System size analysis of the fireball produced in heavy-ion collisions

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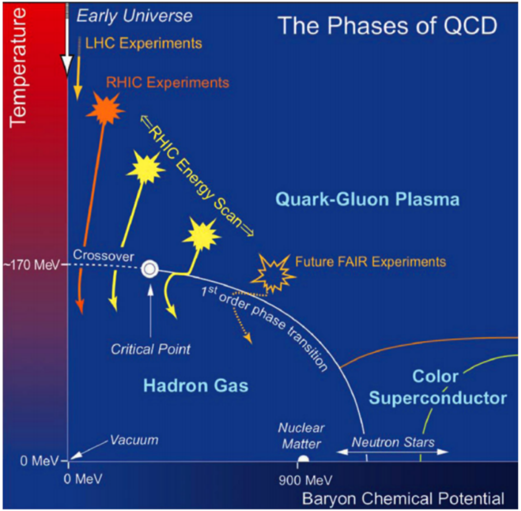
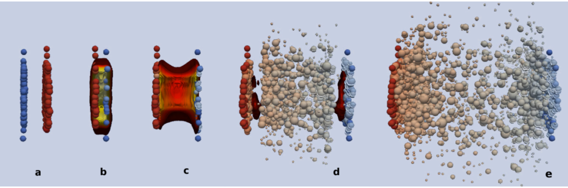
QCD phase diagram

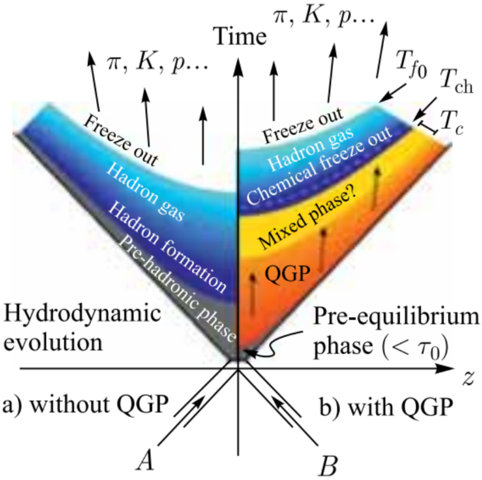
Parameters:

- Temperature T
- Baryon chemical potential μ



Evolution of quark-gluon plasma





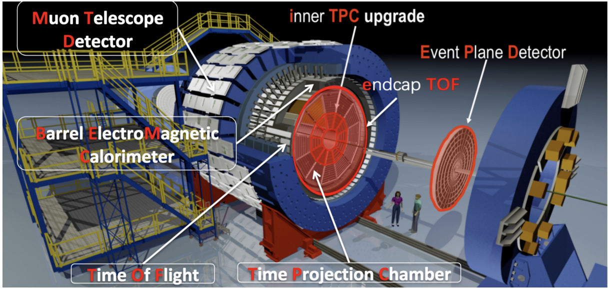
Stages of ultrarelativistic nuclear collision:

a) Nuclei go through each other
b) Expansion
c) Formation of hot matter
d) Hadron gas
e) Final patricles

The STAR experiment

TPC (Time Projection Chamber)

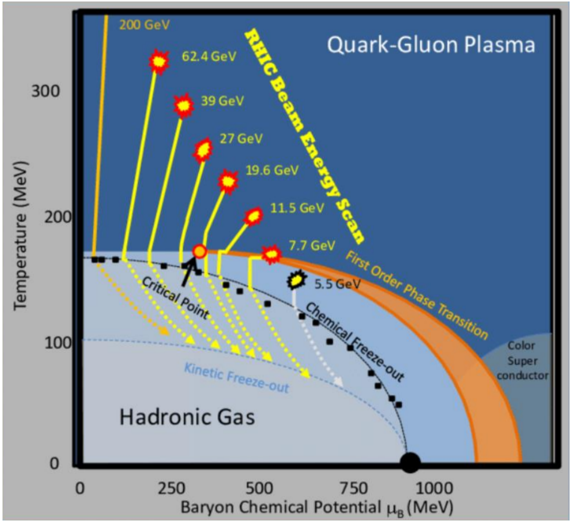
- used for tracking and identification
- length 4.2 m, diameter 4 m (1 m),
- azimuthal angle 2π
- pseudorapidity range $|\eta| < 1$
- in a magnetic field 0.5 Tesla



