

# Casimir effect in magnetized dense QCD

arXiv: 2402.17638

arXiv: 2408.08384



ASRC (JAEA)

先端基礎研究センター（原子力研究開発機構）

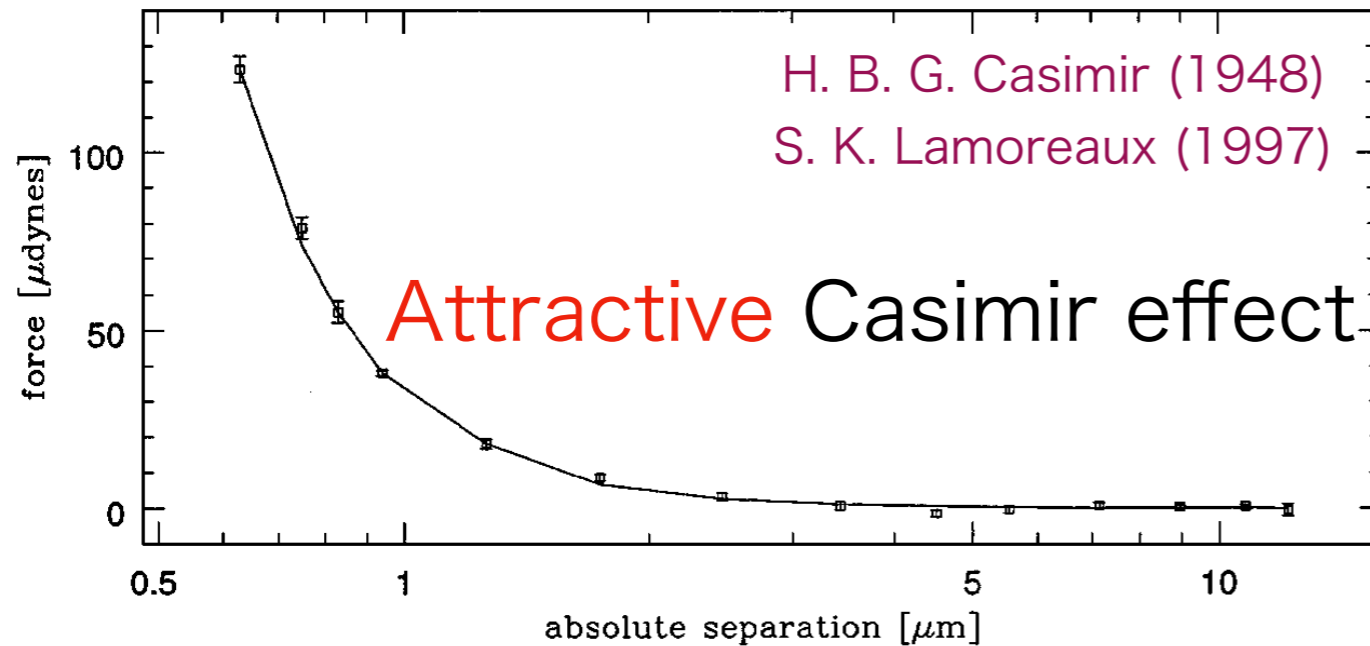
Daisuke Fujii



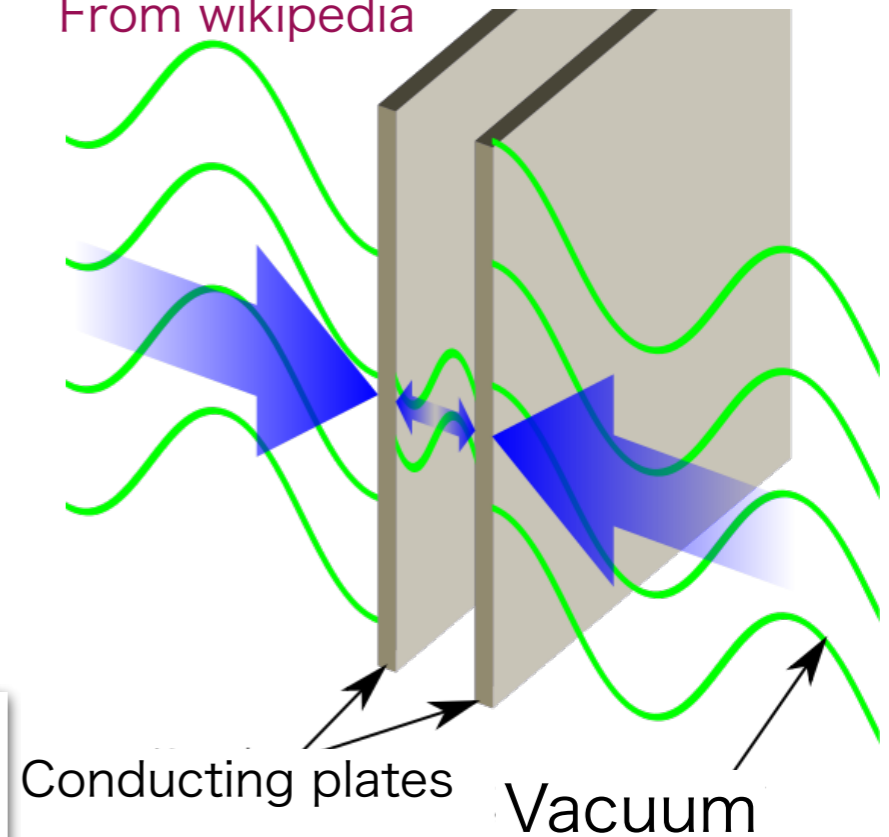
Collaborators:

K. Nakayama (RIKEN-CCS), K. Suzuki (ASRC, JAEA)

# Casimir effect

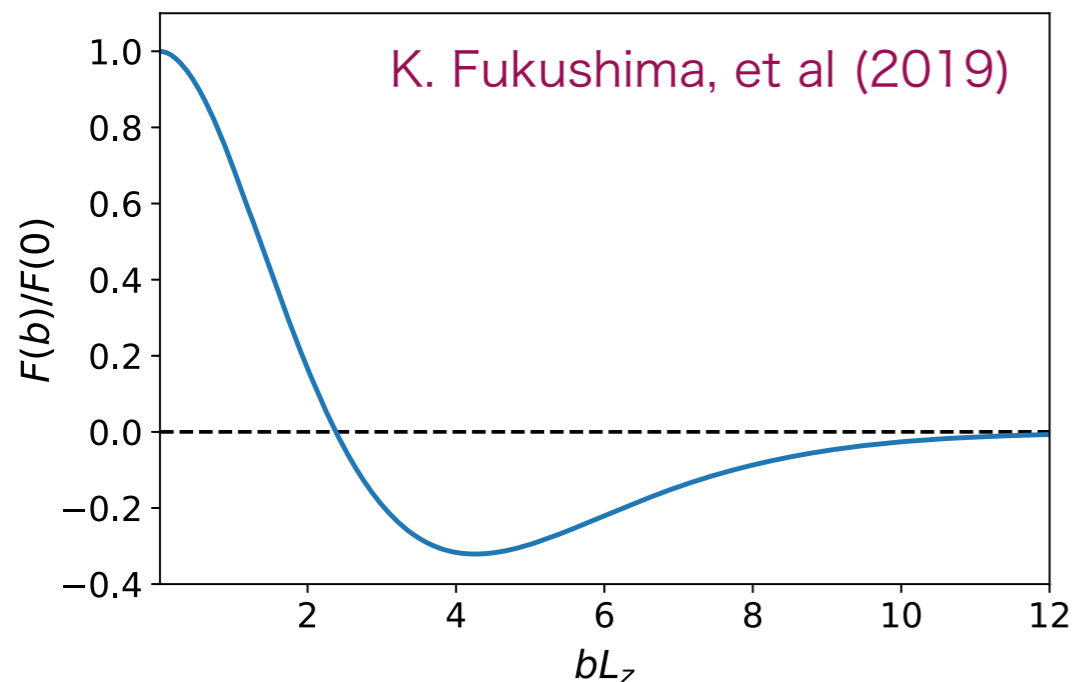


From wikipedia

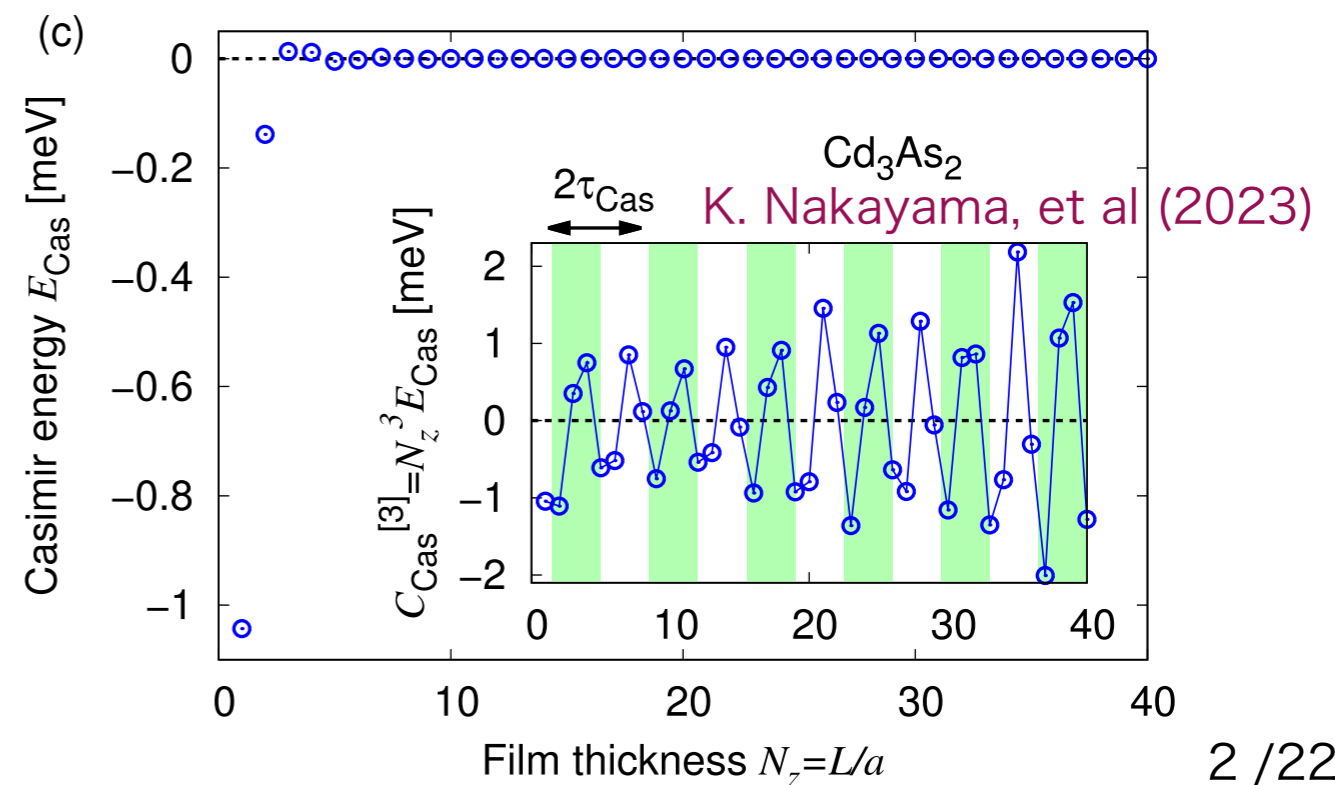


## Some types of Casimir effect

► Sign-flipping Casimir effect



► Oscillating Casimir effect

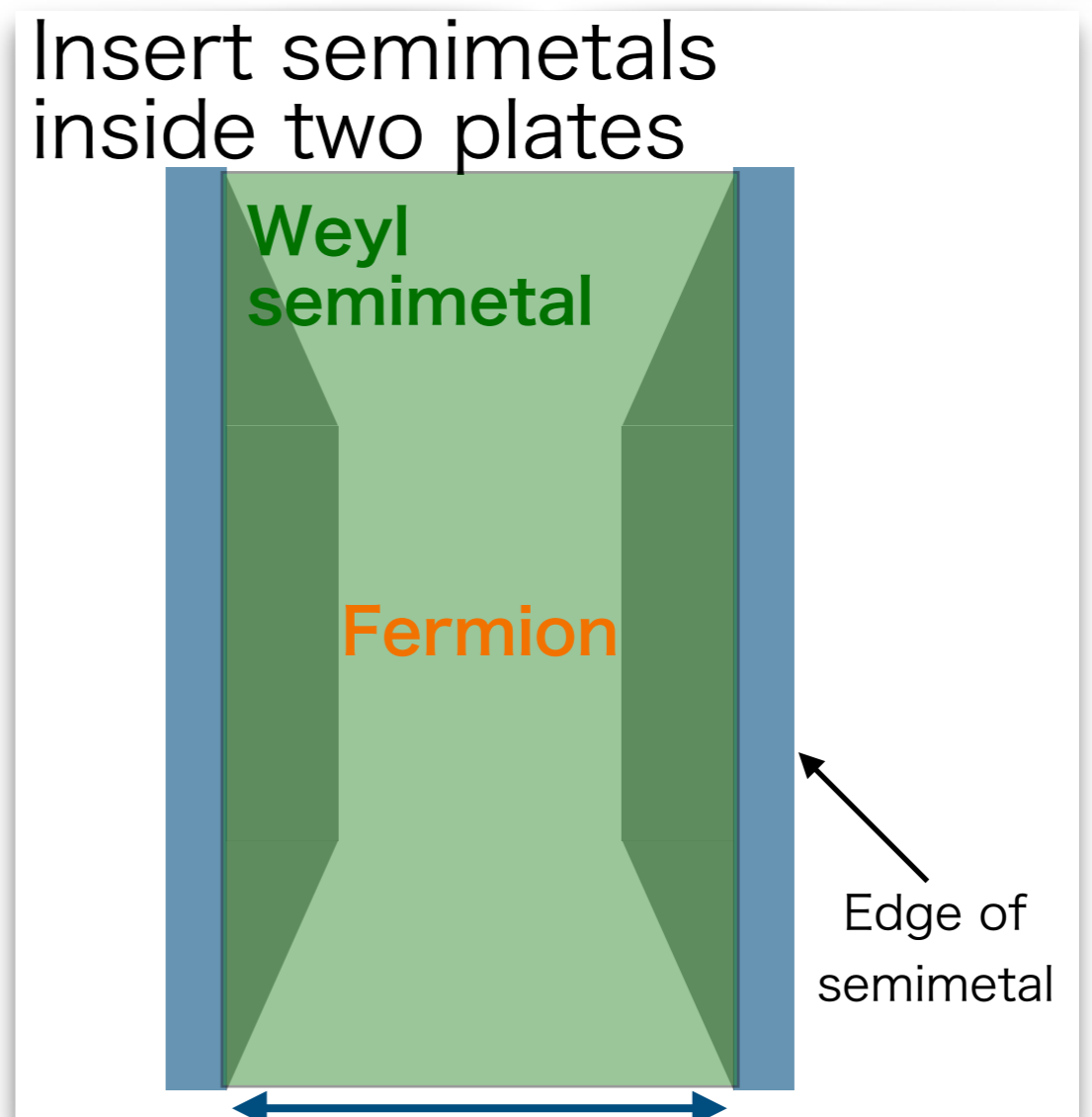
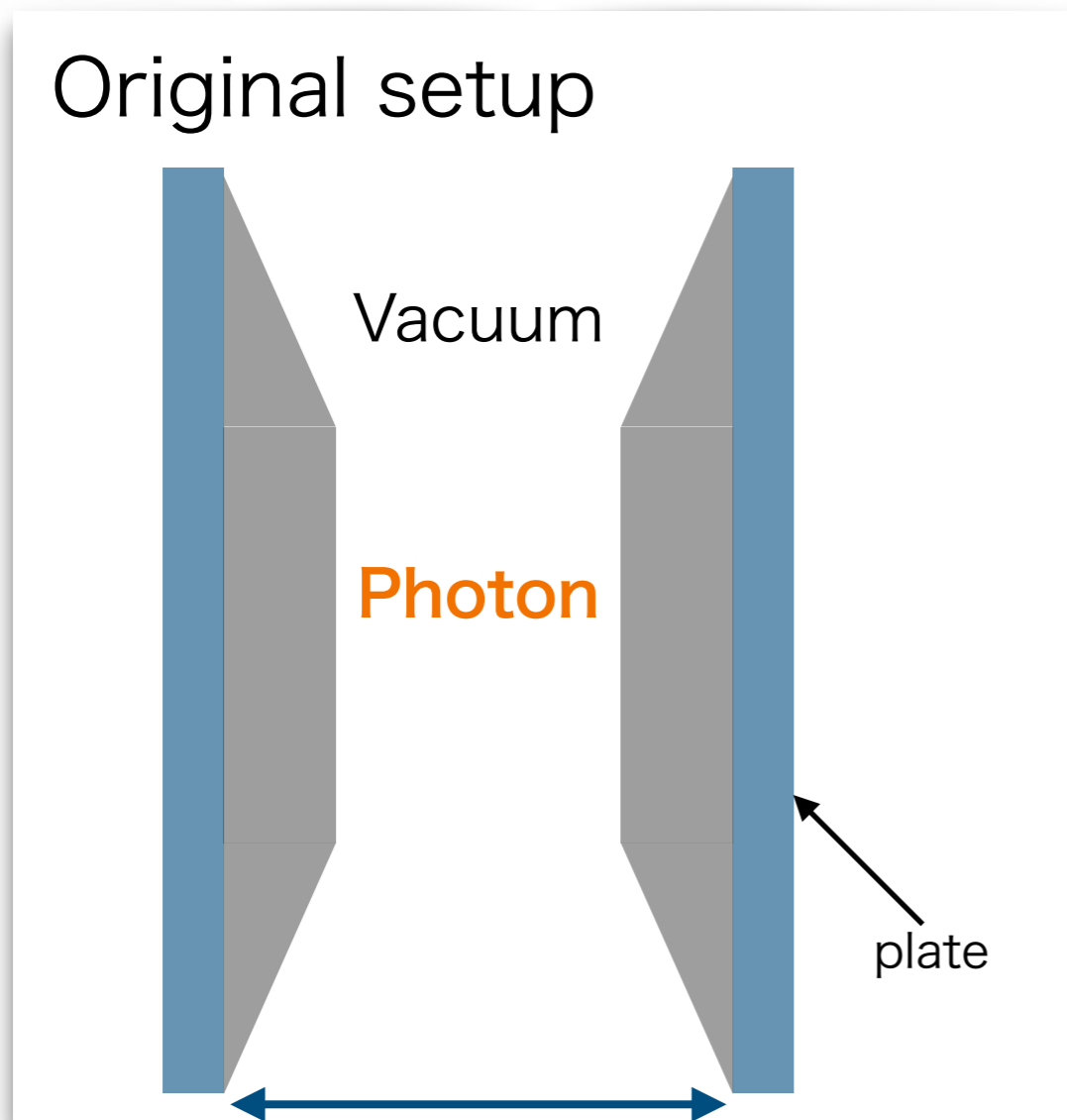


