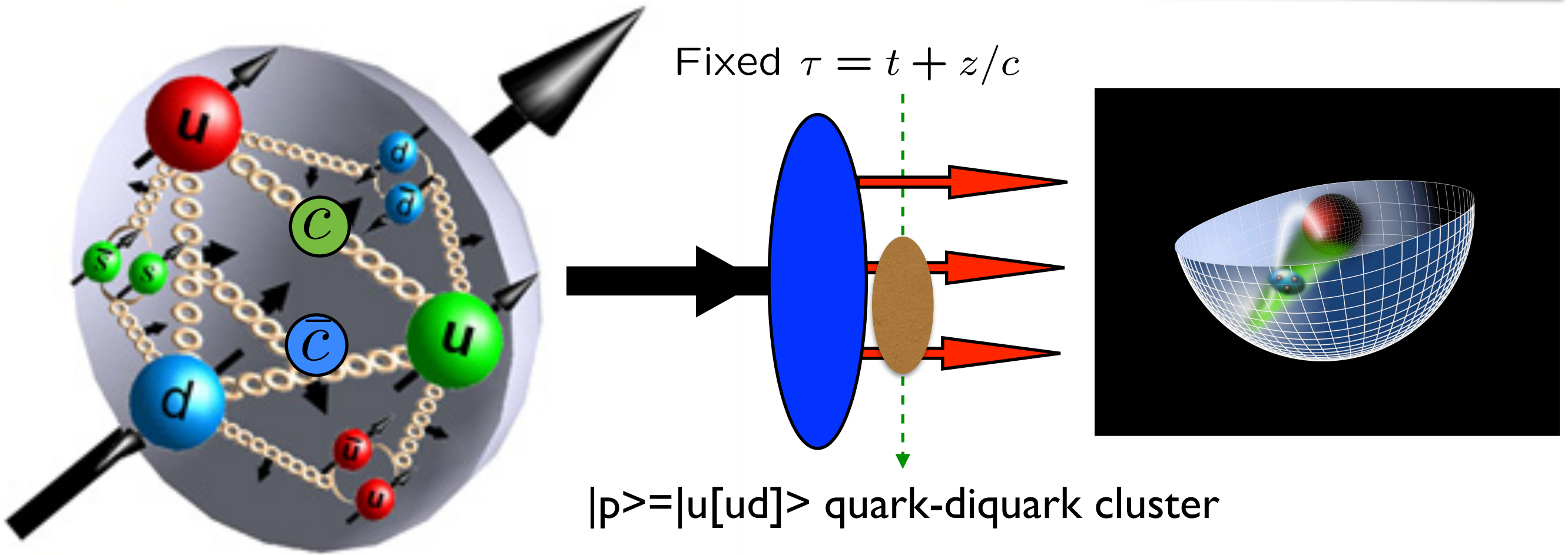


# New Perspectives for Hadron Spectroscopy and Dynamics and the QCD Coupling from Color-Confining Holographic Light-Front QCD



with Guy de Tèramond, Hans Günter Dosch, Cèdric Lorcè, Alexandre Deur, and Joshua Erlich

13th International Conference  
On New Frontiers in Physics  
Half a Century of Quantum Chromodynamics  
Crete  
August 26-September 4 2024

Stan Brodsky

SLAC NATIONAL ACCELERATOR LABORATORY



August 29, 2024

# *Challenge: Compute Hadron Structure, Spectroscopy, and Dynamics from QCD!*

- **Color Confinement**
- **Origin of the QCD Mass Scale**
- **Meson and Baryon Spectroscopy**
- **Exotic States: Tetraquarks, Pentaquarks, Gluonium,**
- **Universal Regge Slopes:  $n$ ,  $L$ , Mesons and Baryons**
- **Almost Massless Pion: GMOR Chiral Symmetry Breaking**  
$$M_\pi^2 f_\pi^2 = -\frac{1}{2}(m_u + m_d) \langle \bar{u}u + \bar{d}d \rangle + \mathcal{O}((m_u + m_d)^2)$$
- **QCD Coupling at all Scales  $\alpha_s(Q^2)$**
- **Eliminate Scale Uncertainties and Scheme Dependence**
- **BLM/PMC (Principle of Maximum Conformality)**



# New Insights into Color-Confining QCD Dynamics

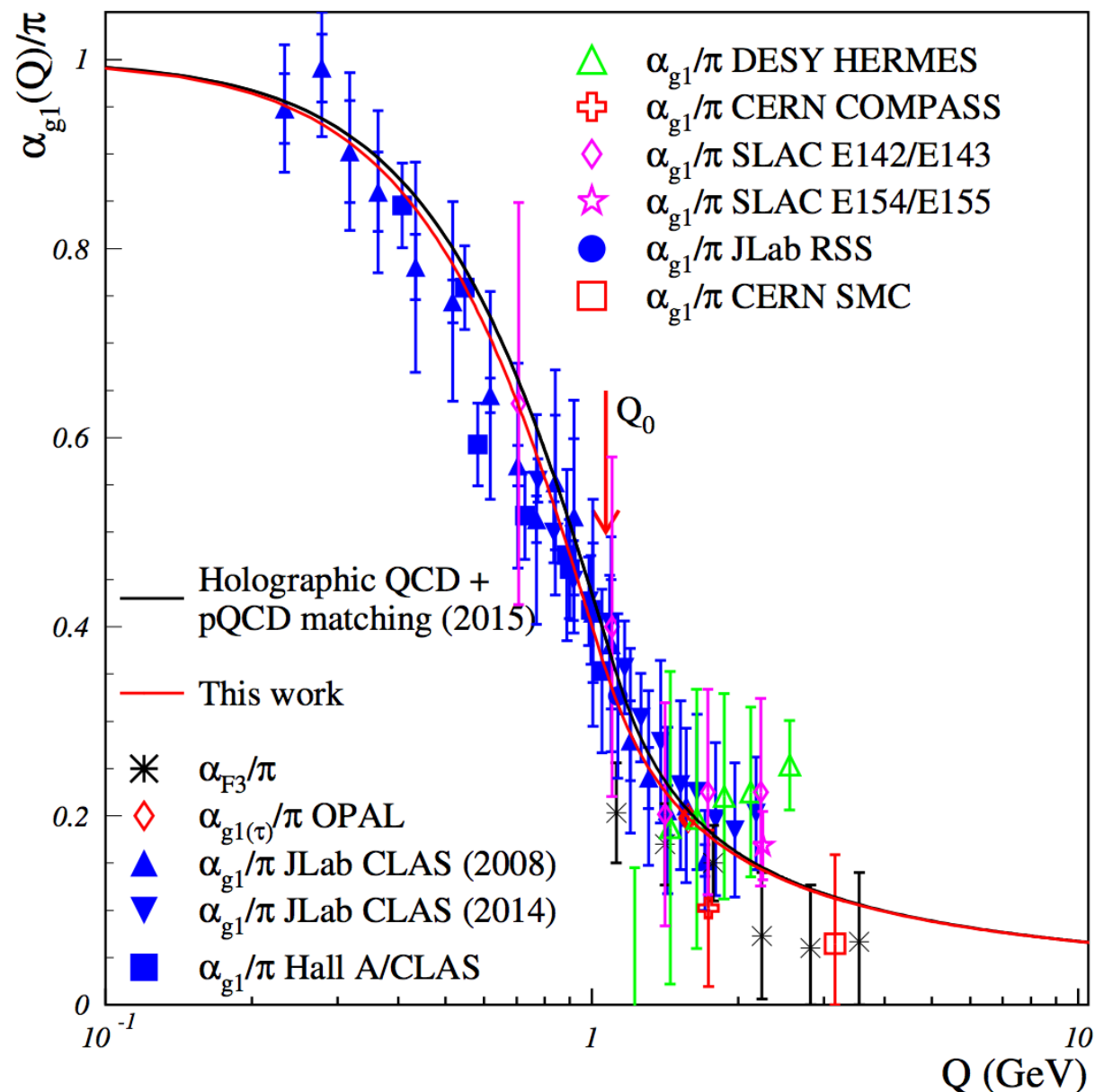
- Light-Front Holography
- Color Confinement from AdS Space-Time
- QCD Coupling at all Scales!  $\alpha_s(Q^2)$
- Light-Front: Frame-Independent and Causal
- Hadron Spectroscopy: Meson-Baryon-Tetraquark  
Supersymmetry
- Standard PQCD Results: Counting Rules, Factorization  
Theorems: all preserved in Short-Distance Regime

Dirac: Front Form

*Supersymmetric Features of Hadron Physics  
from Superconformal Algebra  
and Light-Front Holography*

Fixed  $\tau = t + z/c$

# Running Coupling from AdS/QCD



Bjorken sum rule:

$$\frac{\alpha_{g_1}(Q^2)}{\pi} = 1 - \frac{6}{g_A} \int_0^1 dx g_1^{p-n}(x, Q^2)$$

Effective coupling in LFHQCD  
(valid at low- $Q^2$ )

$$\alpha_{g_1}^{AdS}(Q^2) = \pi \exp(-Q^2/4\kappa^2)$$

Imposing continuity for  $\alpha$   
and its first derivative

A. Deur, S.J. Brodsky, G.F. de Téramond,  
Phys. Lett. B 750, 528 (2015); J. Phys. G 44, 105005 (2017).

**Analytic, defined at all scales, IR Fixed Point**



Fixed  $\tau = t + z/c$

# Light-Front QCD

Physical gauge:  $A^+ = 0$

Exact frame-independent formulation of nonperturbative QCD!

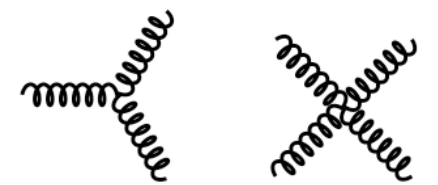
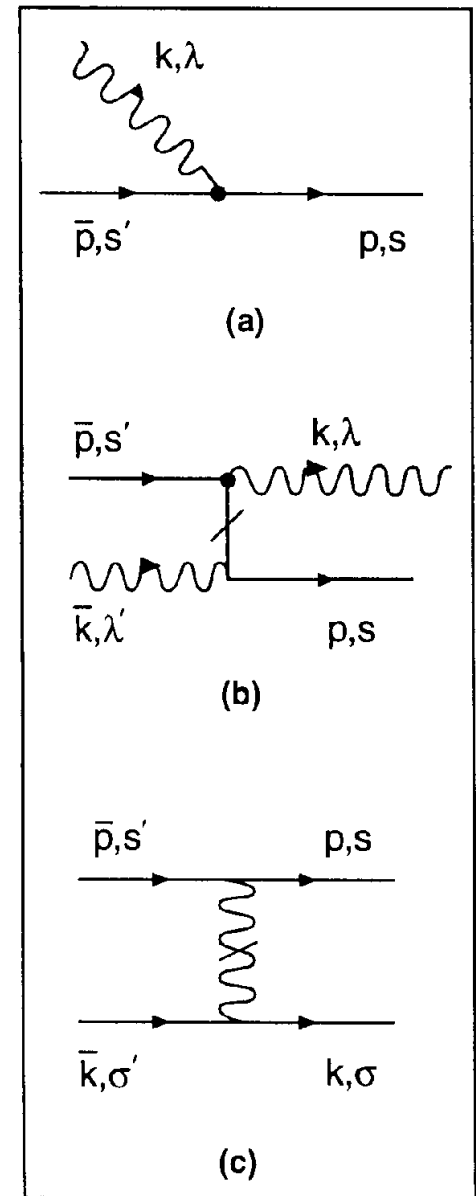
$$L^{QCD} \rightarrow H_{LF}^{QCD}$$

$$H_{LF}^{QCD} = \sum_i \left[ \frac{m^2 + k_{\perp}^2}{x} \right]_i + H_{LF}^{int}$$

$H_{LF}^{int}$ : Matrix in Fock Space

$$H_{LF}^{QCD} |\Psi_h\rangle = \mathcal{M}_h^2 |\Psi_h\rangle$$

$$|p, J_z\rangle = \sum_{n=3} \psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) |n; x_i, \vec{k}_{\perp i}, \lambda_i\rangle$$



$H_{LF}^{int}$

Eigenvalues and Eigensolutions give Hadronic Spectrum and Light-Front wavefunctions

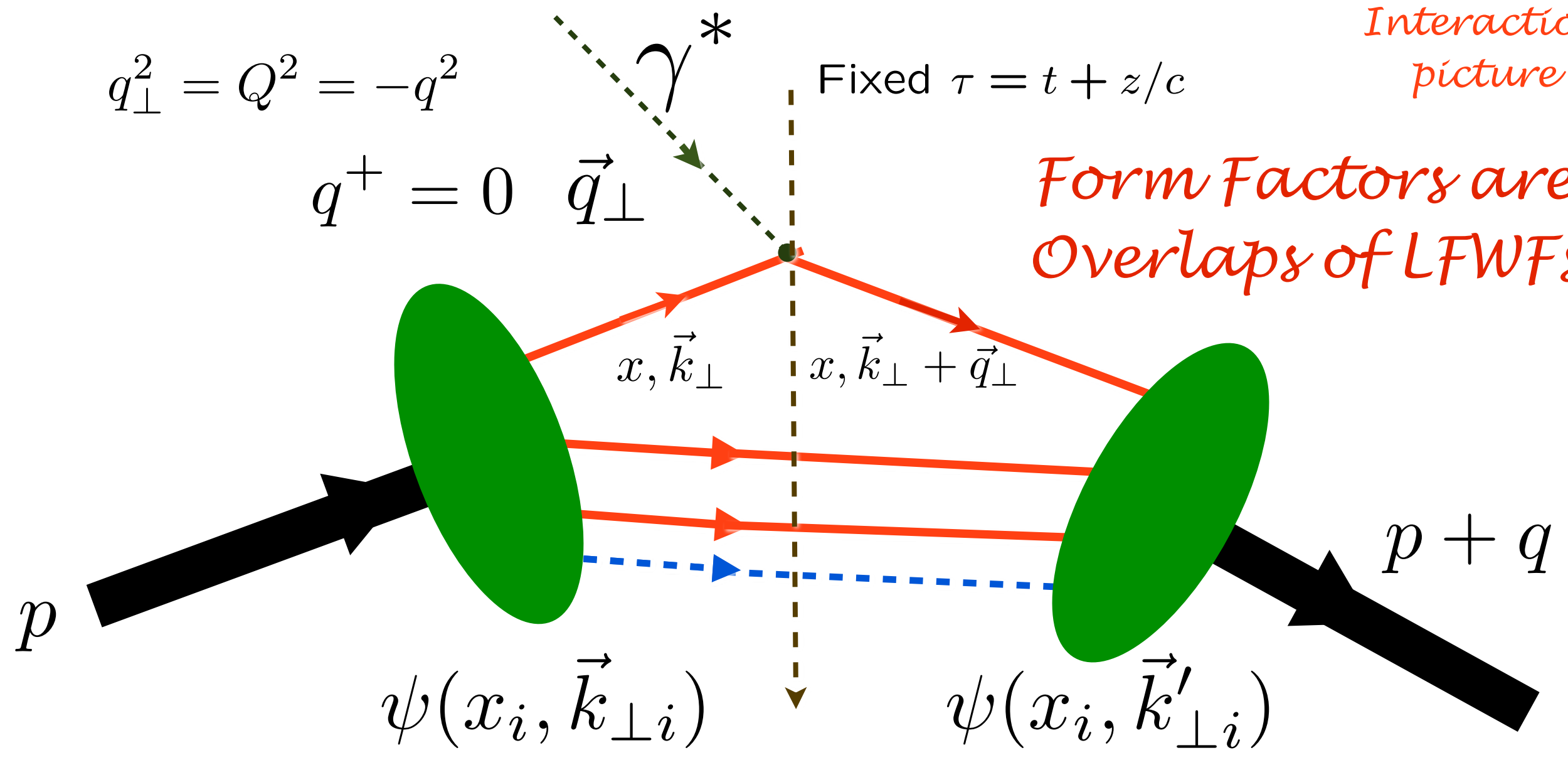
**LFWFs: Off-shell in P- and invariant mass**

Solve nPQCD by matrix diagonalization: Hornbostel, Pauli, sjb

Front Form

Interaction picture

Form Factors are Overlaps of LFWFs



struck  $\vec{k}'_{\perp i} = \vec{k}_{\perp i} + (1 - x_i)\vec{q}_{\perp}$

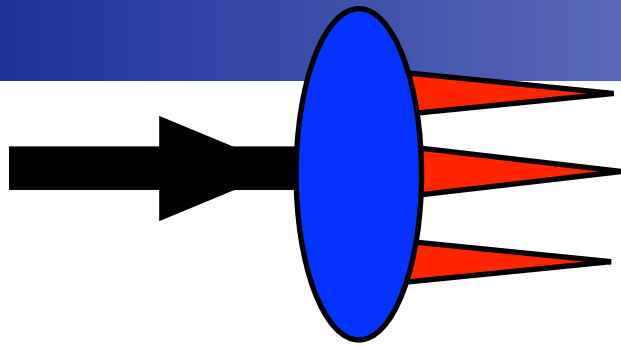
spectators  $\vec{k}'_{\perp i} = \vec{k}_{\perp i} - x_i\vec{q}_{\perp}$

Drell & Yan, West  
Exact LF formula!

Drell, sjb

causal, frame-independent

Light-Front Wavefunctions  
underly hadronic observables

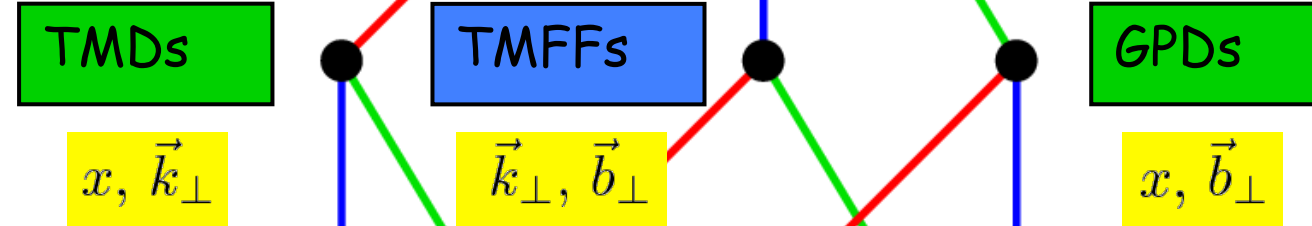


$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

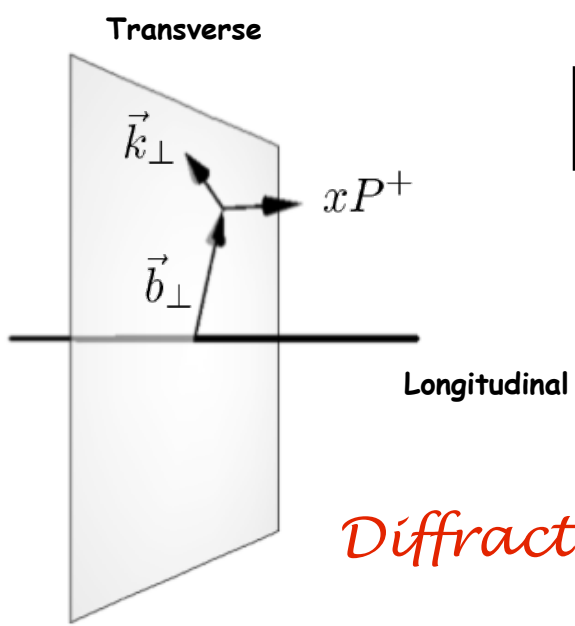
Transverse density in  
momentum space

Momentum space  $\vec{k}_{\perp} \leftrightarrow \vec{z}_{\perp}$  Position space  
 $\vec{\Delta}_{\perp} \leftrightarrow \vec{b}_{\perp}$   
 Transverse density in position  
space

Weak transition  
form factors



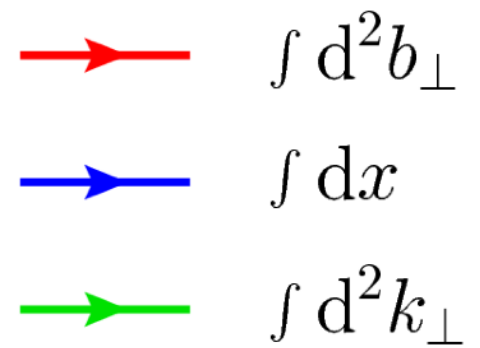
*DGLAP, ERBL Evolution  
Factorization Theorems*



*Diffractive DIS from FSI*

*Sivers, T-odd from lensing*

**Charges**





## Exclusive processes in perturbative quantum chromodynamics

G. Peter Lepage

*Laboratory of Nuclear Studies, Cornell University, Ithaca, New York 14853*

Stanley J. Brodsky

*Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305*

(Received 27 May 1980)



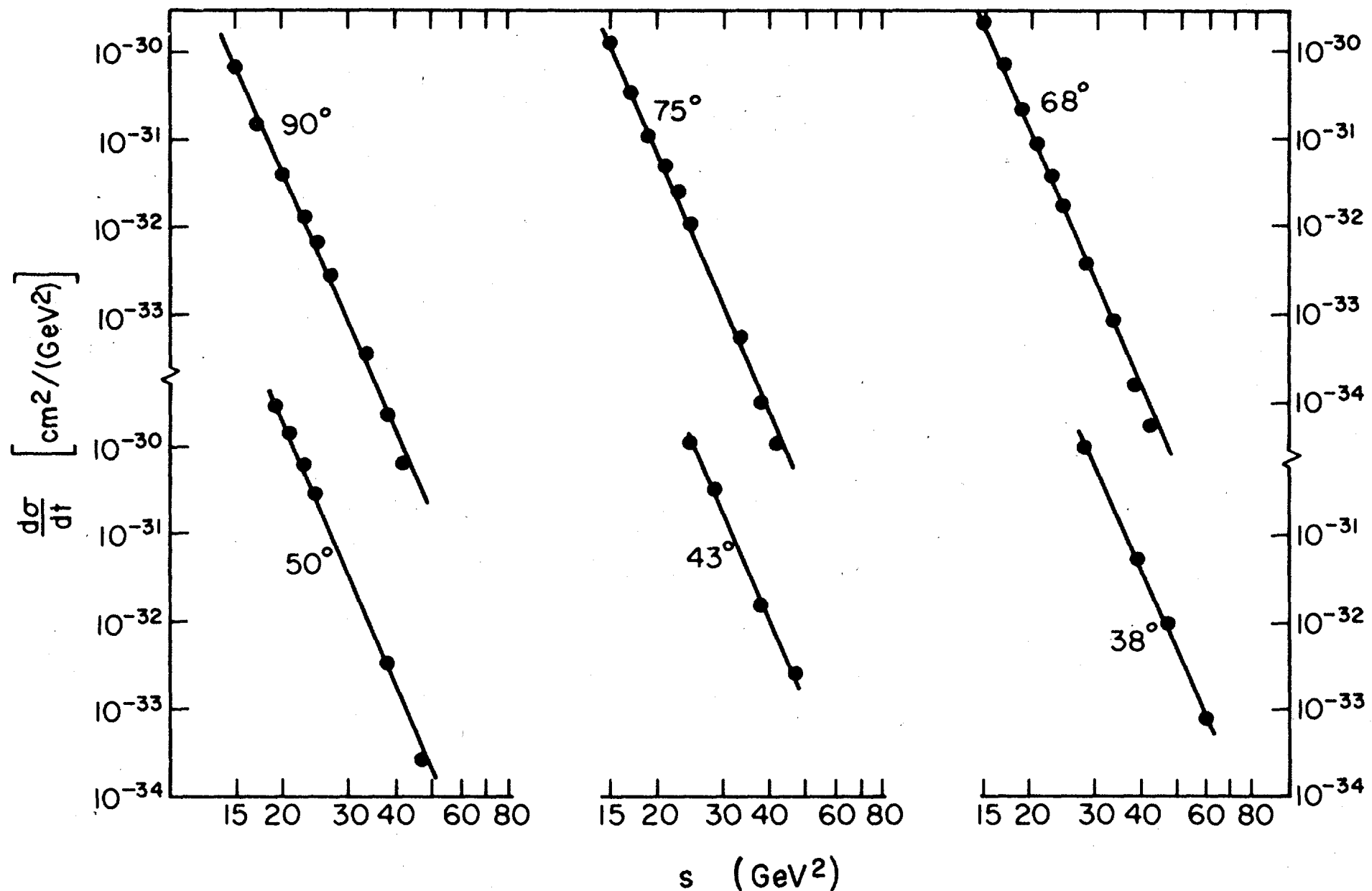
We present a systematic analysis in perturbative quantum chromodynamics (QCD) of large-momentum-transfer exclusive processes. Predictions are given for the scaling behavior, angular dependence, helicity structure, and normalization of elastic and inelastic form factors and large-angle exclusive scattering amplitudes for hadrons and photons. We prove that these reactions are dominated by quark and gluon subprocesses at short distances, and thus that the dimensional-counting rules for the power-law falloff of these amplitudes with momentum transfer are rigorous predictions of QCD, modulo calculable logarithmic corrections from the behavior of the hadronic wave functions at short distances. These anomalous-dimension corrections are determined by evolution equations for process-independent meson and baryon “distribution amplitudes”  $\phi(x_i, Q)$  which control the valence-quark distributions in high-momentum-transfer exclusive reactions. The analysis can be carried out systematically in powers of  $\alpha_s(Q^2)$ , the QCD running coupling constant. Although the calculations are most conveniently carried out using light-cone perturbation theory and the light-cone gauge, we also present a gauge-independent analysis and relate the distribution amplitude to a gauge-invariant Bethe-Salpeter amplitude.

**Rigorous QCD analysis of exclusive reactions**  
**Hadron Distribution amplitudes**  
**ERBL Evolution**

Also: Efremov and Radyshkin

# Scaling of Hard Exclusive reactions: Fixed t/s

EXCLUSIVE PROCESSES IN PERTURBATIVE QUANTUM...



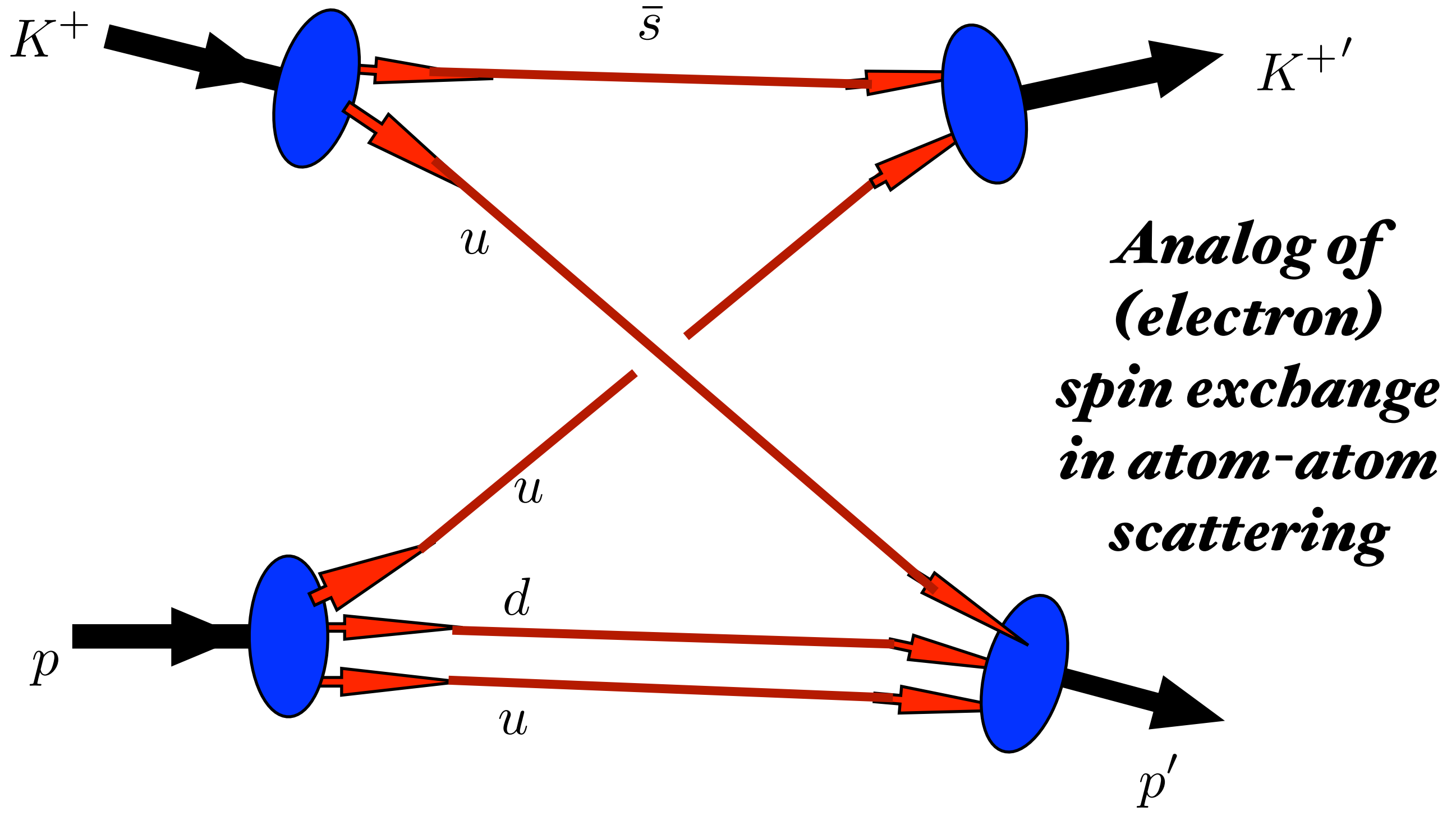
Cross sections for  $pp \rightarrow pp$  at wide angles

The straight lines correspond to a falloff of  $1/s^{10}$ .

$$\frac{d\sigma}{dt}(p + p \rightarrow p + p) = \frac{F(\theta_{CM})}{s^{10}}$$

Manifestation of Asymptotic Freedom

$$K^+ p \rightarrow K^+ p$$



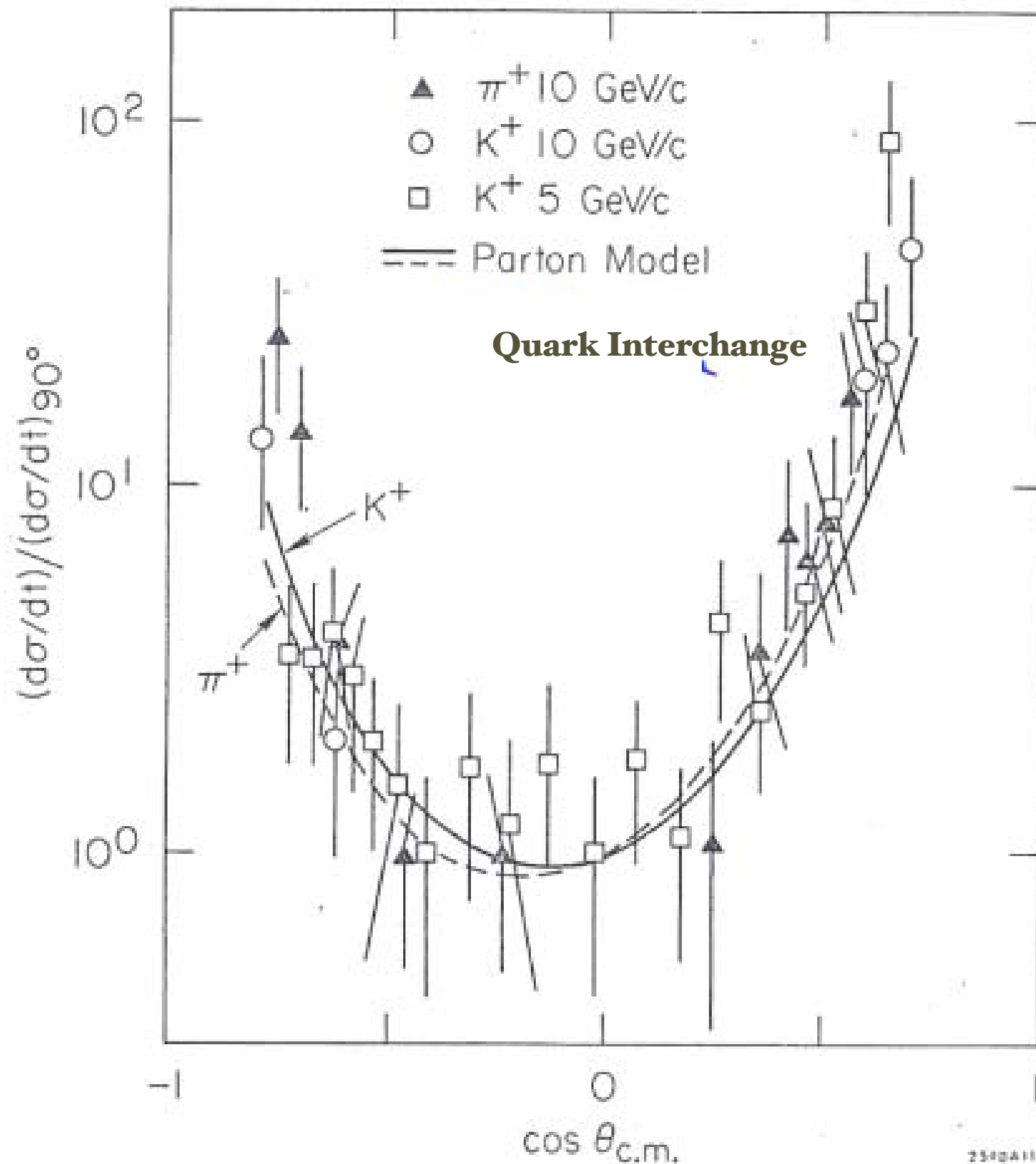
## Quark Interchange

*Blankenbecler, Gunion, sjb*

Interactions between exchanged quarks suppressed at high momentum transfer



# Quark Interchange Blankenbecler, Gunion, sjb



$$M(t, u) \text{ interchange} \propto \frac{1}{ut^2}$$

$$\frac{d\sigma}{dt} (K^+ p \rightarrow K^+ p) = \frac{F(t/s)}{s^8}$$

**Non-linear Regge behavior:**

$$\alpha_R(t) \rightarrow -1$$

$$\frac{d\sigma}{dt} = \frac{f(t/s)}{s^{N-2}} \quad N-2 = \# \text{ fundamental constituents} - 2 = 2+3+2+3-2=8$$

**“Counting Rules”** Farrar and sjb; Muradyan, Matveev, Tavkelidze

# Scaling: manifestation of asymptotically free hadronic interactions

From dimensional arguments at high energies in binary reactions:

## CONSTITUENT COUNTING RULE

Brodsky and Farrar, Phys. Rev. Lett. 31 (1973) 1153  
Matveev et al., Lett. Nuovo Cimento, 7 (1973) 719

Counting Rules:

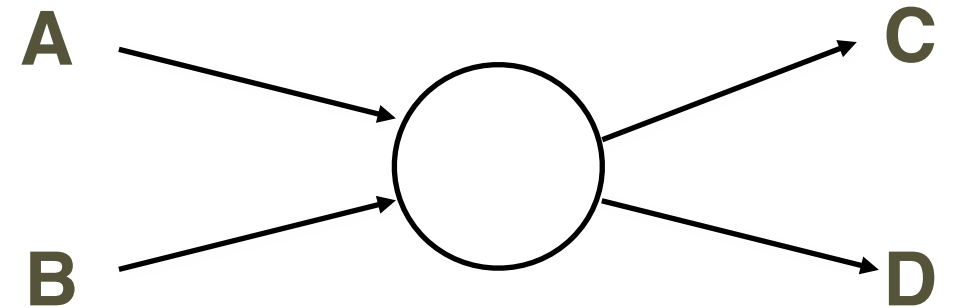
$$q(x) \sim (1-x)^{2n_{spect}-1} \text{ for } x \rightarrow 1$$

$$F(Q^2) \sim \left(\frac{1}{Q^2}\right)^{(n-1)}$$

$$\frac{d\sigma}{dt}(AB \rightarrow CD) \sim \frac{F(t/s)}{s^{(n_{participants}-2)}}$$

$$n_{participants} = n_A + n_B + n_C + n_D$$

$$\frac{d\sigma}{d^3p/E}(AB \rightarrow CX) \sim F(\hat{t}/\hat{s}) \times \frac{(1-x_R)^{(2n_{spectators}-1)}}{(p_T^2)^{(n_{participants}-2)}}$$



helicity  
conservation

Farrar, Jackson;  
Lepage, sjb;  
Burkardt,  
Schmidt, Sjb

Fixed  $\tau = t + z/c$

# Light-Front QCD

Physical gauge:  $A^+ = 0$

Exact frame-independent formulation of nonperturbative QCD!

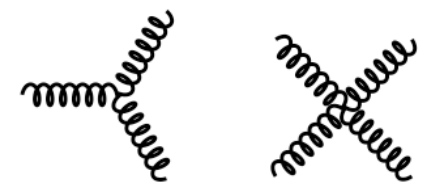
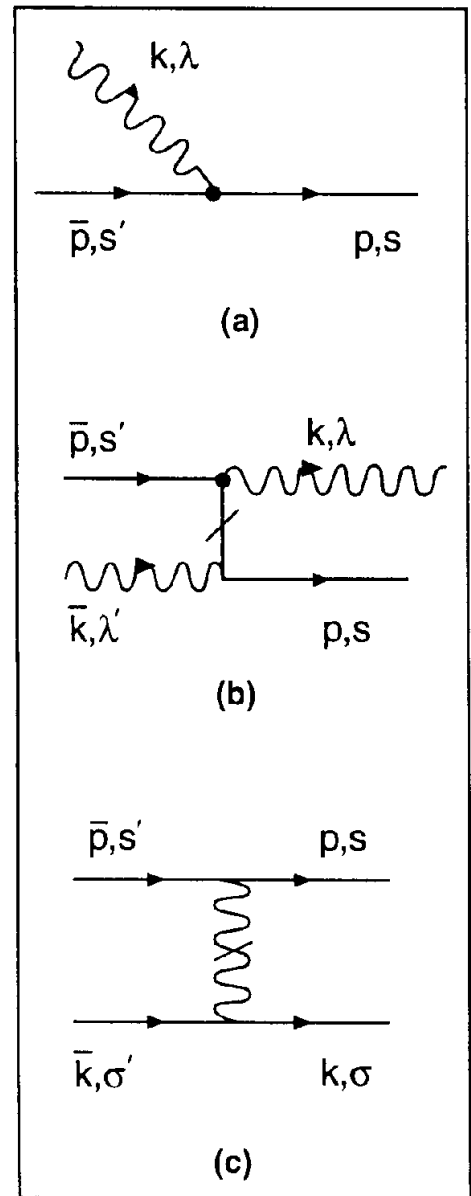
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$H_{LF}^{int}$

Eigenvalues and Eigensolutions give Hadronic Spectrum and Light-Front wavefunctions

**LFWFs: Off-shell in P- and invariant mass**

Solve nPQCD by matrix diagonalization: Hornbostel, Pauli, sjb

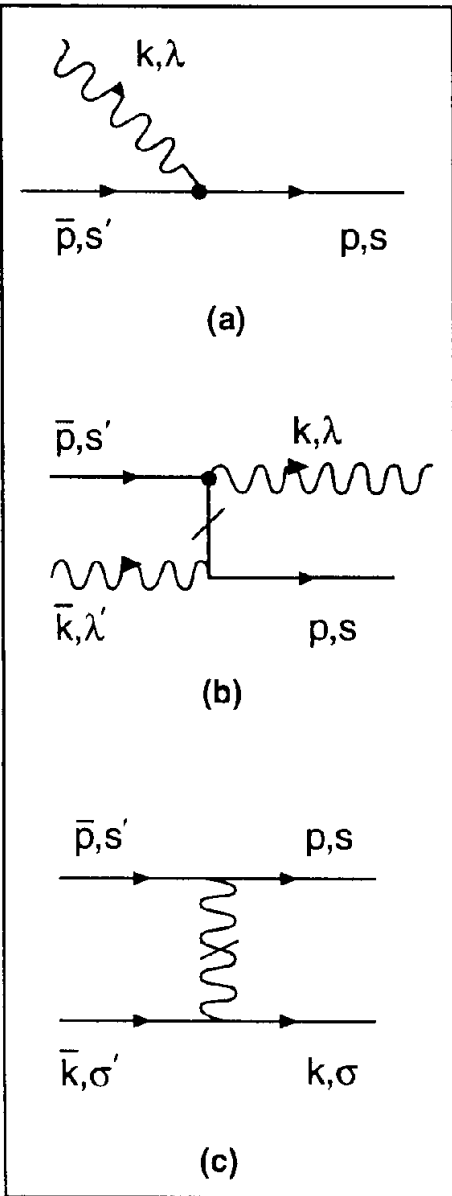


Light-Front QCD  
Heisenberg Equation

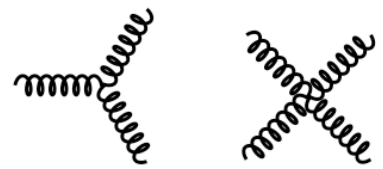
$$H_{LC}^{QCD} |\Psi_h\rangle = \mathcal{M}_h^2 |\Psi_h\rangle$$

DLCQ: Solved QCD(1+1) for any quark mass and flavors

Hornbostel, Pauli, sjb



n	Sector	1	2	3	4	5	6	7	8	9	10	11	12	13
	q $\bar{q}$	q $\bar{q}$	gg	q $\bar{q}$ g	q $\bar{q}$ q $\bar{q}$	gg g	q $\bar{q}$ gg	q $\bar{q}$ q $\bar{q}$ g	q $\bar{q}$ q $\bar{q}$ q $\bar{q}$	gg gg	q $\bar{q}$ gg g	q $\bar{q}$ q $\bar{q}$ gg	q $\bar{q}$ q $\bar{q}$ q $\bar{q}$ g	q $\bar{q}$ q $\bar{q}$ q $\bar{q}$ q $\bar{q}$
1	q $\bar{q}$					.		.	.	.	.	.	.	.
2	gg				.			.	.		.	.	.	.
3	q $\bar{q}$ g								.	.		.	.	.
4	q $\bar{q}$ q $\bar{q}$		.			.				.	.		.	.
5	gg g	.			.			.	.			.	.	.
6	q $\bar{q}$ gg								.				.	.
7	q $\bar{q}$ q $\bar{q}$ g	.	.			.				.				.
8	q $\bar{q}$ q $\bar{q}$ q $\bar{q}$	.	.	.		.	.			.	.			
9	gg gg	.		.	.			.	.			.	.	.
10	q $\bar{q}$ gg g	.	.		.				.				.	.
11	q $\bar{q}$ q $\bar{q}$ gg	.	.	.		.				.				.
12	q $\bar{q}$ q $\bar{q}$ q $\bar{q}$ g	.	.	.	.	.	.			.	.			
13	q $\bar{q}$ q $\bar{q}$ q $\bar{q}$ q $\bar{q}$	.	.	.	.	.	.	.		.	.	.		



Minkowski space; frame-independent; no fermion doubling; no ghosts

# Discretized LF Quantization

DLCQ: Diagonalize QCD Hamiltonian, periodic LF BC

BLFQ (Vary et al)  
Use LF Holographic Basis

# Solve QCD by Matrix Diagonalization

Diagonalize the LF Hamiltonian on an Orthonormal Basis

Lorentz Frame-Independent,

Minkowski Causal LF Time

Compute Hadron masses, LF Wavefunctions

Successful applications to QCD(1+1)

Use advanced computer resources

Competitive with LGTh?

