Simulating collectivity in dense baryon matter with multiple fluids

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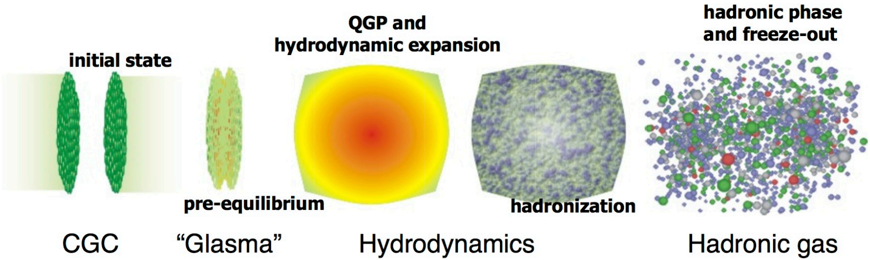
Jakub Cimerman, IK, Boris Tomasik, Pasi Huovinen,
Phys.Rev. C **107** (2023) 4, 044902 [2301.11894]
IK, Jakub Cimerman, arXiv:2312.11325

Take-aways from this talk:

- Modelling HIC at lower RHIC BES/FXT/FAIR energies with fluid dynamics is challenging NOT because of the EoS → slide 5
- directed flow observable seems to be sensitive to many details of the modelling (and not necessarily to the EoS only) → slide 17
- Magnetic field is not the only force that can split Λ-$\bar{\Lambda}$ polarizations → slide 22

Status quo at high energies (LHC or top RHIC)

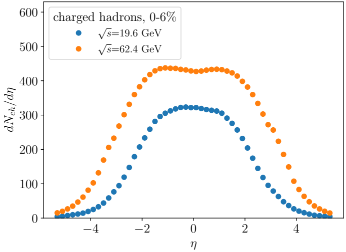
- a relatively clear separation between the initial state and the fluid stage.
- longitudinal boost invariance: 3D → 2D
- equation of state at $n_{\mathrm{B}} = 0$



Foraging into lower energies - RHIC BES

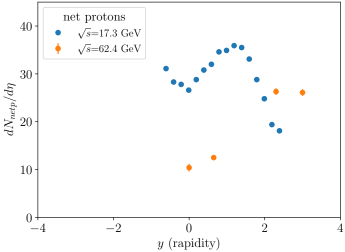
There is no boost invariance

- 3D initial state
- 3D hydro evolution



Baryon and electric charge densities are significant

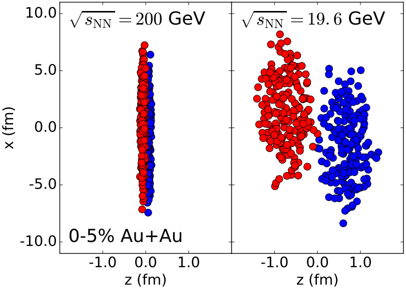
- EoS at finite n_B for hydro evolution



Foraging into *even* lower energies - RHIC FXT, FAIR

The paradigm of “thin pancakes” gradually loses its applicability.

- **Nuclei pass through each other slowly**
  (the passage can last as long as subsequent fluid stage)
- There is no clear separation of the initial state and the fluid stage.



picture credit: C. Shen, B. Schenke, Phys. Rev. C 97, 024907 (2018)

In order to see the effects of the EoS at high densities,

One must start hydro description early!

Dynamical fluidization (1 fluid)

Regions of fluid phase are created dynamically, where (and when) the density is large enough.

Difficulty: how to treat non-fluid and fluid phase together (in the intial state)?

Multi-fluid dynamics

Hydrodynamic description starts from the very beginning of the collision.

Difficulty: reasonability of fluid description at the very start of heavy ion collision?

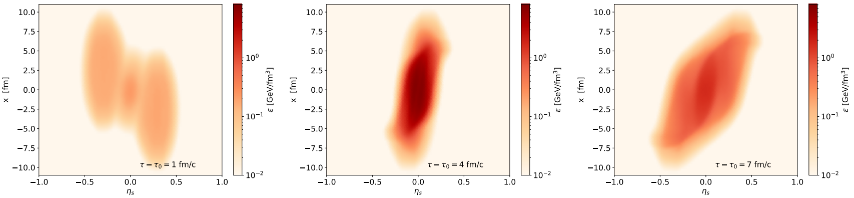
Multi-fluid model discussed in this talk:
MUFFIN: MUlti Fluid simulation for Fast IoN collisions

Think of it as a reincarnation of multi-fluid model for ion-ion collisions.