# Oscillating Casimir effect

- **Casimir effect** from **Dirac/Weyl** electron fields in **Semimetals**

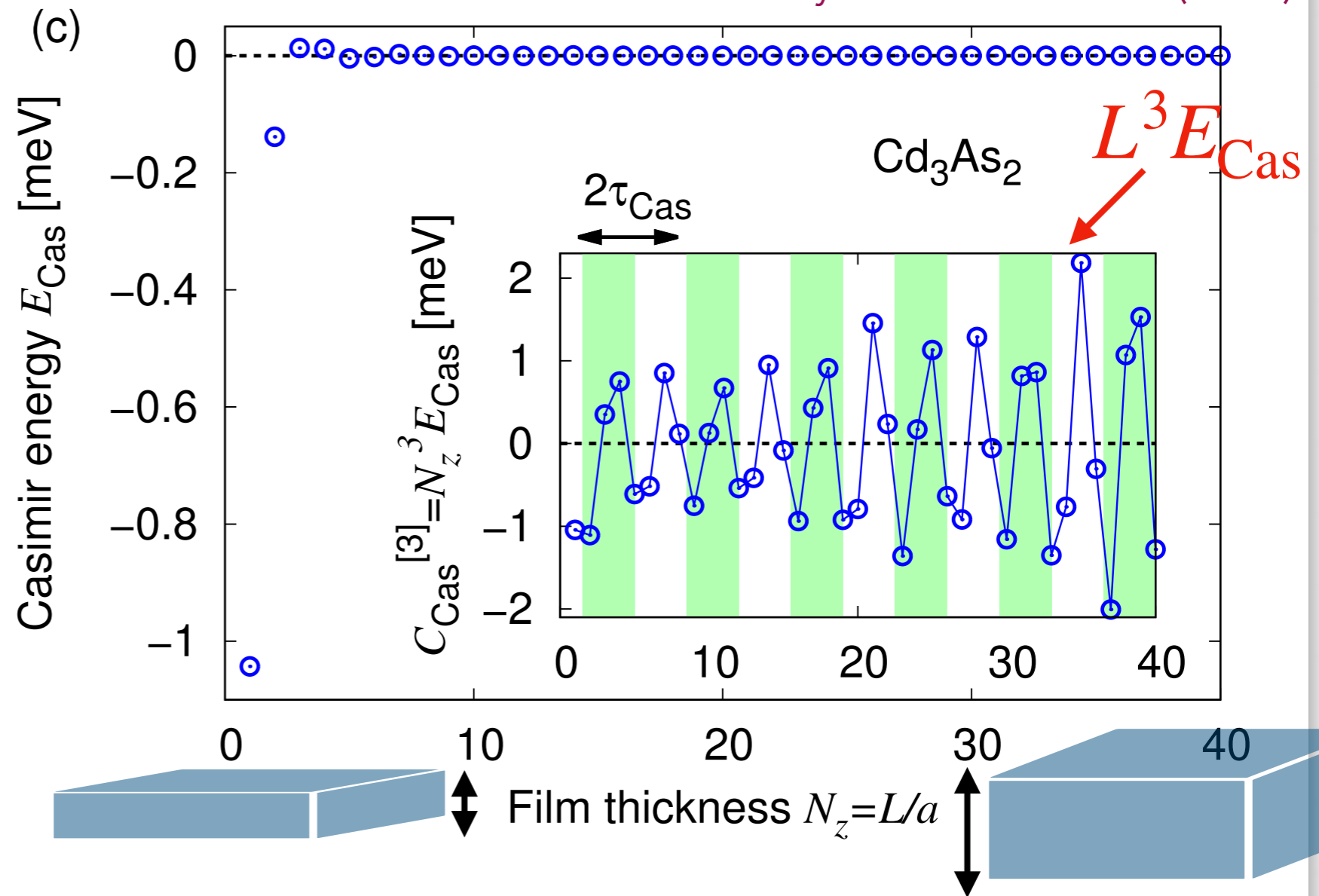
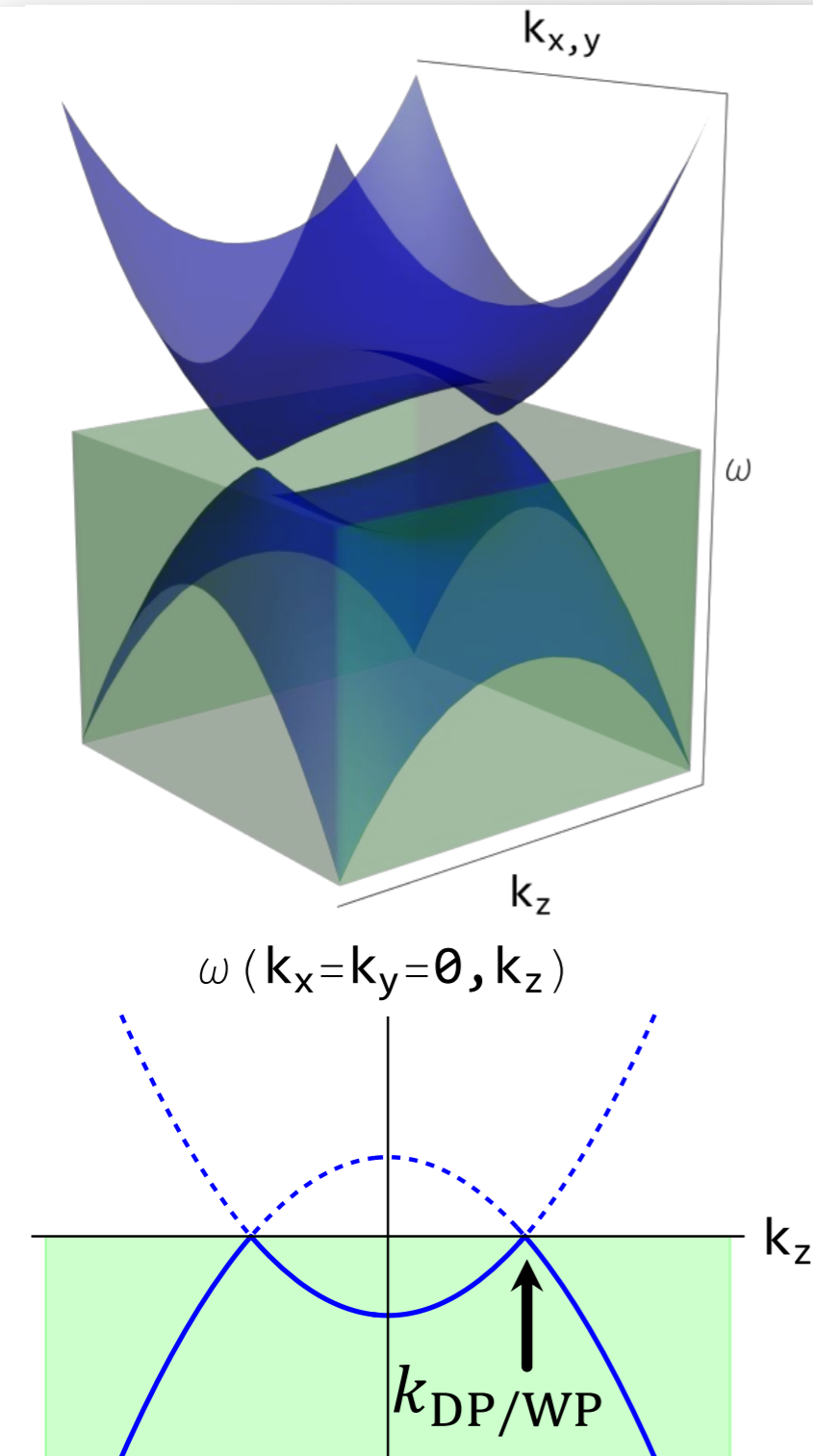
- ▶ Thin films of Dirac semimetal  $\text{Cd}_3\text{As}_2$  and  $\text{Na}_3\text{Bi}$

- ▶ **Oscillating** attractive/zero/repulsive Casimir forces



# Oscillating Casimir effect

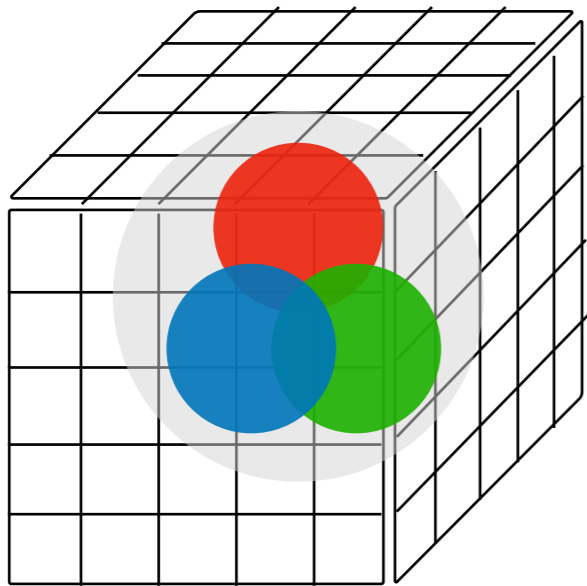
K. Nakayama and K. Suzuki (2023)



