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Quantum Solution of Classical Turbulence

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Using the loop equation, we reduce the problem of decaying turbulence in the $3+1$ dimensional Navier-Stokes equation to the quantum mechanics of N Fermi particles on a ring in one dimension, interacting with an Euler ensemble of random fractions $\frac{p}{q}$ with denominator $q < N$. We find the solution of this system in the statistical limit $N \to \infty$ and compute the energy spectrum, dissipation rate, and velocity correlation function in decaying turbulence without approximations and fitted parameters. We find the whole spectrum of critical indexes, some of which are real, but others are complex numbers related to zeros of the Riemann ζ function. Grid turbulence experimental data and the recent large-scale DNS verify our predictions for the energy decay curve and the energy spectrum. **All scaling laws -K41, multifractal and Heisenberg – are ruled out.**

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Duality of Fluid Mechanics

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Our generation of theoretical physicists is accustomed to **building and solving models** based on **existing theories**.

These basic theories –**Classical Mechanics, Statistical Physics, Relativity, Quantum Mechanics, Quantum Field Theory**– were all built for us in previous centuries.

Statistical Theory of Turbulence? Still missing.

The phenomenological models like K41 of multifractal scaling laws all fall short of the microscopic theory we seek.

This theory must be built from the Navier-Stokes equations, like Gibbs Statistics was built from Newton's mechanics or the field theory was quantized by Dirac-Feynman sum over histories of classical field.

Turbulent statistics must emerge spontaneously from the (unknown) NS internal symmetry without ad hoc stochastic forces.

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This talk presents a new perspective on fluid mechanics, leading to such a statistical theory of turbulence.

By employing the loop equation, we reformulate fluid mechanics in arbitrary spatial dimensions as a singular one-dimensional problem.

This transformation is based on the concept of rough initial conditions in the Cauchy problem for the Navier-Stokes equation.

These rough initial conditions arise from thermal fluctuations and are inherent in any physical fluid. Consequently, physical fluid dynamics can be viewed as the evolution of a statistical distribution.

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The loop average is defined as the Fourier transform of the PDF for velocity circulation

$$\Psi[\gamma, C] = \left\langle \exp\left(\frac{\imath\gamma}{\nu}\Gamma_C\right)\right\rangle; \quad (1)$$

$$\Gamma_C = \oint d\vec{C}(\theta) \cdot \vec{v}(\vec{C}(\theta)); \quad (2)$$

This is a particular case of the Hopf functional

$$\Psi[\gamma, C] = \left\langle \exp\left(\int_{\vec{r}\in\mathbb{R}_d} \vec{J}_C(\vec{r}) \cdot \vec{v}(\vec{r})\right)\right\rangle \quad (3)$$

with an imaginary source $\vec{J}(\vec{r})$ concentrated on a fixed loop in space

$$\vec{J}_C(\vec{r}) = \frac{\imath\gamma}{\nu} \oint d\vec{C}(\theta)\delta\left(\vec{r} - \vec{C}(\theta)\right) \quad (4)$$

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We derived a closed functional equation for the loop average in incompressible Navier-Stokes equation**M93, M23PR**

$$\imath\nu\partial_t\Psi[\gamma, C] = \left\langle \gamma \oint d\vec{C}(\theta) \cdot \left(-\nu\vec{\nabla}\times\vec{\omega} + \vec{v}\times\vec{\omega}\right) \exp\left(\frac{\imath\gamma}{\nu}\Gamma_C\right)\right\rangle = \oint d\vec{C}(\theta)\cdot\vec{L}\left[\frac{\delta}{\delta\vec{C}(.)}\right]\Psi[\gamma, C] \quad (5)$$

The operator $\vec{L}\left[\frac{\delta}{\delta\vec{C}(.)}\right]$ only depends on the functional derivative, but does not depend on the coordinate $\vec{C}(.)$ in loop space.

This independence (translation invariance) is the key to the solution. External forces would break it.

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This equation is equivalent to the Schrödinger equation in loop space with Hamiltonian $\hat{H}_C = \oint d\vec{C}(\theta) \cdot \vec{L}\left[\frac{\delta}{\delta \vec{C}(.)}\right]$.

