Symmetries of QCD

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Chiral symmetry of quantum chromodynamics

Importance of chiral symmetry for strong interactions was realized long before the advent of QCD:

Gell-Mann – Levy linear sigma model: 1960
Nambu and Jona-Lasinio model: 1960-1961
Nonlinear sigma model of Weinberg: 1968
Current algebra: 1960th

If two fermions are massless the Dirac Lagrangian is $SU(2)_R \times SU(2)_L$ invariant: independent isospin symmetry for the right- and left-handed fermions (it is in addition invariant under $U(1)_V \times U(1)_A$). If the interaction respects the chiral symmetry, the whole theory is chirally symmetric.

This symmetry is spontaneously (dynamically) broken: the ground state - the vacuum - is not chiral invariant. A fundamental consequence: the lowest excitation with the pion quantum numbers is massless - the Nambu - Goldstone boson.

The chiral symmetry breaking gives the basis for a systematic expansion in hadron physics near the chiral limit - the chiral perturbation theory.

One of the implications from the Nambu–Jona-Lasinio and the linear sigma model - the hadron mass is generated via spontaneous breaking of chiral symmetry (the quark condensate) - strongly influenced the development of hadron physics for many years. It is misleading!

QCD Lagrangian:

$$\mathcal{L} = \bar{q}(i\gamma^\mu \partial_\mu - m)q + g\bar{q}\gamma^\mu \mathbf{T} q \cdot \mathbf{A}_\mu - \frac{1}{4}(\mathbf{F}_{\mu\nu} \cdot \mathbf{F}^{\mu\nu})$$

In the chiral limit the Lagrangian can be separated into the right- and left-handed parts of quarks $q = R + L$:

$$\mathcal{L} = \bar{R} i\gamma^\mu \partial_\mu R + \bar{L} i\gamma^\mu \partial_\mu L + g\bar{R}\gamma^\mu \mathbf{T} R \cdot \mathbf{A}_\mu + g\bar{L}\gamma^\mu \mathbf{T} L \cdot \mathbf{A}_\mu - \frac{1}{4}(\mathbf{F}_{\mu\nu} \cdot \mathbf{F}^{\mu\nu})$$

This Lagrangian is invariant under the $U(1)_A$ transformation

$$q \longrightarrow e^{i\theta\gamma_5} q$$

The $U(1)_A$ is broken by quantization.
It is also invariant under $SU(2)_R \times SU(2)_L$ chiral transformation:

$$\begin{pmatrix} u_R \\ d_R \end{pmatrix} \rightarrow \begin{pmatrix} u'_R \\ d'_R \end{pmatrix} = \exp\left(i \frac{\varepsilon^n \sigma^n}{2} \right) \begin{pmatrix} u_R \\ d_R \end{pmatrix}$$

$$\begin{pmatrix} u_L \\ d_L \end{pmatrix} \rightarrow \begin{pmatrix} u'_L \\ d'_L \end{pmatrix} = \exp\left(i \frac{\varepsilon^n \sigma^n}{2} \right) \begin{pmatrix} u_L \\ d_L \end{pmatrix}$$

There are two independent isospin symmetries.

Symmetries of electric and magnetic interactions in electrodynamics and QCD

$$
\begin{aligned}
div\mathbf{E} &= 4\pi\rho \\
rot\mathbf{B} - \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} &= \frac{4\pi}{c}\mathbf{j} \\
rot\mathbf{E} + \frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} &= 0 \\
div\mathbf{B} &= 0
\end{aligned}
$$

How do we define **E** and **B** in a given Lorentz frame?

$$\mathbf{F} = q\mathbf{E} + q\frac{\mathbf{v}}{c} \times \mathbf{B}$$

Possible to measure directly **F** in electrodynamics but not possible in quantum chromodynamics. Is there another method to distinguish **E** and **B**? Yes!