***H. C. Pauli, K. Hornbostel, sjb***



# BLM Renormalization Scale Setting

## On the elimination of scale ambiguities in perturbative quantum chromodynamics



Stanley J. Brodsky  
*Institute for Advanced Study, Princeton, New Jersey 08540  
and Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305\**

G. Peter Lepage  
*Institute for Advanced Study, Princeton, New Jersey 08540  
and Laboratory of Nuclear Studies, Cornell University, Ithaca, New York 14853\**

Paul B. Mackenzie  
*Fermilab, Batavia, Illinois 60510  
(Received 23 November 1982)*

We present a new method for resolving the scheme-scale ambiguity that has plagued perturbative analyses in quantum chromodynamics (QCD) and other gauge theories. For Abelian theories the method reduces to the standard criterion that only vacuum-polarization insertions contribute to the effective coupling constant. Given a scheme, our procedure automatically determines the coupling-constant scale appropriate to a particular process. This leads to a new criterion for the convergence of perturbative expansions in QCD. We examine a number of well known reactions in QCD, and find that perturbation theory converges well for all processes other than the gluonic width of the  $\Upsilon$ . Our analysis calls into question recent determinations of the QCD coupling constant based upon  $\Upsilon$  decay.



All orders: PMC (Principle of Maximum Conformality)  
Satisfies all principles of renormalization theory  
Eliminates  $n!$  renormalons  
Commensurate scale relations between observables  
Abelian limit: Standard QED Scale-Setting

M. Mojaza, sjb  
L. di Giustino, Xing-Gang Wu

Kataev, Mikhailov



# *Challenge: Compute Hadron Structure, Spectroscopy, and Dynamics from QCD!*

- **Color Confinement**
- **Origin of the QCD Mass Scale**
- **Meson and Baryon Spectroscopy**
- **Exotic States: Tetraquarks, Pentaquarks, Gluonium,**
- **Universal Regge Slopes:  $n$ ,  $L$ , Mesons and Baryons**
- **Almost Massless Pion: GMOR Chiral Symmetry Breaking**  
$$M_\pi^2 f_\pi^2 = -\frac{1}{2}(m_u + m_d) \langle \bar{u}u + \bar{d}d \rangle + \mathcal{O}((m_u + m_d)^2)$$
- **QCD Coupling at all Scales  $\alpha_s(Q^2)$**
- **Eliminate Scale Uncertainties and Scheme Dependence**
- **BLM/PMC (Principle of Maximum Conformality); Lepage Mackenzie, sjb, Di Giustino, Kataev, Mikhailov**

*Need a First Approximation to QCD*

*Comparable in simplicity to  
Schrödinger Theory in Atomic Physics*

**Relativistic, Frame-Independent, Color-Confining**

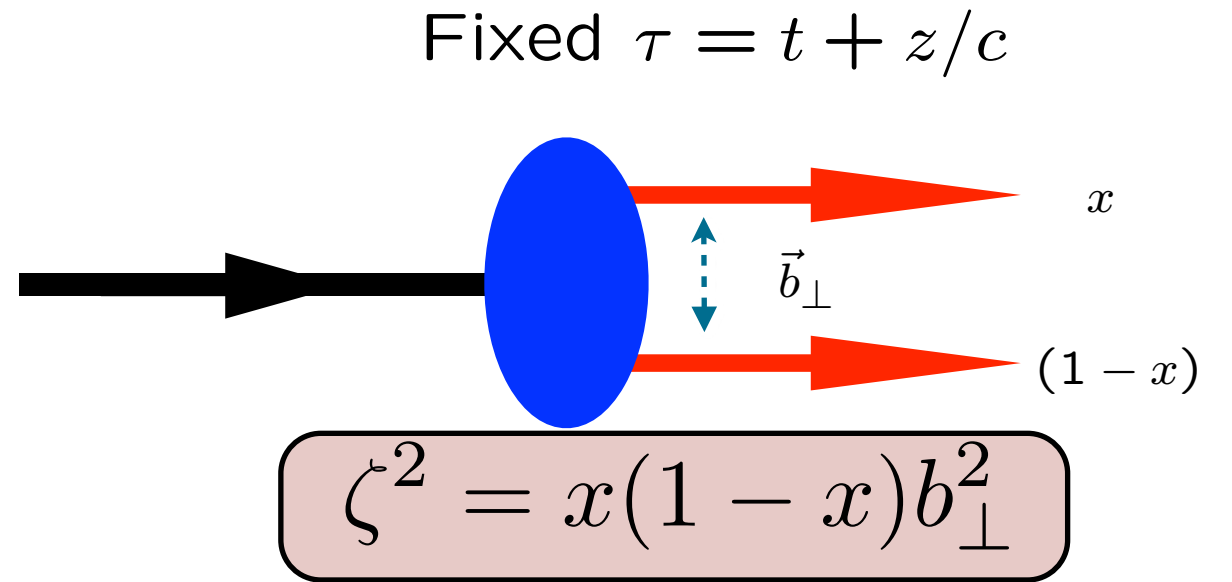
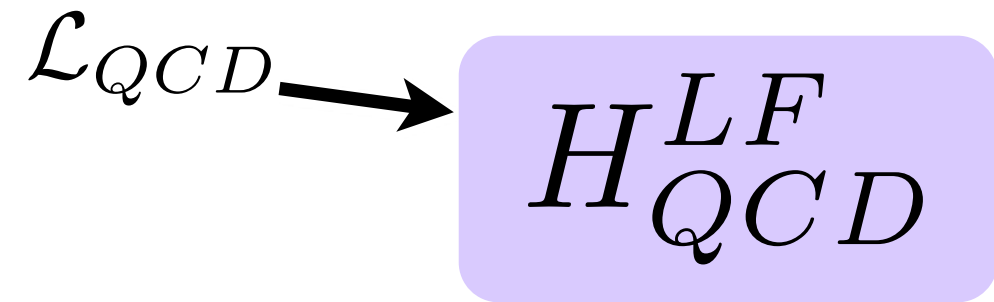
**Origin of hadronic mass scale if  $m_q=0$**

Semi-Classical Approximation to QCD

*de Téramond, Dosch, Lorcé, sjb*

**AdS/QCD**  
**Light-Front Holography**

# Light-Front QCD



$$(H_{LF}^0 + H_{LF}^I) |\Psi\rangle = M^2 |\Psi\rangle$$

*Coupled Fock states*

*Eliminate higher Fock states and retarded interactions*

$$\left[ \frac{\vec{k}_{\perp}^2 + m^2}{x(1-x)} + V_{\text{eff}}^{LF} \right] \psi_{LF}(x, \vec{k}_{\perp}) = M^2 \psi_{LF}(x, \vec{k}_{\perp})$$

*Effective two-particle equation*

$$\left[ -\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U(\zeta) \right] \psi(\zeta) = \mathcal{M}^2 \psi(\zeta)$$

*Azimuthal Basis  $\zeta, \phi$*

**Single variable Equation**

$$m_q = 0$$

*Confining AdS/QCD potential!*

**AdS/QCD: LF Holography**

$$U(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2 (L + S - 1)$$

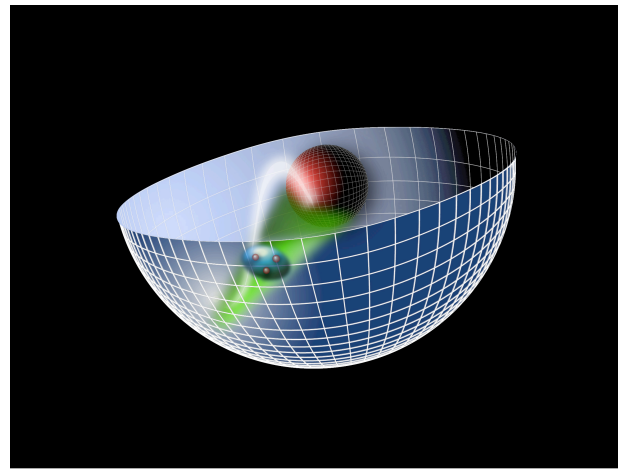
*Sums an infinite # diagrams*

*Semiclassical first approximation to QCD*

*de Téramond, Dosch, Lorcé, sjb*

*AdS/QCD  
Soft-Wall Model*

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$



$$\zeta^2 = x(1-x)b_{\perp}^2$$

*Light-Front Holography*

$$\left[ -\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U(\zeta) \right] \psi(\zeta) = M^2 \psi(\zeta)$$



***Light-Front Schrödinger Equation***

$$U(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2 (L + S - 1)$$

*Single variable  $\zeta$*

***Unique  
Confinement Potential!***  
*Conformal Symmetry  
of the AdS action*

***Confinement scale:***

$$\kappa \simeq 0.5 \text{ GeV}$$

- **de Alfaro, Fubini, Furlan:**
- **Fubini, Rabinovici:**

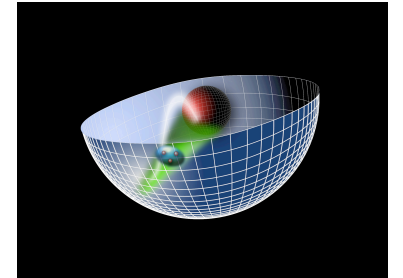
**Scale can appear in Hamiltonian and EQM  
without affecting conformal invariance of AdS action!**

*GeV units external to QCD: Ratios of Masses Determined*



## *Dilaton-Modified AdS*

$$ds^2 = e^{\varphi(z)} \frac{R^2}{z^2} (\eta_{\mu\nu} x^\mu x^\nu - dz^2)$$



- **Soft-wall dilaton profile breaks**

- **Color Confinement in  $z$**   $e^{\varphi(z)} = e^{+\kappa^2 z^2}$

- **Introduces confinement scale  $\kappa$**

- **Uses  $AdS_5$  as template for conformal theory**

AdS/CFT

**D. Gross: duality of QCD with string theory**

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$

**Positive-sign dilaton**

• de Teramond, sjb

*AdS Soft-Wall Schrödinger Equation for bound state of two scalar constituents:*

$$\left[ -\frac{d^2}{dz^2} - \frac{1 - 4L^2}{4z^2} + U(z) \right] \Phi(z) = \mathcal{M}^2 \Phi(z)$$

$$U(z) = \kappa^4 z^2 + 2\kappa^2 (L + S - 1)$$

*Derived from variation of Action for Dilaton-Modified AdS<sub>5</sub>*

***Identical to Single-Variable Light-Front Bound State Equation in  $\zeta$ !***

$$z \quad \longleftrightarrow \quad \zeta = \sqrt{x(1-x)} \vec{b}_\perp^2$$

***Light-Front Holography***

# Introduce "Dilaton" to simulate confinement analytically

- Nonconformal metric dual to a confining gauge theory

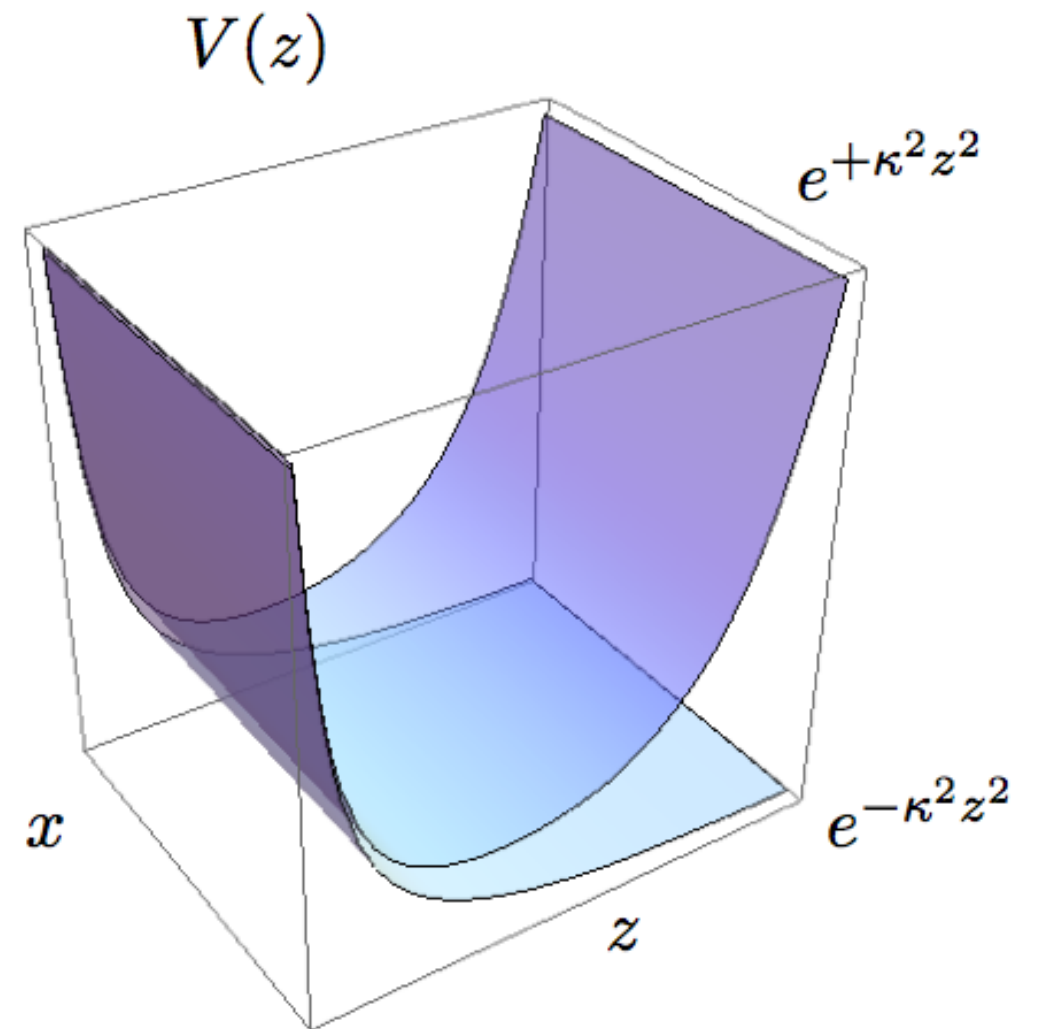
$$ds^2 = \frac{R^2}{z^2} e^{\varphi(z)} (\eta_{\mu\nu} dx^\mu dx^\nu - dz^2)$$

where  $\varphi(z) \rightarrow 0$  at small  $z$  for geometries which are asymptotically AdS<sub>5</sub>

- Gravitational potential energy for object of mass  $m$

$$V = mc^2 \sqrt{g_{00}} = mc^2 R \frac{e^{\varphi(z)/2}}{z}$$

- Consider warp factor  $\exp(\pm\kappa^2 z^2)$
- Plus solution:  $V(z)$  increases exponentially confining any object in modified AdS metrics to distances  $\langle z \rangle \sim 1/\kappa$



*Klebanov and Maldacena*

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$

**Positive-sign dilaton**

- de Te'ramond, sjb

$$\left( -\partial_{\zeta}^2 + \kappa^4 \zeta^2 + 2\kappa^2(L_B + 1) + \frac{4L_B^2 - 1}{4\zeta^2} \right) \psi_J^+ = M^2 \psi_J^+ \quad \left. \vphantom{\psi_J^+} \right\}$$

$$\left( -\partial_{\zeta}^2 + \kappa^4 \zeta^2 + 2\kappa^2 L_B + \frac{4(L_B + 1)^2 - 1}{4\zeta^2} \right) \psi_J^- = M^2 \psi_J^- \quad \left. \vphantom{\psi_J^-} \right\}$$

$$M^2(n, L_B) = 4\kappa^2(n + L_B + 1) \quad \mathbf{S=1/2, P=+}$$

## Meson Equation

$$\lambda = \kappa^2$$

$$\left( -\partial_{\zeta}^2 + \kappa^4 \zeta^2 + 2\kappa^2(J - 1) + \frac{4L_M^2 - 1}{4\zeta^2} \right) \phi_J = M^2 \phi_J$$

$$M^2(n, L_M) = 4\kappa^2(n + L_M)$$

$\mathbf{S=0, P=+}$   
*Same  $\kappa$ !*

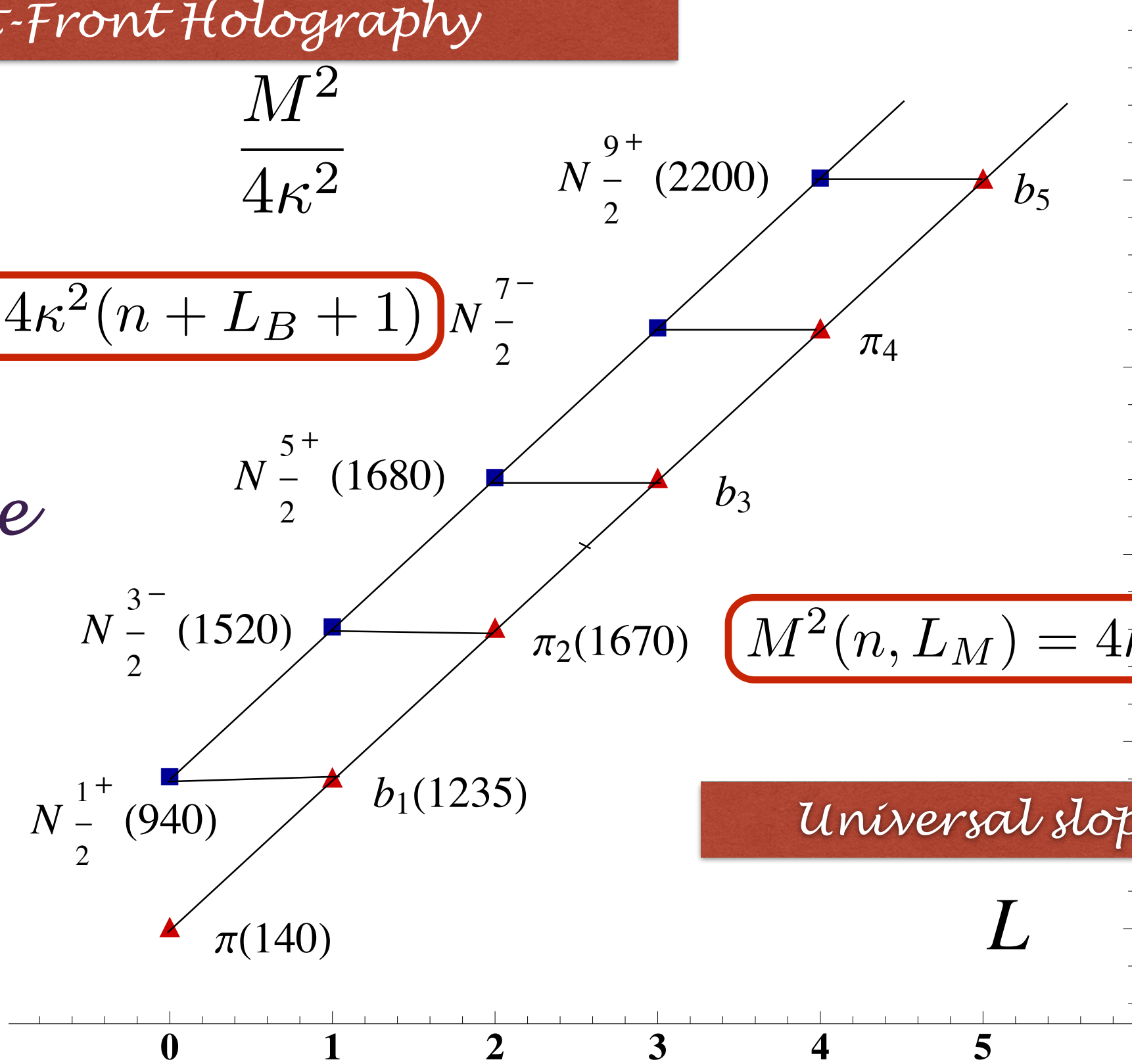
**$S=0, I=I$  Meson is superpartner of  $S=1/2, I=I$  Baryon**

**Meson-Baryon Degeneracy for  $L_M=L_B+1$**



$$M^2(n, L_B) = 4\kappa^2(n + L_B + 1)$$

*Same slope*



$$M^2(n, L_M) = 4\kappa^2(n + L_M)$$

*Universal slopes in  $n, L$*

$$\frac{M_{meson}^2}{M_{nucleon}^2} = \frac{n + L_M}{n + L_B + 1}$$

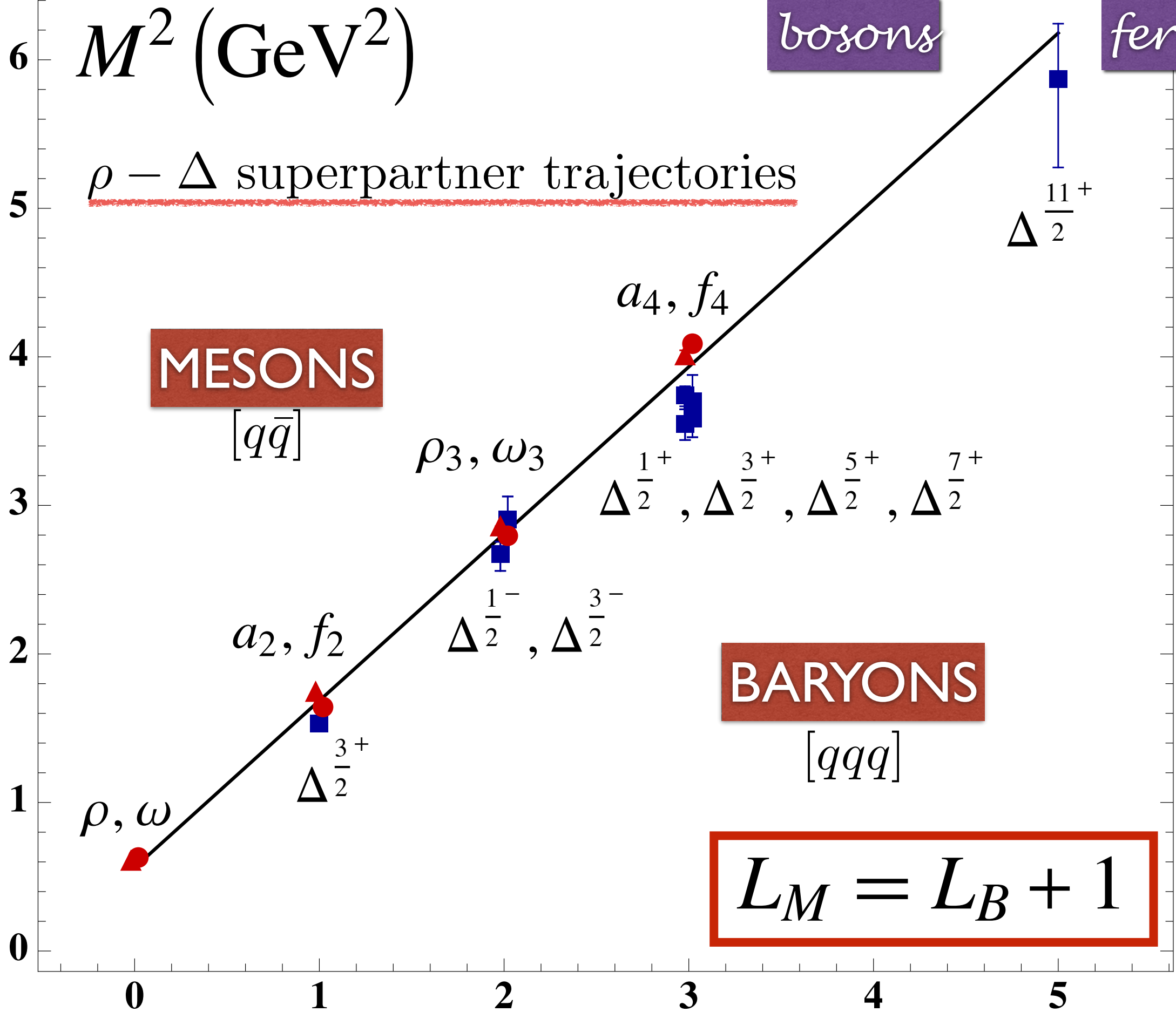
**Meson-Baryon  
Mass Degeneracy  
for  $L_M=L_B+1$**

$M^2$  (GeV<sup>2</sup>)

bosons

fermions

$\rho - \Delta$  superpartner trajectories



BARYONS

MESONS

$L_M = L_B + 1$

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$

**Positive-sign dilaton**

• de Teramond, sjb

*AdS Soft-Wall Schrödinger Equation for bound state of two scalar constituents:*

$$\left[ -\frac{d^2}{dz^2} - \frac{1 - 4L^2}{4z^2} + U(z) \right] \Phi(z) = \mathcal{M}^2 \Phi(z)$$

$$U(z) = \kappa^4 z^2 + 2\kappa^2 (L + S - 1)$$

*Derived from variation of Action for Dilaton-Modified AdS<sub>5</sub>*

***Identical to Single-Variable Light-Front Bound State Equation in  $\zeta$ !***

$$z \quad \longleftrightarrow \quad \zeta = \sqrt{x(1-x)} \vec{b}_\perp^2$$

***Light-Front Holography***

## Holographic Mapping of AdS Modes to QCD LFWFs

*Drell-Yan-West: Form Factors are  
Convolution of LFWFs*

- Integrate Soper formula over angles:

$$F(q^2) = 2\pi \int_0^1 dx \frac{(1-x)}{x} \int \zeta d\zeta J_0 \left( \zeta q \sqrt{\frac{1-x}{x}} \right) \tilde{\rho}(x, \zeta),$$

with  $\tilde{\rho}(x, \zeta)$  QCD effective transverse charge density.

- Transversality variable

$$\zeta = \sqrt{x(1-x)} \vec{b}_\perp^2$$

- Compare AdS and QCD expressions of FFs for arbitrary  $Q$  using identity:

$$\int_0^1 dx J_0 \left( \zeta Q \sqrt{\frac{1-x}{x}} \right) = \zeta Q K_1(\zeta Q),$$

the solution for  $J(Q, \zeta) = \zeta Q K_1(\zeta Q)$  !

**de Te'ramond, sjb**

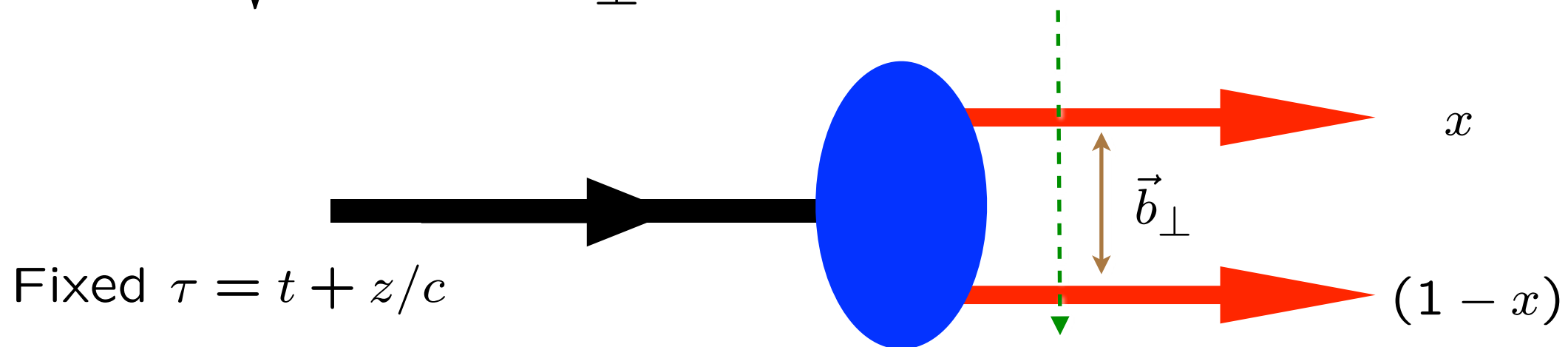
*Identical to Polchinski-Strassler Convolution of AdS Amplitudes*

*LF(3+1) ↔ AdS<sub>5</sub>*



$LF(3+1) \longleftrightarrow AdS_5$ 

# Light-Front Holographic Dictionary

 $\psi(x, \vec{b}_\perp) \longleftrightarrow \phi(z)$ 
 $\zeta = \sqrt{x(1-x)b_\perp^2} \longleftrightarrow z$ 


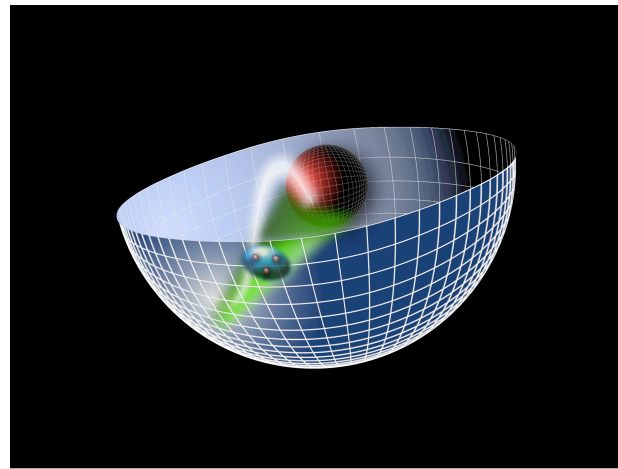
$$\psi(x, \zeta) = \sqrt{x(1-x)} \zeta^{-1/2} \phi(\zeta)$$

$$(\mu R)^2 = L^2 - (J - 2)^2$$

**Light-Front Holography:** Unique mapping derived from equality of LF and AdS formula for EM and gravitational current matrix elements and identical equations of motion

*AdS/QCD  
Soft-Wall Model*

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$



$$\zeta^2 = x(1-x)b_{\perp}^2$$

*Light-Front Holography*

$$\left[ -\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U(\zeta) \right] \psi(\zeta) = M^2 \psi(\zeta)$$



***Light-Front Schrödinger Equation***

$$U(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2 (L + S - 1)$$

*Single variable  $\zeta$*

***Unique  
Confinement Potential!***  
*Conformal Symmetry  
of the action*

***Confinement scale:***

$$\kappa \simeq 0.5 \text{ GeV}$$

- **de Alfaro, Fubini, Furlan:**
- **Fubini, Rabinovici:**

**Scale can appear in Hamiltonian and EQM  
without affecting conformal invariance of action!**

*GeV units external to QCD: Ratios of Masses Determined*

$$\left( -\partial_{\zeta}^2 + \kappa^4 \zeta^2 + 2\kappa^2(L_B + 1) + \frac{4L_B^2 - 1}{4\zeta^2} \right) \psi_J^+ = M^2 \psi_J^+ \quad \left. \vphantom{\psi_J^+} \right\}$$

$$\left( -\partial_{\zeta}^2 + \kappa^4 \zeta^2 + 2\kappa^2 L_B + \frac{4(L_B + 1)^2 - 1}{4\zeta^2} \right) \psi_J^- = M^2 \psi_J^- \quad \left. \vphantom{\psi_J^-} \right\}$$

$$M^2(n, L_B) = 4\kappa^2(n + L_B + 1) \quad \mathbf{S=1/2, P=+}$$

## Meson Equation

$$\lambda = \kappa^2$$

$$\left( -\partial_{\zeta}^2 + \kappa^4 \zeta^2 + 2\kappa^2(J - 1) + \frac{4L_M^2 - 1}{4\zeta^2} \right) \phi_J = M^2 \phi_J$$

$$M^2(n, L_M) = 4\kappa^2(n + L_M)$$

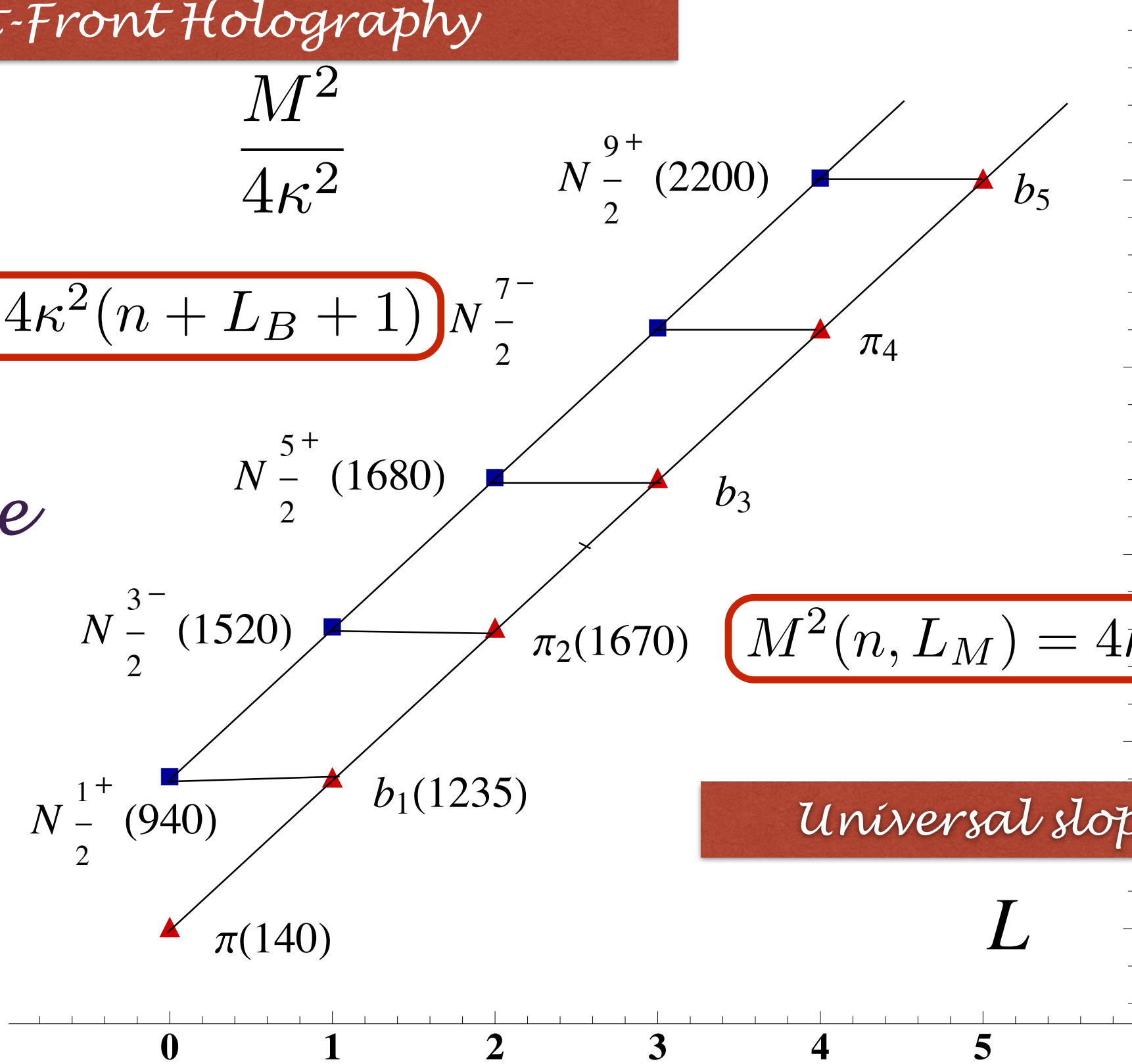
$\mathbf{S=0, P=+}$   
*Same  $\kappa$ !*

**$S=0, I=I$  Meson is superpartner of  $S=1/2, I=I$  Baryon**

**Meson-Baryon Degeneracy for  $L_M=L_B+1$**

$M^2(n, L_B) = 4\kappa^2(n + L_B + 1)$

*Same slope*



$M^2(n, L_M) = 4\kappa^2(n + L_M)$

*Universal slopes in  $n, L$*

$\frac{M_{meson}^2}{M_{nucleon}^2} = \frac{n + L_M}{n + L_B + 1}$

**Meson-Baryon  
Mass Degeneracy  
for  $L_M=L_B+1$**



# Massless pion!

## Meson Spectrum in Soft Wall Model

$$m_\pi = 0 \text{ if } m_q = 0$$

*Pion: Negative term for  $J=0$  cancels positive terms from LFKE and potential*



- Effective potential:  $U(\zeta^2) = \kappa^4 \zeta^2 + 2\kappa^2(J - 1)$

- LF WE

$$\left( -\frac{d^2}{d\zeta^2} - \frac{1 - 4L^2}{4\zeta^2} + \kappa^4 \zeta^2 + 2\kappa^2(J - 1) \right) \phi_J(\zeta) = M^2 \phi_J(\zeta)$$

- Normalized eigenfunctions  $\langle \phi | \phi \rangle = \int d\zeta \phi^2(z)^2 = 1$

$$\phi_{n,L}(\zeta) = \kappa^{1+L} \sqrt{\frac{2n!}{(n+L)!}} \zeta^{1/2+L} e^{-\kappa^2 \zeta^2 / 2} L_n^L(\kappa^2 \zeta^2)$$

- Eigenvalues

$$M_{n,J,L}^2 = 4\kappa^2 \left( n + \frac{J+L}{2} \right)$$

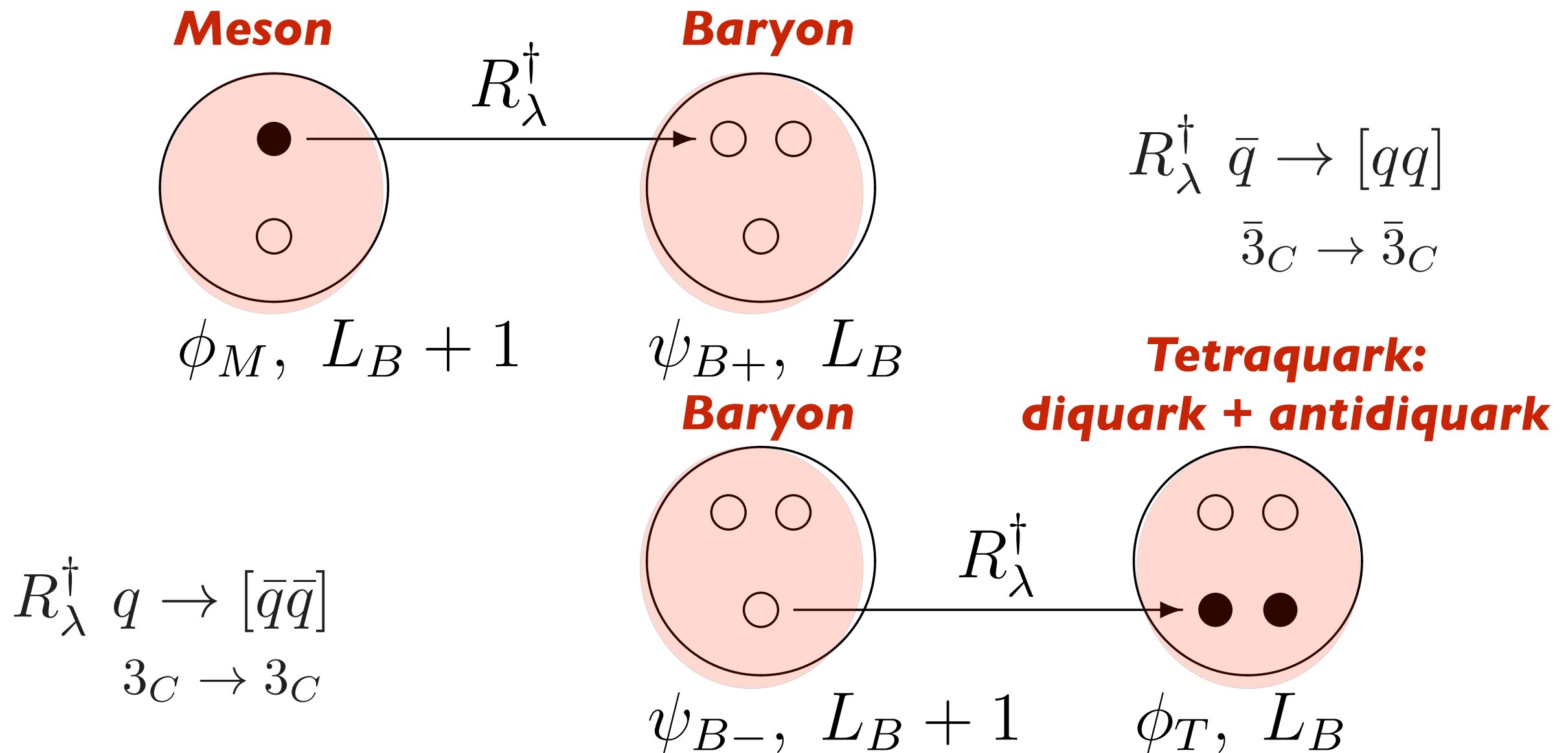
$$\vec{\zeta}^2 = \vec{b}_\perp^2 x(1-x)$$

**G. de Te'ramond, H. G. Dosch, sjb**

# Superconformal Algebra

## Four-Plet Representations

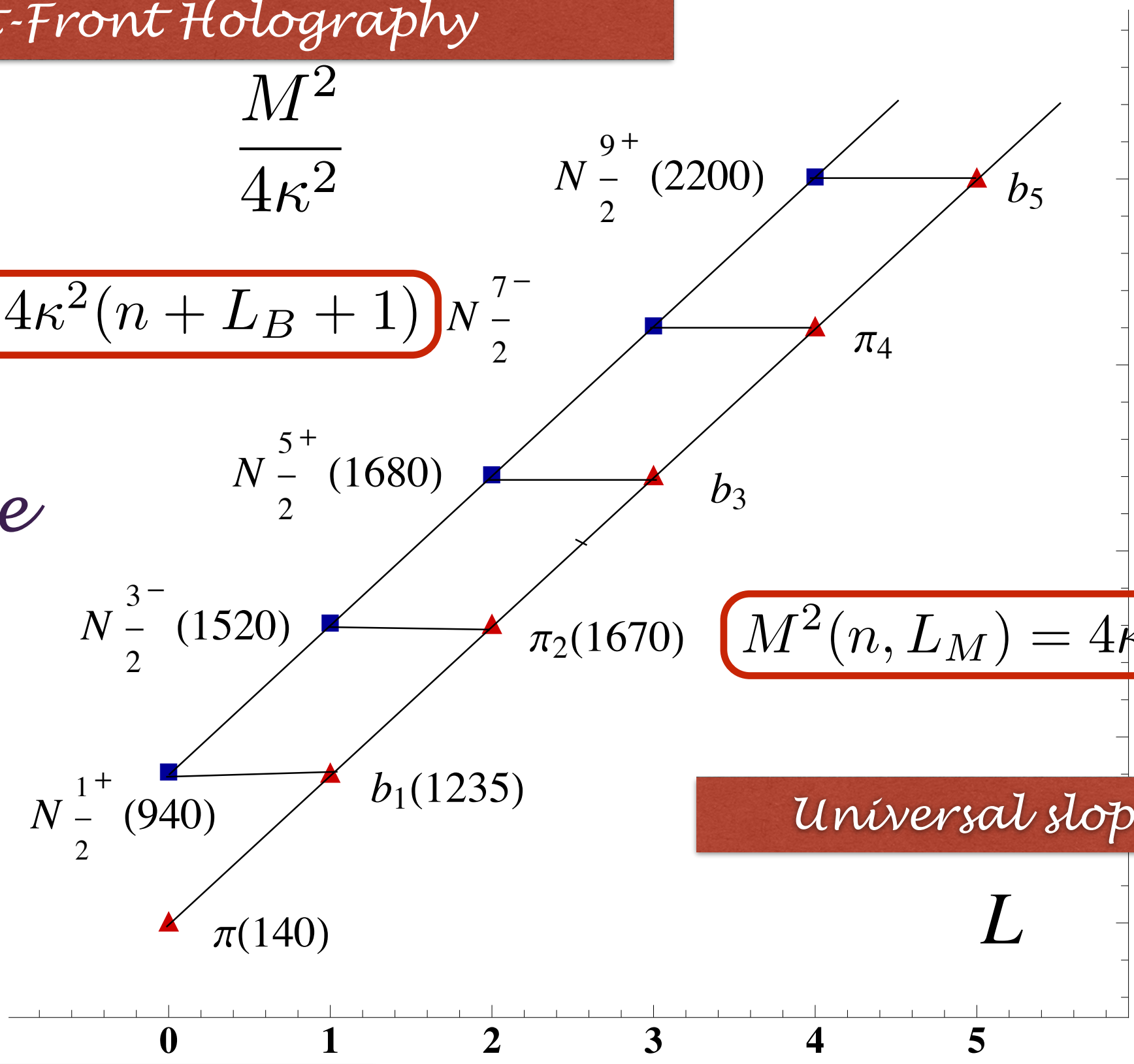
Bosons, Fermions with Equal Mass!