Beam Energy Scan program on RHIC

Used data:

- RHIC BES-I, 2010-2011
- $Au + Au \sqrt{S_{NN}} = 7.7 - 39$ GeV.
- Phys.Rev.C 96 (2017) 044904, Phys.Rev.C 92 (2015) 014904.



Tsallis-3 statistics (J.Phys.G:Nucl.Part.Phys.50 2023 125002)

- Tsallis entropy:

$$S = \sum_i \frac{p_i^q - p_i}{1-q}, \ \sum_i p_i = 1,$$

где p_i - probability ith microscopic state of the system, $q \in [0, \infty]$.

- In the Gibbs limit $q \to 1$ the entropy recovers the Boltzmann-Gibbs entropy:

$$S = \sum_i p_i \ln p_i$$

- In Grand Canonical Ensemble thermodynamic potential Ω takes the form:

$$\Omega = \langle H \rangle - TS - \mu \langle N \rangle, \text{ where}$$

$$\langle H \rangle = \frac{1}{\theta} \sum_i p_i^q E_i, \ \langle N \rangle = \frac{1}{\theta} \sum_i p_i^q N_i, \ \theta = \sum_i p_i^q.$$

Tsallis-3 statistics

Let's consider a relativistic ideal gas in the Grand Canonical Ensemble. From the principle of thermodynamic equilibrium (the principle of maximum entropy), normalization expressions for the parameters $\Lambda = -\theta T S + \langle H \rangle - \mu \langle N \rangle$ и $\theta = \sum_i p_i^q$ can be derived:

$$\begin{cases} 1 = \sum\limits_{n=0}^{n_0} \dfrac{\omega^n}{n!\,\Gamma\left(\frac{1}{q-1}\right)} \int\limits_0^\infty t^{\frac{2-q}{q-1}-n} e^{-t+\beta'(\Lambda+\mu n)} (\mathrm{K}_2(\beta' m))^n \, \mathrm{d}t \,, \\ \theta = \sum\limits_{n=0}^{n_0} \dfrac{\omega^n}{n!\,\Gamma\left(\frac{q}{q-1}\right)} \int\limits_0^\infty t^{\frac{1}{q-1}-n} e^{-t+\beta'(\Lambda+\mu n)} (\mathrm{K}_2(\beta' m))^n \, \mathrm{d}t \,, \end{cases} .$$

where $\omega = \dfrac{gV}{2\pi^2} \dfrac{m^2 T \theta^2}{q-1}$, $\beta' = \dfrac{-t(1-q)}{T\theta^2}$,

n_0 - the number of terms to be taken into account, counting from zero.

