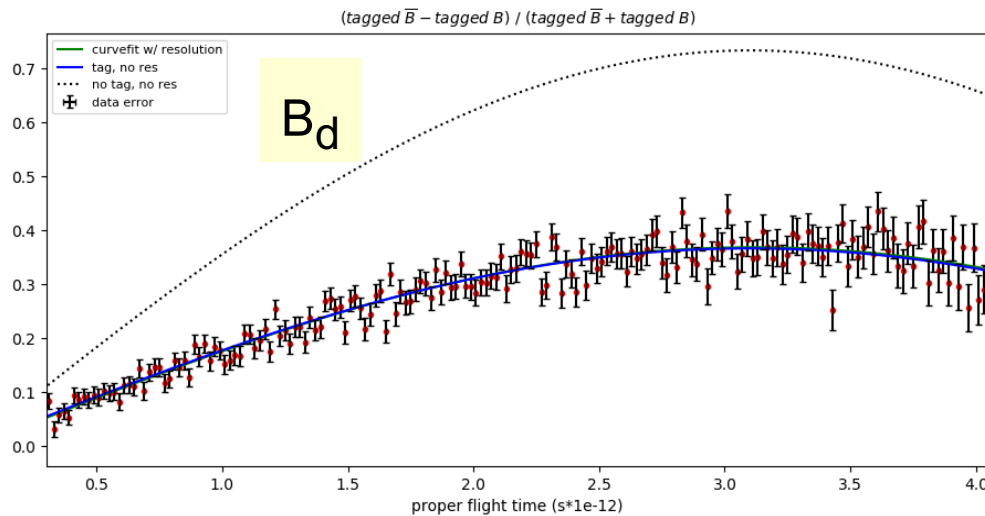
Expected sensitivity to CP parameters from ϕK_S

$$\mathcal{A}_S = \frac{\Gamma(\bar{B}_{tagged}(t) \rightarrow f) - \Gamma(B_{tagged}(t) \rightarrow f)}{\Gamma(\bar{B}_{tagged}(t) \rightarrow f) + \Gamma(B_{tagged}(t) \rightarrow f)}$$

- 150 ab^{-1}
- Number of signal decays: BRs given earlier + efficiency $O(40\%)$
- Mistag rate $\omega = 0.25$
- Neglect bckgd