Proton:  $|u[ud]\rangle$  Quark + Scalar Diquark  
 Equal Weight:  $L=0, L=1$

$M^2(n, L_B) = 4\kappa^2(n + L_B + 1)$

*Same slope*



$M^2(n, L_M) = 4\kappa^2(n + L_M)$

*Universal slopes in  $n, L$*

$\frac{M_{meson}^2}{M_{nucleon}^2} = \frac{n + L_M}{n + L_B + 1}$

**Meson-Baryon  
Mass Degeneracy  
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$$\phi_{n,L}(\zeta) = \kappa^{1+L} \sqrt{\frac{2n!}{(n+L)!}} \zeta^{1/2+L} e^{-\kappa^2 \zeta^2 / 2} L_n^L(\kappa^2 \zeta^2)$$

- Eigenvalues

$$\mathcal{M}_{n,J,L}^2 = 4\kappa^2 \left( n + \frac{J+L}{2} \right)$$

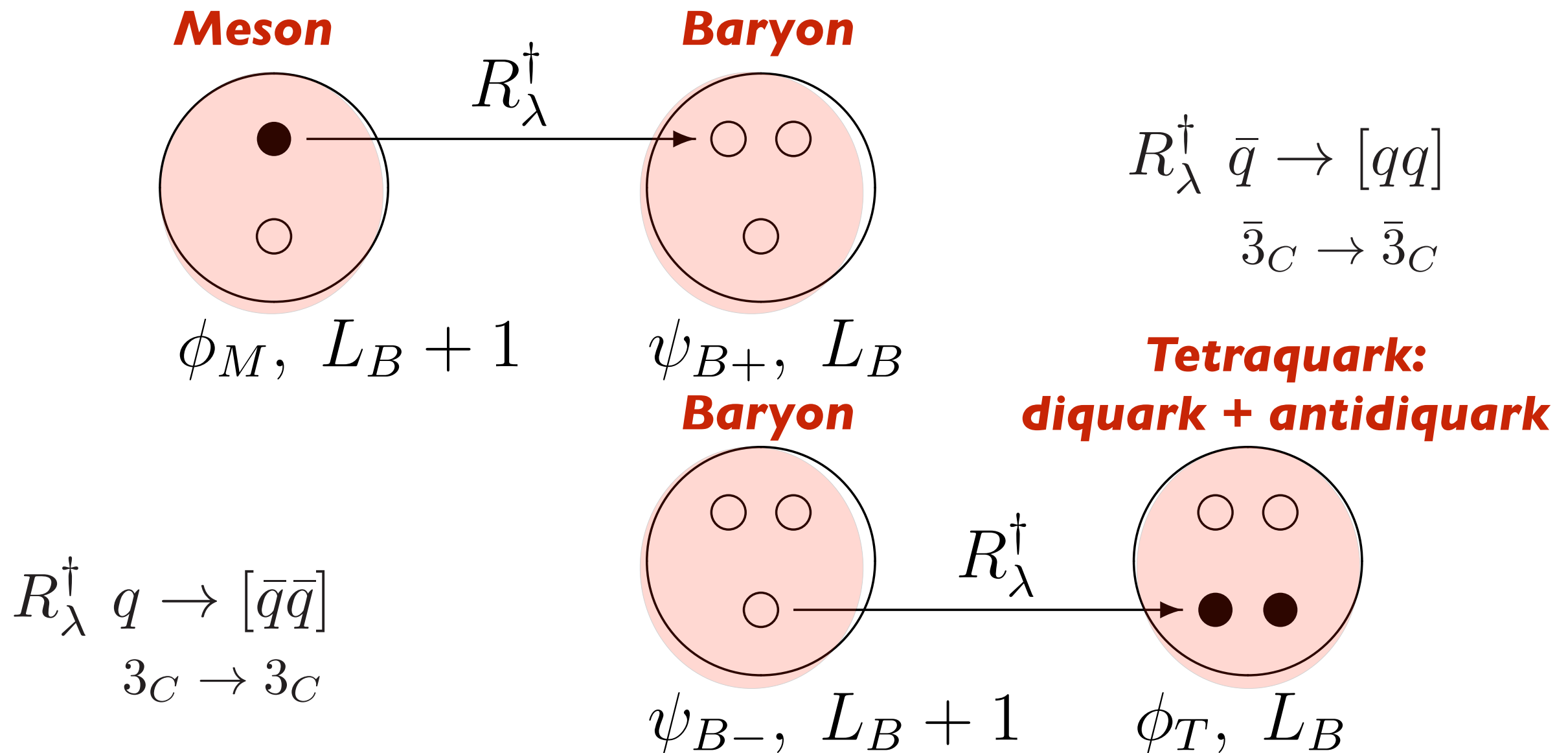
$$\vec{\zeta}^2 = \vec{b}_\perp^2 x(1-x)$$



# Superconformal Algebra

## Four-Plet Representations

Bosons, Fermions with Equal Mass!



Proton:  $|u[ud]\rangle$  Quark + Scalar Diquark  
 Equal Weight:  $L=0, L=1$

# Universal Hadronic Decomposition

$$\frac{\mathcal{M}_H^2}{\kappa^2} = (1 + 2n + L) + (1 + 2n + L) + (2L + 4S + 2B - 2)$$

- **Universal quark light-front kinetic energy**

$$\Delta\mathcal{M}_{LFKE}^2 = \kappa^2(1 + 2n + L)$$

- **Universal quark light-front potential energy**

$$\Delta\mathcal{M}_{LFPE}^2 = \kappa^2(1 + 2n + L)$$

- **Universal Constant Contribution from AdS and Superconformal Quantum Mechanics**

$$\Delta\mathcal{M}_{spin}^2 = 2\kappa^2(L + 2S + B - 1)$$

hyperfine spin-spin

**Equal:  
Virial  
Theorem**

# Supersymmetry in QCD

- A hidden symmetry of Color  $SU(3)_c$  in hadron physics:
- Relates meson and baryon spectroscopy
- QCD: No squarks or gluinos!
- Emerges from Light-Front Holography and Super-Conformal Algebra
- Color Confinement

*de Téramond, Dosch, Lorcé, sjb*

**Input: one fundamental mass scale**

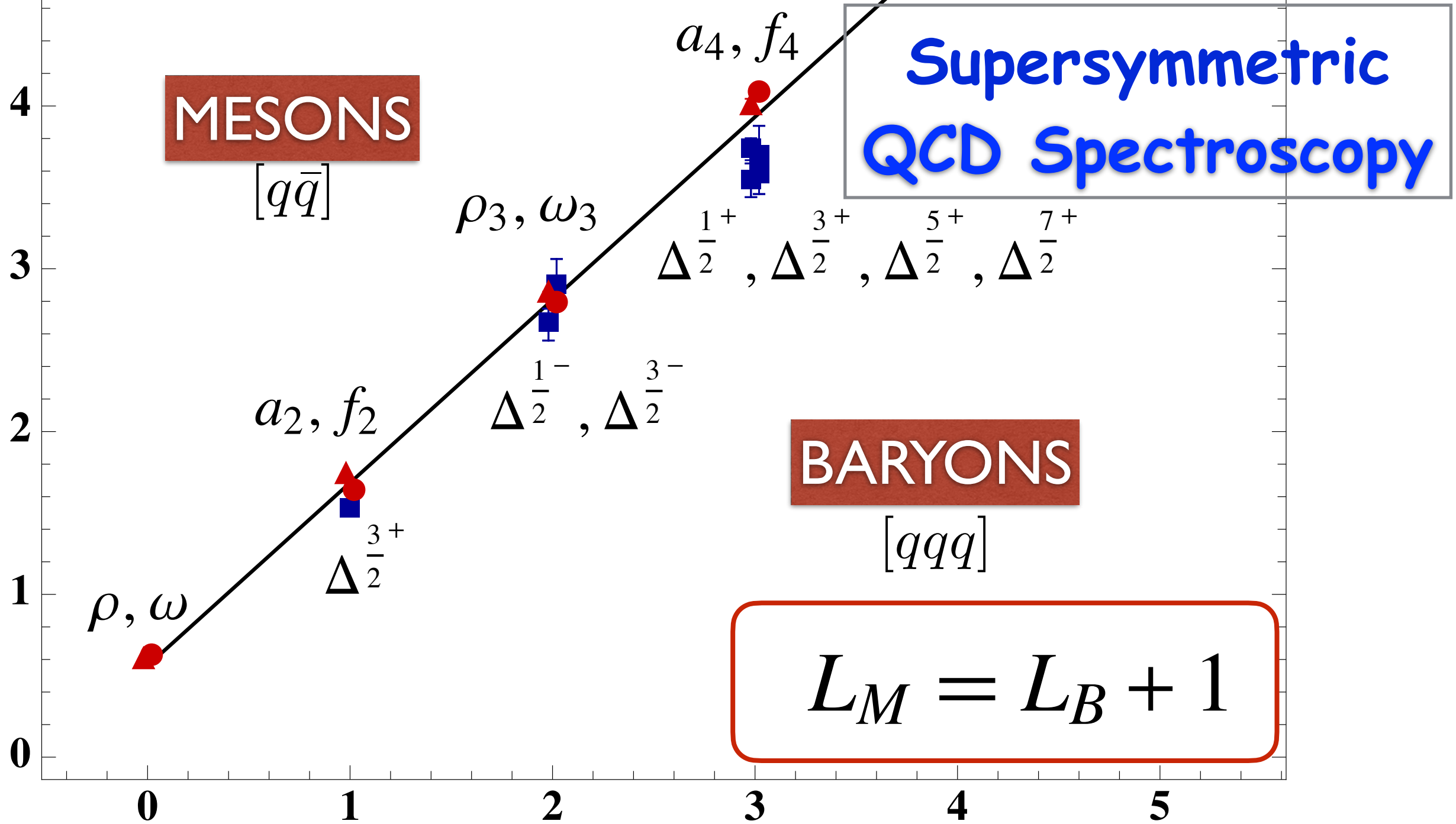
$$\kappa = \sqrt{\lambda} = 0.523 \pm 0.024 \text{ GeV}$$

$M^2$  (GeV<sup>2</sup>)

bosons

fermions

$\rho - \Delta$  superpartner trajectories





# Remarkable Features of Light-Front Schrödinger Equation

## Dynamics + Spectroscopy!

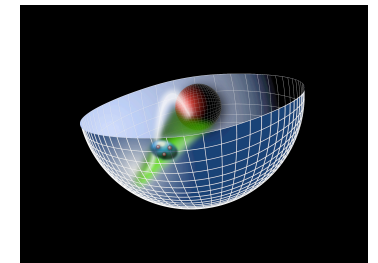
- **Relativistic, frame-independent**
- **QCD scale appears - unique LF potential**
- **Reproduces spectroscopy and dynamics of light-quark hadrons with one parameter**
- **Zero-mass pion for zero mass quarks!**
- **Regge slope same for  $n$  and  $L$  -- not usual HO**
- **Splitting in  $L$  persists to high mass -- contradicts conventional wisdom based on breakdown of chiral symmetry**
- **Phenomenology: LFWFs, Form factors, electroproduction**
- **Extension to heavy quarks**

$$U(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2 (L + S - 1)$$

# LFHQCD: Underlying Principles

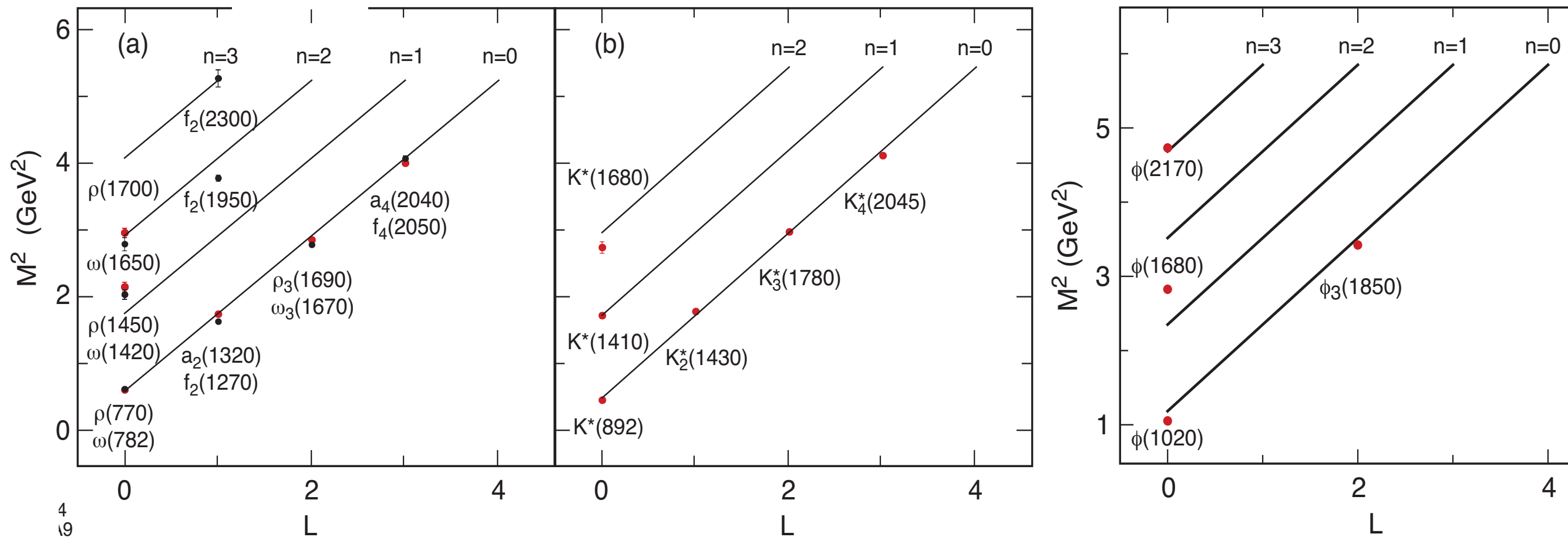
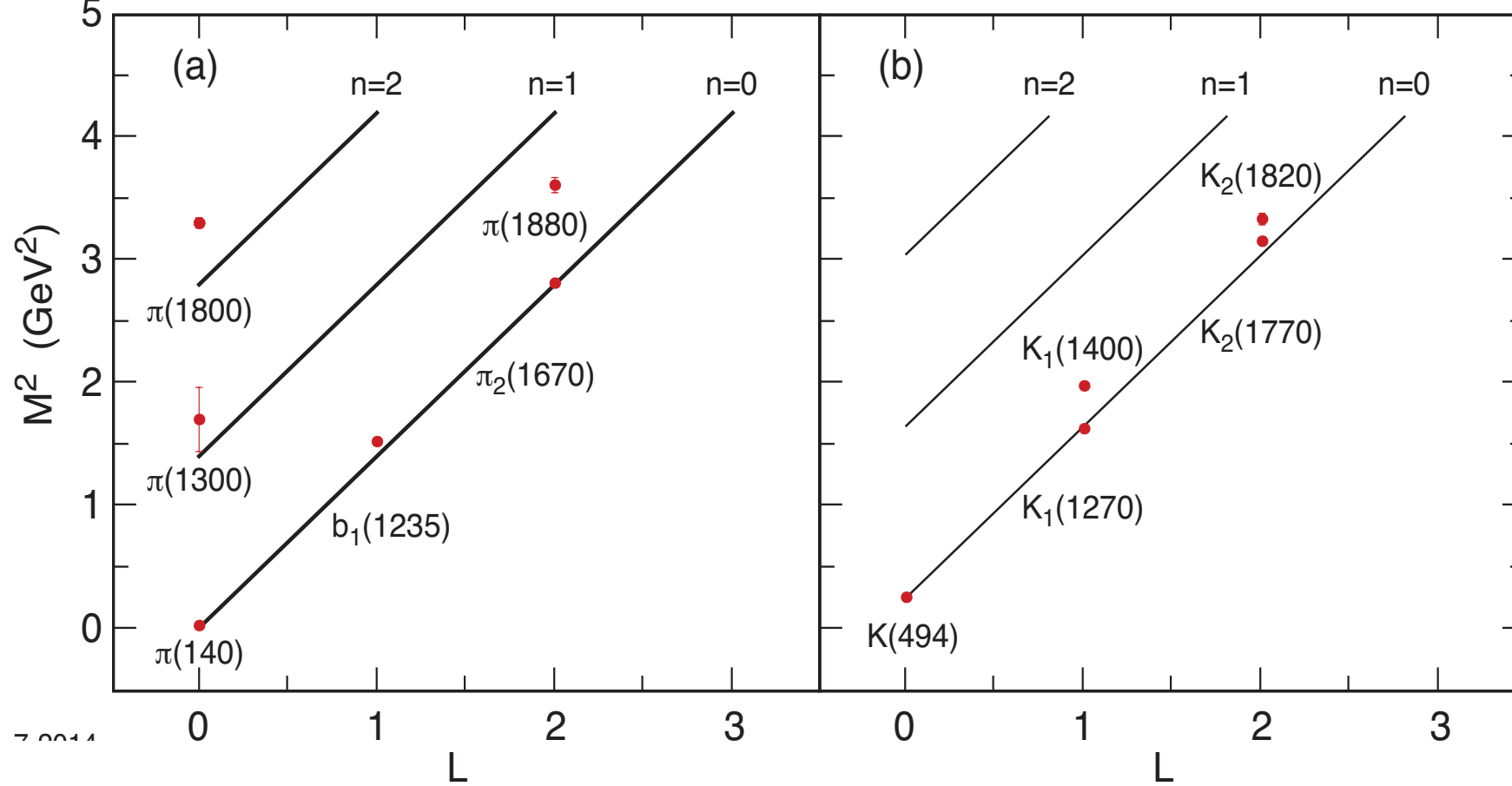
- **Poincarè Invariance: Independent of the observer's Lorentz frame: Quantization at Fixed Light-Front Time  $\tau$**
- **Causality: Information within causal horizon: Light-Front**
- **Light-Front Holography:  $AdS_5 = LF (3+1)$**

$$z \leftrightarrow \zeta \text{ where } \zeta^2 = b_{\perp}^2 x(1-x)$$



- **Introduce Mass Scale  $\kappa$  while retaining the Conformal Invariance of the AdS Action (dAFF)**
- **Unique Dilaton in  $AdS_5$ :  $e^{+\kappa^2 z^2}$**
- **Unique color-confining LF Potential  $U(\zeta^2) = \kappa^4 \zeta^2$**
- **Superconformal Algebra: Mass Degenerate 4-Plet:**

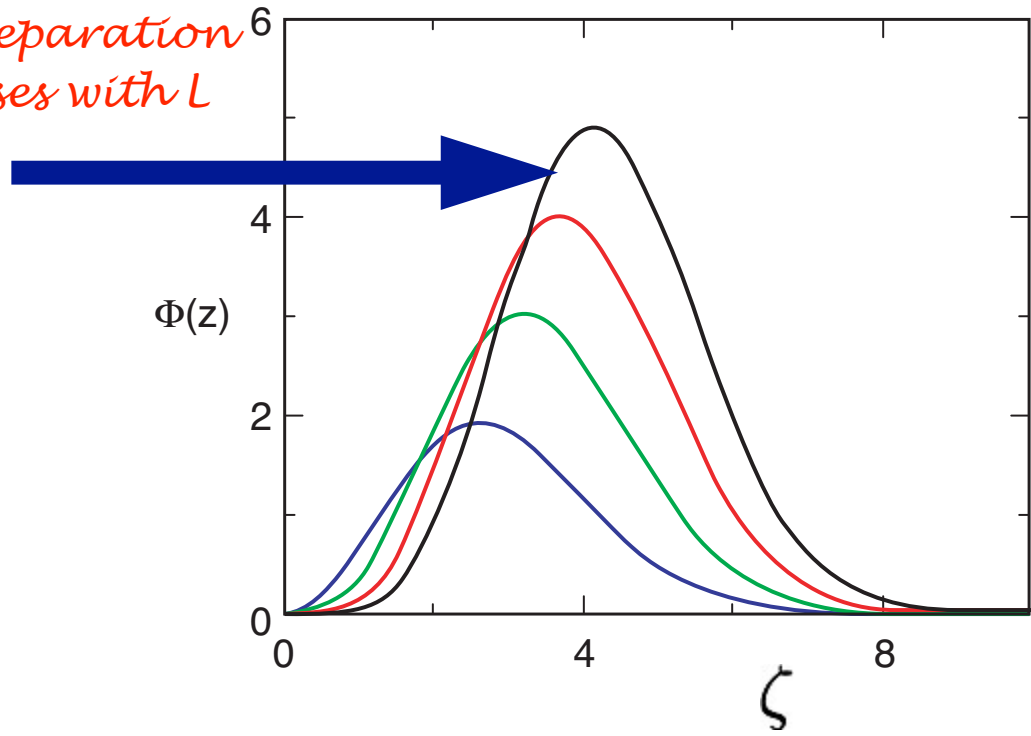
Meson  $q\bar{q} \leftrightarrow$  Baryon  $q[qq] \leftrightarrow$  Tetraquark  $[qq][\bar{q}\bar{q}]$



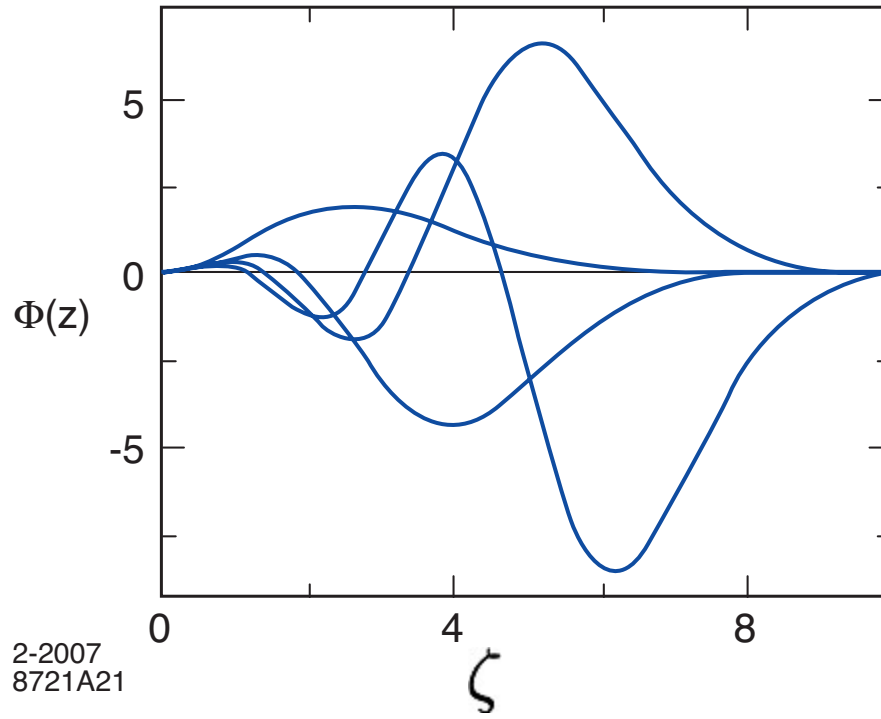
$$M^2(n, L, S) = 4\kappa^2(n + L + S/2)$$

*Equal Slope in  $n$  and  $L$*

Quark separation increases with  $L$



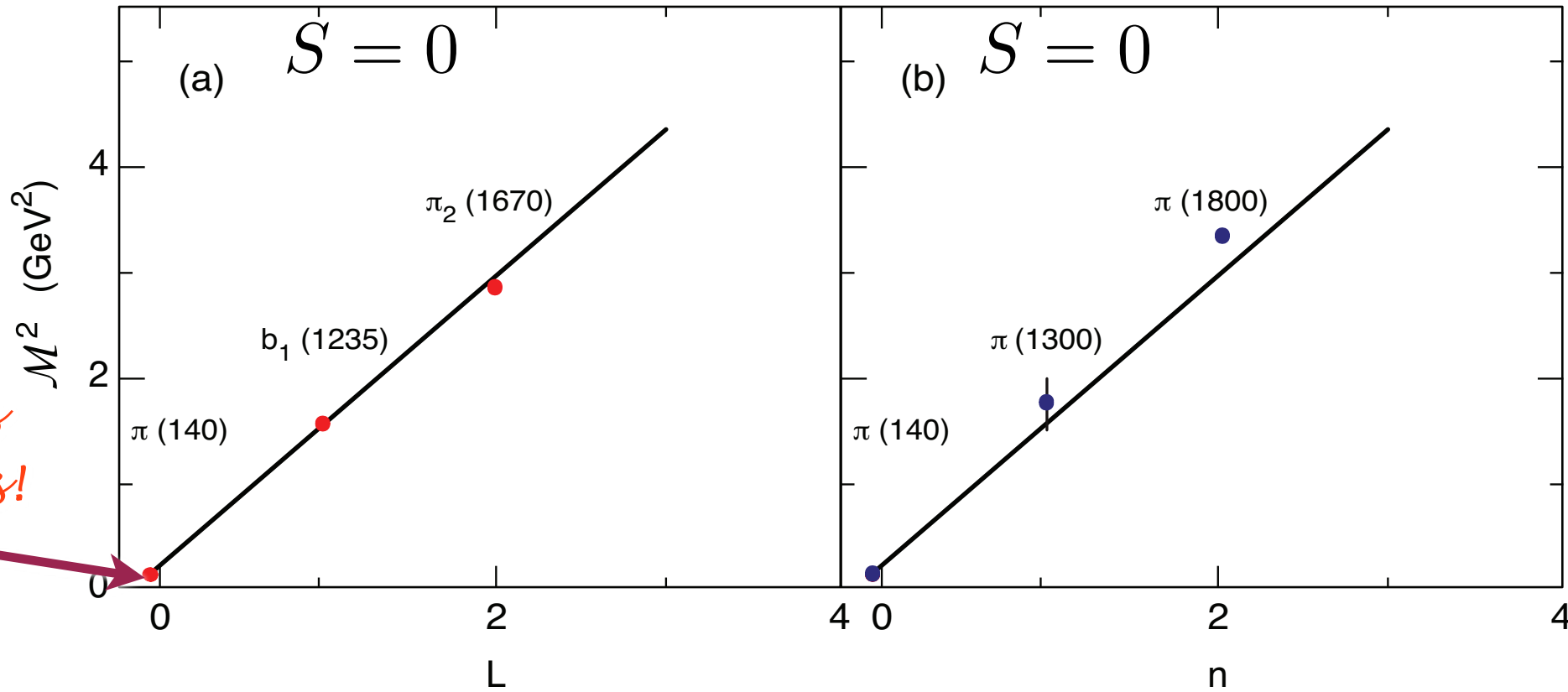
2-2007  
8721A21



Soft Wall Model

Fig: Orbital and radial AdS modes in the soft wall model for  $\kappa = 0.6$  GeV .

Same slope in  $n$  and  $L$ !



Pion has zero mass!

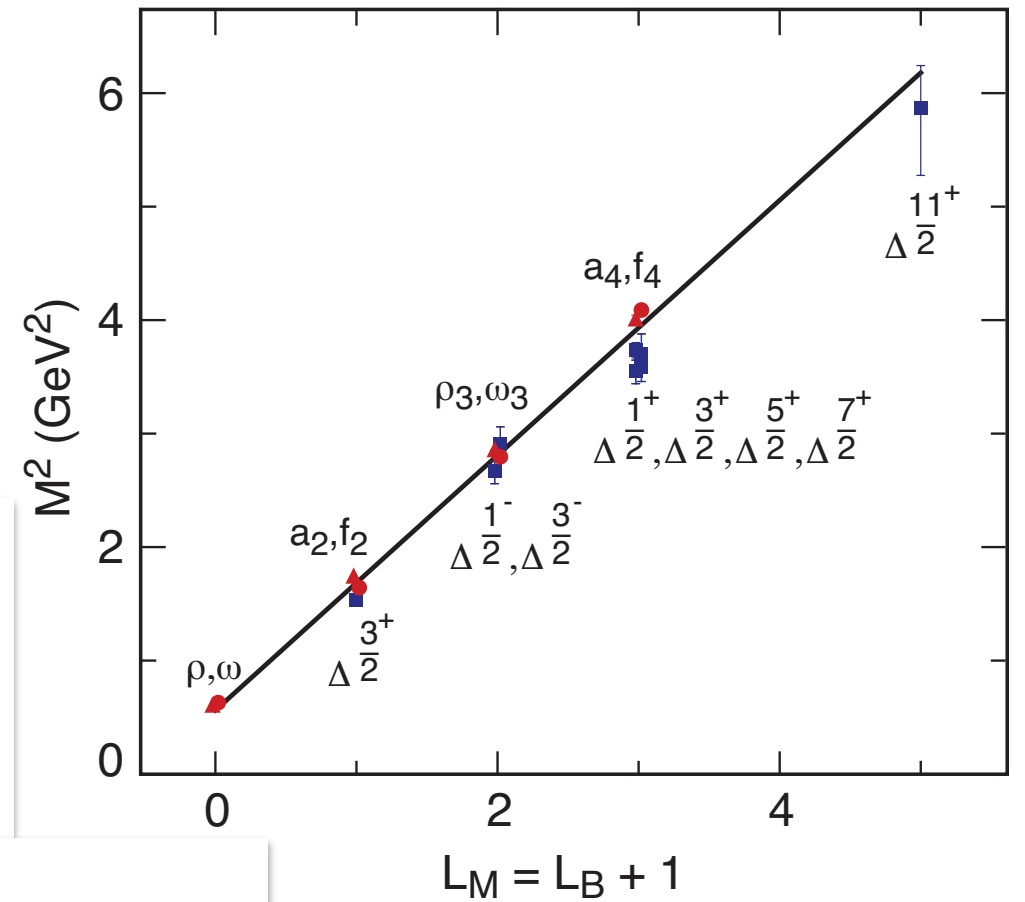
$$m_q = 0$$

Light meson orbital (a) and radial (b) spectrum for  $\kappa = 0.6$  GeV.

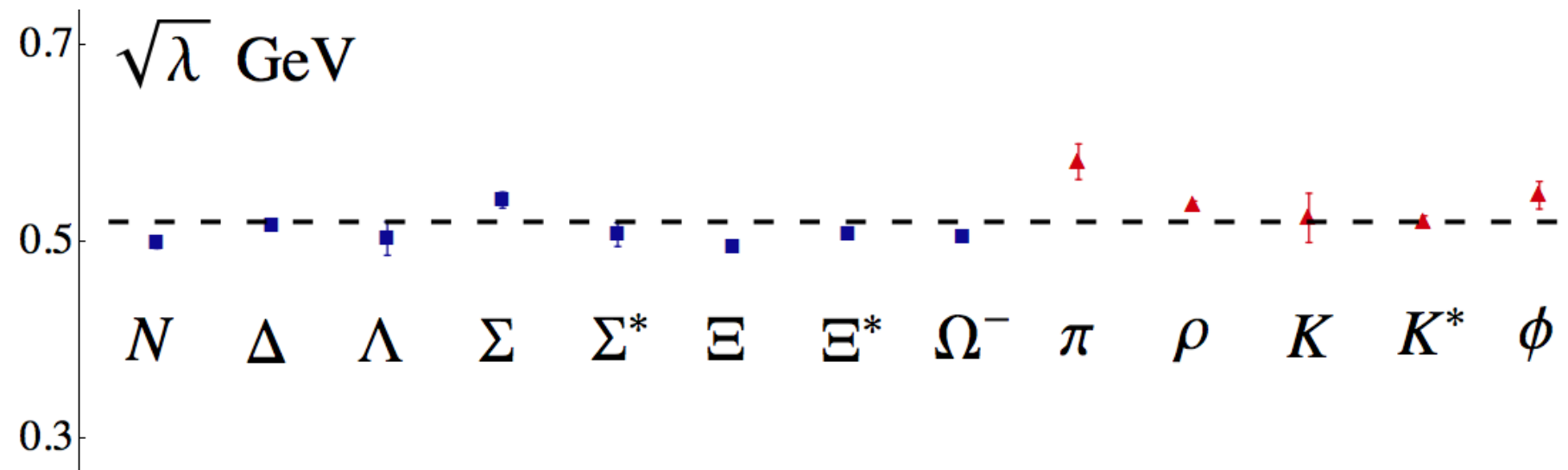


# Universal Regge Slope in L and n

$$\kappa = \sqrt{\lambda} = 0.523 \pm 0.024$$



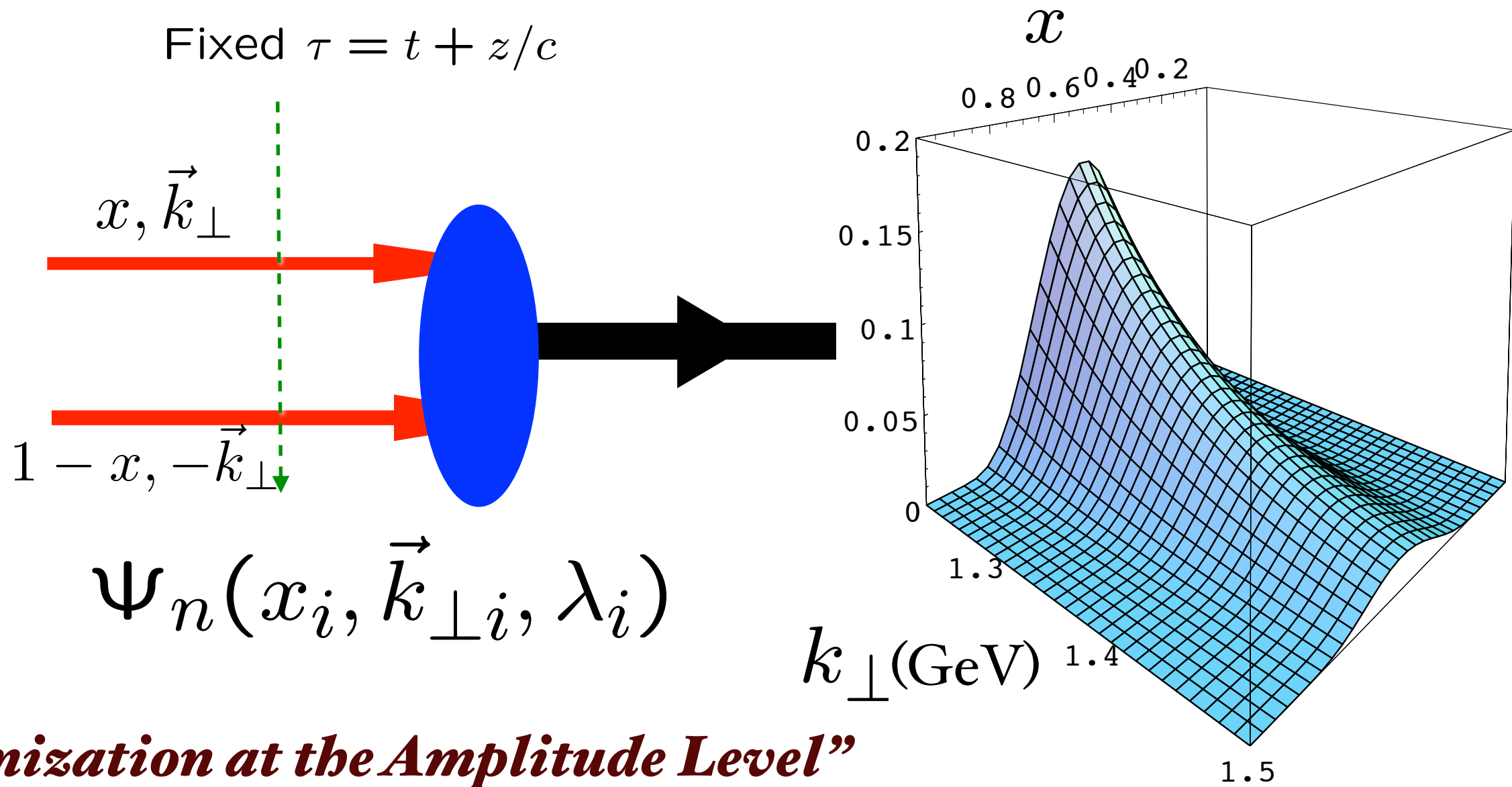
How universal is the semiclassical approximation based on superconformal LFHQCD ?



Best fit for hadronic scale  $\sqrt{\lambda}$  from different light hadron sectors including radial and orbital excitations

• *Light Front Wavefunctions:*  $\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$

off-shell in  $P^-$  and invariant mass  $\mathcal{M}_{q\bar{q}}^2$



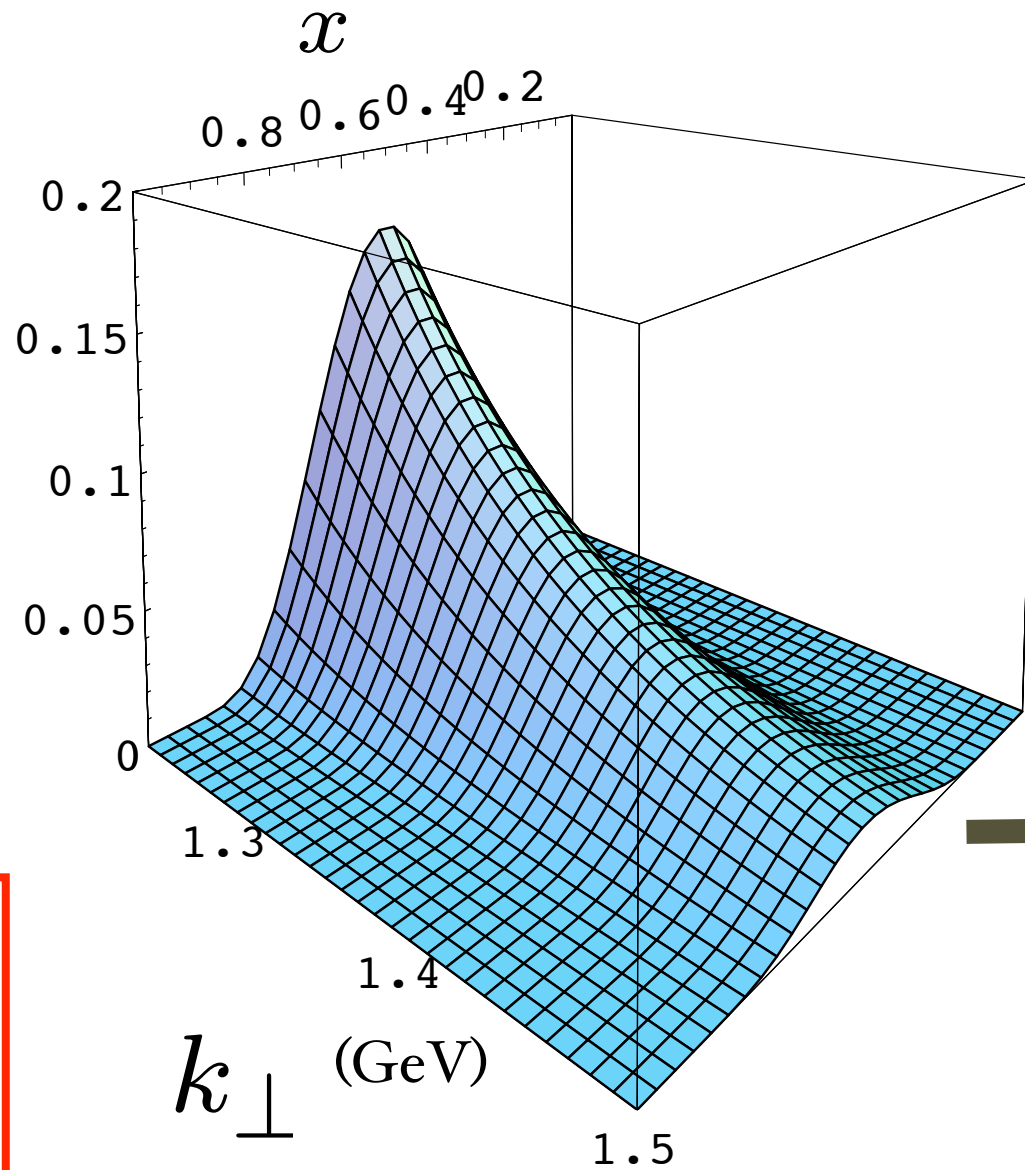
***“Hadronization at the Amplitude Level”***

**Boost-invariant LFWF connects confined quarks and gluons to hadrons**

# Prediction from AdS/QCD: Meson LFWF

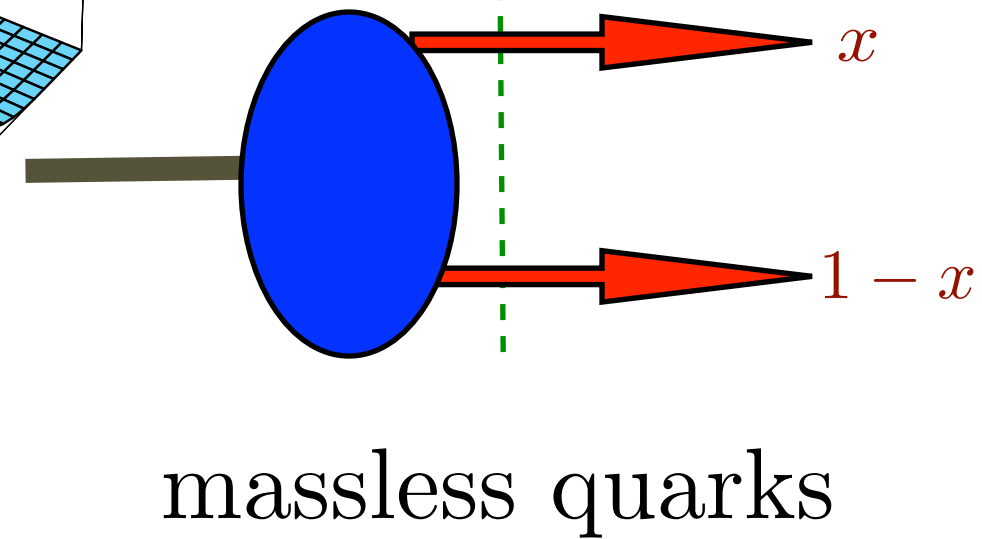
$$e^{\varphi(z)} = e^{+\kappa^2 z}$$

$$\psi_M(x, k_{\perp}^2)$$



de Teramond,  
Cao, sjb

“Soft Wall”  
model



**Note coupling**

$$k_{\perp}^2, x$$

$$\psi_M(x, k_{\perp}) = \frac{4\pi}{\kappa \sqrt{x(1-x)}} e^{-\frac{k_{\perp}^2}{2\kappa^2 x(1-x)}}$$