Consider charges to be particles with $s = 1/2$. They are characterized by helicities (chiralities for massless particles):

$$
\begin{aligned}
R &: \ \mathbf{s}\cdot\mathbf{p} > 0 \\
L &: \ \mathbf{s}\cdot\mathbf{p} < 0
\end{aligned}
$$

$$\begin{pmatrix} R \\ L \end{pmatrix}$$

Consider a $SU(2)_{CS}$ chiral spin transformation that mixes R and L (L.Ya.G., 2015):

$$\begin{pmatrix} R \\ L \end{pmatrix} \rightarrow \begin{pmatrix} R' \\ L' \end{pmatrix} = \exp\left(i\frac{\varepsilon^n \sigma^n}{2}\right)\begin{pmatrix} R \\ L \end{pmatrix}$$

Dirac eq. prohibits such transformation.

What happens with the charge density ρ?

$$R'^{\dagger}R' + L'^{\dagger}L' = R^{\dagger}R + L^{\dagger}L$$

i.e.

$$\rho' = \rho$$

Charge density is invariant under the chiral spin transformation.

What happens with the current density $\mathbf{j} = \rho\mathbf{v}$? Upon the chiral spin transformation $\mathbf{v}$ and $\mathbf{j}$ change.

$$\mathbf{F}_E = q\mathbf{E}$$

$$\mathbf{F}_B = \sim \mathbf{j} \times \mathbf{B}$$

The interaction of a charge with the electric field is invariant under $SU(2)_{CS}$, while the interaction of a current with the magnetic field is not.

We can distinguish the electric and magnetic fields by the chiral spin symmetry. The electric part of the EM theory is more symmetric than the magnetic part. It is a gauge-invariant statement.

$$\mathcal{L} = \mathcal{L}(\mathbf{E}, \mathbf{B}) - \rho\phi + \mathbf{j} \cdot \mathbf{A} + \quad Dirac \quad Lagrangian$$

The Dirac Lagrangian is not invariant under the $SU(2)_{CS}$.

The chromoelectric field of QCD is defined via interaction with the color charge

$$\mathbf{F} = Q^a \mathbf{E}^a; \qquad Q^a = \int d^3x \; q^\dagger(x) T^a q(x), \qquad a = 1, ..., 8$$

It is invariant under $SU(2)_{CS}$:

$$q \to q' = \exp\left(i \frac{\varepsilon^n \Sigma^n}{2}\right) q, \qquad \Sigma = \{\gamma_k, -i\gamma_5\gamma_k, \gamma_5\}$$

$$\overline{q}(x)\gamma^\mu T^a q(x) \; A_\mu^a = q^\dagger(x) T^a q(x) \; A_0^a + \overline{q}(x)\gamma^i T^a q(x) \; A_i^a$$

In a given Lorentz frame interaction of quarks with the electric part of the gluonic field is chiral spin invariant like in electrodynamics.

$SU(2)_{CS} \times SU(N_F) \subset SU(2N_F)$; $SU(2N_F)$ is also a symmetry of the color charge.

The color charge and electric part of the theory have a $SU(2N_F)$ symmetry that is larger than the chiral symmetry of QCD as a whole.

The fundamental vector of $SU(2N_F)$ at $N_F = 2$

$$\Psi = \begin{pmatrix} u_{\mathrm{R}} \\ u_{\mathrm{L}} \\ d_{\mathrm{R}} \\ d_{\mathrm{L}} \end{pmatrix} .$$

L.Ya.G., EPJA, 2015; L.Ya.G., M.Pak,PRD, 2015

The $SU(2)_{CS}$ and $SU(2N_F)$ are explicitly broken by magnetic interaction and by quark kinetic term.

Chiral, CS and $SU(4)$ multiplets

$(0,0)$		$f_1(0,1^{++})$ $\overline{\Psi}(\mathbb{1}_F \otimes \gamma^5\gamma^i)\Psi$		$\omega(0,1^{--})$ $\overline{\Psi}(\mathbb{1}_F \otimes \gamma^i)\Psi$	
$(1/2,1/2)_a$	$U(1)_A$	$b_1(1,1^{+-})$ $\overline{\Psi}(\tau^a \otimes \gamma^5\gamma^4\gamma^i)\Psi$	$\xleftrightarrow{SU(2)_A}$	$\omega(0,1^{--})$ $\overline{\Psi}(\mathbb{1}_F \otimes \gamma^4\gamma^i)\Psi$	$U(1)_A$
$(1/2,1/2)_b$		$\rho(1,1^{--})$ $\overline{\Psi}(\tau^a \otimes \gamma^4\gamma^i)\Psi$	$\xleftrightarrow{SU(2)_A}$	$h_1(0,1^{+-})$ $\overline{\Psi}(\mathbb{1}_F \otimes \gamma^5\gamma^4\gamma^i)\Psi$	
$(1,0)\oplus(0,1)$		$\rho(1,1^{--})$ $\overline{\Psi}(\tau^a \otimes \gamma^i)\Psi$	$\xleftrightarrow{SU(2)_A}$	$a_1(1,1^{++})$ $\overline{\Psi}(\tau^a \otimes \gamma^5\gamma^i)\Psi$	