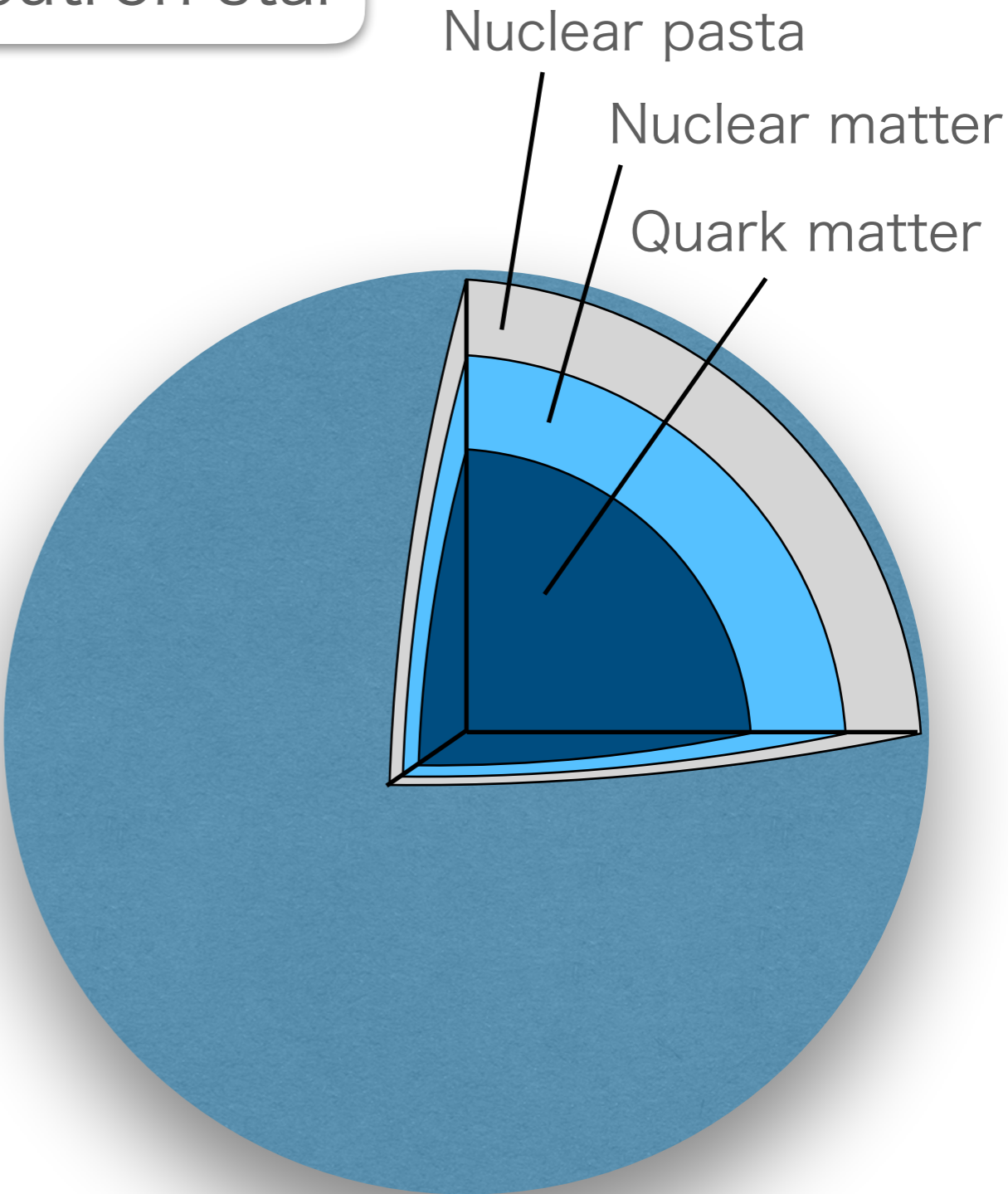
► Oscillating due to the Weyl point

# Small volume physics in QCD

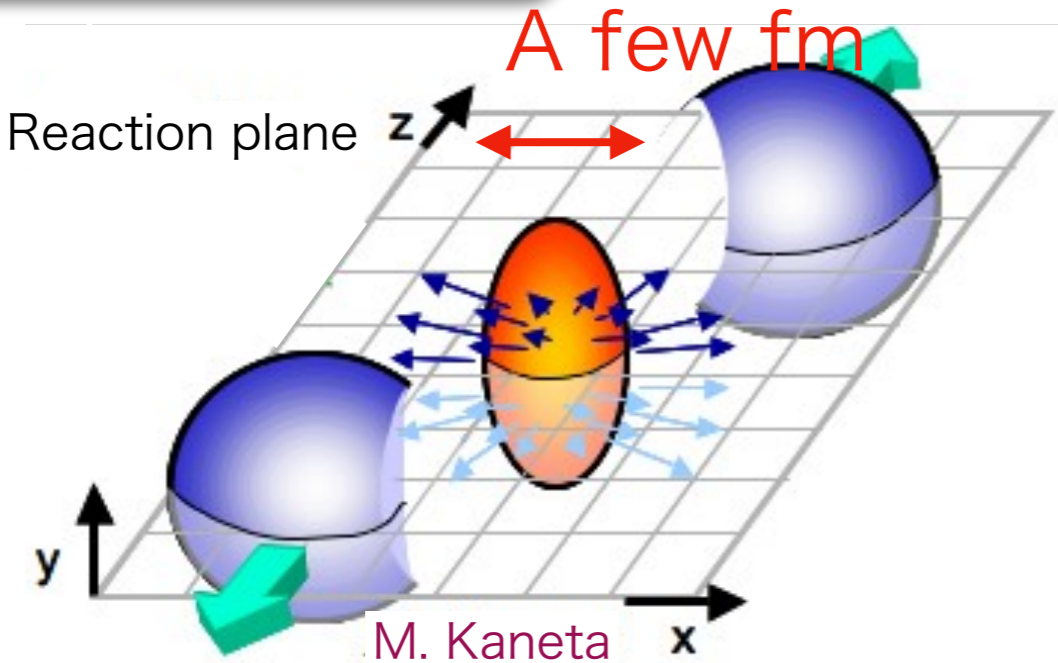
Lattice QCD



Neutron star

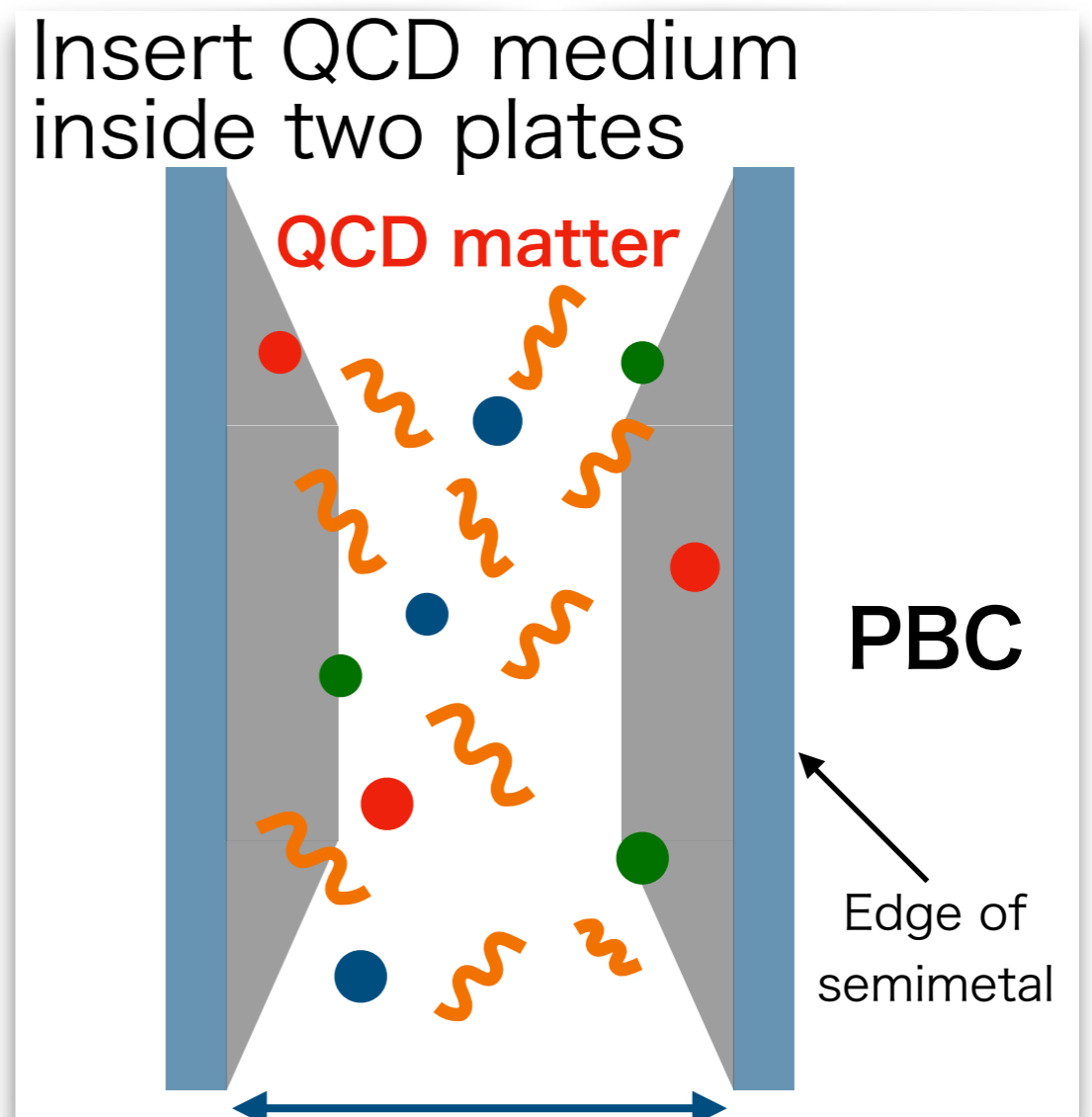
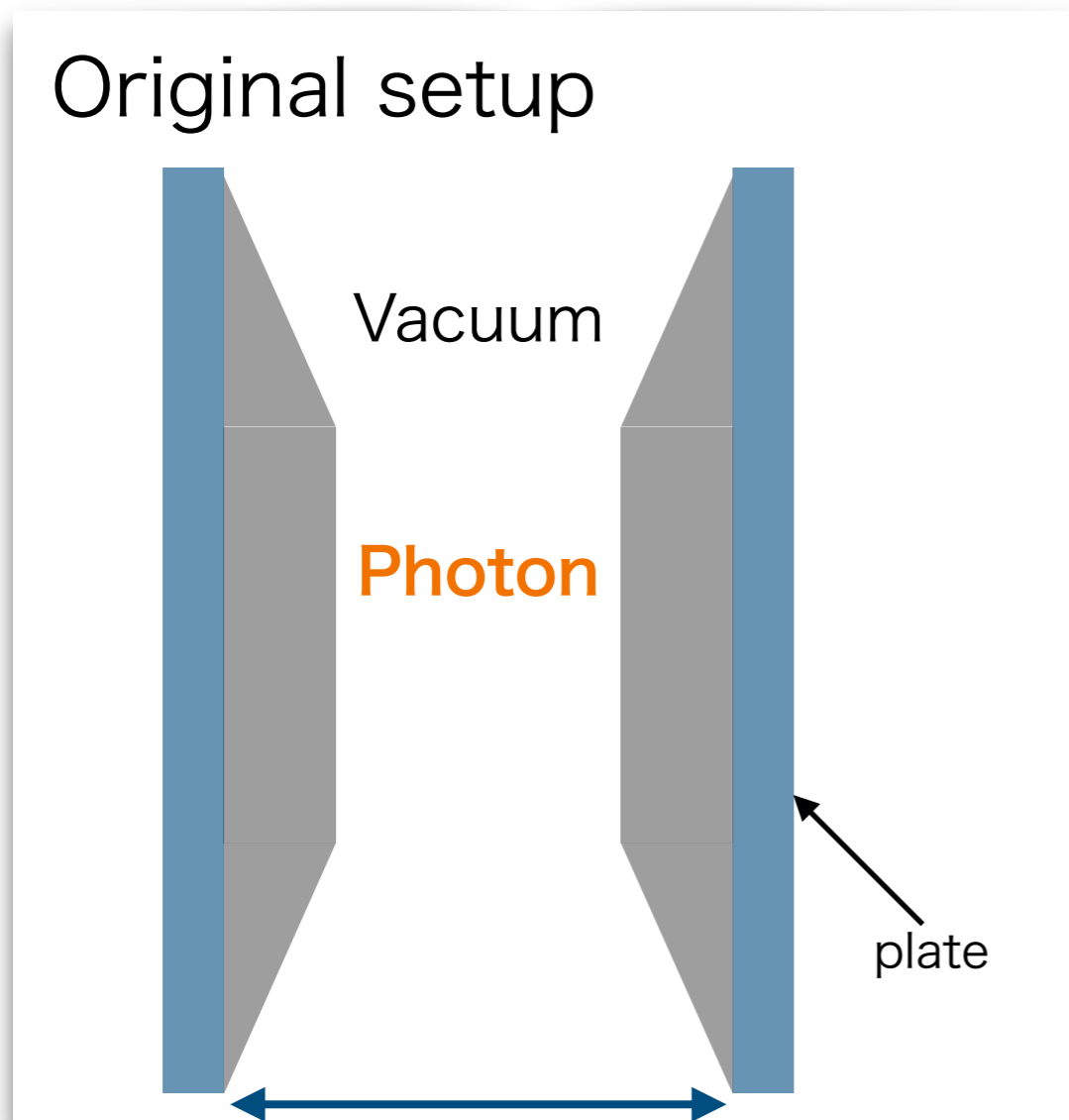


Fire ball of QGP

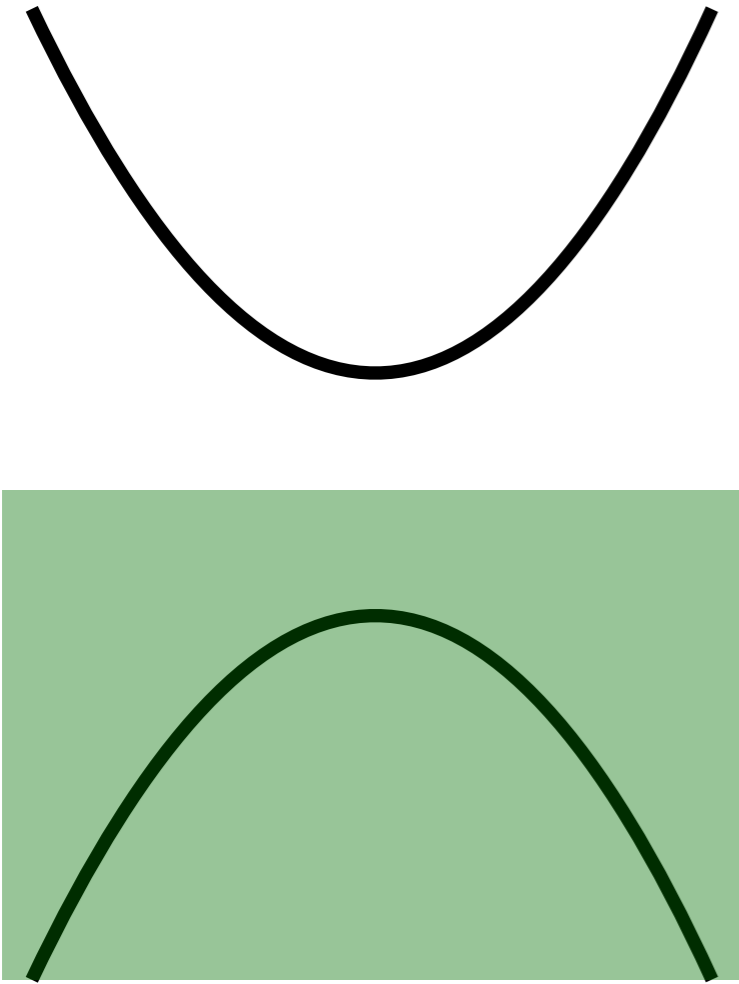
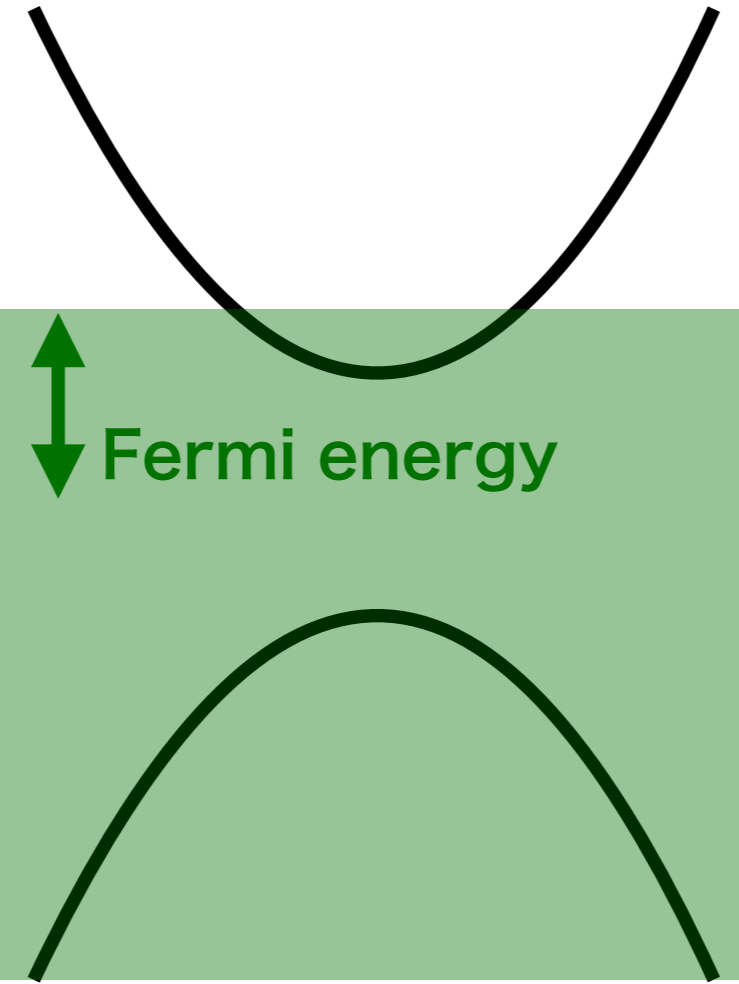
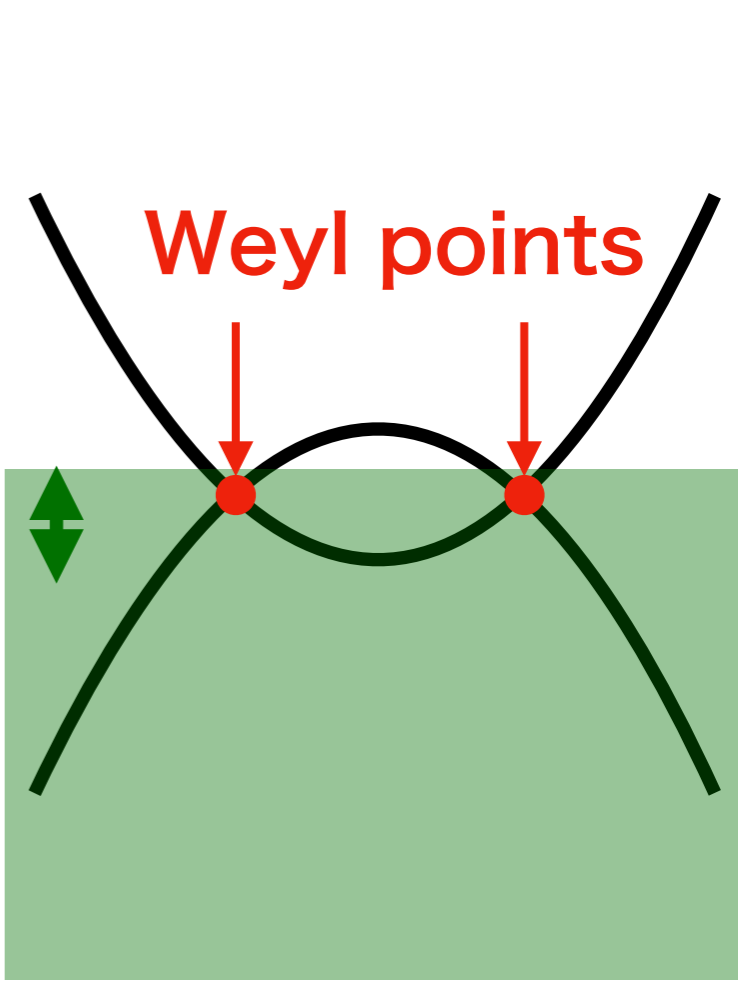