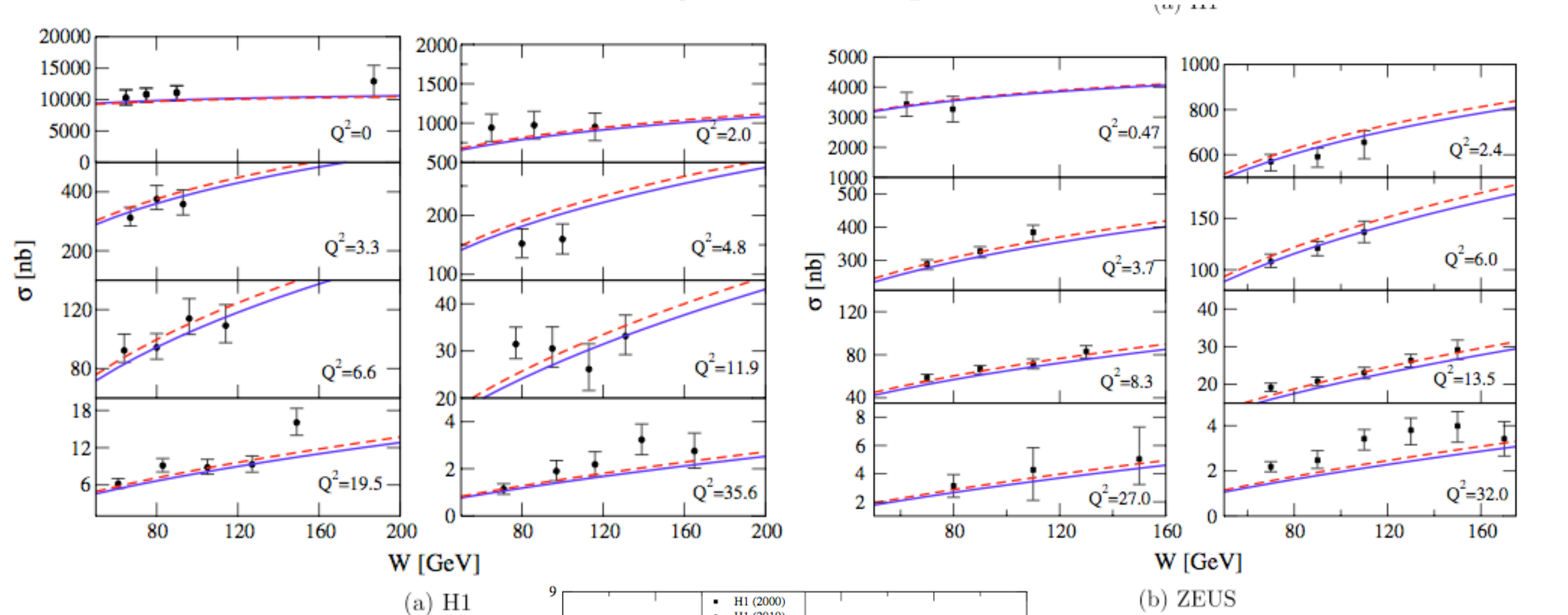
$$\phi_{\pi}(x) = \frac{4}{\sqrt{3}\pi} f_{\pi} \sqrt{x(1-x)}$$

$$f_{\pi} = \sqrt{P_{q\bar{q}}} \frac{\sqrt{3}}{8} \kappa = 92.4 \text{ MeV}$$

**Same as DSE!** C. D. Roberts et al.

*Provides Connection of Confinement to Hadron Structure*

### AdS/QCD Holographic Wave Function for the $\rho$ Meson and Diffractive $\rho$ Meson Electroproduction

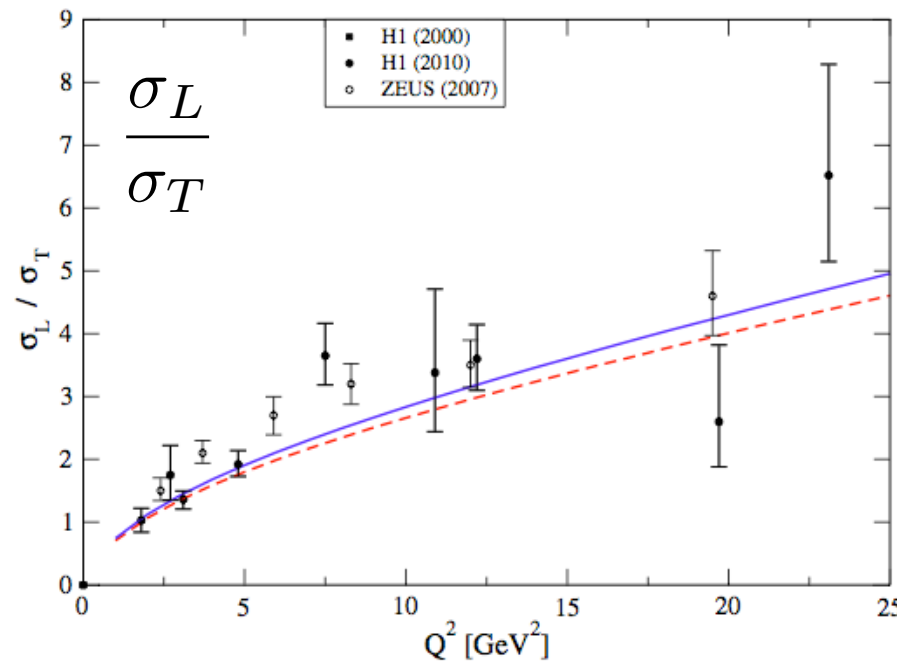


(a) H1

(b) ZEUS

**J. R. Forshaw,  
R. Sandapen**

$$\gamma^* p \rightarrow \rho^0 p'$$



$$\psi_M(x, k_\perp) = \frac{4\pi}{\kappa\sqrt{x(1-x)}} e^{-\frac{k_\perp^2}{2\kappa^2 x(1-x)}}$$

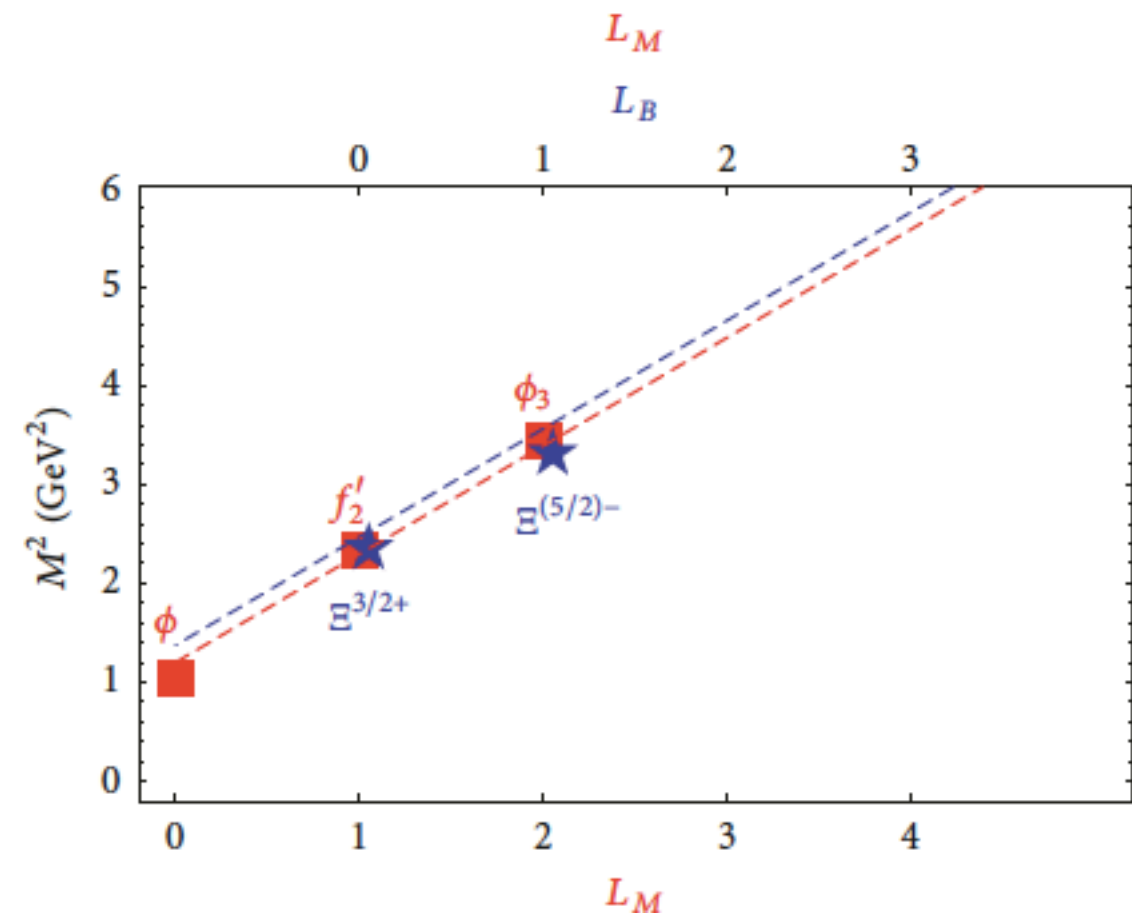
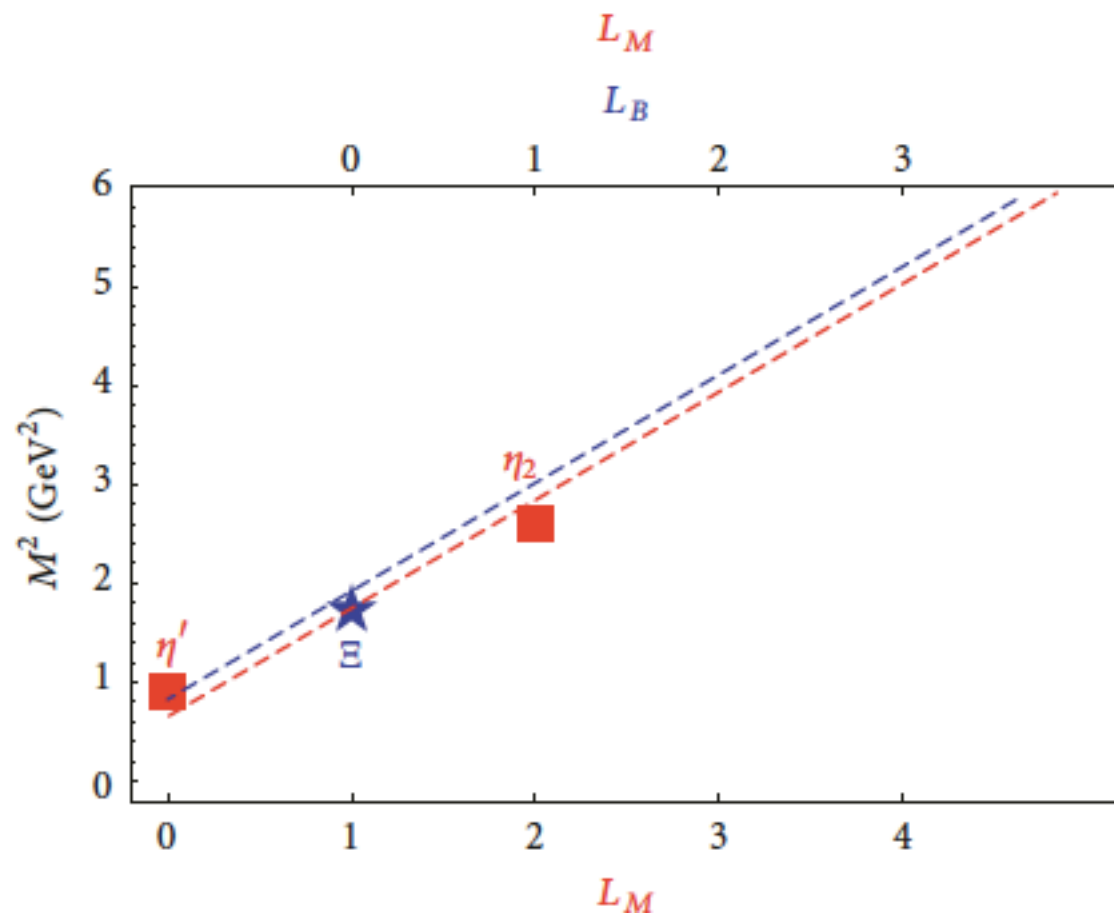
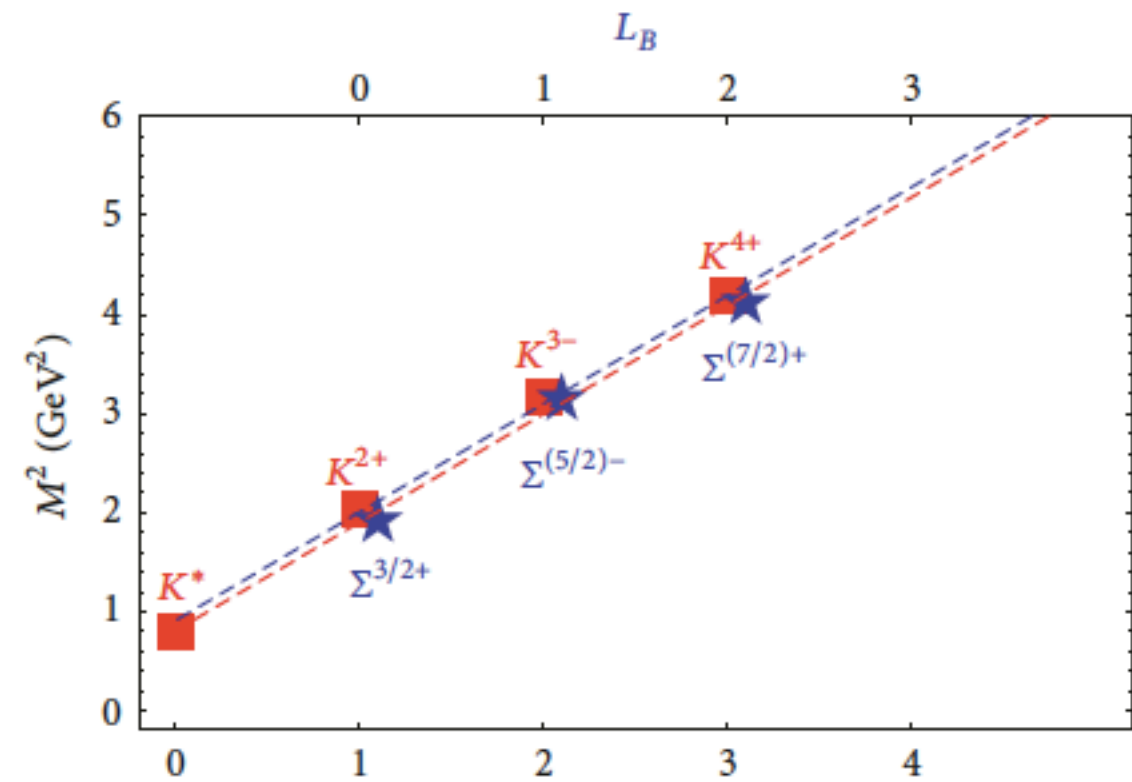
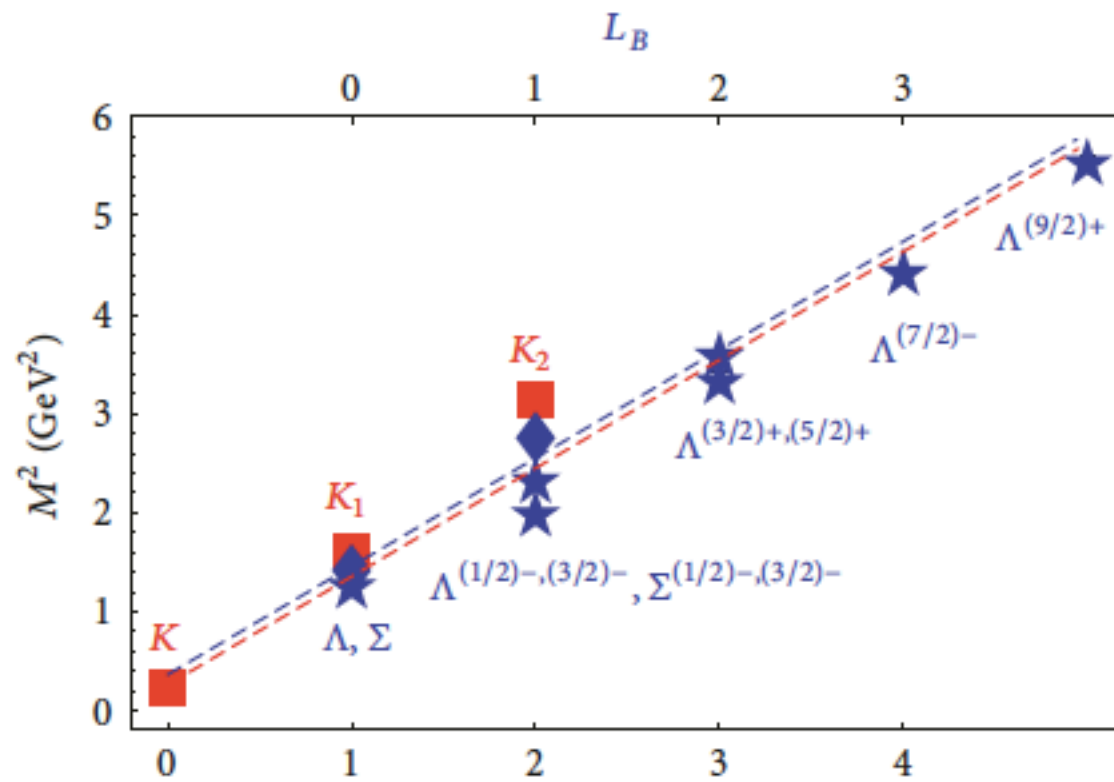


# *Light-Front Holography: First Approximation to QCD*

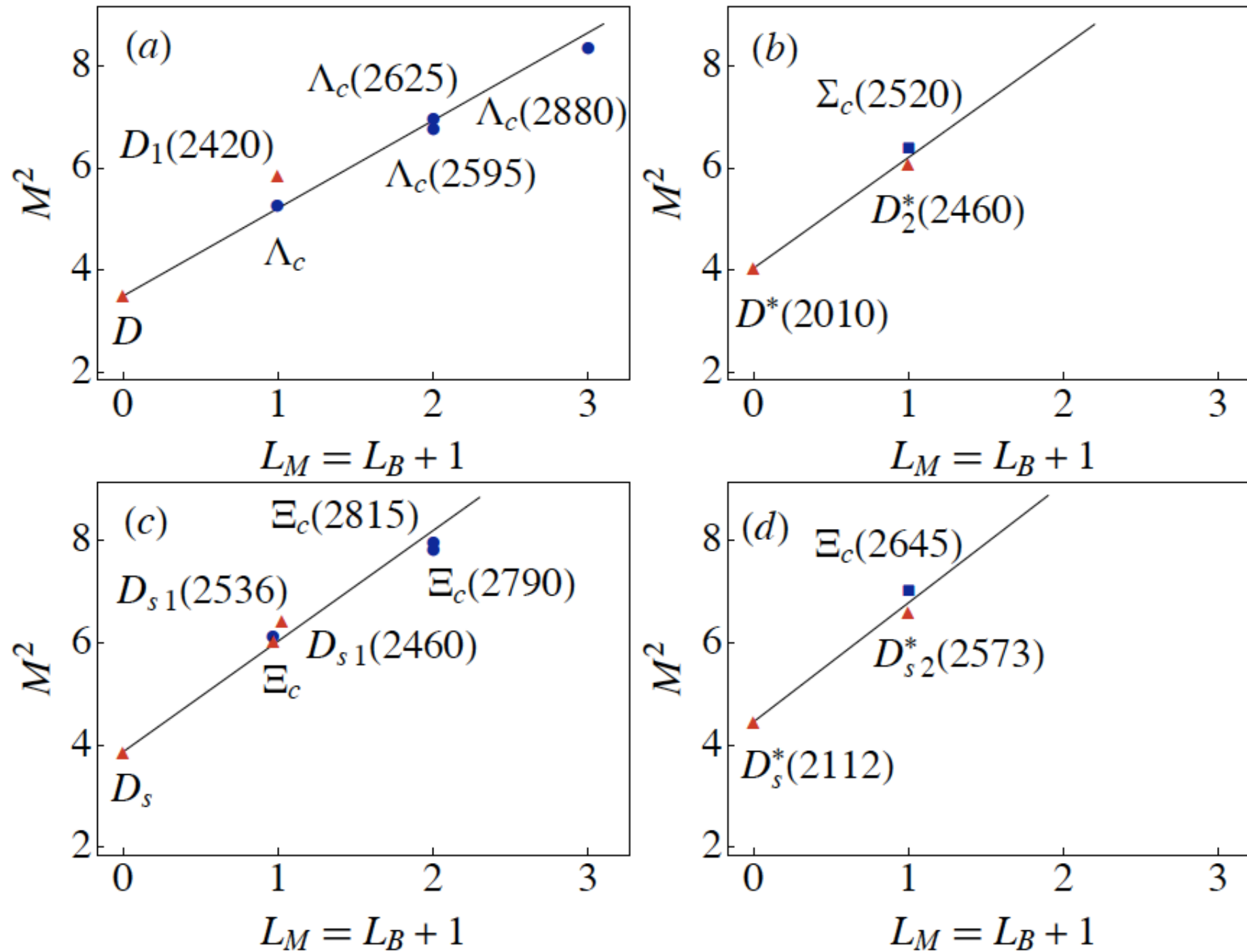
- **Color Confinement, Analytic form of confinement potential**  
*de Téramond, Dosch, Lorcé, sjb*
- **Retains underlying conformal properties of QCD despite mass scale (DeAlfaro-Fubini-Furlan Principle)**
- **Massless quark-antiquark pion bound state in chiral limit, GMOR**
- **QCD coupling at all scales**
- **Connection of perturbative and nonperturbative mass scales**
- **Poincarè Invariant**
- **Hadron Spectroscopy-Regge Trajectories with universal slopes in  $n, L$**
- **Supersymmetric 4-Plet: Meson-Baryon-Tetraquark Symmetry**
- **Light-Front Wavefunctions**
- **Form Factors, Structure Functions, Hadronic Observables**
- **OPE: Constituent Counting Rules**
- **Hadronization at the Amplitude Level: Many Phenomenological Tests**
- **Systematically improvable: Basis LF Quantization (BLFQ)**

*Supersymmetric Features of Hadron Physics  
from Superconformal Algebra  
and Light-Front Holography*

# Supersymmetry across the light and heavy-light spectrum



# Supersymmetry across the light and heavy-light spectrum

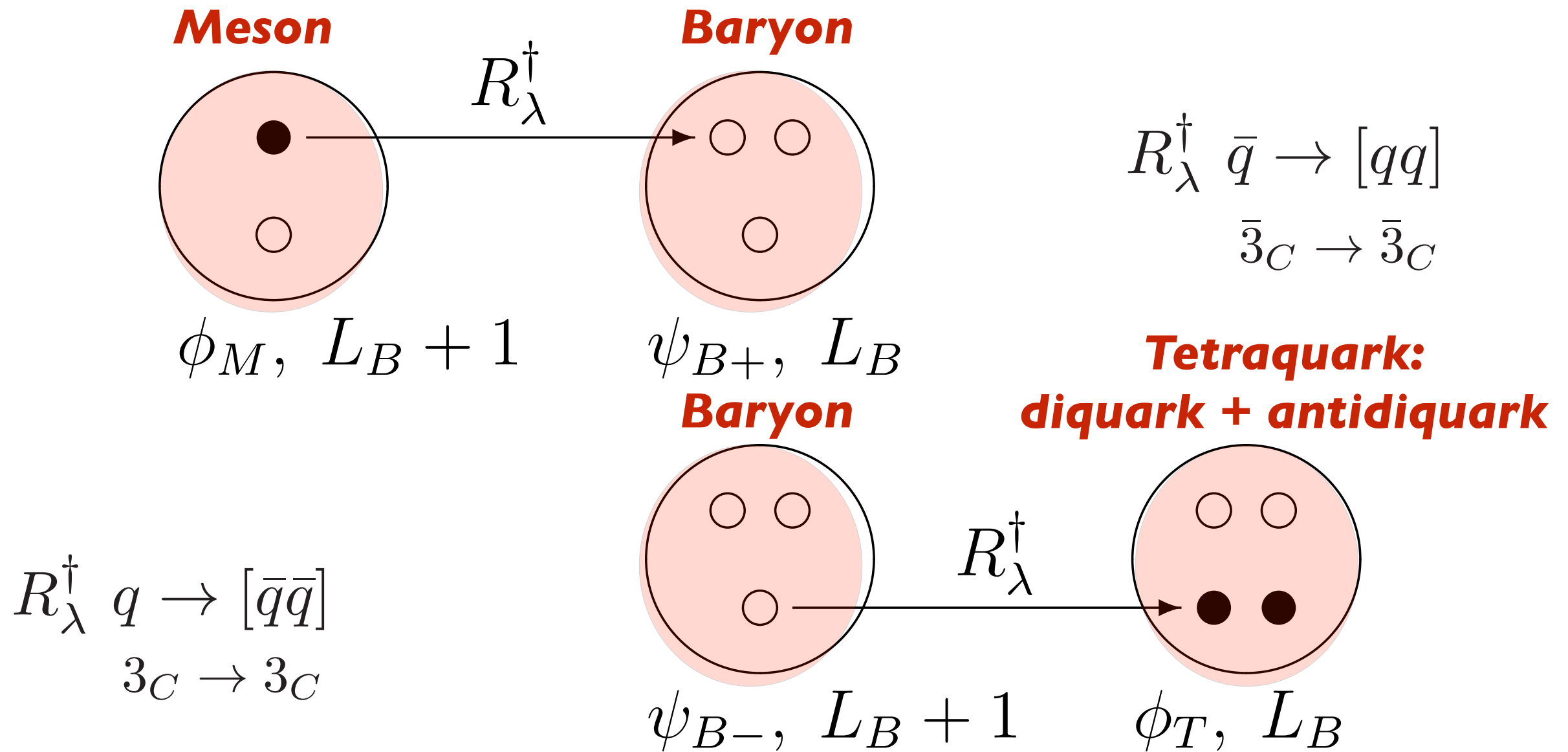


**Heavy charm quark mass does not break supersymmetry**

# Superconformal Algebra

## Four-Plet Representations

Bosons, Fermions with Equal Mass!



Proton:  $|u[ud]\rangle$  Quark + Scalar Diquark  
 Equal Weight:  $L=0, L=1$

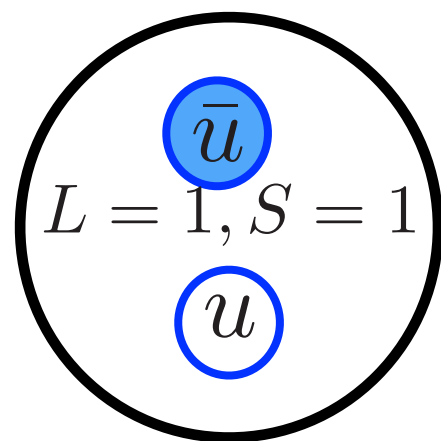


# Superconformal Algebra 4-Plet

$$R_\lambda^\dagger \begin{array}{l} \bar{q} \rightarrow (qq) \\ \bar{3}_C \rightarrow \bar{3}_C \end{array} S = 1$$

Vector ( ) + Scalar [ ] Diquarks

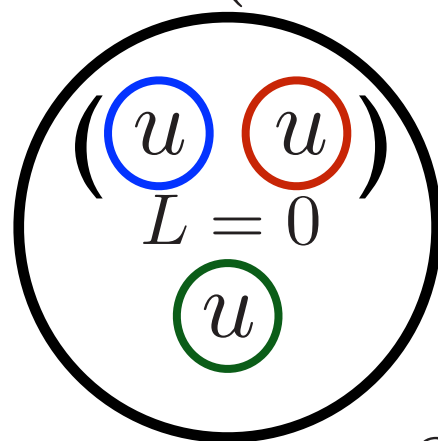
$f_2(1270)$



$$J^{PC} = 2^{++}$$

**Meson**

$\Delta^+(1232)$



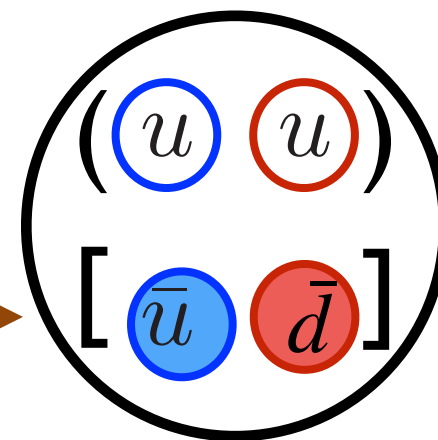
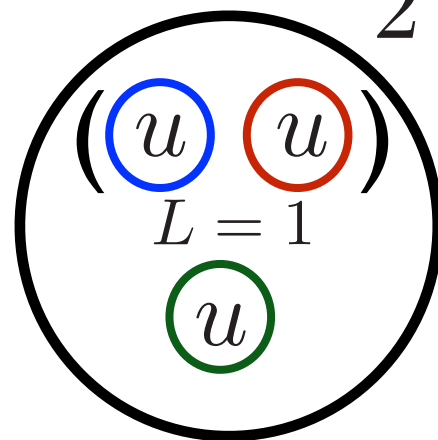
$$J^P = \frac{3}{2}^+$$

**Baryon**

**Tetraquark**

$$J^{PC} = 1^{++}$$

$a_1(1260)$



$$\begin{array}{l} S = 0 \\ L = 0 \end{array}$$

$$R_\lambda^\dagger \begin{array}{l} q \rightarrow [\bar{q}\bar{q}] \\ 3_C \rightarrow 3_C \end{array}$$

# Superconformal meson-baryon-tetraquark symmetry

*H. G. Dosch, G. d-Te'ramond, sjb, PRD 91, 085016 (2015)*

Upon the substitution in the superconformal equations

$$x \mapsto \zeta, \quad E \mapsto M^2,$$

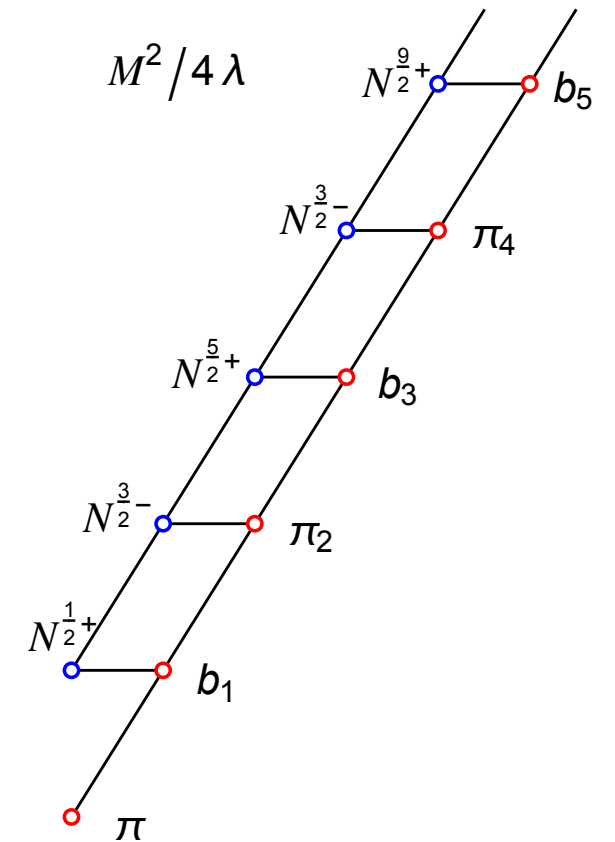
$$\lambda \mapsto \lambda_B = \lambda_M, \quad f \mapsto L_M - \frac{1}{2} = L_B + \frac{1}{2}$$

$$\phi_1 \mapsto \phi_M, \quad \phi_2 \mapsto \phi_B$$

we find the LF meson/baryon bound-state equations

$$\left( -\frac{d^2}{d\zeta^2} + \frac{4L_M^2 - 1}{4\zeta^2} + \lambda_M^2 \zeta^2 + 2\lambda_M(L_M - 1) \right) \phi_M = M^2 \phi_M$$

$$\left( -\frac{d^2}{d\zeta^2} + \frac{4L_B^2 - 1}{4\zeta^2} + \lambda_B^2 \zeta^2 + 2\lambda_B(L_B + 1) \right) \phi_B = M^2 \phi_B$$



$$\Phi = \begin{pmatrix} \phi_M & \phi_B^- \\ \phi_B^+ & \phi_T \end{pmatrix}$$

Superconformal QM imposes the condition  $\lambda = \lambda_M = \lambda_B$  (equality of Regge slopes)

and the remarkable relation  $L_M = L_B + 1$

$L_M$  is the LF angular momentum between the quark and antiquark in the meson and  $L_B$  between the active quark and spectator diquark cluster in the baryon

Full hadron 4-plet: meson-baryon-tetraquark

Meson			Baryon			Tetraquark		
$q\text{-cont}$	$J^{P(C)}$	Name	$q\text{-cont}$	$J^P$	Name	$q\text{-cont}$	$J^{P(C)}$	Name
$\bar{q}q$	$0^{-+}$	$\pi(140)$	—	—	—	—	—	—
$\bar{q}q$	$1^{+-}$	$b_1(1235)$	$[ud]q$	$(1/2)^+$	$N(940)$	$[ud][\bar{u}\bar{d}]$	$0^{++}$	$f_0(980)$
$\bar{q}q$	$2^{-+}$	$\pi_2(1670)$	$[ud]q$	$(1/2)^-$	$N_{\frac{1}{2}}^-(1535)$	$[ud][\bar{u}\bar{d}]$	$1^{-+}$	$\pi_1(1400)$
				$(3/2)^-$	$N_{\frac{3}{2}}^-(1520)$			$\pi_1(1600)$
$\bar{q}q$	$1^{--}$	$\rho(770), \omega(780)$	—	—	—	—	—	—
$\bar{q}q$	$2^{++}$	$a_2(1320), f_2(1270)$	$[qq]q$	$(3/2)^+$	$\Delta(1232)$	$[qq][\bar{u}\bar{d}]$	$1^{++}$	$a_1(1260)$
$\bar{q}q$	$3^{--}$	$\rho_3(1690), \omega_3(1670)$	$[qq]q$	$(1/2)^-$	$\Delta_{\frac{1}{2}}^-(1620)$	$[qq][\bar{u}\bar{d}]$	$2^{--}$	$\rho_2(\sim 1700)?$
				$(3/2)^-$	$\Delta_{\frac{3}{2}}^-(1700)$			
$\bar{q}q$	$4^{++}$	$a_4(2040), f_4(2050)$	$[qq]q$	$(7/2)^+$	$\Delta_{\frac{7}{2}}^+(1950)$	$[qq][\bar{u}\bar{d}]$	$3^{++}$	$a_3(\sim 2070)?$
$\bar{q}s$	$0^{-+}$	$K(495)$	—	—	—	—	—	—
$\bar{q}s$	$1^{+-}$	$\bar{K}_1(1270)$	$[ud]s$	$(1/2)^+$	$\Lambda(1115)$	$[ud][\bar{s}\bar{q}]$	$0^{++}$	$K_0^*(1430)$
$\bar{q}s$	$2^{-+}$	$K_2(1770)$	$[ud]s$	$(1/2)^-$	$\Lambda(1405)$	$[ud][\bar{s}\bar{q}]$	$1^{-+}$	$K_1^*(\sim 1700)?$
				$(3/2)^-$	$\Lambda(1520)$			
$\bar{s}q$	$0^{-+}$	$K(495)$	—	—	—	—	—	—
$\bar{s}q$	$1^{+-}$	$K_1(1270)$	$[sq]q$	$(1/2)^+$	$\Sigma(1190)$	$[sq][\bar{s}\bar{q}]$	$0^{++}$	$a_0(980)$ $f_0(980)$
$\bar{s}q$	$1^{-+}$	$K^*(890)$	—	—	—	—	—	—
$\bar{s}q$	$2^{++}$	$K_2^*(1430)$	$[sq]q$	$(3/2)^+$	$\Sigma(1385)$	$[sq][\bar{q}\bar{q}]$	$1^{++}$	$K_1(1400)$
$\bar{s}q$	$3^{-+}$	$K_3^*(1780)$	$[sq]q$	$(3/2)^-$	$\Sigma(1670)$	$[sq][\bar{q}\bar{q}]$	$2^{-+}$	$K_2(\sim 1700)?$
$\bar{s}q$	$4^{++}$	$K_4^*(2045)$	$[sq]q$	$(7/2)^+$	$\Sigma(2030)$	$[sq][\bar{q}\bar{q}]$	$3^{++}$	$K_3(\sim 2070)?$
$\bar{s}s$	$0^{-+}$	$\eta(550)$	—	—	—	—	—	—
$\bar{s}s$	$1^{+-}$	$h_1(1170)$	$[sq]s$	$(1/2)^+$	$\Xi(1320)$	$[sq][\bar{s}\bar{q}]$	$0^{++}$	$f_0(1370)$ $a_0(1450)$
$\bar{s}s$	$2^{-+}$	$\eta_2(1645)$	$[sq]s$	$(?)^?$	$\Xi(1690)$	$[sq][\bar{s}\bar{q}]$	$1^{-+}$	$\Phi'(1750)?$
$\bar{s}s$	$1^{--}$	$\Phi(1020)$	—	—	—	—	—	—
$\bar{s}s$	$2^{++}$	$f_2'(1525)$	$[sq]s$	$(3/2)^+$	$\Xi^*(1530)$	$[sq][\bar{s}\bar{q}]$	$1^{++}$	$f_1(1420)$
$\bar{s}s$	$3^{--}$	$\Phi_3(1850)$	$[sq]s$	$(3/2)^-$	$\Xi(1820)$	$[sq][\bar{s}\bar{q}]$	$2^{--}$	$\Phi_2(\sim 1800)?$
$\bar{s}s$	$2^{++}$	$f_2(1950)$	$[ss]s$	$(3/2)^+$	$\Omega(1672)$	$[ss][\bar{s}\bar{q}]$	$1^{++}$	$K_1(\sim 1700)?$

Meson

Baryon

Tetraquark

**New Organization of the Hadron Spectrum**

M. Nielsen,  
sjb



# Superpartners for states with one c quark

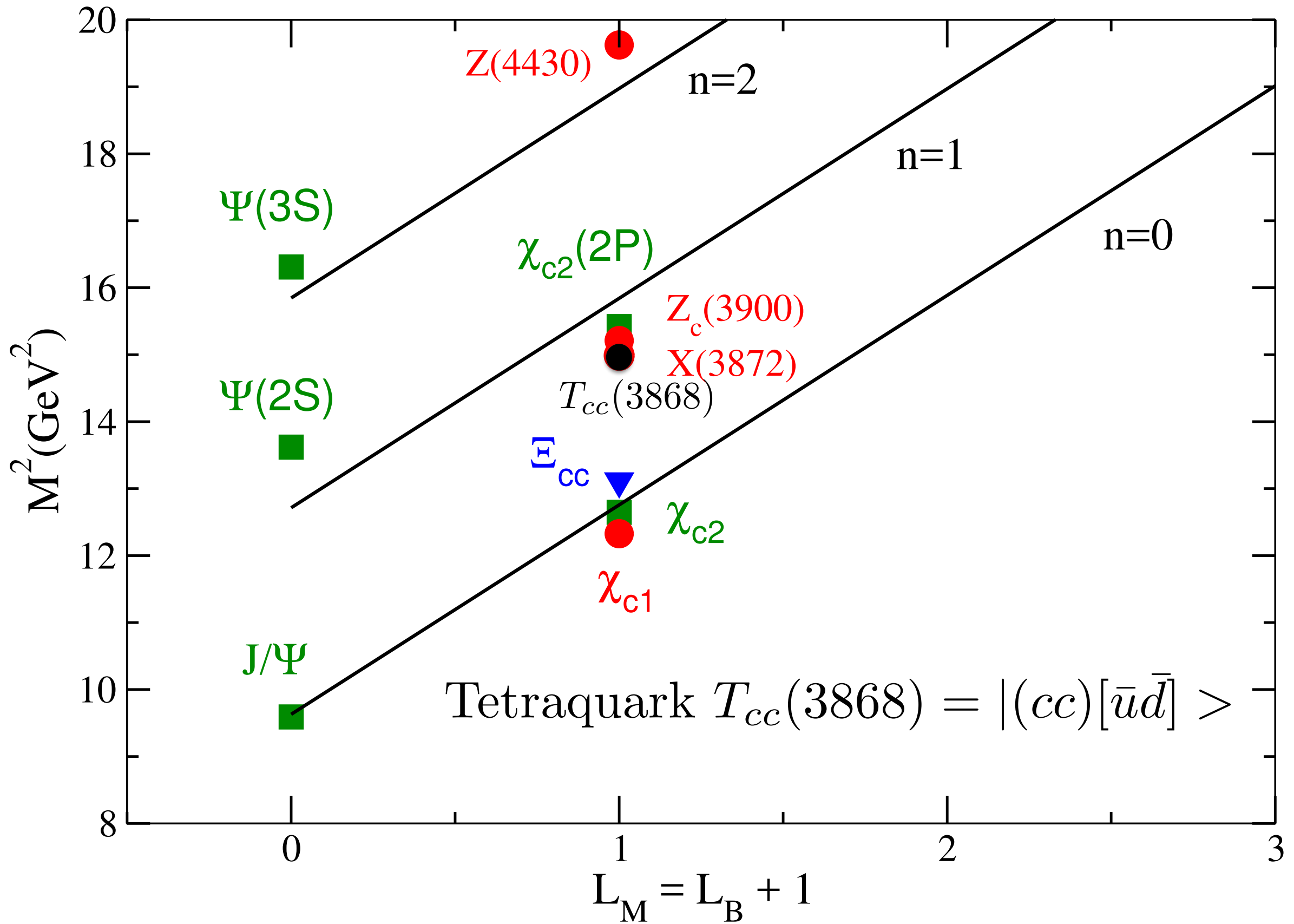
Meson			Baryon			Tetraquark		
$q$ -cont	$J^{P(C)}$	Name	$q$ -cont	$J^P$	Name	$q$ -cont	$J^{P(C)}$	Name
$\bar{q}c$	$0^-$	$D(1870)$	—	—	—	—	—	—
$\bar{q}c$	$1^+$	$D_1(2420)$	$[ud]c$	$(1/2)^+$	$\Lambda_c(2290)$	$[ud][\bar{c}\bar{q}]$	$0^+$	$\bar{D}_0^*(2400)$
$\bar{q}c$	$2^-$	$D_J(2600)$	$[ud]c$	$(3/2)^-$	$\Lambda_c(2625)$	$[ud][\bar{c}\bar{q}]$	$1^-$	—
$\bar{c}q$	$0^-$	$\bar{D}(1870)$	—	—	—	—	—	—
$\bar{c}q$	$1^+$	$\bar{D}_1(2420)$	$[cq]q$	$(1/2)^+$	$\Sigma_c(2455)$	$[cq][\bar{u}\bar{d}]$	$0^+$	$D_0^*(2400)$
$\bar{q}c$	$1^-$	$D^*(2010)$	—	—	—	—	—	—
$\bar{q}c$	$2^+$	$D_2^*(2460)$	$(qq)c$	$(3/2)^+$	$\Sigma_c^*(2520)$	$(qq)[\bar{c}\bar{q}]$	$1^+$	$D(2550)$
$\bar{q}c$	$3^-$	$D_3^*(2750)$	$(qq)c$	$(3/2)^-$	$\Sigma_c(2800)$	$(qq)[\bar{c}\bar{q}]$	—	—
$\bar{s}c$	$0^-$	$D_s(1968)$	—	—	—	—	—	—
$\bar{s}c$	$1^+$	$D_{s1}(2460)$	$[qs]c$	$(1/2)^+$	$\Xi_c(2470)$	$[qs][\bar{c}\bar{q}]$	$0^+$	$\bar{D}_{s0}^*(2317)$
$\bar{s}c$	$2^-$	$D_{s2}(\sim 2860)?$	$[qs]c$	$(3/2)^-$	$\Xi_c(2815)$	$[sq][\bar{c}\bar{q}]$	$1^-$	—
$\bar{s}c$	$1^-$	$D_s^*(2110)$	—	—	—	—	—	—
$\bar{s}c$	$2^+$	$D_{s2}^*(2573)$	$(sq)c$	$(3/2)^+$	$\Xi_c^*(2645)$	$(sq)[\bar{c}\bar{q}]$	$1^+$	$D_{s1}(2536)$
$\bar{c}s$	$1^+$	$\bar{D}_{s1}(\sim 2700)?$	$[cs]s$	$(1/2)^+$	$\Omega_c(2695)$	$[cs][\bar{s}\bar{q}]$	$0^+$	??
$\bar{s}c$	$2^+$	$D_{s2}^*(\sim 2750)?$	$(ss)c$	$(3/2)^+$	$\Omega_c(2770)$	$(ss)[\bar{c}\bar{s}]$	$1^+$	??

