Proton structure through large-scale simulations

University of Cyprus

Constantia Alexandrou

The Cyprus Institute

Extended Twisted Mass Collaboration

AQTIVATE

European Joint Doctorate, grant agreement No. 101072344

Research & Innovation Foundation

EXCELLENCE/0421/0043

XIII International Conference on New Frontiers in Physics

26 Aug - 4 Sep 2024, OAC, Kolymbari, Crete, Greece

Half a century of QCD

Outline

- **Introduction & status of lattice Quantum Chromodynamics (QCD) simulations**
- **3D structure of the nucleon**
  - **Mellin moments**
    - ➡ **Charges**
    - ➡ **Form factors**
    - ➡ **Spin content of the nucleon**
  - **Direct computation of parton distributions**
- **Conclusions**

Quantum ChromoDynamics (QCD)

- QCD: gauge theory of the strong interaction
- Lagrangian: formulated in terms of quarks and gluons

$$\mathcal{L}_{QCD} = -\frac{1}{4}F^{a}_{\mu\nu}F^{a\,\mu\nu} + \sum_{f=u,d,s,c,b,t} \bar{\psi}_f \left(i\gamma^{\mu}D_{\mu} - m_f\right)\psi_f \qquad D_{\mu} = \partial_{\mu} - ig\frac{\lambda^{a}}{2}A^{a}{}_{\mu}$$

Harald Fritzsch

Murray Gell-Mann

Heinrich Leutwyler

Phys. Lett. 47B (1973) 365

Quantum ChromoDynamics (QCD)

- QCD: gauge theory of the strong interaction
- Lagrangian: formulated in terms of quarks and gluons

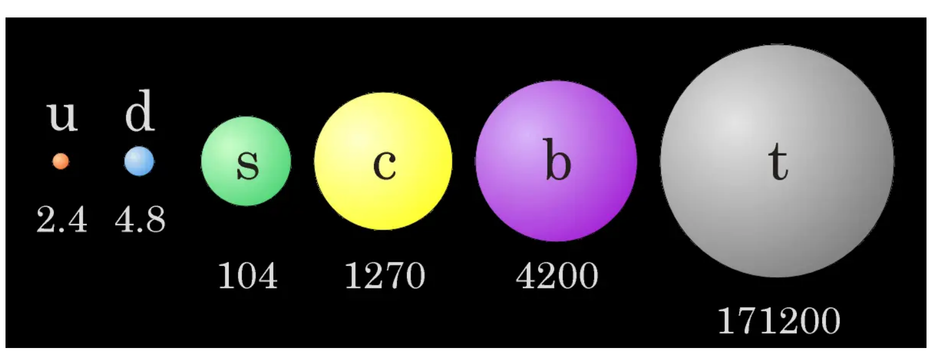
$$\mathcal{L}_{QCD} = -\frac{1}{4} F^a_{\mu\nu} F^{a\,\mu\nu} + \sum_{f=u,d,s,c,b,t} \bar{\psi}_f \left(i\gamma^\mu D_\mu - m_f \right) \psi_f \qquad D_\mu = \partial_\mu - ig\frac{\lambda^a}{2} A^a{}_\mu$$

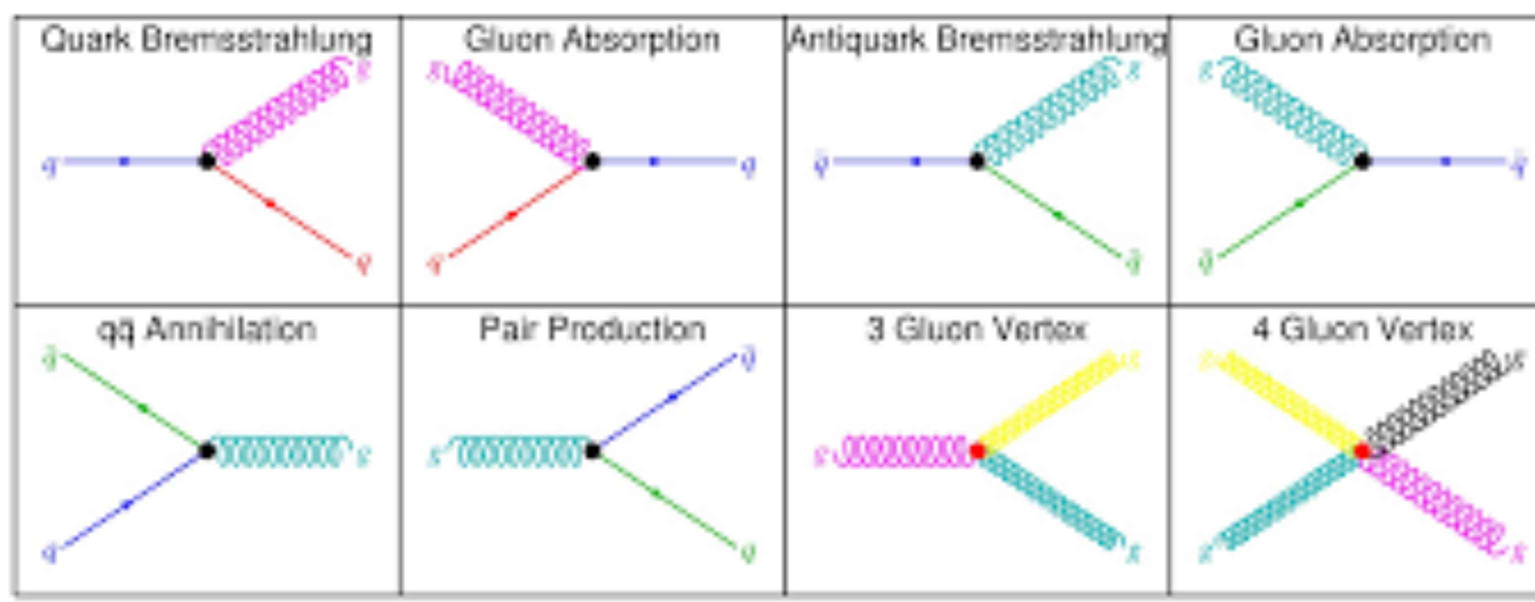

Harald Fritzsch


Murray Gell-Mann


Heinrich Leutwyler

Phys. Lett. 47B (1973) 365





Quantum ChromoDynamics (QCD)

- QCD-Gauge theory of the strong interaction
- Lagrangian: formulated in terms of quarks and gluons

$$\mathcal{L}_{QCD} = -\frac{1}{4} F^{a}_{\mu\nu} F^{a\,\mu\nu} + \sum_{f=u,d,s,c,b,t} \bar{\psi}_f \left(i\gamma^\mu D_\mu - m_f \right) \psi_f \qquad D_\mu = \partial_\mu - ig\frac{\lambda^a}{2} A^a{}_\mu$$

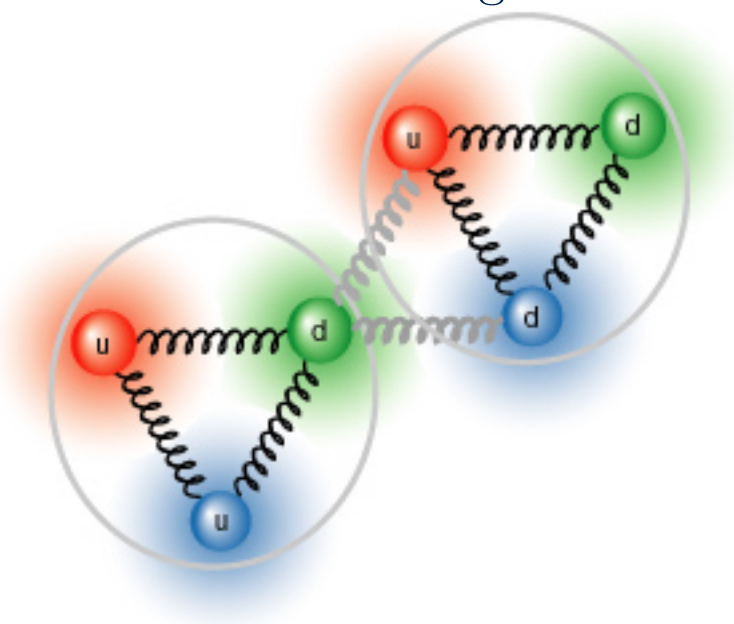

Harald Fritzsch

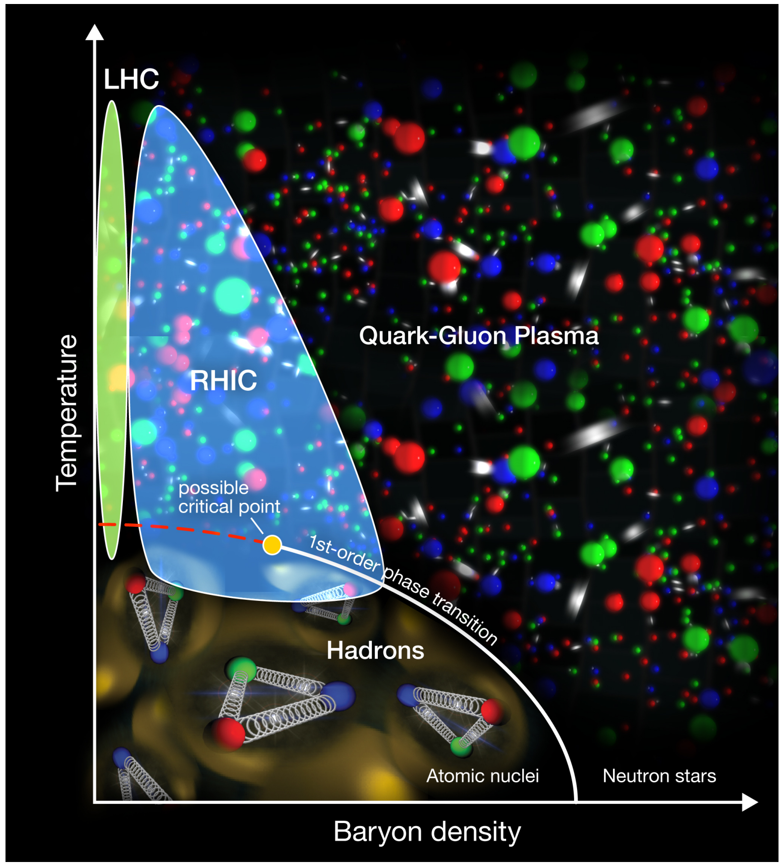

Murray Gell-Mann


Heinrich Leutwyler

Phys. Lett. 47B (1973) 365

- This "*simple*" Lagrangian produces the amazingly rich structure of strongly interacting matter both at the subatomic level and at large scale in the universe

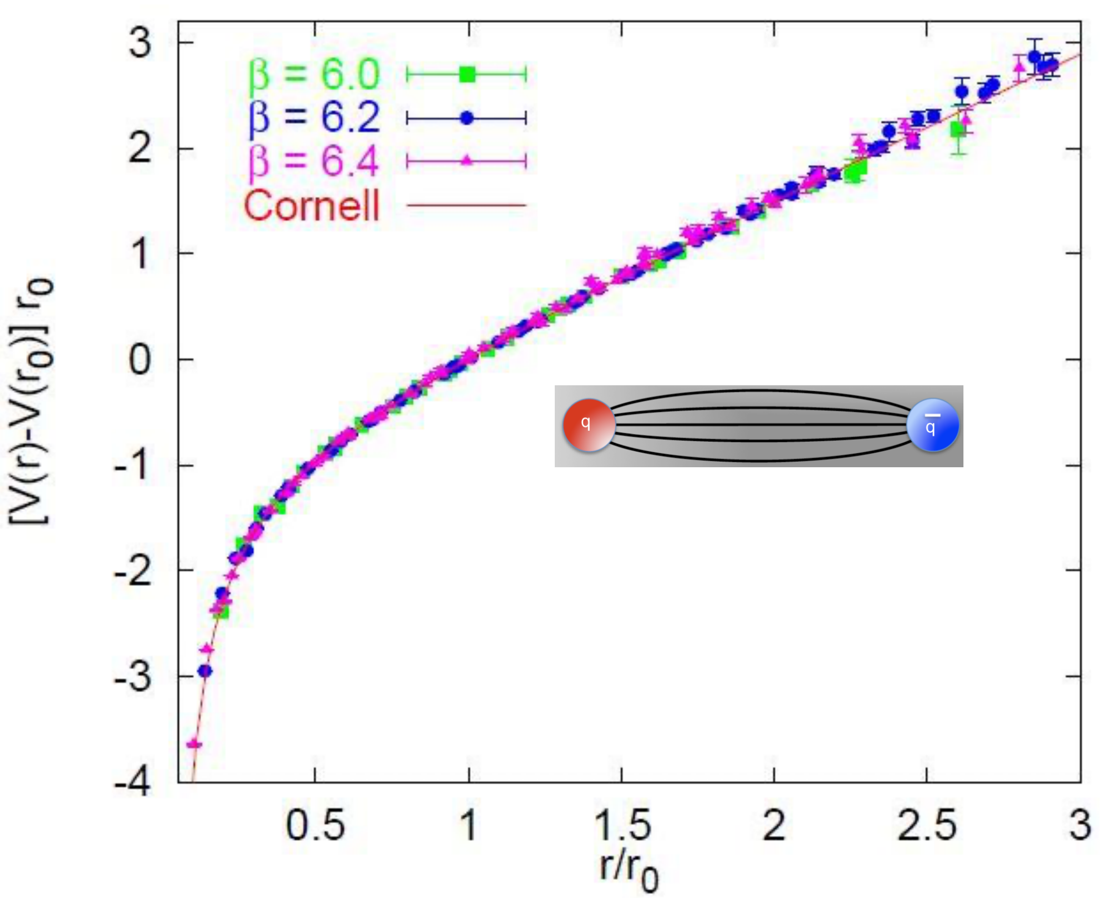


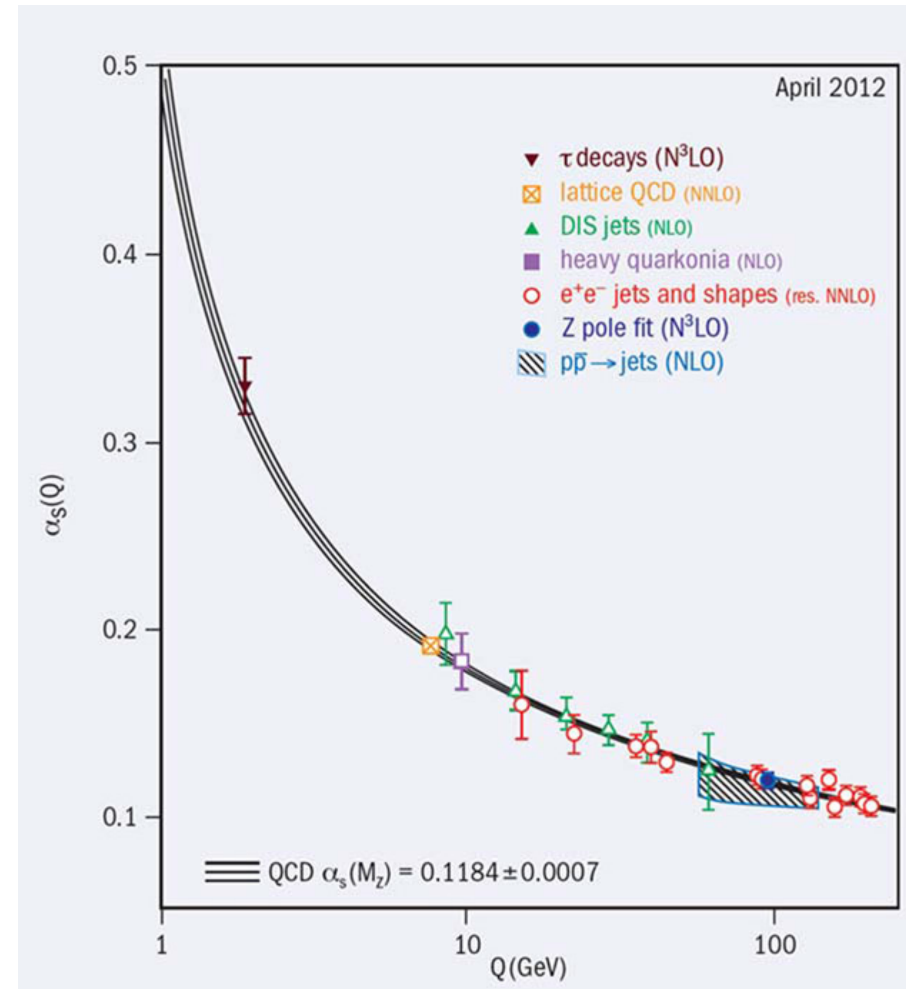


Quantum ChromoDynamics (QCD)

$$\mathcal{L}_{QCD} = -\frac{1}{4}F^{a}_{\mu\nu}F^{a\,\mu\nu} + \sum_{f=u,d,s,c,b,t} \bar{\psi}_f \left(i\gamma^\mu D_\mu - m_f\right)\psi_f$$

- Unique properties:
  - ★Confinement
  - ★Asymptotic freedom





David Gross Frank Wilczek David Politzer

Nobel prize in Physics 2004

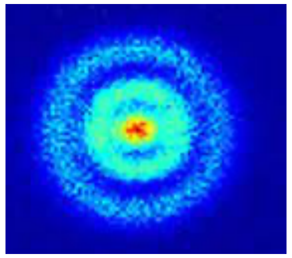
Quantum ChromoDynamics (QCD)

$$\mathcal{L}_{QCD} = -\frac{1}{4} F^a_{\mu\nu} F^{a\,\mu\nu} + \sum_{f=u,d,s,c,b,t} \bar{\psi}_f \left(i\gamma^\mu D_\mu - m_f \right) \psi_f$$

- Unique properties:
  - ★Confinement
  - ★Asymptotic freedom
  - ★Mass generation via interaction

QED

Quantum theory of the electromagnetic force mediated by exchange of photons



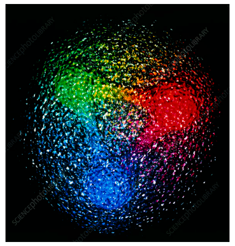
Hydrogen atom

$$m_{\text{Hydrogen}} = \underbrace{0.51 \text{ MeV}}_{m_{e^-}} + \underbrace{938.29 \text{ MeV}}_{m_{p^+}} - \underbrace{13.6 \text{ eV}}_{\text{E}_{\text{binding}}}$$

A. Stodolna et al., PRL 110 (2013) 213001

QCD

Quantum theory of the strong force mediated by exchange of gluons



Artist impression

Proton

$$m_p = \underbrace{2.3 \text{ MeV}}_{2\times m_u} + \underbrace{4.7 \text{ MeV}}_{m_d} + \underbrace{929 \text{ MeV}}_{\text{E}_{\text{binding}}}$$

99% of proton mass from interaction!

Lattice QCD provides an *ab initio* method to study a wide class of strong interaction phenomena

- Lattice QCD uses directly $\mathcal{L}_{QCD}$ or the action $S_{QCD} = \int d^4x \, \mathcal{L}_{QCD}$

Lattice QCD

$$\mathcal{L}_{QCD} = -\frac{1}{4} F^{a}_{\mu\nu} F^{a\,\mu\nu} + \sum_{f=u,d,s,c,b,t} \bar{\psi}_f \left(i\gamma^\mu D_\mu - m_f \right) \psi_f$$

Ken Wilson 1974

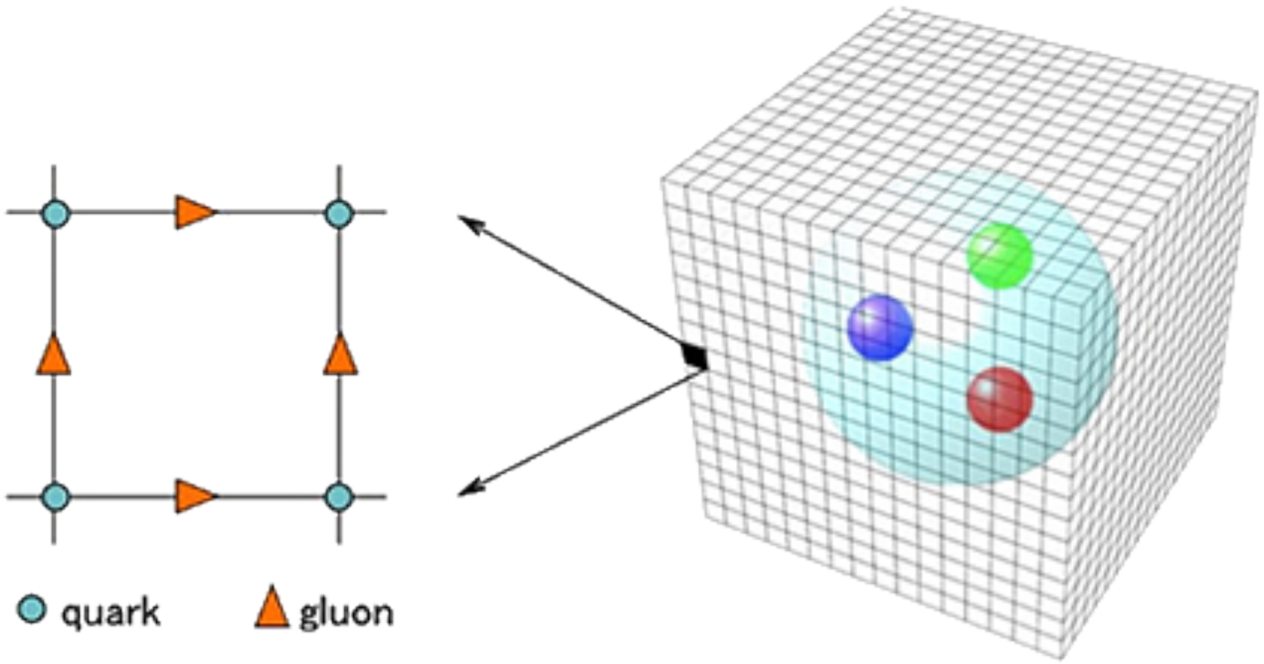
- Formulate in path integrals:

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \mathcal{D}[A]\mathcal{D}[\bar{\psi}]\mathcal{D}[\psi]\mathcal{O}(\bar{\psi}, \psi, A) e^{i(S_g[A]+S_f[A,\bar{\psi},\psi])}$$

- Integrate out fermions:

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \mathcal{D}[A]\mathcal{O}(D_f^{-1}[A], A) \left(\prod_{f=u,d,s,c,b,t} \mathrm{Det}(D_f[A]) \right) e^{iS_g[A]}$$

❋Discretisation of QCD on a 4-D space-time lattice —> provides a non-perturbative regularisation



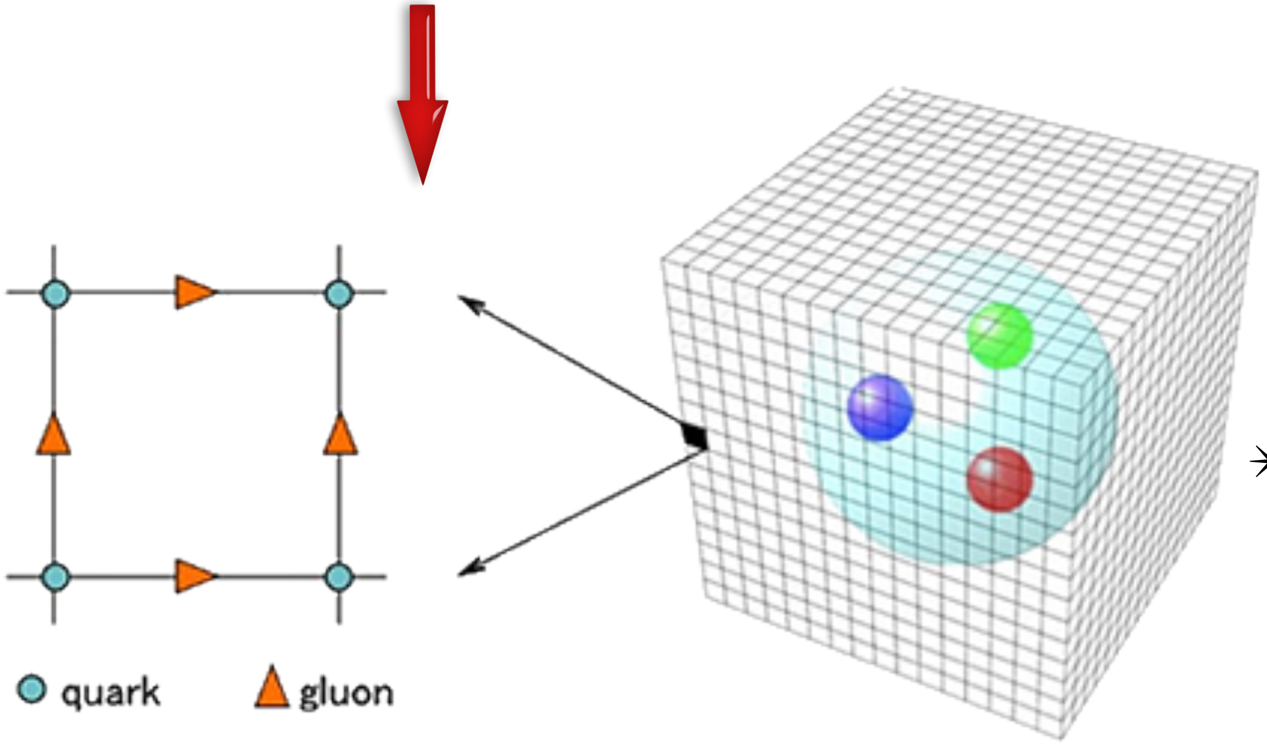
- Quark fields on sites: $\psi(x)$ - b and t quarks too heavy to include
- Gauge fields on links ~ parallel transporters:

$$U_\mu(x) = e^{iagA_\mu}$$

Lattice QCD

$$\mathcal{L}_{QCD} = -\frac{1}{4} F^{a}_{\mu\nu} F^{a\,\mu\nu} + \sum_{f=u,d,s,c,b,t} \bar{\psi}_f \left(i\gamma^\mu D_\mu - m_f \right) \psi_f$$

