Single-pair measurement of the Bell parameter and relativistic independence

Many students, colleagues and:

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XIII ICNFP, September 3 2024

Theory

Relativistic Causality

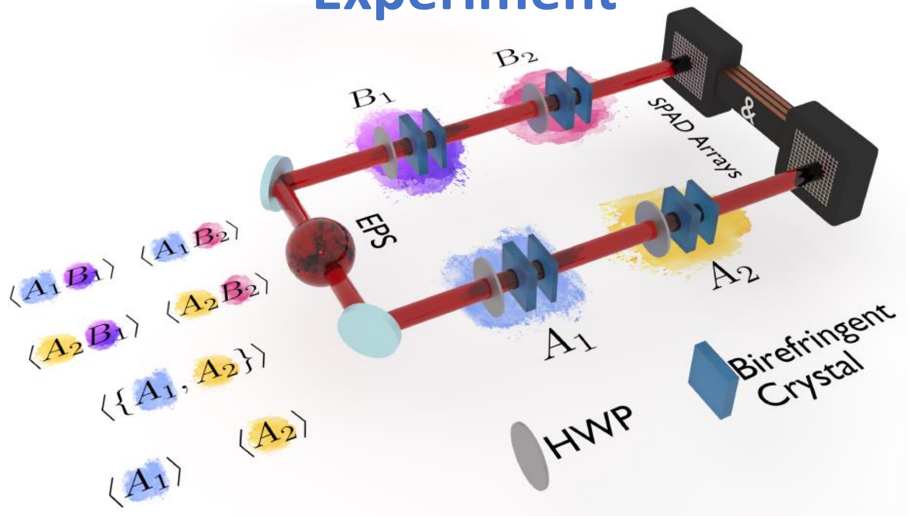
Relativistic Independence (RI)

Carmi A., Cohen E.,
Sci. Adv. 5, eaav8370 (2019)

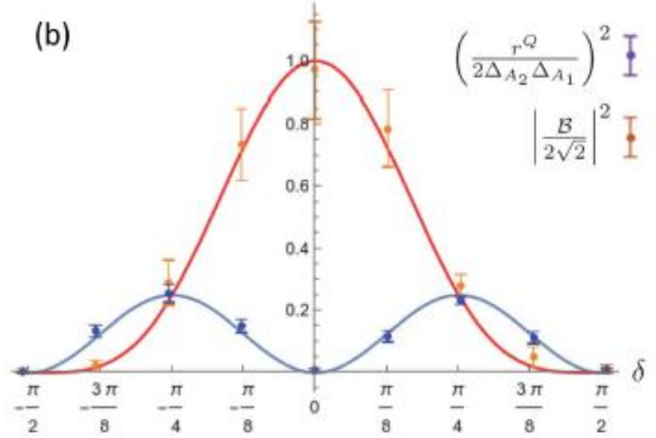
Quantum Nonlocality

Quantum Uncertainty

Experiment



$$\mathcal{RI} = \left|\frac{\mathcal{B}}{2\sqrt{2}}\right|^2 + \left(\mathrm{Re}\left[\frac{r}{2\Delta_{A_2}\Delta_{A_1}}\right]\right)^2 \leq 1$$



Since then: Many new quantitative predictions regarding quantum correlations in space and time

S. Virzi et al., Quant. Sci. Technol. 9, 045027 (2024)+forthcoming

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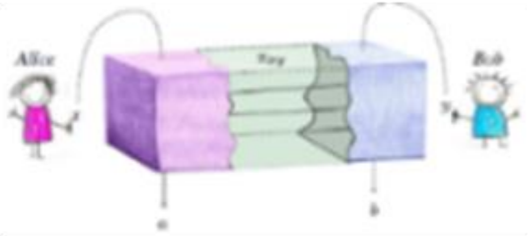
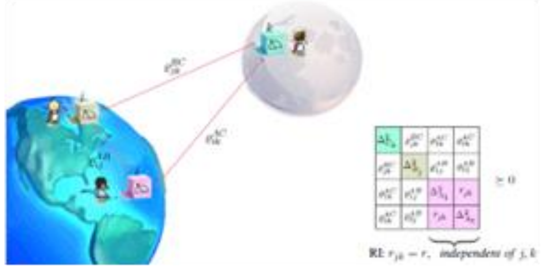
From Quantum Foundations To Quantum Technology

Entanglement&Nonlocality

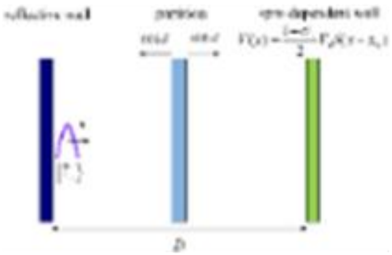




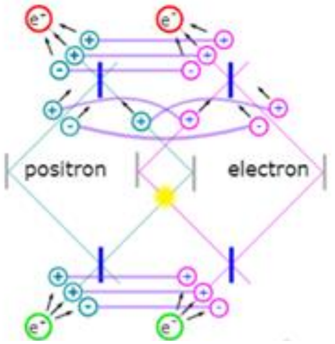
(*Sci. Adv.* 5, eaav8370 (2019)

NJP 21, 073032 (2019), etc.)

Weak values and Weak measurements

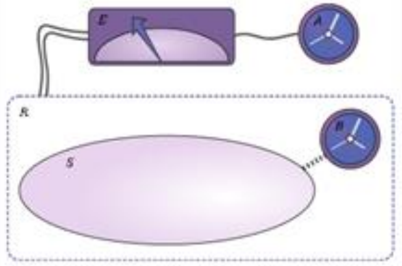


(*Nat. Commun.* 12, 4770 (2021)

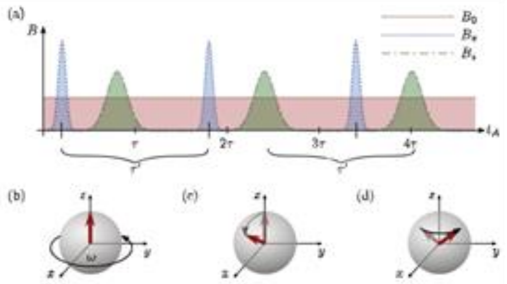


(PNAS 120, e2018437120 (2023))

Quantum time



(Quantum 6, 683 (2022))

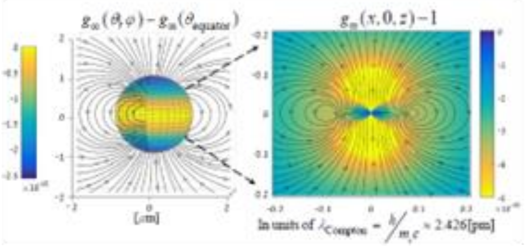


(PRA 105, 042207 (2022))

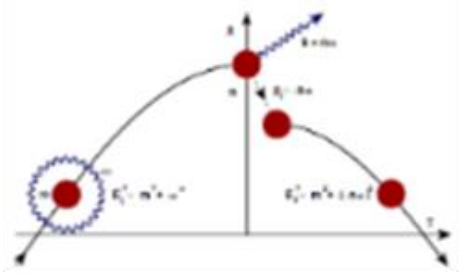
(Comm. Phys. 5, 298 (2022))

(PRA 109, 032205 (2024))

Information-based Quantum Gravity



(Ann. Phys. 534, 2100348 (2022))

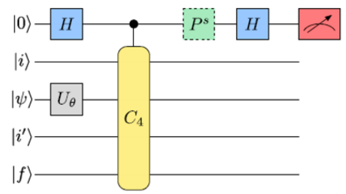


(*PRD* 104,021015 (2021))

(*NJP* 212, 083038 (2019))

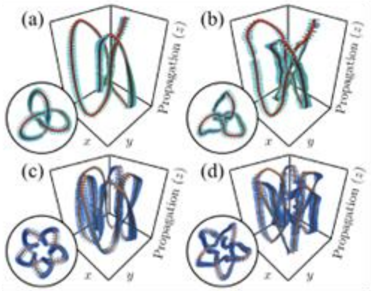
See also: *PNAS* 115, 11730 (2018) – Quantum Emergent Phenomena
PNAS 114, 6480 (2017) – Quantum Interference via nonlocal dynamics

From Quantum Foundations To Quantum Technology



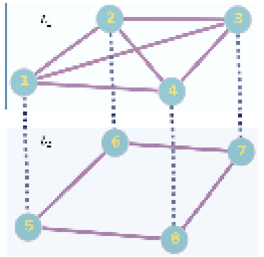
(*Quantum Sci. Technol. 9, 015030 (2024)*)

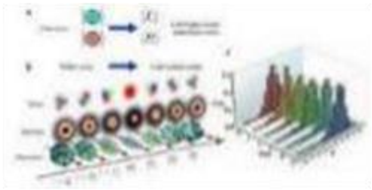
Quantum Computation and Communication



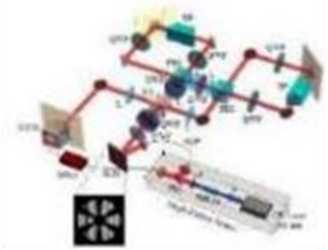
(*Nat. Commun.* 11, 5119 (2020))

Photonic Quantum Walks for Quantum Simulations

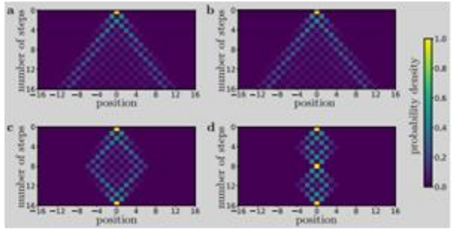




(*Nat. Rev. Phys.* 1, 437-449 (2019))

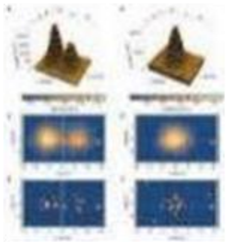


(*Optica* 6, 174-180 (2019))



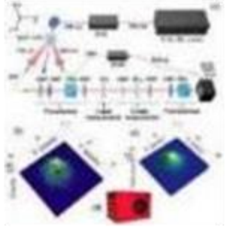
(PRA 105, 032413 (2022))

Quantum Measurements & Quantum Metrology



(*Nat. Phys.* 13, 1191 (2017)

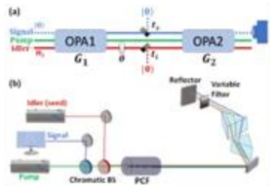
Light: Sci. Appl. 10, 106 (2021)).