M. Nielsen, sjb

predictions

beautiful agreement!





Mesons : *GreenSquare*, Baryons (*BlueTriangle*), Tetraquarks (*RedCircle*)

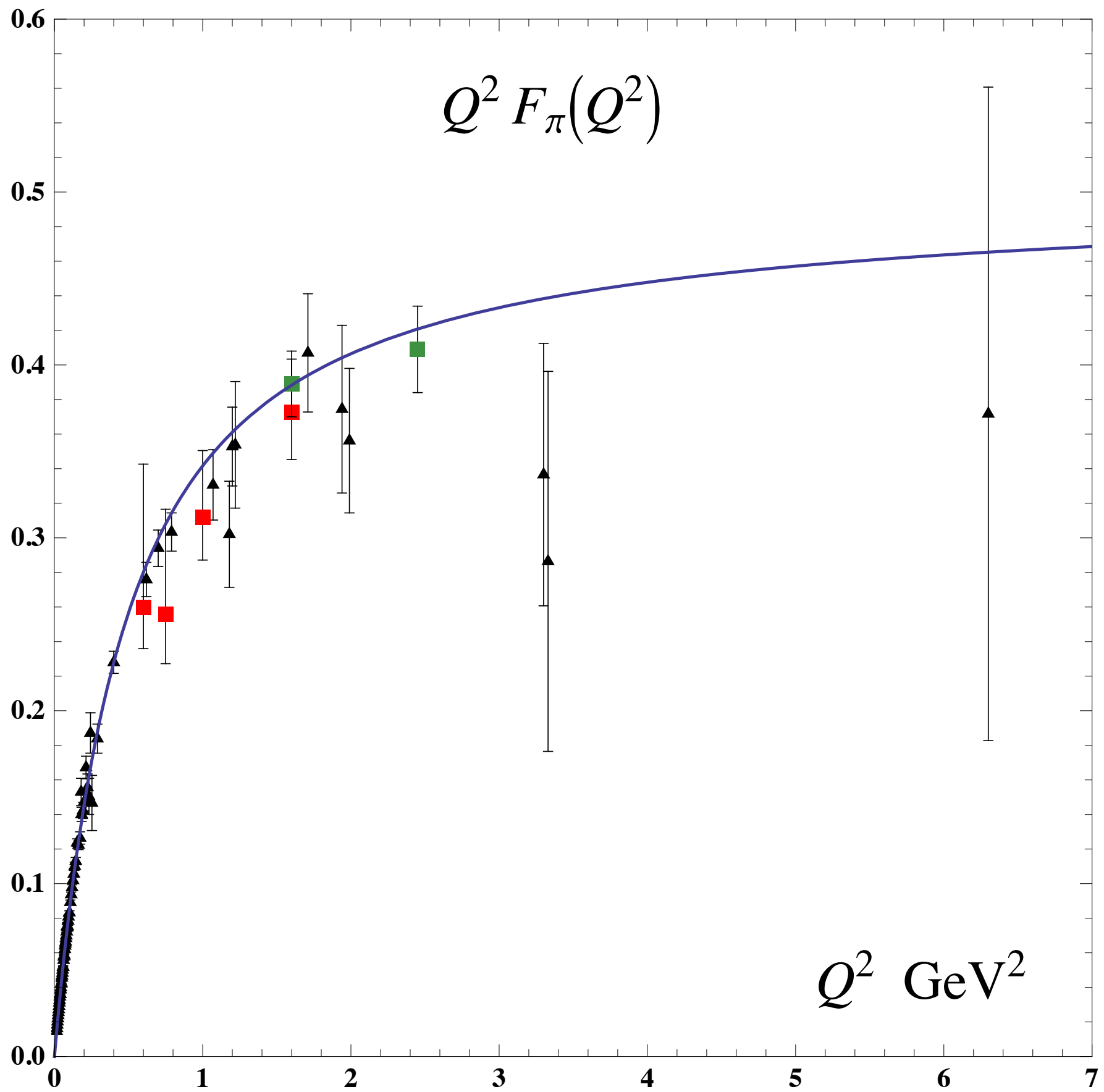
# Connection to the Linear Instant-Form Heavy Quark Potential

Harmonic Oscillator  $U(\zeta) = \kappa^4 \zeta^2$  LF Potential for relativistic light quarks

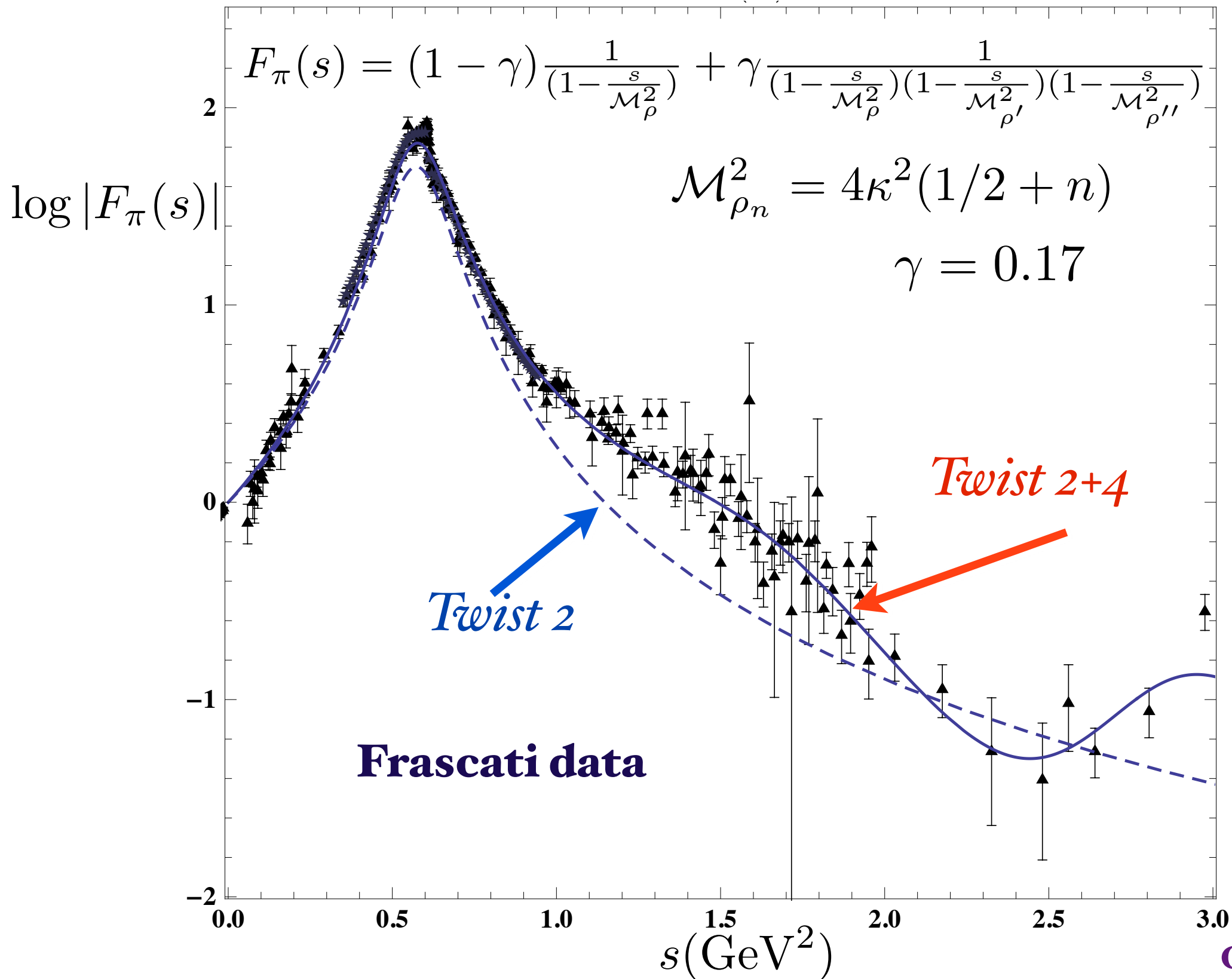


Linear instant nonrelativistic form  $V(r) = Cr$  for heavy quarks

A.P. Trawinski, S.D. Glazek, H. D. Dosch, G. de Teramond, sjb



# Timelike Pion Form Factor from AdS/QCD and Light-Front Holography

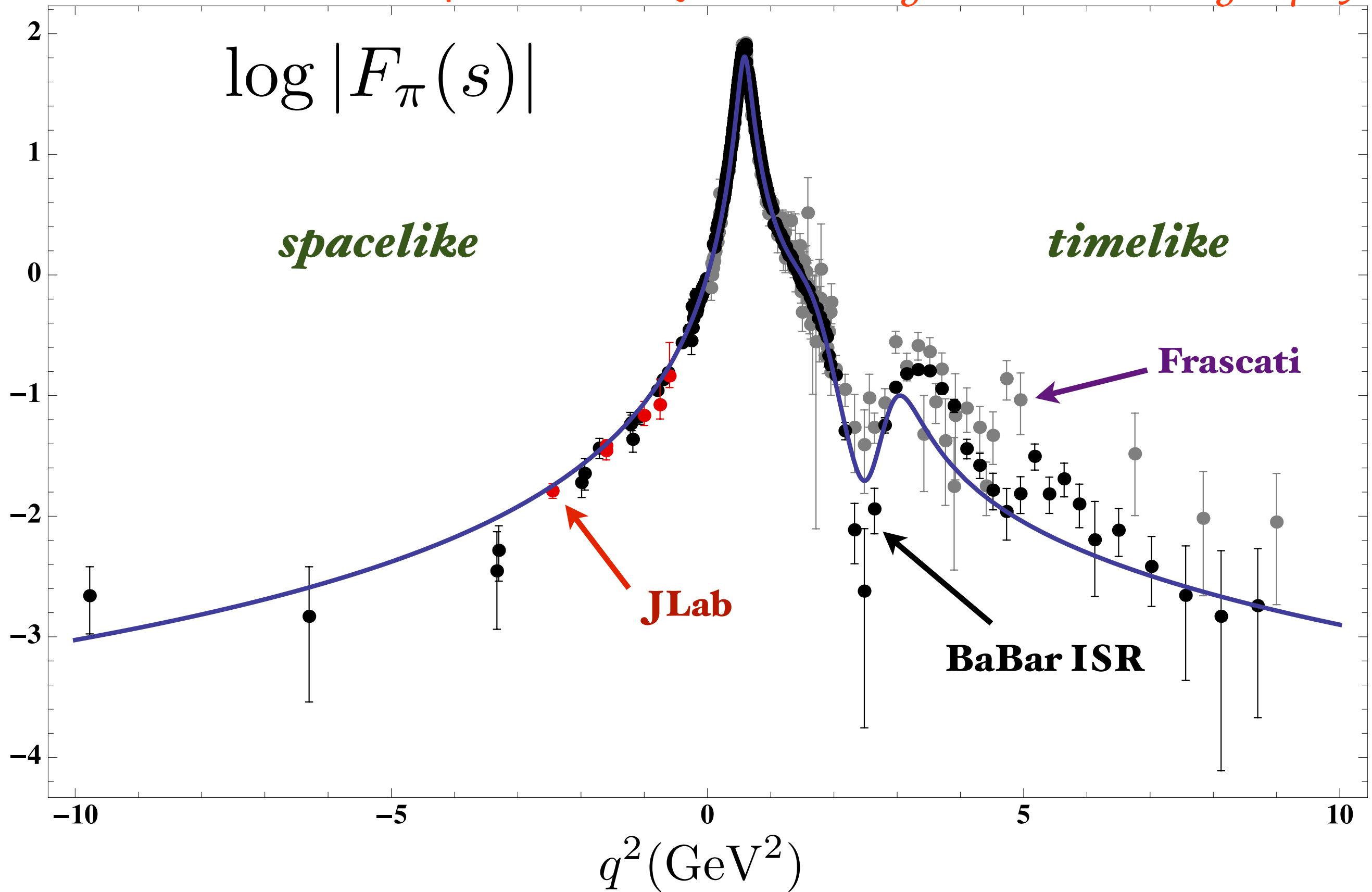


**Prescription for Timelike poles :**

$$\frac{1}{s - M^2 + i\sqrt{s}\Gamma}$$

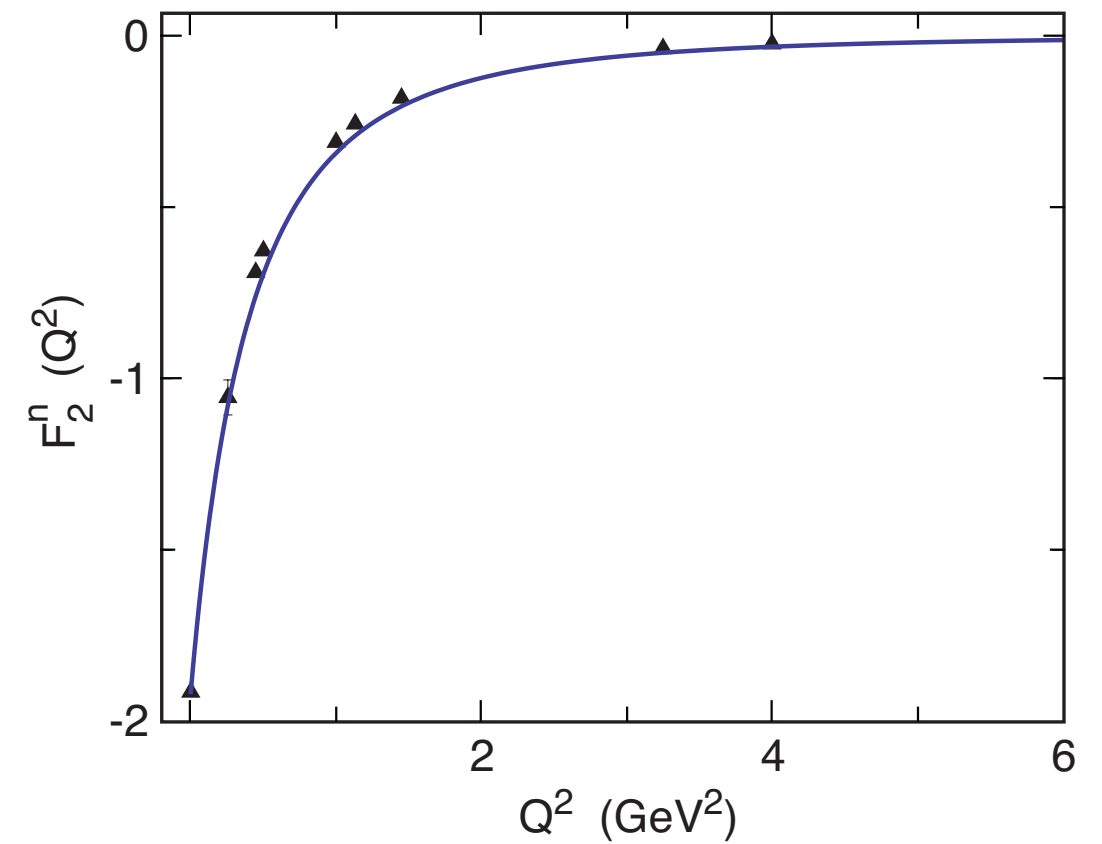
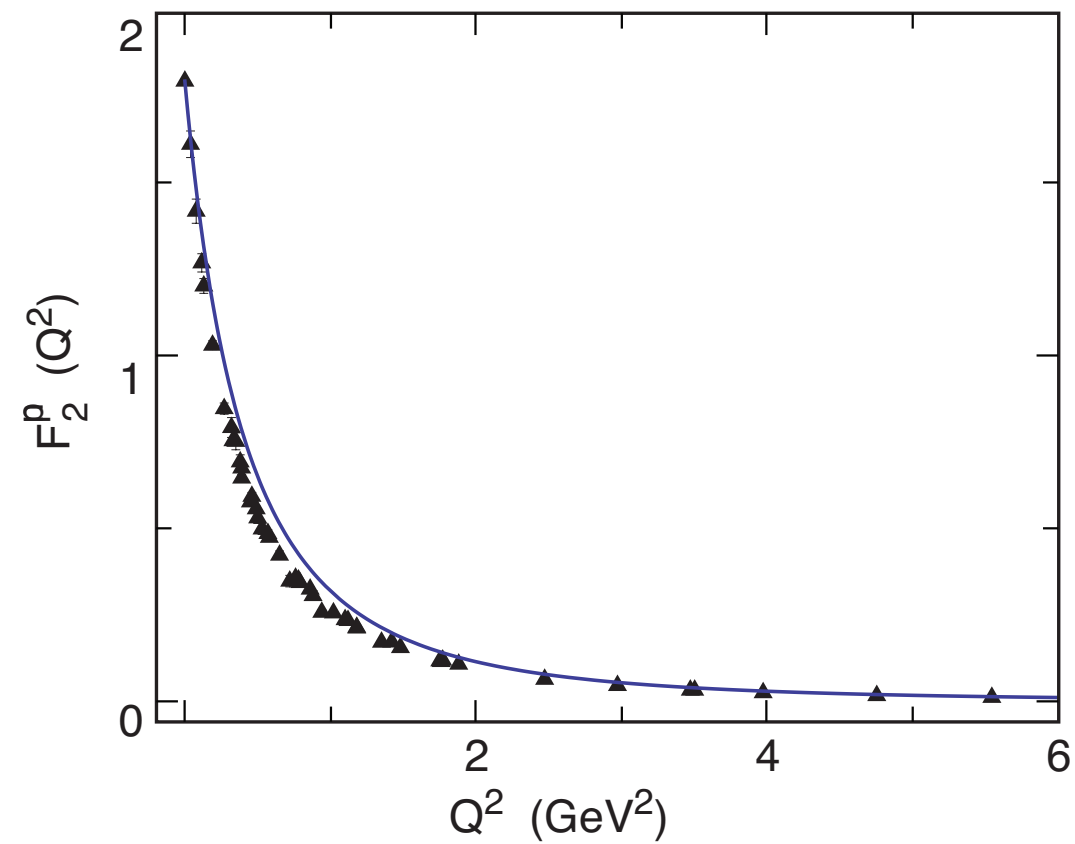
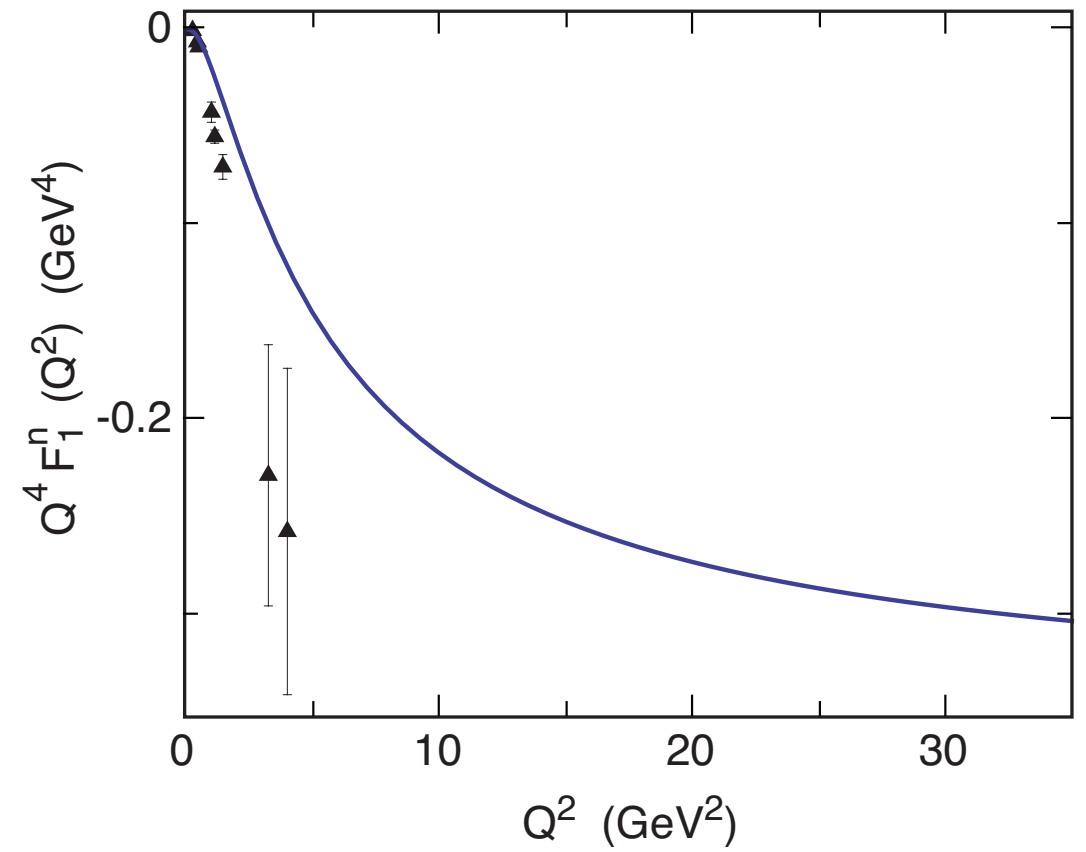
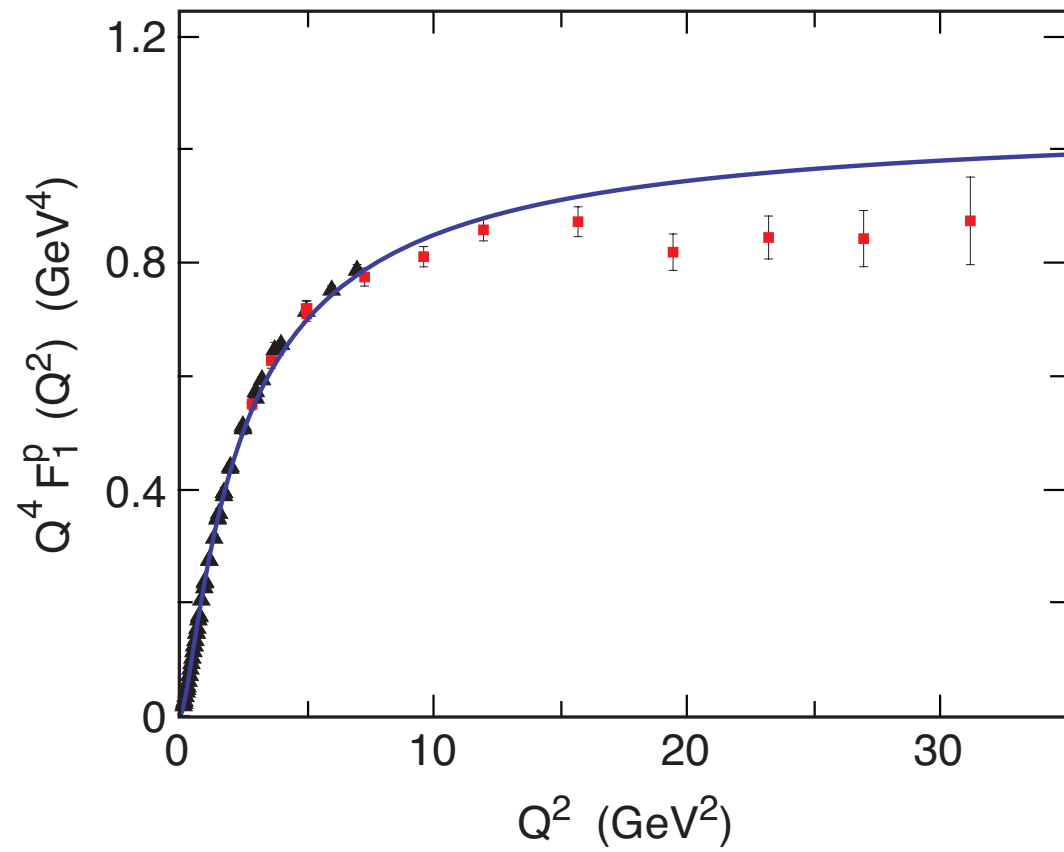
**14% four-quark probability**

# Pion Form Factor from AdS/QCD and Light-Front Holography





Using  $SU(6)$  flavor symmetry and normalization to static quantities



# Exact LF Formula for Pauli Form Factor

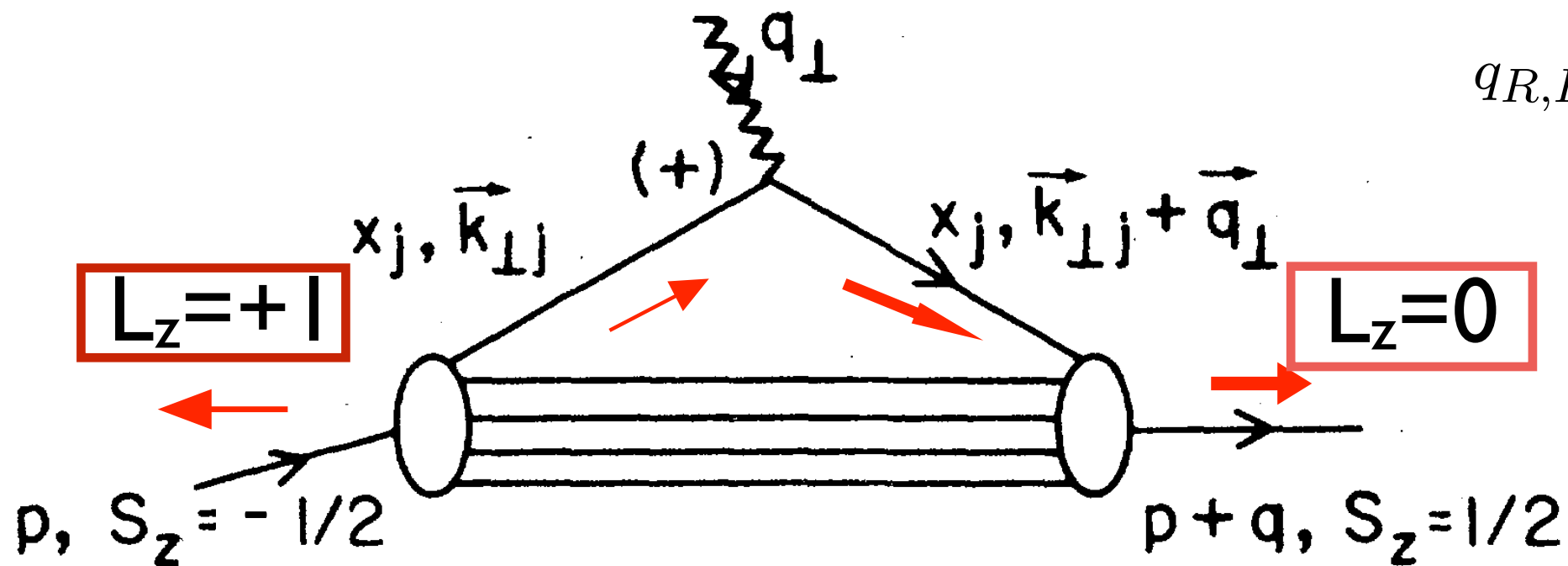
$$\frac{F_2(q^2)}{2M} = \sum_a \int [dx][d^2\mathbf{k}_\perp] \sum_j e_j \frac{1}{2} \times \quad \text{Drell, sjb}$$

$$\left[ -\frac{1}{q^L} \psi_a^{\uparrow*}(x_i, \mathbf{k}'_{\perp i}, \lambda_i) \psi_a^\downarrow(x_i, \mathbf{k}_{\perp i}, \lambda_i) + \frac{1}{q^R} \psi_a^{\downarrow*}(x_i, \mathbf{k}'_{\perp i}, \lambda_i) \psi_a^\uparrow(x_i, \mathbf{k}_{\perp i}, \lambda_i) \right]$$

$$\mathbf{k}'_{\perp i} = \mathbf{k}_{\perp i} - x_i \mathbf{q}_\perp$$

$$\mathbf{k}'_{\perp j} = \mathbf{k}_{\perp j} + (1 - x_j) \mathbf{q}_\perp$$

$$q_{R,L} = q^x \pm iq^y$$

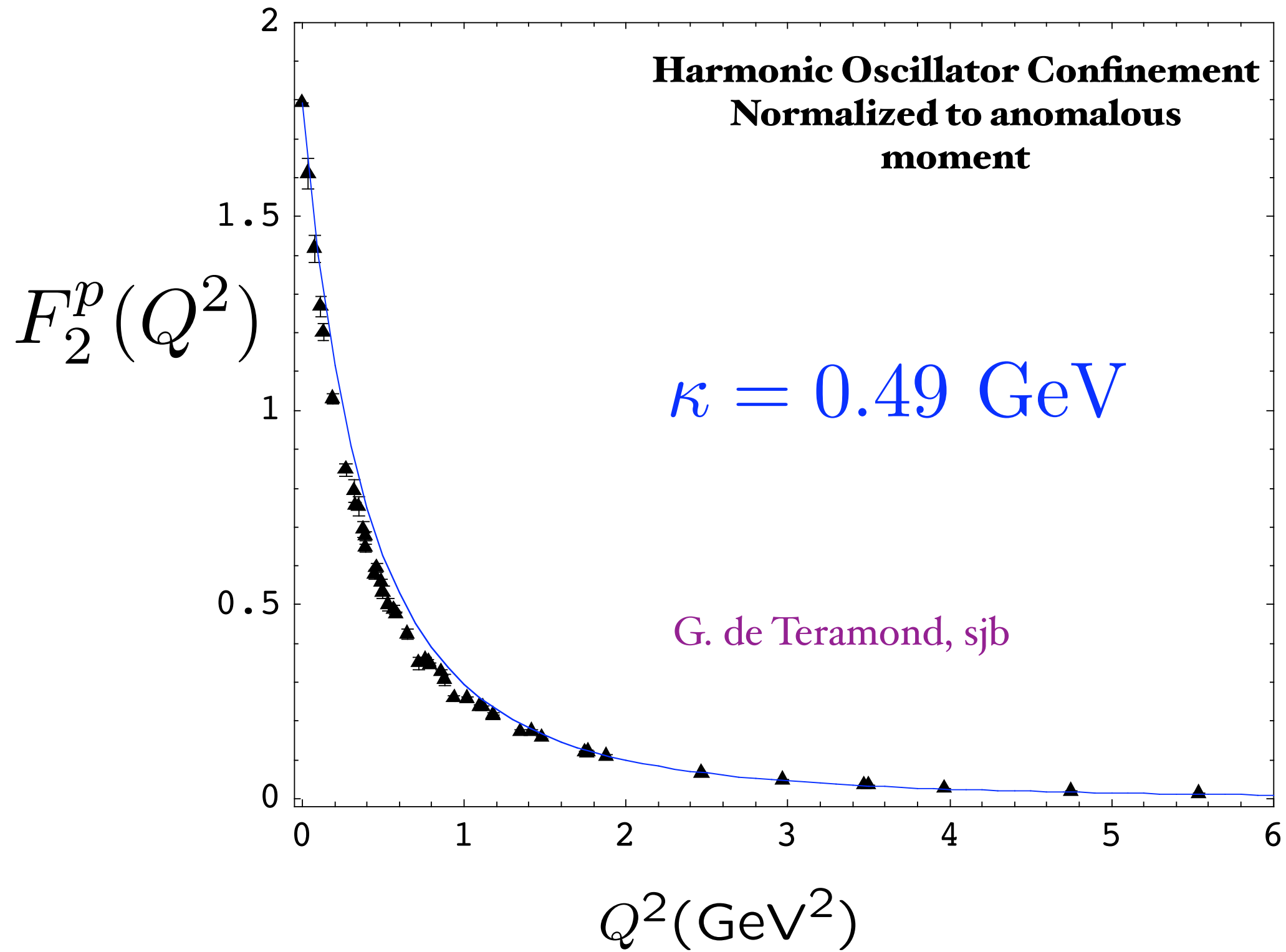


Must have  $\Delta l_z = \pm 1$  to have nonzero  $F_2(q^2)$

Nonzero Proton Anomalous Moment  $\rightarrow$   
Nonzero orbital quark angular momentum

# Spacelike Pauli Form Factor

From overlap of  $L = 1$  and  $L = 0$  LFWFs

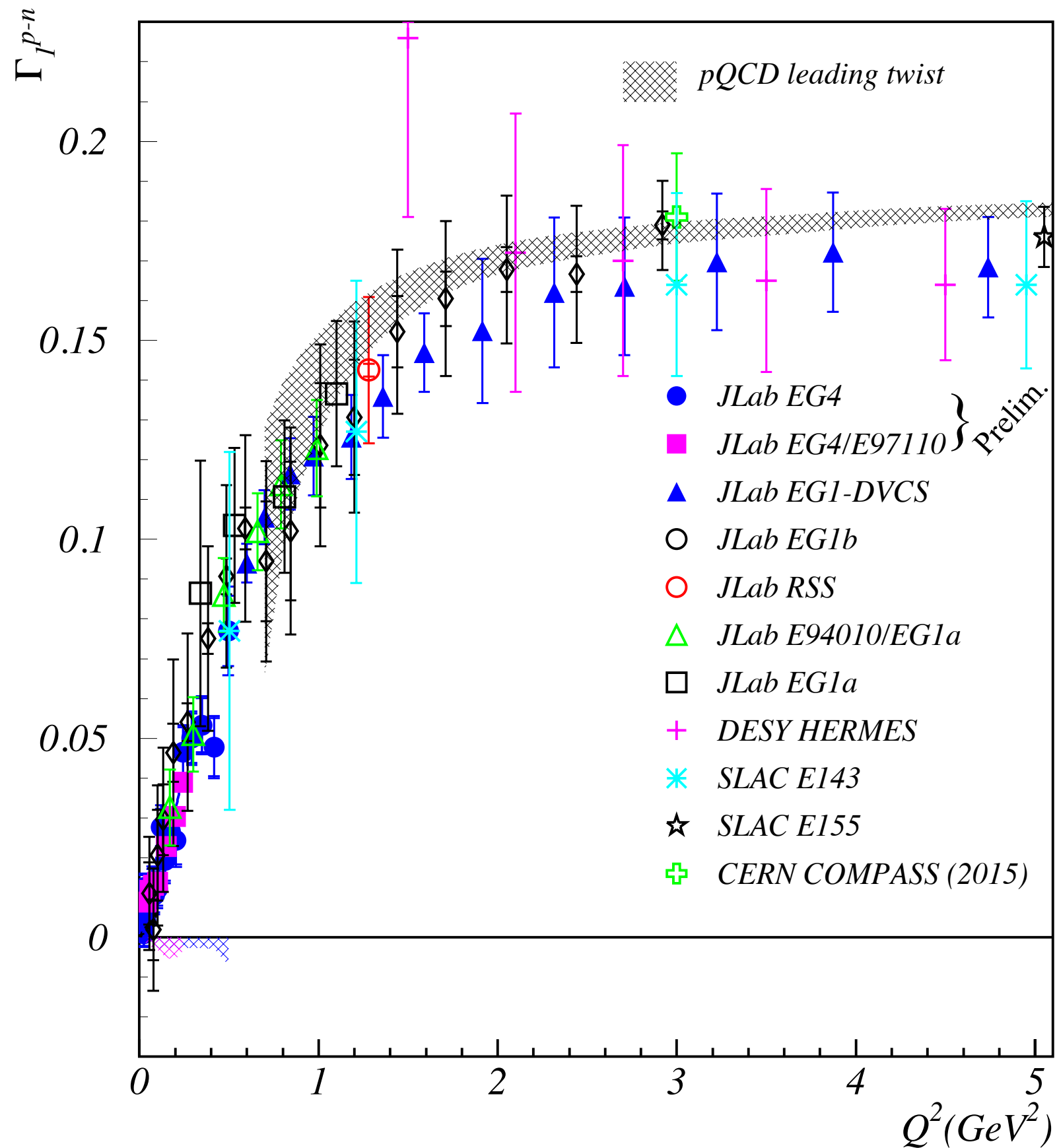


Bjorken sum rule defines effective charge:  $\alpha_{g1}(Q^2)$

$$\int_0^1 dx [g_1^{ep}(x, Q^2) - g_1^{en}(x, Q^2)] \equiv \frac{g_a}{6} \left[ 1 - \frac{\alpha_{g1}(Q^2)}{\pi} \right]$$

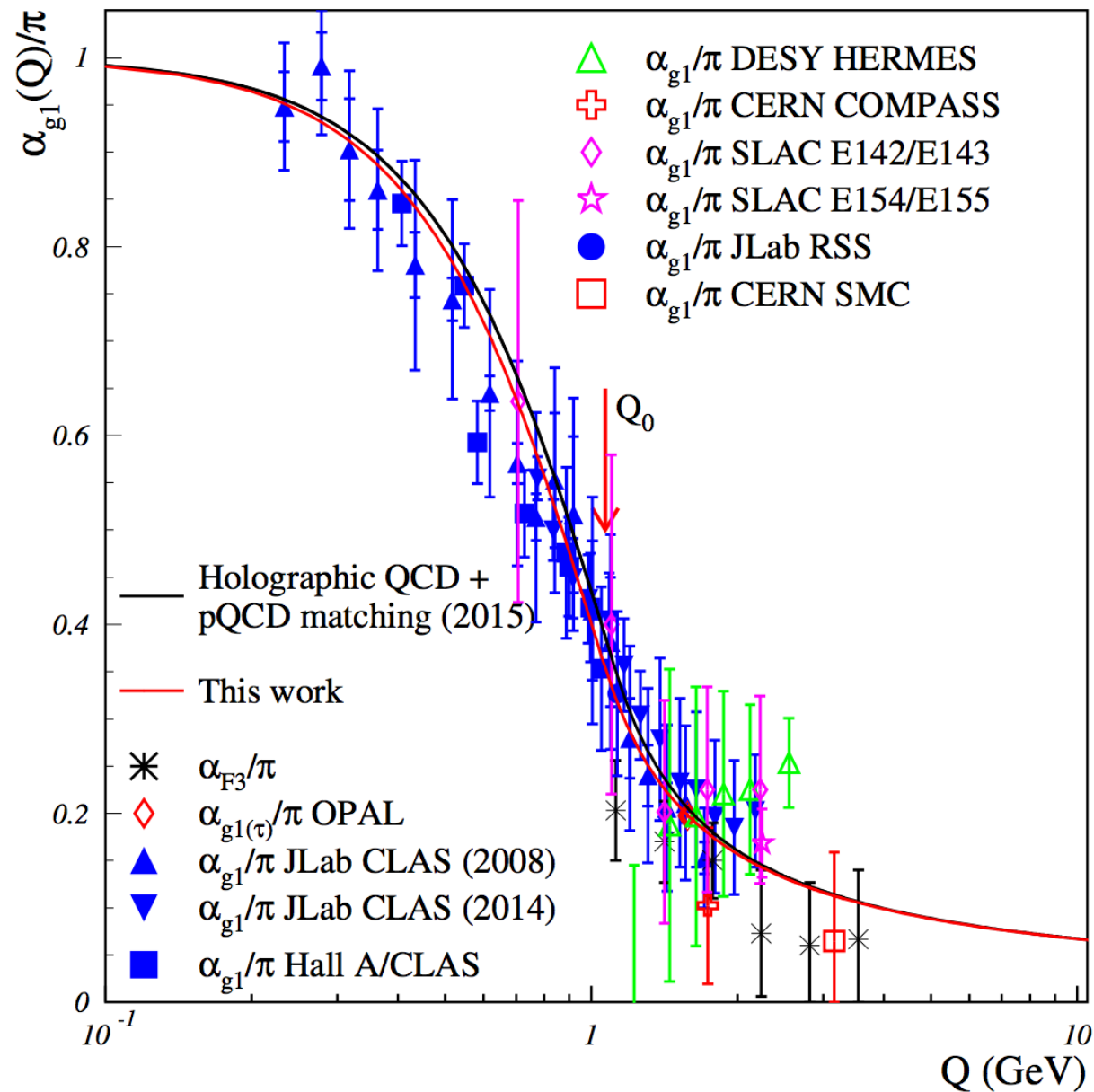
- **Can be used as standard QCD coupling**
- **Well measured**
- **Asymptotic freedom at large  $Q^2$**
- **Computable at large  $Q^2$  in any pQCD scheme**
- **Universal  $\beta_0, \beta_1$**
- **Analytic connection to other schemes:**  
**Commensurate scale relations**

# Bjorken sum $\Gamma_1^{p-n}$ measurements





# Running Coupling from AdS/QCD



Bjorken sum rule:

$$\frac{\alpha_{g_1}(Q^2)}{\pi} = 1 - \frac{6}{g_A} \int_0^1 dx g_1^{p-n}(x, Q^2)$$

Effective coupling in LFHQCD  
(valid at low- $Q^2$ )

$$\alpha_{g_1}^{AdS}(Q^2) = \pi \exp(-Q^2/4\kappa^2)$$

Imposing continuity for  $\alpha$   
and its first derivative

A. Deur, S.J. Brodsky, G.F. de Téramond,  
Phys. Lett. B 750, 528 (2015); J. Phys. G 44, 105005 (2017).

**Analytic, defined at all scales, IR Fixed Point**

$$m_\rho = \sqrt{2}\kappa$$

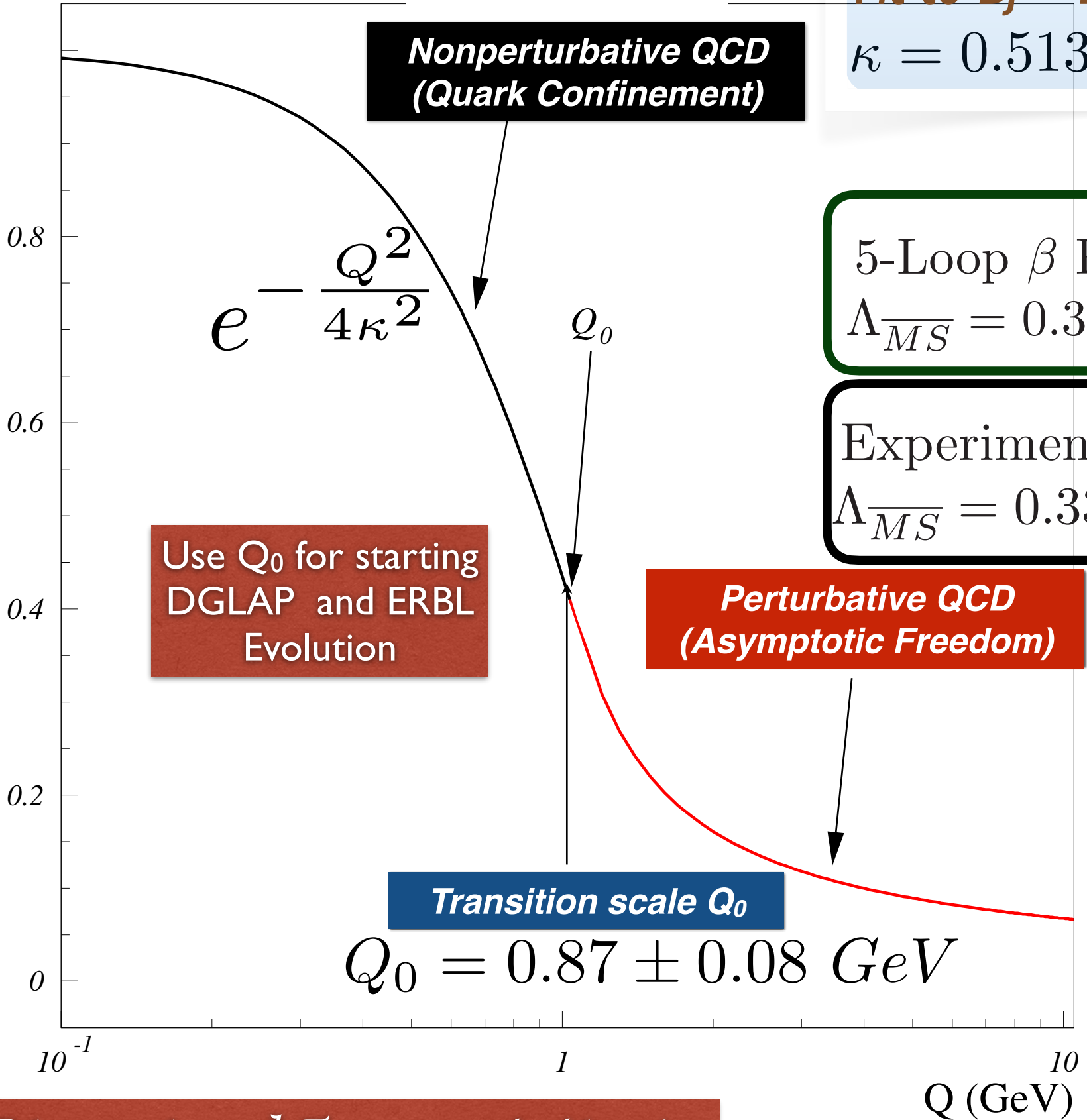
$$m_p = 2\kappa$$

Deur, de Tèramond, sjb

**All-Scale QCD Coupling**

Fit to Bj + DHG Sum Rules:  
 $\kappa = 0.513 \pm 0.007 \text{ GeV}$

$$\frac{\alpha_{g_1}^s(Q^2)}{\pi}$$



5-Loop  $\beta$  Prediction:  
 $\Lambda_{\overline{MS}} = 0.339 \pm 0.019 \text{ GeV}$