# Oscillating Casimir effect

- Consider **counterparts** of several Casimir effect in **NJL model**



# What is a counterpart in QCD (NJL) ?

Insulator	Metals	Weyl (semi)metals
		
<p>QCD vacuum with chiral cond.</p>	<p>Dense QCD with chiral cond.</p>	<p>?</p>

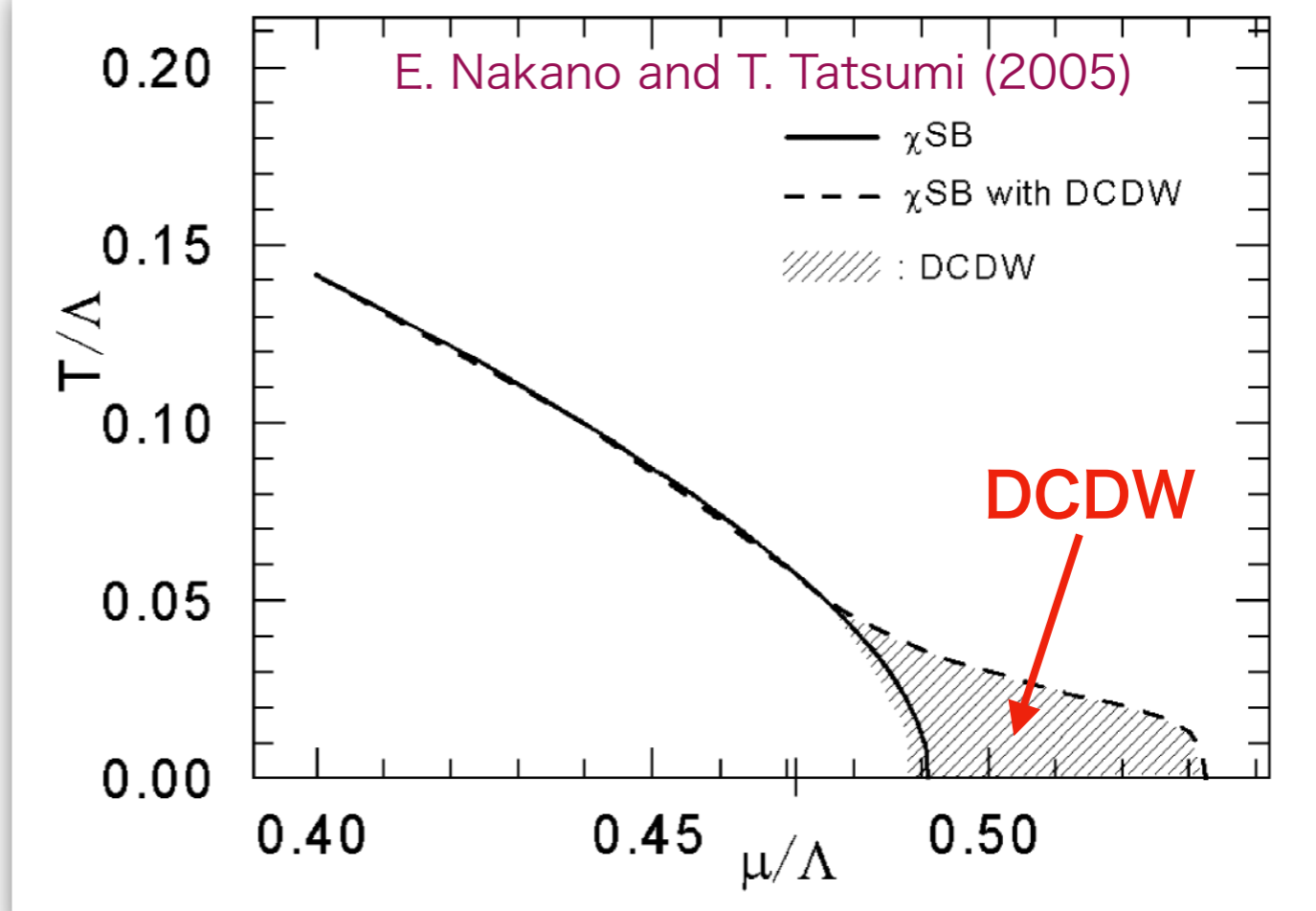
# What is a counterpart in QCD ?

## ● Dual Chiral Density Wave (DCDW)

- ▶ A candidate for **ground state** near **chiral phase transition** of density QCD (NJL)
- ▶ Order parameter with **spatially inhomogeneous** chiral condensate
- ▶ The spiral structure of **scalar** ( $\sigma$ ) and **pseudoscalar** ( $\pi_0$ ) condensates

$$\langle \bar{q}q \rangle \rightarrow M \cos(\vec{q} \cdot \vec{r})$$
$$\langle \bar{q}i\gamma_5 q \rangle \rightarrow M \sin(\vec{q} \cdot \vec{r})$$

### Prediction from NJL





# Dispersion relations in DCDW

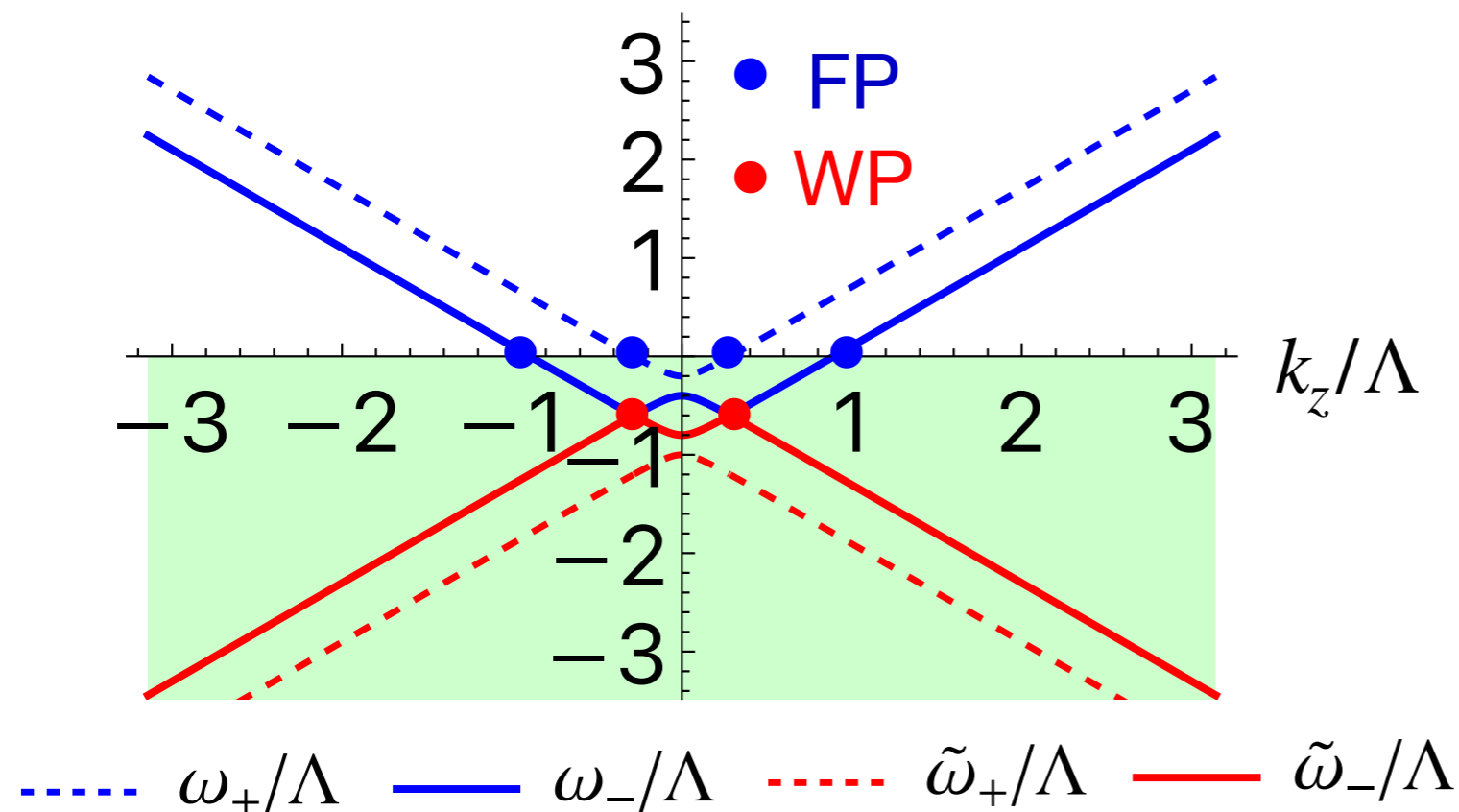
- Four eigenvalues of Dirac field at  $q = (0,0,q)$

$$\omega_{\pm} = \sqrt{k_x^2 + k_y^2 + \left(\sqrt{k_z^2 + M^2} \pm \frac{q}{2}\right)^2} \quad \tilde{\omega}_{\pm} = -\sqrt{k_x^2 + k_y^2 + \left(\sqrt{k_z^2 + M^2} \pm \frac{q}{2}\right)^2}$$

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(a)  $M/\Lambda = 0.1$ ,  $q/\Lambda = 0.6$ ,  $\mu/\Lambda = 0.6$

$\omega/\Lambda$  ( $k_x = k_y = 0, k_z$ )



Wavenumber :  
Splitting of L/R ( $\pm$ )

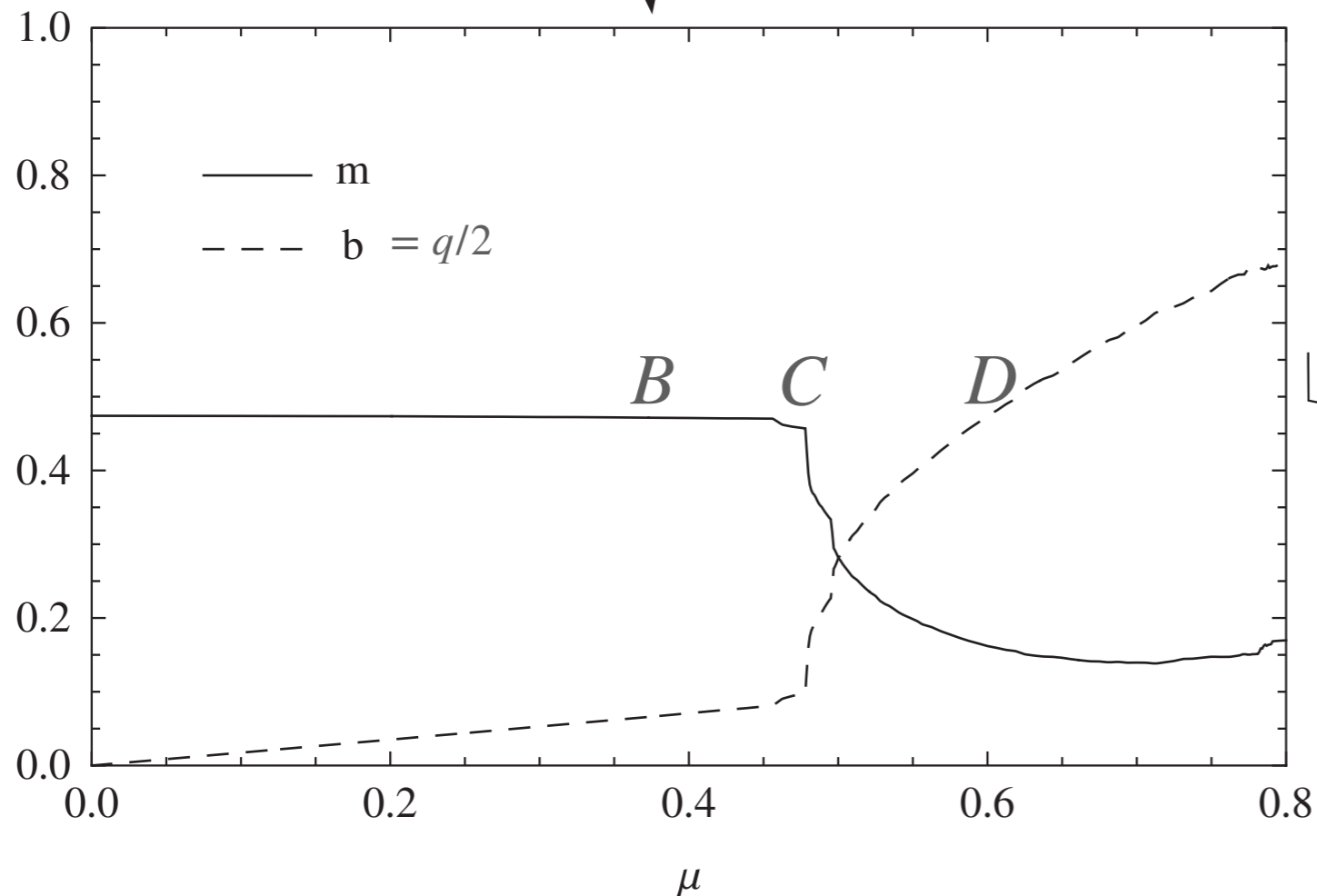
Amplitude of DCDW  
(Chiral cond.) :  
Gap of  $\omega_{\pm}$  and  $\tilde{\omega}_{\pm}$