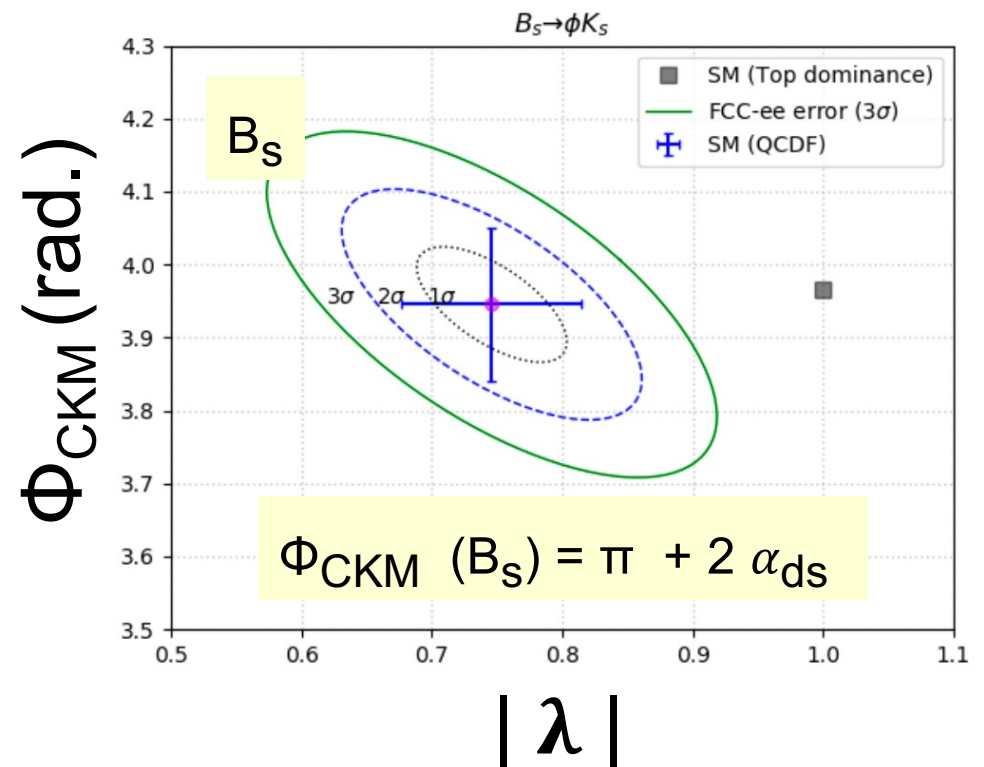
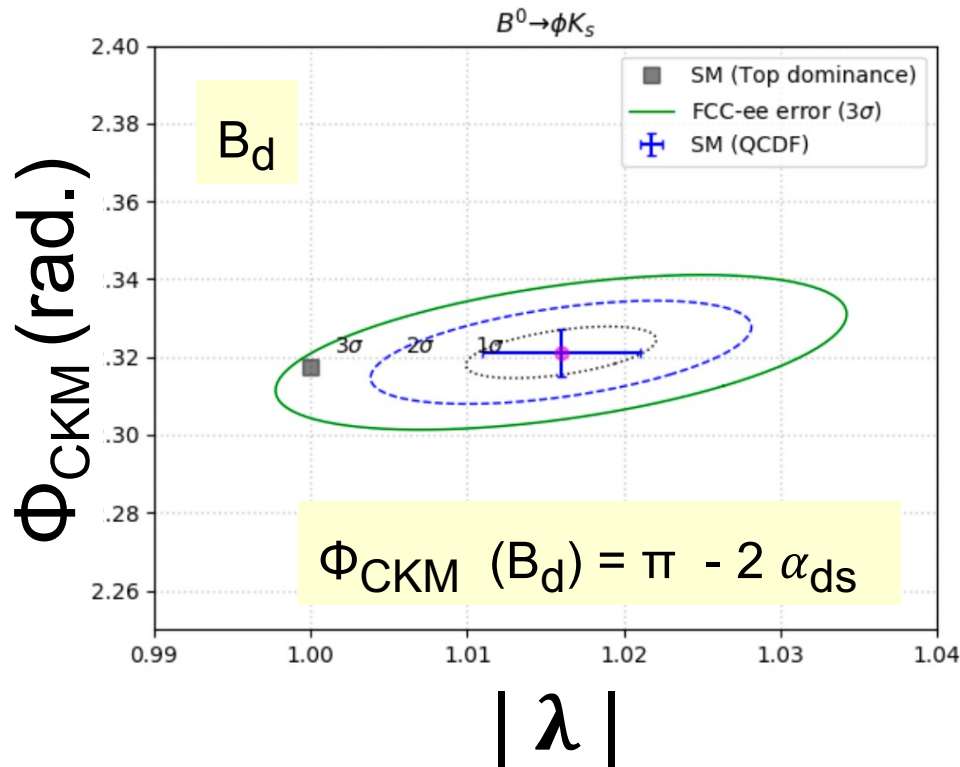
Proper time (ps)



- $\omega = 0$, perfect vertex
- $\omega = 0.25$, perfect vertex
- $\omega = 0.25$, vertex resolution

Expected sensitivity to CP parameters from ϕK_S

Combined fit of Φ_{CKM} and $|\lambda|$ ($|\lambda| \neq 1$ quantifies direct CPV) and Φ_{CKM}