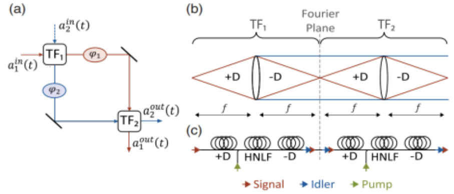
(*PRL* 117, 170402 (2016))

(*Nat. Phys.* 16, 1206-1210 (2020))

(Quant. Sci. Technol. 9, 045027 (2024))

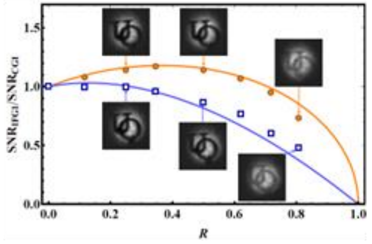


(*PRL* 127, 173603 (2021))



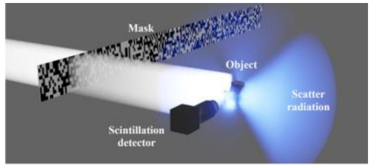
(*PRL 130, 253601 (2023)*)

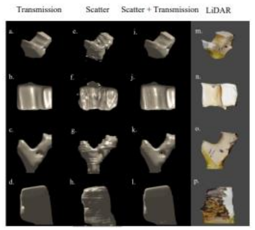
Quantum Imaging



(*Opt. Express* 27, 2212 (2019))

Quantum Inspired X-ray imaging with deep learning tools





(*Comms. Eng. 3, 39 (2024)*)

1

Background

Uncertainty, entanglement, nonlocal correlations and their interrelations

2

Relativistic Independence

Deriving quantum nonlocality from outside the quantum formalism+applications

3

Recent quantum optical experiments

Entanglement preserving measurement of the Bell parameter on each and every pair

Proof of the Relativistic Independence bounds

Uncertainty and nonlocality: An intimate relation

Uncertainty as an axiom

Uncertainty → **Nonlocality**

$$\sigma_A \sigma_B \geq \left| \frac{1}{2i} \langle [\hat{A}, \hat{B}] \rangle \right|$$

$$\sigma_A^2 \sigma_B^2 \geq \left| \frac{1}{2} \langle \{\hat{A}, \hat{B}\} \rangle - \langle \hat{A} \rangle \langle \hat{B} \rangle \right|^2 + \left| \frac{1}{2i} \langle [\hat{A}, \hat{B}] \rangle \right|^2$$

$$\left| c_{00} + c_{10} + c_{01} - c_{11} \right| > 2$$

$$\frac{dA}{dt} = \frac{i}{\hbar} [H, A]$$

$$[a, a^\dagger] = 1 \quad \left[a^{(\mu)}(\mathbf{k}), a^{\dagger\,(\mu')}(\mathbf{k}') \right] = \delta_{\mathbf{k},\mathbf{k}'} \delta_{\mu,\mu'}$$

Uncertainty as an axiom

Uncertainty → **Any quantum phenomenon?**

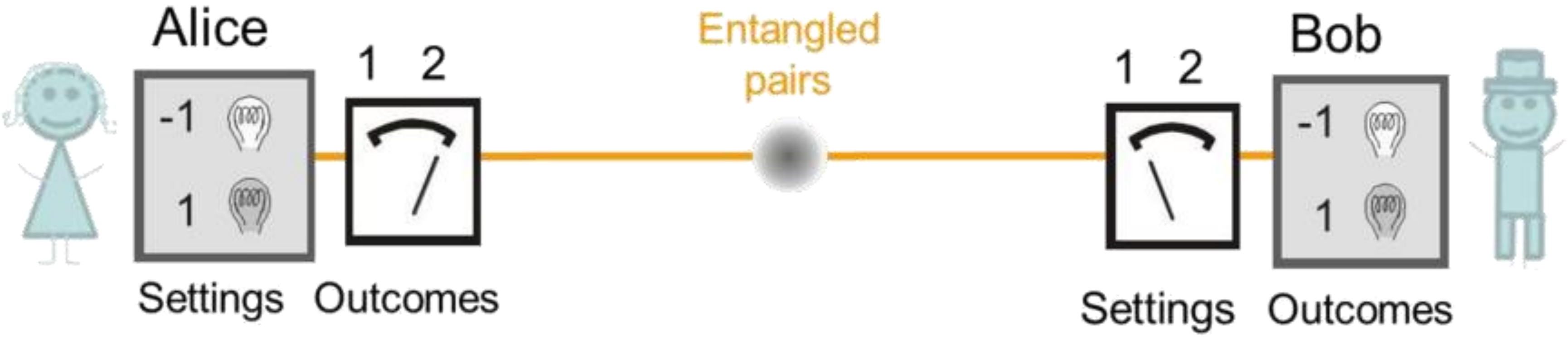
$$\sigma_A \sigma_B \geq \left| \frac{1}{2i} \langle [\hat{A}, \hat{B}] \rangle \right|$$

$$\sigma_A^2 \sigma_B^2 \geq \left| \frac{1}{2} \langle \{\hat{A}, \hat{B}\} \rangle - \langle \hat{A} \rangle \langle \hat{B} \rangle \right|^2 + \left| \frac{1}{2i} \langle [\hat{A}, \hat{B}] \rangle \right|^2$$

$$\frac{dA}{dt} = \frac{i}{\hbar}[H, A]$$

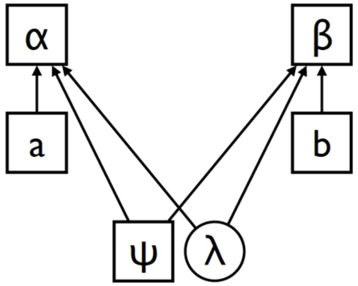
$$[a, a^\dagger] = 1 \quad \left[a^{(\mu)}(\mathbf{k}), a^{\dagger\,(\mu')}(\mathbf{k}')\right] = \delta_{\mathbf{k},\mathbf{k}'} \delta_{\mu,\mu'}$$

Bell-CHSH Experiment – Intro



$$A_0, A_1 = \pm 1 \qquad c_{ij} = E(A_i B_j) \qquad B_0, B_1 = \pm 1$$

LHV:



$P(A_0, A_1, B_0, B_1, \lambda)$ exists

$$B \equiv |c_{00} + c_{10} + c_{01} - c_{11}| \leq 2$$

QM:

$$C \stackrel{\text{def}}{=} M - VV^T,$$

$$M_{ij} \stackrel{\text{def}}{=} \frac{1}{2}\langle\{X_i, X_j\}\rangle \text{ and } V_i \stackrel{\text{def}}{=} \langle X_i \rangle$$

$$B \equiv |c_{00} + c_{10} + c_{01} - c_{11}| \leq 2\sqrt{2}$$

Paneru D., Cohen E., Fickler R., Boyd R.W., Karimi E., "Entanglement: Quantum or Classical?", Rep. Prog. Phys. 83, 064001 (2020).

A. Carmi, E. Cohen, "On the significance of the quantum mechanical covariance matrix", Entropy 20, 500 (2018).

We set to discover: Why are quantum correlations the way they are?

- Why $2\sqrt{2}$ and not 4?

- Models such as PR-boxes seem to have stronger correlations without violating relativistic causality, so what’s the problem?

  PR boxes and other post-quantum models violate either generalized uncertainty or a subtle form of relativistic causality (dubbed together RI)

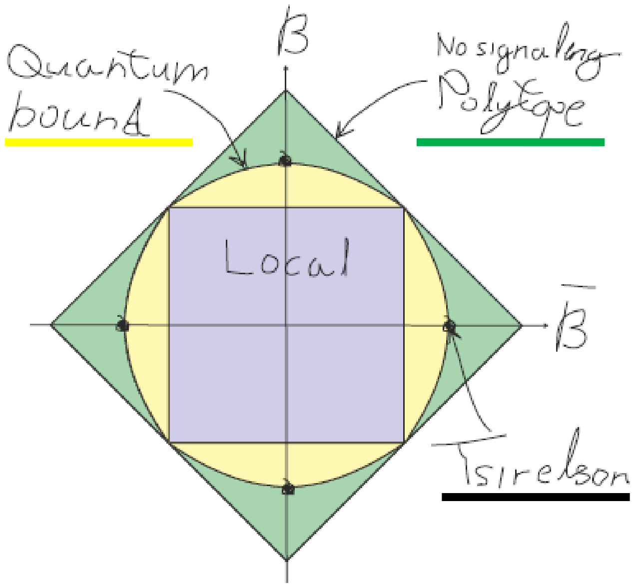
- Along the way we found a strong relation between uncertainty and nonlocality as well as new bounds on spatial and temporal correlations

Quantum bounds beyond Tsirelson's

Uffink: $B^2 + \bar{B}^2 \leq 8$

$$B \equiv \left| c_{00} + c_{10} + c_{01} - c_{11} \right|$$

$$\bar{B} \equiv \left| c_{00} + c_{10} - c_{01} + c_{11} \right|$$



$$\mathcal{L} \subsetneq \mathcal{Q} \subsetneq \mathcal{NS}$$

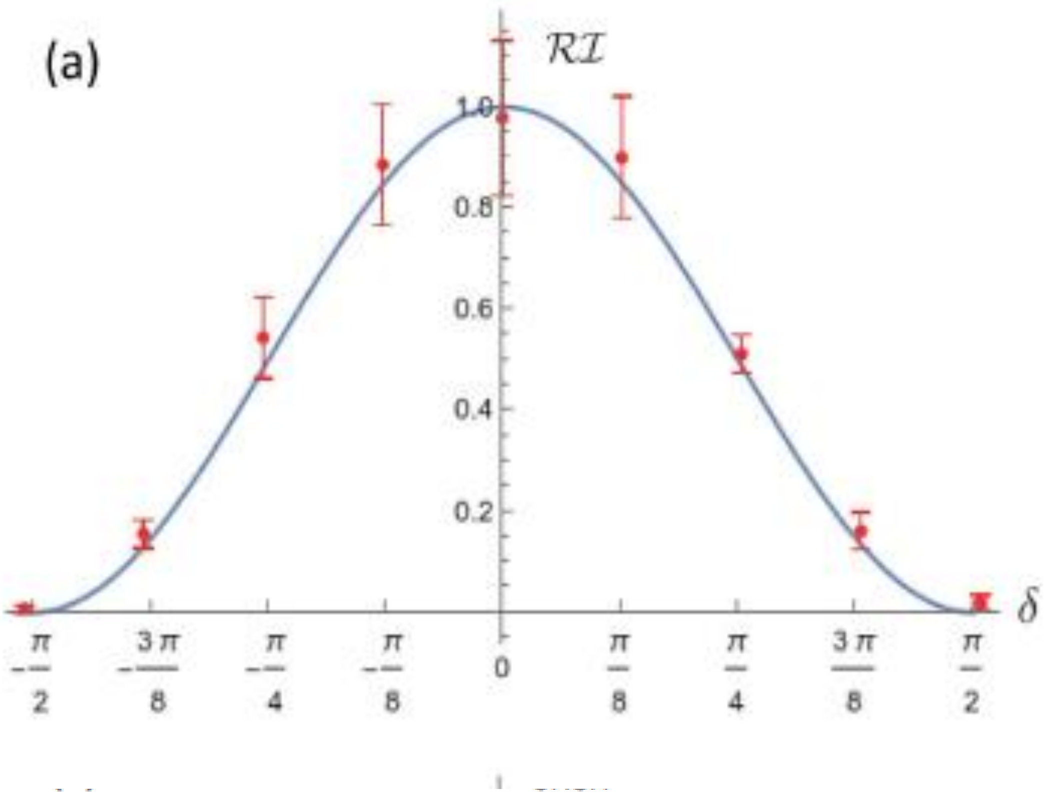
TLM: $\left| c_{00}c_{10} - c_{01}c_{11} \right| \leq \sqrt{1 - c_{00}{}^2}\sqrt{1 - c_{10}{}^2} + \sqrt{1 - c_{01}{}^2}\sqrt{1 - c_{11}{}^2}$