Ken Wilson 1974



✶Discretisation of QCD on a 4-D space-time lattice
- provides a non-perturbative regularisation
- preserves gauge invariance

Mike Creutz 1980

1. Simulation of gauge ensembles {U}:

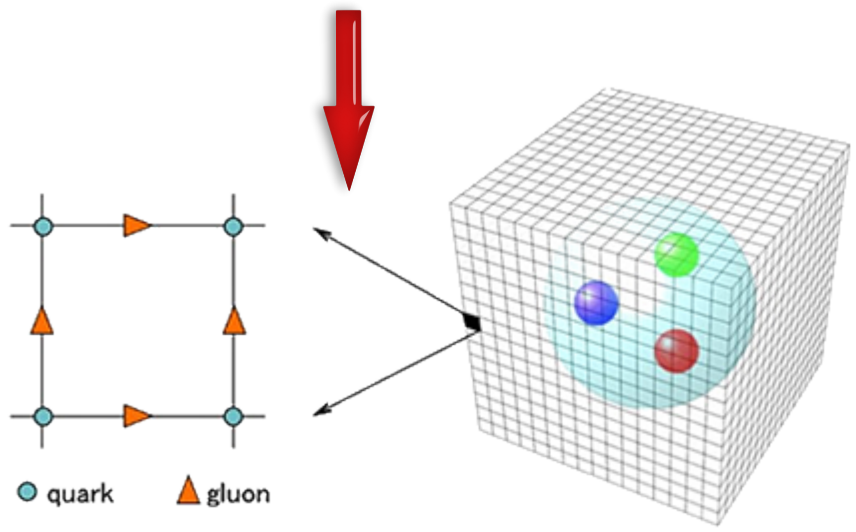
$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \mathcal{D}[U] \mathcal{O}(D_f^{-1}[U], U) \left(\prod_{f=u,d,s,c} \mathrm{Det}(D_f[U]) \right) e^{-S_g[U]}$$

Rotate to imaginary time

$$P[U] = \frac{1}{Z} \left(\prod_{f=u,d,s,c} \mathrm{Det}(D_f[U]) \right) e^{-S_g[U]}$$

Simulations of lattice QCD

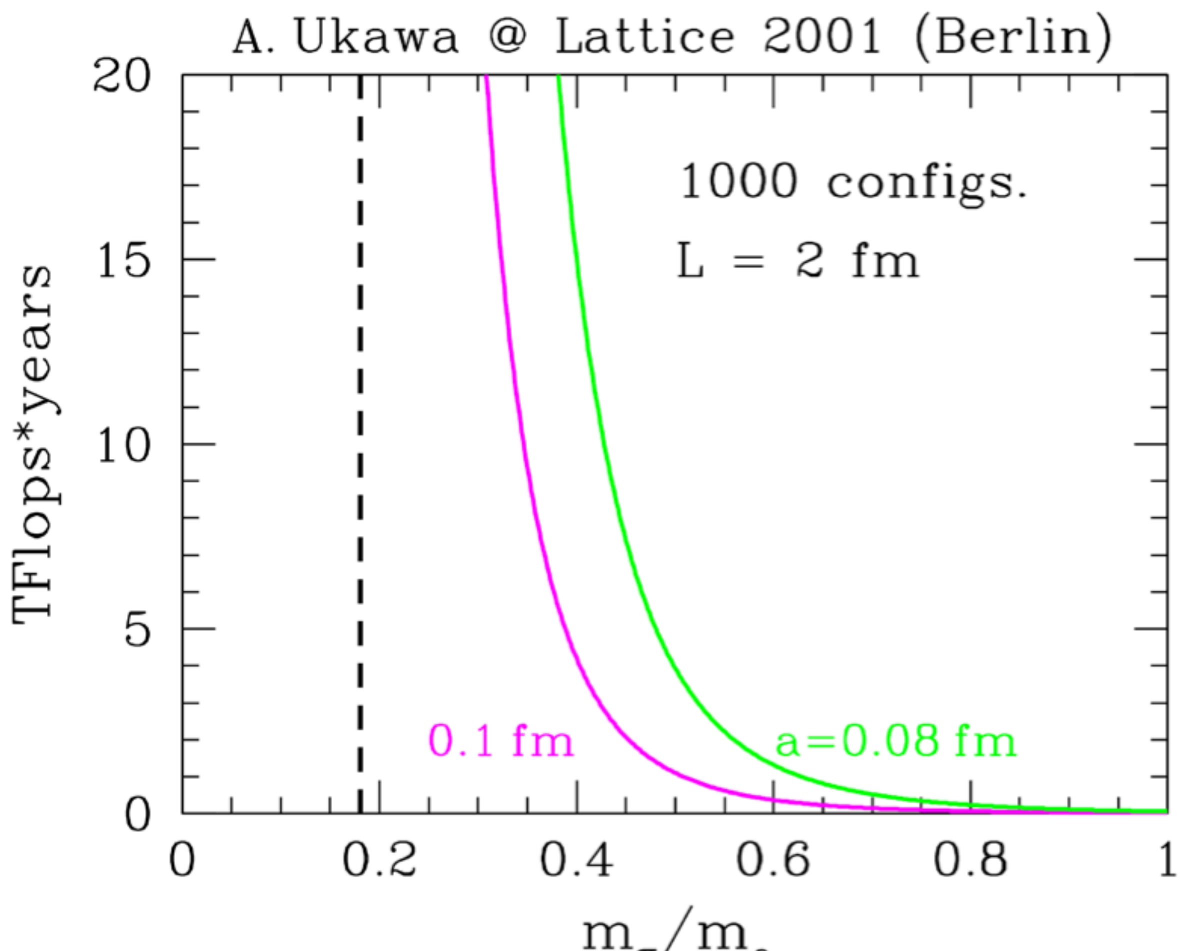
$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \mathcal{D}[U] \mathcal{O}(D_f^{-1}[U], U) \left(\prod_{f=u,d,s,c} \mathrm{Det}(D_f[U]) \right) e^{-S_g[U]}$$



1. **Simulation of gauge ensembles {U}**: Using HMC

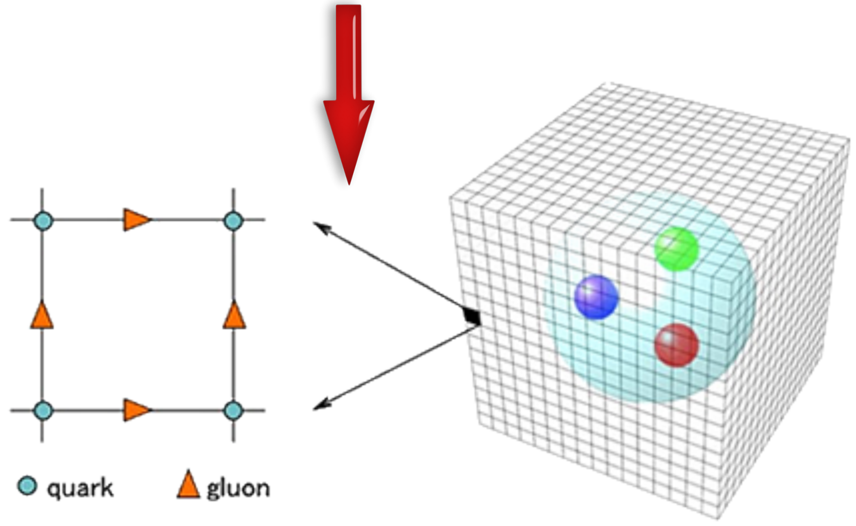
$$P[U] = \frac{1}{Z} \left(\prod_{f=u,d,s,c} \mathrm{Det}(D_f[U]) \right) e^{-S_g[U]}$$

Estimated cost to generate 1000 independent fermion configurations



Simulations of lattice QCD

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \mathcal{D}[U] \mathcal{O}(D_f^{-1}[U], U) \left(\prod_{f=u,d,s,c} \mathrm{Det}(D_f[U]) \right) e^{-S_g[U]}$$

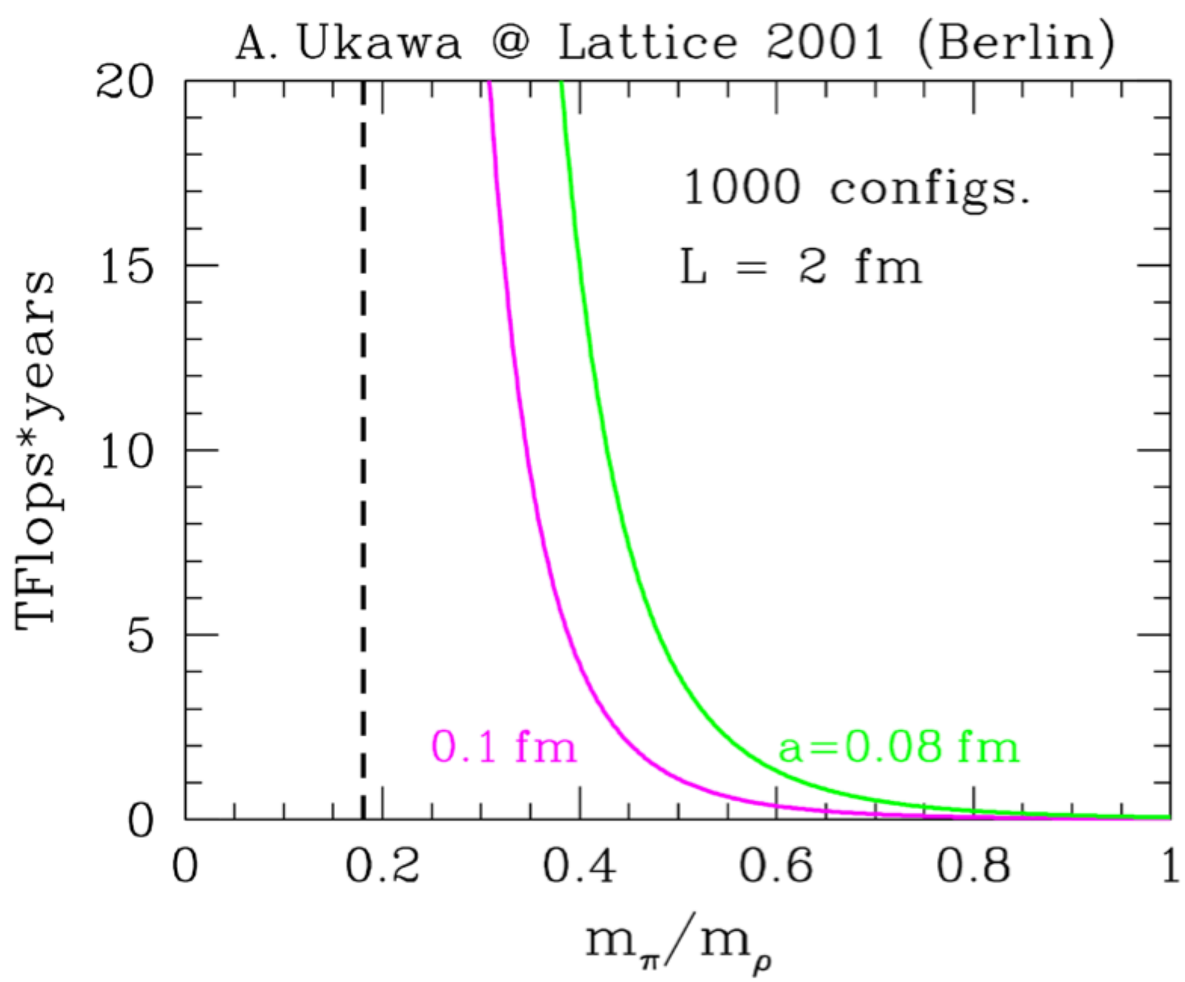


1. **Simulation of gauge ensembles {U}**: Using HMC

$$P[U] = \frac{1}{Z} \left(\prod_{f=u,d,s,c} \mathrm{Det}(D_f[U]) \right) e^{-S_g[U]}$$

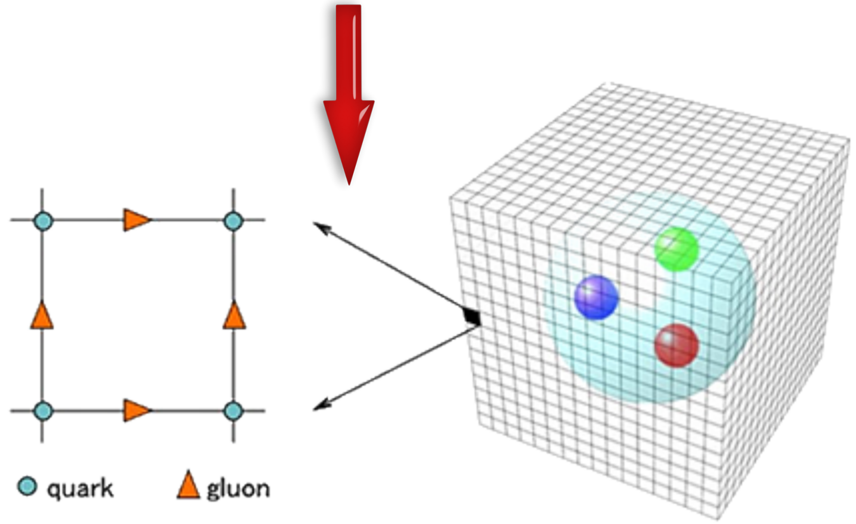
Estimated cost to generate 1000 independent fermion configurations

Many algorithmic improvements:
1. …..
2. Domain decomposition 2004, M. Lüscher
3. …



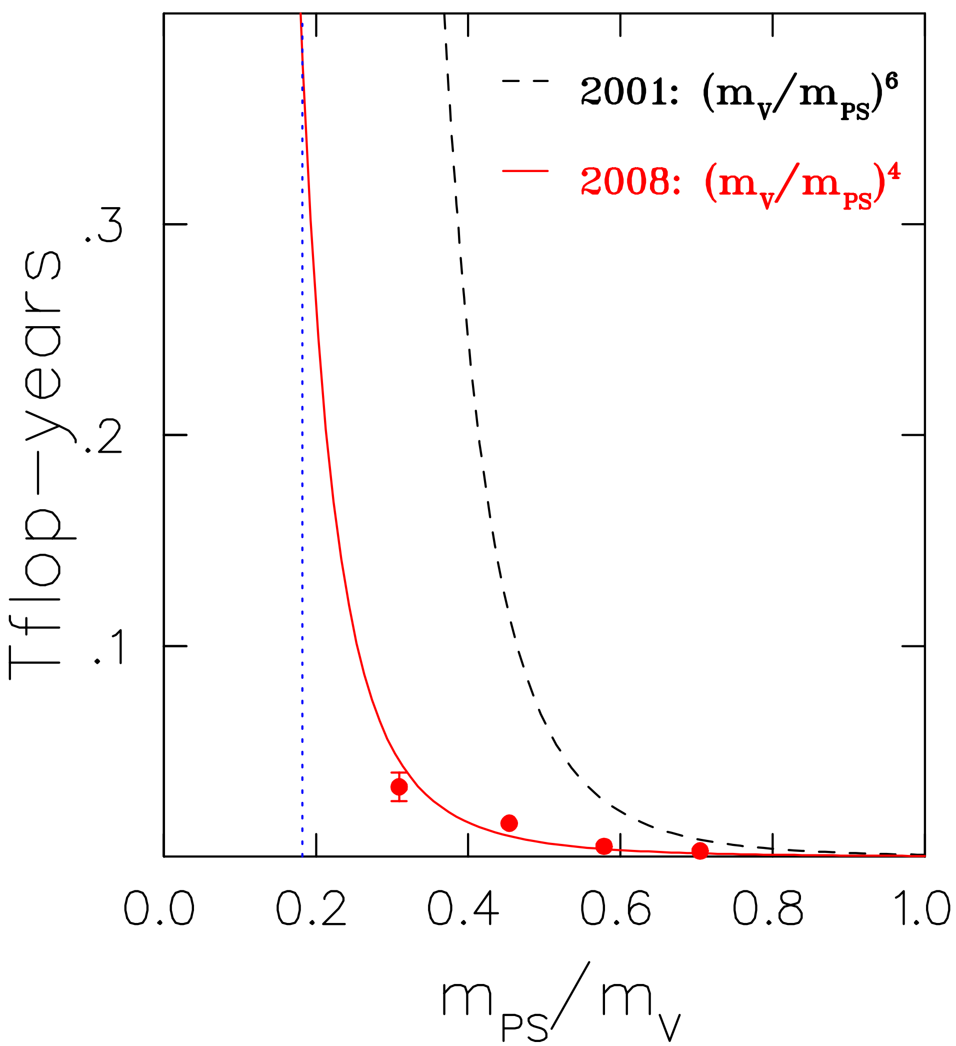
Simulations of lattice QCD

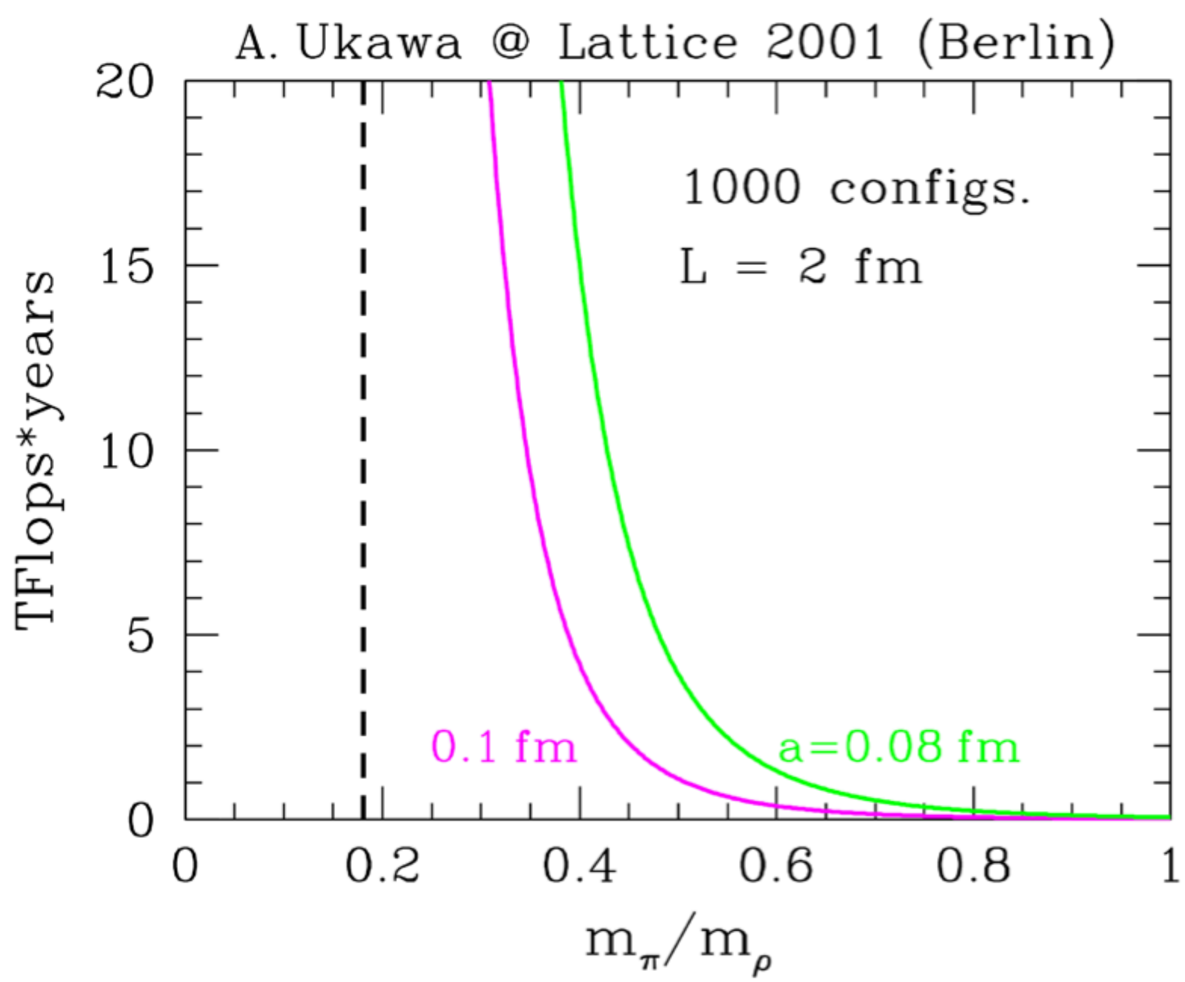
$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \mathcal{D}[U]\mathcal{O}(D_f^{-1}[U], U) \left(\prod_{f=u,d,s,c} \mathrm{Det}(D_f[U]) \right) e^{-S_g[U]}$$



1. **Simulation of gauge ensembles {U}**: Using HMC

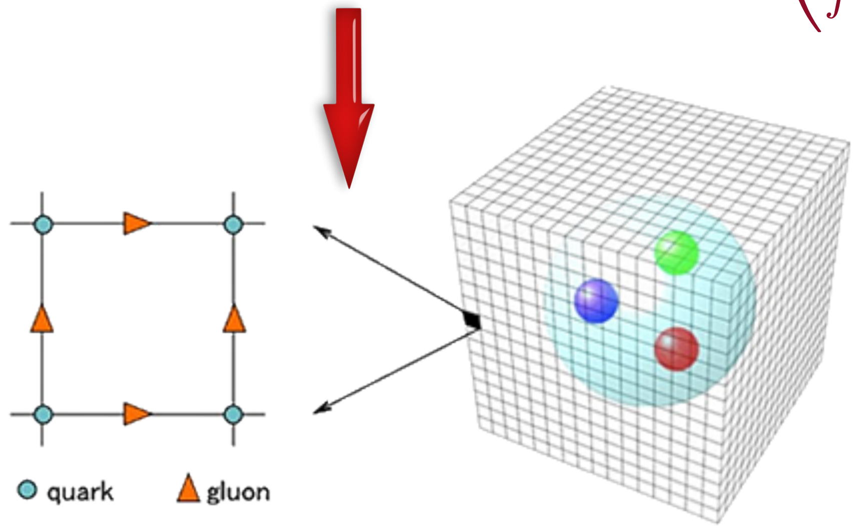
$$P[U] = \frac{1}{Z} \left(\prod_{f=u,d,s,c} \mathrm{Det}(D_f[U]) \right) e^{-S_g[U]}$$



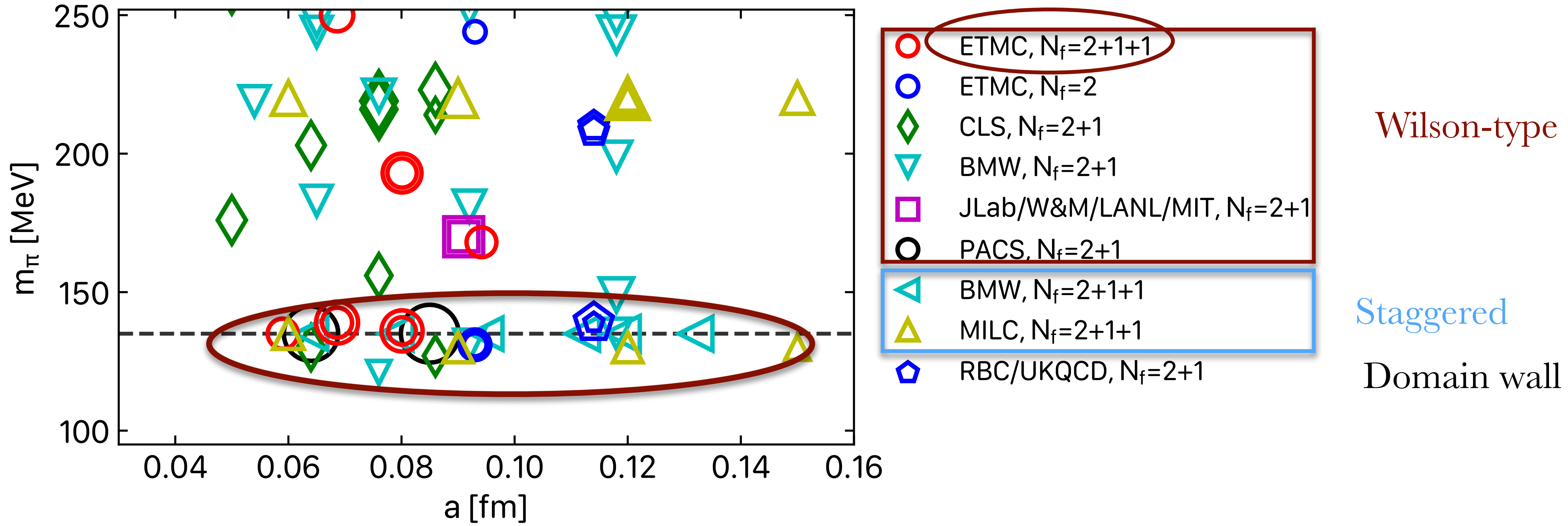


Simulations of lattice QCD

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \mathcal{D}[U] \mathcal{O}(D_f^{-1}[U], U) \left(\prod_{f=u,d,s,c} \mathrm{Det}(D_f[U]) \right) e^{-S_g[U]}$$