A plane wave in loop space solves this Schrödinger equation

$$\Psi[\gamma, C] = \left\langle \exp\left(\frac{\imath\gamma}{\nu} \oint d\vec{C}(\theta) \cdot \vec{P}(t,\theta)\right)\right\rangle; \tag{6}$$

$$\imath\gamma\partial_t \vec{P} = \vec{L}\left[-\imath\frac{\gamma}{\nu}\partial_\theta \vec{P}(t,\theta)\right]; \tag{7}$$

$$\nu\partial_t \vec{P} = -\gamma^2 (\Delta\vec{P})^2 \vec{P} + \Delta\vec{P}\left(\gamma^2 \vec{P}\cdot\Delta\vec{P} + \imath\gamma\left(\frac{(\vec{P}\cdot\Delta\vec{P})^2}{\Delta\vec{P}^2} - \vec{P}^2\right)\right); \tag{8}$$

with $\Delta\vec{P} = \vec{P}(\theta+0) - \vec{P}(\theta-0), \bar{P} = \frac{\vec{P}(\theta+0)+\vec{P}(\theta-0)}{2}$.

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The sum goes over the initial data distribution for $\vec{P}(0, \theta)$ (which is induced by thermal fluctuations of initial velocity).

In Newton's mechanics, the trajectory eventually covers the energy surface (ergodicity). We expect the loop momentum trajectory $\vec{P}(t, \theta)$ to cover some universal manifold (decaying turbulence trajectory).

This analogy with QM is not a computational trick nor a poetic metaphor: this is an exact mathematical equivalence leading to observable quantum oscillations in the decaying energy spectrum.

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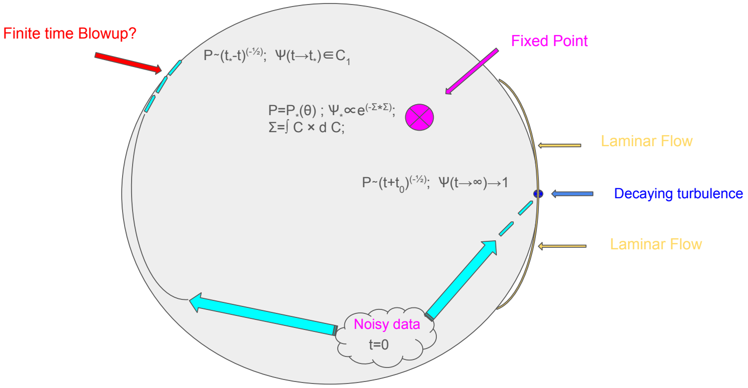
Asymptotic regimes of the loop equation

Loop Functional Evolution



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Decaying turbulence solution

Exact solution for decaying turbulence

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Surprisingly, an infinite family of analytic solutions of this singular nonlinear equation was found.

$$\frac{\gamma \vec{P}(t,\theta)}{\nu} = \frac{1}{\sqrt{2\nu(t+t_0)}}\hat{\Omega}\cdot\vec{F}(\theta);\ \hat{\Omega}\in O(3); \tag{9}$$

$$\vec{F}_k = \frac{\left\{\cos(\alpha_k), \sin(\alpha_k), i\cos\left(\frac{\beta}{2}\right)\right\}}{2\sin\left(\frac{\beta}{2}\right)}; \tag{10}$$

$$\theta_k = \frac{2\pi k}{N};\ \beta = \frac{2\pi p}{q};\ N\to\infty; \tag{11}$$

$$\alpha_{k+1} = \alpha_k + \sigma_k\beta;\ \sigma_k = \pm 1,\ \beta\sum\sigma_k = 2\pi p r; \tag{12}$$

The parameters $\hat{\Omega}, N, q, r, \sigma_0 \dots \sigma_{N-1}$ are arbitrary, making this solution for $\vec{F}(\theta)$ a universal ensemble of integer numbers (Euler ensemble).

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The statistical limit of the Euler ensemble $N \to \infty, \nu \to 0, \tilde{\nu} = \nu N^2 = \textit{ const }$ can be computed in quadrature.