Experiment:  
 $\Lambda_{\overline{MS}} = 0.332 \pm 0.017 \text{ GeV}$

Use  $Q_0$  for starting  
 DGLAP and ERBL  
 Evolution

**Perturbative QCD  
 (Asymptotic Freedom)**

**Transition scale  $Q_0$**

$$Q_0 = 0.87 \pm 0.08 \text{ GeV}$$

$$\lambda \equiv \kappa^2$$

*Reverse Dimensional Transmutation!*

Initial DGLAP evolution scale from IR-UV  
matching of QCD coupling

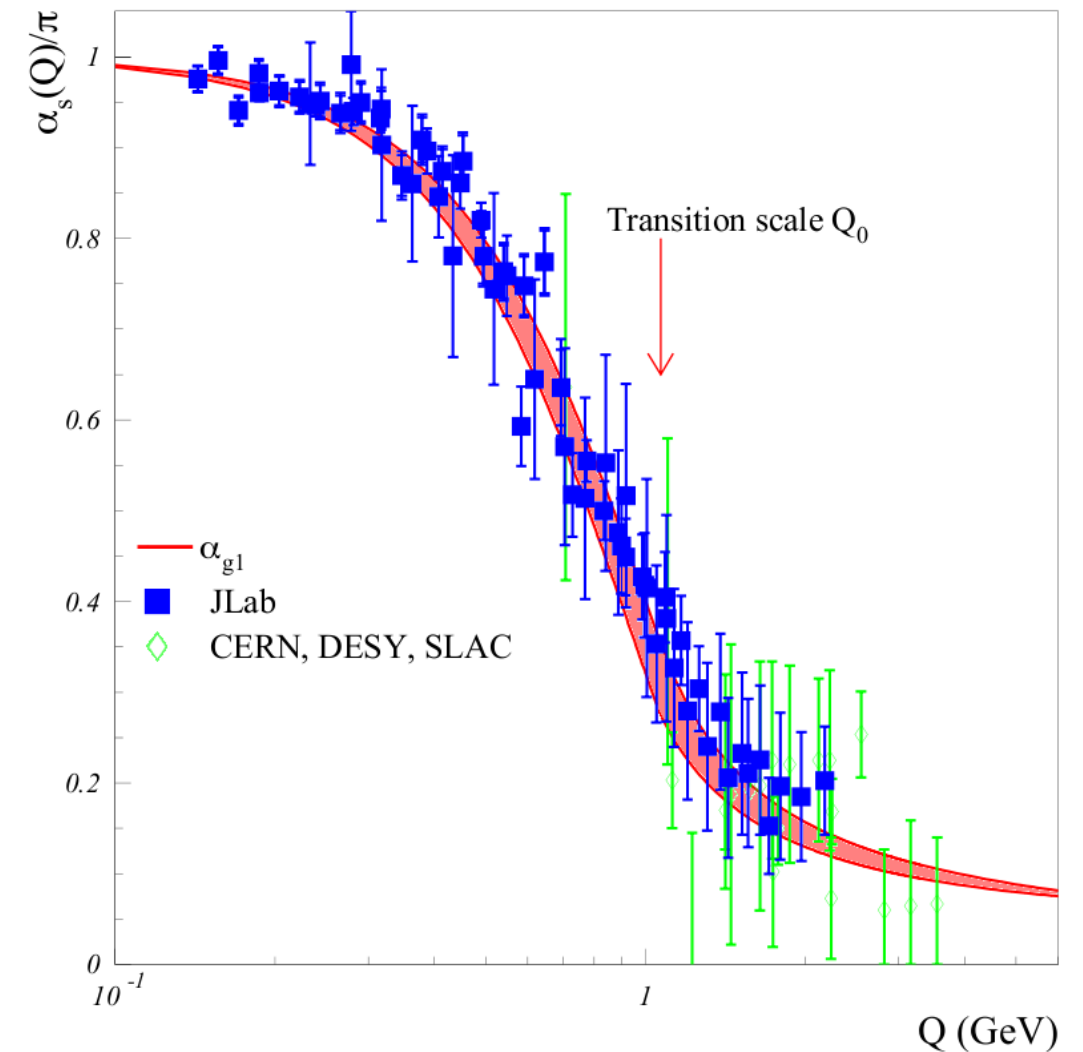
IR behavior of strong coupling in LFHQCD

$$\alpha_s^{IR}(Q^2) = \alpha_s^{IR}(0)e^{-Q^2/4\lambda}$$

$\Lambda_{QCD}$  and transition scale  $Q_0$  from matching  
perturbative (5-loop) and nonperturbative  
regimes for  $\sqrt{\lambda} = 0.534 \pm 0.05$  GeV

Transition scale:  $Q_0^2 \simeq 1$  GeV<sup>2</sup>

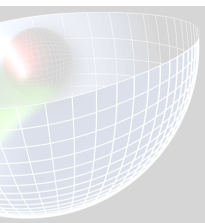
Connection between proton mass,  $M_p^2 = 4\lambda$ ,  
the  $\rho$  mass,  $M_\rho^2 = 2\lambda$ , and the perturbative  
QCD scale  $\Lambda_{QCD}$  in any RS !



IR QCD strong coupling from Bjorken  
sum-rule vs HLFQCD prediction (red)

Similar behavior of the IR coupling was obtained from the DSE

D. Binosi *et al.* (2017) and Z. F. Cui, *et al.* Chin. Phys. C **44**, 083102 (2020)



# Generalized Crewther Relation

$$\left[1 + \frac{\alpha_R(s^*)}{\pi}\right] \left[1 - \frac{\alpha_{g_1}(q^2)}{\pi}\right] = 1$$

$$\sqrt{s^*} \simeq 0.52Q$$

*Conformal relation true to all orders in  
perturbation theory!*

*No radiative corrections to axial anomaly*

*Nonconformal terms set relative scales (BLM)*

*No renormalization scale ambiguity!*

**Both observables go through new quark thresholds  
at commensurate scales!**

$$R_{e^+e^-}(Q^2) \equiv 3 \sum_{\text{flavors}} e_q^2 \left[ 1 + \frac{\alpha_R(Q)}{\pi} \right].$$

$$\int_0^1 dx [g_1^{ep}(x, Q^2) - g_1^{en}(x, Q^2)] \equiv \frac{1}{3} \left| \frac{g_A}{g_V} \right| \left[ 1 - \frac{\alpha_{g_1}(Q)}{\pi} \right]$$

$$\frac{\alpha_{g_1}(Q)}{\pi} = \frac{\alpha_R(Q^*)}{\pi} - \left( \frac{\alpha_R(Q^{**})}{\pi} \right)^2 + \left( \frac{\alpha_R(Q^{***})}{\pi} \right)^3$$

*Geometric Series in Conformal QCD*

*Generalized Crewther Relation*

Lu, Kataev, Gabadadze, Sjb



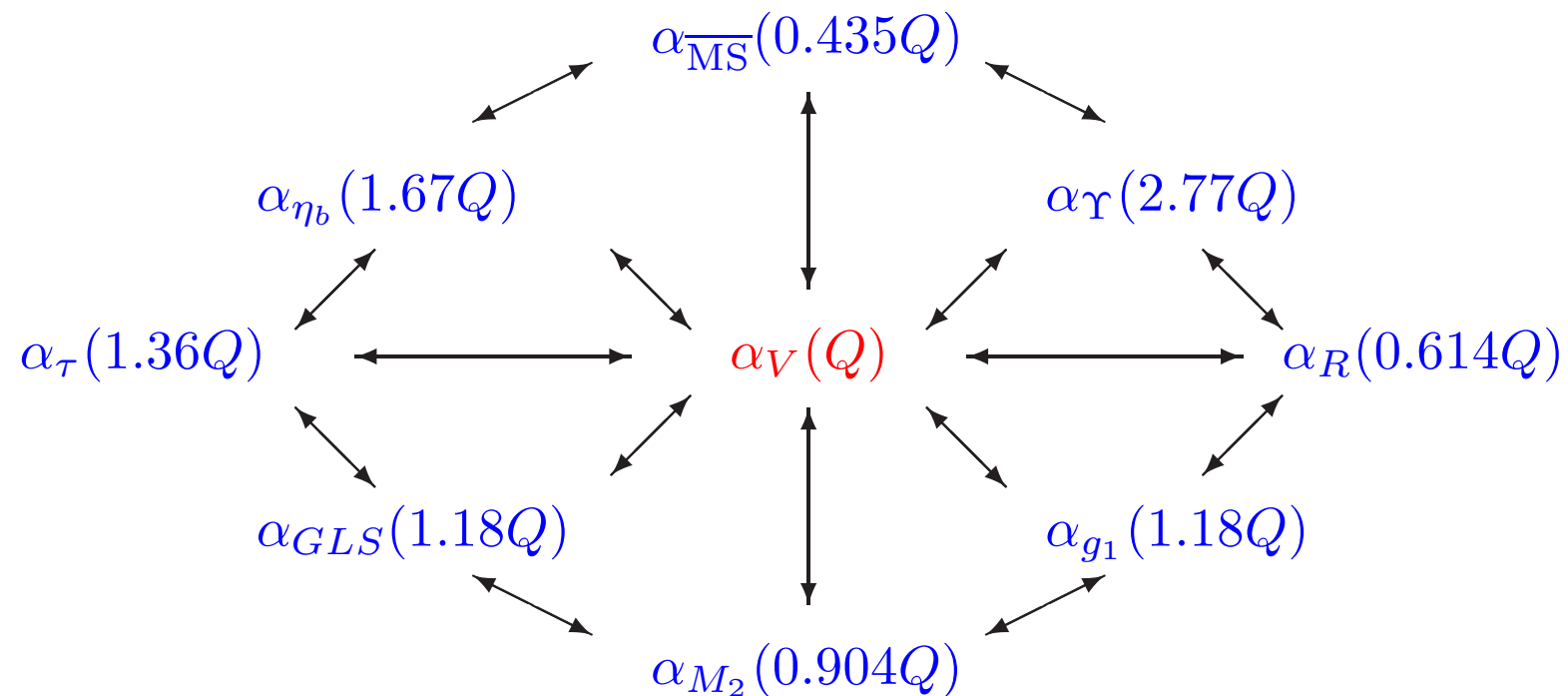
# Commensurate Scale Relations (CSR)

Special degeneracy holds for any scheme (see exercise)

PMC scales in physical schemes  $\Rightarrow$  CSR between physical observables

$$a_A(Q) = a_B(Q_1[Q]) + r_{2,0}^{AB} a_B(Q_2[Q])^2 + r_{3,0}^{AB} a_B(Q_3[Q])^3 + \dots$$

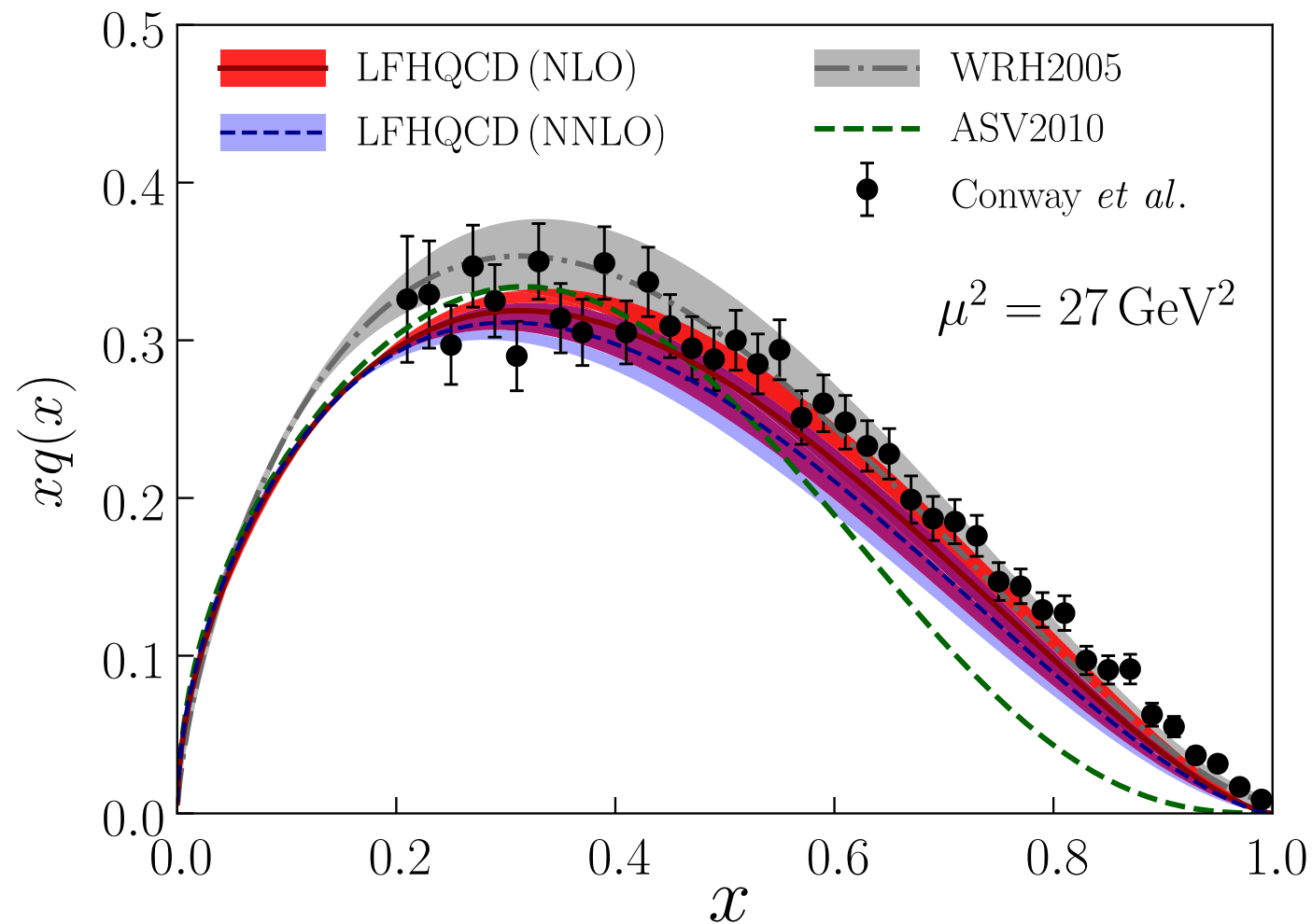
Measuring  $A$  at a scale  $Q$  predicts value of  $B$  to leading order at the scale  $Q_1[Q]$



Exact in special case, e.g.:  $\alpha_{\tau \rightarrow \nu_\tau + \mathbf{h}}(M_\tau^2) = \alpha_{e^+e^- \rightarrow \mathbf{h}}(Q_1^2)$ .

CSR: 
$$\ln \frac{Q_1^2}{M_\tau^2} = -\frac{19}{12} - \frac{169}{64} \frac{\alpha_{e^+e^- \rightarrow \mathbf{h}}(M_\tau^2)}{\pi} - \frac{83273}{3072} \frac{\alpha_{e^+e^- \rightarrow \mathbf{h}}(M_\tau^2)^2}{\pi^2} + \dots$$

Highly non-trivial QCD prediction free of scheme- and scale-ambiguities!



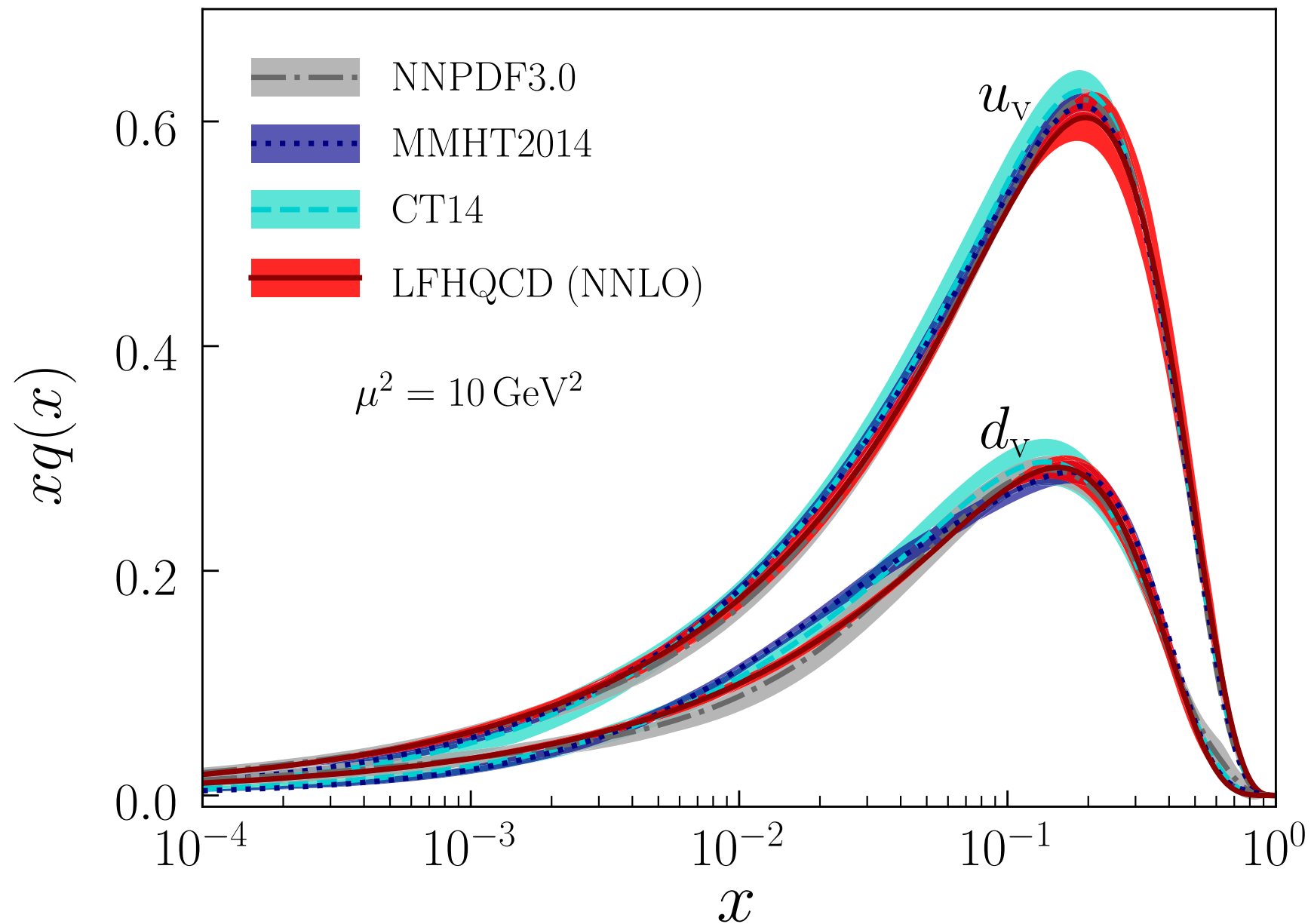
Comparison for  $xq(x)$  in the pion from LFHQCD (red band) with the NLO fits [82,83] (gray band and green curve) and the LO extraction [84]. NNLO results are also included (light blue band). LFHQCD results are evolved from the initial scale  $\mu_0 = 1.1 \pm 0.2 \text{ GeV}$  at NLO and the initial scale  $\mu_0 = 1.06 \pm 0.15 \text{ GeV}$  at NNLO.

### *Universality of Generalized Parton Distributions in Light-Front Holographic QCD*

*Guy F. de Téramond, Tianbo Liu, Raza Sabbir Sufian, Hans Günter Dosch, Stanley J. Brodsky, and Alexandre Deur* *PHYSICAL REVIEW LETTERS* 120, 182001 (2018)

# Features of BLM/PMC

- **Predictions are scheme-independent at every order**
- **Matches conformal series**
- **Commensurate Scale Relations between observables: Generalized Crewther Relation** ([Kataev, Lu, Rathsmann, sjb](#))
- **No  $n!$  Renormalon growth**
- **New scale appears at each order;  $n_F$  determined at each order - matches virtuality of quark loops**
- **Multiple Physical Scales Incorporated** ([Hoang, Kuhn, Tuebner, sjb](#))
- **Rigorous: Satisfies all Renormalization Group Principles**
- **Realistic Estimate of Higher-Order Terms**
- **Same as Gell-Mann Low for QED**
- **GUT: Must use the same scale setting procedure for QED, QCD**
- **Eliminates unnecessary theory error**
- **Maximal sensitivity to new physics**



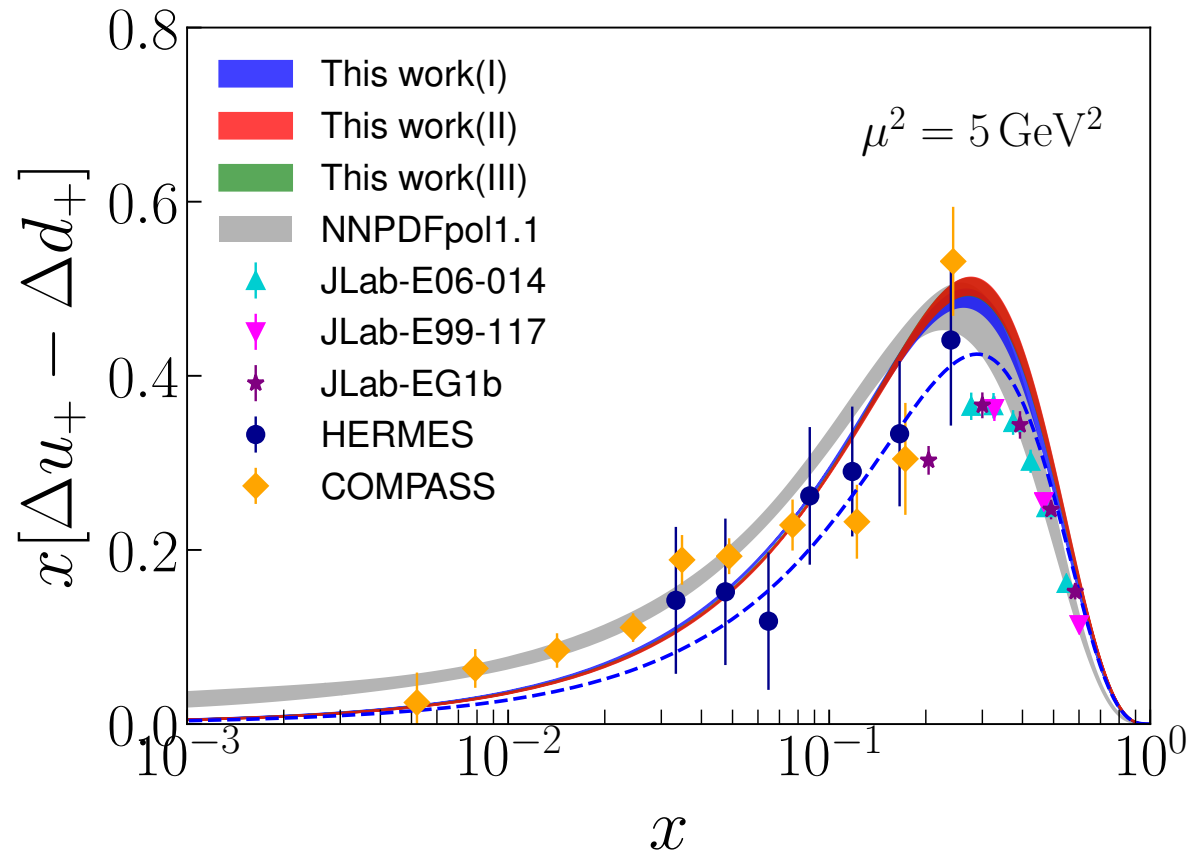
Comparison for  $xq(x)$  in the proton from LFHQCD (red bands) and global fits: MMHT2014 (blue bands) [5], CT14 [6] (cyan bands), and NNPDF3.0 (gray bands) [77]. LFHQCD results are evolved from the initial scale  $\mu_0 = 1.06 \pm 0.15$  GeV.

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*PHYSICAL REVIEW LETTERS 120, 182001 (2018)*

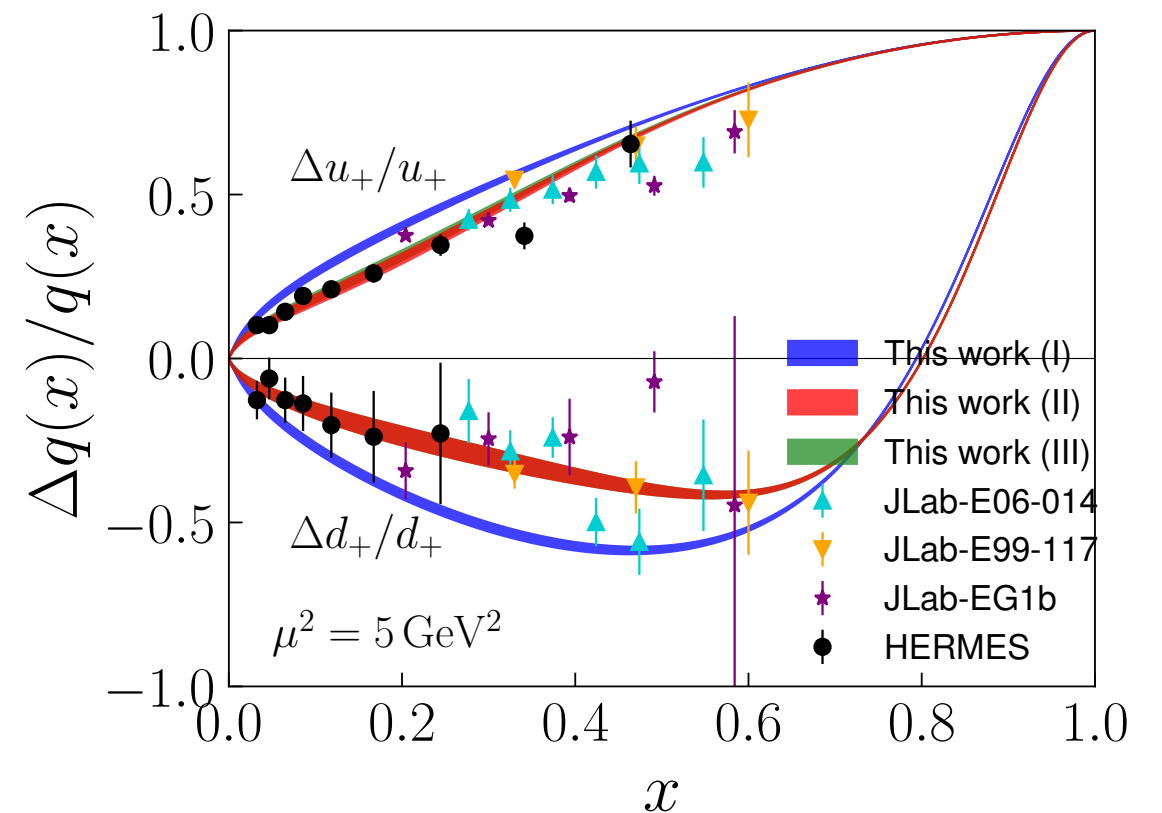
# Tianbo Liu, Raza Sabbir Sufian, Guy F. de Téramond, Hans Gunter Dösch, Alexandre Deur, sjb



Polarized distributions for the  
isovector combination  $x[\Delta u_+(x) - \Delta d_+(x)]$

$$d_+(x) = d(x) + \bar{d}(x) \quad u_+(x) = u(x) + \bar{u}(x)$$

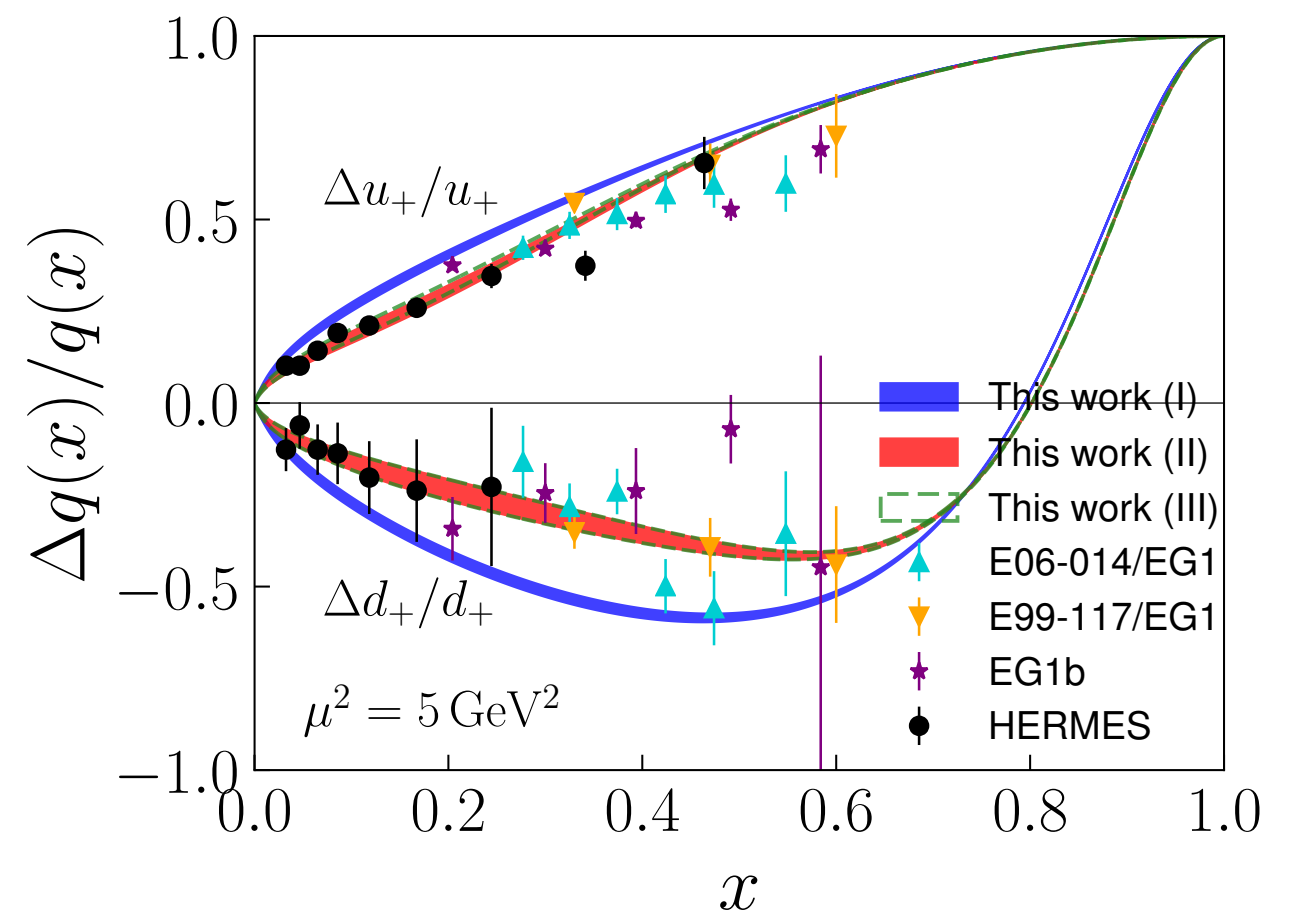
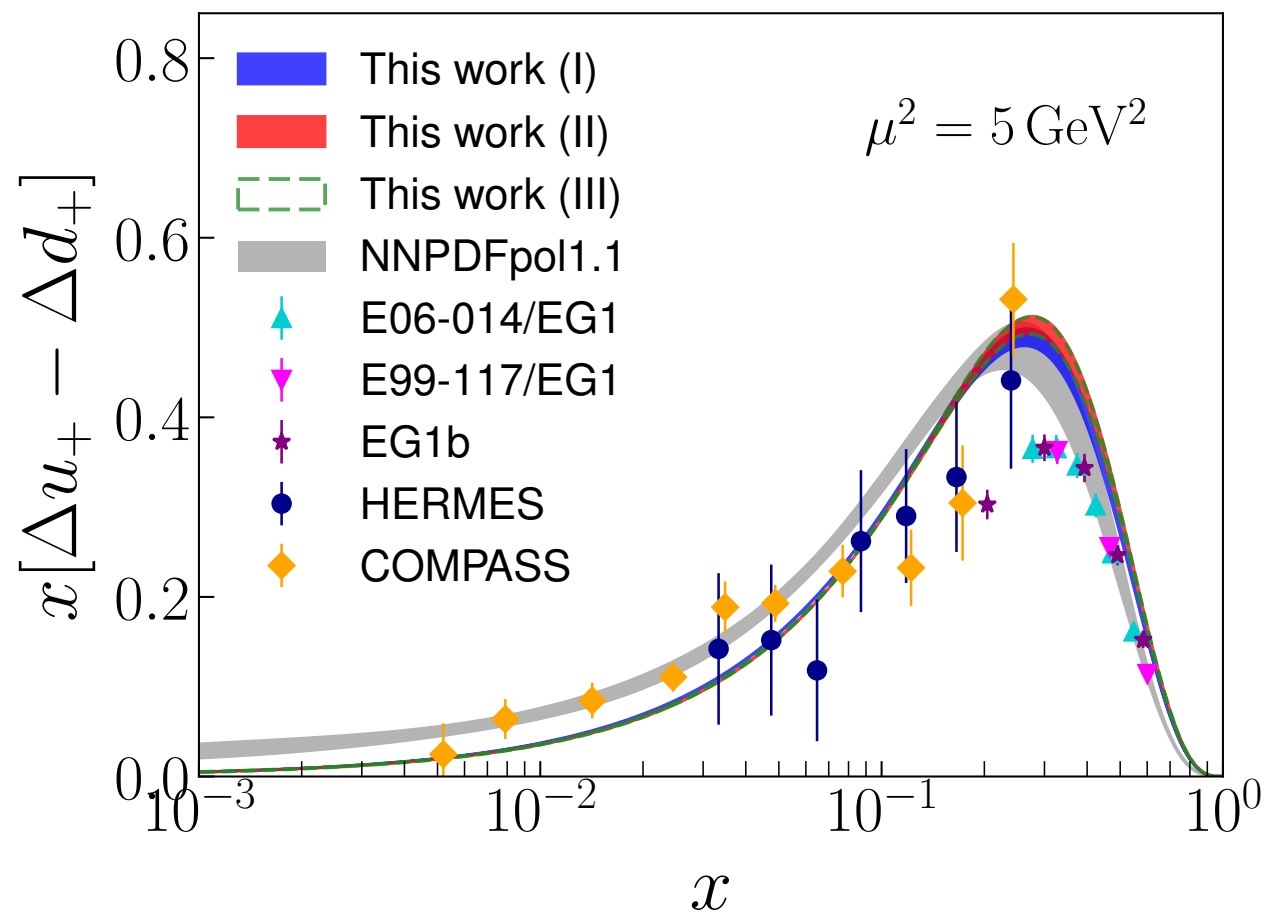
$$\Delta q(x) = q_{\uparrow}(x) - q_{\downarrow}(x)$$

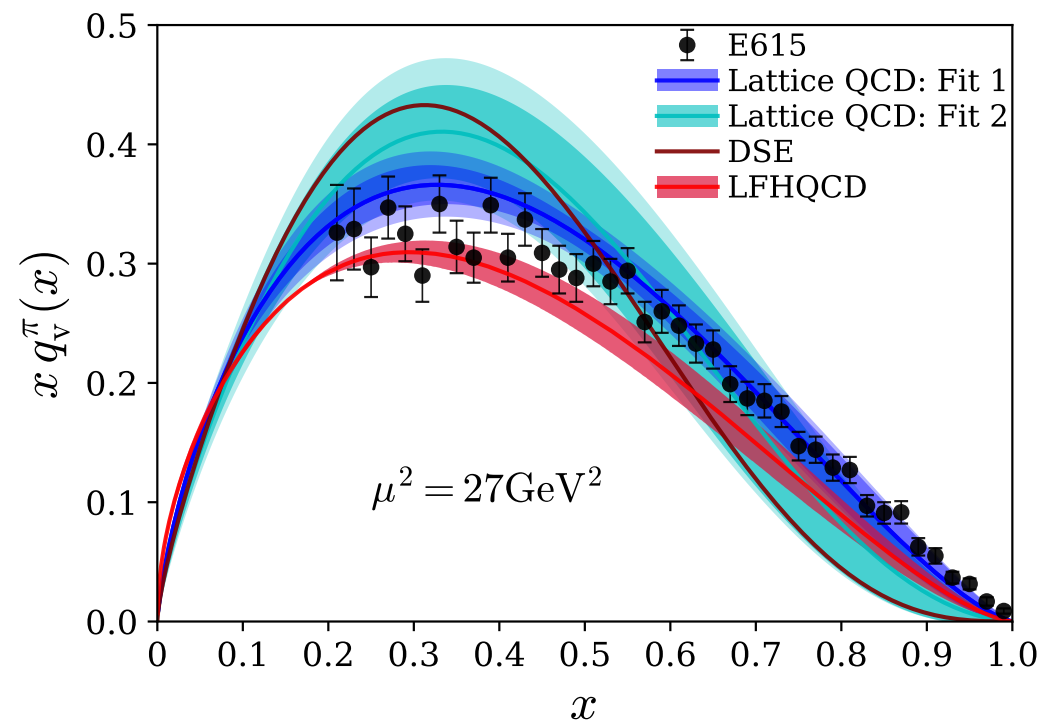
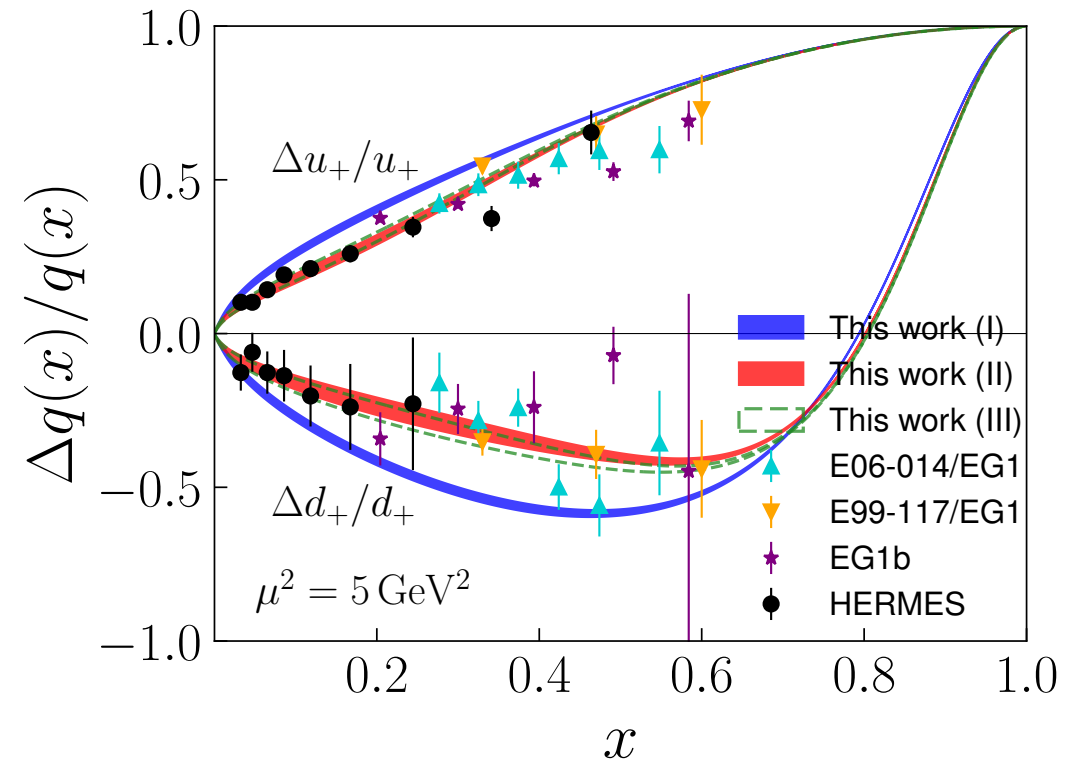
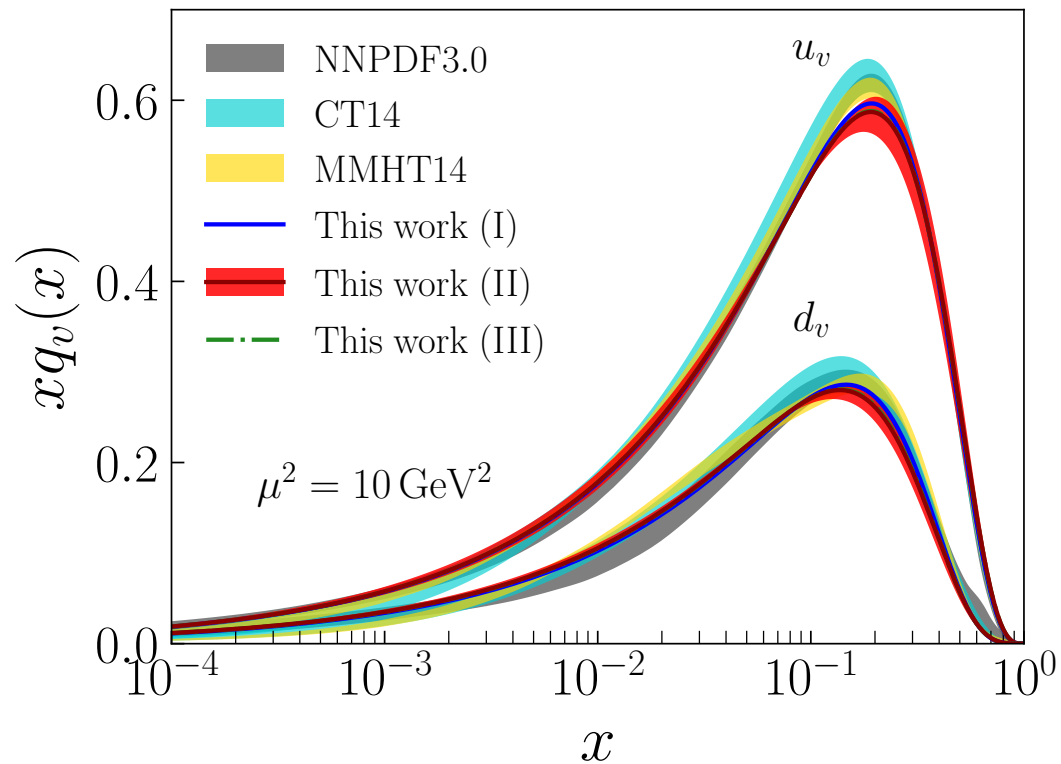