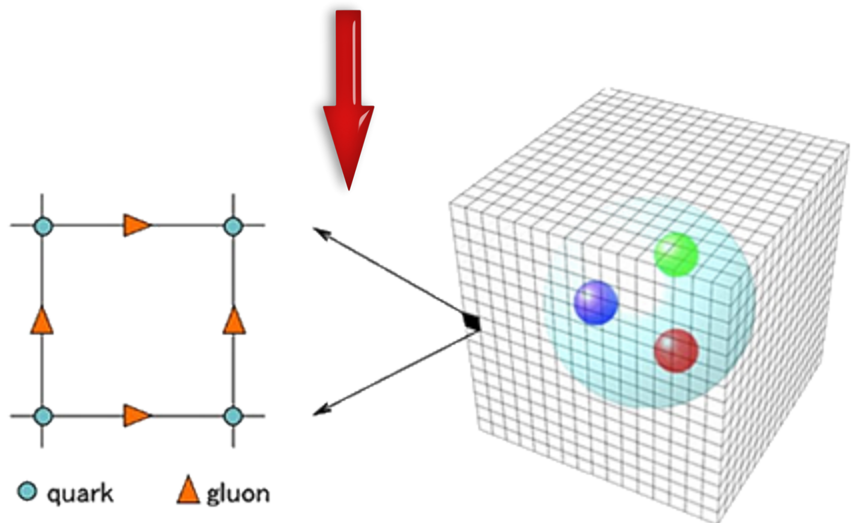


1. Simulation of gauge ensembles {U}: $P[U] = \frac{1}{Z} \left(\prod_{f=u,d,s,c} \mathrm{Det}(D_f[U]) \right) e^{-S_g[U]}$



Simulations of lattice QCD

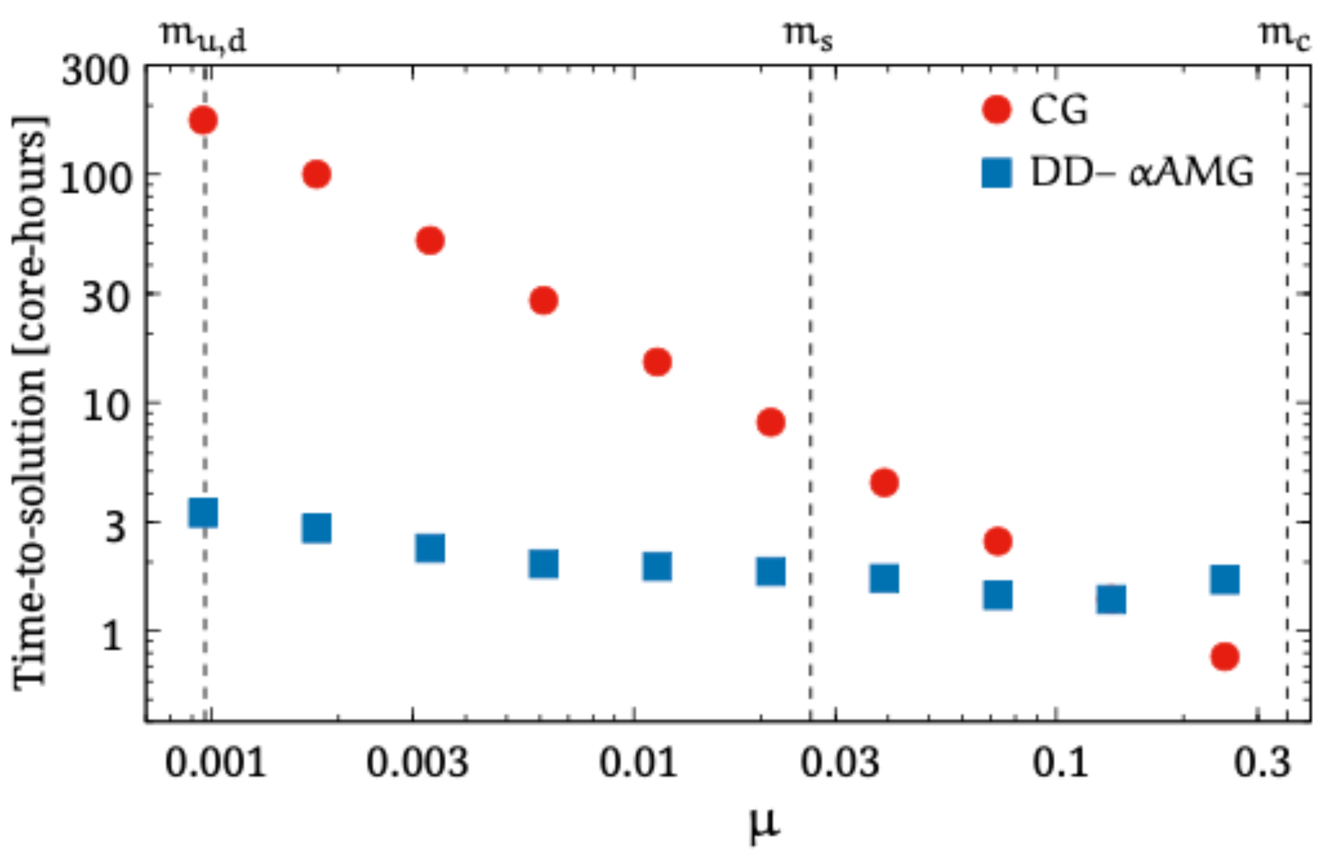
$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \mathcal{D}[U] \mathcal{O}(D_f^{-1}[U], U) \left(\prod_{f=u,d,s,c} \mathrm{Det}(D_f[U]) \right) e^{-S_g[U]}$$



1. **Simulation of gauge ensembles {U}:**

$$P[U] = \frac{1}{Z} \left(\prod_{f=u,d,s,c} \mathrm{Det}(D_f[U]) \right) e^{-S_g[U]}$$

2. **Quark propagators or inverse of Dirac matrix $D_f[U]$:** Multi-grid solvers



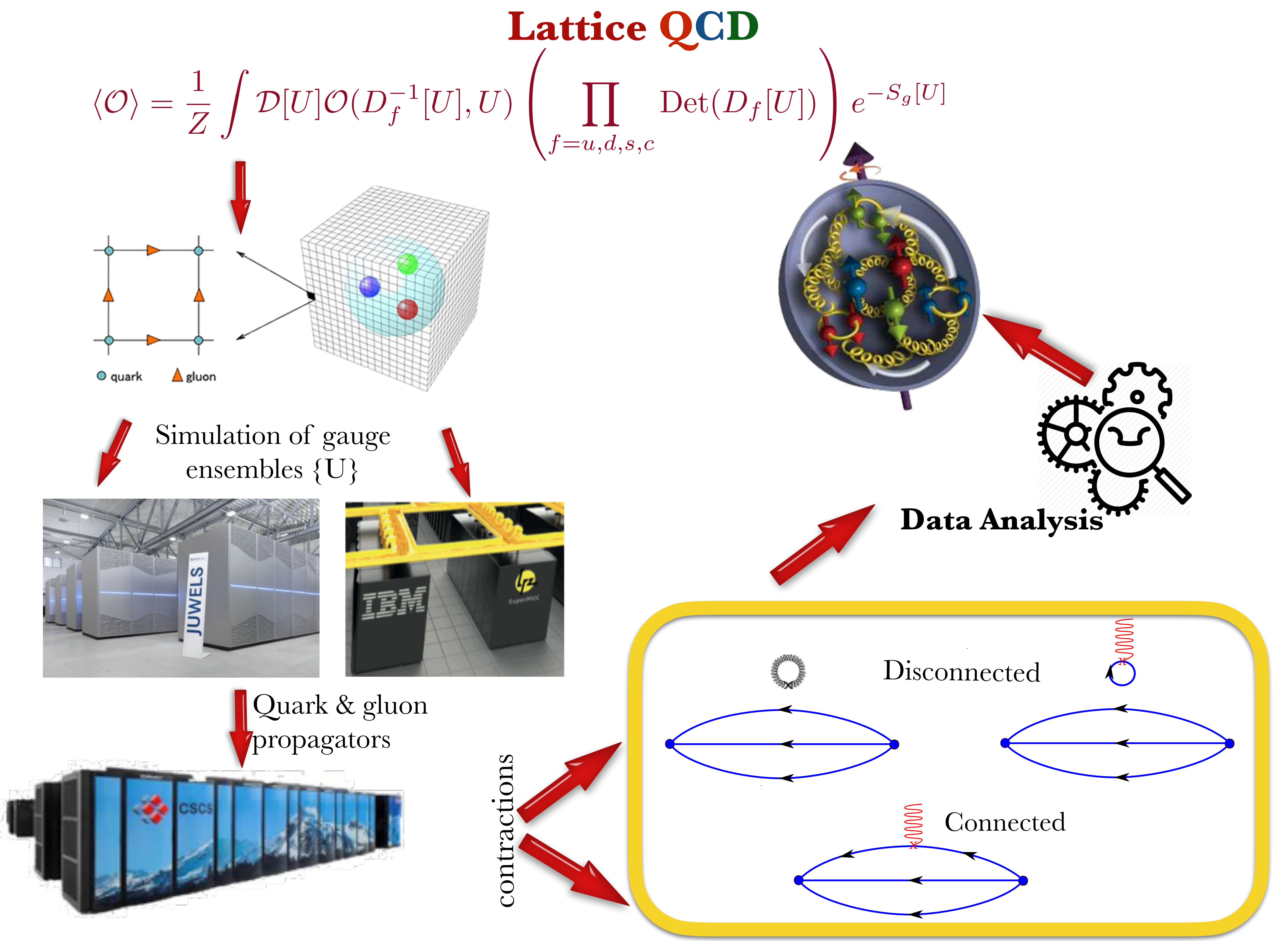
Linear system to be solved:

$$D_f[U, \mu] v = b$$

Use an adaptive aggregation-based domain decomposition multi-grid approach

A. Frommer, *et al.*, SIAM J.Sci.Comput. 36 (2014) 4, A1581

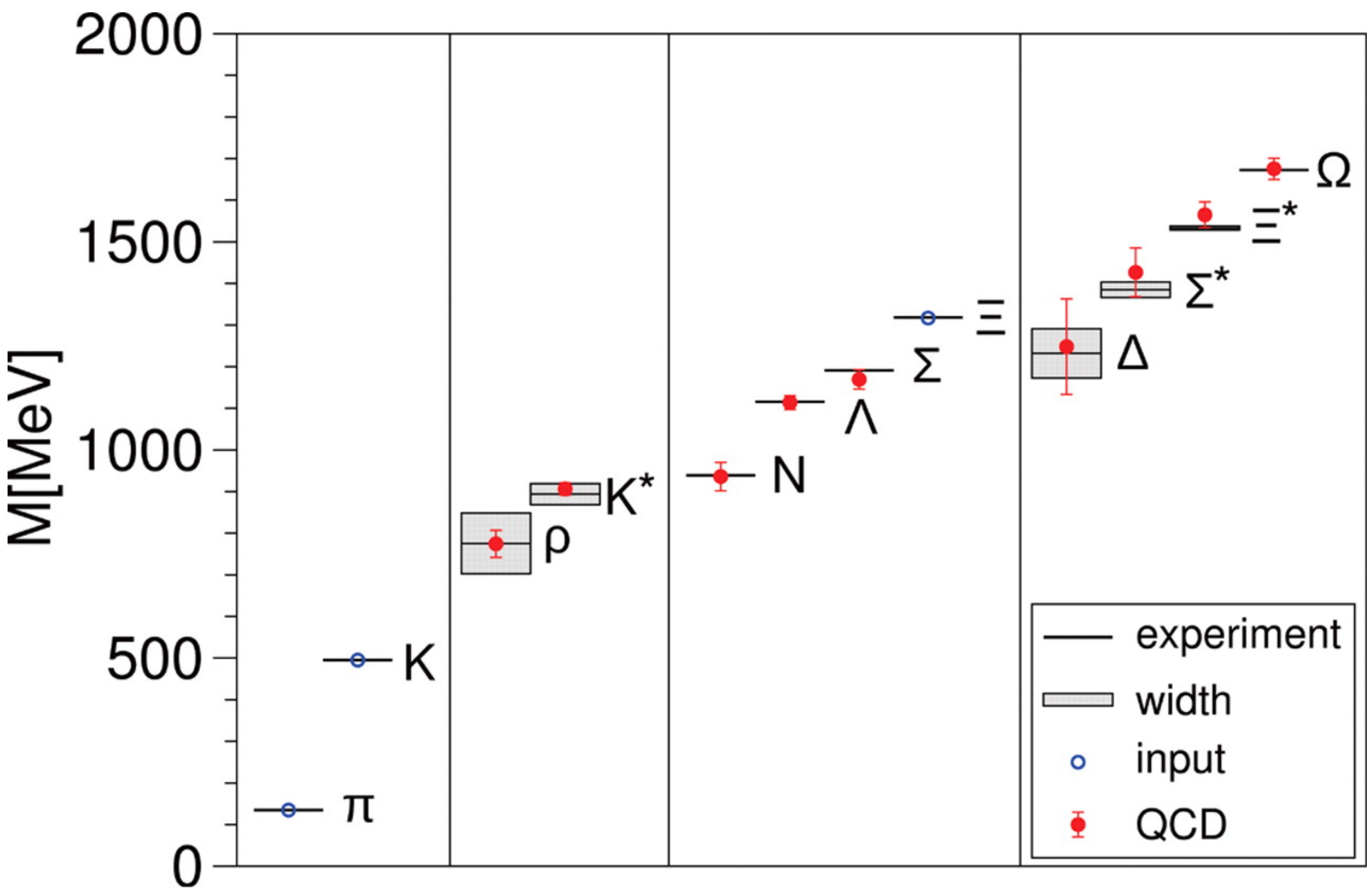
ETMC: C. A. *et al.* Phys.Rev.D 94 (2016) 11, 114509, arXiv:1610.02370

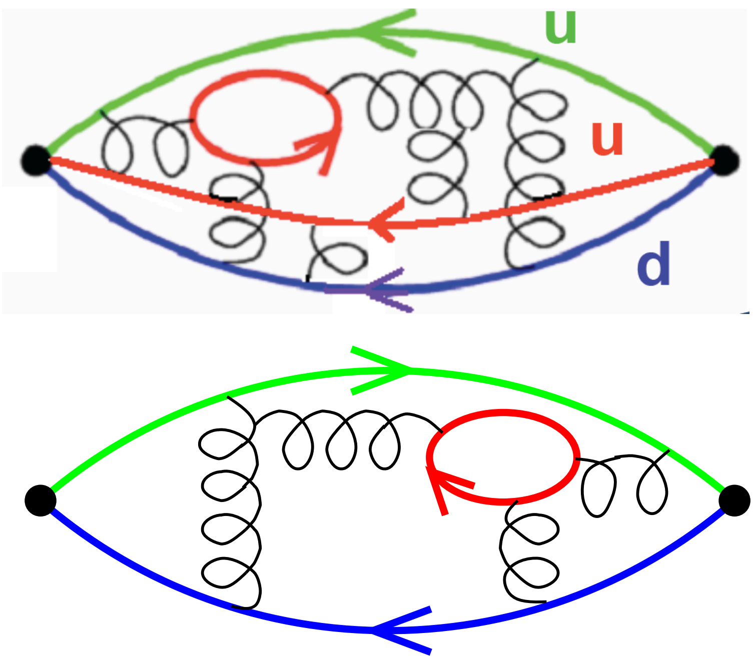

Lattice QCD
$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \mathcal{D}[U] \mathcal{O}(D_f^{-1}[U], U) \left(\prod_{f=u,d,s,c} \mathrm{Det}(D_f[U]) \right) e^{-S_g[U]}$
quark
gluon
Simulation of gauge ensembles {U}
JUWELS
IBM
SuperMUC
Quark & gluon propagators
CSCS
contractions
Disconnected
Connected
Data Analysis

Low-lying hadron spectrum

- BMW collaboration determined the low-lying hadron masses

S. Durr *et al.*, *Science* 322 (2008) 1224





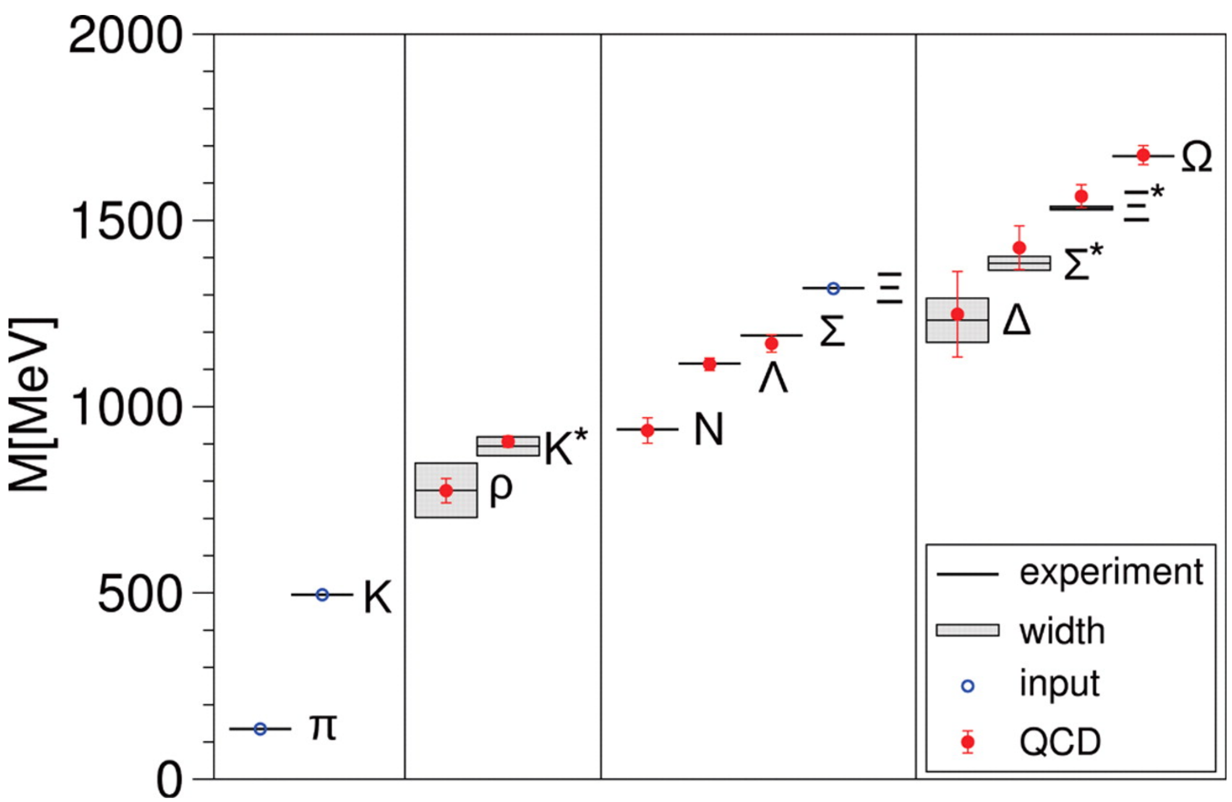
(B) Full QCD

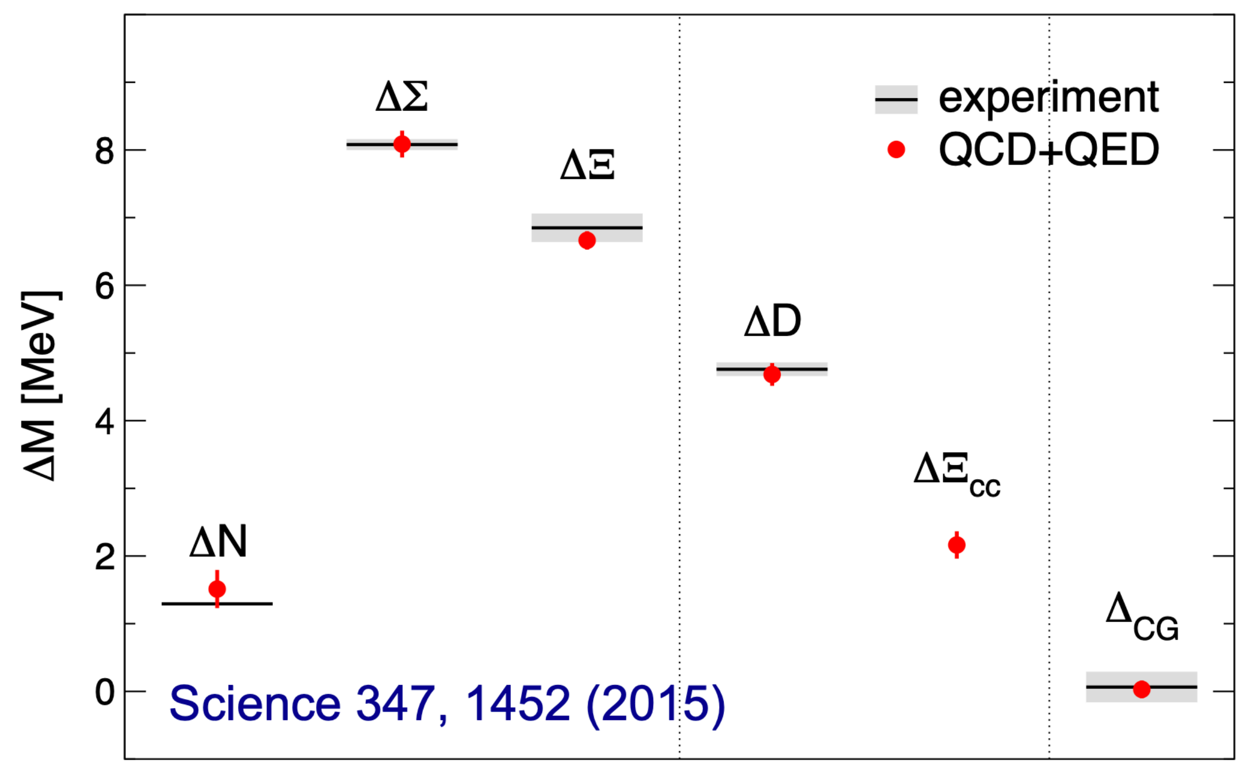
Low-lying hadron spectrum

- BMW collaboration determined the low-lying hadron masses, as well as the mass splittings