# DCDW under Magnetic fields

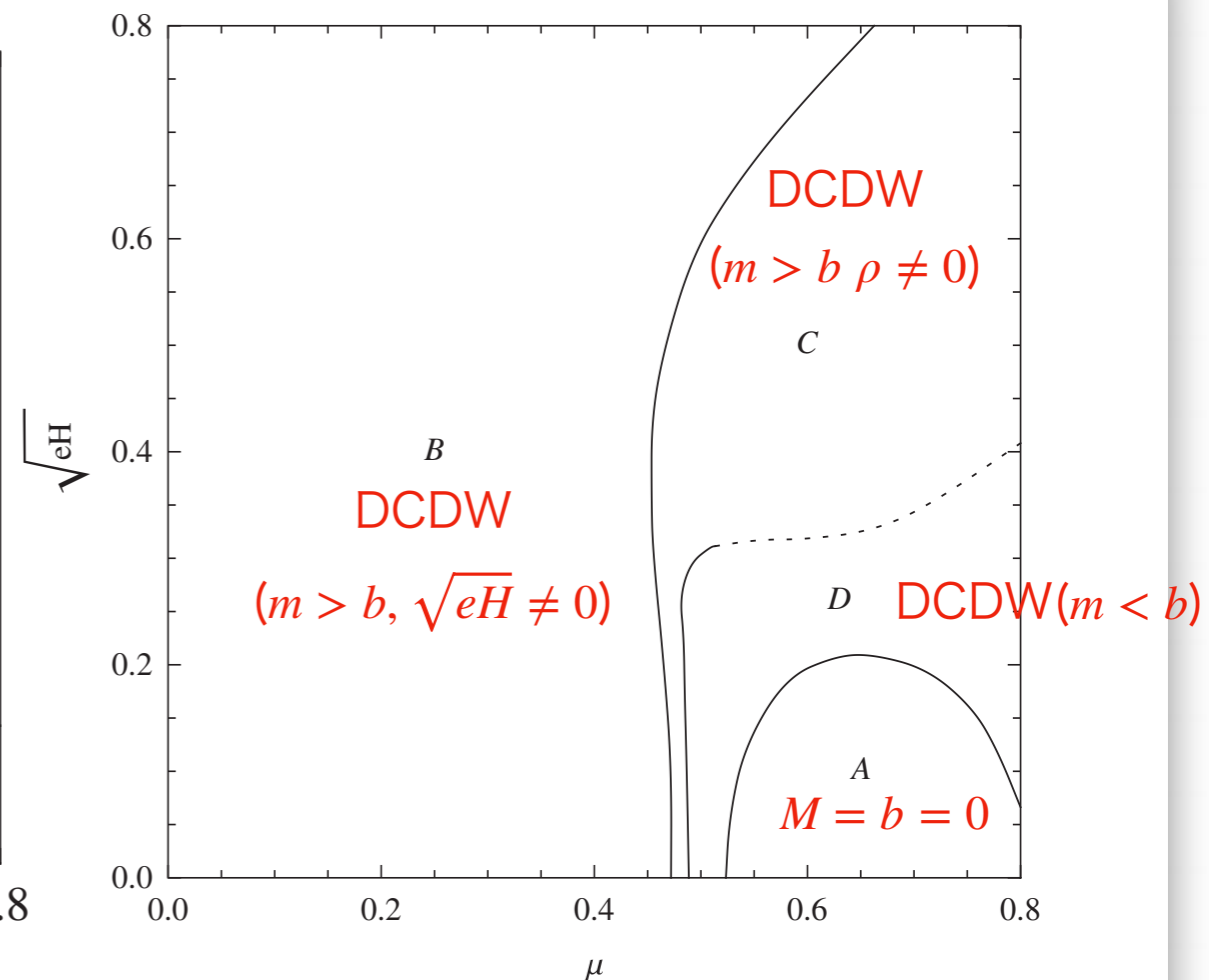
## ● Magnetic Dual Chiral Density Wave (MDCDW)

- ▶ The **magnetic field** makes the DCDW phase more **robust** because of the effective 1dim space.
- ▶ MDCDW is realized in the interior of a **neutron star**

(c)  $\sqrt{eH} = 0.3$



I. E. Frolov and V. Ch. Zhukovsky, (2010)



# Dispersion relations in MDCDW

- Four eigenvalues of Dirac field at  $q = (0,0,q)$

## Lowest Landau Levels (LLLs)

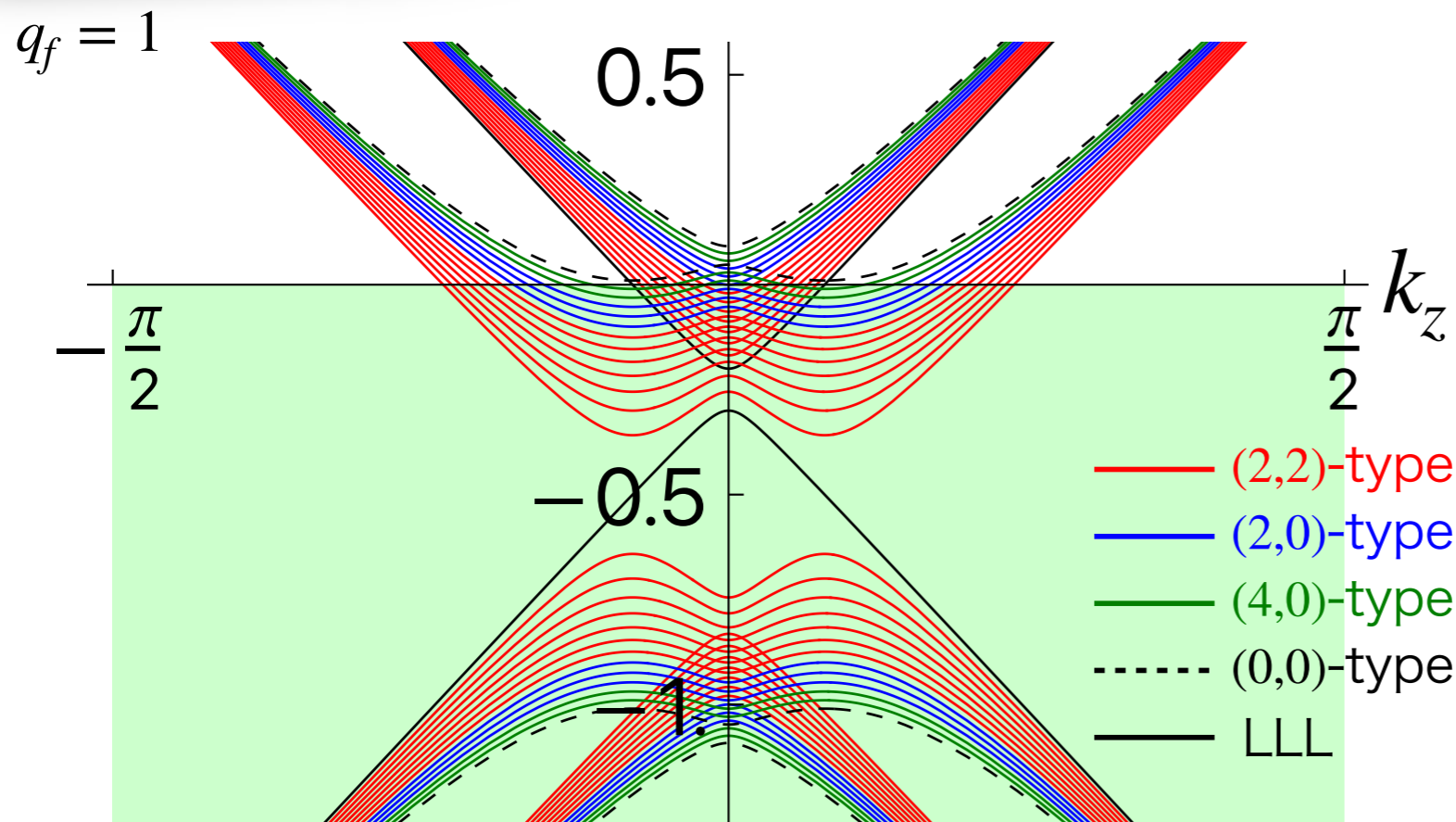
$$\omega_{l=0} = \sqrt{M^2 + k_z^2} + \frac{q}{2} \quad \tilde{\omega}_{l=0} = -\sqrt{M^2 + k_z^2} + \frac{q}{2}$$

## Higher Landau Levels (HLLs)

u, d quark charge

Magnetic fields

$$\omega_{\pm,l} = \sqrt{2|q_f B|l + (\sqrt{k_z^2 + M^2} \pm \frac{q}{2})^2} \quad \tilde{\omega}_{\pm,l} = -\sqrt{2|q_f B|l + (\sqrt{k_z^2 + M^2} \pm \frac{q}{2})^2} \quad l = 1,2,\dots$$

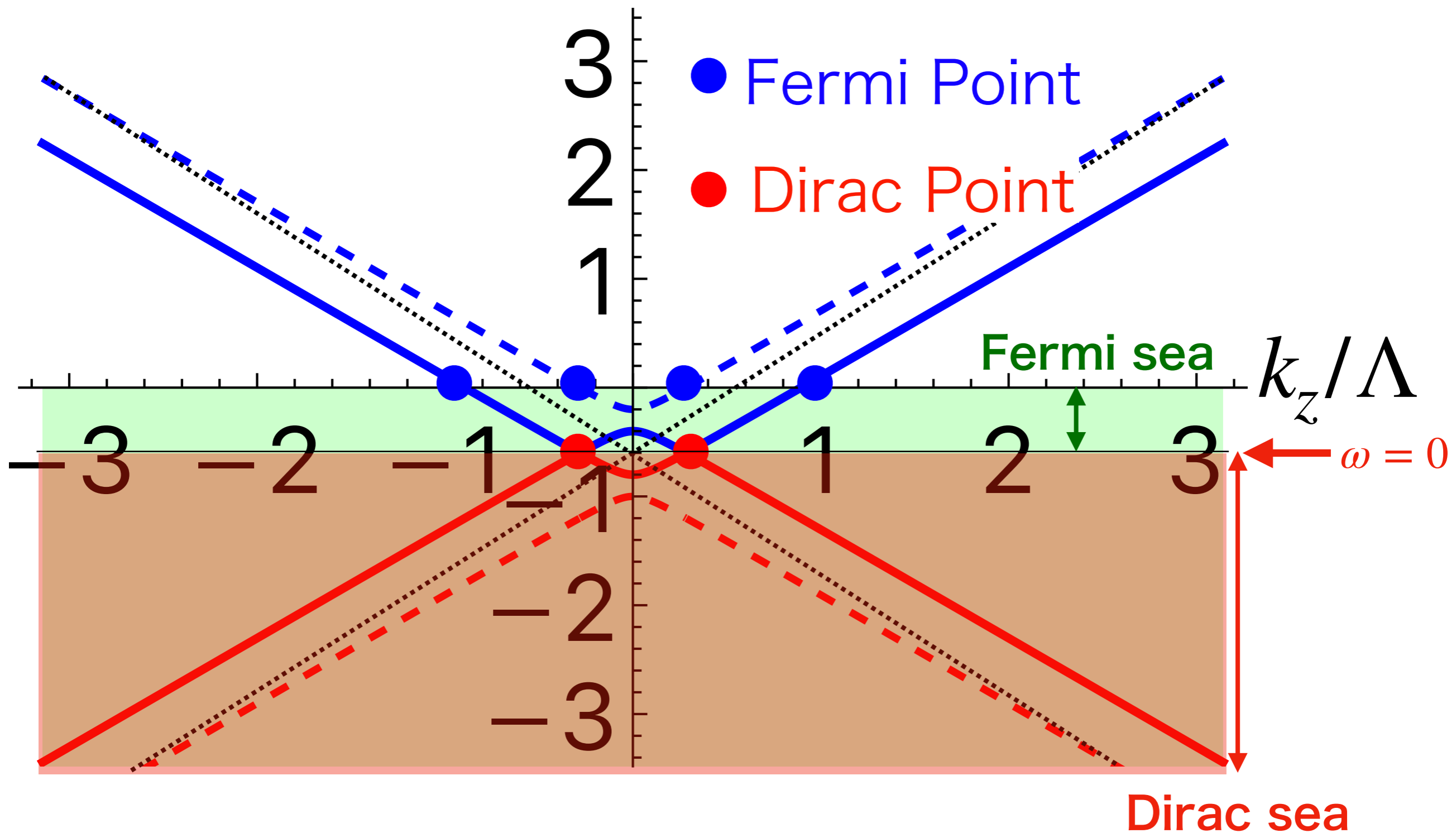


Wavenumber :  
Splitting of L/R ( $\pm$ )

Amplitude of DCDW  
(Chiral cond. like) :  
Gap of  $\omega_{\pm}$  and  $\tilde{\omega}_{\pm}$