Equations of motion in multi-fluid dynamics

- Incoming nuclei = two fluids labelled as projectile and target
- Interaction of the fluids (slowing down) via "friction" terms
- Friction transports energy and momentum into the third fluid labelled as fireball
- It is a minimal setup to reproduce baryon stopping at low $\sqrt{s}$ and baryon transparency at high $\sqrt{s}$.

$$
\begin{aligned}
\partial_\mu T_{\mathrm{p}}^{\mu\nu}(x) &= -F_{\mathrm{p}}^{\nu}(x) + F_{\mathrm{fp}}^{\nu}(x),\\
\partial_\mu T_{\mathrm{t}}^{\mu\nu}(x) &= -F_{\mathrm{t}}^{\nu}(x) + F_{\mathrm{ft}}^{\nu}(x),\\
\partial_\mu T_{\mathrm{f}}^{\mu\nu}(x) &= F_{\mathrm{p}}^{\nu}(x) + F_{\mathrm{t}}^{\nu}(x) - F_{\mathrm{fp}}^{\nu}(x) - F_{\mathrm{ft}}^{\nu}(x),
\end{aligned}
$$



Snapshots of multi-fluid evolution in x-η_s plane, Au-Au collision at $\sqrt{s_{\mathrm{NN}}} = 7.7$ GeV

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\end{aligned}$$

The total energy of all 3 fluids is conserved:

$$\partial_\mu \left[T_p^{\mu\nu}(x) + T_t^{\mu\nu}(x) + T_f^{\mu\nu}(x) \right] = 0.$$

- The model captures essential non-equilibrium at the early stages of heavy-ion collisions

Friction terms

Projectile-target friction [Ivanov, Russkikh, Toneev, Phys.Rev.C 73 (2006) 044904]:
Derived based on average energy-momentum transfer in NN scattering [L.M. Satarov, Sov. J. Nucl. Phys. 52, 264 (1990)]

$$F_{\alpha}^{\nu} = \vartheta^2 \rho_p^{\xi} \rho_t^{\xi} m_N V_{\mathrm{rel}}^{pt} [(u_{\alpha}^{\nu} - u_{\bar{\alpha}}^{\nu}) \sigma_{\mathrm{P}}(s_{pt}) + (u_p^{\nu} + u_t^{\nu}) \sigma_{\mathrm{E}}(s_{pt})]$$

where:

- ϑ^2 is a unification factor - which suppresses the friction further when the fluids slow down with respect to each other,
- $\rho_p^{\xi}, \rho_t^{\xi}$ are generalised densities of constituents in the projectile and target fluids,
- V_{rel}^{pt} is a relative velocity of the $p-$ and $t-$ fluid cells,
- m_N is nucleon mass,
- u_{α} , $\alpha = p,t$, $\bar{\alpha} = t,p$ are 4-velocities of the fluid cells,
- $\sigma_{\mathrm{P}}, \sigma_{\mathrm{E}}$ are cross-sections for momentum and energy transfer, respectively.

Friction terms (2)

Fireball-projectile/target friction [same reference]:

$$F^{\nu}_{f\alpha} = \rho^{b}_{\alpha}\xi_{f\alpha}(s_{f\alpha})V^{f\alpha}_{\mathrm{rel}}\frac{T^{0\nu}_{f(eq)}}{u^{0}_{f}}\sigma^{N\pi\to R}_{\mathrm{tot}}(s_{f\alpha}),$$

where:

- $\rho^{b}_{p},\rho^{b}_{t}$ are baryon densities of of the projectile and target fluids,
- $V^{f\alpha}_{\mathrm{rel}}$ is a relative velocity of the fireball and baryon-rich fluid cells,
- $T^{0\nu}_{f(eq)}$ is energy-momentum tensor of the fireball fluid,
- $\sigma^{N\pi\to R}_{\mathrm{tot}}$ is a pion-nucleon cross-section.
- $\xi_{f\alpha}$ is a “K-factor” (a fitting factor) for the friction term, which is intended to compensate for all the missing/incorrect physics therein, and to lead to better agreement with the data

Equations of state in the fluid stage

Chiral model

J. Steinheimer, et al, J. Phys. G 38, 035001 (2011)

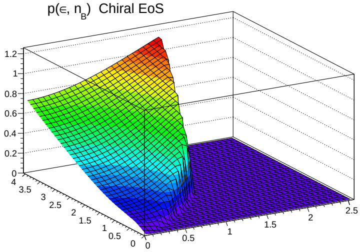
- good agreement with lattice QCD at $\mu_B = 0$
- crossover type PT between confined and deconfined phases at all μ_B