$(0,0)$		$f_1(0,1^{++})$ $\overline{\Psi}(\mathbb{1}_F \otimes \gamma^5\gamma^i)\Psi$	$SU(4)$	$\omega(0,1^{--})$ $\overline{\Psi}(\mathbb{1}_F \otimes \gamma^i)\Psi$	$SU(2)_{CS}$
$(1/2,1/2)_a$		$b_1(1,1^{+-})$ $\overline{\Psi}(\tau^a \otimes \gamma^5\gamma^4\gamma^i)\Psi$	$\leftrightarrow$	$\omega(0,1^{--})$ $\overline{\Psi}(\mathbb{1}_F \otimes \gamma^4\gamma^i)\Psi$	
$(1/2,1/2)_b$	$SU(2)_{CS}$	$\rho(1,1^{--})$ $\overline{\Psi}(\tau^a \otimes \gamma^4\gamma^i)\Psi$	$\leftrightarrow$	$h_1(0,1^{+-})$ $\overline{\Psi}(\mathbb{1}_F \otimes \gamma^5\gamma^4\gamma^i)\Psi$	
$(1,0)\oplus(0,1)$		$\rho(1,1^{--})$ $\overline{\Psi}(\tau^a \otimes \gamma^i)\Psi$	$\leftrightarrow$	$a_1(1,1^{++})$ $\overline{\Psi}(\tau^a \otimes \gamma^5\gamma^i)\Psi$	

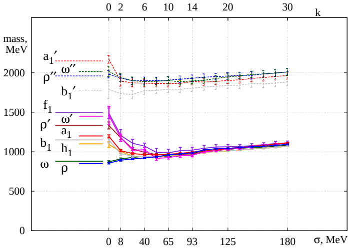
Observation of the chiral spin symmetry and its implications for hadrons

Banks-Casher:

$$i\gamma_\mu D_\mu \psi_n(x) = \lambda_n \psi_n(x), \qquad < \bar{q}q >= -\pi\rho(0).$$

Low mode truncation, M.Denissenya, L.Ya.G., C.B.Lang, 2014-2015:

$$S = S_{Full} - \sum_{i=1}^{k} \frac{1}{\lambda_i} |\lambda_i\rangle\langle\lambda_i|.$$



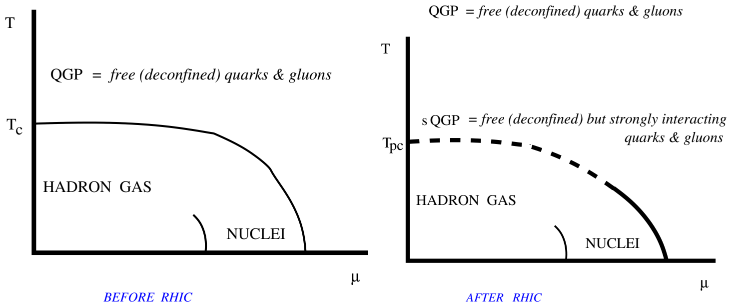
SU(2)$_{CS}$ and *SU*(4) symmetries.

The magnetic interaction is located predominantly in the near zero modes while the confining electric interaction is distributed among all modes. Confinement and chiral symmetry breaking are not directly related.

A clear implication for the genesis of hadron spectra: the *SU*(4) degeneracy of the electric string is lifted by chiral symmetry breaking

Hot QCD. Before and after RHIC

What happens with hadrons in the medium upon increasing T?



2006-2010 Budapest-Wuppertal and HotQCD (Bielefeld-BNL) lattice collaborations: The chiral restoration crossover is observed at $T = 120 - 180$ MeV with the pseudocritical temperature at $T_{ch} \sim 155$ MeV. The deconfinement as observed from the Polyakov loop is approximately at the same temperature. The latter statement turned out to be technically wrong.