The solution is based on the transformation of this ensemble to the quantum trace of N fermions on a ring coupled with random Euler fractions $\frac{p}{q}$.

Here is the resulting formula for the second moment of velocity difference in decaying turbulence:

$$\left\langle \Delta \vec{v}^2 \right\rangle (r) = \frac{\tilde{\nu}^2}{\nu t} \int_{\epsilon - \imath\infty}^{\epsilon + \imath\infty} \frac{dp}{2\pi\imath} V(p) \left(\frac{|\vec{r}|}{\sqrt{\tilde{\nu} t}} \right)^p ; \tag{13}$$

$$V(p) = -\frac{f(-1-p)\zeta\left(\frac{13}{2} - p\right)\csc\left(\frac{\pi p}{2}\right)}{16\pi^2 (p+1)(2p-15)(2p-5)\zeta\left(\frac{15}{2} - p\right)} \tag{14}$$

Here $f(z)$ is an entire function computed using Mellin integrals of elementary functions. $V(p)$ is meromorphic. ν is physical viscosity and turbulent viscosity $\tilde{\nu}$ is a free parameter of our solution.

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The spectrum p_n of poles $V(p)$ determines the scaling dimensions in the correlation function's expansion in $\left(|\vec{r}|/\sqrt{\tilde{\nu}t}\right)^{p_n}$.

indexes of velocity correlation
-1
0
$2n$ if $n \in \mathbb{Z} \wedge n \geq 1$
$5/2$
$11/2$
$7 \pm \imath t_n$ if $n \in \mathbb{Z} \wedge n > 0$
$1/2(15 + 4n)$ if $n \in \mathbb{Z} \wedge n \geq 0$

Here $\pm t_n$ are Riemann zeros of $\zeta(1/2 + \imath t)$.

Imaginary parts of dimensions lead to quantum oscillations.

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The singularities of the Mellin transform $h(p)$ for the energy spectrum are defined as follows:

$$E(k,t) = \frac{\tilde{\nu}^{5/2}}{\nu\sqrt{t}} \int_{-\epsilon-\imath\infty}^{-\epsilon+\imath\infty} \frac{dp}{2\pi\imath} h(p) \left(k\sqrt{\tilde{\nu}t}\right)^p;$$

These singularities are given by the following table of simple poles:

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$$E(t) = \int_{-1-\epsilon-\imath\infty}^{-1-\epsilon+\imath\infty} \frac{dq}{2\pi\imath} e(q) \left(k_0^2 \tilde{\nu} t\right)^q ;$$

$$e(q) = 2\pi k_0^2 \tilde{\nu}^2 \frac{h(2q-1)}{q(q+1)}$$

The table of complex poles of this function is:

indexes of energy decay
$-\,{}^{5}\!/_{4}$
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$-{}^{7}\!/_{2} \pm {}^{\imath t_n}\!/_{2}$ if $n \in \mathbb{Z} \wedge n > 0$
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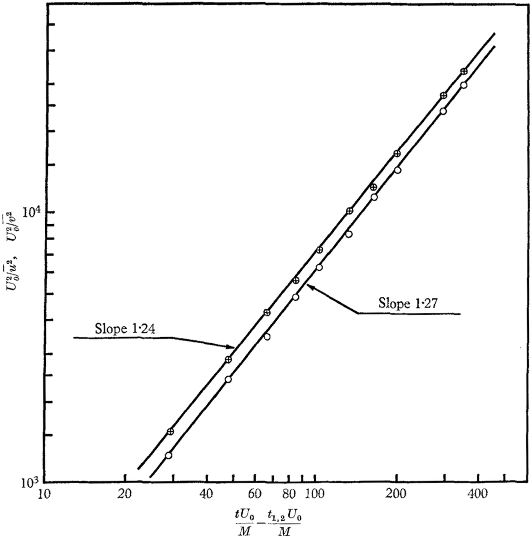