NPA: $Q \subseteq \ldots \subseteq Q^3 \subseteq Q^2 \subseteq Q^{1+AB} \subseteq Q^1$

Quantum bounds beyond Tsirelson's

Uffink: $B^2 + \overline{B}^2 \leq 8$

Examples of ours:



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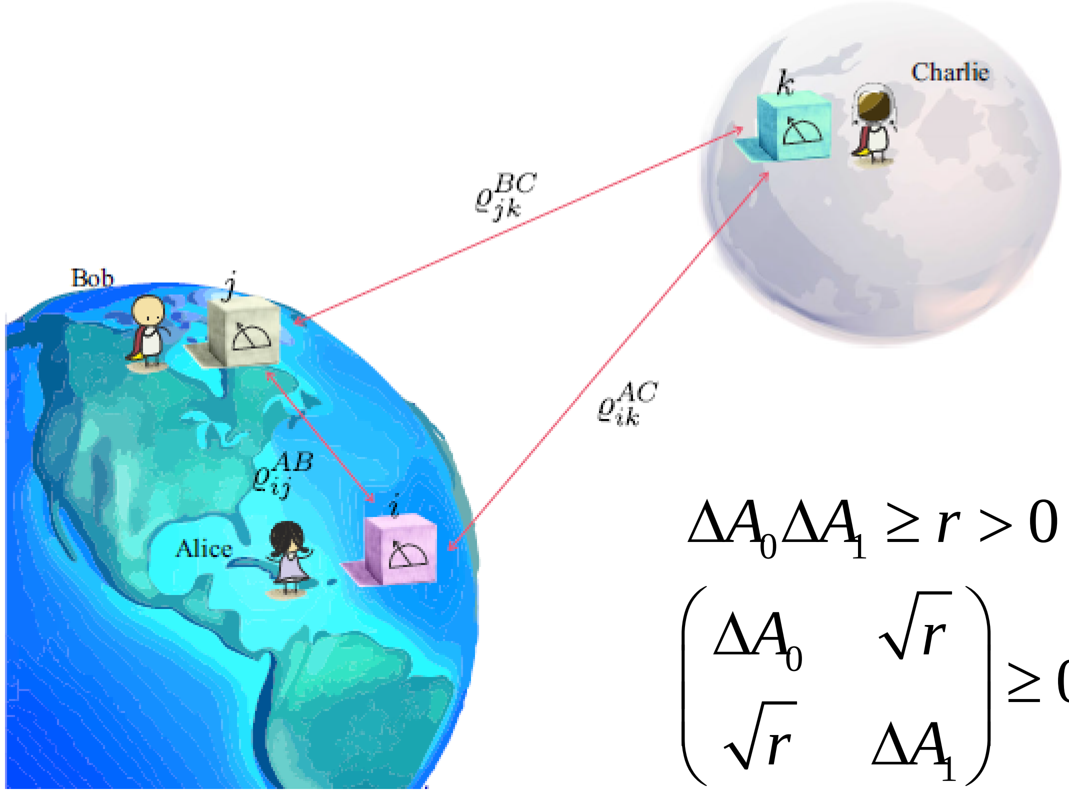
Relativistic Independence

Relativistic Independence

We proved that relativistic causality and generalized uncertainty alone yield all the aforementioned bounds, and even some new ones



$$\Delta A_0 \Delta A_1 \geq r > 0$$

$$\begin{pmatrix} \Delta A_0 & \sqrt{r} \\ \sqrt{r} & \Delta A_1 \end{pmatrix} \geq 0$$

$$\begin{pmatrix} \Delta^2_{C_k} & \varrho^{BC}_{jk} & \varrho^{AC}_{1k} & \varrho^{AC}_{0k} \\ \varrho^{BC}_{jk} & \Delta^2_{B_j} & \varrho^{AB}_{1j} & \varrho^{AB}_{0j} \\ \varrho^{AC}_{1k} & \varrho^{AB}_{1j} & \Delta^2_{A_1} & r_{jk} \\ \varrho^{AC}_{0k} & \varrho^{AB}_{0j} & r_{jk} & \Delta^2_{A_0} \end{pmatrix} \succeq 0$$

RI: $r_{jk} = r$, independent of j, k

Carmi A., Cohen E., *Sci. Adv.* 5, eaav8370 (2019)

Local correlations constrain nonlocal ones!

Local (Alice) **Nonlocal**

$$\begin{bmatrix} 1 & \eta_A \\ \eta_A^* & 1 \end{bmatrix} \succeq \begin{bmatrix} \varrho_{0j} \\ \varrho_{1j} \end{bmatrix} \begin{bmatrix} \varrho_{0j} & \varrho_{1j} \end{bmatrix}$$

Local (Bob) **Nonlocal**

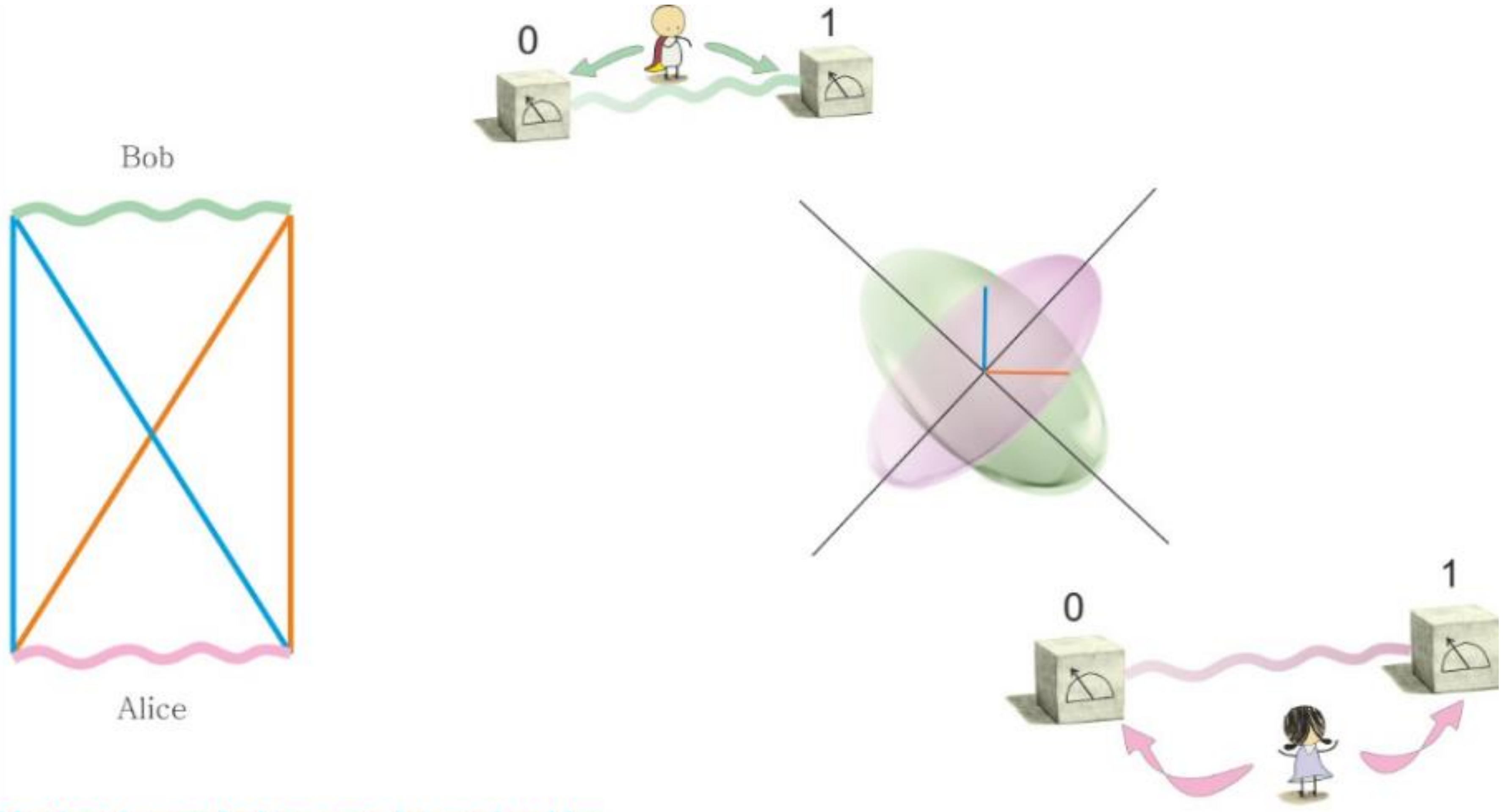
$$\begin{bmatrix} 1 & \eta_B \\ \eta_B^* & 1 \end{bmatrix} \succeq \begin{bmatrix} \varrho_{i0} \\ \varrho_{i1} \end{bmatrix} \begin{bmatrix} \varrho_{i0} & \varrho_{i1} \end{bmatrix}$$

$$\varrho_{ij} \stackrel{\text{def}}{=} \frac{\langle A_i B_j \rangle - \langle A_i \rangle \langle B_j \rangle}{\Delta_{A_i} \Delta_{B_j}}$$

$$\eta_A \stackrel{\text{def}}{=} \frac{\langle A_0 A_1 \rangle - \langle A_0 \rangle \langle A_1 \rangle}{\Delta_{A_0} \Delta_{A_1}}, \quad \eta_B \stackrel{\text{def}}{=} \frac{\langle B_0 B_1 \rangle - \langle B_0 \rangle \langle B_1 \rangle}{\Delta_{B_0} \Delta_{B_1}}$$

Local correlations determine the extent of nonlocal correlations!