Tsallis-3 statistics

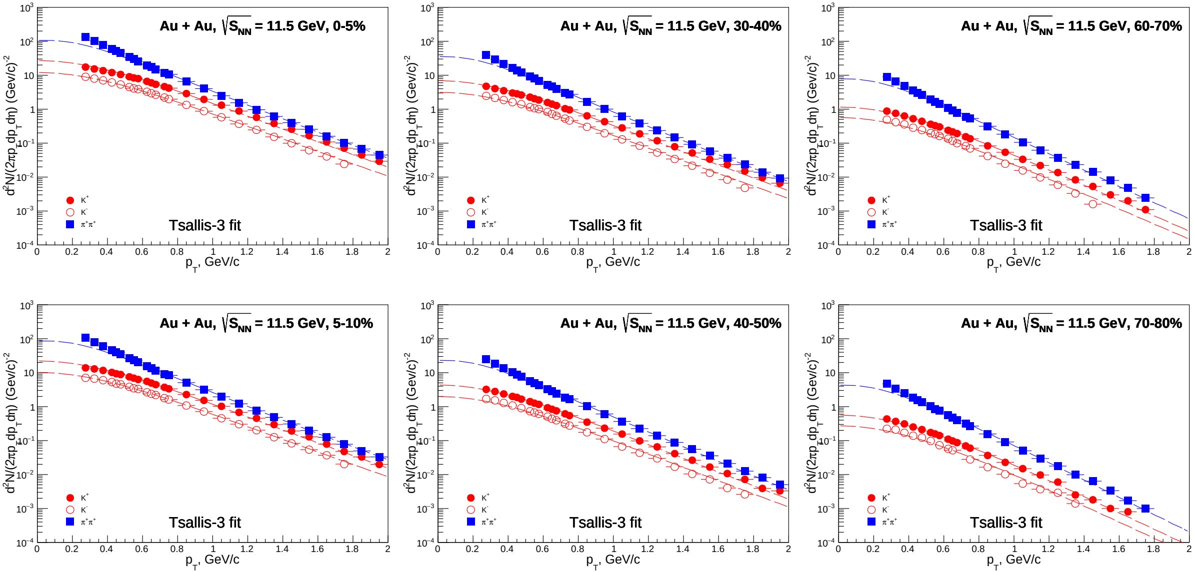
- The expression for transverse momentum distribution of particles of relativistic ideal gas in the grand canonical ensemble in rapidity range $y \in [y_{\text{min}}, y_{\text{max}}]$ takes the form:

$$\left.\frac{\mathrm{d}^2 N}{p_T\,\mathrm{d}p_T\,\mathrm{d}y}\right|_{y_{\text{min}}}^{y_{\text{max}}} = \frac{gV}{(2\pi)^2} m_T \int\limits_{y_{\text{min}}}^{y_{\text{max}}} \mathrm{d}y \cosh y \times$$

$$\times \frac{1}{\theta} \sum_{n=0}^{n_0} \frac{\omega^n}{n!\,\Gamma\left(\frac{q}{q-1}\right)} \int\limits_0^\infty t^{\frac{1}{q-1}-n} e^{-t+\beta'(\Lambda - m_T \cosh y + \mu(n+1))} (\mathrm{K}_2(\beta' m))^n \,\mathrm{d}t$$

- In this work: $n_0 = 1$, $\mu = 0$.

Tsallis-3 statistics

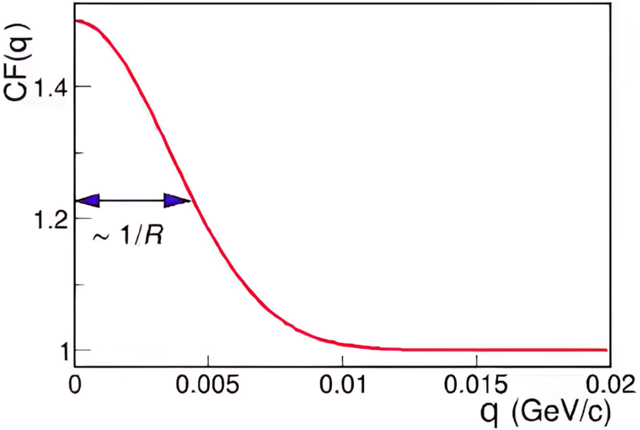


Femtoscopic approach

- Correlation function:

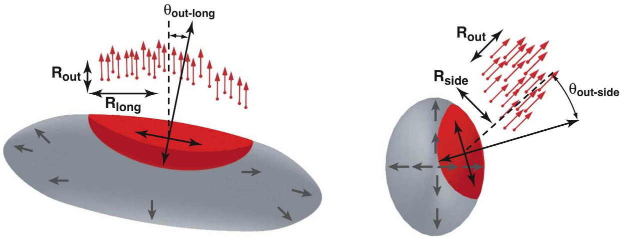
$$C(\vec{q}) = \frac{N(\vec{q})}{D(\vec{q})},$$

  - $N(\vec{q})$ - uses pairs from the same event, contains quantum statistic correlations and interactions,
  - $D(\vec{q})$ - uses pairs from different events, contains only background.