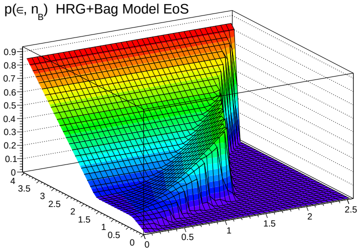
Hadron resonance gas + Bag Model

P.F. Kolb, et al, Phys.Rev. C 62, 054909 (2000)

(a.k.a. `EoS Q`)

- hadron resonance gas made of u, d quarks including repulsive meanfield
- Maxwell construction resulting in 1^{st} order PT at all μ_B





Both EoS are fairly outdated, therefore we are looking for modern alternatives.

Hydrodynamic algorithm: `vHLLE`

`https://github.com/yukarpenko/vhlle`
Comput. Phys. Commun. 185 (2014), 3016 [arXiv:1312.4160]
(this reference paper is outdated!)

✓ shear and bulk viscosity in "Israel-Stewart" with cross-terms

✓ $\tau - \eta$ (hyperbolic), as well as Cartesian coordinate frames (separate branches of the code)

✓ grid resize to optimize CPU time

✓ several initial state, EoS modules. All realized via classes ⇒ easy to plug in new IS/EoS

✓ multi-fluid evolution added with very little overhead ⇒ see a fork by Jakub Cimerman

✓ Multi-threading possible for 3-fluid evolution

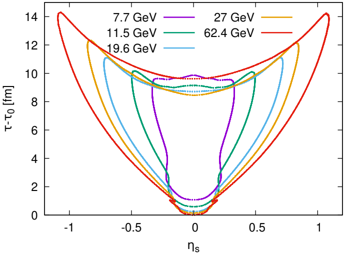
✓ using vHLLE as a library: possible (WIP)

If you are interested to run the complete model yourself:
`https://github.com/jakubcimerman/run-MUFFIN`

Fluid-to-particle transition (particlization)

- Diagonalize $T_p^{\mu\nu}(x) + T_t^{\mu\nu}(x) + T_f^{\mu\nu}(x)$

  $\Rightarrow$ extract energy density $\varepsilon_{\rm eff}$
- construct a surface of fixed $\varepsilon_{\rm eff} = \varepsilon_{\rm sw}$ using `CORNELIUS`.



- On such surface, $\int d\Sigma_\mu T^{0\mu} = 0$ (Gauss theorem)

  $\Rightarrow$ we use it to check the accuracy of the simulations

- Exclude parts of hypersurface which corresponds to matter flowing in
- Hadron sampling according to Cooper-Frye, with

$$f(x,p) = f_p(x,p) + f_t(x,p) + f_f(x,p)$$

using `SMASH-hadron-sampler`

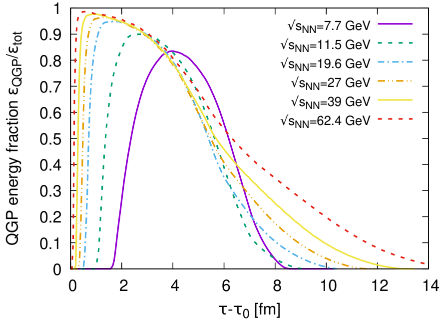
- Sampled hadrons +spectator nucleons

$\Downarrow$

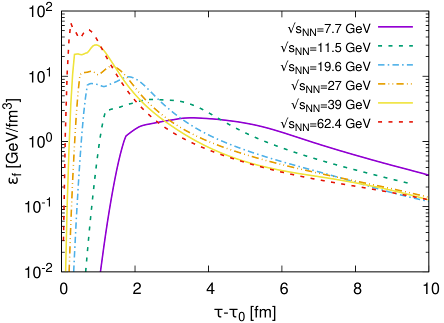
`SMASH` for rescatterings and resonance decays

When do we witness QGP creation in MUFFIN?

QGP fraction as a function of time at different $\sqrt{s_{\rm NN}}$:



Energy density in central cell of fireball fluid as a function of time at different $\sqrt{s_{\rm NN}}$:



⇒ Significant fraction of medium in QGP phase exists down to $\sqrt{s_{\rm NN}} = 7.7$ GeV.