New Approach to Quantum Nonlocality



Nonlocal correlations are determined by

Alice uncertainty relations Bob uncertainty relations

Relativity (causality): uncertainty relations are local ⟶ quantum correlations

New bounds on nonlocal correlations

- Quantum bounds on correlations are uncertainty principles
- We thus propose new, richer quantum bounds

$$|\varrho_{00}\varrho_{10} - \varrho_{01}\varrho_{11}| \leq \sum_{j=0,1} \sqrt{(1-\varrho_{0j}^2)(1-\varrho_{1j}^2) - \mathrm{Im}(\eta_A)^2}$$

$$|\varrho_{00}\varrho_{01} - \varrho_{10}\varrho_{11}| \leq \sum_{i=0,1} \sqrt{(1-\varrho_{i0}^2)(1-\varrho_{i1}^2) - \mathrm{Im}(\eta_B)^2}$$

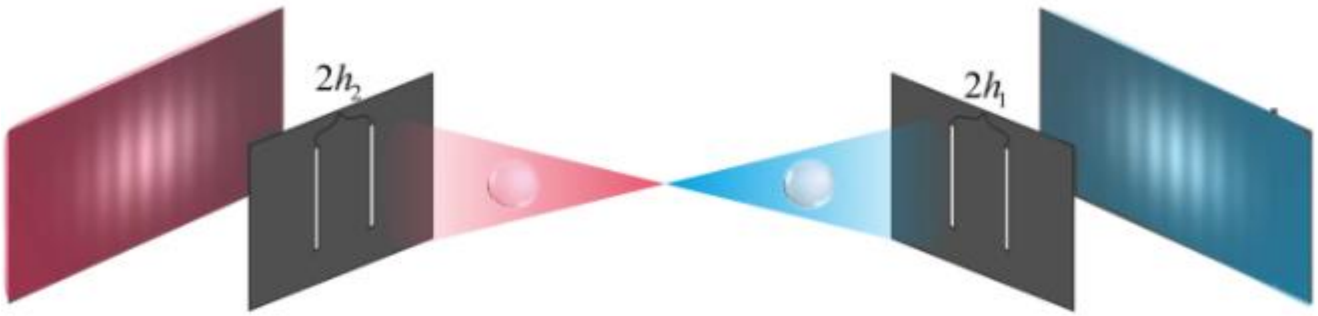
$$|\mathscr{B}| \leq \min\left\{\sqrt{2}\left(\sqrt{1+\mathrm{Re}(\eta_A)} + \sqrt{1-\mathrm{Re}(\eta_A)}\right),\ 2\sqrt{2}\sqrt{1-\mathrm{Im}(\eta_A)^2}\right\} \leq 2\sqrt{2}$$

$$|\mathscr{B}| \leq \min\left\{\sqrt{2}\left(\sqrt{1+\mathrm{Re}(\eta_B)} + \sqrt{1-\mathrm{Re}(\eta_B)}\right),\ 2\sqrt{2}\sqrt{1-\mathrm{Im}(\eta_B)^2}\right\} \leq 2\sqrt{2}$$

$$\varrho_{ij} \stackrel{\text{def}}{=} \frac{\langle A_i B_j\rangle - \langle A_i\rangle\langle B_j\rangle}{\Delta_{A_i}\Delta_{B_j}} \qquad \eta_A \stackrel{\text{def}}{=} \frac{\langle A_0 A_1\rangle - \langle A_0\rangle\langle A_1\rangle}{\Delta_{A_0}\Delta_{A_1}}, \quad \eta_B \stackrel{\text{def}}{=} \frac{\langle B_0 B_1\rangle - \langle B_0\rangle\langle B_1\rangle}{\Delta_{B_0}\Delta_{B_1}}$$

$$\mathscr{B} \stackrel{\text{def}}{=} \varrho_{00} + \varrho_{10} + \varrho_{01} - \varrho_{11}$$

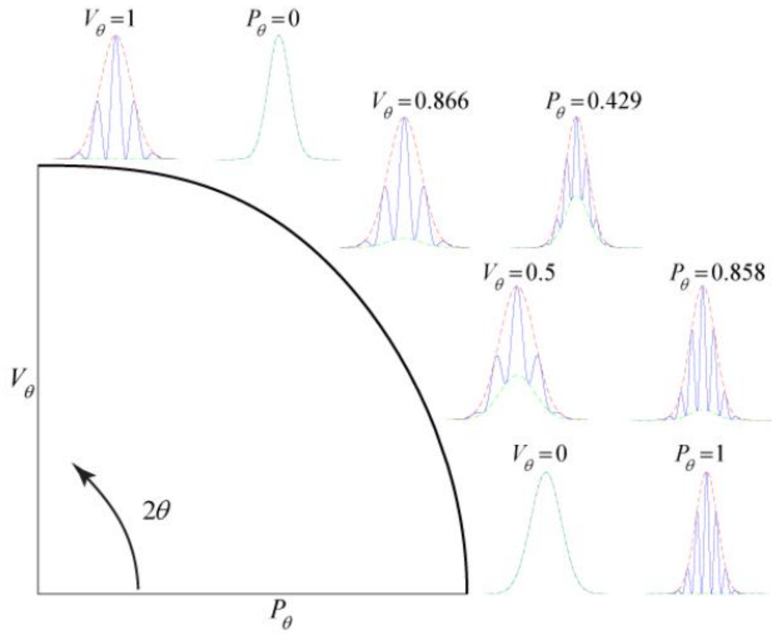
Complementarity relations



$$V^2 + D^2 = 1$$

$$P_\theta^2 + V_\theta^2 \leq 1$$

$$\left(\frac{\mathscr{B}}{2\sqrt{2}}\right)^2 + |\eta|^2 \leq 1$$



Peled, B. Y., Te'eni, A., Georgiev, D., Cohen, E., & Carmi, A. (2020). Double Slit with an Einstein–Podolsky–Rosen Pair. *Applied Sciences*, *10*(3), 792.

Georgiev D., Bello L., Carmi A., Cohen E., "Quantum complementarity of one- and two-particle visibilities: direct evaluation method for continuous variables", Phys. Rev. A 103, 062211 (2021).

Further Generalization

$$\forall i, j. \quad A_i^\dagger \neq A_i, \quad B_j^\dagger \neq B_j, \quad [A_i, B_j] \neq 0$$

$$C(X_i, X_j) = \frac{\langle X_i X_j^\dagger \rangle - \langle X_i \rangle \langle X_j^\dagger \rangle}{\Delta_{X_i} \Delta_{X_j}} \qquad C \succeq 0$$

$$|\mathscr{B}| = \sqrt{\frac{1}{4}\left(\mathscr{B} + \mathscr{B}^\dagger\right)^2 - \frac{1}{4}\left(\mathscr{B} - \mathscr{B}^\dagger\right)^2} \leq \sqrt{2}\left[\sqrt{1+\eta} + \sqrt{1-\eta}\right] \leq 2\sqrt{2}$$

How to measure? Weak measurements!

The quantum formalism is so restricting that even correlations between non-Hermitian, “signaling” operators cannot go beyond $2\sqrt{2}$

A. Carmi A., Y. Herasymenko, E. Cohen, K. Snizhko, NJP 21, 073032 (2019)

A fundamental quantitative relation

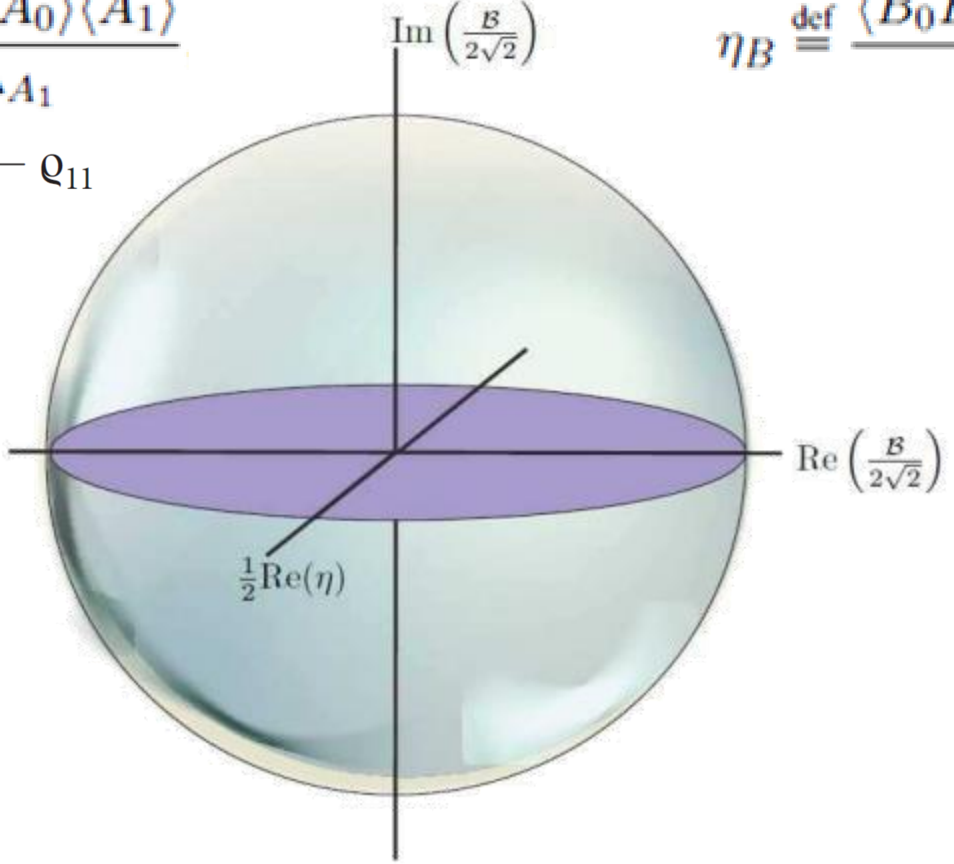
local **nonlocal** **signaling**

$$\left(\frac{\mathrm{Re}(\eta)}{2}\right)^2 + \left(\frac{\mathrm{Re}(\mathcal{B})}{2\sqrt{2}}\right)^2 + \left(\frac{\mathrm{Im}(\mathcal{B})}{2\sqrt{2}}\right)^2 \leqslant 1$$

$$\eta_A \stackrel{\text{def}}{=} \frac{\langle A_0 A_1\rangle - \langle A_0\rangle\langle A_1\rangle}{\Delta_{A_0}\Delta_{A_1}}$$

$$\eta_B \stackrel{\text{def}}{=} \frac{\langle B_0 B_1\rangle - \langle B_0\rangle\langle B_1\rangle}{\Delta_{B_0}\Delta_{B_1}}$$

$$\mathscr{B} \stackrel{\text{def}}{=} \varrho_{00} + \varrho_{10} + \varrho_{01} - \varrho_{11}$$



A. Carmi A., Y. Herasymenko, E. Cohen, K. Snizhko, NJP 21, 073032 (2019)

Temporal correlations: Leggett-Garg inequalities

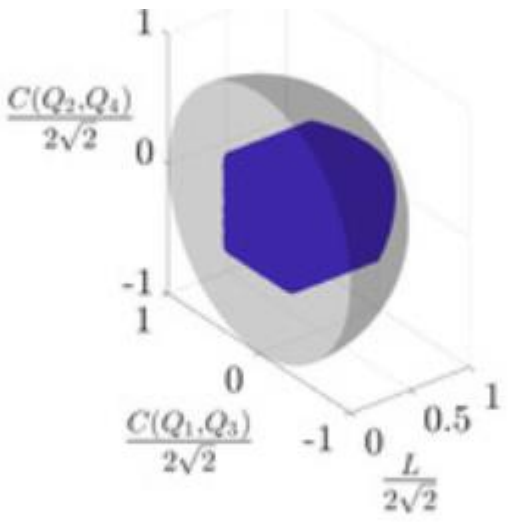
Theorem 1. *Elaborate Leggett–Garg-like inequality.*