Femtoscopic approach

- Bertsch-Pratt out-side-long coordinate system:
  - q_{long} - along the beam direction
  - q_{out} - along the transverse momentum of the pair
  - q_{side} - perpendicular to longitudinal and outward directions



- Gaussian parameterization (Bowler-Sinyukov):

$$C(\vec{q}) = (1-\lambda) + K_{\text{Coul}}(q_{\text{inv}})\lambda \exp\left(-q_o^2 R_o^2 - q_s^2 R_s^2 - q_l^2 R_l^2 - 2q_o q_s R_{os}^2 - 2q_o q_l R_{ol}^2\right),$$

  - λ - correlation strength
  - $K_{\text{Coul}}(q_{\text{inv}})$ - Coulomb correction factor
  - $R_{\text{long}} \sim$ source lifetime
  - $R_{\text{out}} \sim$ - geometric size and emission duration
  - $R_{\text{side}} \sim$ - geometric size
  - $R_{\text{os,ol}}$ - crossterms

Femtoscopic approach

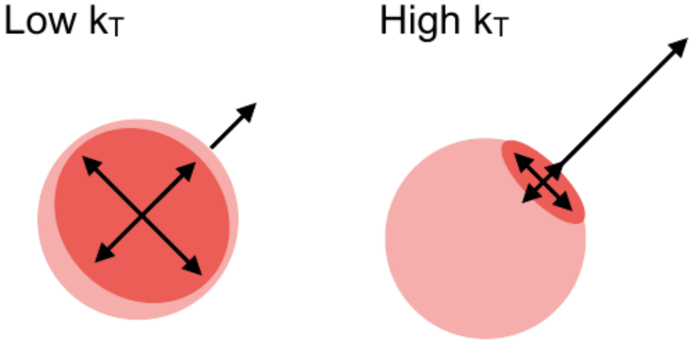
Parameterizations:

- m_T scaling: $R_i \propto \frac{1}{\sqrt{m_T}}$
- Power law: $R_i \propto m_T^{-\alpha}$
- Blast-Wave:

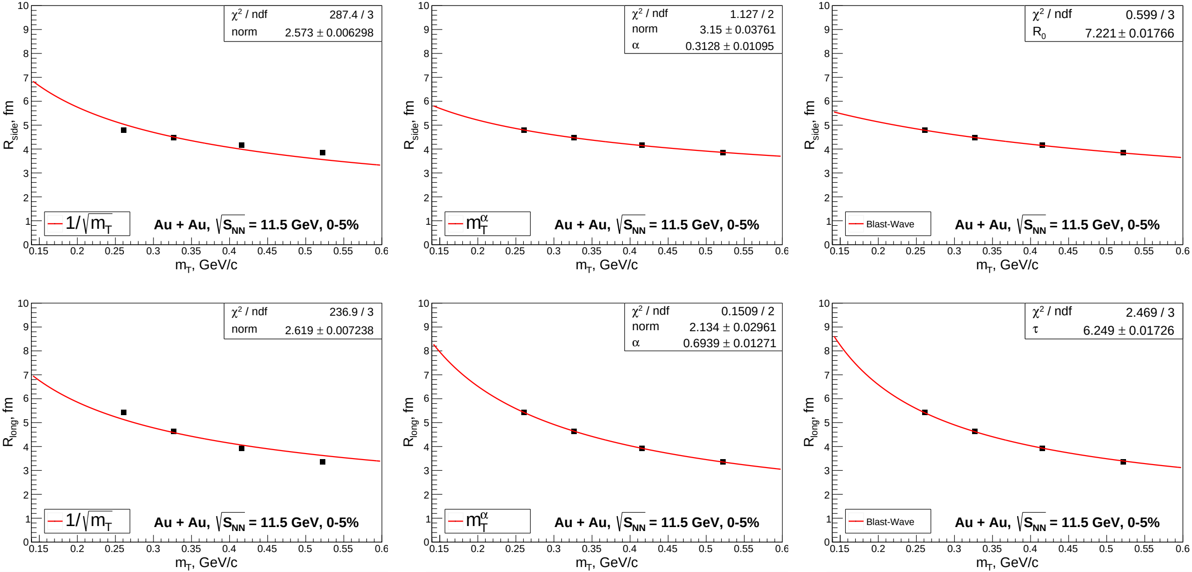
$$R_{\text{side}} = \frac{R_0}{\sqrt{1+\rho_0^2(m_T/T)}},$$