S. Durr *et al.*, Science 322 (2008) 1224

Sz. Borsanyi *et al.*, Science 347 (2015) 1452

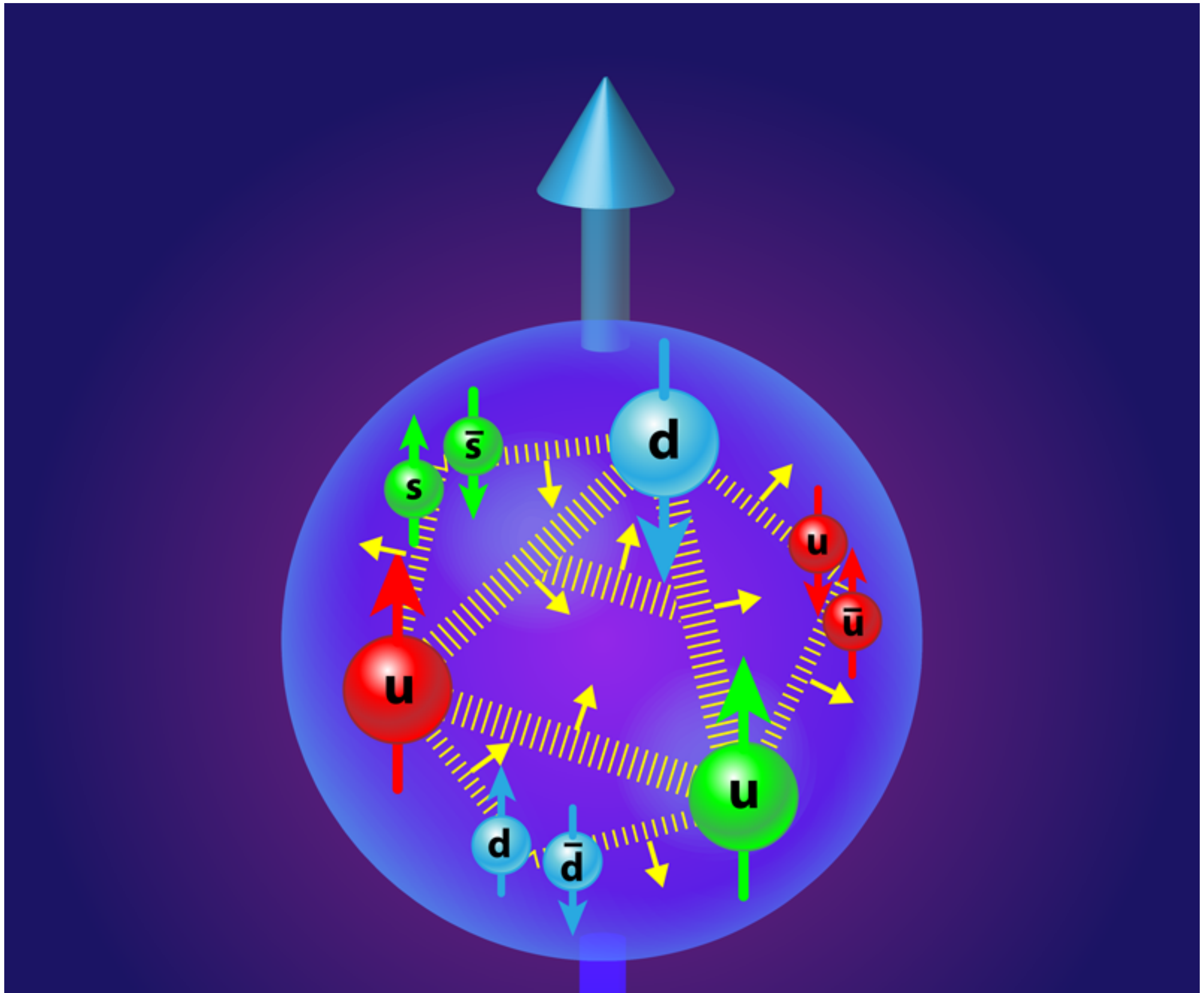




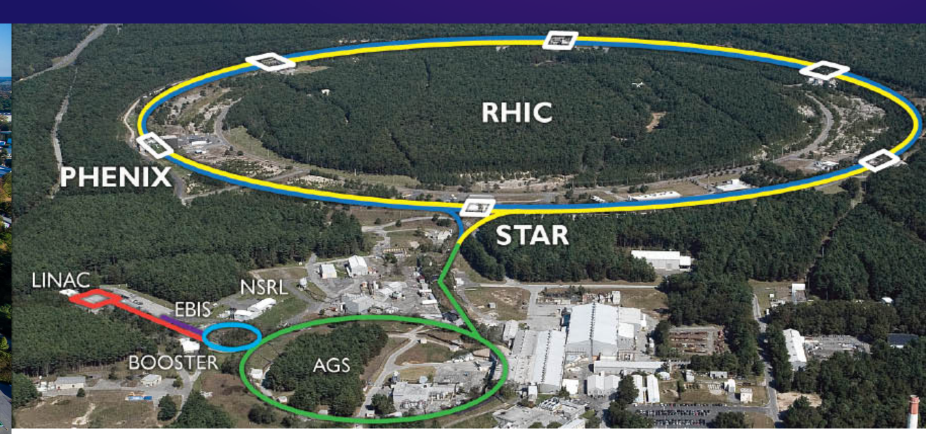
	mass splitting [MeV]	QCD [MeV]	QED [MeV]
$\Delta N = n - p$	1.51(16)(23)	2.52(17)(24)	-1.00(07)(14)
$\Delta\Sigma = \Sigma^- - \Sigma^+$	8.09(16)(11)	8.09(16)(11)	0
$\Delta\Xi = \Xi^- - \Xi^0$	6.66(11)(09)	5.53(17)(17)	1.14(16)(09)
$\Delta D = D^\pm - D^0$	4.68(10)(13)	2.54(08)(10)	2.14(11)(07)
$\Delta\Xi_{cc} = \Xi_{cc}^{++} - \Xi_{cc}^{+}$	2.16(11)(17)	-2.53(11)(06)	4.69(10)(17)
$\Delta_{\rm CG} = \Delta N - \Delta\Sigma + \Delta\Xi$	0.00(11)(06)	-0.00(13)(05)	0.00(06)(02)

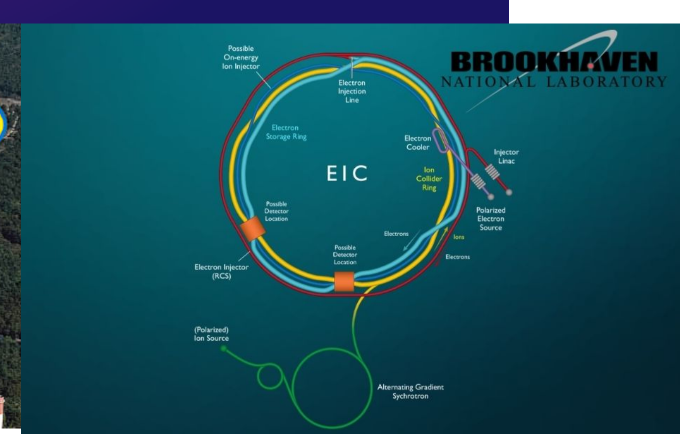
1. Lattice QCD reproduces the low-lying hadron masses and mass splittings

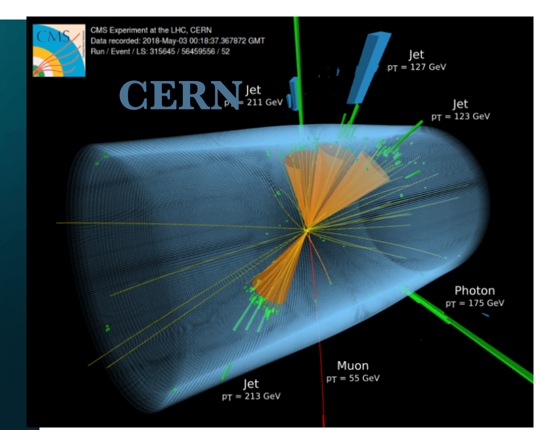
3D structure of the nucleon



Relevant for interpreting and providing input for on-going and future experiments

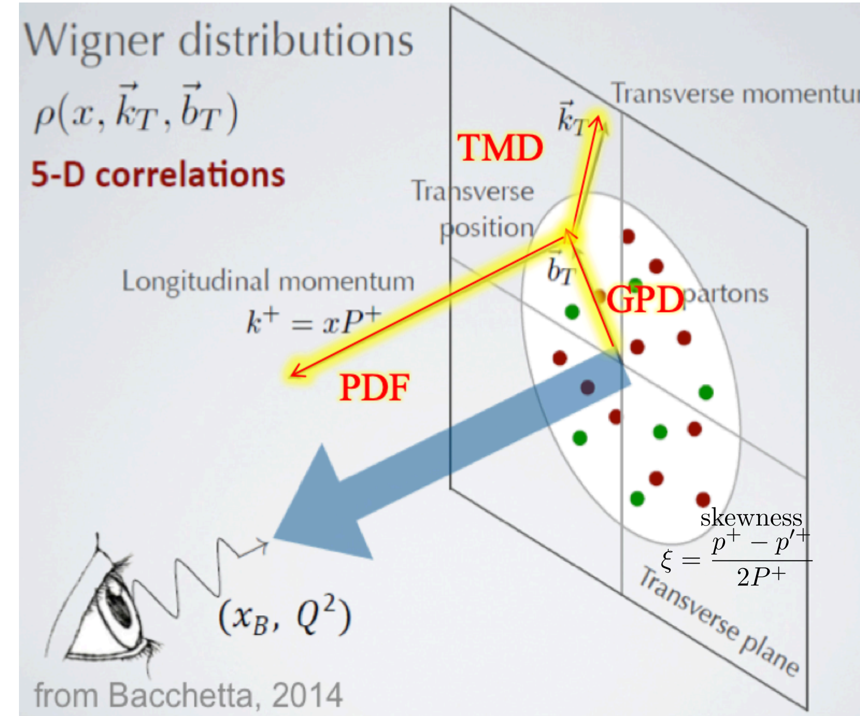




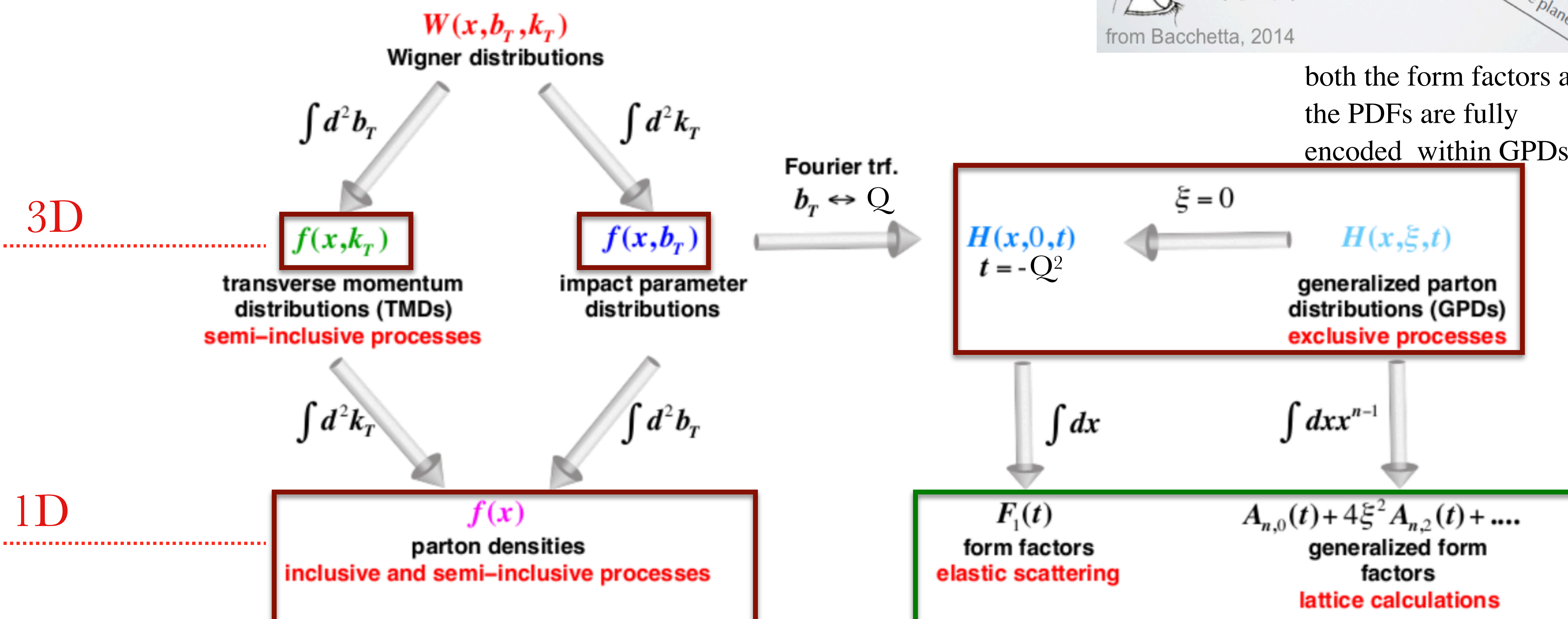


3D structure of the nucleon

- The 3D-structure of the nucleon is a major goal of on-going experiments and the future EIC
- Lattice QCD can contribute towards this goal - many recent developments to compute Mellin moments but also directly parton distributions



both the form factors and the PDFs are fully encoded within GPDs

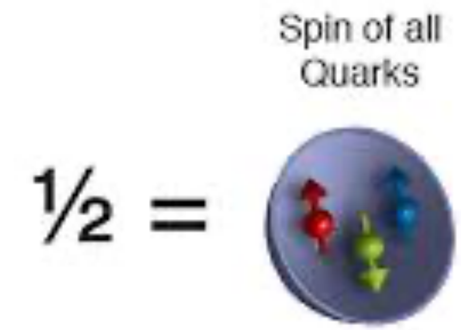


Studies in lattice QCD since the 1980s

EIC white paper, arXiv:1212.1701

Proton spin puzzle

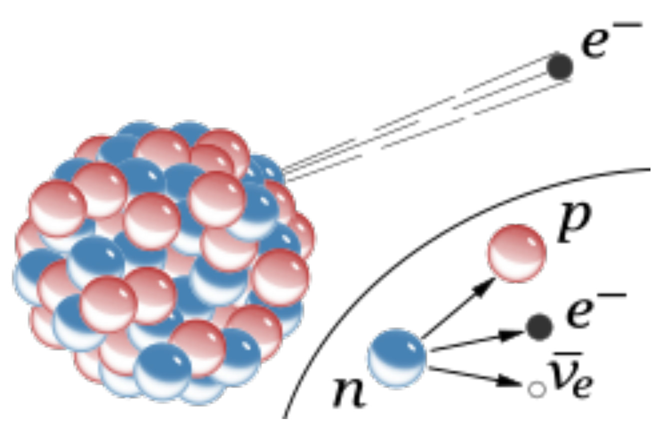
* Proton is made of 3 quarks (valence) of spin 1/2 adding to give total spin of



* Deep inelastic scattering experiments with a polarised proton target first by the European Muon Collaboration (EMC) at CERN in 1988 and followup experiments at SLAC and DESY found that only a small fraction of the proton spin is due to the valence quarks —> **proton spin puzzle**

* The axial charge of the proton measures the intrinsic spin of quarks in the proton $\Delta\Sigma_{q^+} = g_A^q$

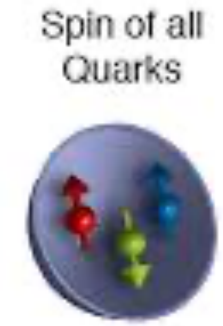
* The isovector (u-d) axial charge is accurately known from neutron beta decay



Nuclear transmutation

Proton spin puzzle

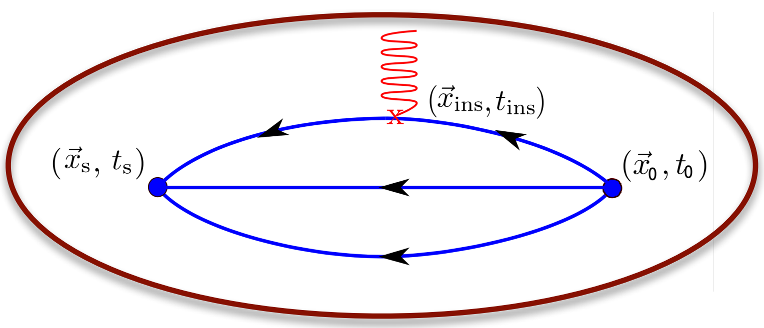
- Proton is made of 3 quarks (valence) of spin adding to give the proton a total spin of ½ =

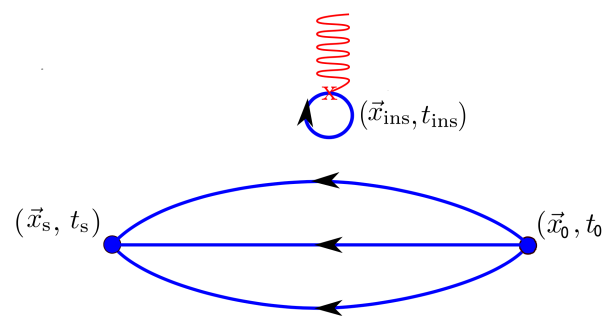


- Deep inelastic scattering experiments with a polarised proton target first by the European Muon Collaboration (EMC) at CERN in 1988 and followup experiments at SLAC and DESY found that only a small fraction of the proton spin (25%) is due to the valence quarks —> proton spin puzzle

- The axial charge of the proton measures the spin of quarks in the proton $\Delta\Sigma_{q^+} = g_A^q$

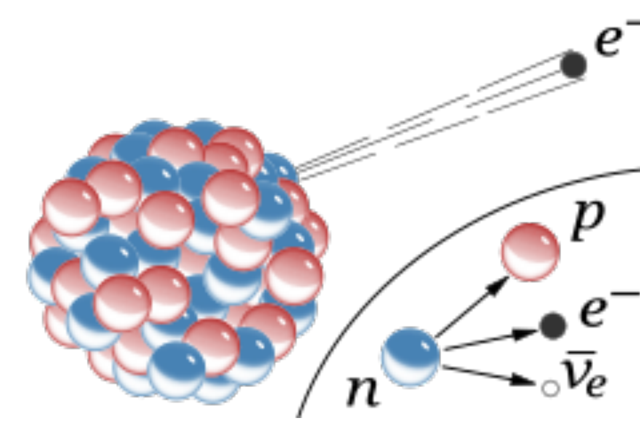
- Can be compute from the proton matrix element of the axial-vector current



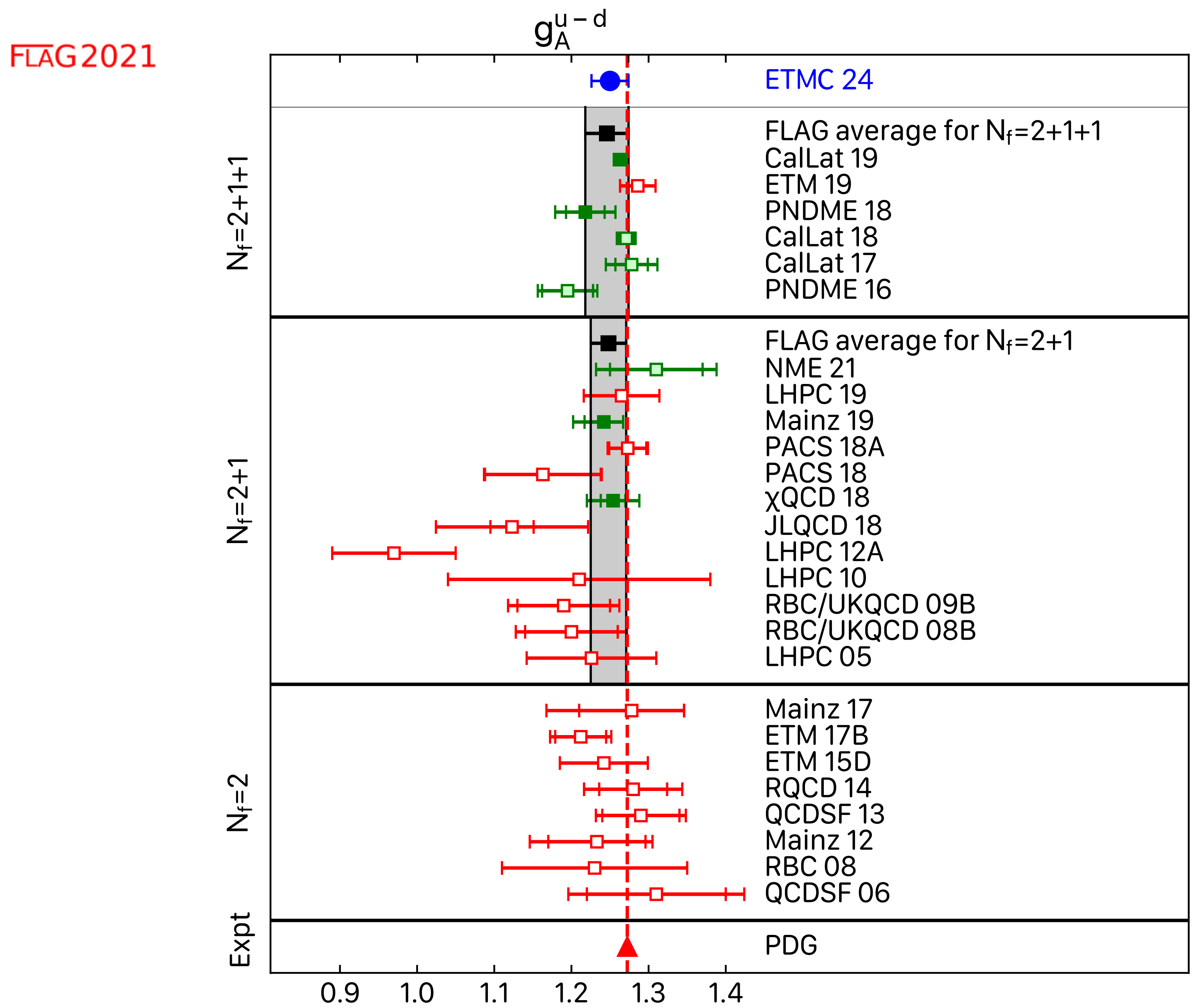


- The isovector axial charge is accurately known from neutron double beta decay

- Easier to compute in lattice QCD since only the connected contribution is needed



Proton isovector axial charge



Lattice QCD results on g_A consistent with experimental value

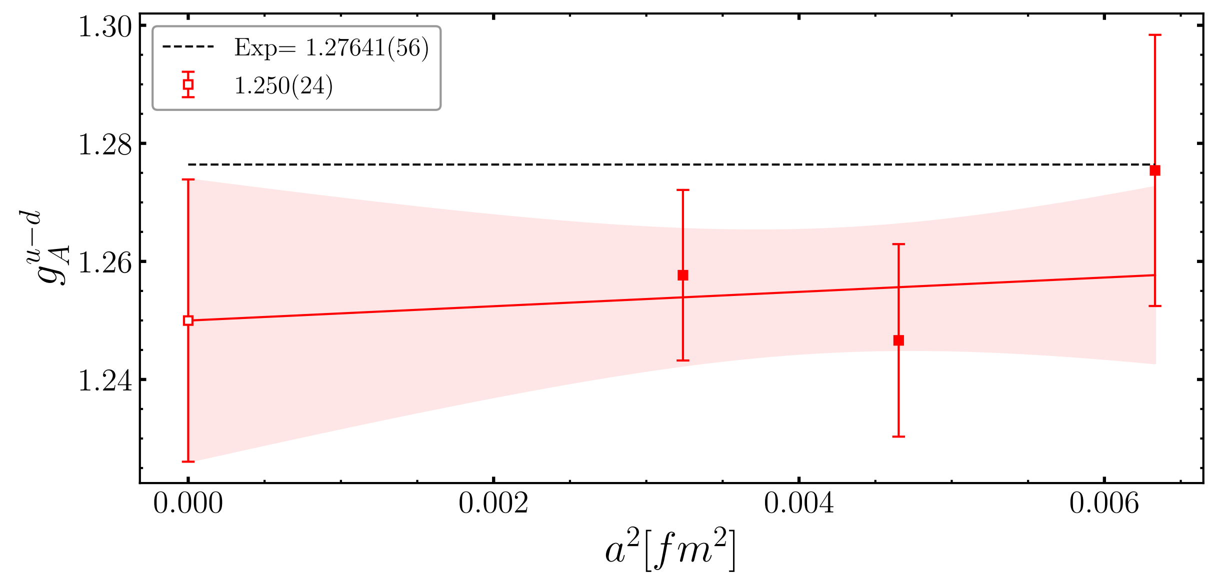
$$\langle N|\bar{q}\Gamma^{\mu\nu}\tau^3 q|N\rangle, \ \ \Gamma^{\mu\nu} = \gamma^\mu, \ \gamma^\mu\gamma_5, \ \frac{1}{2}[\gamma^\mu, \gamma^\nu]$$

- $g_V = 1$
- $g_A = 1.2723 \pm 0.0023$ ☜ reproduce
- $g_T = 0.53 \pm 0.25$ M. Radici and A. Bacchetta. PRL 120 (2018) 192001