We need objective information. Can be obtained in computer simulations on the lattice.

Correlation functions

The most detailed information about QCD is encoded in correlation functions

$$C_\Gamma(t,x,y,z) = \langle O_\Gamma(t,x,y,z)\, O_\Gamma(0,\mathbf{0})^\dagger \rangle \ .$$

They carry the full spectral information $\rho_\Gamma(\omega,\mathbf{p})$

$$C_\Gamma(t,\mathbf{p}) = \int_0^\infty \frac{d\omega}{2\pi}\, K(t,\omega)\rho_\Gamma(\omega,\mathbf{p}), \quad K(t,\omega) = \frac{\cosh(\omega(t-1/2T))}{\sinh(\omega/2T)} \ .$$

The spatial and temporal correlators are defined as

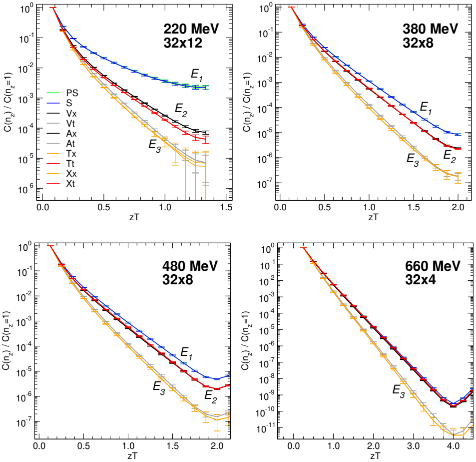
$$C_\Gamma^s(z) = \sum_{x,y,t} C_\Gamma(t,x,y,z) \ ,$$

$$C_\Gamma^t(t) = \sum_{x,y,z} C_\Gamma(t,x,y,z) \ .$$

Spatial correlators above T_{ch}

C. Rohrhofer, Y. Aoki, G. Cossu, H. Fukaya, C. Gattringer, L.Ya.G., S. Hashimoto, C.B. Lang, S. Prelovsek , 2017 - 2019

$N_f = 2$ QCD with the chirally symmetric Dirac operator.



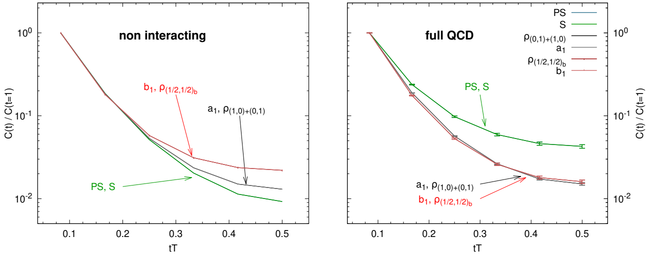
$E1$ - $U(1)_A$ symmetry; $E2$ - chiral spin and $SU(4)$ symmetries; $E3$ consistent with both chiral symmetry and chiral spin ($SU(4)$) symmetry.

$SU(2)_{CS}$ and $SU(4)$ symmetries persist up to $T \sim 500$ MeV.

Temporal correlators above T_{ch}

C. Rohrhofer, Y. Aoki, L.Ya.G., S. Hashimoto, 2020

$N_F = 2$ QCD at $T = 220$ MeV

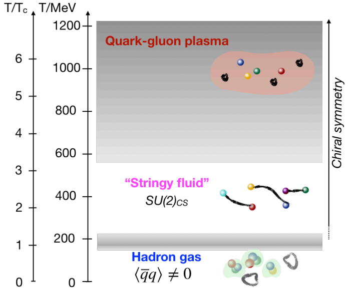


Free quarks: $SU(2)_L \times SU(2)_R$ and $U(1)_A$ multiplets.

Full QCD at $T = 220$ MeV: $U(1)_A$, $SU(2)_L \times SU(2)_R$, $SU(2)_{CS}$ and $SU(2N_F)$ multiplets.

Above T_{ch} QCD is approximately $SU(2)_{CS}$ and $SU(4)$ symmetric.