- 4 mrad on α_{ds} from B_d
- 10x better sensitivity with B_d than with B_S

Addition of ϕK^* ($K^* \rightarrow K_S \pi^0$) decays

Efficiency for ϕK^* : assumes x2 lower than ϕK_S to account for π^0

$B \rightarrow VV$ decay : CP asymmetries are diluted if one does not separate the different polarisations of the vector mesons

ϕK_S

Decay	$\sigma(\lambda_L)$	$\sigma(\alpha_{ds})$
$\bar{B}_d \rightarrow \phi \bar{K}^0$	0.005	0.004
$\bar{B}_s \rightarrow \phi K^0$	0.039	0.045

ϕK^* (no amplitude analysis)

Decay	$\sigma(\lambda_L)$	$\sigma(\alpha_{ds})$
$\bar{B}_d \rightarrow \phi \bar{K}^{*0}$	0.012	0.022
$\bar{B}_s \rightarrow \phi K^{*0}$	0.07	0.14

Amplitude analysis can improve the sensitivity by O(2)

NB: An excellent ECAL resolution is mandatory (e.g. $3\% / \sqrt{E}$) for B_S to ϕK^* , otherwise the B_S signal is polluted by the B_d

Conclusions

- The angle α_{dS} of the “flattest” unitarity angle can be measured at FCC-ee via $B_{(S)}$ to ϕK_S and $B_{(S)}$ to ϕK^*
- Very interesting sensitivities expected !
 - α_{dS} to 4 mrad
- Importance of :
 - A good reconstruction of K_S decays up to large flight distance
 - Hence a large tracking volume
 - Excellent mass and vertex resolutions
 - Light tracker and highly performant vertex detector
 - PID crucial for the B_S
 - Mode with K^* demands in addition a powerful π^0 reconstruction and, for the B_S , an excellent ECAL resolution
- Precise determination of the background for the B_S would require 10 – 100x more Monte-Carlo statistics !
- Write-up is well advanced

Backup

Oscillation formulae including direct CPV

$$\begin{aligned}
 \Gamma(\overline{B}_q(t) \rightarrow f) &= N_{q,f} |A_{q,f}|^2 \left[\frac{1+\rho_q^2}{2} \right] e^{-\Gamma_q t} \times \\
 &\left[\cosh \frac{\Delta\Gamma_q t}{2} - A_{CP}^{dir} \cos(\Delta m_q t) + A_{\Delta\Gamma_q} \sinh \frac{\Delta\Gamma_q t}{2} - A_{CP}^{mix} \sin(\Delta m_q t) \right] \\
 \Gamma(B_q(t) \rightarrow f) &= N_{q,f} |A_{q,f}|^2 \left[\frac{1+\rho_q^2}{2} \right] e^{-\Gamma_q t} \times \\
 &\left[\cosh \frac{\Delta\Gamma_q t}{2} + A_{CP}^{dir} \cos(\Delta m_q t) + A_{\Delta\Gamma_q} \sinh \frac{\Delta\Gamma_q t}{2} + A_{CP}^{mix} \sin(\Delta m_q t) \right]
 \end{aligned} \tag{11}$$