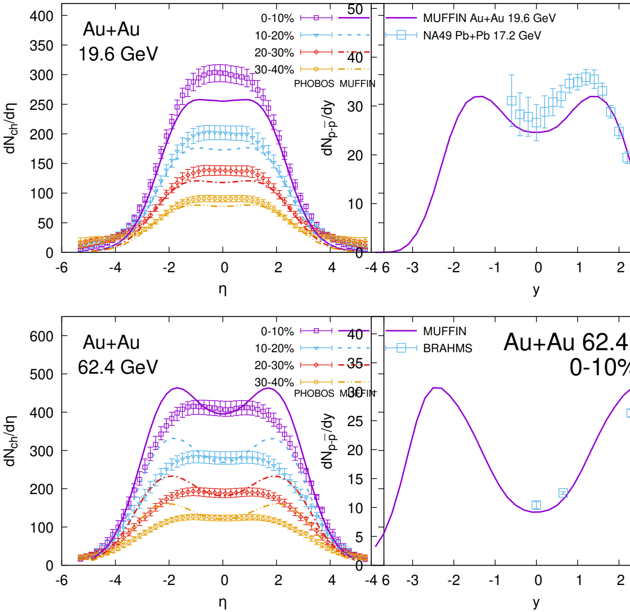
Basic observables vs. the data: $dN/d\eta$, net protons

Fitting parameters in the model:

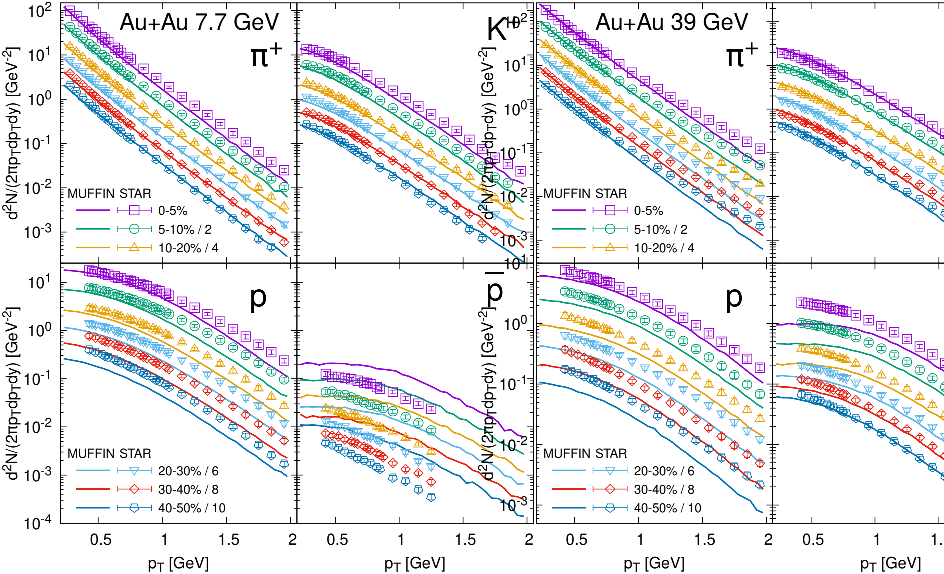
Friction Terms

(both their functional form and the amplitudes)

We fix the functional form and vary the $\xi_{pt}(\sqrt{s_{\rm NN}})$, $\xi_{f\alpha}(\sqrt{s_{\rm NN}})$ to get overall agreement with the data ⇒