## Polarized GPDs and PDFs (HLFHS Collaboration, 2019)

- Separation of chiralities in the AdS action allows computation of the matrix elements of the axial current including the correct normalization, once the coefficients  $c_T$  are fixed for the vector current
- Helicity retention between quark and parent hadron (pQCD prediction):  $\lim_{x \rightarrow 1} \frac{\Delta q(x)}{q(x)} = 1$
- No spin correlation with parent hadron:  $\lim_{x \rightarrow 0} \frac{\Delta q(x)}{q(x)} = 0$





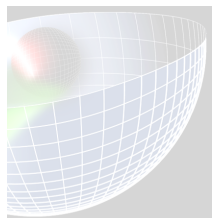
Separation of chiralities from the axial current  
Coefficients  $c_\tau$  are fixed from the vector current

Regge trajectory from HLFQCD

$$\alpha_A(t) = \frac{t}{4\lambda}$$

$$\lim_{x \rightarrow 1} \frac{\Delta q(x)}{q(x)} = 1, \quad \lim_{x \rightarrow 0} \frac{\Delta q(x)}{q(x)} = 0$$

DGLAP NNLO evolution from initial scale  $\mu \simeq 1 \text{ GeV}$  from soft-hard matching in  $\alpha_s$



# Gravitational form factors and gluon distribution functions

G. de Téramond, H. G. Dosch, T. Liu, A. Deur, *sjb PRD* 104 (2021)

Spin-2 gluon gravitational FF  $A(t)$  from the coupling of the metric fluctuations induced by the spin-two Pomeron with the energy momentum tensor in AdS

$$\int d^4x dz \sqrt{g} h_{MN} T^{MN}$$

$$A_\tau^g(t) \sim B(\tau - 1, 2 - \alpha_P(t))$$

with Pomeron Regge trajectory

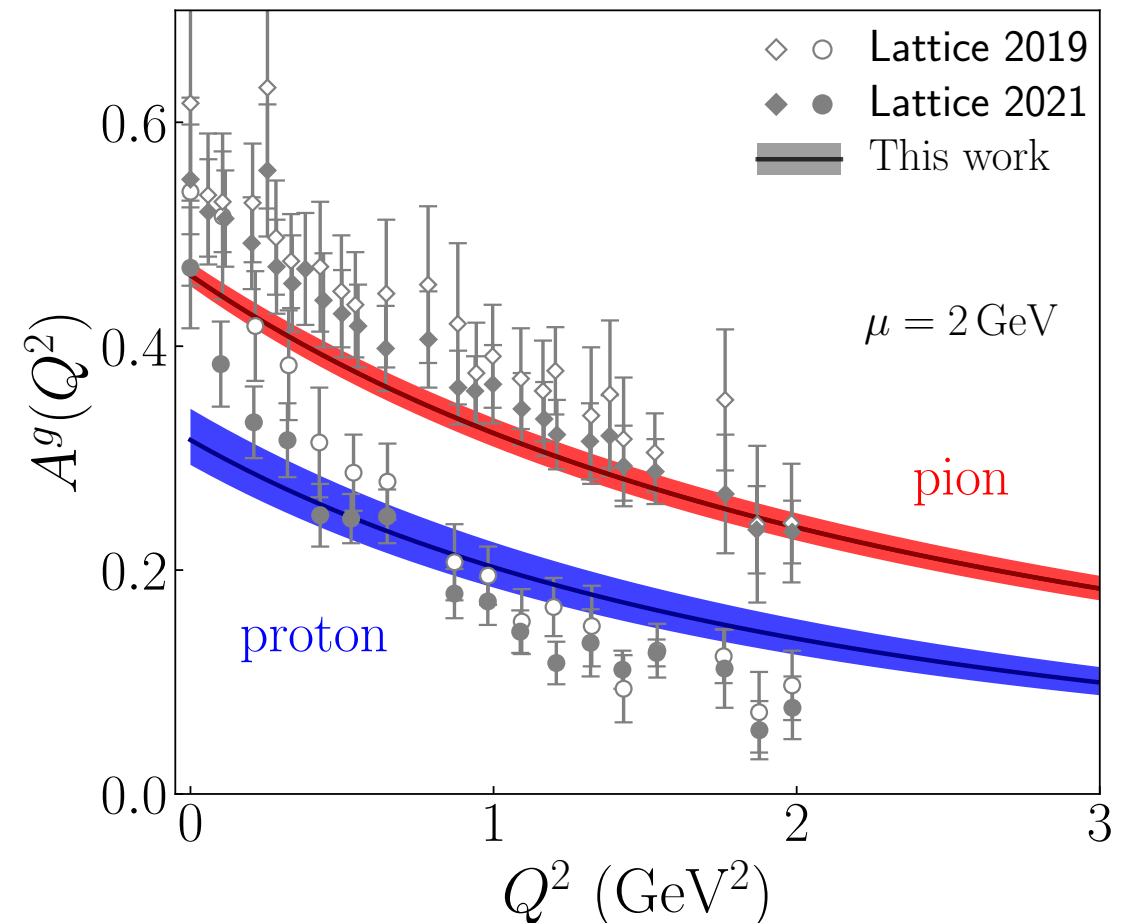
$$\alpha_P(t) = \alpha_P(0) + \alpha'_P t$$

where  $\alpha_P(0) \simeq 1.08$  and  $\alpha' = 0.25 \text{ GeV}^{-2}$

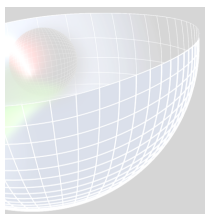
Radial spectrum from  $t$ -channel poles in the  $2^{++}$  trajectory

$$-Q^2 = M_n^2 = \frac{1}{\alpha'} (n + 2 - \alpha(0))$$

with  $M_0 \simeq 1.92 \text{ GeV}$



Lattice data from Shanahan *et al.* (2018) and Pefkou *et al.* (2021)



# Gravitational form factors and gluon distribution functions

Gluon GPD  $H_T^g(x, t) = g_T(x)e^{tf(x)}$

$$f(x) = \alpha'_P \log\left(\frac{1}{w(x)}\right),$$

$$g_T(x) = \frac{1}{N_T} \frac{w'(x)}{x} [1 - w(x)]^{\tau-2} w(x)^{1-\alpha_P(0)}$$

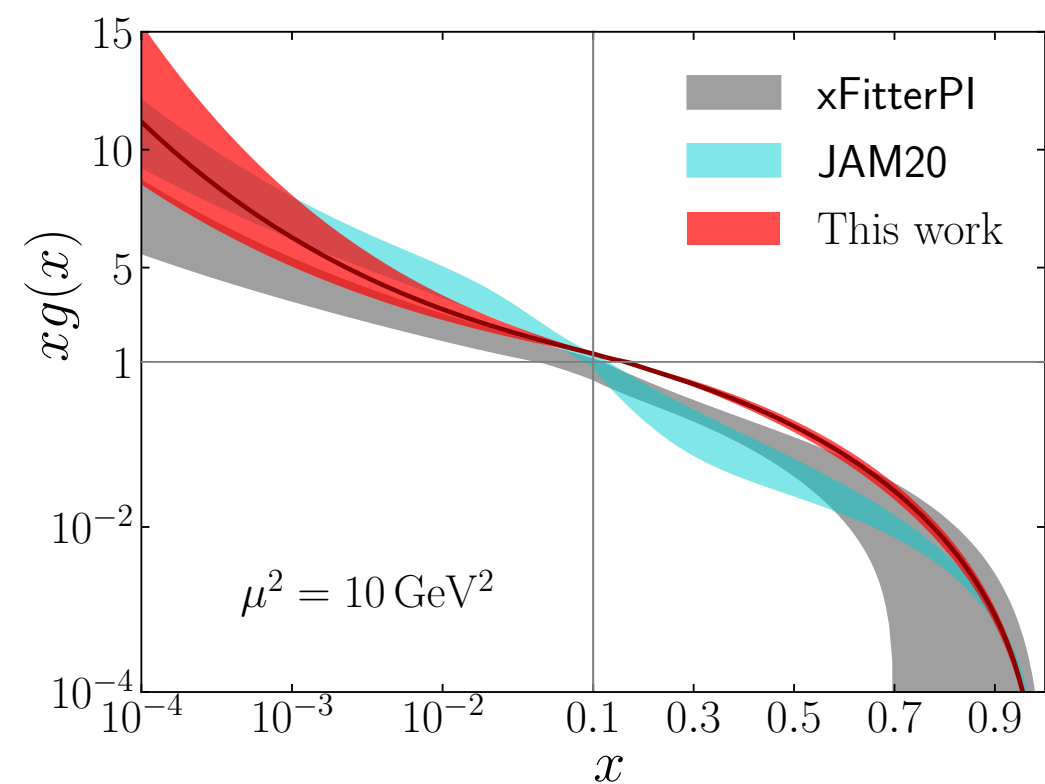
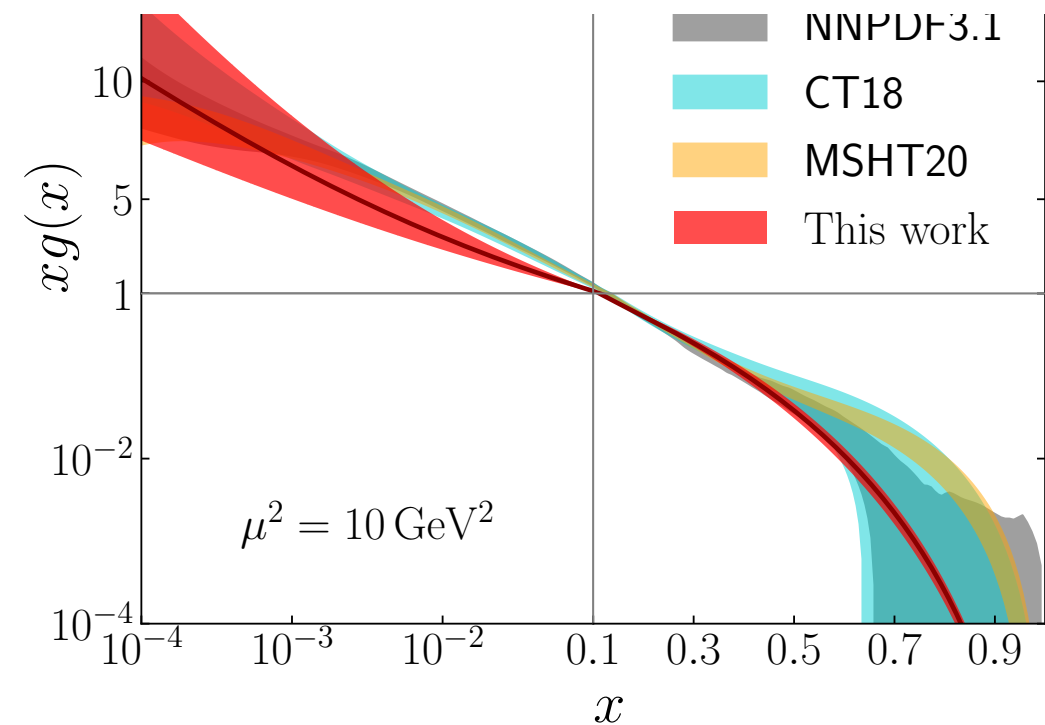
Normalization of  $A^g(0)$  determined from the sum rule:

$$\sum_q \langle x \rangle_q + \sum_{\bar{q}} \langle x \rangle_{\bar{q}} + \langle x \rangle_g = 1$$

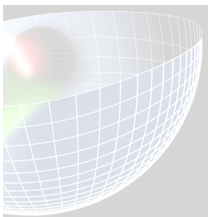
Basic parameters fixed in quark sector:  
No adjustable parameters

Single Pomeron (HLFHS 2022))

Hard Pomeron from the evolution of the nonperturbative gluon distribution function



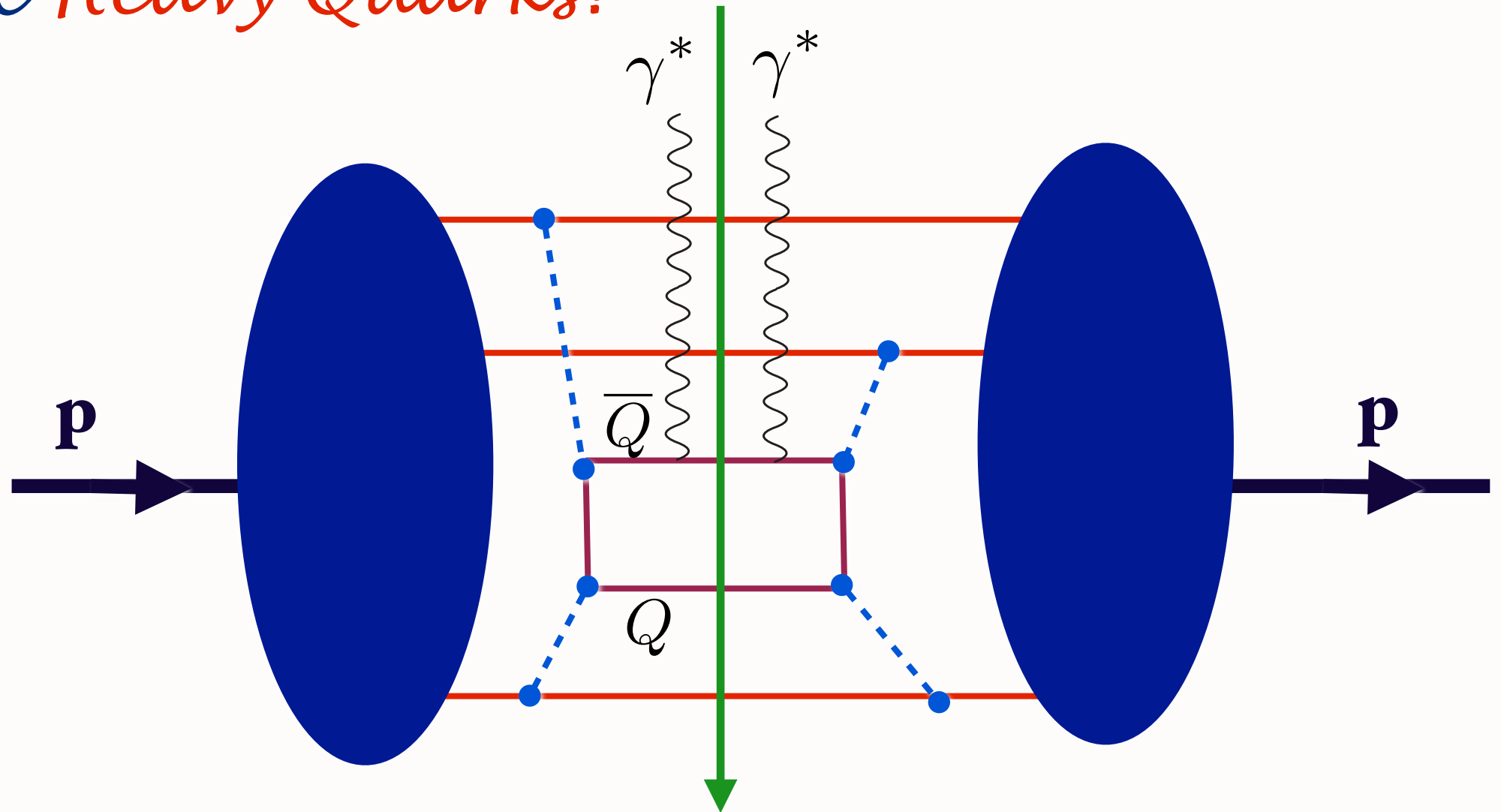
Gluon PDF: Proton upper figure and pion lower figure



*Cut of Proton Self Energy:*

*QCD predicts*

*Intrinsic Heavy Quarks!*



$$\text{Probability (QED)} \propto \frac{1}{M_\ell^4}$$

$$\text{Probability (QCD)} \propto \frac{1}{M_Q^2}$$

$$x_Q \propto (m_Q^2 + k_\perp^2)^{1/2}$$

**Hoyer, Peterson, Sakai, Collins, Ellis, Gunion, Mueller, sjb  
Polyakov, et al.**

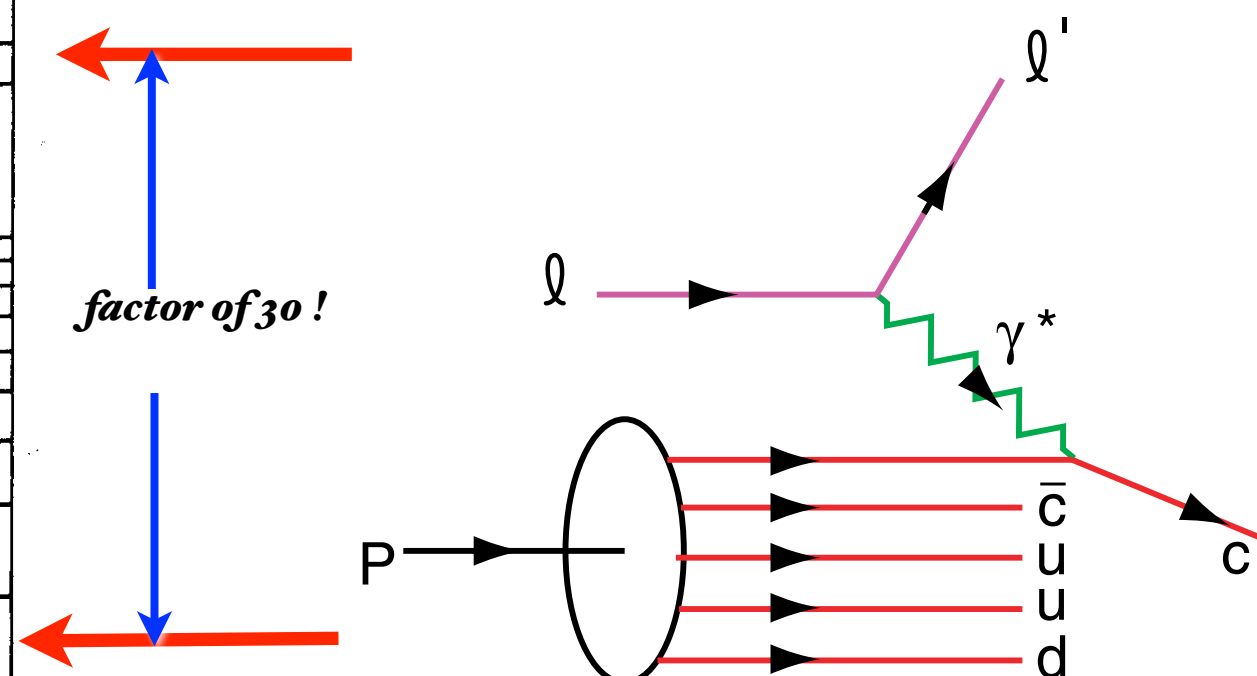
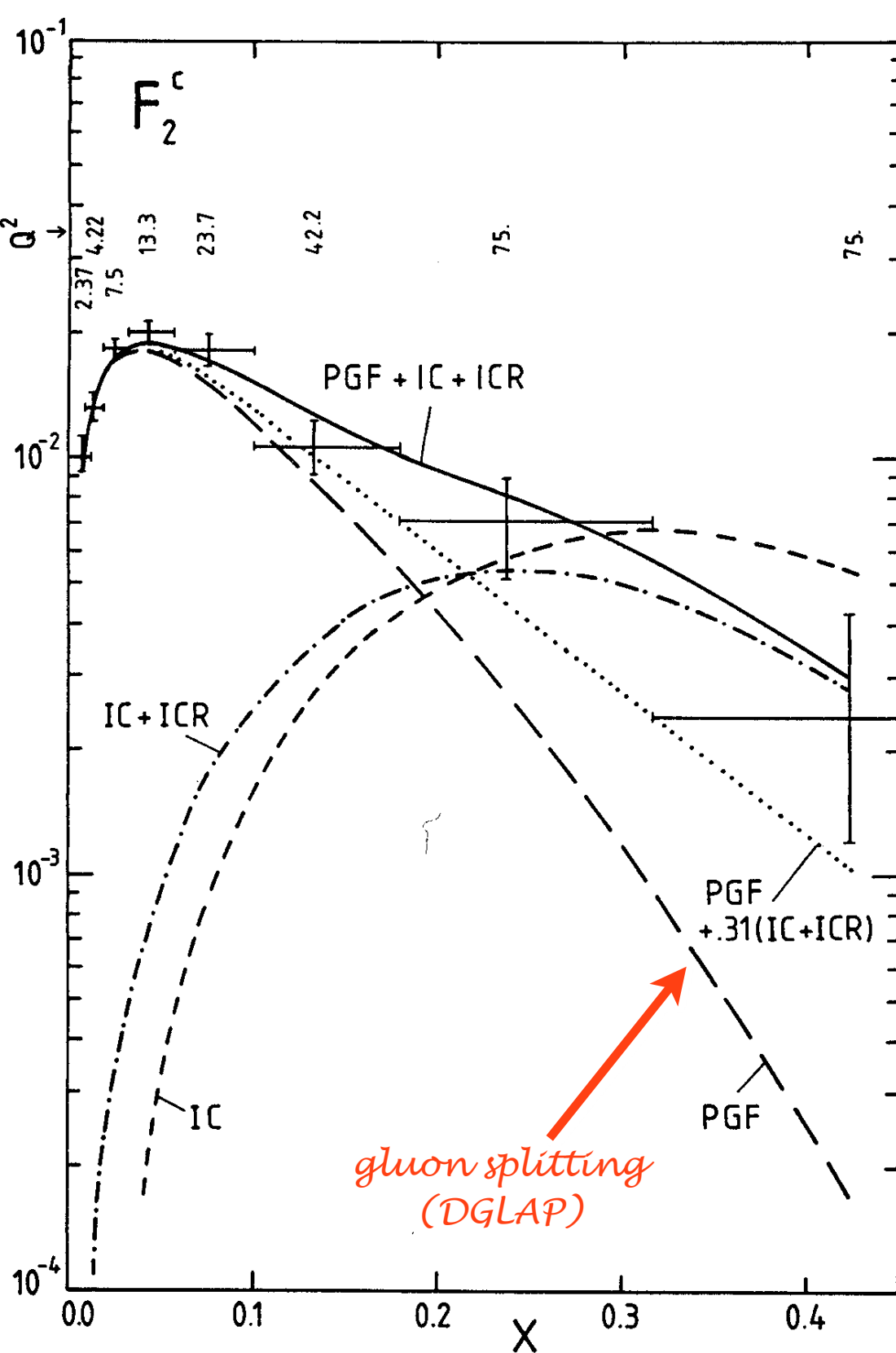


# Measurement of Charm Structure Function

J. J. Aubert et al. [European Muon Collaboration], "Production Of Charmed Particles In 250-GeV Mu+ - Iron Interactions," Nucl. Phys. B 213, 31 (1983).

## First Evidence for Intrinsic Charm

Hoyer, Peterson, Sakai, sjb



**DGLAP / Photon-Gluon Fusion: factor of 30 too small**

*Two Components (separate evolution):*

$$c(x, Q^2) = c(x, Q^2)_{\text{extrinsic}} + c(x, Q^2)_{\text{intrinsic}}$$

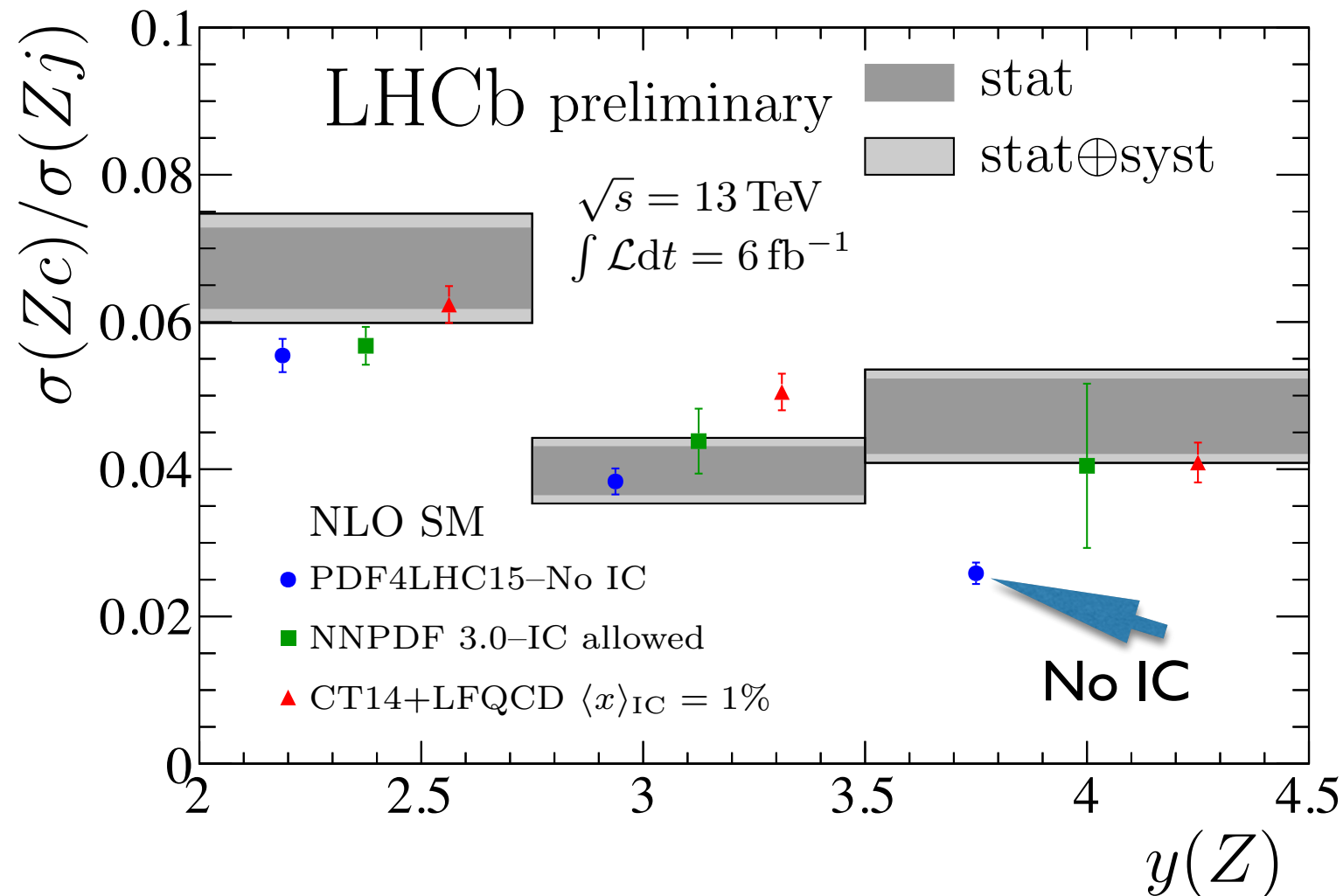
$$pp \rightarrow Z + c + X$$

$$g + c \rightarrow Z + c$$

## Z + c: results



LHCb-PAPER-2021-029



► Clear enhancement in highest- $y$  bin

► Consistent with expected effect from  $|uudc\bar{c}\rangle$  component predicted by LFQCD

► Inconsistent with No-IC theory at  $\sim 3$  standard deviations

► Global PDF analysis required to determine true significance

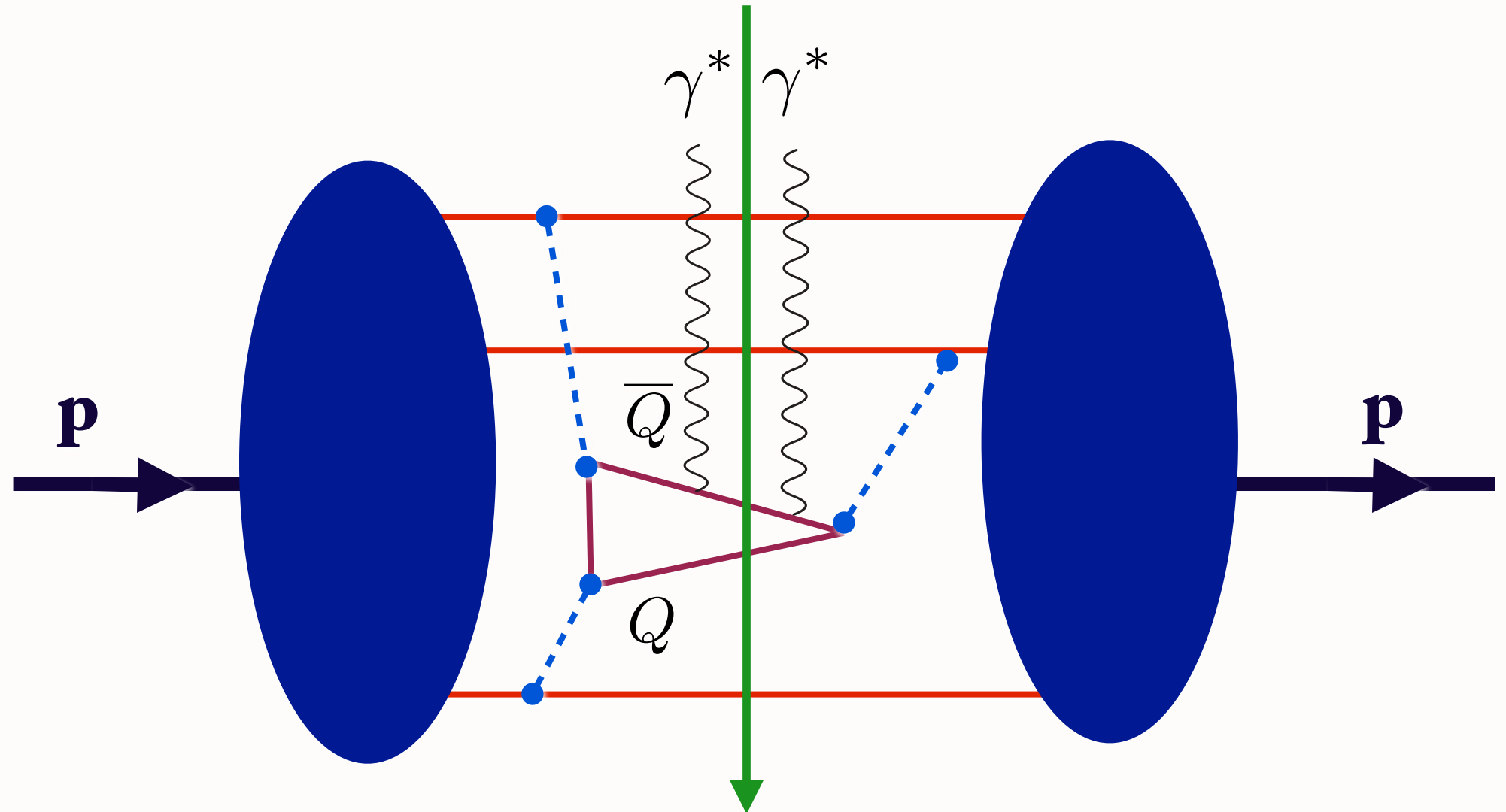
QCD physics measurements at the LHCb experiment

BOOST 2021

Daniel Craik  
on behalf of the LHCb collaboration



## Interference of Intrinsic and Extrinsic Heavy Quark Amplitudes



Interference predicts  $Q(x) \neq \bar{Q}(x)$   
 $\frac{d\sigma}{dydp_T^2}(pp \rightarrow D^+ cdX) \neq \frac{d\sigma}{dydp_T^2}(pp \rightarrow D^- \bar{c}dX)$

QED Analog: J. Gillespie, sjb (1968)

# Constraints on charm-anticharm asymmetry in the nucleon from lattice QCD

Raza Sabbir Sufian<sup>a</sup>, Tianbo Liu<sup>a</sup>, Andrei Alexandru<sup>b,c</sup>, Stanley J. Brodsky<sup>d</sup>, Guy F. de Téramond<sup>e</sup>,  
Hans Günter Dosch<sup>f</sup>, Terrence Draper<sup>g</sup>, Keh-Fei Liu<sup>g</sup>, Yi-Bo Yang<sup>h</sup>

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<sup>b</sup>Department of Physics, The George Washington University, Washington, DC 20052, USA

<sup>c</sup>Department of Physics, University of Maryland, College Park, MD 20742, USA

<sup>d</sup>SLAC National Accelerator Laboratory, Stanford University, Stanford, CA 94309, USA

<sup>e</sup>Laboratorio de Física Teórica y Computacional, Universidad de Costa Rica, 11501 San José, Costa Rica

<sup>f</sup>Institut für Theoretische Physik der Universität, D-69120 Heidelberg, Germany

<sup>g</sup>Department of Physics and Astronomy, University of Kentucky, Lexington, Kentucky 40506, USA

<sup>h</sup>CAS Key Laboratory of Theoretical Physics, Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing 100190, China

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## Abstract

We present the first lattice QCD calculation of the charm quark contribution to the nucleon electromagnetic form factors  $G_{E,M}^c(Q^2)$  in the momentum transfer range  $0 \leq Q^2 \leq 1.4 \text{ GeV}^2$ . The quark mass dependence, finite lattice spacing and volume corrections are taken into account simultaneously based on the calculation on three gauge ensembles including one at the physical pion mass. The nonzero value of the charm magnetic moment  $\mu_M^c = -0.00127(38)_{\text{stat}}(5)_{\text{sys}}$ , as well as the Pauli form factor, reflects a nontrivial role of the charm sea in the nucleon spin structure. The nonzero  $G_E^c(Q^2)$  indicates the existence of a nonvanishing asymmetric charm-anticharm sea in the nucleon. Performing a non-perturbative analysis based on holographic QCD and the generalized Veneziano model, we study the constraints on the  $[c(x) - \bar{c}(x)]$  distribution from the lattice QCD results presented here. Our results provide complementary information and motivation for more detailed studies of physical observables that are sensitive to intrinsic charm and for future global analyses of parton distributions including asymmetric charm-anticharm distribution.

*Keywords:* Intrinsic charm, Form factor, Parton distributions, Lattice QCD, Light-front holographic QCD, JLAB-THY-20-3155, SLAC-PUB-17515

# Intrinsic charm-anticharm asymmetry in the proton

R. S. Sufian, T. Liu, Alexandru, G. de T'era mond, Dosch, Draper, K. F. Liu, Y. B. Yang, sjb (2020)

Intrinsic charm in the proton introduced by Brodsky, Hoyer, Peterson, Sakai, sjb (1980)

Charm FF normalization computed with with three gauge ensembles in LGTH (one at the physical pion mass) and charm distribution from HLFQCD

Intrinsic charm asymmetry  $c(x) - \bar{c}(x)$ ,

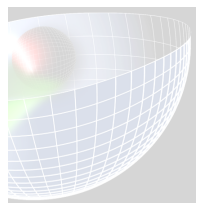
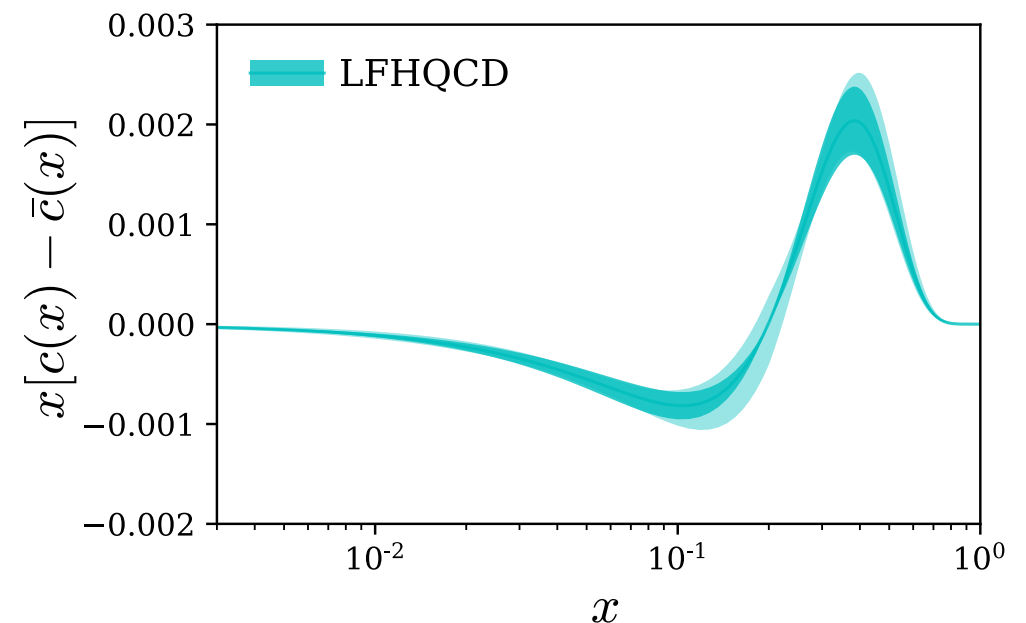
$$c(x) - \bar{c}(x) = \sum_{\tau} c_{\tau} (q_{\tau}(x) - q_{\tau+1}(x))$$

with  $\int_0^1 dx [c(x) - \bar{c}(x)] = 0$

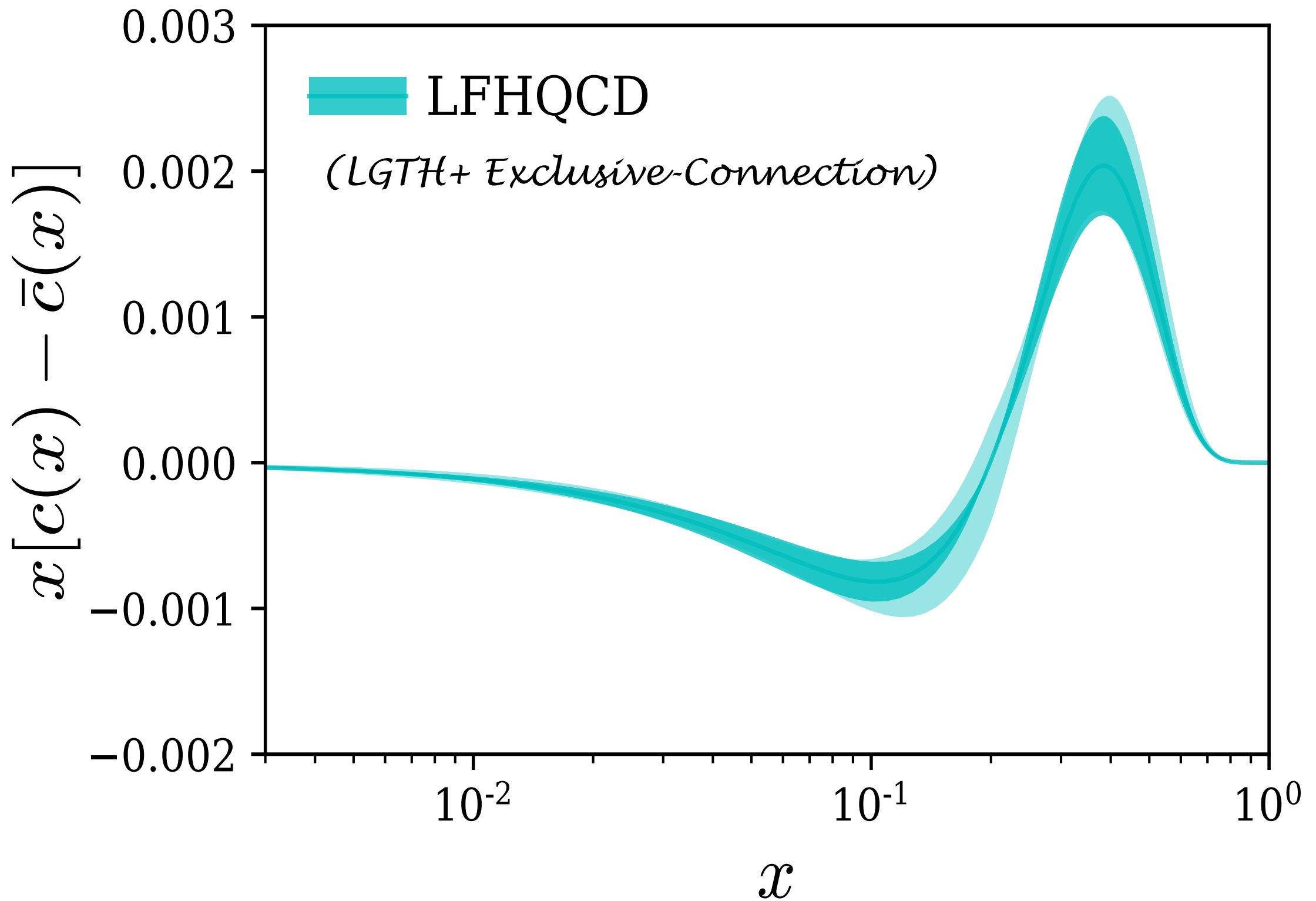
$J/\psi$  Regge trajectory

$$\alpha(t)_{J/\psi} = -2.066 + \frac{t}{4\lambda_c}, \quad \lambda_c = 0.874 \text{ GeV}^2$$

from HLFQCD and HQET

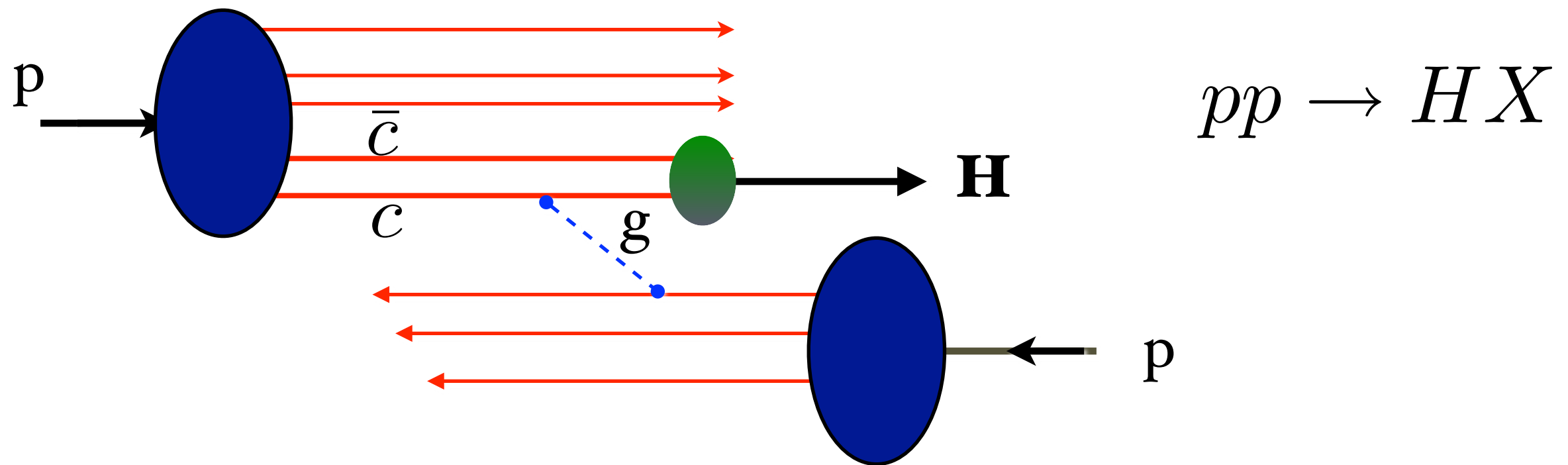






The distribution function  $x[c(x) - \bar{c}(x)]$  obtained from the LFHQCD formalism using the lattice QCD input of charm electromagnetic form factors  $G_{E,M}^c(Q^2)$ . The outer cyan band indicates an estimate of systematic uncertainty in the  $x[c(x) - \bar{c}(x)]$  distribution obtained from a variation of the hadron scale  $\kappa_c$  by 5%.