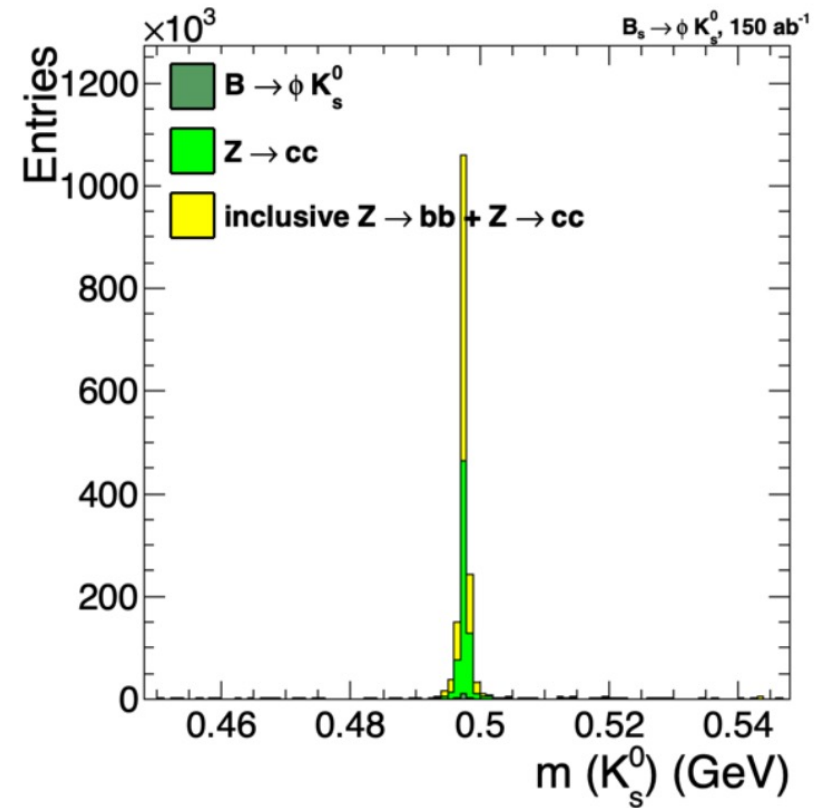
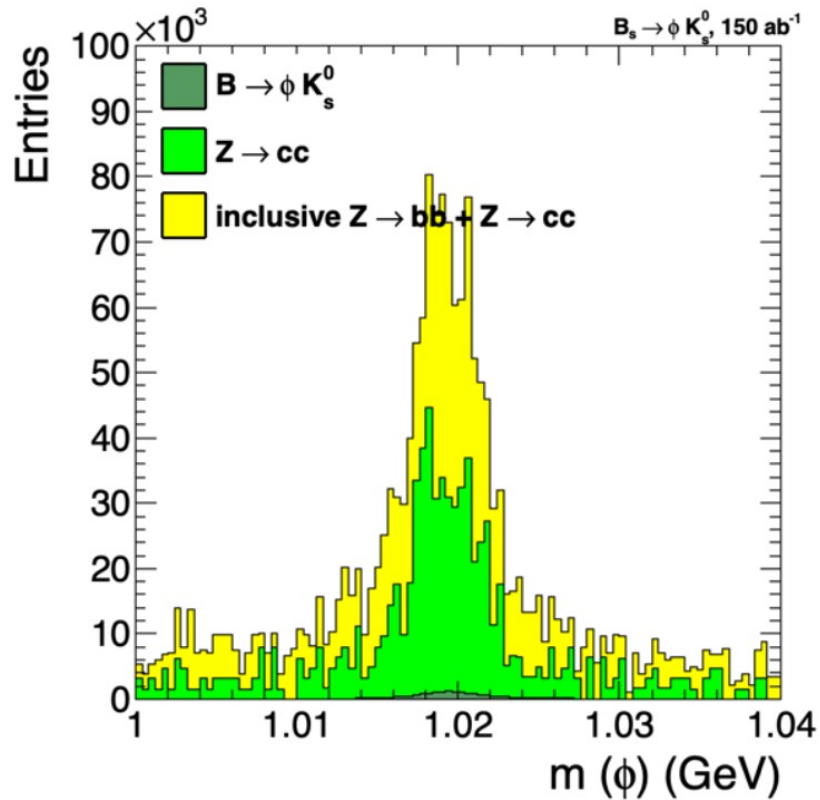
with

$$A_{CP}^{dir} = \frac{1-\rho_q^2}{1+\rho_q^2} \quad , \quad A_{\Delta\Gamma_q} = -\frac{2\text{Re}\lambda_{q,f}}{1+\rho_q^2} \quad , \quad A_{CP}^{mix} = -\frac{2\text{Im}\lambda_{q,f}}{1+\rho_q^2} \tag{12}$$

$$\Gamma(\overline{B}_{tagged}(t) \rightarrow f) = (1 - \omega)\Gamma(\overline{B}_q(t) \rightarrow f) + \omega\Gamma(B_q(t) \rightarrow f)$$

$$\Gamma(B_{tagged}(t) \rightarrow f) = (1 - \omega)\Gamma(B_q(t) \rightarrow f) + \omega\Gamma(\overline{B}_q(t) \rightarrow f)$$