# Results

# Definition of Fermi/Dirac sea

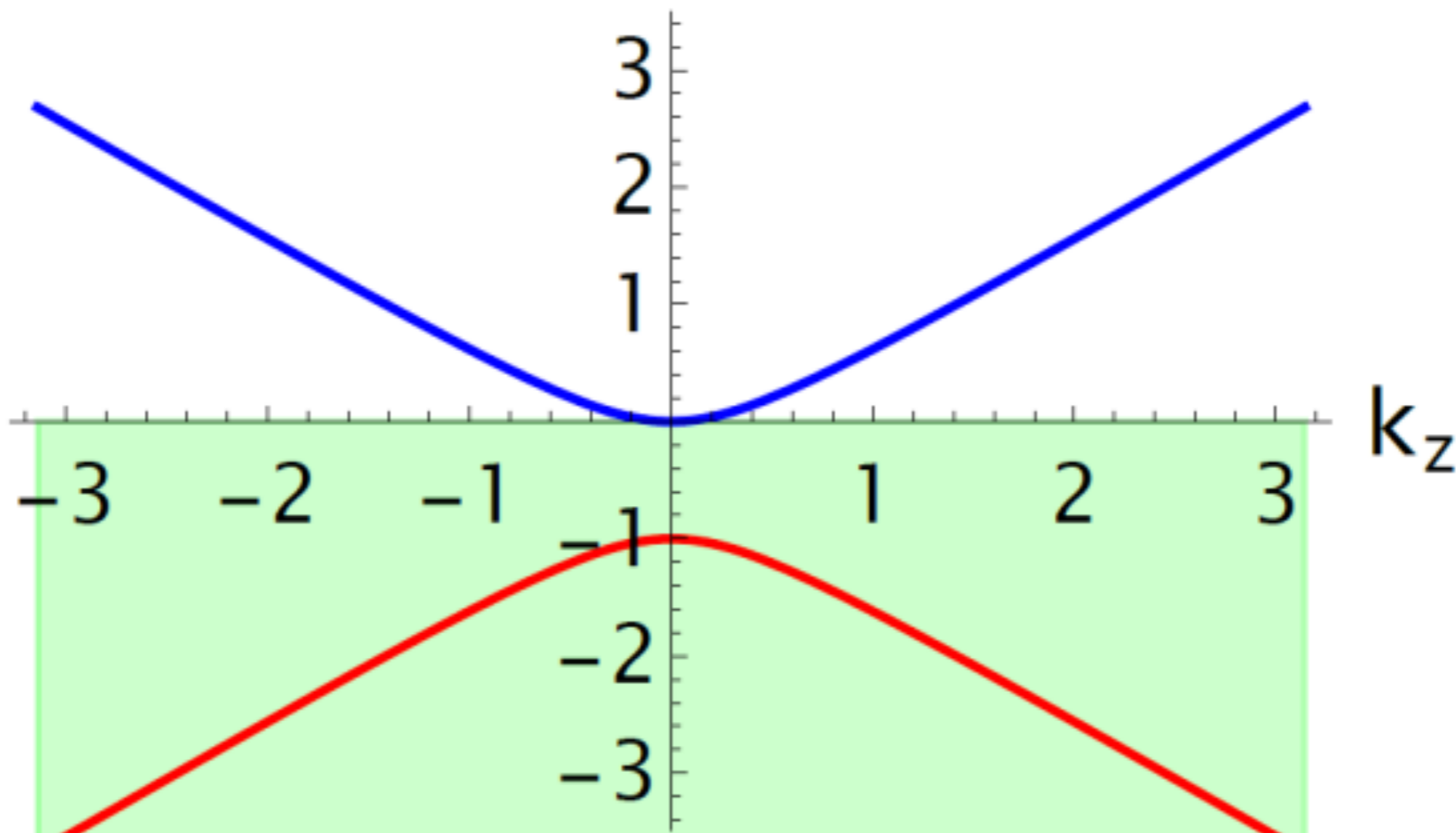


# Zero baryon number density $\rho_B$ (Low $\mu$ )

No Dirac and Fermi point

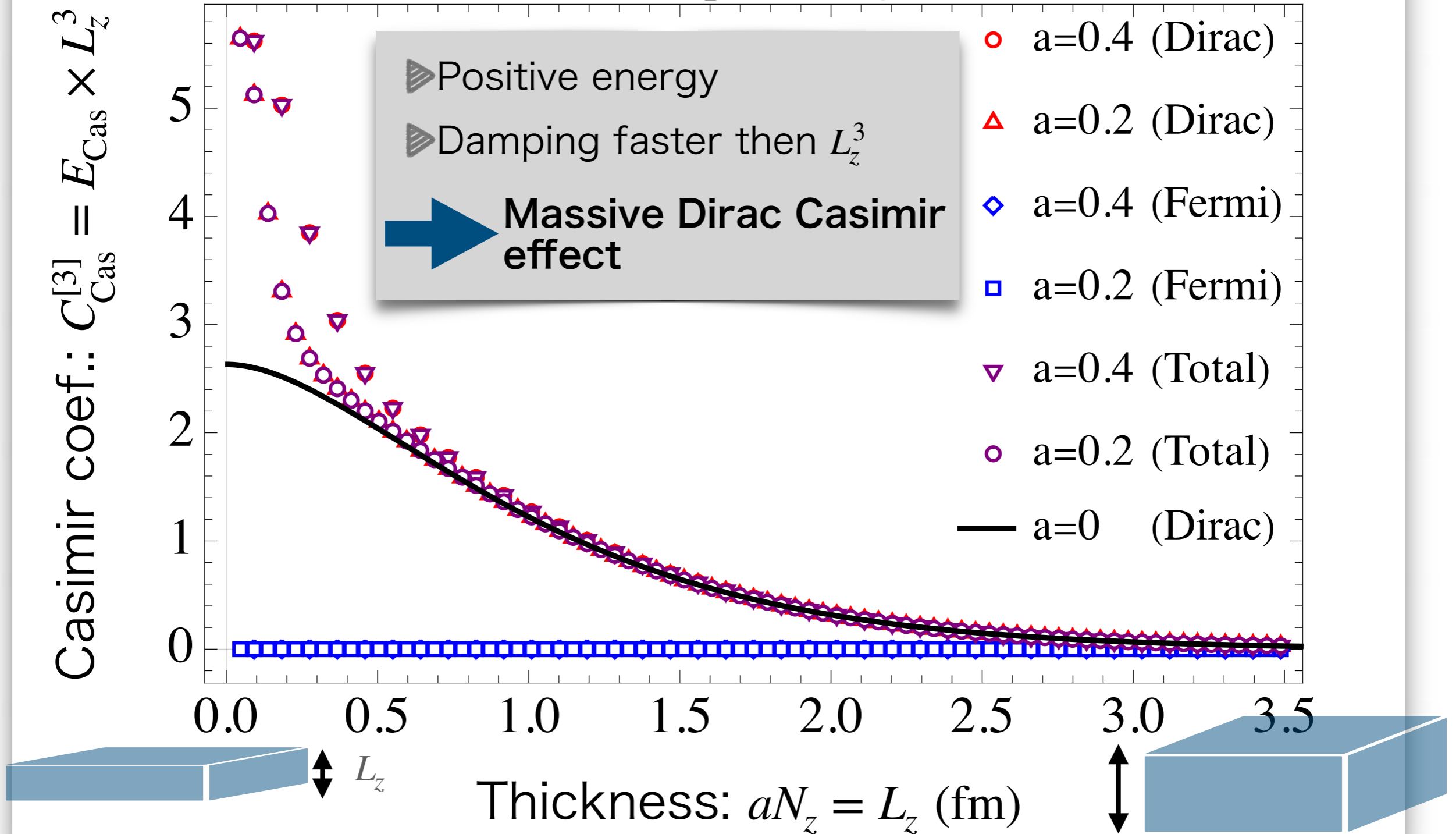
$$M/\Lambda = 0.47, q/\Lambda = 0, \mu/\Lambda = 0.46$$

$$\omega(k_x=k_y=0, k_z)$$



# Zero baryon number density $\rho_B$ (Low $\mu$ )

$$M/\Lambda = 0.47, q/\Lambda = 0, \mu/\Lambda = 0.46$$



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# Finite baryon number density $\rho_B$ (Intermediate $\mu$ )

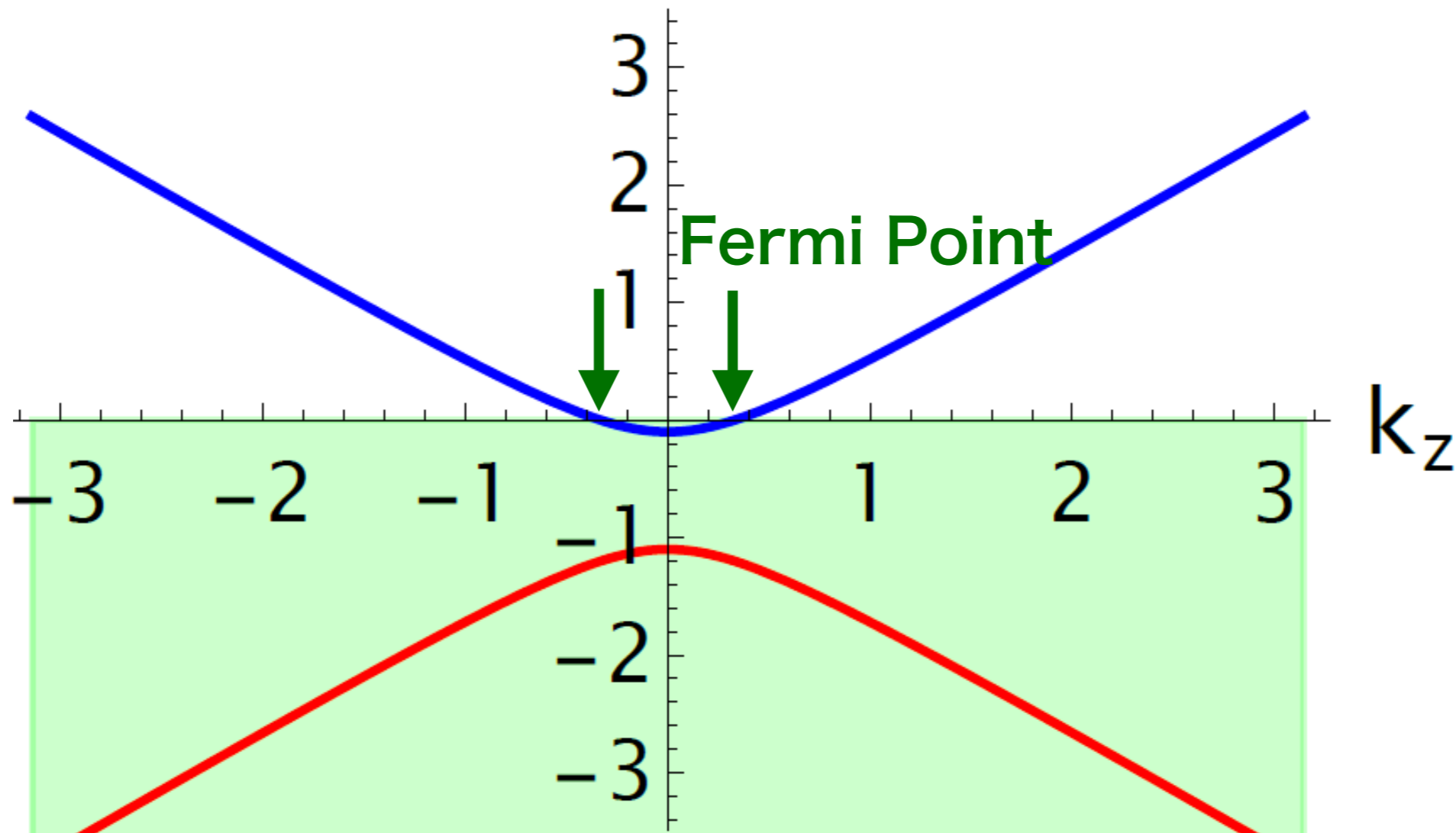
- Oscillation occur by Fermi point

$$M/\Lambda = 0.46, q/\Lambda = 0, \mu/\Lambda = 0.485$$

$$\omega(k_x = k_y = 0, k_z)$$

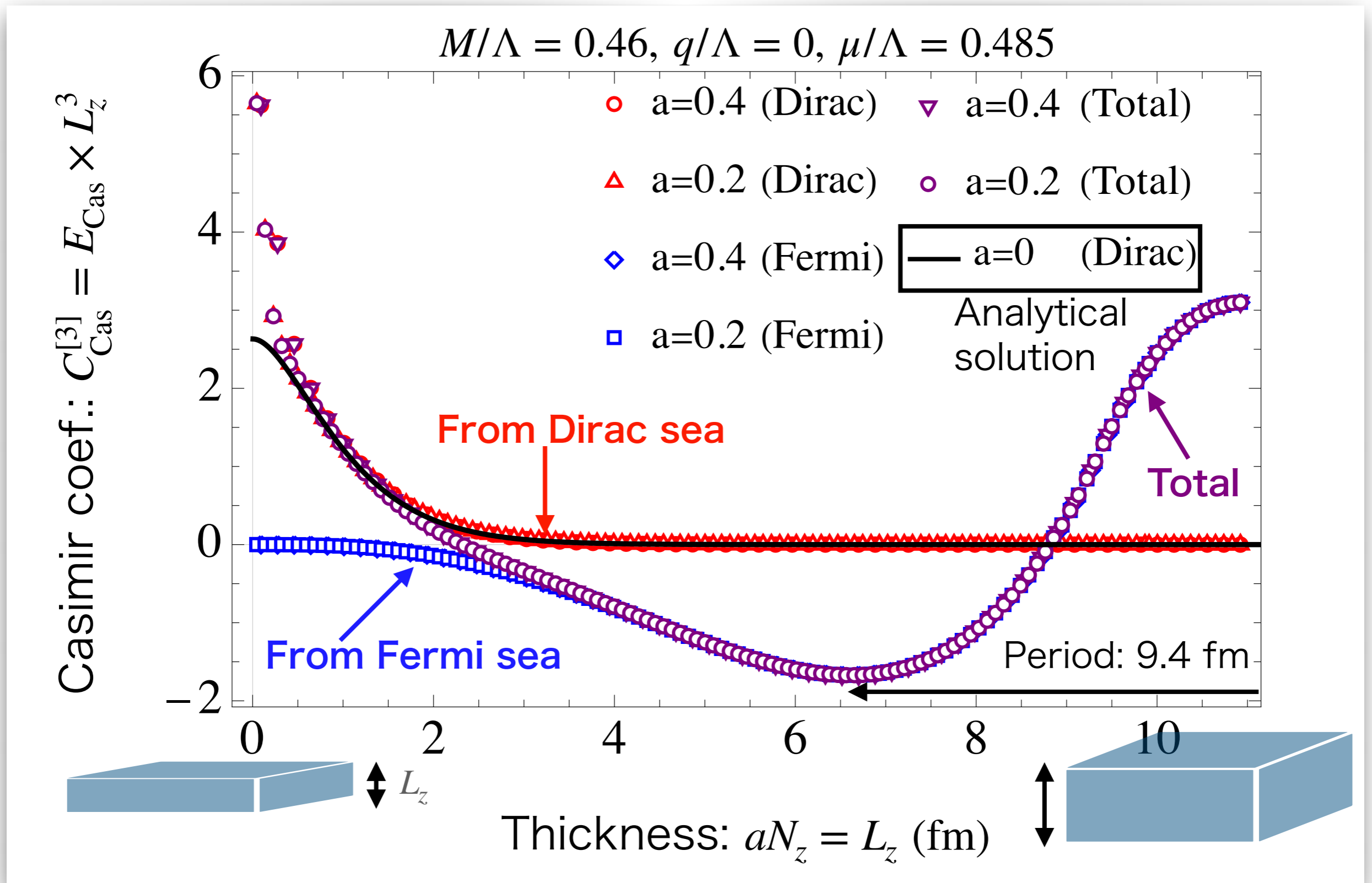
Oscillation period:

$$L_z = \frac{\pi}{k_{\text{FP}}} \sim \frac{2\pi}{0.154} \sim 9.4\text{fm}$$





# Finite baryon number density $\rho_B$ (Intermediate $\mu$ )



# DCDW phase (high $\mu$ )

- Oscillations by DCDW phase

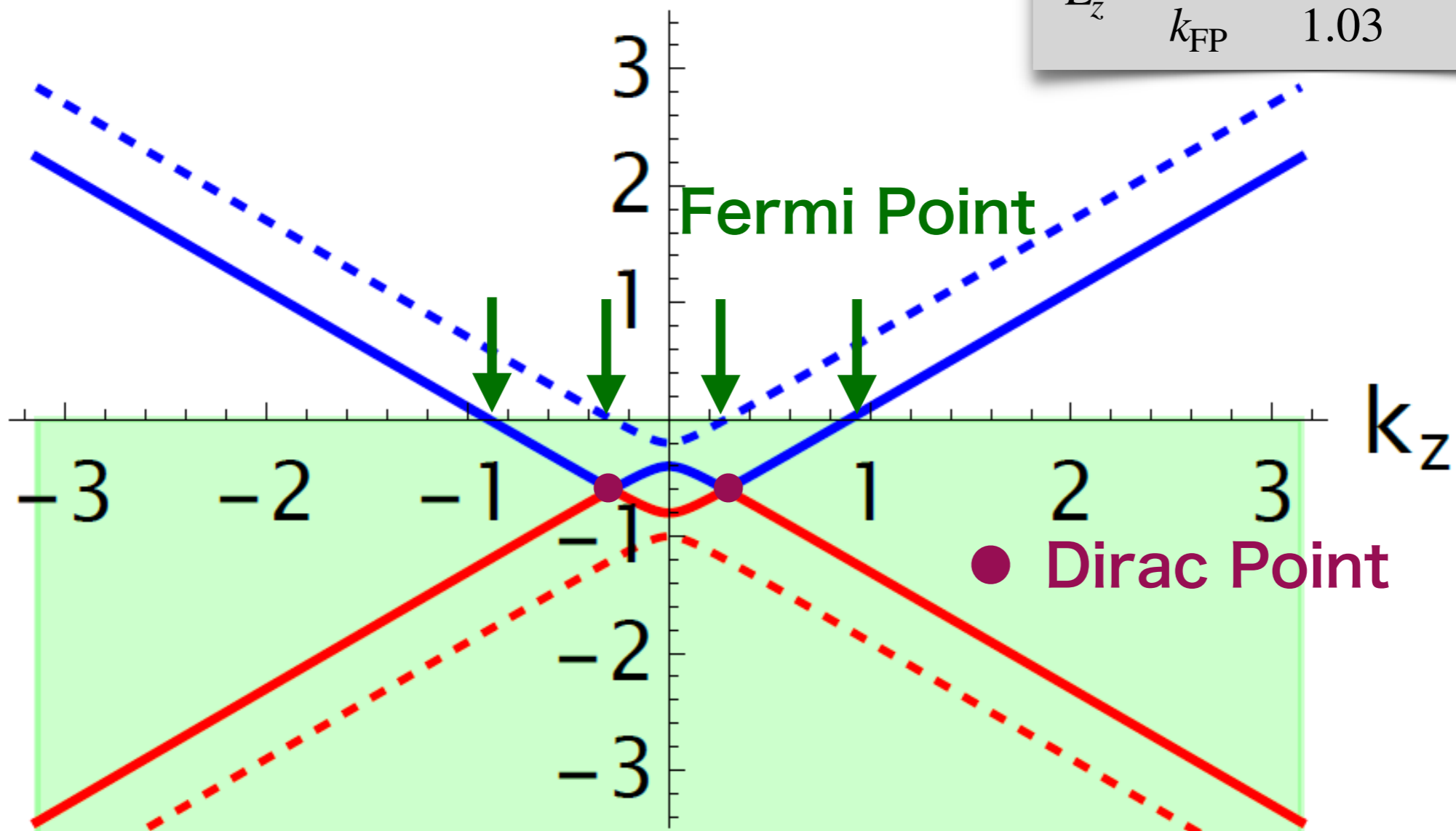
$$M/\Lambda = 0.09, q/\Lambda = 0.62, \mu/\Lambda = 0.72$$