Given four consecutive measurements, we define the generalized LGI parameter as

$$L = |C(Q_1, Q_2) + C(Q_2, Q_3) + C(Q_3, Q_4) - C(Q_1, Q_4)|.$$

The following holds

$$L \leq 2\sqrt{1 + \sqrt{1 - \max\left\{C(Q_1, Q_3)^2, C(Q_2, Q_4)^2\right\}}}.$$

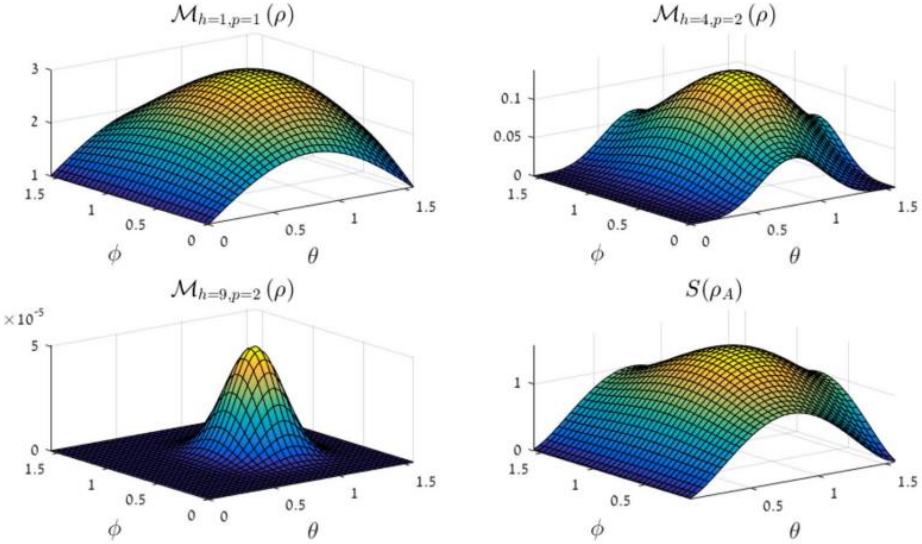


Ben Porath, D. Cohen, E., Leggett-Garg-like Inequalities from a Correlation Matrix Construction, Quantum Reports 5, 398-406 (2023).

Correlation Minor Norm and entanglement

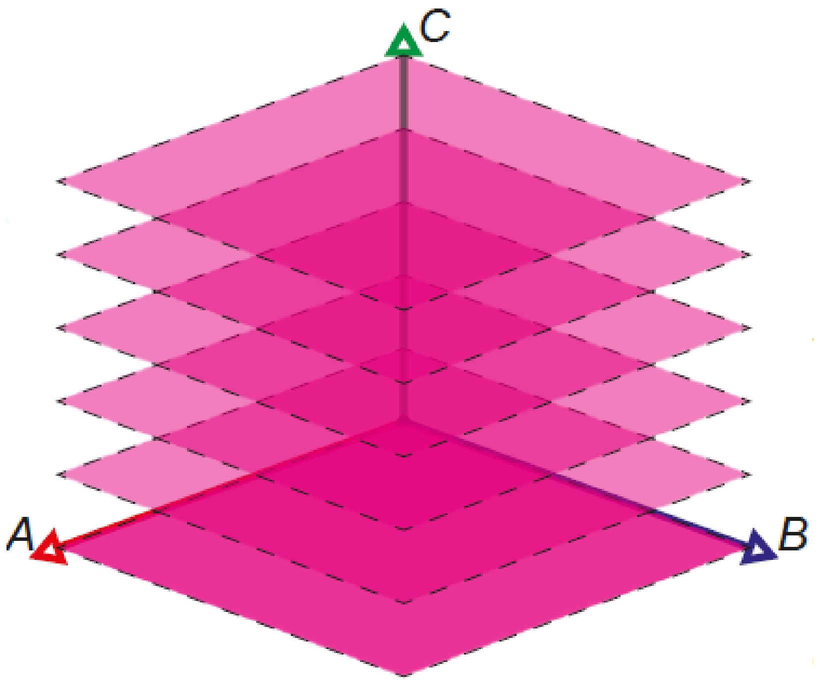
$$\mathcal{M}_{h,p}(\vec{\sigma}) = \left(\sum_{|I|=h} \prod_{k \in I} \sigma_k^p \right)^{\frac{1}{p}}$$

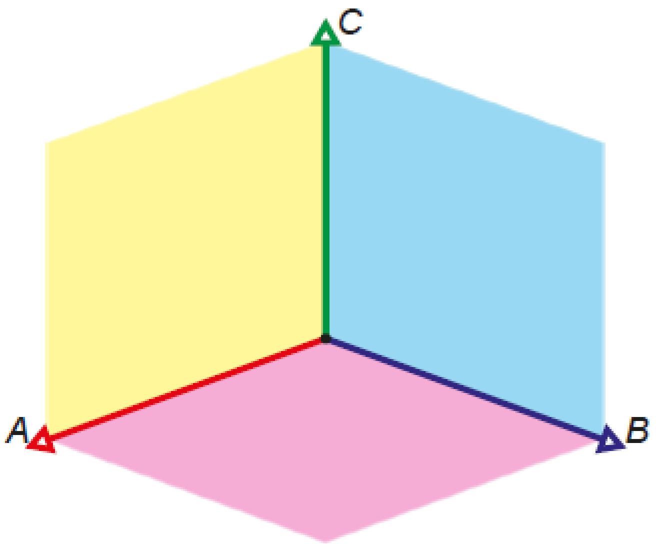
- The CMNs can be used to detect entanglement: ∃ some bound B such that $\mathcal{M}_{h,p} \leq B$ for all separable states
- For example, $\mathcal{M}_{1,1}(\vec{\sigma}) > 1$ implies that ρ is entangled
- But for other h and p values it yields a better characterization



Peled B.Y., Te'eni A., Carmi A., Cohen E., "Correlation Minor Norm as a Detector and Quantifier of Entanglement", Sci. Rep. 11, 2849 (2021).

Correlation Minor Norm – Multipartite Case





$$\mathcal{M}_{h,p=\infty} \leq \alpha \left(\frac{\beta}{h-1} \right)^{h-1}$$

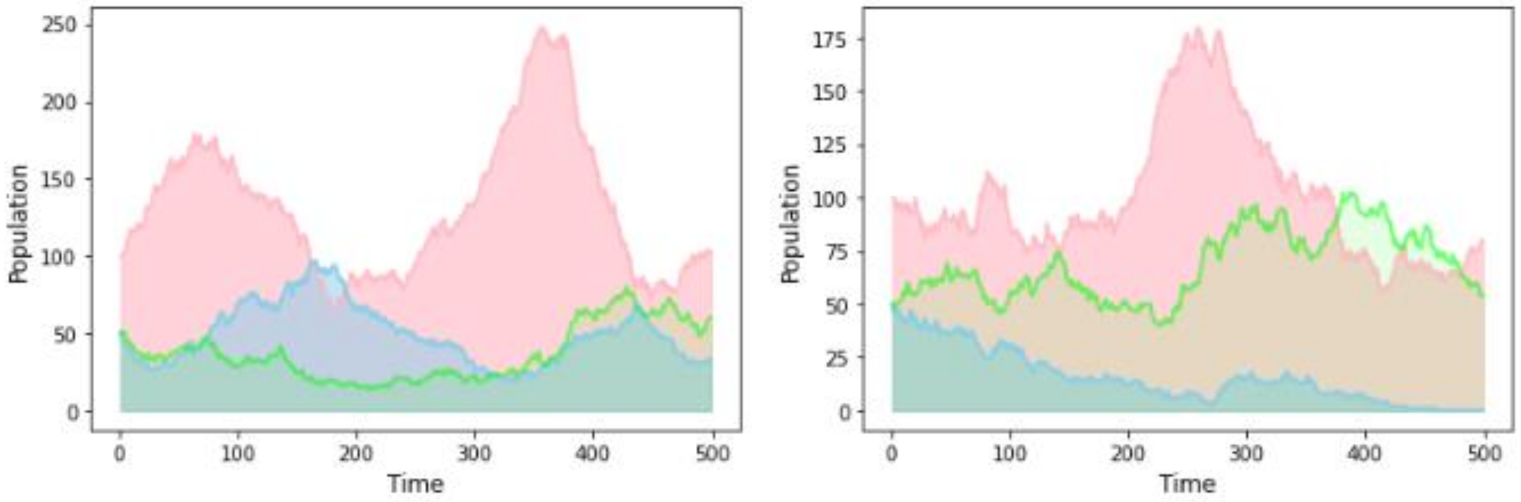
$$\mathcal{M}_{h,p=1} \leq S_h \left(\alpha, \frac{\beta}{d^2-1}, \dots, \frac{\beta}{d^2-1} \right)$$

Wherein:

$$\alpha = \prod_i \frac{1}{\sqrt{d_i}}, \ \beta = \prod_i \sqrt{\frac{d_i - 1}{d_i}}$$

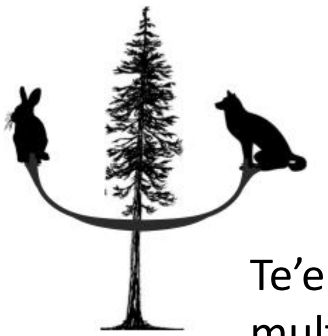
Lenny, R., Te'eni, A., Peled, B. Y., Carmi, A., & Cohen, E. (2023). Multipartite entanglement detection via correlation minor norm. Quantum Information Processing, 22(7), 292.

Networks and "population dynamics"



$$(\mathscr{B}cv_1)^2 + (\mathscr{B}cv_2)^2 \leq 8$$

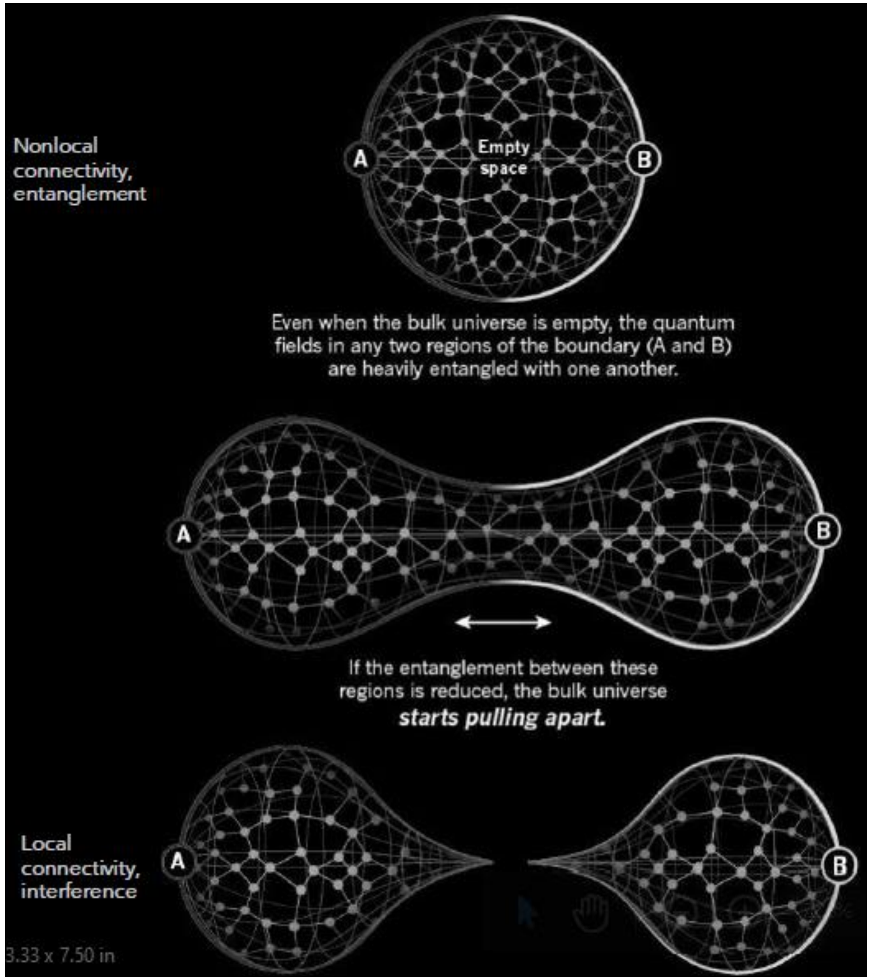
$$\sum_{k=1}^{n} |\mathscr{B}cv_k| \leq 2\sqrt{2n}.$$



Te'eni, A., Peled, B. Y., Cohen, E., Carmi, A. Study of entanglement via a multi-agent dynamical quantum game, *Plos One* 18, e0280798 (2023).