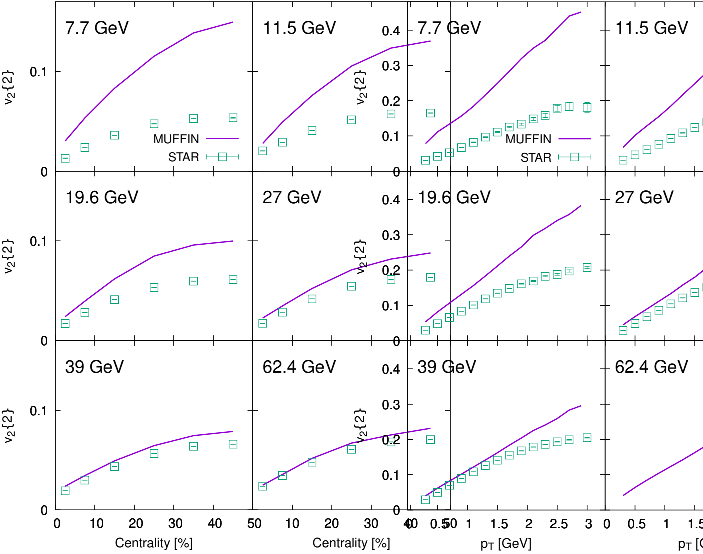
Basic observables vs. the data: dN/dp_T



Basic observables vs. the data: v_2

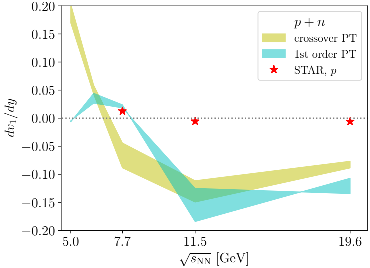
v_2 as a function of centrality

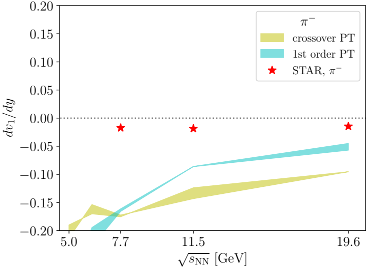
v_2 as a function of p_T



Here a probable culprit is ideal fluid evolution: we haven't switched the viscosity on yet.

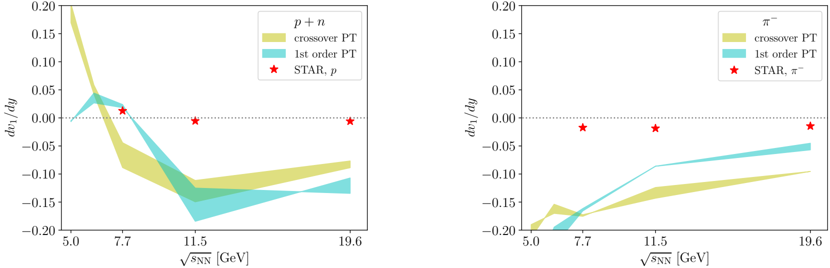
Directed flow in MUFFIN



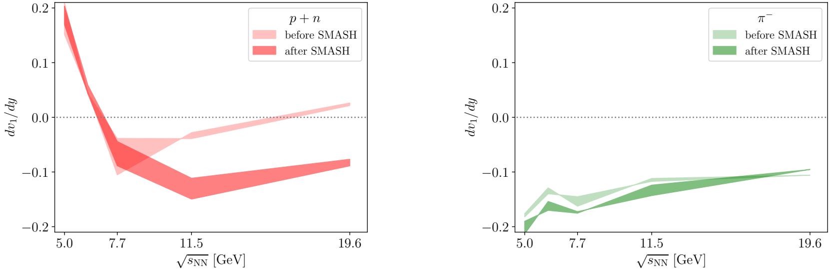


- The directed flow is **much stronger** than what STAR measured.
- There is no clear trend in the EoS dependece.

Directed flow in MUFFIN



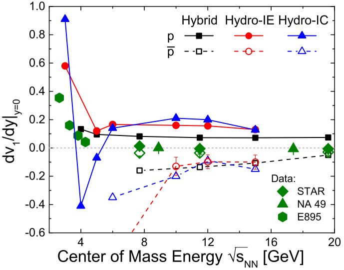
- The directed flow is **much stronger** than what STAR measured.
- Final-state hadronic cascade has a strong influence on v_1.

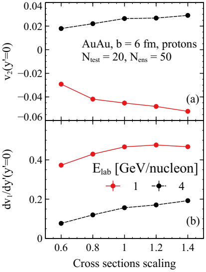


The latter point is compartible with results from other transport studies

hybrid UrQMD: $\Longrightarrow$
Steinheimer, Auvinen, Petersen, Bleicher, Stöcker, Phys. Rev. C 89 (2014) 054913

A strong influence of particlization procedure (fixed time vs. fixed energy density)





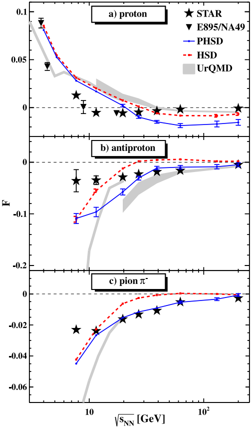
$\Longleftarrow$ SMASH: Oliinychenko, Sorensen, Koch, McLerran, PRC 108, 034908 [2208.11996]:

Scaling cross sections by a factor 0.6 can entirely compensate for the increase of c_s^2 by 0.2.