Mass of Phi and Ks candidates, basic reconstruction



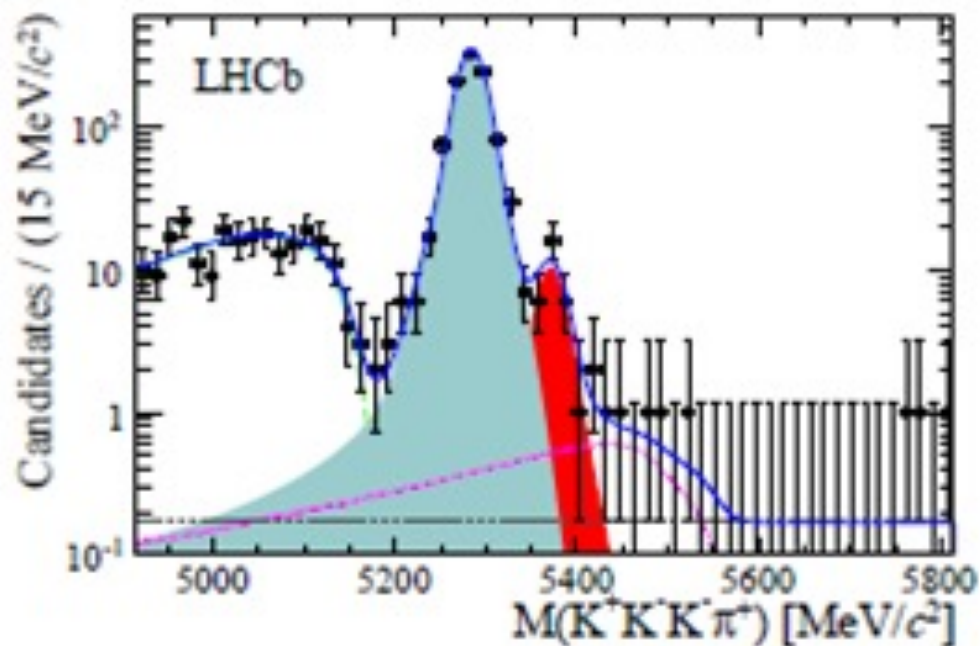
Full list of selection cuts

K_s leg:	$448 < m < 548$ MeV Vertex $\chi^2 < 10$ flight distance > 1 mm and < 1.5 m $p > 1.5$ GeV
ϕ leg:	$1.01 < m < 1.03$ GeV Vertex $\chi^2 < 10$ $p > 1$ GeV
perfect charged hadron PID for the K_s and ϕ tracks	
$B_{d,s}$ candidate:	the four tracks belong to the same hemisphere vertex $\chi^2 < 7.5$ $5.24 < m < 5.32$ GeV for B_d $5.33 < m < 5.41$ GeV for B_s $p > 10$ GeV
Energy in signal hemisphere: (without the B candidate)	below 28 GeV
no reconstructed $D_s^\pm \rightarrow \phi + \text{track} + X$ in signal hemisphere.	

Prospects before FCC ?

BELLE: with 50ab-1, 14x less B/Bbar than FCC with 150 ab-1.
With same efficiencies, uncertainty = 3.7 x larger than FCC

LHCb: hard to tell, have not reco'ed phi Ks
(have reco'ed phi K* with K* -> K+ pi-)



Yield of 1000 B0 and 30 Bs for 1 fb⁻¹.

Also, flavour tagging more difficult...