Generalized Bekenstein Bound

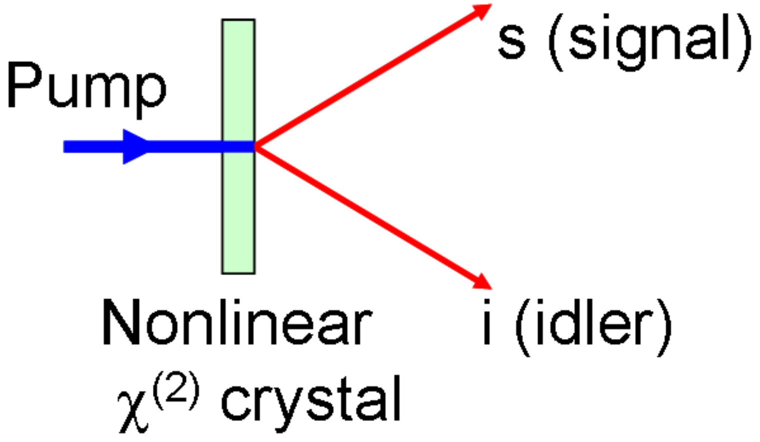
$$H \leq \frac{2\pi R E}{\hbar c \ln 2} \quad \Longrightarrow \quad \mathcal{M}_{h,p}\left(\sqrt{CC^{\dagger}}\right)^{2} \leq \mathcal{M}_{h,p}(P)\,\mathcal{M}_{h,p}(M'_{x})$$



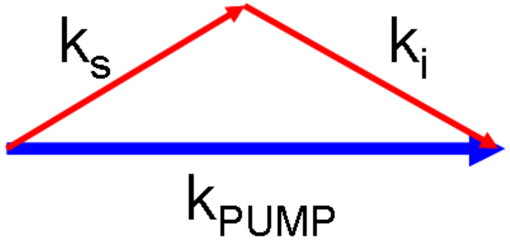
Recent Experiments

Generation of entangled photon pairs

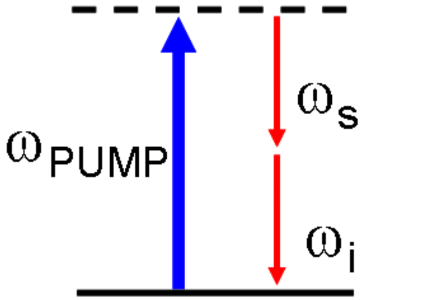
Spontaneous
Parametric
Downconversion



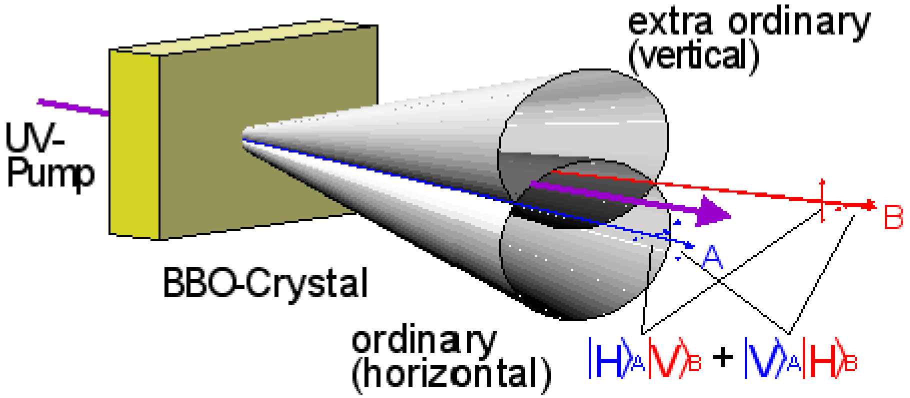
Momentum Conservation



Energy conservation



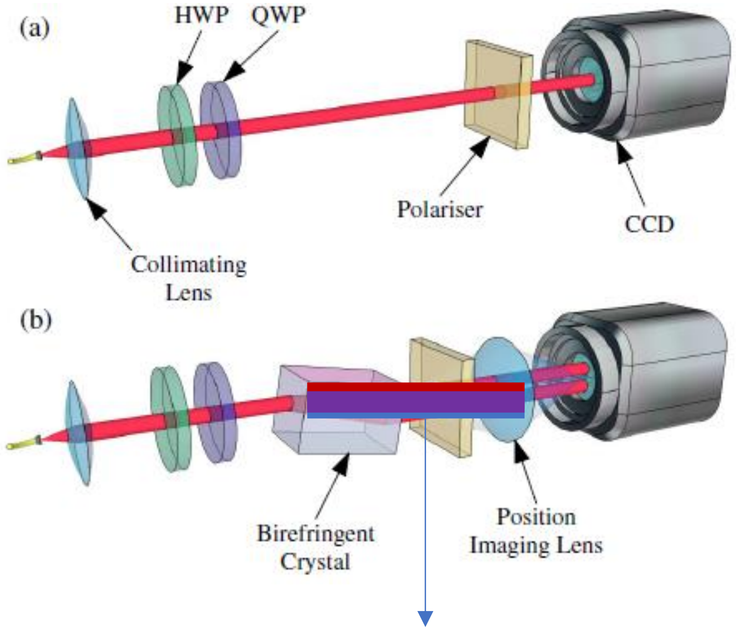
$$\varphi_{PUMP} = \varphi_s + \varphi_i$$



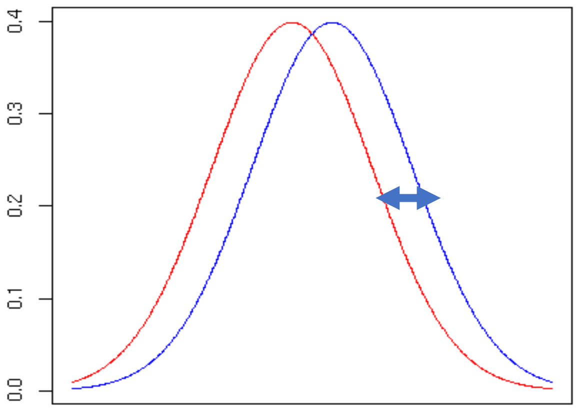
Weak measurements

Pre- and Post-selection

Pre- and Post-selection+
Weak Measurement



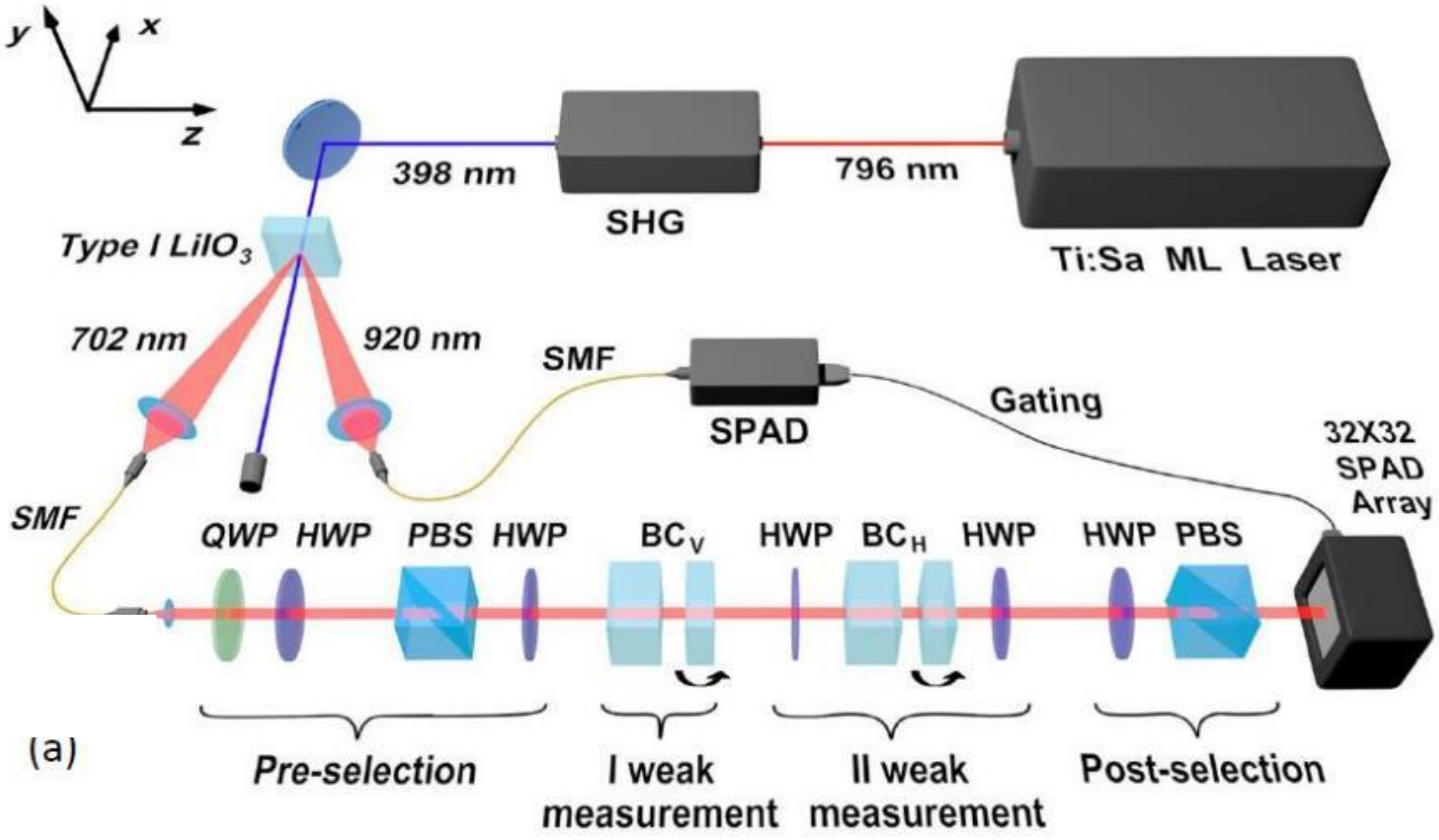
Strong or Weak depends on the spatial overlap between the beams



$$A_w \equiv \frac{\langle \Psi_{fin} | A | \Psi_{in} \rangle}{\langle \Psi_{fin} | \Psi_{in} \rangle}$$