$$\omega(k_x = k_y = 0, k_z)$$

Oscillation period:

$$L_z = \frac{\pi}{k_{\text{WP}}} \sim \frac{2\pi}{0.297} \sim 4.9\text{fm}$$

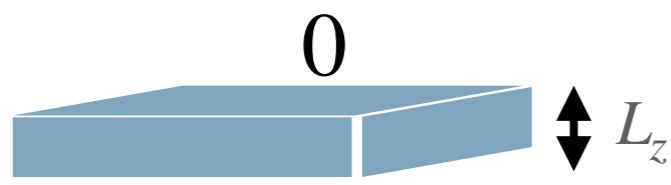
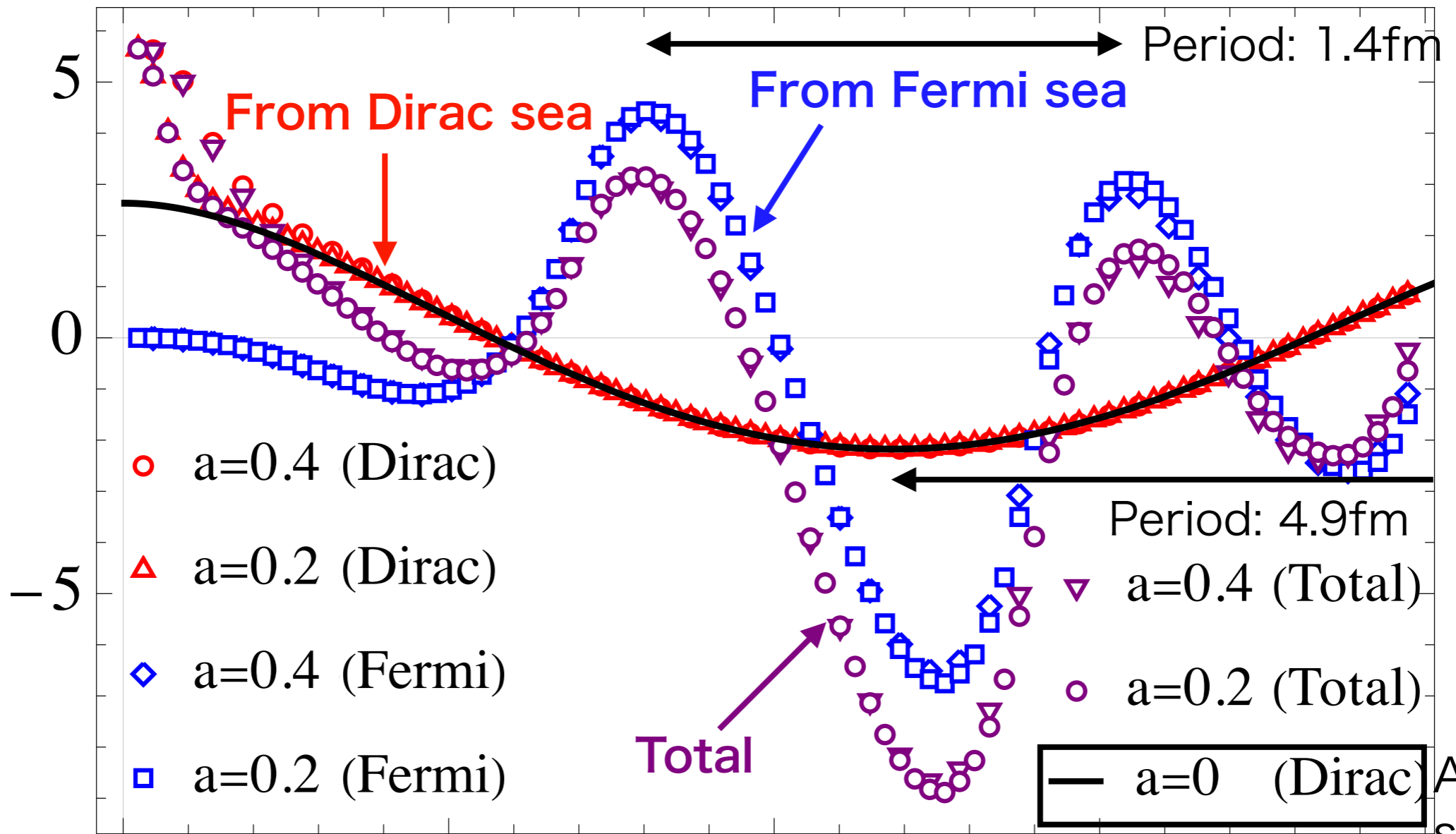
$$L_z = \frac{\pi}{k_{\text{FP}}} \sim \frac{2\pi}{1.03} \sim 1.4\text{fm}$$



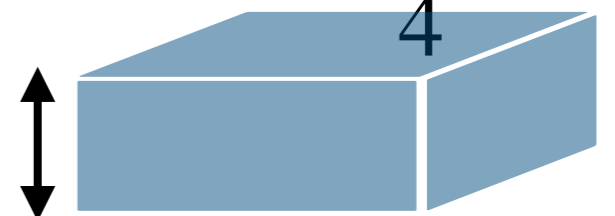
# DCDW phase (high $\mu$ )

$$M/\Lambda = 0.09, q/\Lambda = 0.62, \mu/\Lambda = 0.72$$

Casimir coef.:  $C_{\text{Cas}}^{[3]} = E_{\text{Cas}} \times L_z^3$



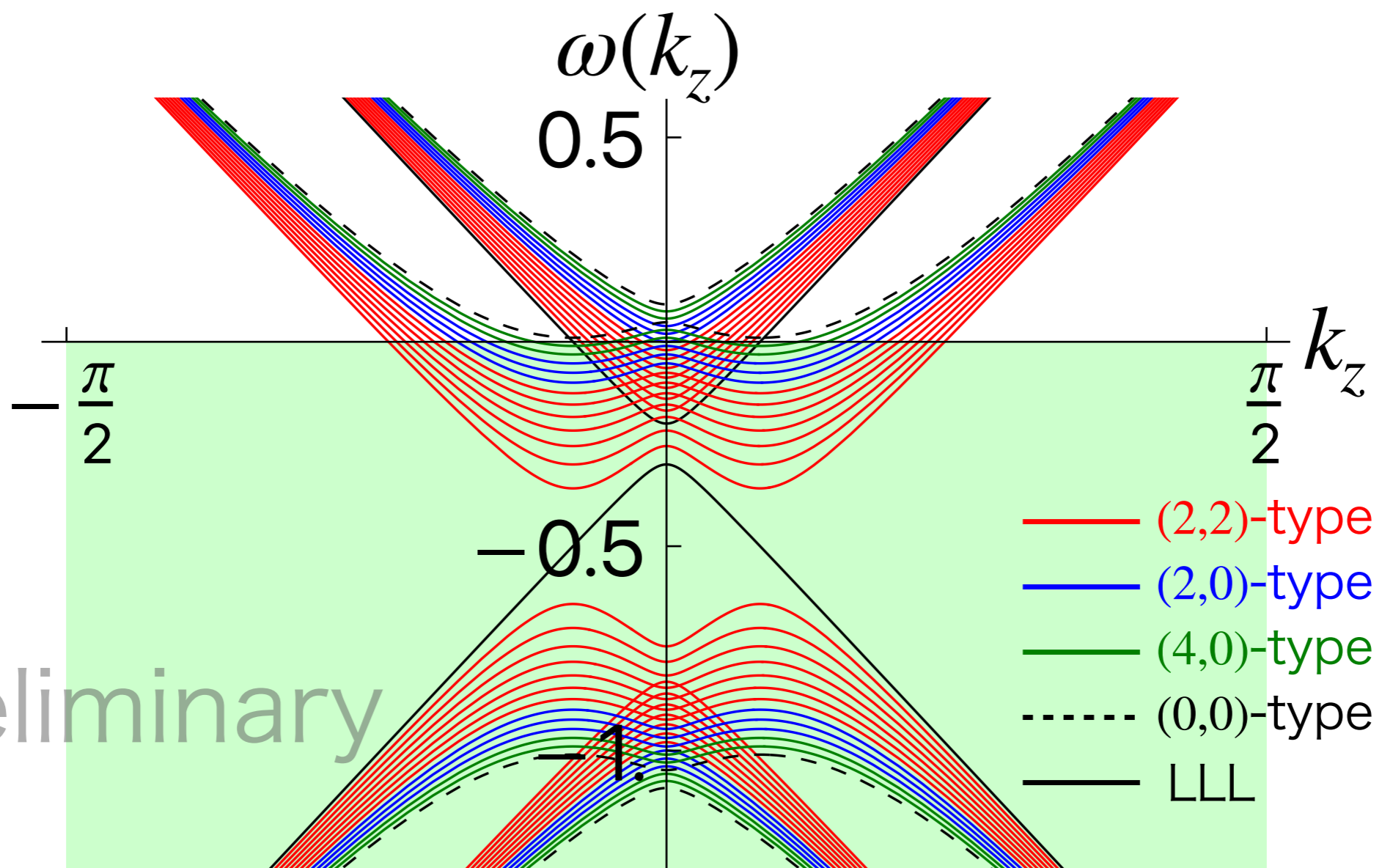
Thickness:  $aN_z = L_z$  (fm)



# DCDW phase (high $\mu$ )

- Oscillations by MDCDW phase

$$q_f = 1 \quad M/\Lambda = 0.15, \quad q/\Lambda = 0.6, \quad \mu/\Lambda = 0.5, \quad eB/\Lambda = (0.05)^2$$



# MDCDW phase

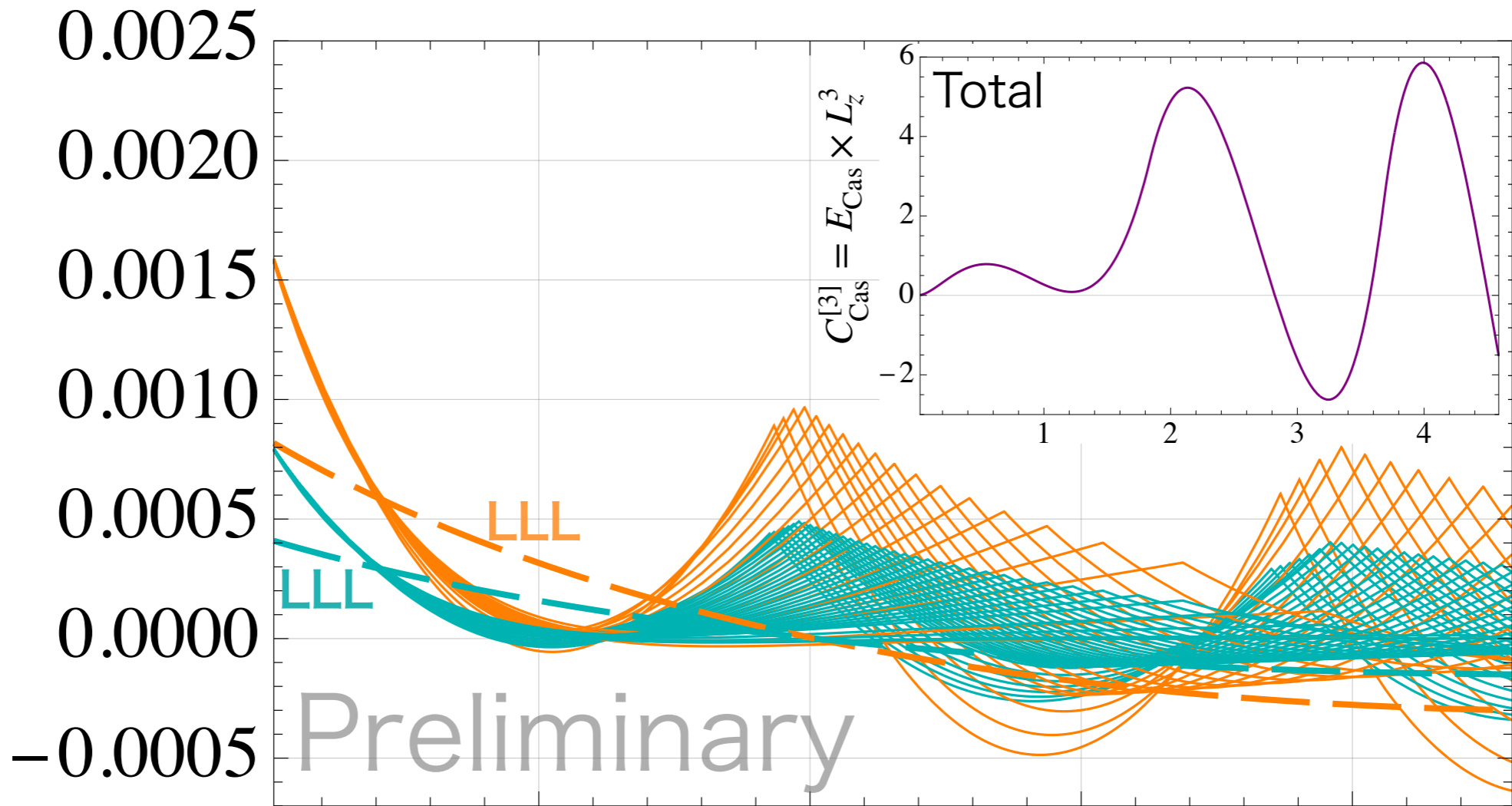
Orange : u-quark

arXiv: 2408.08384

Green : d-quark

$$M/\Lambda = 0.15, q/\Lambda = 0.6, \mu/\Lambda = 0.5, eB/\Lambda = (0.05)^2$$

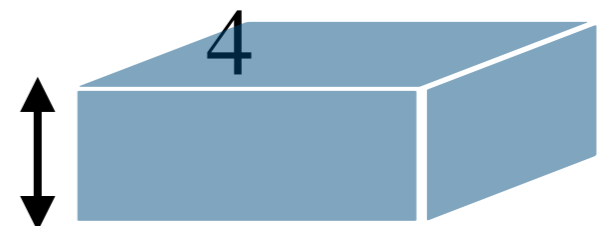
Casimir coef.:  $C_{\text{Cas}}^{[1]} = E_{\text{Cas}} \times L_z$



Preliminary



Thickness:  $L_z$  (fm)