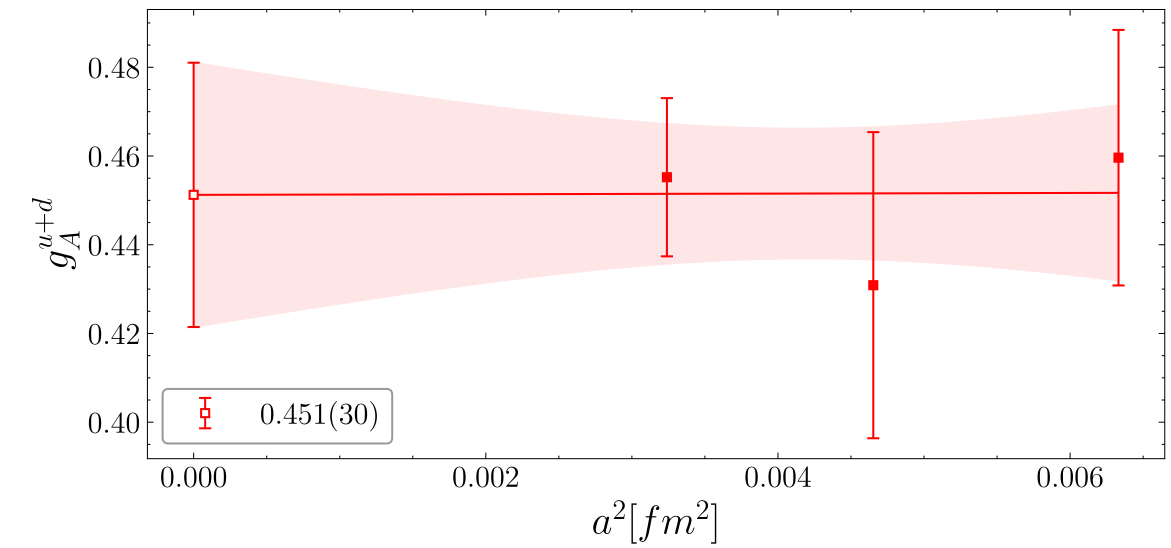
Axial charge

- Axial charges extracted directly from the forward matrix element

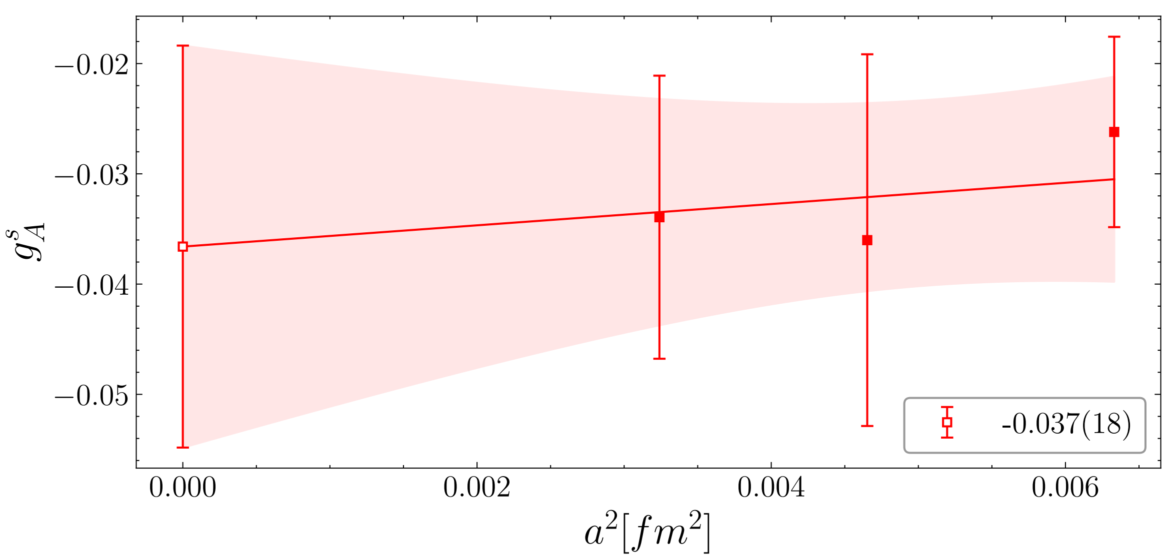
Isovector



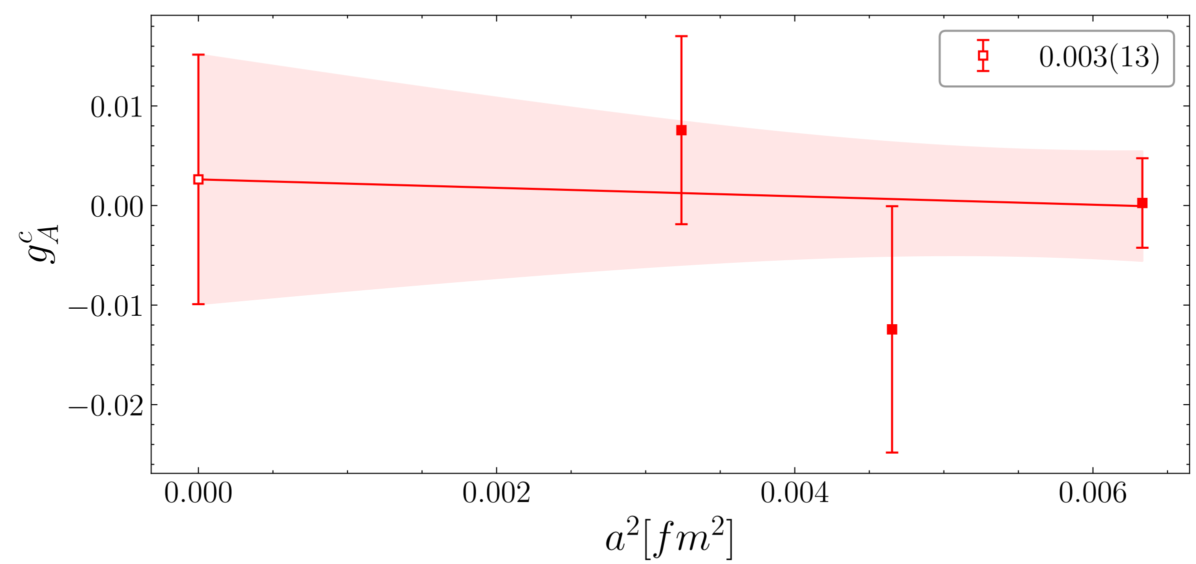
Isoscalar including disconnected



Strange



Charm

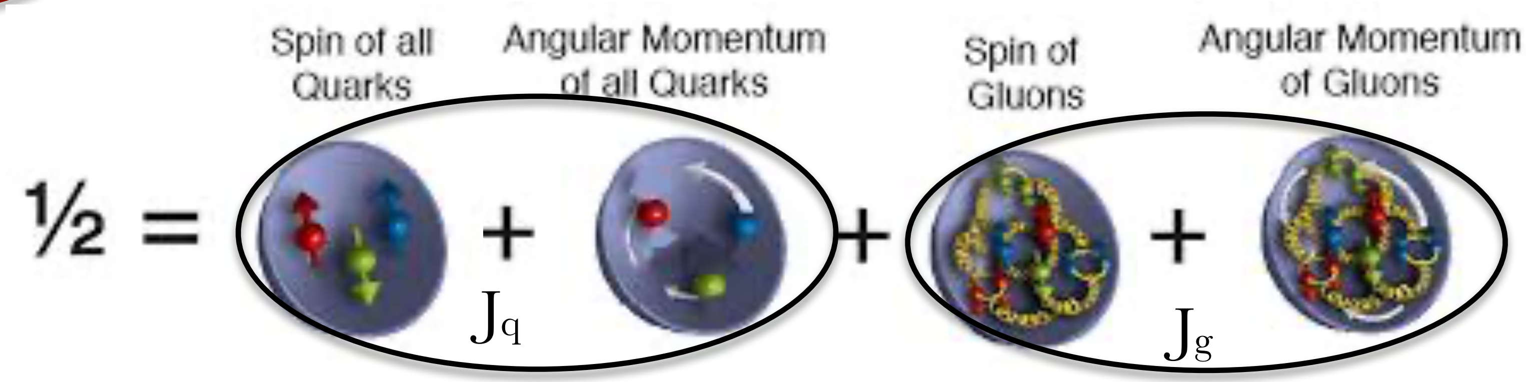


- Non-zero strangeness, upper limit on charmness of 0.013
- $\sum_{q=u,d,s,c} g_A^q = 0.382(70)$ —> intrinsic spin carried by valence quarks: $\frac{1}{2}\sum_q \Delta\Sigma_q = 0.191(35)$

Proton momentum and spin decomposition

- Spin sum: $\sum_q \left[\frac{1}{2}\Delta\Sigma_q + L_q\right] + J_g = \frac{1}{2}$

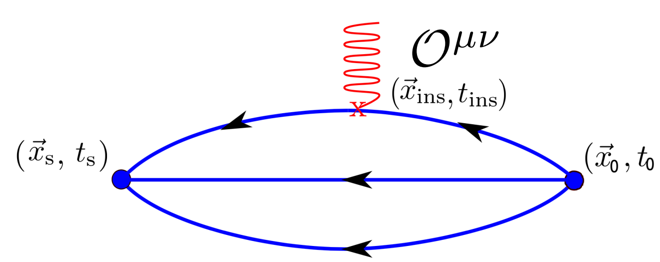
  J_q

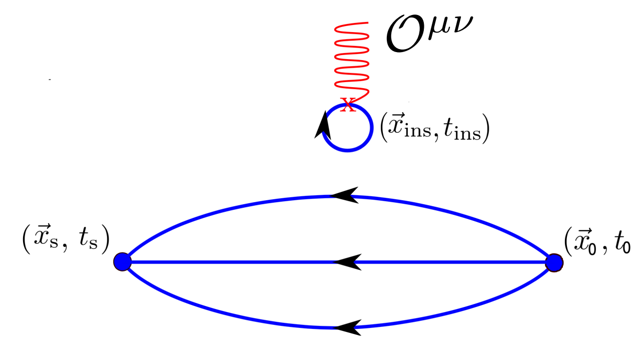


- Quark unpolarised moment: $\mathcal{O}^{\mu\nu,q} = \bar{q}\gamma^{\{\mu}iD^{\nu\}}q$

$$\langle N(p',s')|\mathcal{O}^{\mu\nu,q}|N(p,s)\rangle = \bar{u}_N(p',s')\left[A_{20}^q(q^2)\gamma^{\{\mu}P^{\nu\}} + B_{20}^q(q^2)\frac{i\sigma^{\{\mu\alpha}q_\alpha P^{\nu\}}}{2m} + C_{20}^q(q^2)\frac{q^{\{\mu}q^{\nu\}}}{m}\right]u_N(p,s)$$

Momentum fraction carried by a quark $\langle x\rangle_q = A_{20}^q(0)$ $\quad J_q = \frac{1}{2}\left[A_{20}^q(0) + B_{20}^q(0)\right]$

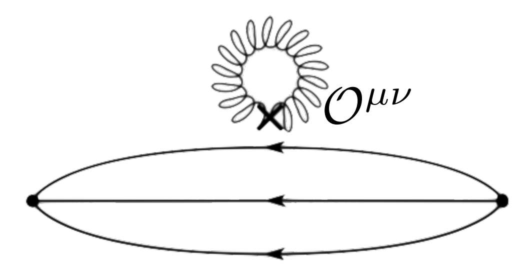




- Gluon unpolarised moment lead to an equivalent expression $\mathcal{O}^{\mu\nu,g} = F^{\{\mu\rho}F_\rho^{\nu\}}$

$\langle x\rangle_g = A_{20}^g(0)$ $\quad J_g = \frac{1}{2}\left[A_{20}^g(0) + B_{20}^g(0)\right]$

Field strength tensor

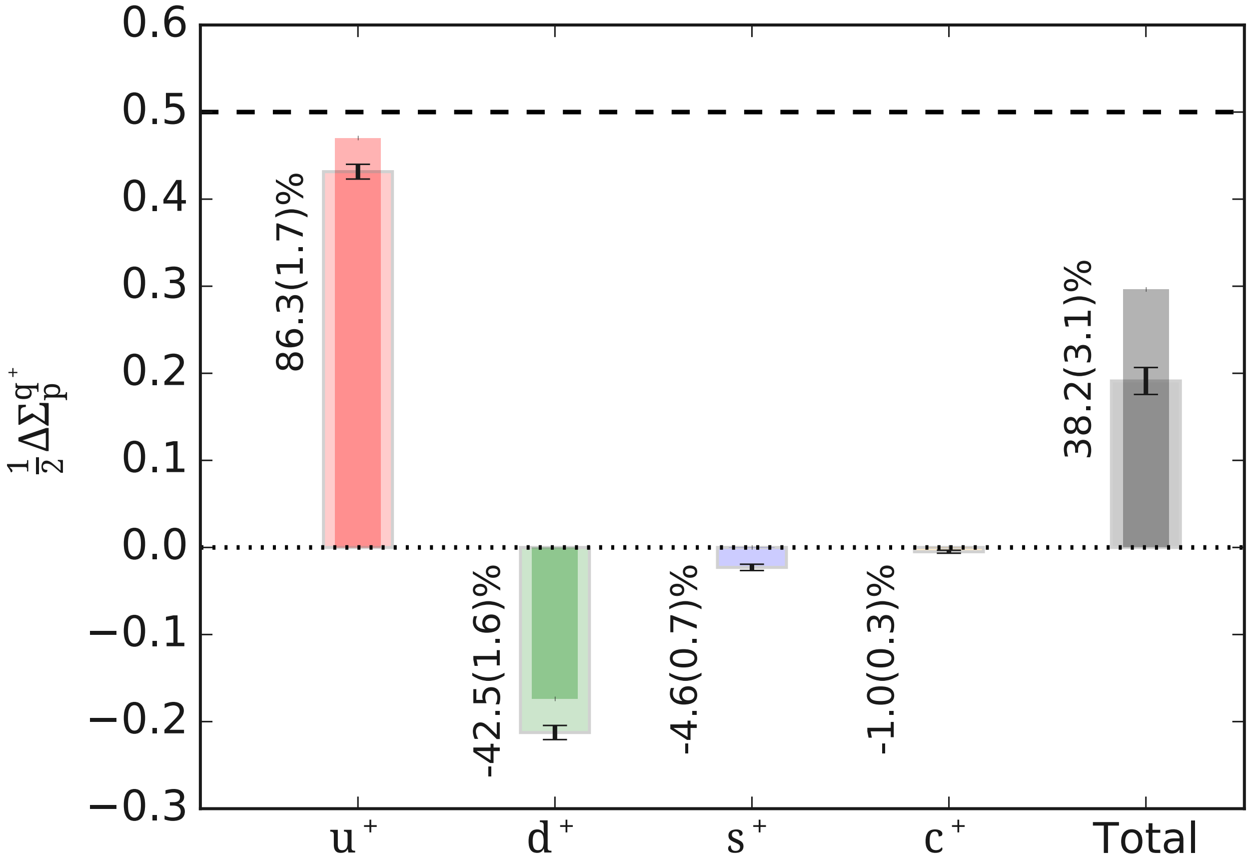


➡ Momentum sum: $\sum_q \langle x\rangle_q + \langle x\rangle_g = 1$

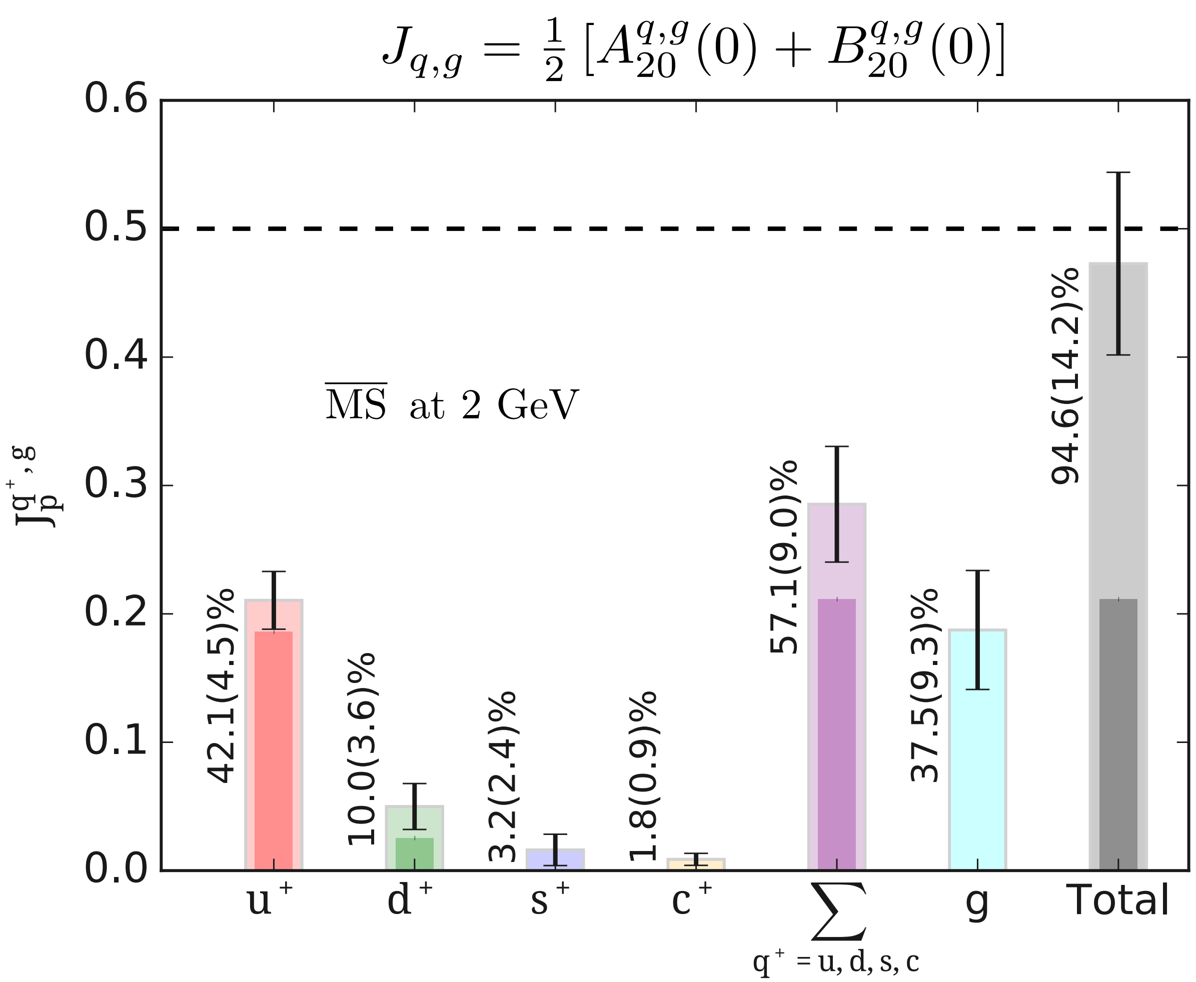
Intrinsic spin of quarks and momentum sum

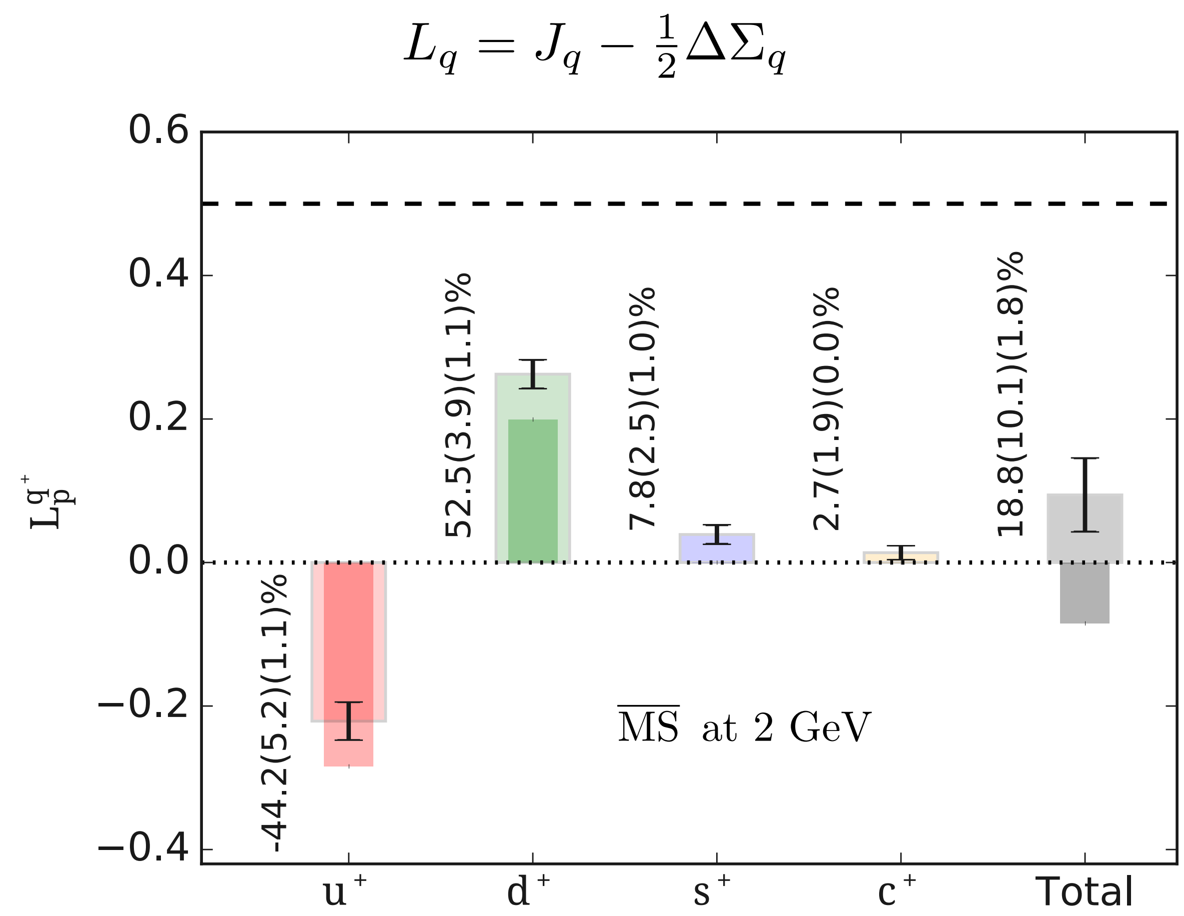
- Axial charge determines intrinsic spin carried by each quark

$$\Delta\Sigma_{q_+}(\mu^2) = \int_0^1 dx\,\left[\Delta q(x,\mu^2) + \Delta\bar{q}(x,\mu^2)\right] = g_A^q$$



Spin sum





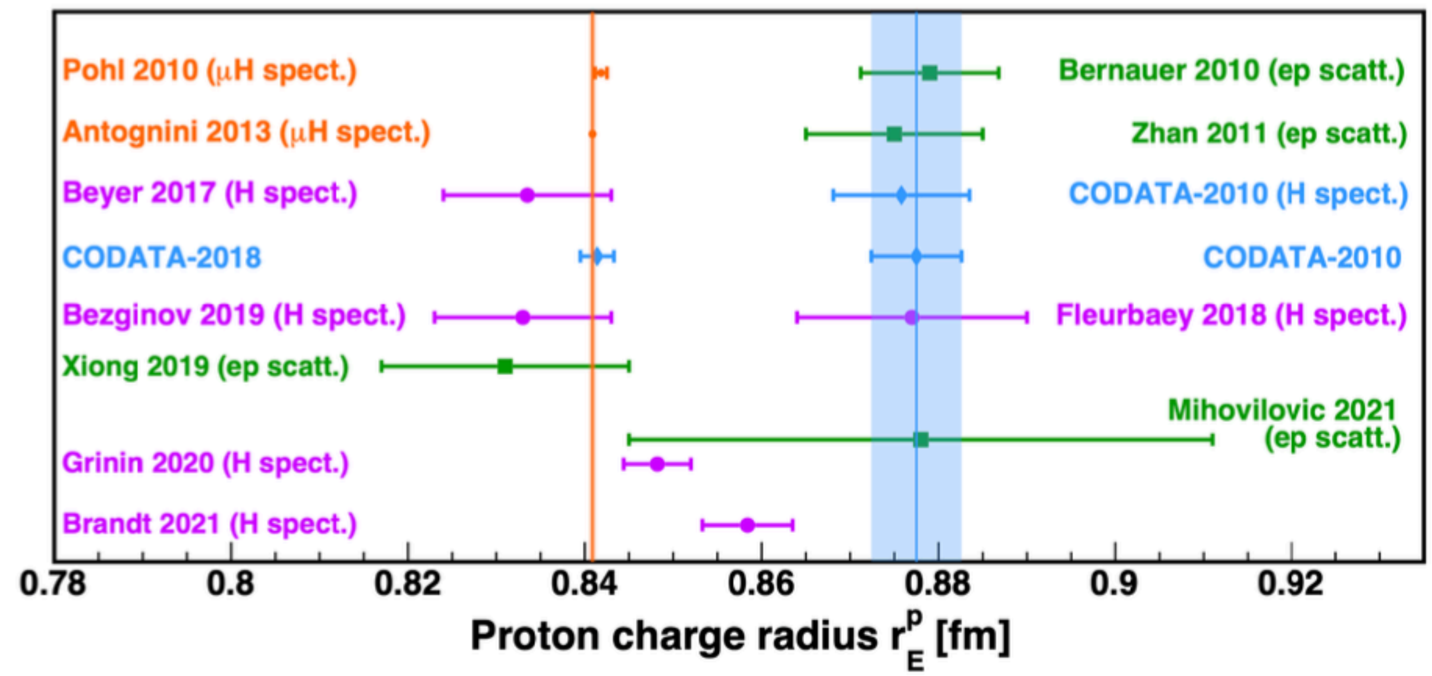
- Spin sum: $\sum_q J_q + J_g = 0.48(7)$

2. Nucleon spin sum verified - lattice QCD solves a 30 year puzzle

C. A. *et al.* (ETMC) Phys. Rev. Lett. **119**, 142002, 1909.00485

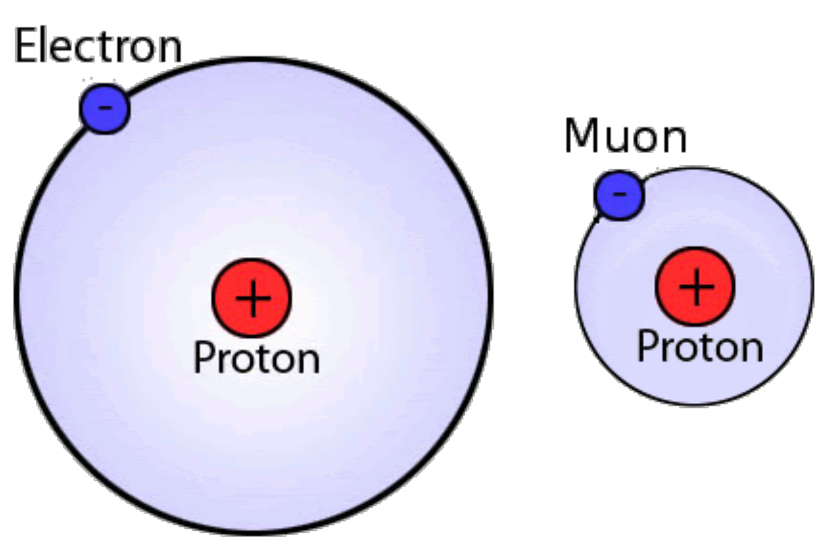
C. A. *et al.* (ETMC) Phys.Rev.D **101** (2020) 9, 094513, 2003.08486