$$R_{\text{long}} = \tau\sqrt{\frac{T}{m_t}\frac{\mathrm{K}_2(m_T/T)}{\mathrm{K}_2(m_T/T)}}$$

(Phys.Rev.C51:328-338 1995)



Femtoscopic approach



Femtoscopic approach

Parameterizations:

- Recalculate gaussian radii to uniform distribution:

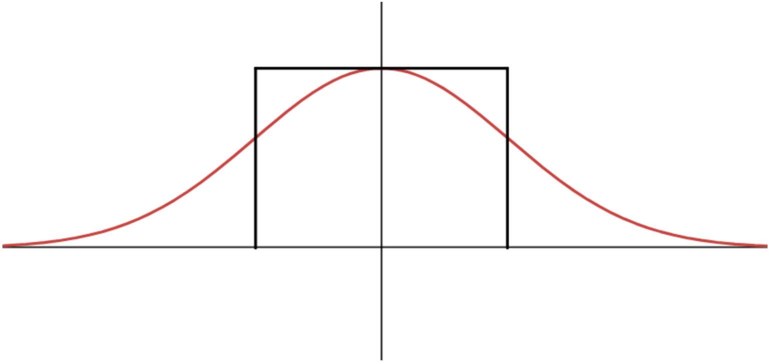
$$\sigma_{\text{gauss}} = \sigma_{\text{uniform}}$$

$$R_{\text{gauss}} = \sqrt{\frac{1}{12}} R_{\text{uniform}}$$

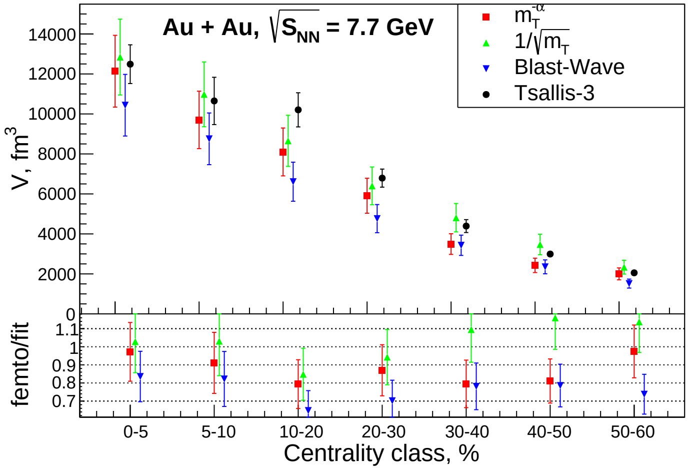
- Calculate the volume:

$$V = R_{\text{side}}^2 R_{\text{long}}$$

(assumption: $R_{\text{out}} \approx R_{\text{side}}$)