# Summary and Outlook

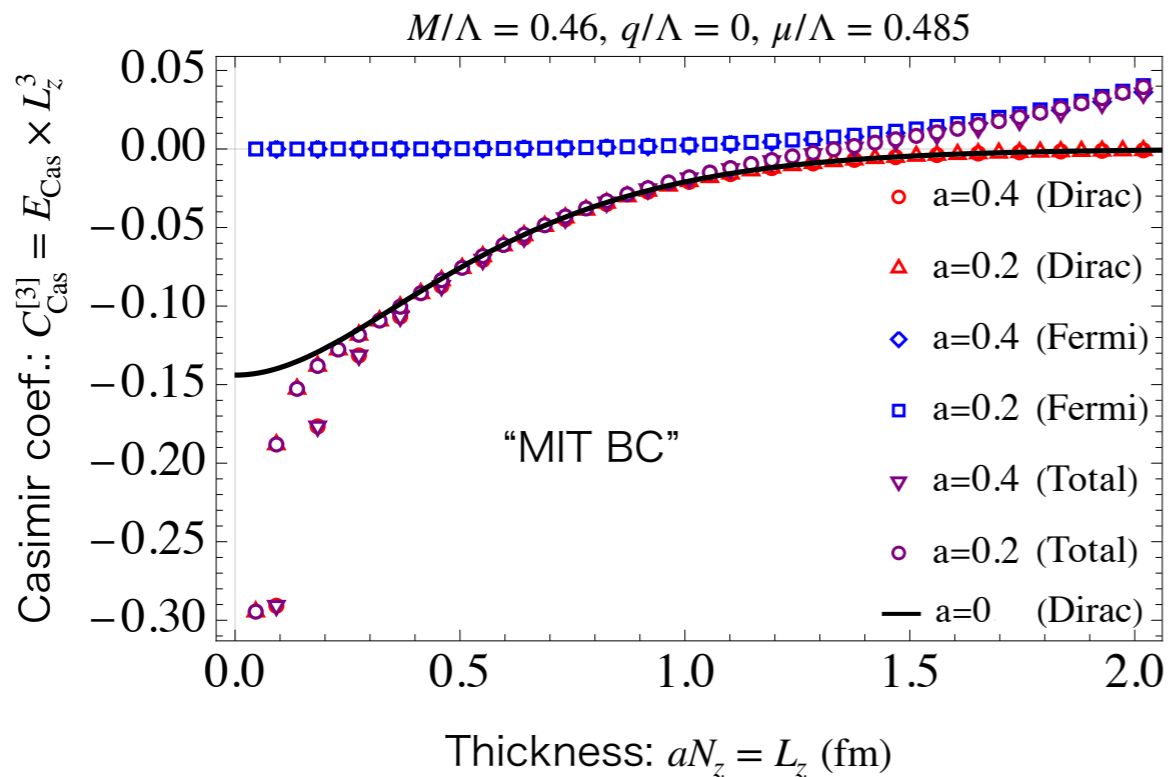
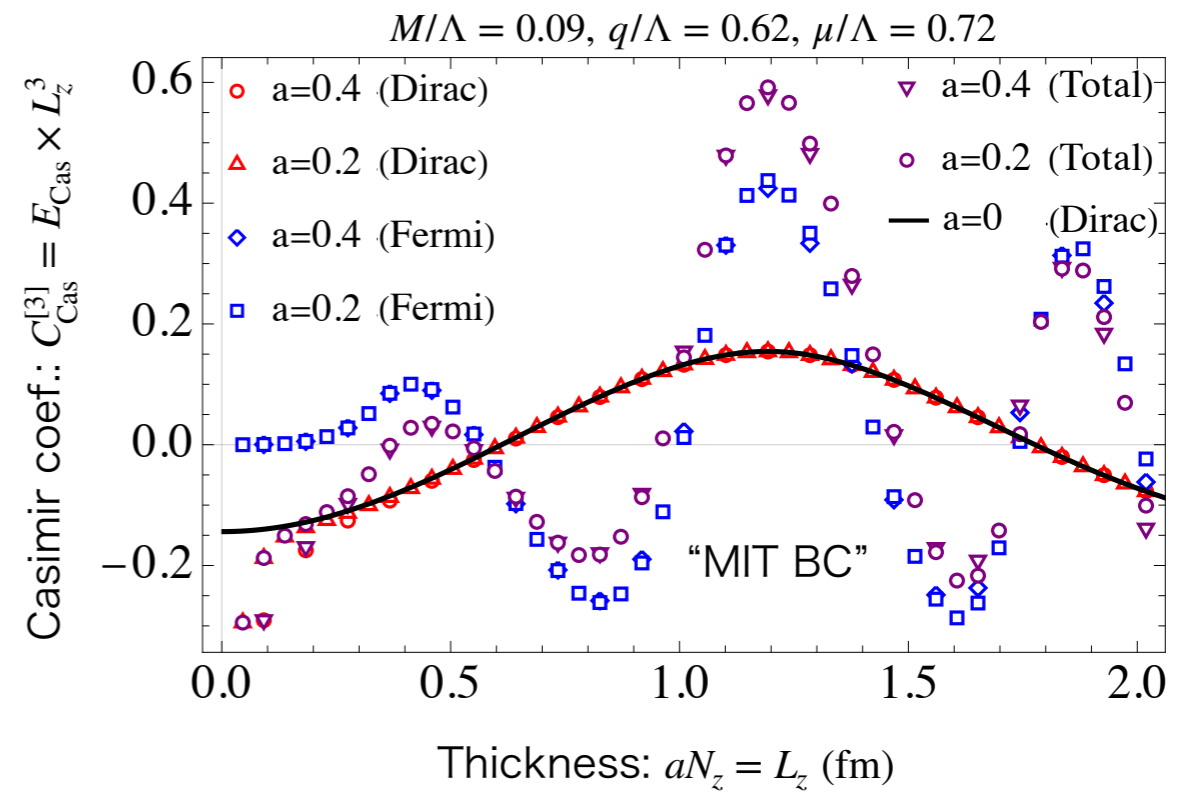
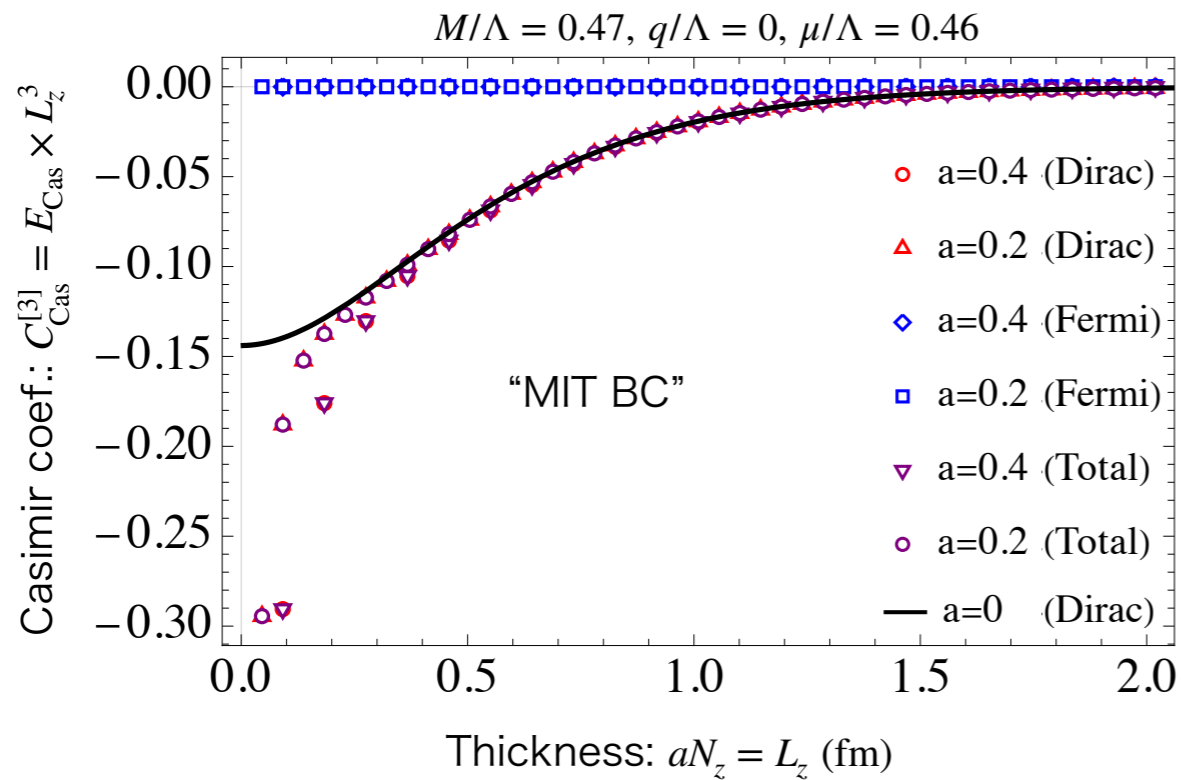
- Casimir effect in dense QCD (NJL)
  - ▶ Low  $\mu$  : **Massive Dirac** Casimir effect
  - ▶ Intermediate  $\mu$  : **Oscillating** Casimir effect from **Fermi sea**
  - ▶ High  $\mu$  : **Oscillating** Casimir effect by **DCDW**
  - ▶ Under  $B$  : **Oscillating** Casimir effect from **each LL**

- Color superconducting phase or kink crystal phase
- Lattice QCD simulations
- . . . .

Thank you for your attention

**Buck up**

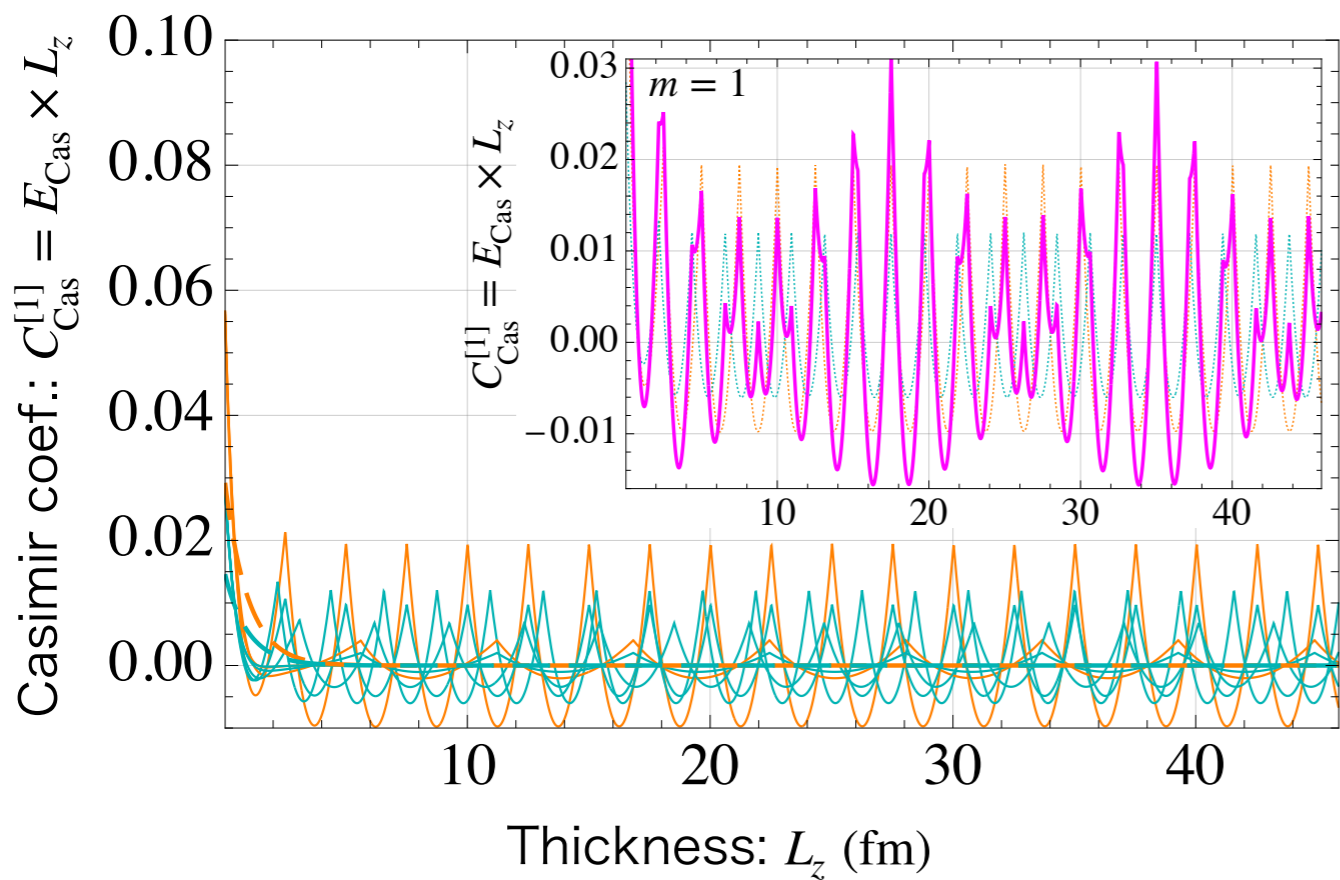
# Buck up



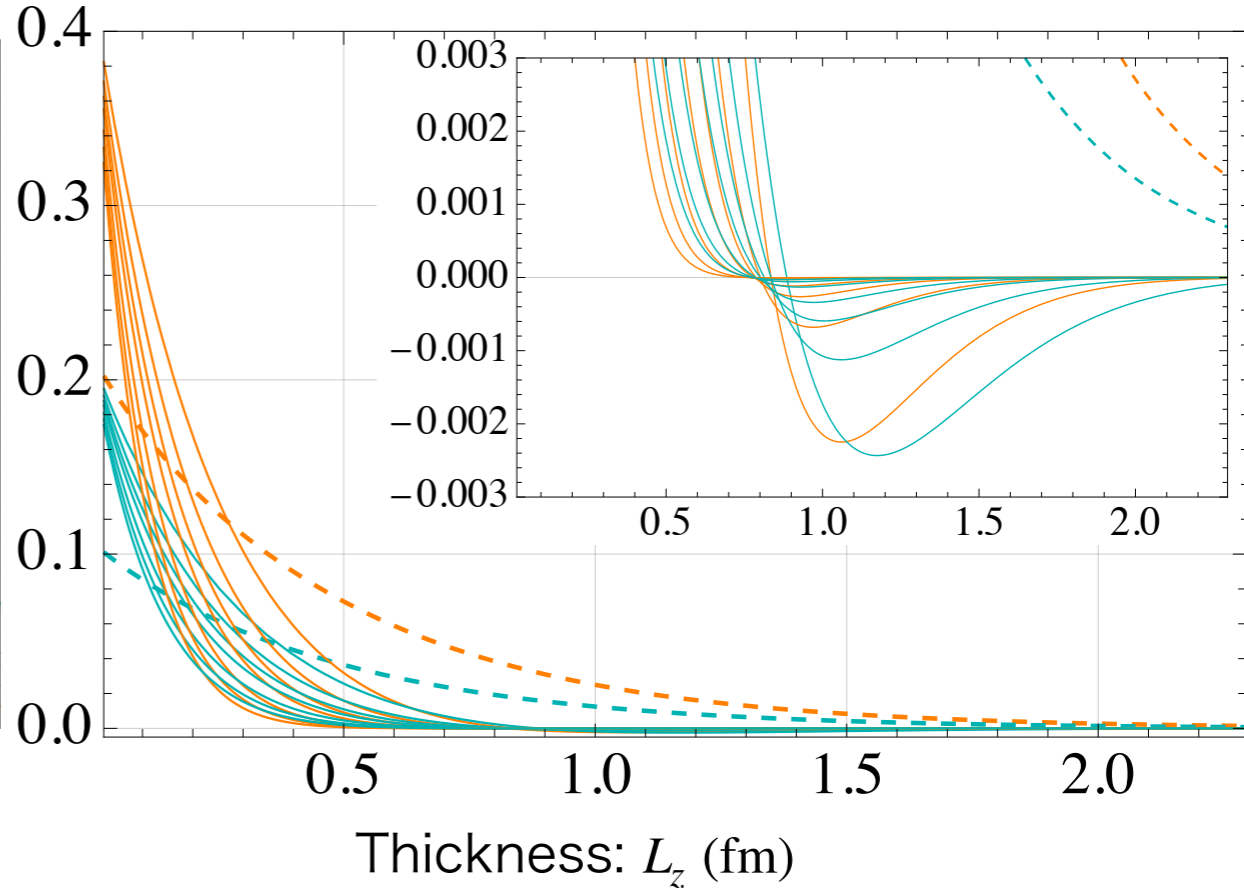


# Buck up

### Intermediate magnetic fields



### Strong magnetic fields



# Buck up

E. Nakano and T. Tatsumi (2005)