Another selection of theory world data on v_1

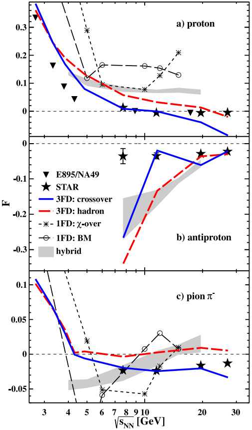
PHSD/HSD/UrQMD:

Konchakovski, Cassing, Ivanov, Toneev,



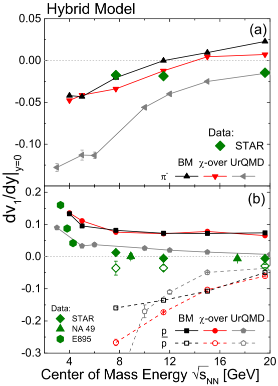
3-fluid/1-fluid models:

Phys. Rev. C 90, 014903 (2014)



hybrid UrQMD:

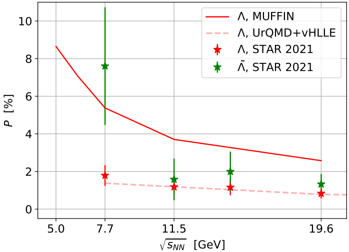
Steinheimer, Auvinen, Petersen, Bleicher, Stöcker, Phys. Rev. C 89 (2014) 054913



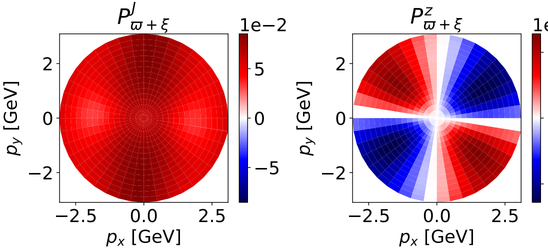
Models with fluid stage generally struggle to reproduce the v_1

Hyperon polarization

Global polarization in 20-50% central Au-Au: Mean hyperon polarization is much stronger in MUFFIN as compared to STAR data



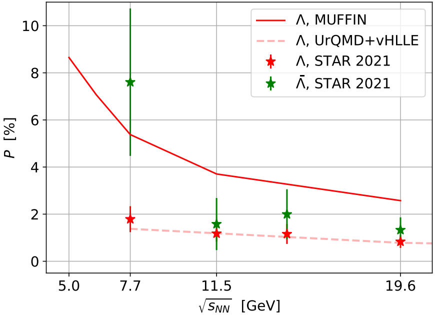
Local polarization in 20-50% central Au-Au at $\sqrt{s_{\rm NN}} = 7.7$ GeV: same patterns as observed at high energies



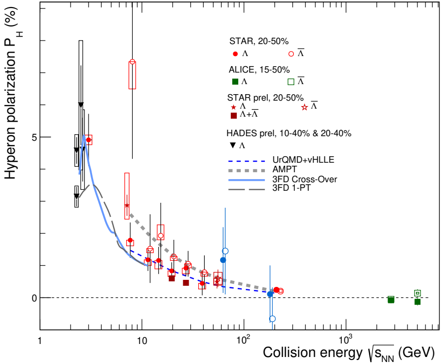
Hyperon polarization

MUFFIN compared to other models

MUFFIN:

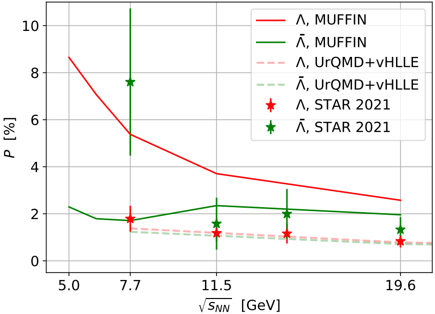


Compilation by Subhash Singha @ SQM 2022:



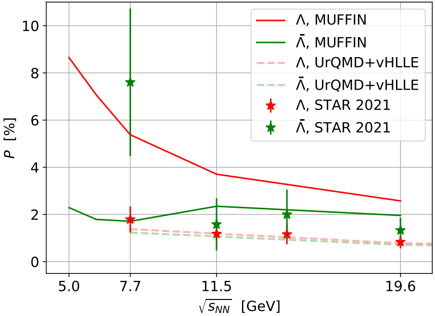
⇒ Λ polarization is stronger in MUFFIN as compared to other hydro-based models

Polarization of $\bar{\Lambda}$ vs. Λ



- MUFFIN produces strong $\Lambda - \bar{\Lambda}$ splitting but with a wrong sign!
- There was a similar but much weaker trend with UrQMD+vHLLE
- Same trend in AMPT+MUSIC, Baochi Fu et al, Phys. Rev. C 103, 024903 (2021) [arXiv:2011.03740]
- The splitting is only due to finite μ_B