Proton charge radius puzzle



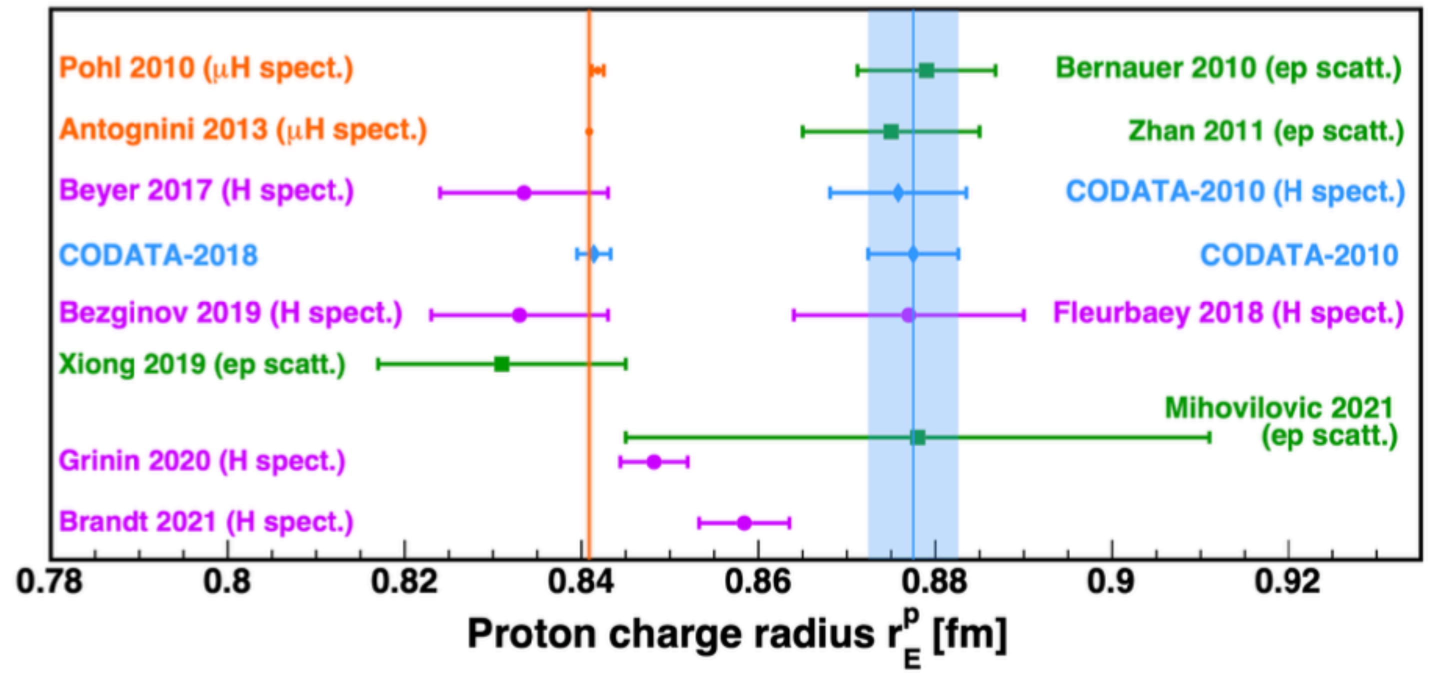
- Using muonic hydrogen the proton radius was found to be smaller than what expected

R. Pohl *et al.*, Nature 466 (2010) 213

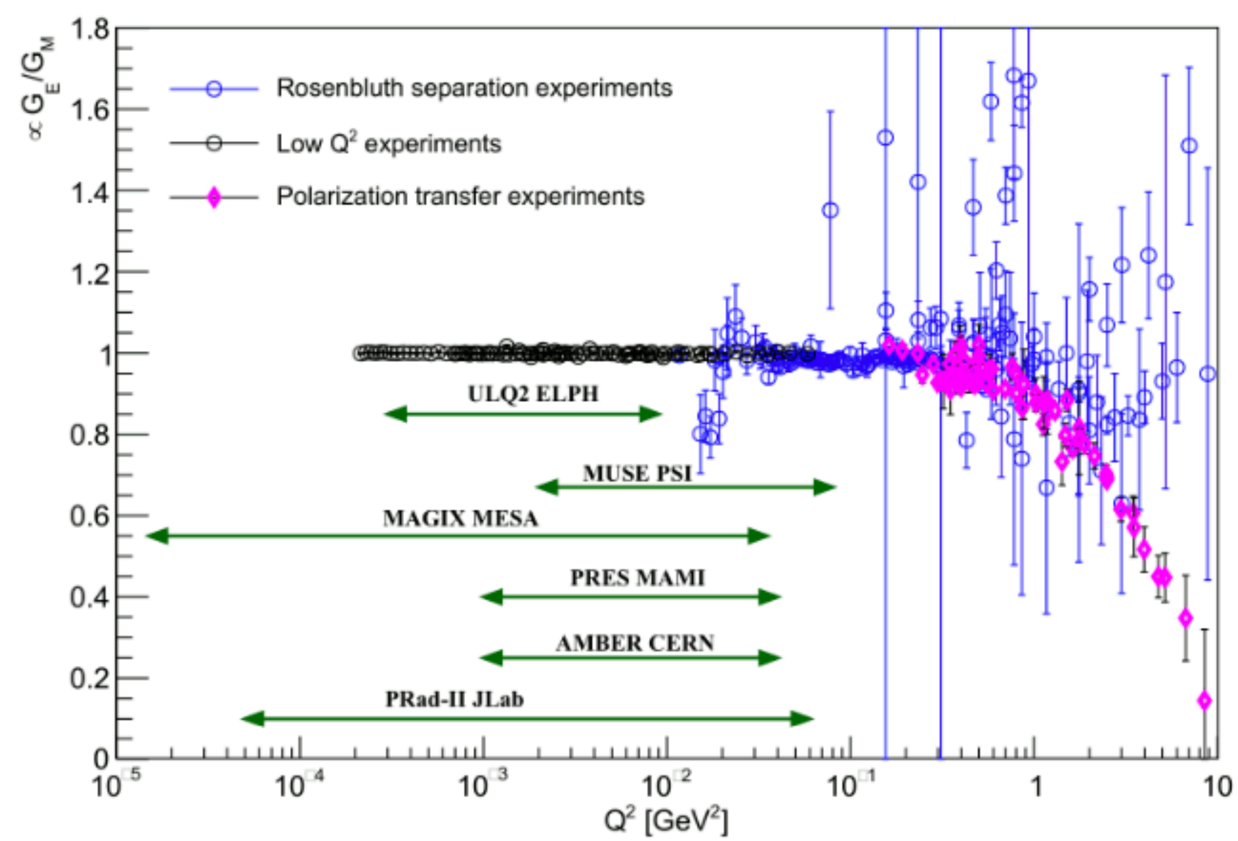


$2S_{1/2}(F = 1) – 2P_{3/2}(F = 2)$ energy difference

Proton charge radius puzzle



- Using muonic hydrogen the radius was found to be smaller than what expected —> led to many theoretical and experimental investigations



Electromagnetic form factors

- Proton matrix element of the vector current

$$\langle N(p',s')|j^{\mu}(0)|N(p,s)\rangle = \bar{u}_N(p',s')\left[\gamma^{\mu}F_1(Q^2) + \frac{i\sigma^{\mu\nu}q_{\nu}}{2m}F_2(Q^2)\right]u_N(p,s)$$

F_1: Dirac; F_2: Pauli

$$G_E(Q^2) = F_1(Q^2) + \frac{Q^2}{4m_N^2}F_2(Q^2),\quad G_M(Q^2) = F_1(Q^2) + F_2(Q^2),\quad Q^2 = -q^2$$

$$\langle r^2\rangle_{E,M} = -\frac{6}{G_{E,M}(0)}\frac{\partial G_{E,M}(Q^2)}{\partial Q^2}|_{Q^2\to 0}$$

Electromagnetic form factors

- Proton matrix element of the vector current

$$\langle N(p',s')|j_\mu|N(p,s)\rangle = \bar{u}_N(p',s')\left[\gamma_\mu F_1(q^2) + \frac{i\sigma_{\mu\nu}q^\nu}{2m_N}F_2(q^2)\right]\gamma_5 u_N(p,s)$$

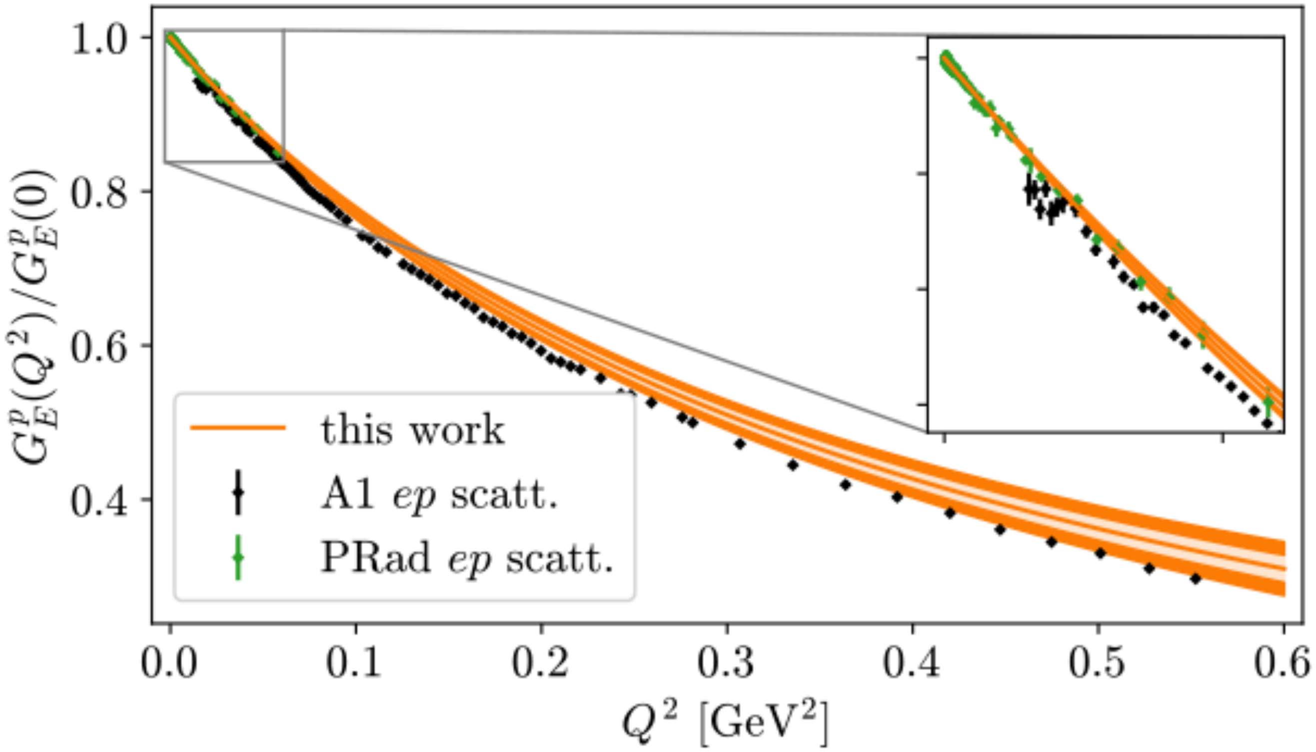
Dirac (F_1), Pauli (F_2)

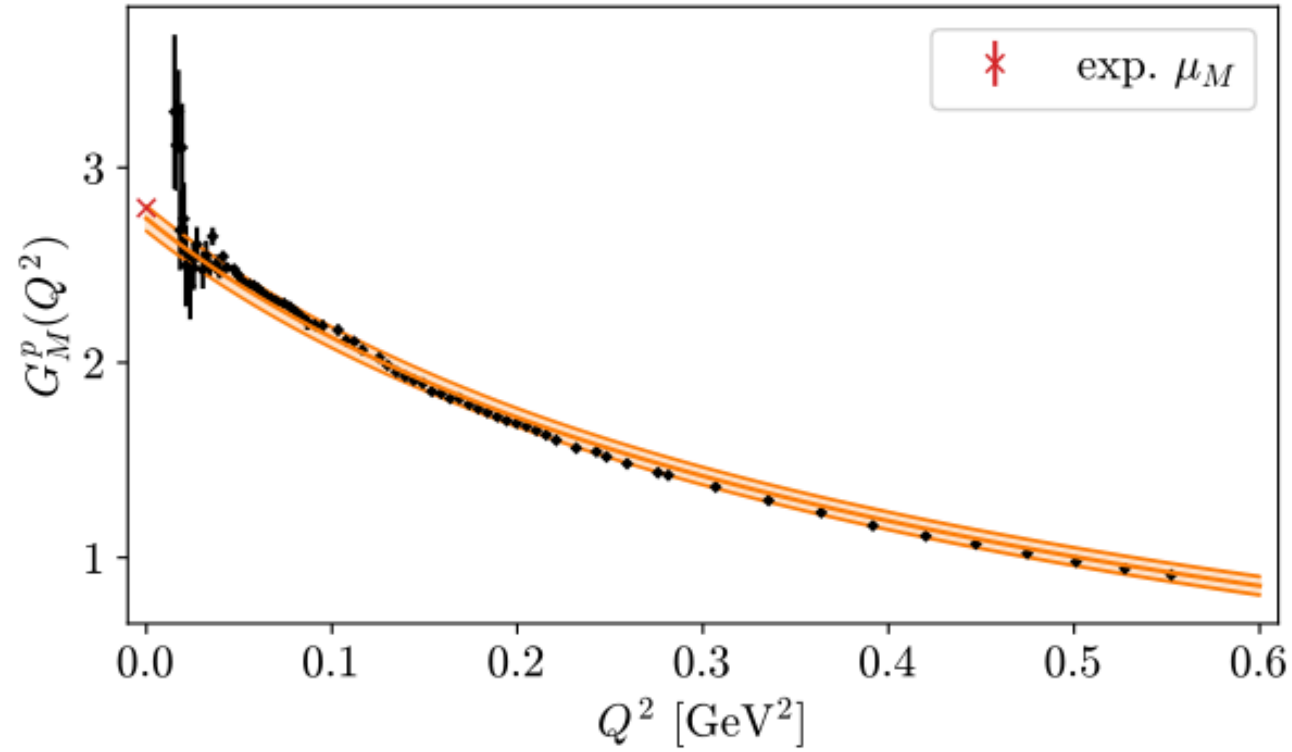
$$G_E(Q^2) = F_1(Q^2) + \frac{Q^2}{4m_N^2}F_2(Q^2), \quad G_M(Q^2) = F_1(Q^2) + F_2(Q^2), \quad Q^2 = -q^2$$

$$\langle r^2\rangle_{E,M} = -\frac{6}{G_{E,M}(0)}\frac{\partial G_{E,M}(Q^2)}{\partial Q^2}|_{Q^2\to 0}$$

- Analysis of 6 CLS ensembles with 4 lattice spacings ranging fro, 0.050 fm to 0.086 fm and pion masses ranging from 300 MeV to physical

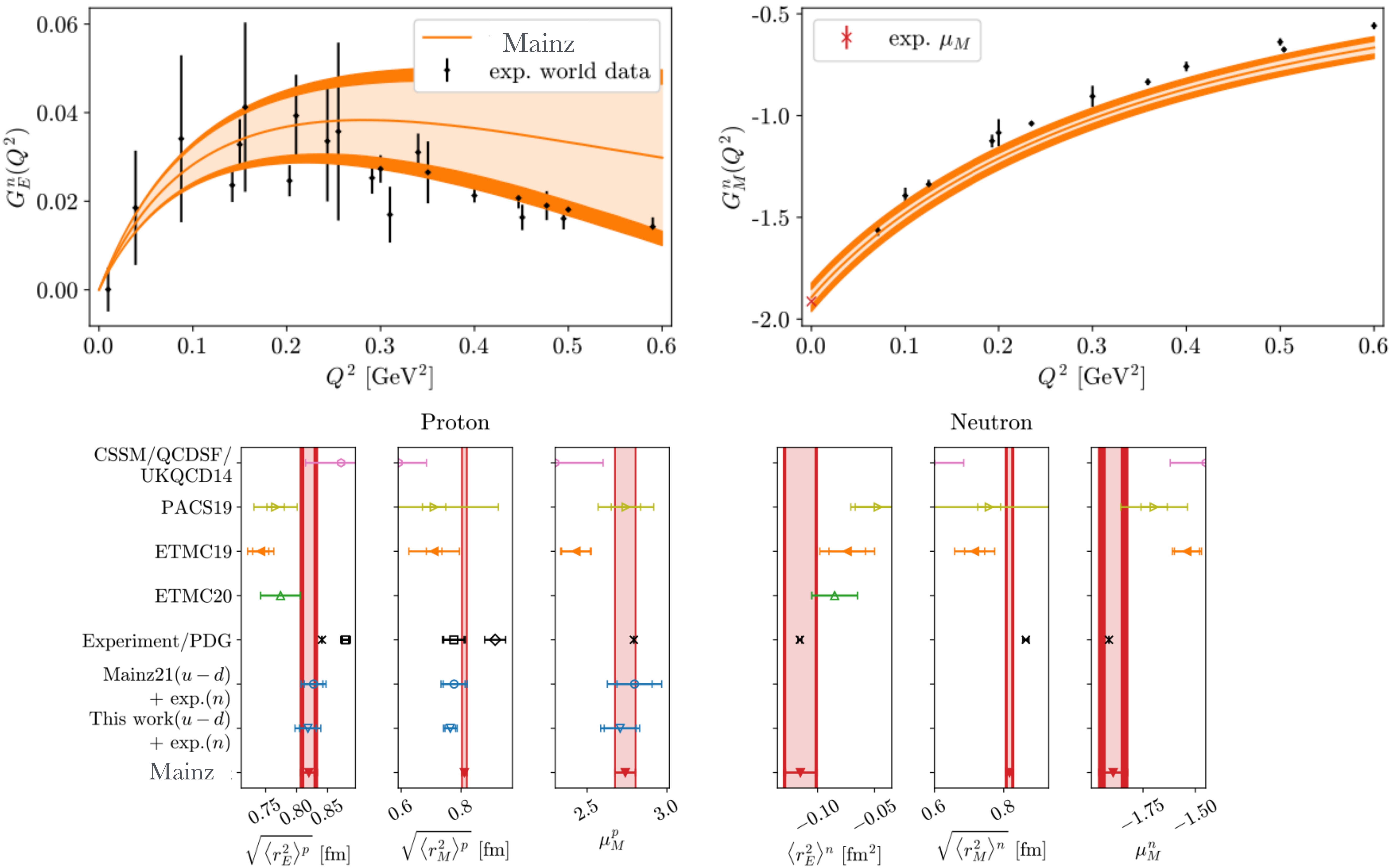
- Only one ensemble at physical pion mass —> chiral extrapolation needed





D. Djukanovic *et al.* (Mainz), Phys. Rev. Lett. 132 (2024) 21,arXiv:2309.07491; Phys. Rev. D 109 (2024) 9, arXiv:2309.06590

Electromagnetic form factors

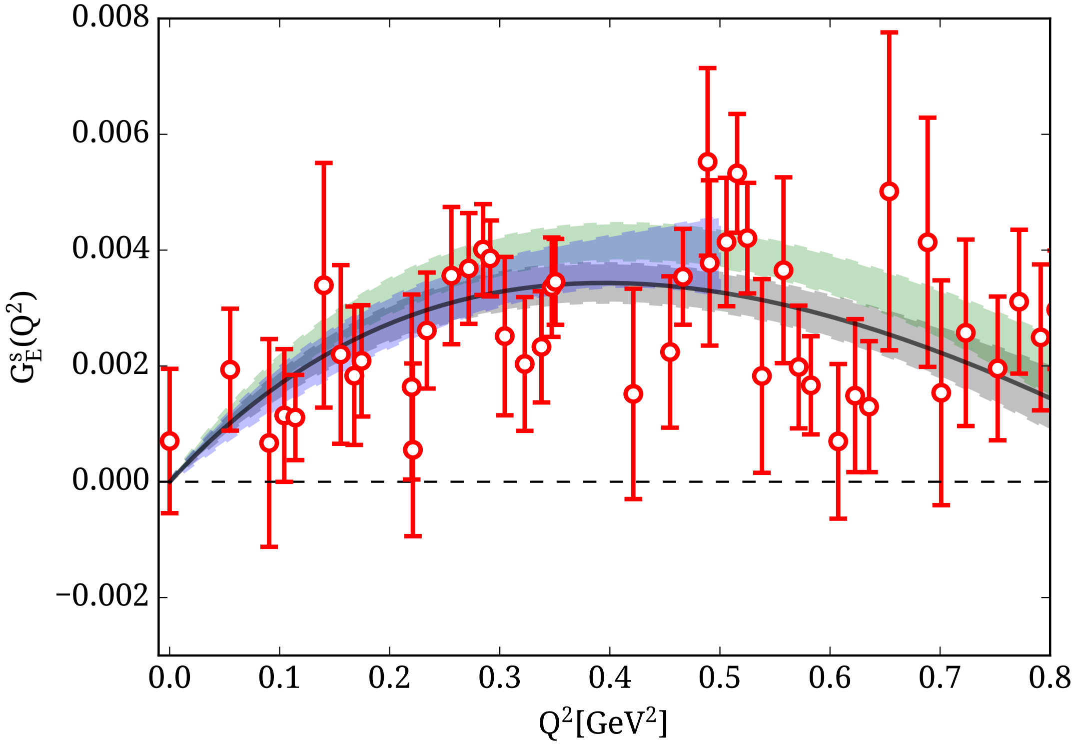


D. Djukanovic *et al.* (Mainz), Phys. Rev. Lett. 132 (2024) 21,arXiv:2309.07491; Phys. Rev. D 109 (2024) 9, arXiv:2309.06590

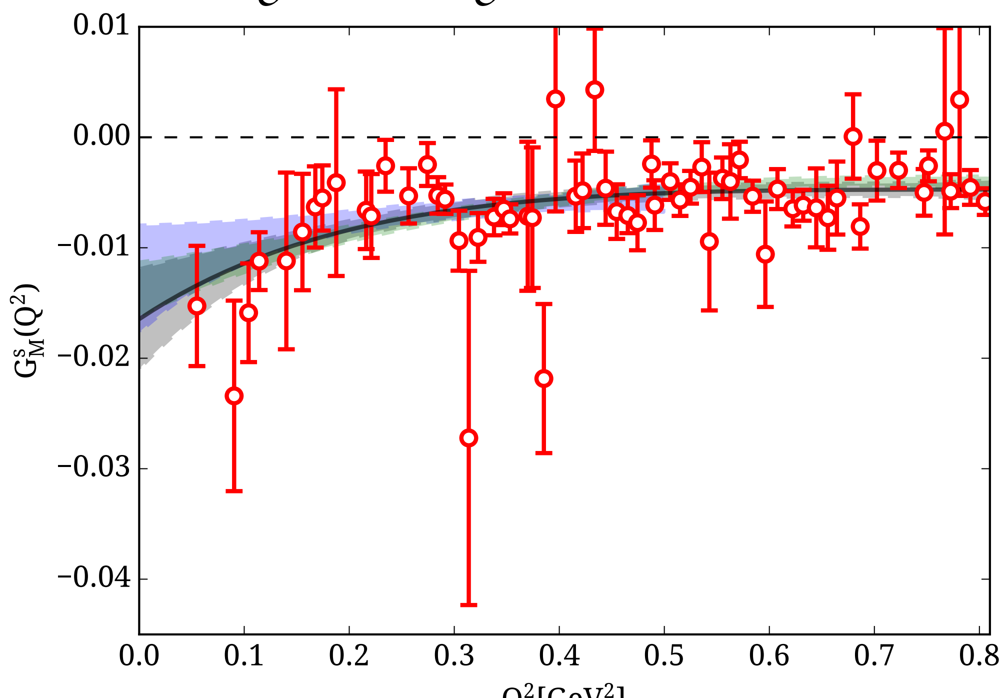
Strangeness of the nucleon

- Sea quark effects can be accurately determined for EM form factors —> provide precise input to experiments