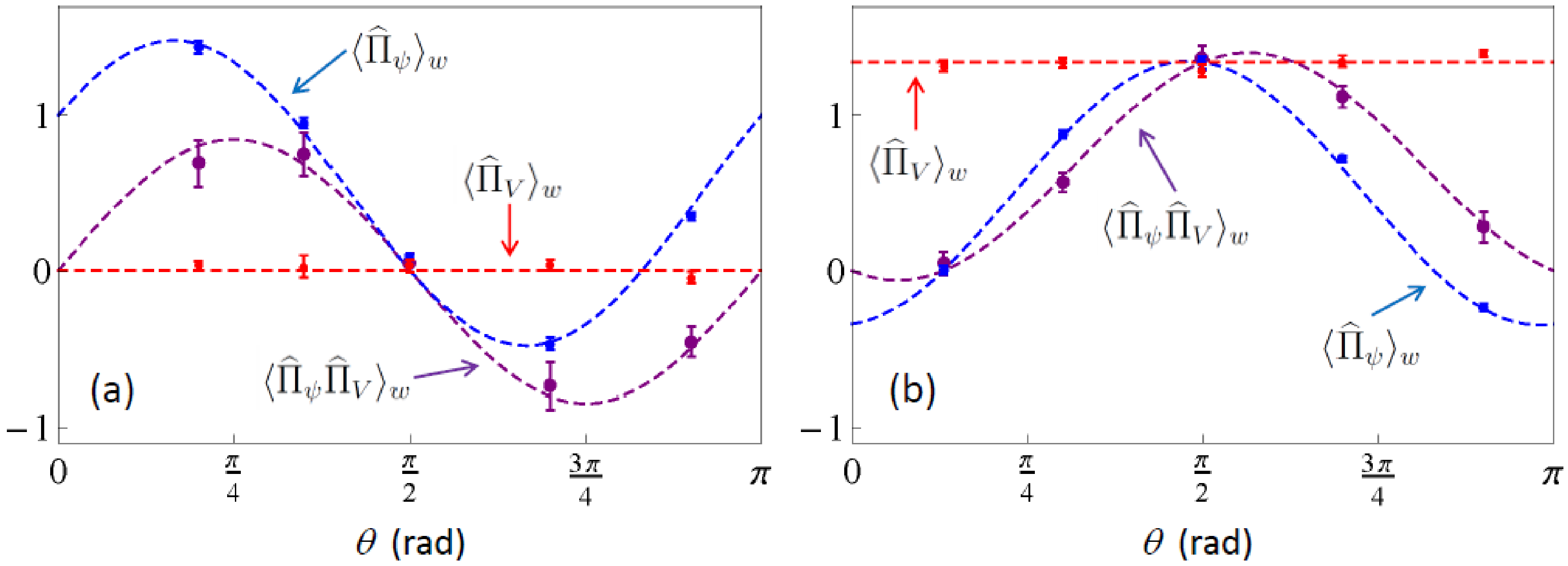
Reviews: J. Dressel *et al.*, *Rev. Mod. Phys.* **86**, 307 (2014), B. Tamir and E. Cohen, *Quanta* **2**, 7 (2013)

Laboratory realization of Sequential weak measurements



F. Piacentini et al., Phys. Rev. Lett. 117, 170402 (2016)

Sequential weak measurements - Results



$$|\psi_i\rangle = 0.588|H\rangle + 0.809|V\rangle$$

$$|\psi_f\rangle = |H\rangle$$

$$|\psi_i\rangle = 0.509|H\rangle + 0.861|V\rangle$$

$$|\psi_f\rangle = -0.397|H\rangle + 0.918|V\rangle$$

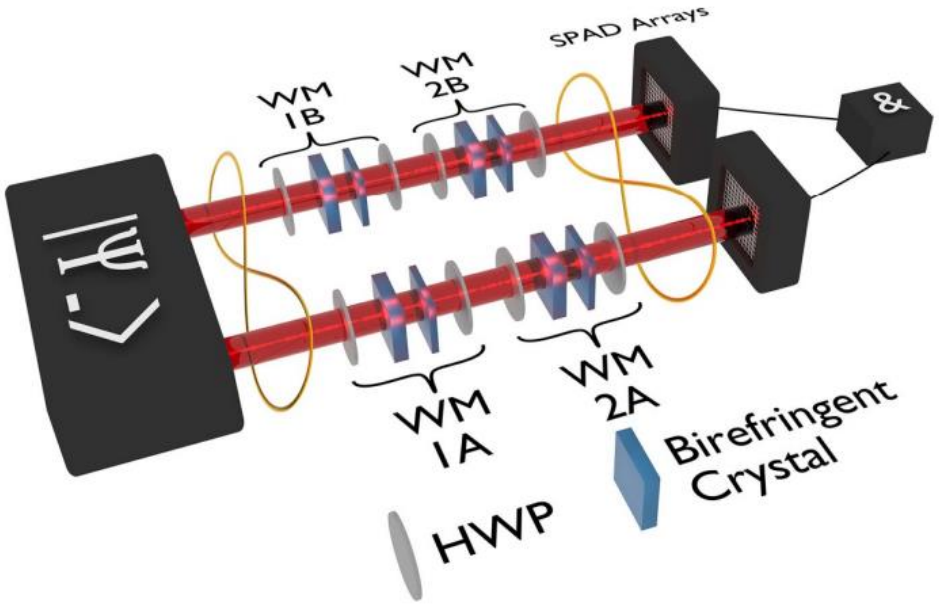
$$\langle A\rangle_w \equiv \frac{\langle\Psi_{fin}|A|\Psi_{in}\rangle}{\langle\Psi_{fin}|\Psi_{in}\rangle}$$

$$\widehat{\Pi}_\psi = |\psi\rangle\langle\psi|$$

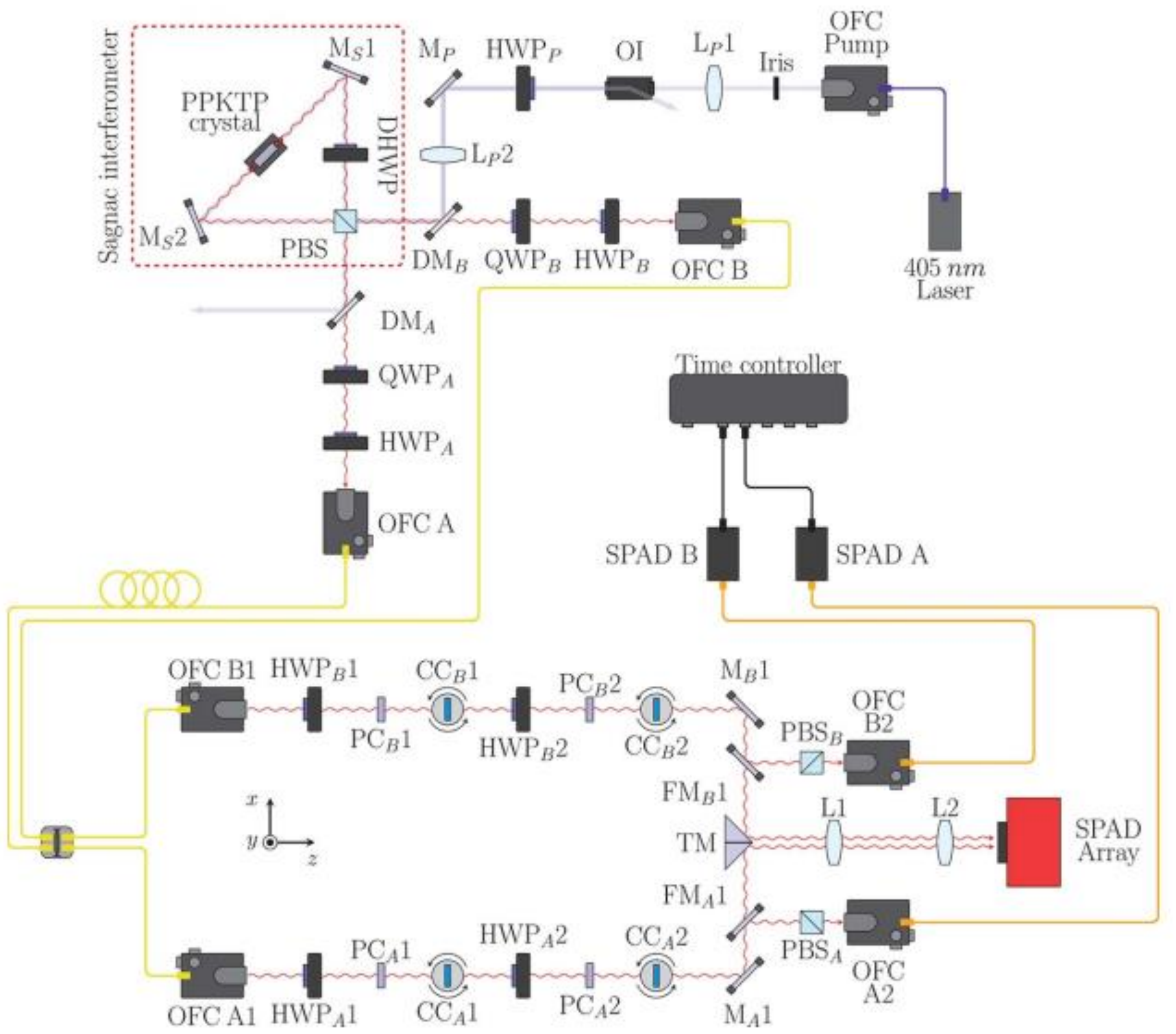
$$|\psi\rangle = \cos\theta|H\rangle + \sin\theta|V\rangle).$$