Reynolds number decay in DNS

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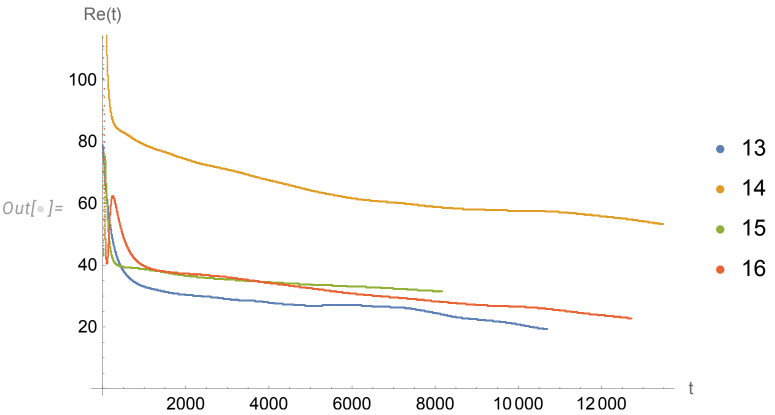

The time decay of Reynolds numbers for each of the four samples from **SreeniDecaying, GDSM24**

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The next test is the effective length scale, which we define as

$$L(t) = \frac{\int E(k,t) k dk}{\int E(k,t) k^2 dk}; \tag{15}$$

The log-log plot of $L(t)$ is shown in Fig.21. In the turbulent region, it perfectly matches our theory $L(t) \propto \sqrt{t}$.

We plot $E(t)$ as a function of $L(t)$

$$E(t) = \int E(k,t) dk; \tag{16}$$

$$\log E(t) \approx a + f(b + \log L(t)) \tag{17}$$

The parameters a, b were fitted by nonlinear regression using "NonlinearModelFit" in *Mathematica®*. The resulting log-log plot is shown in(Fig. 22).

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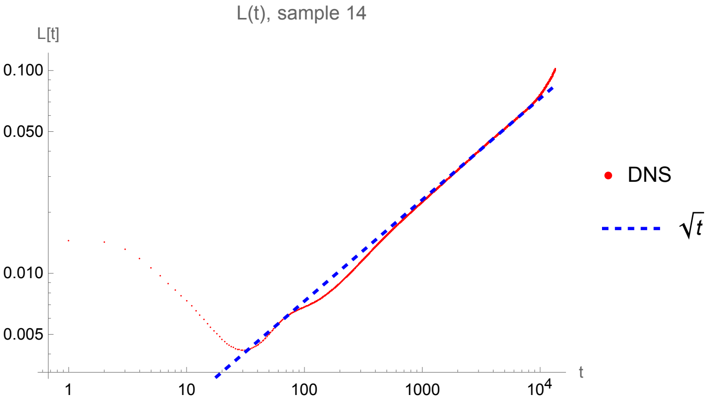

The log-log plot of the effective length $L(t)$ for the sample 14 from **SreeniDecaying, GDSM24**. The turbulent part, $1000 < t < 8000$, closely fits our theory $L(t) \propto \sqrt{t}$.

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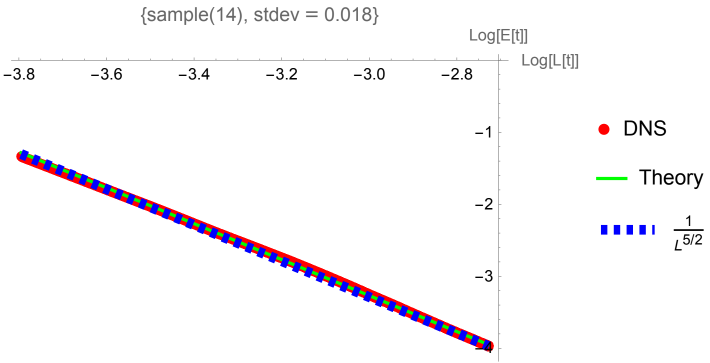

The log-log plot of the decaying energy as a function of decaying length scale $L(t)$. Red dots are the DNS data, the green curve is an exact theoretical curve, and the dashed blue line is its asymptotic limit $E(t) \propto L(t)^{-5/2}$ corresponding to $E \propto t^{-5/4}$.

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We have a scaling law $E(k,t) = \frac{H(k\sqrt{t+t_0})}{\sqrt{t+t_0}}$, which means that the two-dimensional array of the data for $E(k,t)$ must collapse at one-dimensional subset.