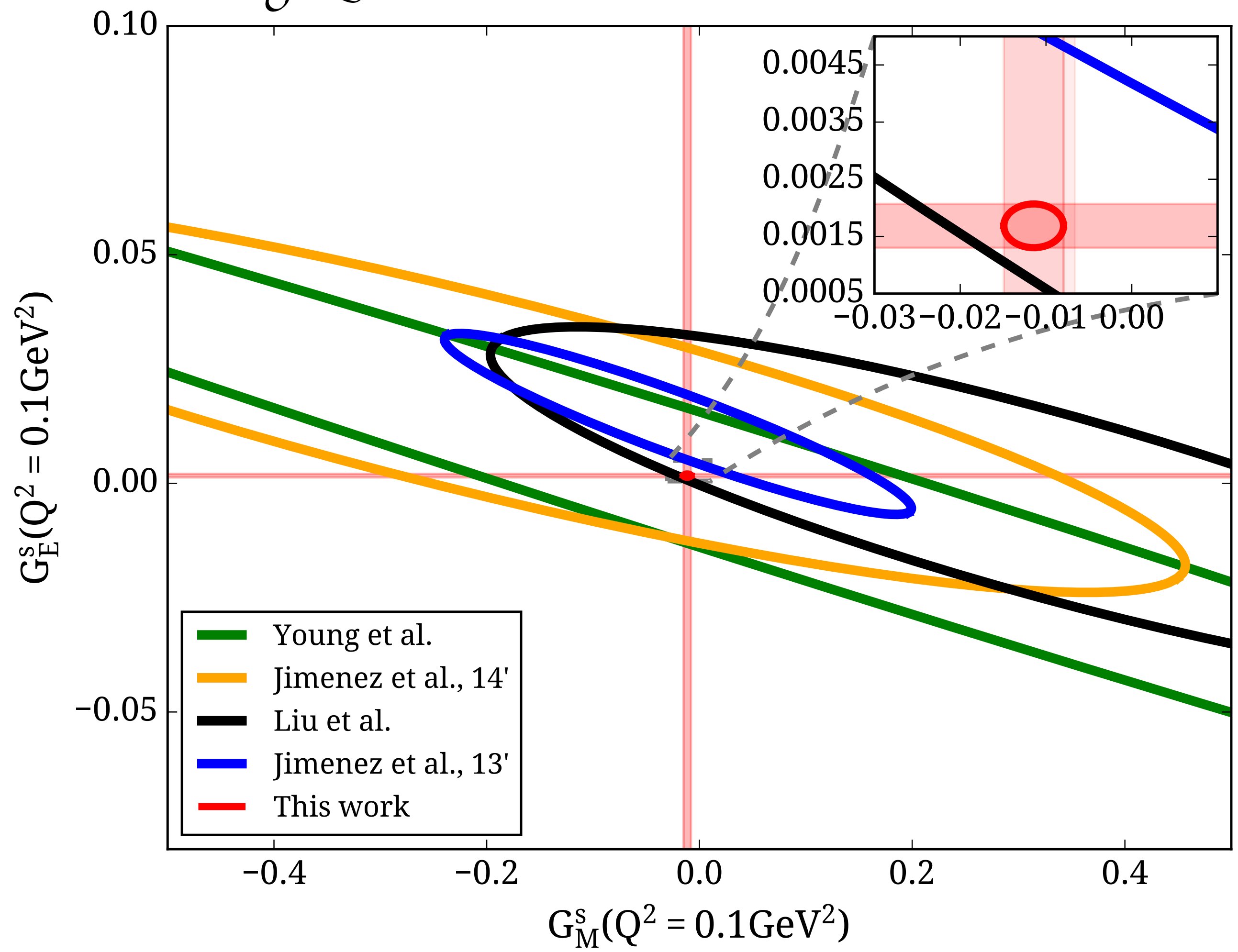
B-ensemble: 64^3 x 128, a~0.08 fm



Negative magnetic moment

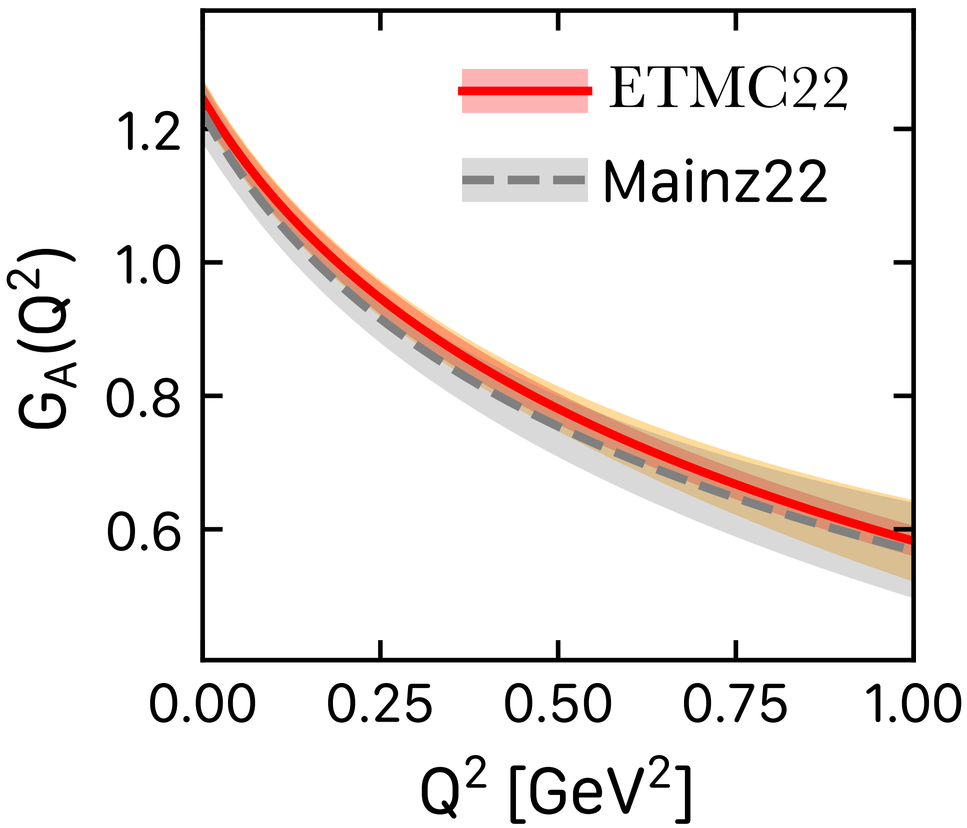


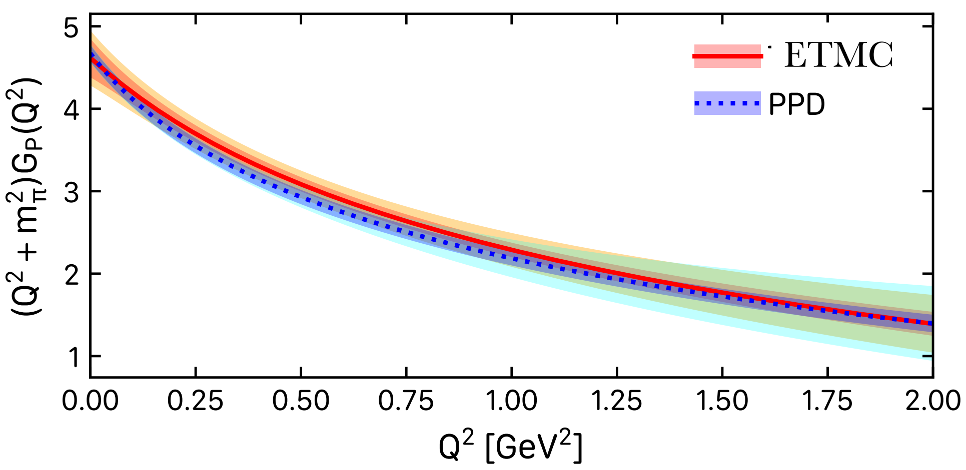
Significant input to experimental searches e.g Q-Weak, SolID, etc

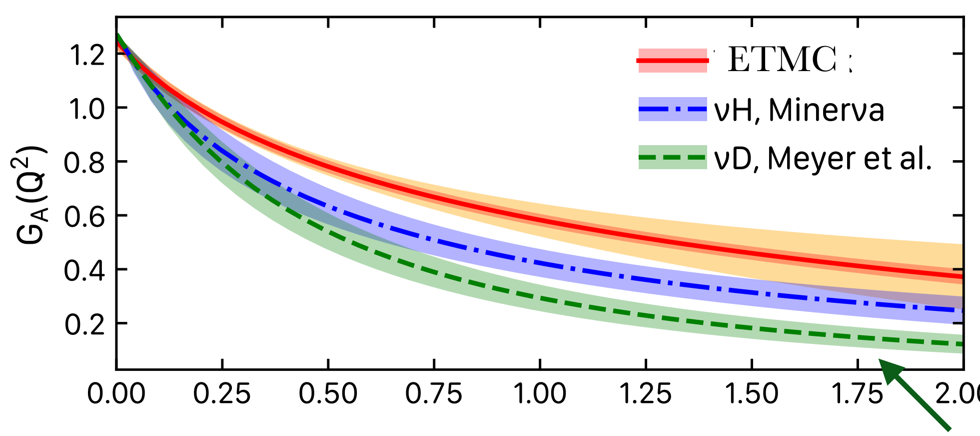


C. A. *et al.*, Phys. Rev. D 101 (2020) 031501, arXiv:1909.10744

Recent results on $G_A(Q^2)$ and $G_P(Q^2)$







- PCAC satisfied in the continuum limit
- Pion pole dominance satisfied for induced pseudoscalar
- Lattice QCD results closer to the new Minerva antineutrino-hydrogen data
- Agreement between our results and those of Mainz

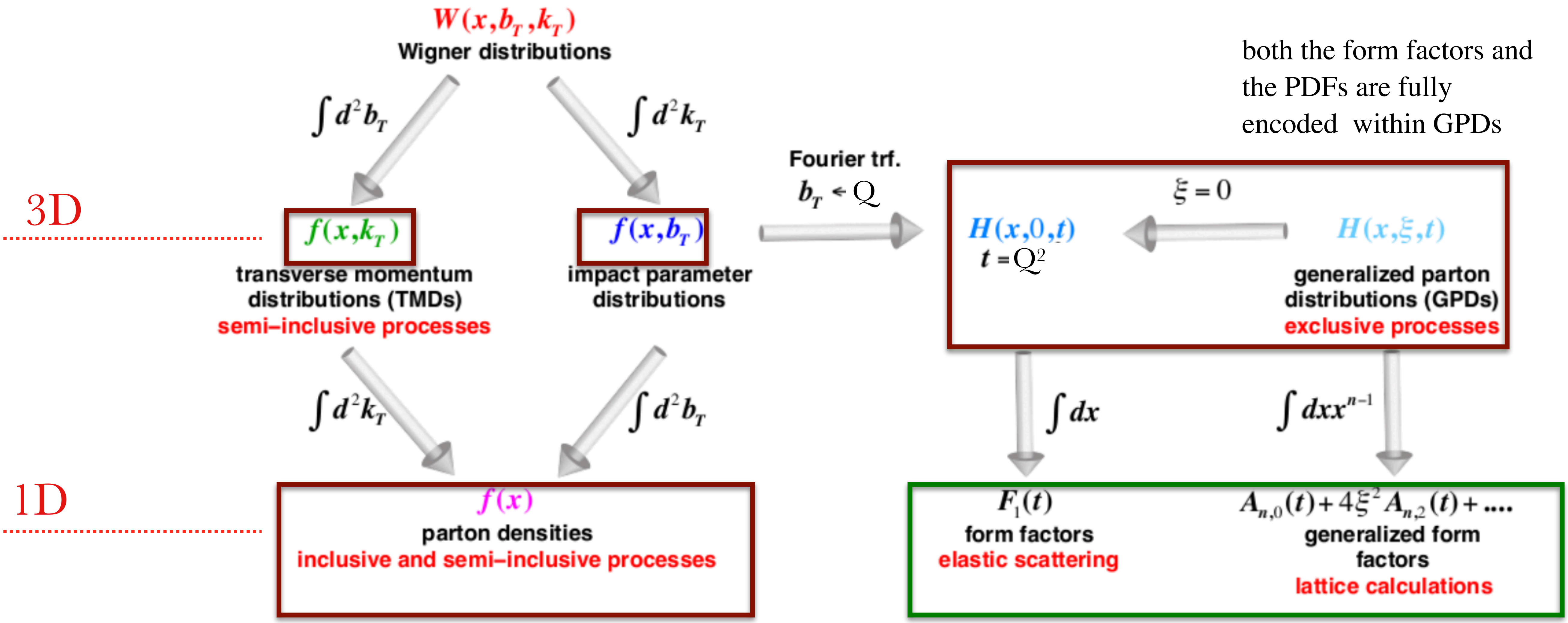
T. Cai *et al.*, Nature 614, 48 (2023)

D. Djukanovic et al. PRD 106, 074503 (2022), arXiv: 2207.03440

3. Lattice QCD reaches unprecedented accuracy in the evaluation of nucleon form factors

Generalised parton distributions (GPDs)

both the form factors and the PDFs are fully encoded within GPDs

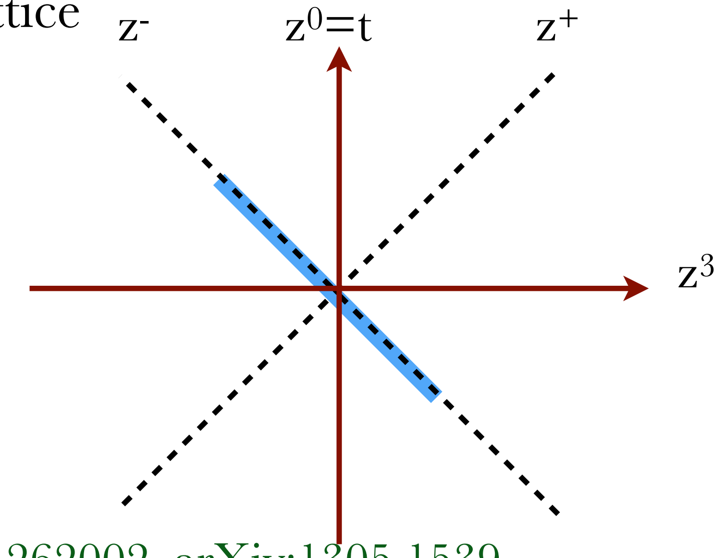


EIC white paper, arXiv:1212.1701

Direct computation of parton distributions

- PDFs light-cone correlation matrix elements - cannot be computed on a Euclidean lattice

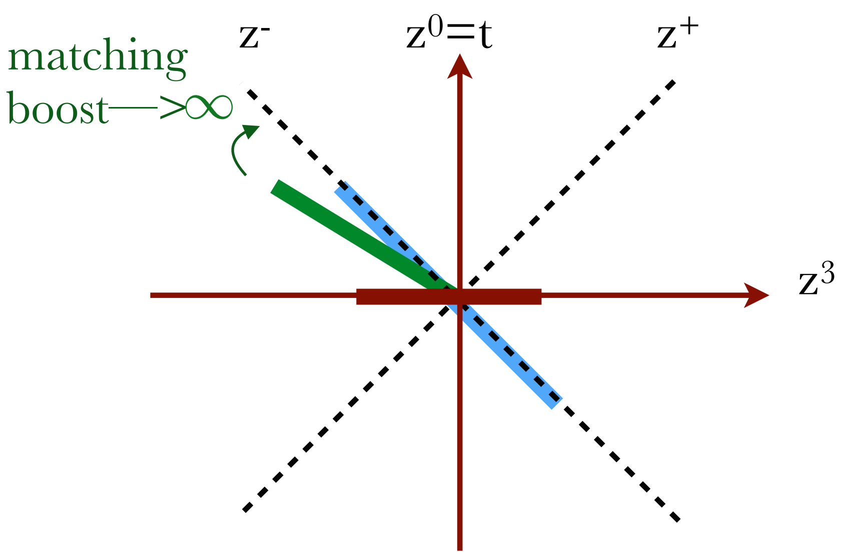
$$F_\Gamma(x) = \frac{1}{2}\int \frac{dz^-}{2\pi}\, e^{ixP^+z^-}\langle N(p)|\bar{\psi}(-z/2)\Gamma W(-z/2,z/2)\psi(z/2)|N(p)\rangle|_{z^+=0,\vec{z}=0}$$



- Define spatial correlators e.g. along z^3 and boost nucleon state to large momentum

X. Ji, Phys. Rev. Lett. 110 (2013) 262002, arXiv:1305.1539

- Match to the infinite momentum frame using the matching kernel computed in perturbation theory (large momentum effective theory - LaMET)



Computation of quasi-PDFs

- Compute space-like matrix elements for boosted nucleon states and take the large boost limit

$$\tilde{F}_\Gamma(x, P_3, \mu) = 2P_3 \int_{-\infty}^{\infty} \frac{dz}{4\pi} e^{-ixP_3 z} \langle P_3 | \overline{\psi}(0)\, \Gamma W(0,z)\, \psi(z) | P_3 \rangle|_\mu$$

Renormalise non-perturbatively, $Z(z,\mu)$

Need to eliminate both UV and exponential divergences

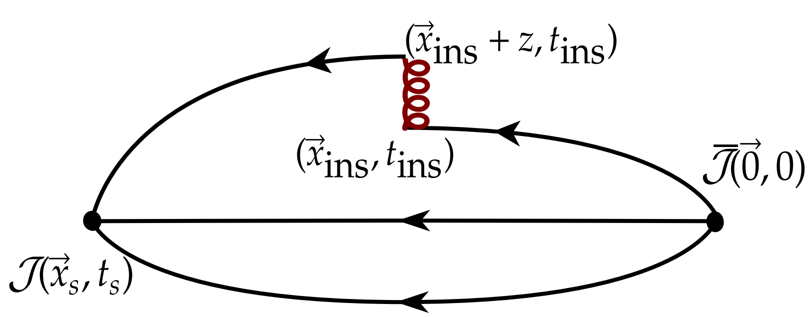
- Match using LaMET

Perturbative kernel

$$\tilde{F}_\Gamma(x, P_3, \mu) = \int_{-1}^{1} \frac{dy}{|y|} C\left(\frac{x}{y}, \frac{\mu}{yP_3}\right) F_\Gamma(y, \mu) + \mathcal{O}\left(\frac{m_N^2}{P_3^2}, \frac{\Lambda_{QCD}^2}{P_3^2}\right)$$

X. Ji, Phys. Rev. Lett. 110 (2013) 262002, arXiv:1305.1539

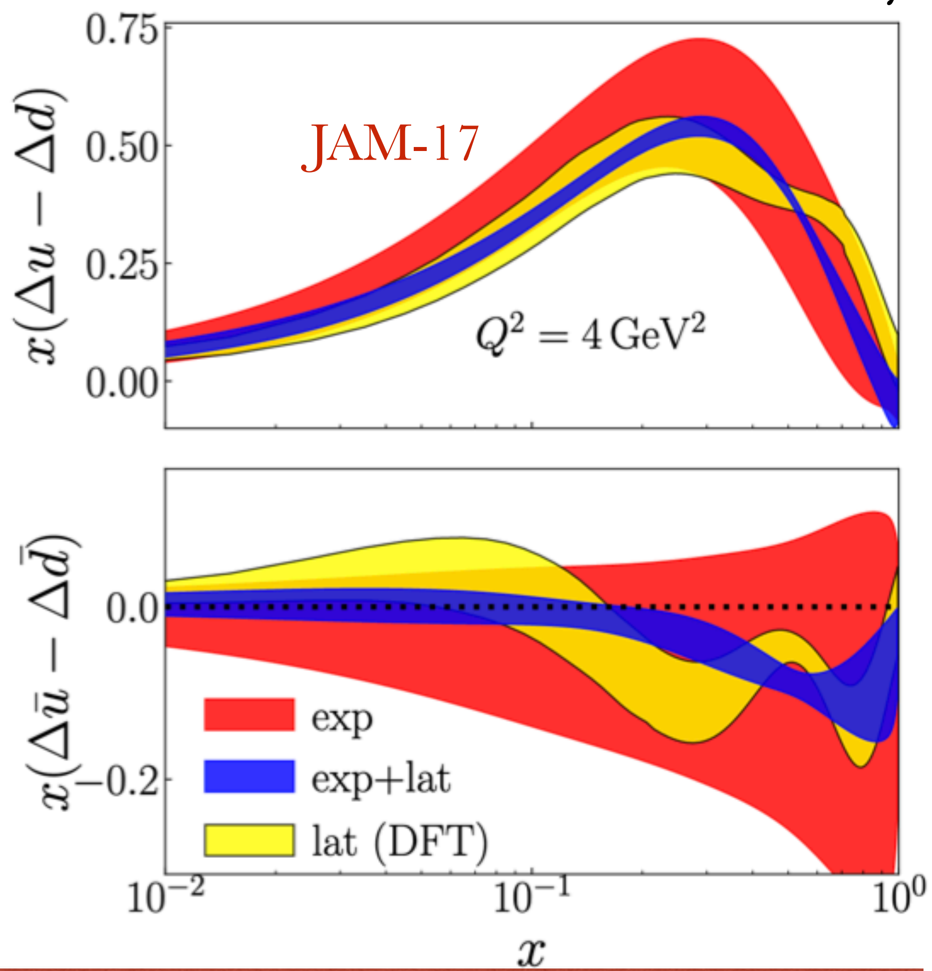
Isovector (u-d)



$\Gamma =$	γ_0	unpolarised
	$\gamma_5\gamma_3$	helicity
	$\sigma_{3i}, i = 1, 2$	transversity

C.A. et al. (ETMC) Phys. Rev. Lett. **121**, 112001 (2018)

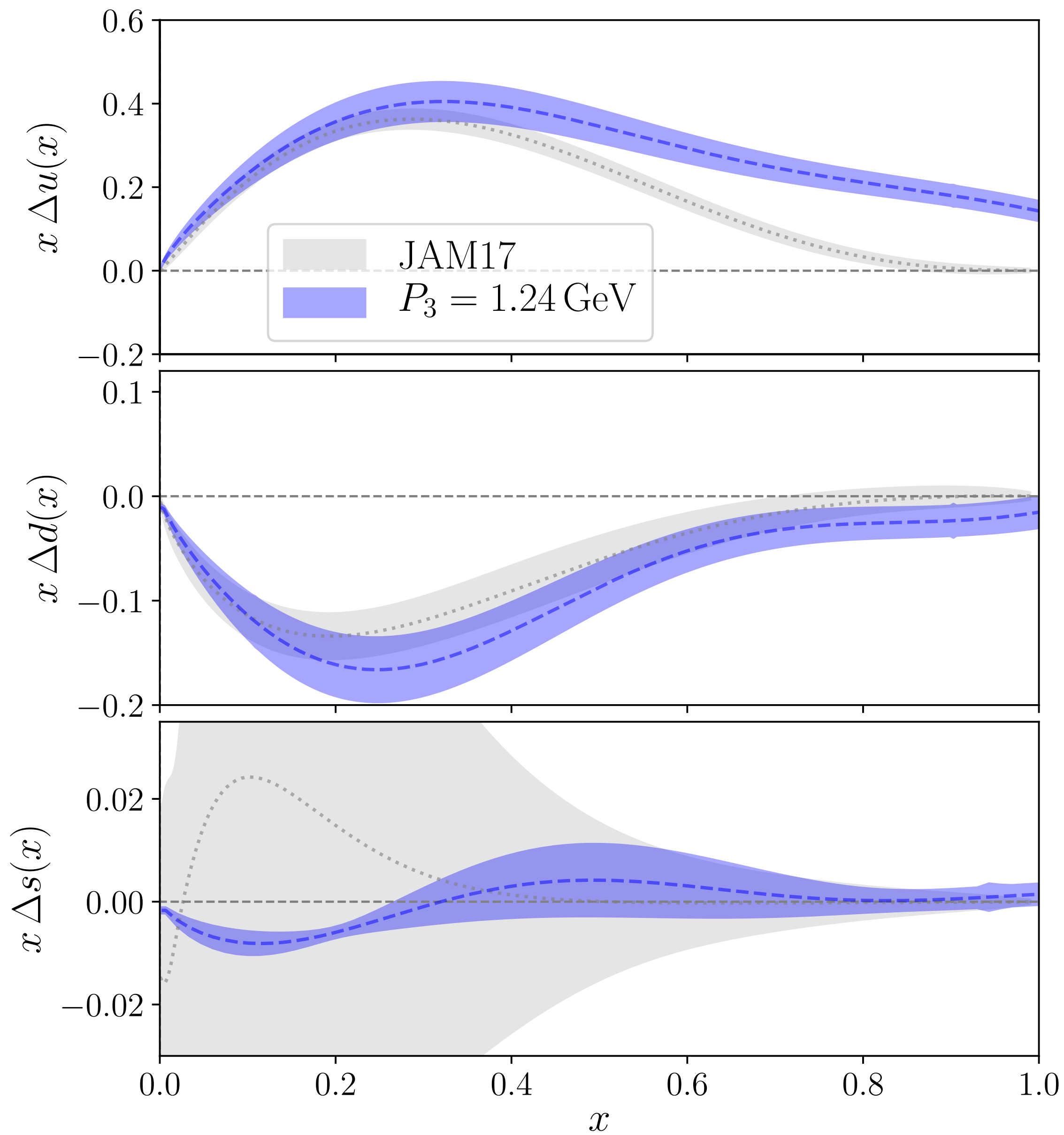
State-of-the-art results on helicity



4. Parton distribution functions can be computed directly in lattice QCD

Helicity distributions

$32^3 \times 64$	a=0.0938(3)(2) fm	$m_N = 1.050(8)$ GeV
$L = 3.0$ fm	$m_\pi \approx 260$ MeV	$m_\pi L \approx 4.0$