Polarization of $\bar{\Lambda}$ vs. Λ



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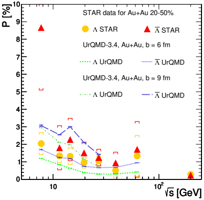
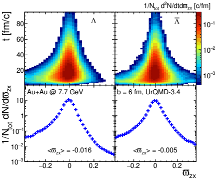
- The splitting is only due to finite μ_B

Correct sign of splitting in UrQMD 3.4 + coarse graining:

O. Vitiuk, L. Bravina, E. Zabrodin, Phys. Lett. B 803 (2020), 135298





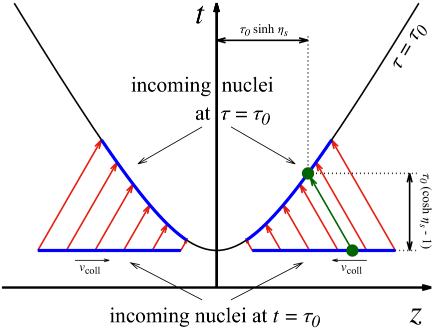
explanation?

Conclusions

- We present the next incarnation of 3-fluid model for relativistic heavy-ion collisions at RHIC BES/FAIR/... energies.
- Different from the existing model by Ivanov,Toneev,Soldatov, there is fluctuating initial state, shear and bulk viscosities (implemented but not enabled yet), Monte Carlo hadron sampling and hadronic afterburner (SMASH). Equation of state can be easily swapped.
- We fit the dN/dy and p_T distributions of hadrons from RHIC BES.
- v_2 is overestimated, which presumably happens due to ideal hydro evolution
- Directed flow is much stronger than the data (same as in other models), and there is no clear EoS trend
- Global polarization is stronger than the data; splitting between $\bar{\Lambda}$ and Λ is strong but has a wrong sign.
- **Outlook:** construct different friction terms based on different underlying assumptions; explore viscous fluid evolution,
- plug in different equations of state to explore sensitivity to the EoS (EoS currently in use are outdated).

Backup slides

Coordinate frame and setup for multi-fluid evolution



- Nucleons from the incoming nuclei are sampled at $t = t_0$ surface (fixed Cartesian time).
- The nucleons are then propagated according to free-flying trajectories onto $\tau = \tau_0$ hypersurface.
- The nucleons are then melted into the fluids: their energies and momenta are distributed to nearby fluid cells using a smearing kernel:

$$T^{0\mu}(x_{\text{cell}}, y_{\text{cell}}, \eta_{\text{cell}}) = \sum_{i \in \text{nucleons}} p_i^\mu K(\Delta x, \Delta y, \Delta \eta_s)$$

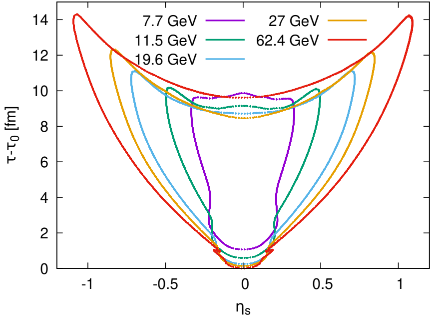
$$N^0_{b,q}(x_{\text{cell}}, y_{\text{cell}}, \eta_{\text{cell}}) = \sum_{i \in \text{nucleons}} \{B_i, Q_i\} K(\Delta x, \Delta y,$$

with a smearing kernel: $K(\Delta x, \Delta y, \Delta \eta_s) = A \exp\left(-\frac{\Delta x^2 + \Delta y^2 + \Delta \eta_s^2 \tau^2 \cosh^2 \eta_s \cosh^2 y}{2\sigma^2}\right)$

Fluid-to-particle transition (particlization) part 1

- Diagonalize $T_p^{\mu\nu}(x) + T_t^{\mu\nu}(x) + T_f^{\mu\nu}(x)$

  $\Rightarrow$ extract energy density ε_{sw}
- construct a hypersurface of fixed $\varepsilon_{\text{sw}} = 0.5$ GeV/fm^3 using `CORNELIUS`.