F. Piacentini et al., Phys. Rev. Lett. 117, 170402 (2016)

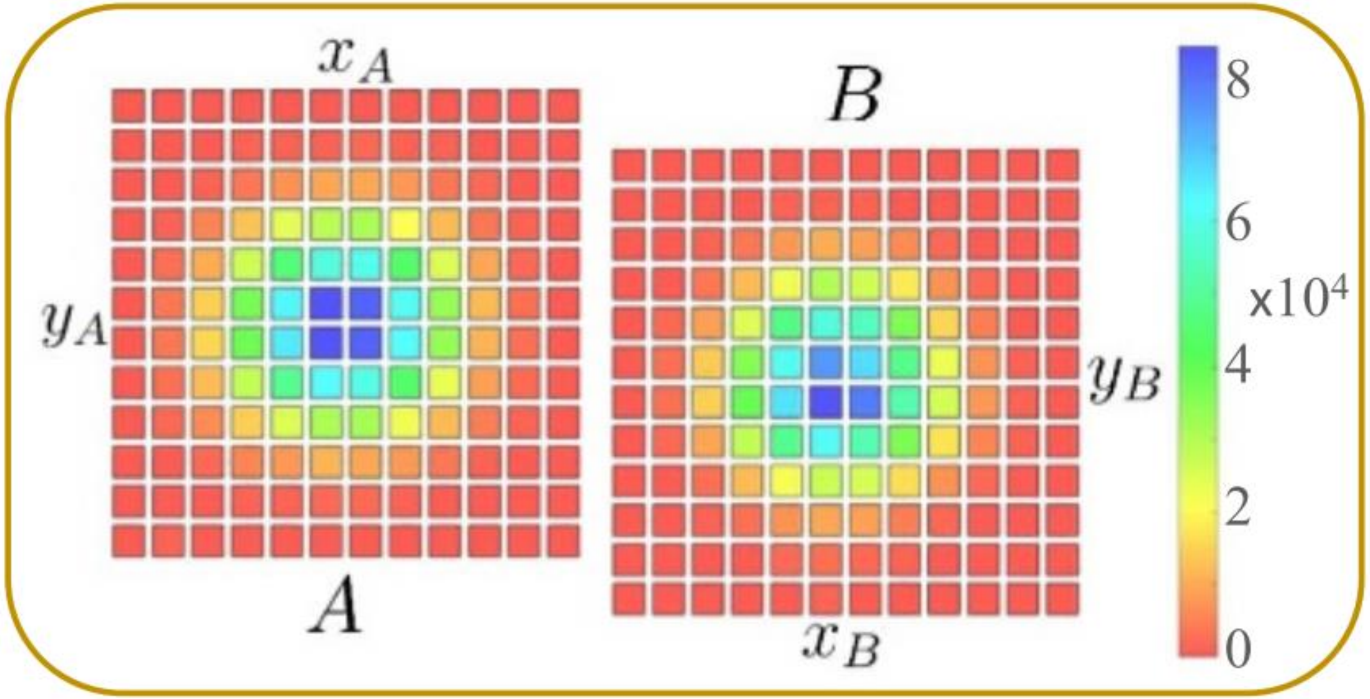
Measurements of the Bell parameter for each pair

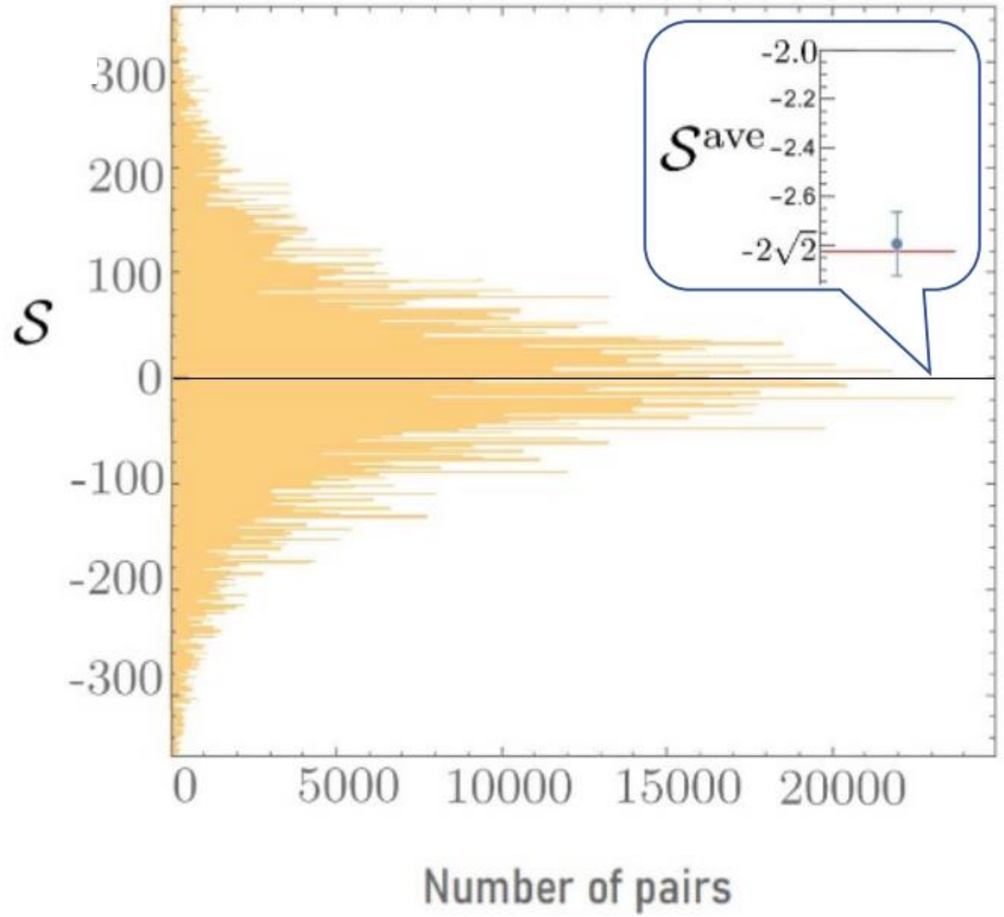


Measurements of the Bell parameter for each pair

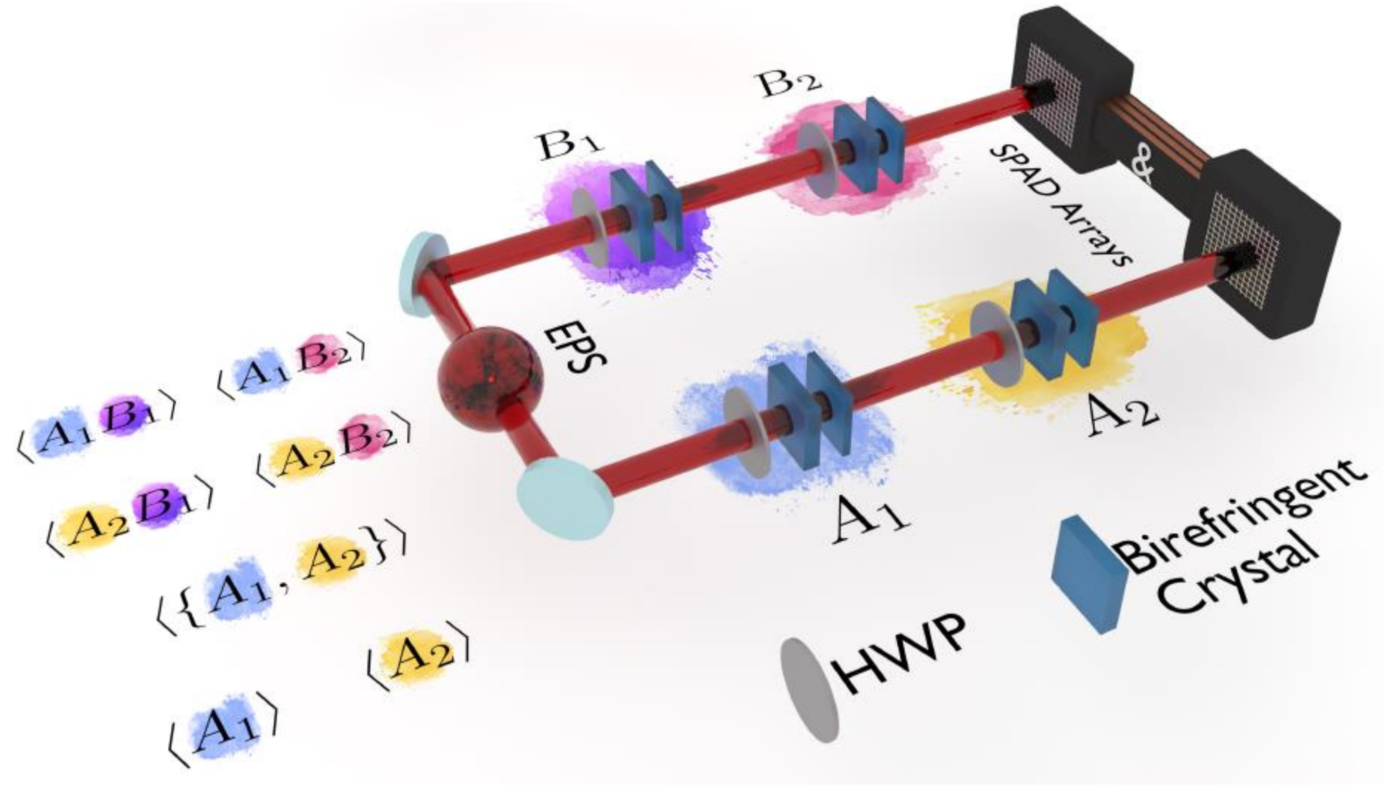


Measurements of the Bell parameter for each pair

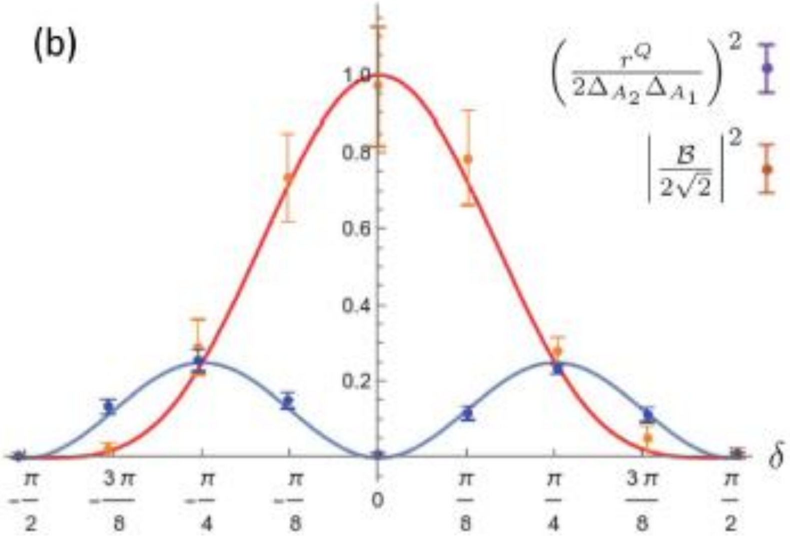




Testing relativistic independence



$$\mathcal{RI} = \left|\frac{\mathcal{B}}{2\sqrt{2}}\right|^2 + \left(\mathrm{Re}\left[\frac{r}{2\Delta_{A_2}\Delta_{A_1}}\right]\right)^2 \leq 1$$



$$r^Q = \frac{\langle \hat{A}_1 \hat{A}_2 + \hat{A}_2 \hat{A}_1 \rangle}{2} - \langle \hat{A}_1 \rangle \langle \hat{A}_2 \rangle$$

Thank you

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Positions available!