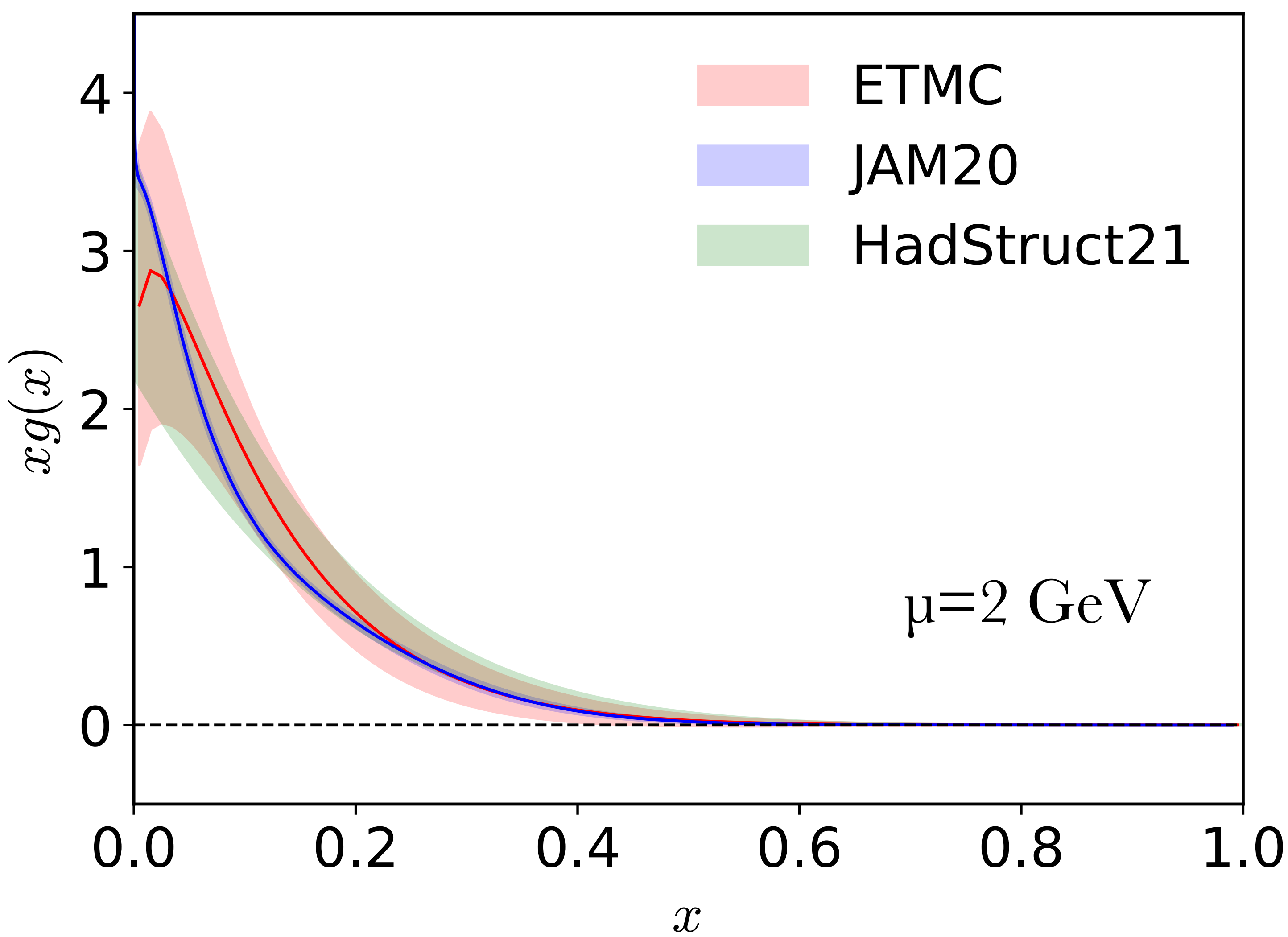


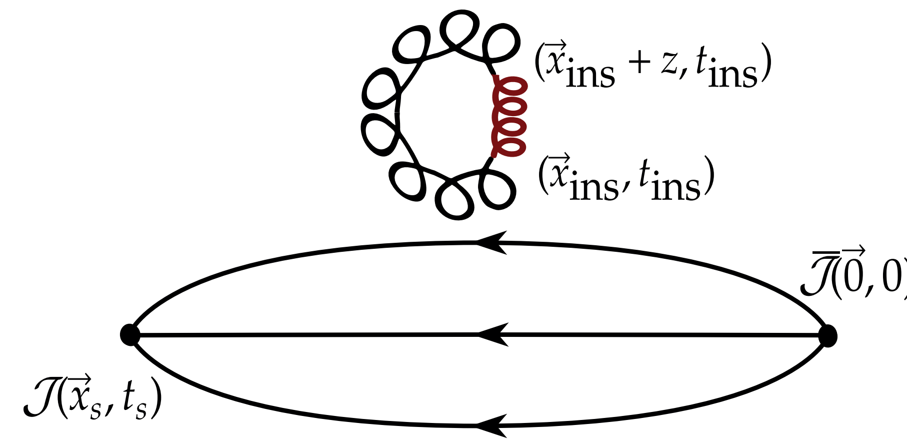
C. A., M. Constantinou, K. Jansen, F. Manigrasso, Phys. Rev. Lett. 126 (2021) 10, 102003, arXiv:2009.1306

C.A., G. Iannelli, K. Jansen, F. Manigrasso, Phys. Rev. D 102 (2020) 9, 094508, arXiv:2007.13800

Unpolarized gluon PDF

- Calculate the matrix elements of a spin-averaged nucleon for two gluon fields connected by a Wilson line
- Use Wilson flow to reduce ultraviolet fluctuations
- Pseudo-PDF approach with pion mass 358 MeV





T. Khan, *et al.* (HadStruc Collaboration) Phys. Rev. D 101 (2021) 094516, 2107.08960
J. Delmar et al. (ETMC) PoS LATTICE2022 (2023) 099, 2212.11399

Generalised parton distributions

- Compute space-like matrix element with different initial and final nucleon boosts in the Breit frame

$$h_\Gamma(z,\tilde{\xi},Q^2,P_3) = \langle N(P_3\hat{e}_z + \vec{Q}/2)|\,\overline{\psi}(z)\,\Gamma W(z,0)\,\psi(0)|\,N(P_3\hat{e}_z - \vec{Q}/2)\rangle$$

$$\tilde{\xi} = -\frac{Q_3}{2P_3}\ :\ \mathrm{quasi-skewness} \quad \tilde{\xi} = \xi + \mathcal{O}(\frac{1}{P_3^2})$$

- Rest of the steps are the same as for quasi-PDFs: i.e. renormalise, take the Fourier transform and match and final nucleon boosts

$$\tilde{F}_\Gamma(z,\tilde{\xi},Q^2,P_3,\mu^0,\mu_3^0) = \int_{-1}^{1} \frac{dy}{y} C_\Gamma\left(\frac{x}{y},\frac{\mu}{yP_3},\frac{\mu_3^0}{yP_3},\frac{(\mu^0)^2}{(\mu_3^0)^2}\right) F_\Gamma(y,Q^2,\xi,\mu) + \mathcal{O}\left(\frac{m^2}{P_3^2},\frac{Q^2}{P_3^2},\frac{\Lambda^2_{\mathrm{QCD}}}{x^2P_3^2}\right)$$

RI-scale

$\overline{\mathrm{MS}}$ – scale

Reduces to the matching kernel for ξ=0
Does not depend on Q^2

X.Ji *et al.*, Phys.Rev. D92 (2015) 014039
X.Xiong, J-H. Zhang, Phys.Rev. D92 (2015) 054037
Y-S. Liu *et al.*, Phys.Rev. D100 (2019), 034006

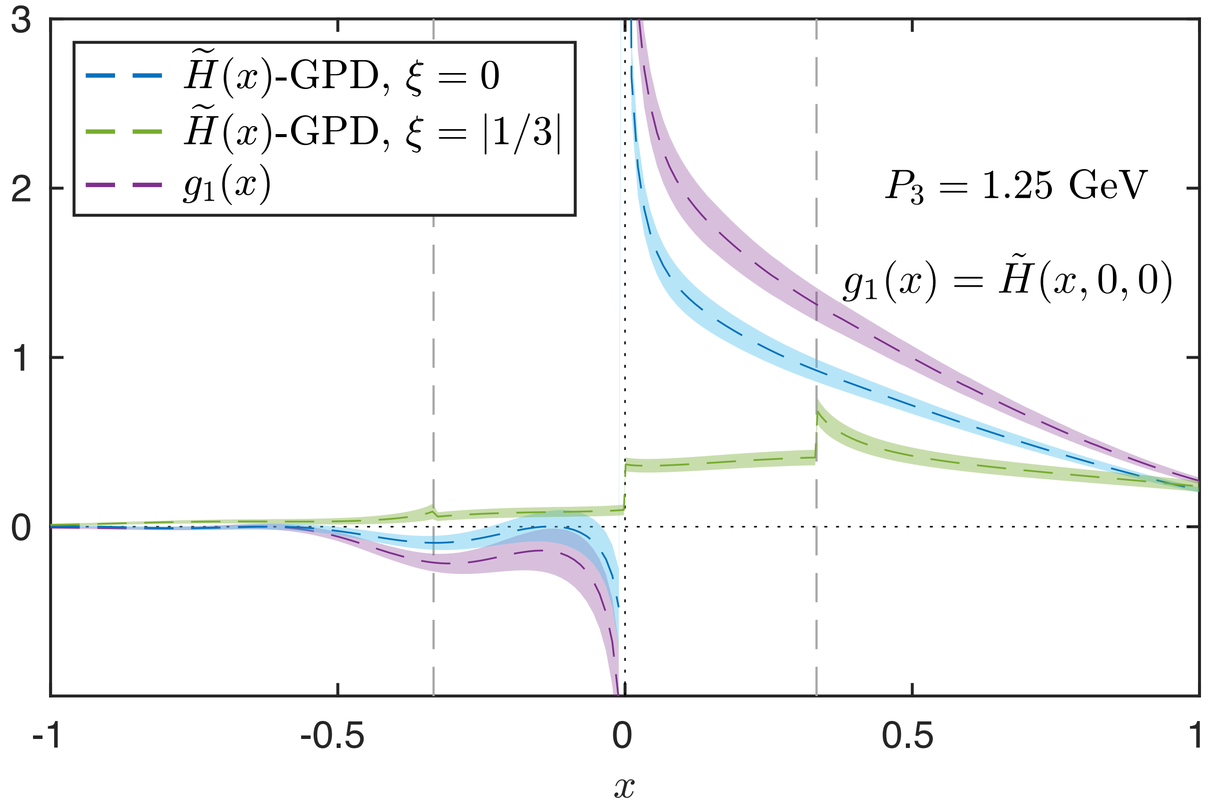
- First studies for pion and nucleon GPDs

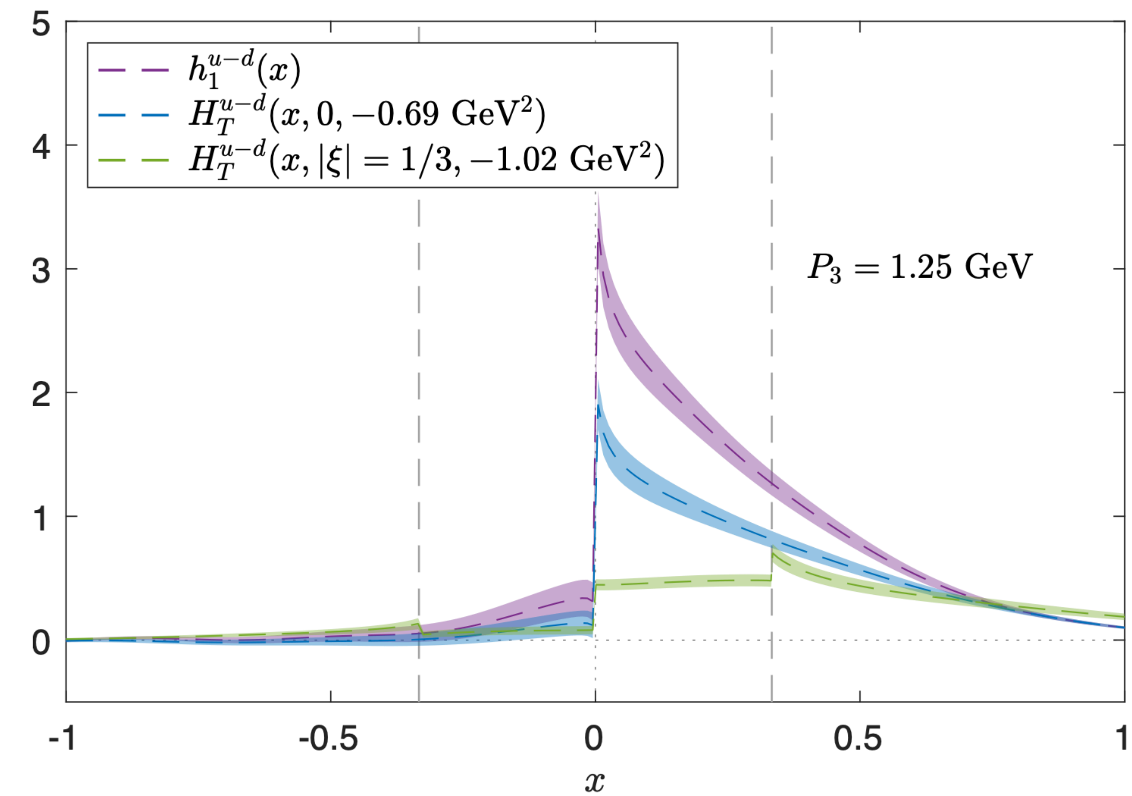
J.W. Chen, H.W. Lin, J.H. Zhang, Nucl. Phys. B 952, 114940 (2020), 1904.12376
C. A. *et al.*, Phys.Rev.Lett. 125 (2020) 26, 262001, 2008.10573
H.-W. Lin, Phys. Rev. Lett. 127, 182001 (2021), 2008.12474
H.-W. Lin, Phys. Lett. B 824, 136821 (2022), 2112.07519

Helicity & transversity GPDs

$32^3 \times 64$	a=0.0938(3)(2) fm	$m_N = 1.050(8)$ GeV
$L = 3.0$ fm	$m_\pi \approx 260$ MeV	$m_\pi L \approx 4.0$

Q^2=0.69 GeV^2





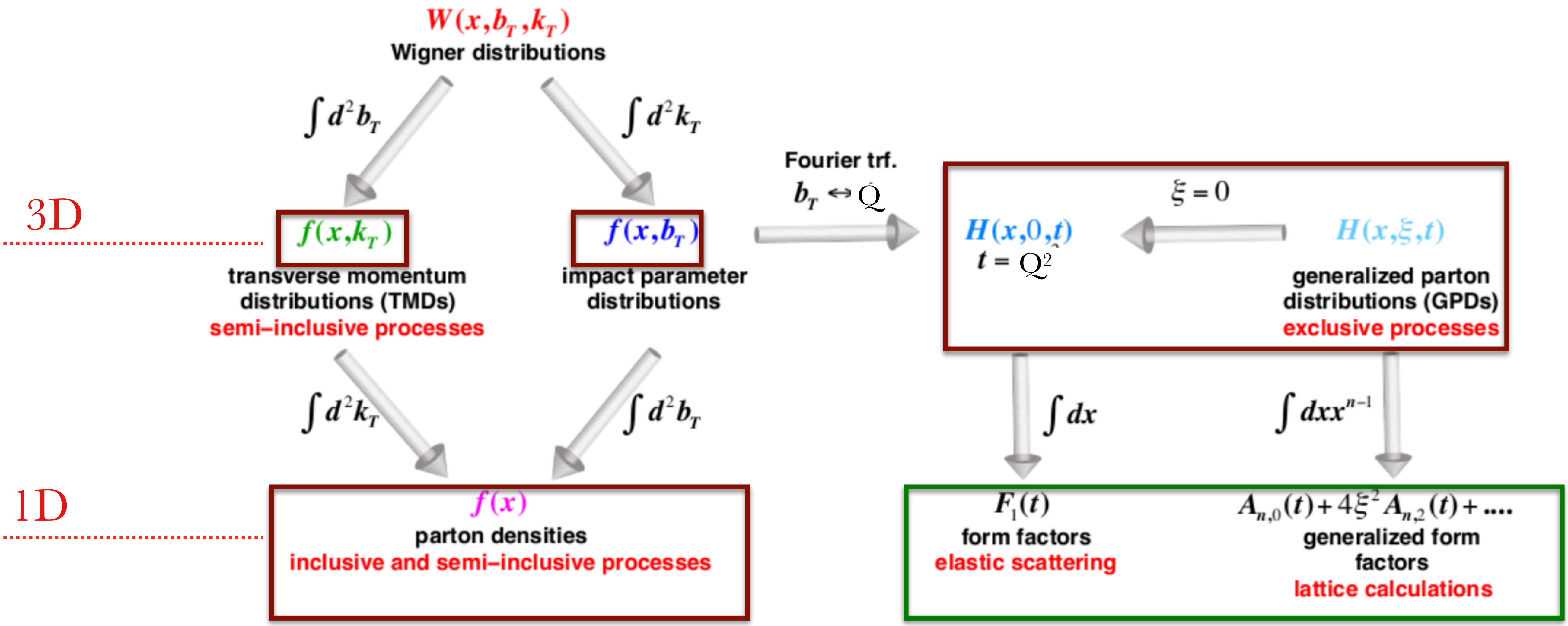
C. A. *et al.* (ETMC) Phys. Rev. Lett. 125 (2020) 262001,2008.10573
C.A. *et al.*(ETMC) , Phys.Rev.D 105 (2022) 3, 034501, 2108.10789

New developments of expressing GPDs in terms of Lorentz invariant amplitudes allows easier access to a range of momentum transfers in lattice QCD calculations

S. Bhattacharya *et al.* Phys. Rev. D 106 (2022) 114512, 2209.05373 for unpolarized
S. Bhattacharya *et al.* Phys. Rev. D 109 (2024) 034508, 2310.13114 for helicity

5. GDPs can be computed directly in lattice QCD

Transverse momentum distributions (TMDs)



EIC white paper, arXiv:1212.1701

Towards TMD PDFs in lattice QCD

X. Ji, *et al.* Phys. Rev. D 99 (2019) 114006, 1801.05930

M. A. Ebert, I. W. Stewart, Y. Zhao, Phys.Rev.D 99 (2019) 3, 034505, 1811.00026; JHEP 09 (2019) 037, 1901.03685; JHEP 03 (2020) 099 ,1910.08569

- Quasi-TMDs formulated in the LaMET approach

- First results obtained for the unpolarised nucleon TMD PDF by the Lattice Parton Collaboration (LPC)

X. Ji *et al.* (LPC) 2211.02340

$$f^{\rm TMD}(x,\vec{b}_T,\mu,\zeta) = H(\frac{\zeta_z}{\mu^2})\, e^{-\ln(\frac{\zeta_z}{\zeta})K(b_T,\mu)}\, \tilde{f}(x,\vec{b}_T,\mu,\zeta_z)\sqrt{S_r(b_T,\mu)}+\mathcal{O}\left(\frac{\Lambda^2_{\rm QCD}}{\zeta_z},\frac{M^2}{(P^z)^2},\frac{1}{b_T^2\zeta_z}\right)$$

perturbative matching kernel

Collins-Soper kernel, which is non-perturbative for $q_T \sim 1/b_T \sim \Lambda_{\rm QCD}$

Rapidity independent reduced soft function

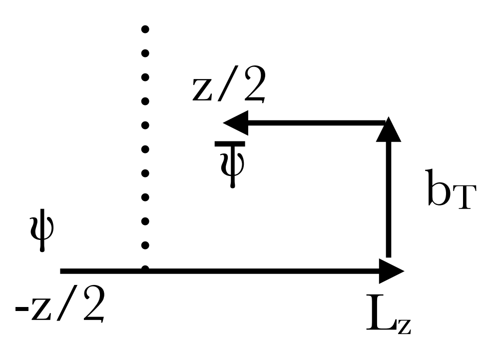
- $\zeta_z=(2xP^z)^2$ is the Collins-Soper scale of the quasi-TMD

- Quasi-TMD PDF is given as $\tilde{f}^{\rm TMD}(x,\vec{b}_T,\mu,\zeta_z) = \int \frac{dz}{2\pi}\, e^{-iz\zeta_z}\, \frac{P^z}{E_{\vec{P}}} B_\Gamma(z,\vec{b}_T,\mu,P^z)$

Renormalised beam function obtained from the bare

$$\tilde{B}_{0,\Gamma}(z,\vec{b}_T,L,P^z;1/a) = \langle N(P^z)|\bar{\psi}(z/2,\vec{0}_T)\Gamma\mathcal{W}(z,\vec{b}_T,L\hat{z})q(-z/2,\vec{b}_T)|N(P^z)\rangle$$

$\mathcal{W}(z,\vec{b}_T,L\hat{z}) =$

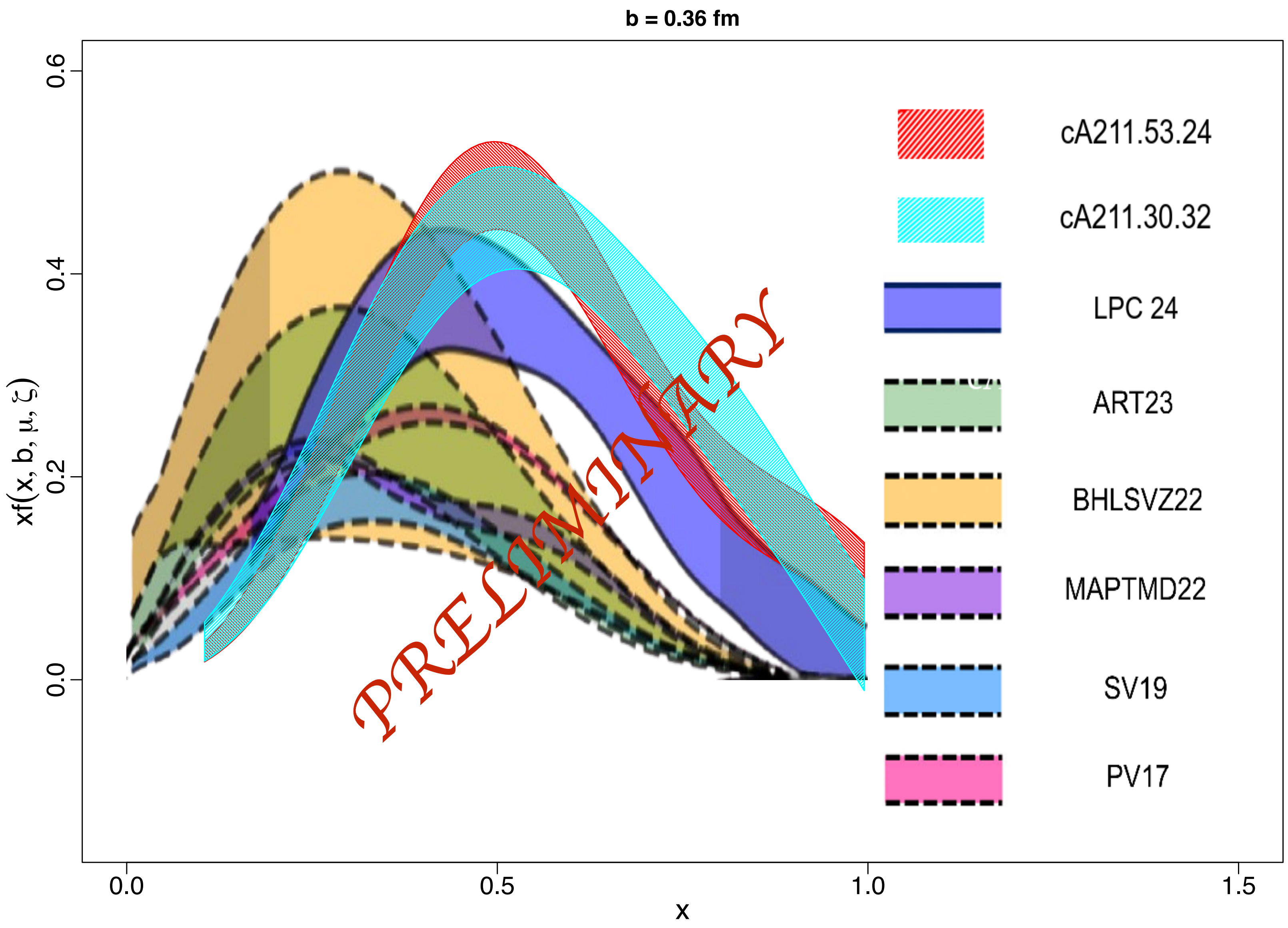


Nucleon unpolarised isovector TMD PDF

- LPC published the first results modelling the momentum dependence and taking the chiral and continuum limits

Jin-Chen He et al. (LPC) arXiv:2211.02340

- ETMC has preliminary results at one lattice spacing (0.093 fm) and heavier than physical pion mass (350 MeV), renormalised with the ratio scheme



Conclusions

- ❋ Lattice QCD provides precision results on low-lying hadron masses and their splitting including isospin and EM effects

- ❋ Nucleon charges and form factors are computed to unprecedented accuracy reproducing known quantities such as the axial charge and EM form factors and providing precise results on e.g. tensor charges and axial form factors

- ❋ The so called proton spin and radius puzzles can be understood from lattice QCD computations

- ❋ Direct computation of generalised parton distributions is shown to be feasible as is the transverse momentum distributions which is also underway

- ❋ Many other results are emerging, such as muon g_μ-2, properties of resonances and exotics, charm physics, etc.

Extended Twisted Mass Collaboration

EuroHPC Joint Undertaking

Computational resources

PRACE

Extended Twisted Mass Collaboration