Three regimes of QCD



$0 - T_{ch}$ - Hadron Gas (broken chiral symmetry);

$T_{ch} - 3T_{ch}$ - Stringy Fluid (chiral, $SU(2)_{CS}$ and $SU(4)$ symmetries; electric confinement)

Stringy fluid is mostly populated with $J = 0, 1$ states. It is a densely packed system of the color-singlet clusters that interact strongly. Quark interchanges between the clusters are significant.

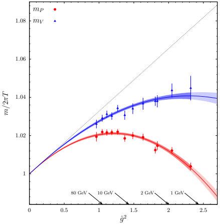
$T > 3T_{ch}$ - a smooth approach to QGP (chiral symmetry)

Screening masses and stringy fluid. L.Ya.G., O. Philipsen, R. Pisarski, 2022

$$
\begin{aligned}
e^{pV/T} = Z &= \mathrm{Tr}(e^{-aHN_\tau}) \\
&= \mathrm{Tr}(e^{-aH_z N_z}) = \sum_{n_z} e^{-E_{n_z} N_z} \ ,
\end{aligned}
$$

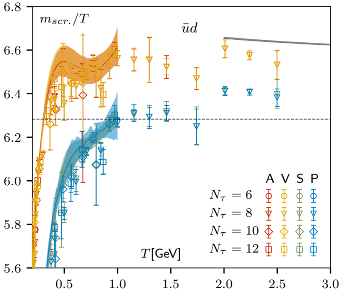
QGP - (quasi)parton dynamics drives observables.

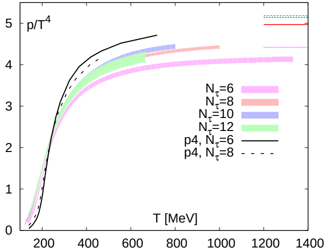
Lattice at $T \sim 1$ - 160 GeV (M.D. Brida et al, 2022) :



$$
\begin{aligned}
\frac{m_{PS}}{2\pi T} &= 1 + p_2\,\hat{g}^2(T) + p_3\,\hat{g}^3(T) + p_4\,\hat{g}^4(T)\ , \\
\frac{m_V}{2\pi T} &= \frac{m_{PS}}{2\pi T} + s_4\,\hat{g}^4(T)\ ,
\end{aligned}
$$

From A. Bazavov et al, 2018,2019:

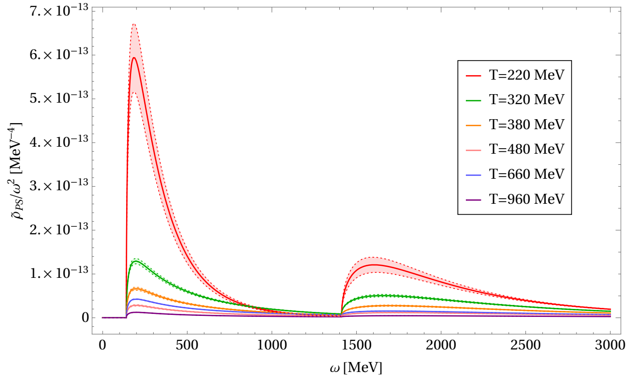




An independent demonstration of the existence of a temperature window $T_{ch} < T < 3T_{ch}$, in which chiral symmetry is restored but the dynamics is inconsistent with a (quasi)partonic description.

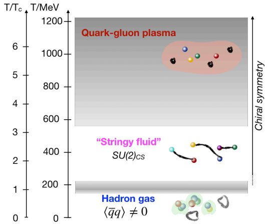
π spectral function

From spatial correlators via generalised Lehmann representation to spectral functions; P. Lowdon and O. Philipsen, 2022



Existence of a pion state and its first radial excitations above T_{ch}. A clear demonstration that above T_{ch} the degrees of freedom are hadron-like.