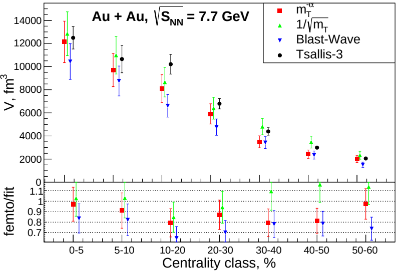


Results



Au + Au, $\sqrt{S_{NN}}$ = 7.7 GeV
$m_T^{-\alpha}$
$1/\sqrt{m_T}$
Blast-Wave
Tsallis-3
V, fm^3
14000
12000
10000
8000
6000
4000
2000
0
femto/fit
1.1
1
0.9
0.8
0.7
0-5
5-10
10-20
20-30
30-40
40-50
50-60
Centrality class, %

Results



Au + Au, $\sqrt{S_{NN}}$ = 7.7 GeV
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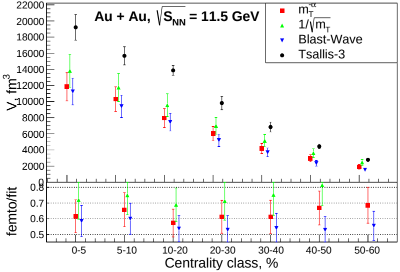
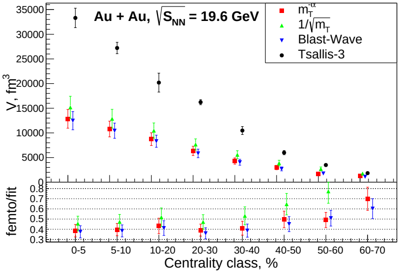
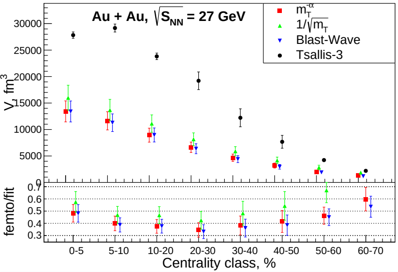


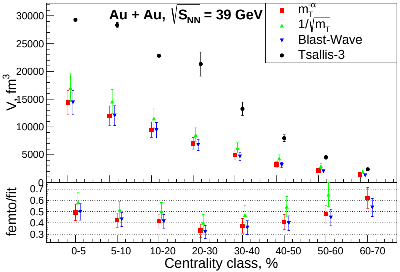
Au + Au, $\sqrt{S_{NN}}$ = 11.5 GeV
$m_T^{-\alpha}$
$1/\sqrt{m_T}$
Blast-Wave
Tsallis-3
V, fm^3
femto/fit
Centrality class, %



Au + Au, $\sqrt{S_{NN}}$ = 19.6 GeV
$m_T^{-\alpha}$
$1/\sqrt{m_T}$
Blast-Wave
Tsallis-3
V, fm^3
femto/fit
Centrality class, %



Au + Au, $\sqrt{S_{NN}}$ = 27 GeV
$m_T^{-\alpha}$
$1/\sqrt{m_T}$
Blast-Wave
Tsallis-3
V, fm^3
femto/fit
Centrality class, %



Au + Au, $\sqrt{S_{NN}}$ = 39 GeV
$m_T^{-\alpha}$
$1/\sqrt{m_T}$
Blast-Wave
Tsallis-3
V, fm^3
femto/fit
Centrality class, %

Discussions

- Correlations measure only the region of homogeneity. If the collective expansion is strong, then the region of homogeneity can be smaller than the entire source volume.
- Correlation function can deviate from Gaussian by exponential tails caused by resonance decay contributions. This may lead to underestimation of femtoscopic radii.
- In the Tsallis-3 statistics, the system is considered as an ideal gas, so particle interactions and collective effects, including rescattering, are not taken into account.

Conclusion

- Two approaches: femtoscopy and statistical model.
- In the first approach the fireball is considered as a thermodynamic system in the Grand Canonical Ensemble.
- Second method allows the measurement of the spatio-temporal properties of the region of homogeneity by using two-particle correlations of final particles.
- We observe that at high energies the volume values diverge significantly, while at low energies, they are more consistent.

Thanks to my supervisor, my scientific group and to Richard Lednicky and Alexandru Parvan

Thank you for your attention!