- On such hypersurface, $\int d\Sigma_\mu T^{0\mu} = 0$ (Gauss theorem)

  $\Rightarrow$ we use it to check the accuracy of the simulations

Particlization part 2

- Exclude parts of hypersurface which corresponds to matter flowing in:

$$\mathrm{d}\Sigma^\mu \mathrm{d}\Sigma_\mu > 0 \quad \text{and} \quad \mathrm{d}\Sigma_0 < 0,$$

$$\mathrm{d}\Sigma^\mu \mathrm{d}\Sigma_\mu < 0 \quad \text{and} \quad \mathrm{d}\Sigma_\mu T^{\mu 0} < 0$$

- distribution function on the particlization surface:

$$f(x,p) = f_p(x,p) + f_t(x,p) + f_f(x,p)$$

- Hadron sampling according to Cooper-Frye, using `SMASH-hadron-sampler`:

$$N = \int \frac{\mathrm{d}^3 p}{E_p} \int \mathrm{d}\Sigma_\mu(x) p^\mu f(p, T(x), \mu_i(x))$$

(grand canonical sampling with $T(x)$, $\mu_i(x)$)

- Sampled hadrons +spectator nucleons

$$\Downarrow$$

`SMASH` for rescatterings and resonance decays

Centrality determination in MUFFIN vs. "Monte Carlo Glauber"

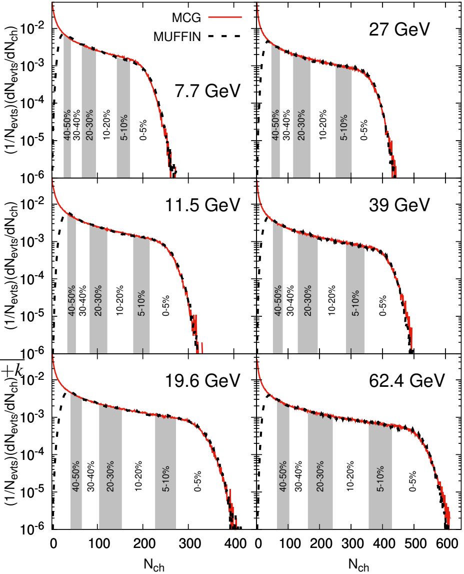
We make a comparison between:

- a semi-minimum-bias MUFFIN simulation ($0 < b < 12$ fm) and
- a two-component model for particle production, where N_{part} and N_{coll} come from a Monte Carlo Glauber sampling:

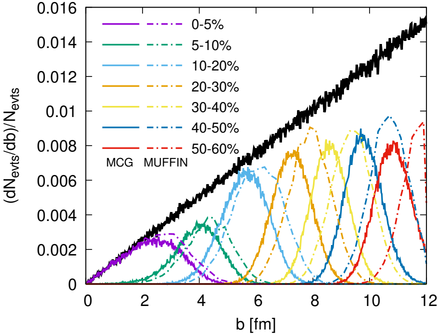
$$\frac{dN_{\text{ch}}}{d\eta} = n_{pp}\left[(1-x)\frac{\langle N_{\text{part}}\rangle}{2} + x\langle N_{\text{coll}}\rangle\right]$$

$$P_{\text{NBD}}(n_{pp},k;n) = \frac{\Gamma(n+k)}{\Gamma(n+1)\Gamma(k)}\,\frac{(n_{pp}/k)^n}{(n_{pp}/k+1)^{n+k}}$$

- "MCG" fits the N_{ch} distribution from a semi-minbias MUFFIN simulation with $b = 0-12$ fm



- we bin the generated events in centrality classes based on $dN_{\mathrm{ch}}/d\eta$ at mid-rapidity:

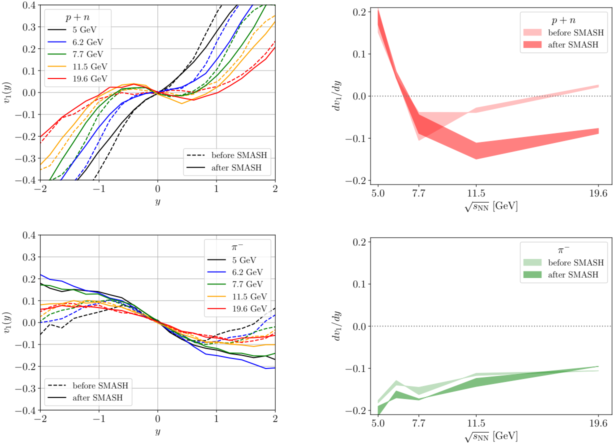


- For each centrality class, the mean impact parameter in MUFFIN has a larger value as compared to the "Monte Carlo Glauber"

Similar findings: arXiv:2303.07919 by Kuttan, Steinheimer, Zhou, Bleicher and Stoecker

Directed flow in MUFFIN

effects of hadronic cascade



Final-state hadronic cascade drives the directed flow further away from the data.