We already saw the consequence of that collapse in the scaling law for $L(t)$. However, the low k part of the spectrum is discrete, which introduces lattice artifacts.

We found the following method to avoid choosing the range of discrete wavelengths and suppress statistical errors and lattice artifacts.

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Verifying second velocity moment by the DNS data

Now, consider the second moment of velocity, related to the energy spectrum by Fourier transform

$$\left\langle \Delta \vec{v}^2 \right\rangle (r) = \int \frac{d^3 k}{4\pi^3} \frac{\left(1 - e^{\imath \vec{k}\cdot\vec{r}}\right) E(k,t)}{4\pi \vec{k}^2}; \tag{18}$$

There is a sharper test, namely the effective index $\xi_2(r,t)$ defined as a log-log derivative of this second moment

$$\xi_2(r,t) = r\partial_r \log \left\langle \Delta v^2 \right\rangle (r) = \frac{\int_0^\infty dk \left(-\cos(kr) + \frac{\sin(kr)}{kr}\right) E(k,t)}{\int_0^\infty dk \left(1 - \frac{\sin(kr)}{kr}\right) E(k,t)}; \tag{19a}$$

$$\left\langle \xi_2(xL(t),t)\right\rangle_t = f(x); \tag{19b}$$

This universal function $f(x)$ is numerically well defined as the integration over the spectrum suppresses the DNS noise.

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Verifying second velocity moment by the DNS data

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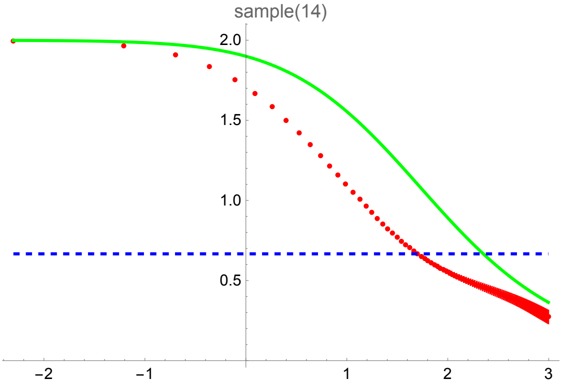

The plot of $f(x)$ in (19) as a function of $\log x$. The red dots with error bars are the DNS data. The green curve is the theoretical index without adjustment of the coordinate scale, and the dashed blue line is the K41 constant value $\frac{2}{3}$.

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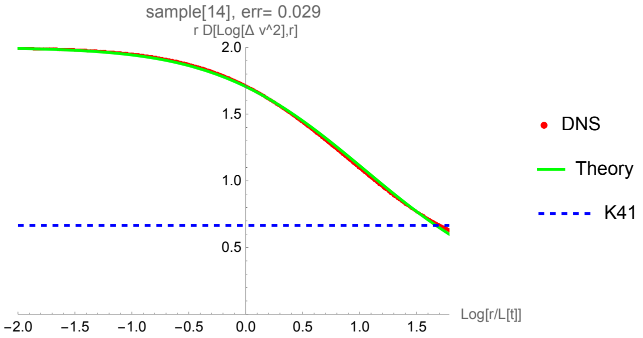

The plot of the effective index $f(x)$ in (19) as a function of $\log x$ in the turbulent range. Red dots with error bars are the DNS data. The K41 $\frac{2}{3}$ law (blue dashed line) is off the charts, our theory (green curve) fits perfectly.

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- We have unveiled a duality between decaying classical turbulence in 3+1 dimensions and the one-dimensional quantum theory of $N \to \infty$ Fermi particles on a ring.
- The density fluctuations of these Fermi particles disappear in the turbulent limit $\nu \sim 1/N^2 \to 0$, leading to the exact WKB (instanton) solution for this density.
- This establishes a new relation between classical nonlinear dynamics and quantum theory: the Fourier transform of classical probability adheres to the QM evolution with the interference of alternative histories.
- The spectrum of the turbulence scaling dimensions is related to the zeros of the Riemann ζ function.
- The grid-turbulence experiments and recent DNS rule out K41, multifractal, and Heisenberg scaling laws but confirm this theory.

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