Large Nc QCD phase diagram. T.D. Cohen, L.Ya.G., 2023



In combined large N_c and chiral limit three regimes connected by smooth crossovers might become distinct phases separated by phase transitions:

$$\epsilon_{\mathrm{HG}} \sim N_c^0 \ , \ \ P_{\mathrm{HG}} \sim N_c^0 \ , \ s_{\mathrm{HG}} \sim N_c^0 \ ,$$

$$\epsilon_{\mathrm{str}} \sim N_c^1 \ , \ \ P_{\mathrm{str}} \sim N_c^1 \ , \ s_{\mathrm{str}} \sim N_c^1 \ .$$

$$\epsilon_{\mathrm{QGP}} \sim N_c^2 \ , \ \ P_{\mathrm{QGP}} \sim N_c^2 \ , \ s_{\mathrm{QGP}} \sim N_c^2 \ .$$

$T_{ch} \sim 130$ MeV; $T_d \sim 300$ MeV

N_c^1 scaling of conserved charges. T.D. Cohen, L.Ya.G., 2024

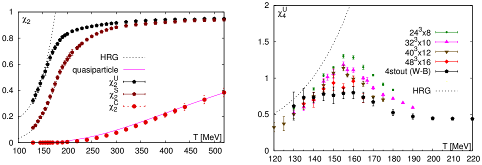


Figure: Fluctuations of conserved net u, d and strange quark numbers in 2+1 QCD at physical quark masses. Source: R. Bellwied et al, PRD 92 (2015) 114505.

Below $T_{ch} \sim 155$ MeV the fluctuations are well reproduced by the Hadron Gas. In this regime structure of hadrons is frosen. All thermodynamical quantities scale as N_c^0.

$$N_q \equiv \int d^3x \; n_q(x) \quad \text{with} \quad n_q(x) = \bar{q}(x)\gamma^0 q(x), \quad q = u, d, s$$

$$N_q \sim N_c^1.$$

Deviation of fluctuations from HRG at $T_{ch} \sim 155$ MeV indicates that the quark degrees of freedom get relevant. In the intermediate stringy fluid regime thermodynamical quantities scale as N_c^1.

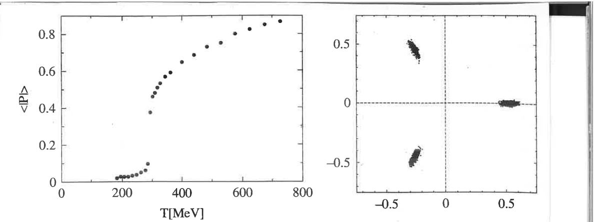
Deconfinement and center symmetry

In the QGP regime both quark and gluon degrees of freedom should be relevant and thermodynamical quantities scale as N_c^2. An observable that is sensitive to $N_c^2 - 1$ gluons is Polyakov loop:

$$P(\mathbf{x}) = \frac{1}{N_c} Tr \left[T \exp \left(i \int_0^\beta d\tau A_0(\mathbf{x}, \tau) \right) \right].$$

The Polyakov loop is the order parameter for Z_3 center symmetry and deconfinement in pure glue theory (or in QCD with heavy quarks). At $T_d \sim 300$ MeV the center symmetry gets spontaneously broken.

The first order deconfinement phase transition around $T_d \sim 300$ MeV:



In QCD with light quarks the center symmetry is explicitly broken by quark loops: the first order phase transition is replaced by a very smooth crossover.

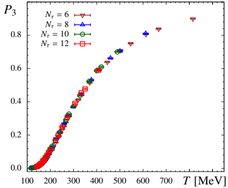
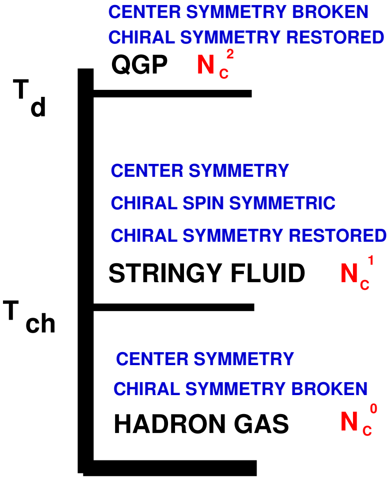


Figure: Temperature evolution of the normalized Polyakov loop in 2+1 QCD at physical quark masses. Source: P. Petreczky, H. P. Schadler, PRD 92 (2015) 094517

One observes a very smooth crossover around $T_d \sim 300$ MeV.

Just above $T_{ch} \sim 155$ MeV P it is very small: confinement and stringy fluid.

Large Nc phase diagram and QCD symmetries



T_{ch} ~ 130 MeV T_d ~ 300 MeV