*Intrinsic Charm Mechanism for Inclusive  
High- $x_F$  Higgs Production*



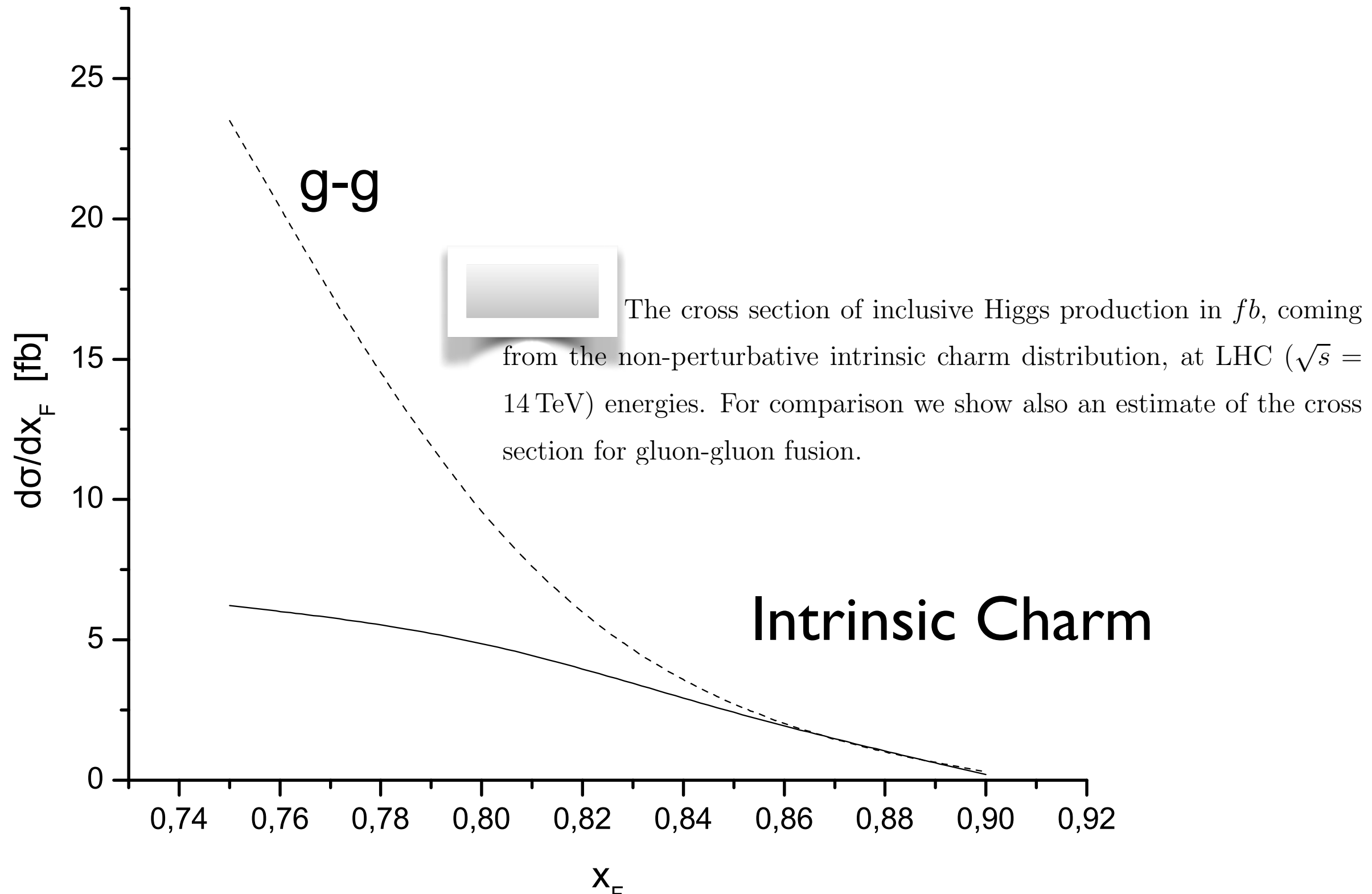
**Also: intrinsic strangeness, bottom, top**

**Higgs can have > 80% of Proton Momentum!**

*New production mechanism for Higgs at the LHC*

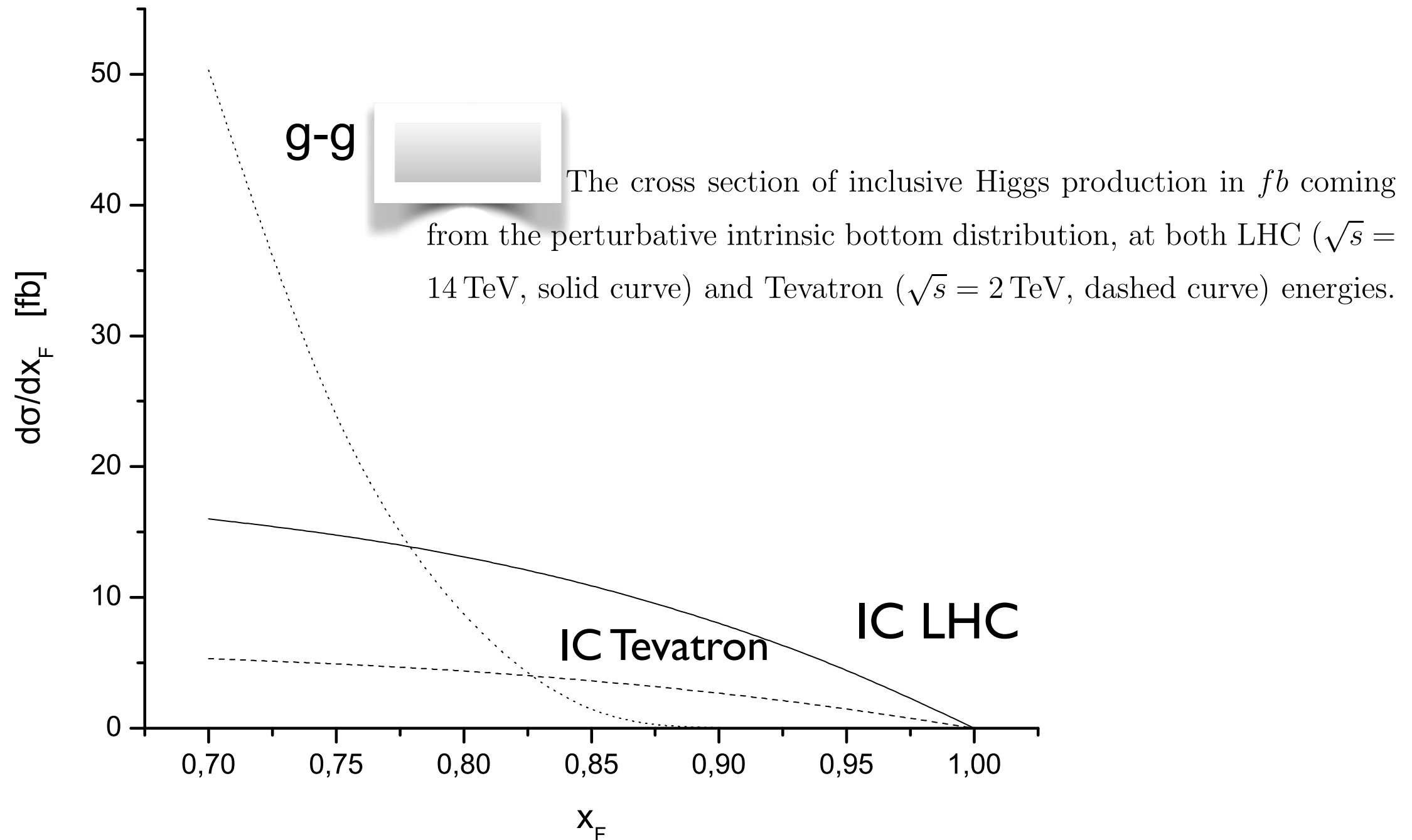
# Higgs Hadroproduction at Large Feynman $x$

A. S. Goldhaber, B. Z. Kopeliovich, I. Schmidt, *sjb*



# Higgs Hadroproduction at Large Feynman $x$

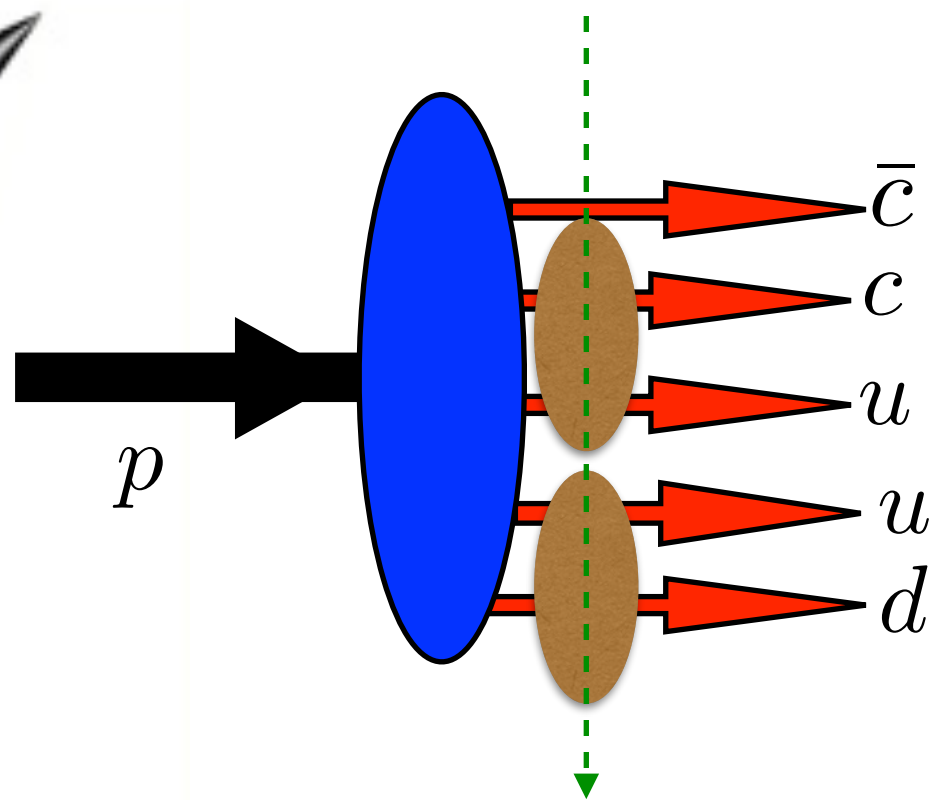
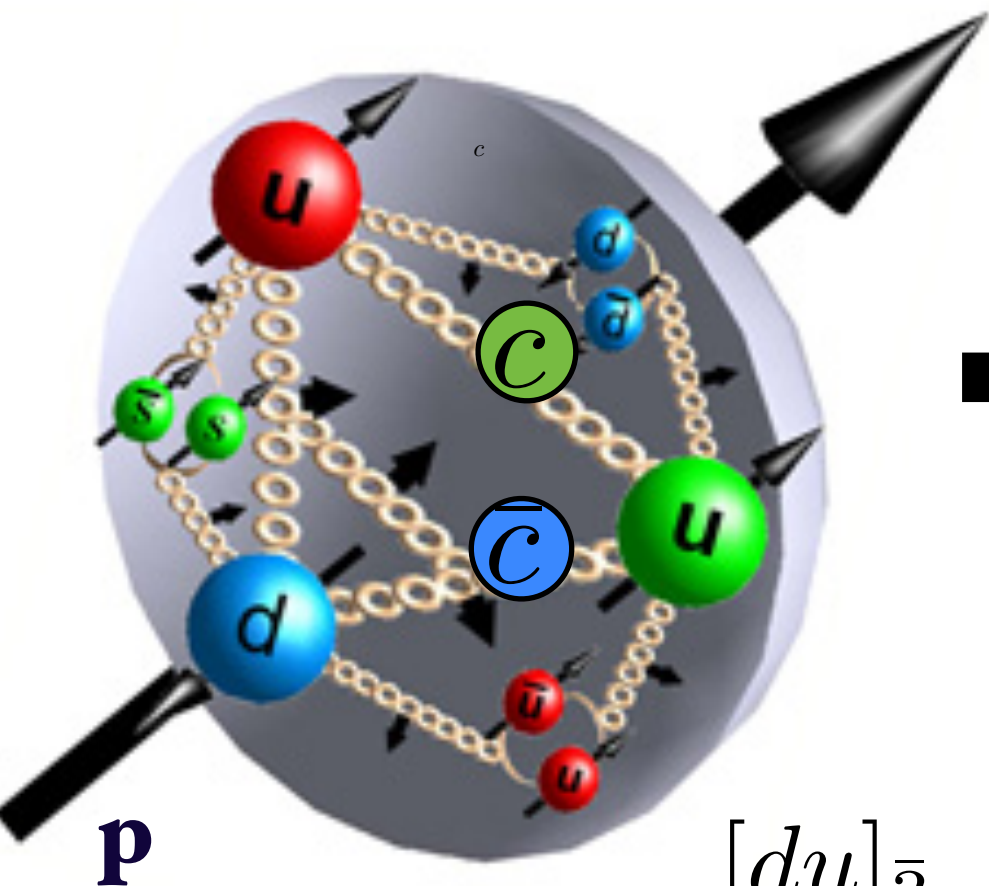
A. S. Goldhaber, B. Z. Kopeliovich, I. Schmidt, *sjb*



# Color confinement potential from AdS/QCD

$$U(\zeta^2) = \kappa^4 \zeta^2, \zeta^2 = b_{\perp}^2 x(1-x)$$

Fixed  $\tau = t + z/c$



Intrinsic Charm

$$|\bar{c}[cu][ud]\rangle$$

$[du]_{\bar{3}_C}$  and  $[cu]_{\bar{3}_C}$   $J = 0$  diquark dominance

$$\psi_n(\vec{k}_{\perp i}, x_i) \propto \frac{1}{\kappa^{n-1}} e^{-\mathcal{M}_n^2/2\kappa^2} \prod_{j=1}^n \frac{1}{\sqrt{x_j}}$$

$$\mathcal{M}_n^2 = \sum_{i=1}^n \left( \frac{k_{\perp}^2 + m^2}{x} \right)_i$$



# An analytic first approximation to QCD

## *AdS/QCD + Light-Front Holography*

- **As Simple as Schrödinger Theory in Atomic Physics**
- **LF radial variable  $\zeta$  conjugate to invariant mass squared**
- **Relativistic, Frame-Independent, Color-Confining**
- **Unique confining potential!**
- **QCD Coupling at all scales: Essential for Gauge Link phenomena**
- **Hadron Spectroscopy and Dynamics from one parameter**
- **Wave Functions, Form Factors, Hadronic Observables, Constituent Counting Rules**
- **Insight into QCD Condensates**
- **Systematically improvable with DLCQ-BLFQ Methods**

*Supersymmetric Features of Hadron Physics  
from Superconformal Algebra  
and Light-Front Holography*

# *Light-Front Holography: First Approximation to QCD*

- **Color Confinement, Analytic form of confinement potential**  
*de Téramond, Dosch, Lorcé, sjb*
- **Retains underlying conformal properties of QCD despite mass scale (DeAlfaro-Fubini-Furlan Principle)**
- **Massless quark-antiquark pion bound state in chiral limit, GMOR**
- **QCD coupling at all scales**
- **Connection of perturbative and nonperturbative mass scales**
- **Poincarè Invariant**
- **Hadron Spectroscopy-Regge Trajectories with universal slopes in  $n, L$**
- **Supersymmetric 4-Plet: Meson-Baryon-Tetraquark Symmetry**
- **Light-Front Wavefunctions**
- **Form Factors, Structure Functions, Hadronic Observables**
- **OPE: Constituent Counting Rules**
- **Hadronization at the Amplitude Level: Many Phenomenological Tests**
- **Systematically improvable: Basis LF Quantization (BLFQ)**

*Supersymmetric Features of Hadron Physics  
from Superconformal Algebra  
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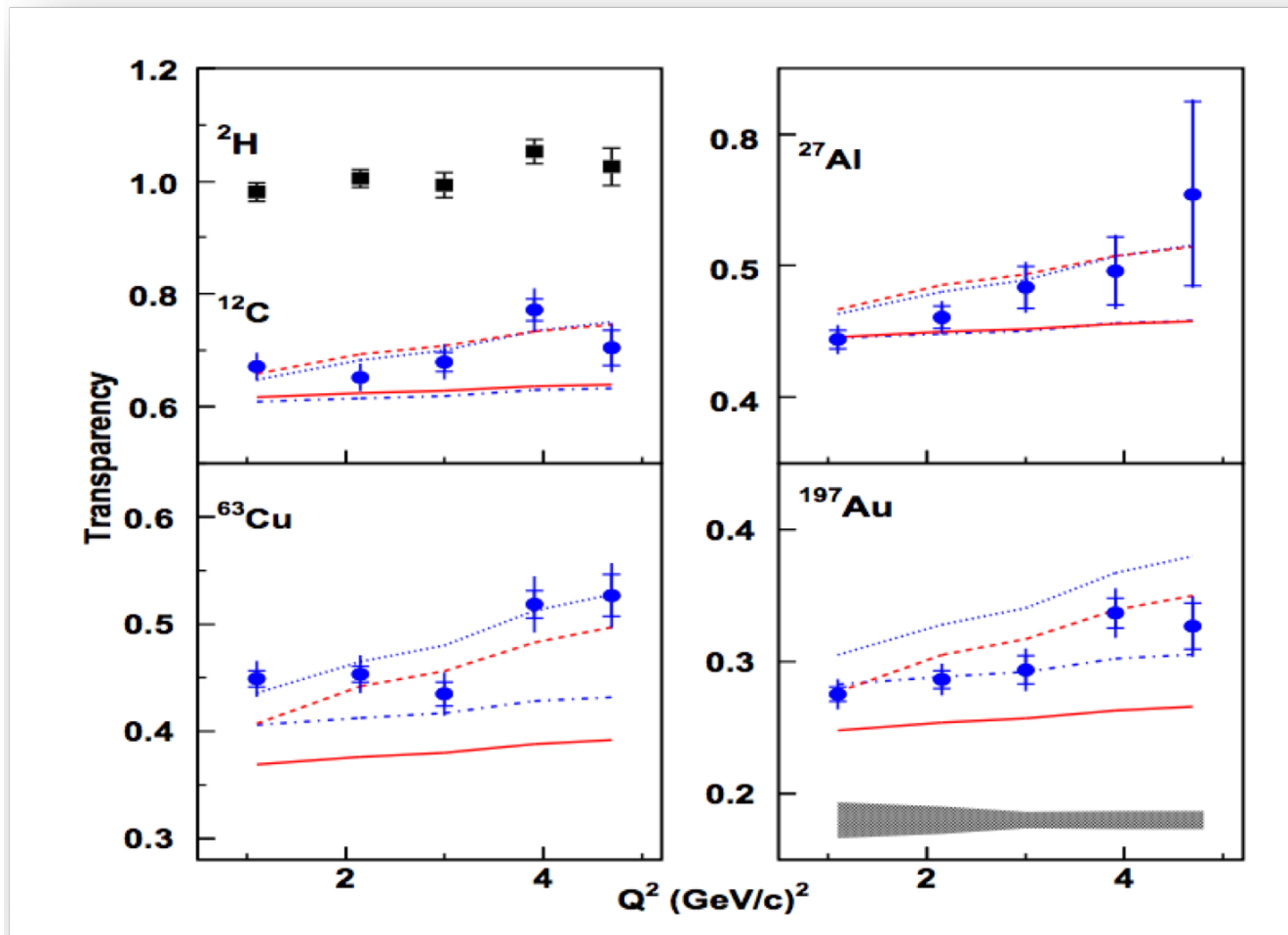
# Color Transparency verified for $\pi^+$ and $\rho$ electroproduction

Hall C E01-107 pion electro-production

$A(e, e' \pi^+)$

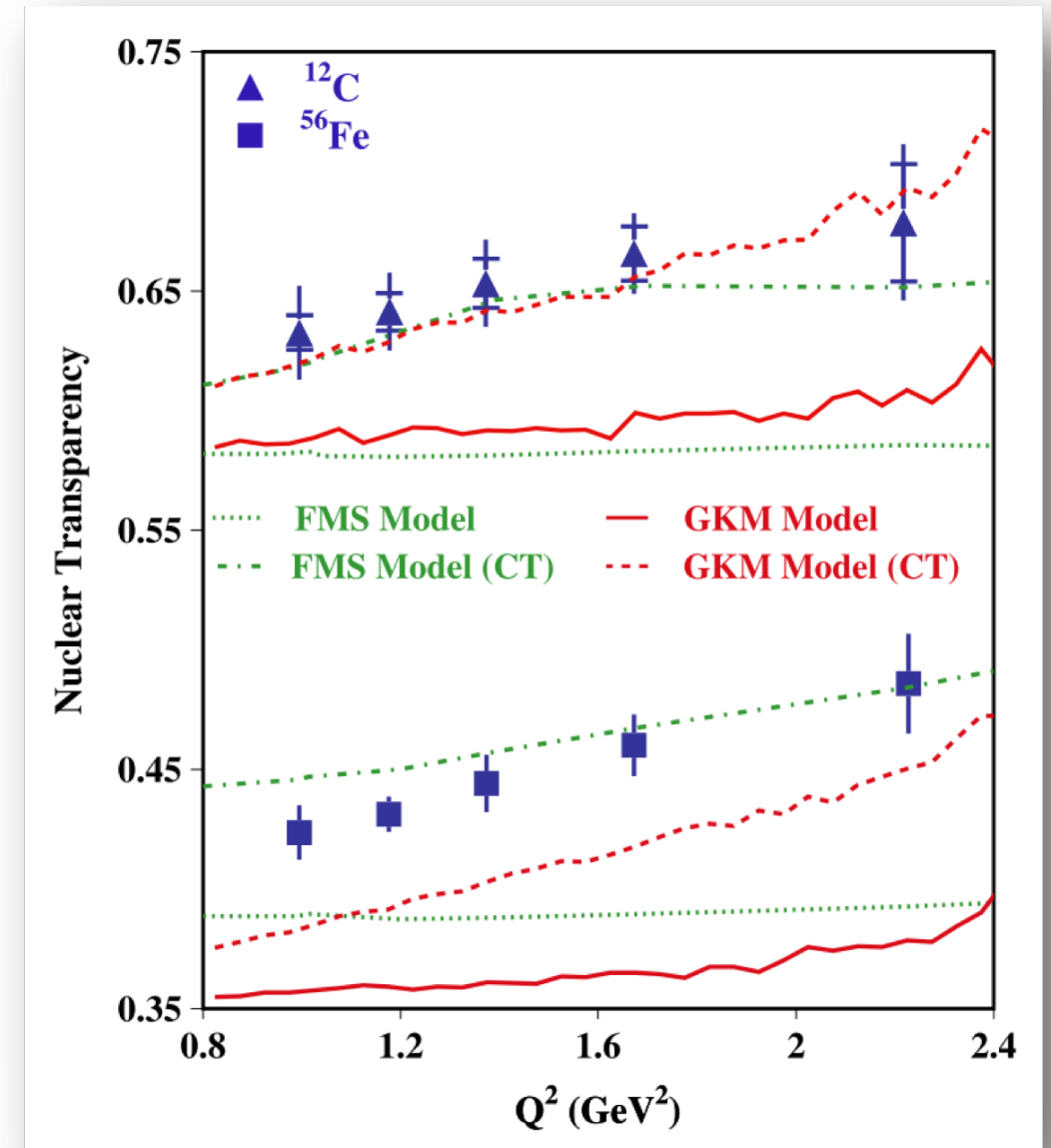
CLAS E02-110 rho electro-production

$A(e, e' \rho^0)$



B. Clasie *et al.* PRL 99:242502 (2007)

X. Qian *et al.* PRC81:055209 (2010)

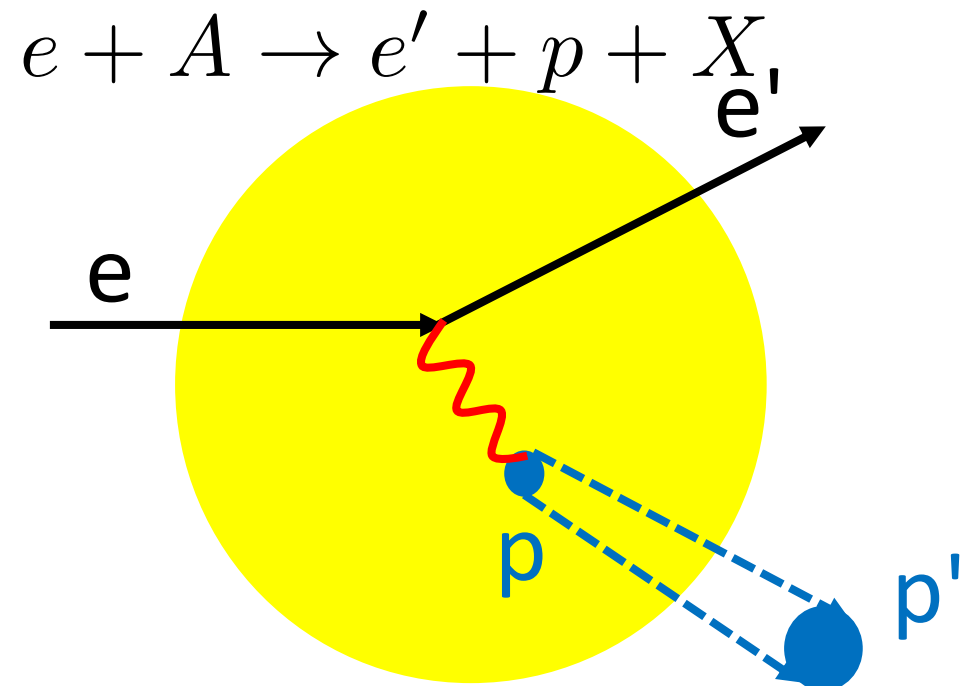


L. El Fassi *et al.* PLB 712,326 (2012)

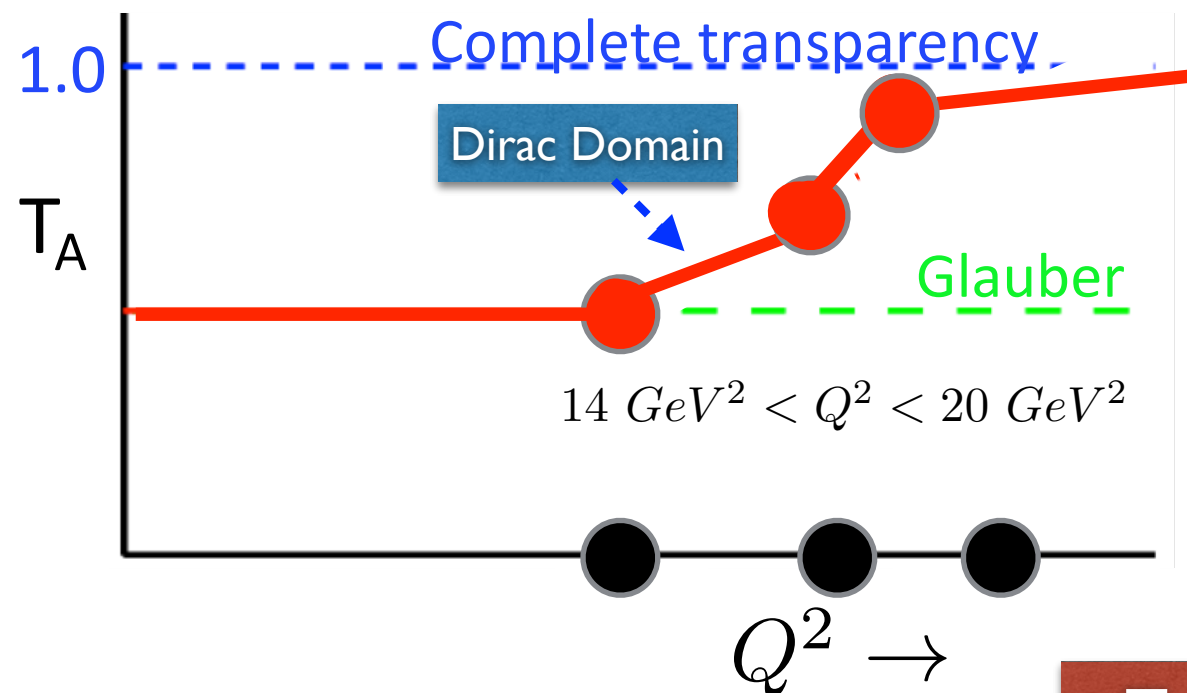
$$T_A = \frac{\frac{d\sigma}{dQ^2} (pA \rightarrow \pi^+ X)}{\frac{d\sigma}{dQ^2} (pp \rightarrow \pi^+ X)}$$

$$T_A = \frac{\frac{d\sigma}{dQ^2} (pA \rightarrow \rho^0 X)}{\frac{d\sigma}{dQ^2} (pp \rightarrow \rho^0 X)}$$

# Color transparency: fundamental prediction of QCD



- Not predicted by strongly interacting hadronic picture  $\rightarrow$  arises in picture of quark-gluon interactions
- QCD: color field of singlet objects vanishes as size is reduced
- Signature is a rise in nuclear transparency,  $T_A$ , as a function of the momentum transfer,  $Q^2$



$$T_A = \frac{\sigma_A \text{ (nuclear cross section)}}{A \sigma_N \text{ (free nucleon cross section)}}$$

G. de Teramond, sjb

Two-Stage Color Transparency for Proton

## Drell-Yan-West Formula in Impact Space

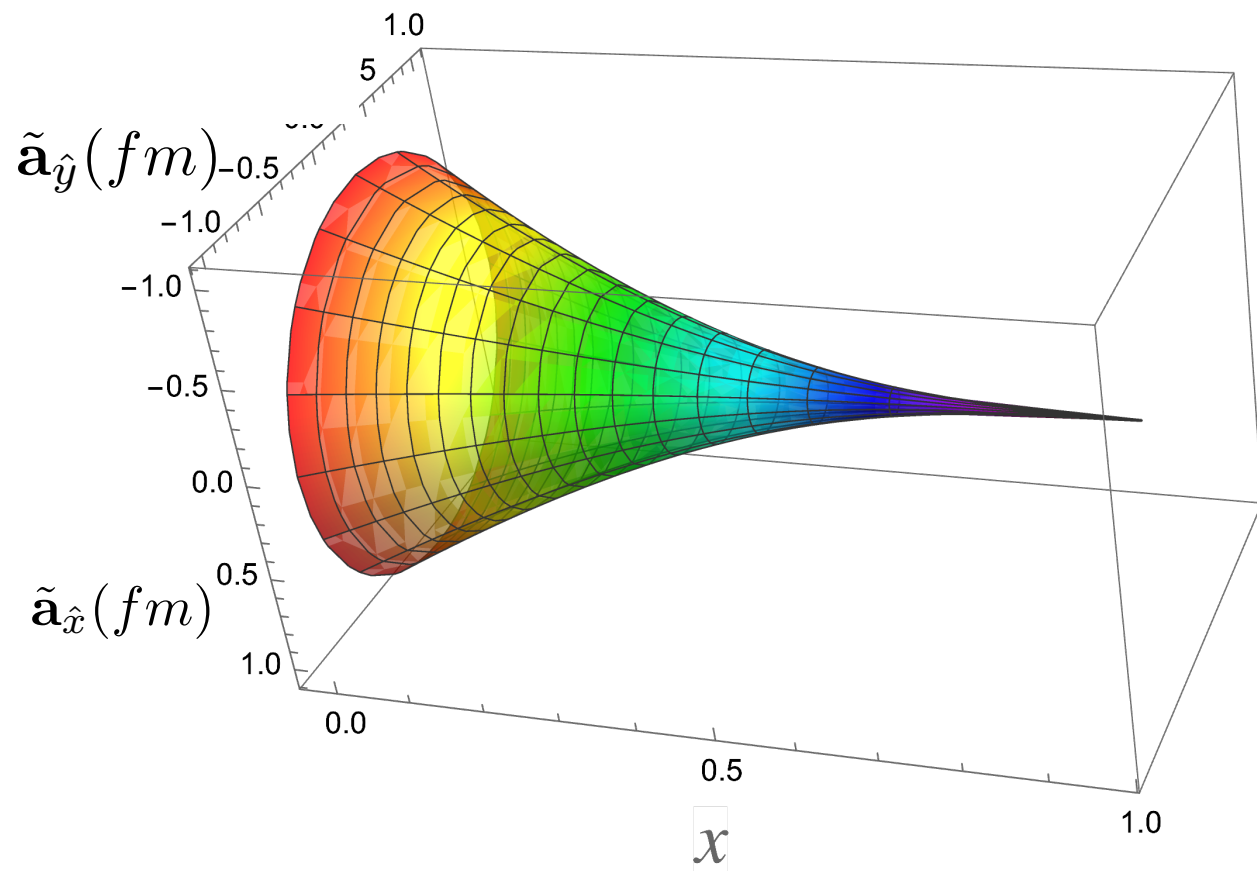
$$\begin{aligned}
 F(q^2) &= \sum_n \prod_{i=1}^n \int dx_i \int \frac{d^2 \mathbf{k}_{\perp i}}{2(2\pi)^3} 16\pi^3 \delta\left(1 - \sum_{j=1}^n x_j\right) \delta^{(2)}\left(\sum_{j=1}^n \mathbf{k}_{\perp j}\right) \\
 &\quad \sum_j e_j \psi_n^*(x_i, \mathbf{k}'_{\perp i}, \lambda_i) \psi_n(x_i, \mathbf{k}_{\perp i}, \lambda_i), \\
 &= \sum_n \prod_{i=1}^{n-1} \int dx_j \int d^2 \mathbf{b}_{\perp j} \exp\left(i \mathbf{q}_{\perp} \cdot \sum_{i=1}^{n-1} x_j \mathbf{b}_{\perp j}\right) |\psi_n(x_j, \mathbf{b}_{\perp j})|^2
 \end{aligned}$$

$$\sum_{i=1}^n x_i = 1 \text{ and } \sum_{i=1}^n \mathbf{b}_{\perp i} = 0.$$

$$F(q^2) = \int_0^1 dx \int d^2 \mathbf{a}_{\perp} e^{i \mathbf{a}_{\perp} \cdot \mathbf{q}_{\perp}} q(x, \mathbf{a}_{\perp}),$$

where  $\mathbf{a}_{\perp} = \sum_{j=1}^{n-1} x_j \mathbf{b}_{\perp j}$  is the  $x$ -weighted transverse position coordinate of the  $n - 1$  spectators.





$$\langle \tilde{\mathbf{a}}_{\perp}^2(x) \rangle = \frac{\int d^2 \mathbf{a}_{\perp} \mathbf{a}_{\perp}^2 q(x, \mathbf{a}_{\perp})}{\int d^2 \mathbf{a}_{\perp} q(x, \mathbf{a}_{\perp})}$$

At large light-front momentum fraction  $x$ , and equivalently at large values of  $Q^2$ , the transverse size of a hadron behaves as a point-like color-singlet object. This behavior is the origin of color transparency in nuclei.

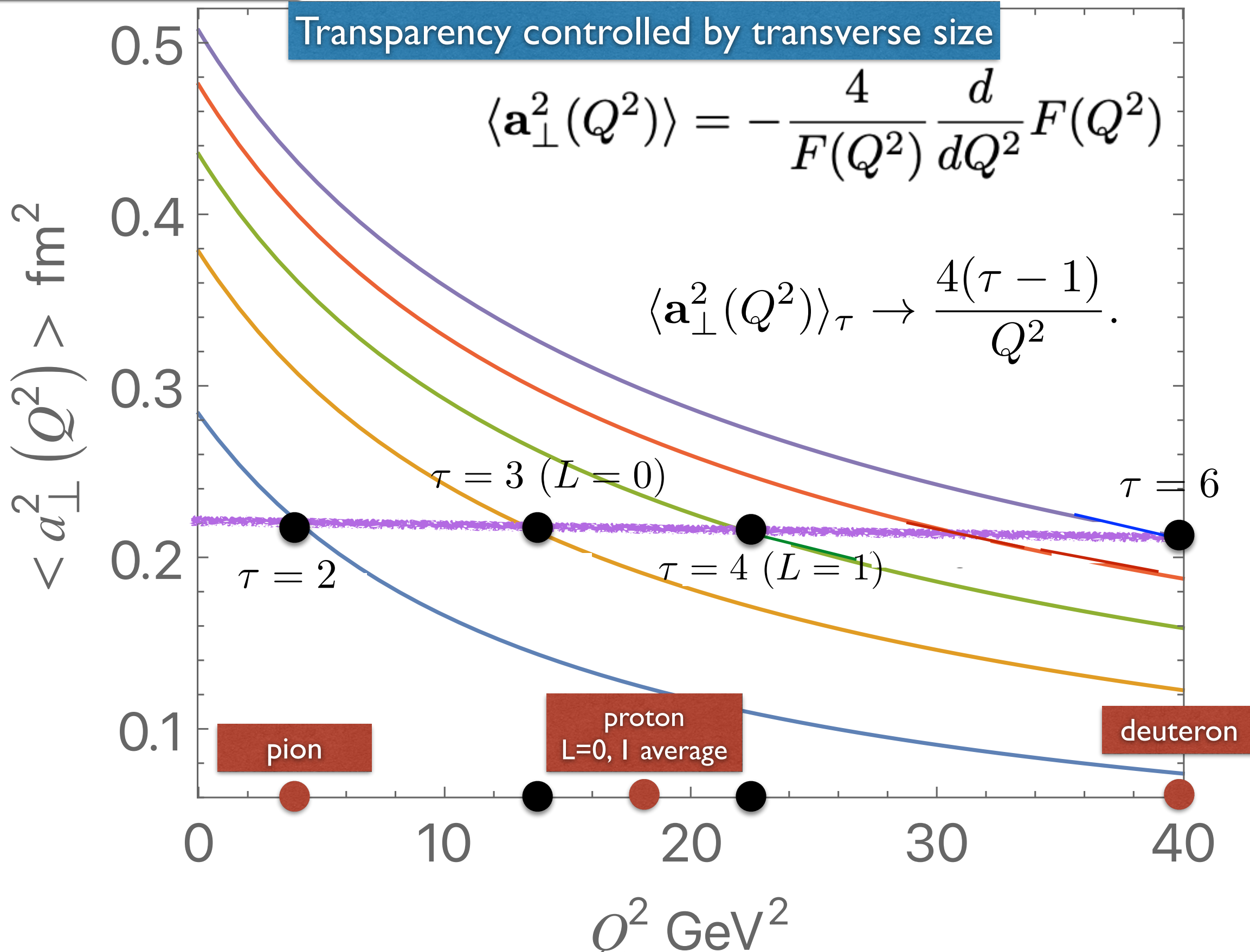
Although the dependence of the transverse impact area as a function of  $x$  is universal, the behavior in  $Q^2$  depends on properties of the hadron, such as its twist.

$$\langle \mathbf{a}_{\perp}^2(Q^2) \rangle_{\tau} \rightarrow \frac{4(\tau - 1)}{Q^2}.$$

*Mean transverse size  
as a function of  $Q$  and Twist*

Transparency scale  $Q$   
increases with twist

Transparency controlled by transverse size



Proton has equal probability for  $\tau = 3$  and  $\tau = 4$

$$F(q^2) = \sum_n \prod_{j=1}^{n-1} \int dx_j \int d^2 \mathbf{b}_{\perp j} \exp\left(i \mathbf{q}_{\perp} \cdot \sum_{j=1}^{n-1} x_j \mathbf{b}_{\perp j}\right) |\psi_n(x_j, \mathbf{b}_{\perp j})|^2$$

$$\sum_i x_i = 1$$

$$\vec{a}_{\perp} \equiv \sum_{j=1}^{n-1} x_j \vec{b}_{\perp j}$$

$$\vec{a}_{\perp}^2(Q^2) = -4 \frac{\frac{d}{dQ^2} F(Q^2)}{F(Q^2)}$$

Proton radius squared at  $Q^2 = 0$

Color Transparency is controlled by the transverse-spatial size  $\vec{a}_{\perp}^2$  and its dependence on the momentum transfer  $Q^2 = -t$  :  
The scale  $Q_{\tau}^2$  required for Color Transparency grows with twist  $\tau$

Light-Front Holography:

For large  $Q^2$  :

$$\langle \mathbf{a}_{\perp}^2(t) \rangle_{\tau} = \frac{1}{\lambda} \sum_{j=1}^{\tau-1} \frac{1}{j - \alpha(t)}$$

$$\langle \mathbf{a}_{\perp}^2(Q^2) \rangle_{\tau} \rightarrow \frac{4(\tau - 1)}{Q^2}.$$

# Two-Stage Color Transparency

$$14 \text{ GeV}^2 < Q^2 < 20 \text{ GeV}^2$$

If  $Q^2$  is in the intermediate range, then the twist-3 state will propagate through the nuclear medium with minimal absorption, and the protons which survive nuclear absorption will only have  $L = 0$  (twist-3).

The twist-4  $L = 1$  state which has a larger transverse size will be absorbed.

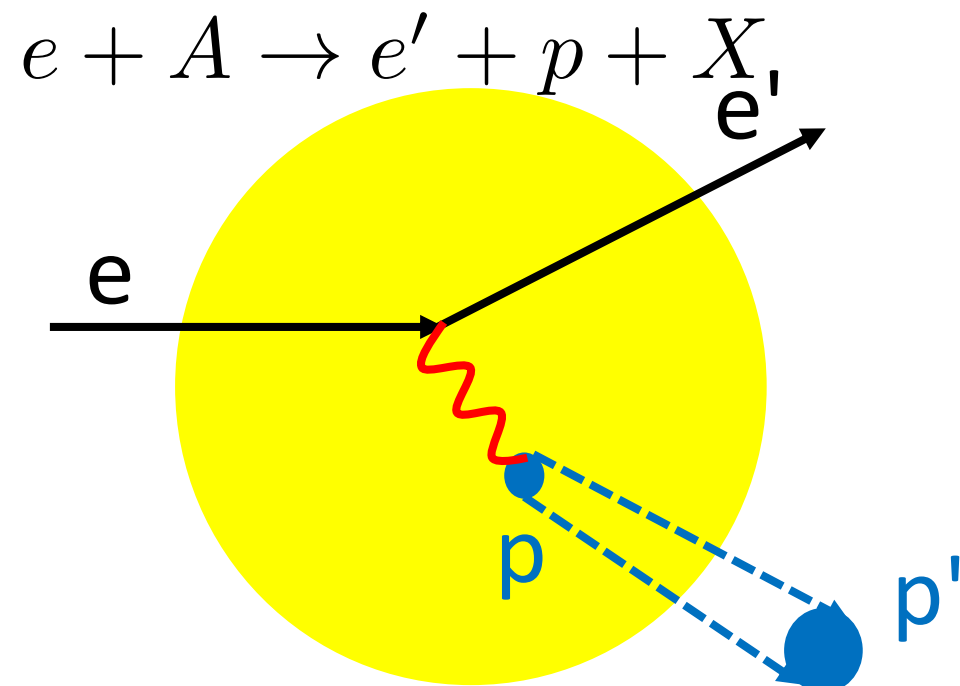
Thus 50% of the events in this range of  $Q^2$  will have full color transparency and 50% of the events will have zero color transparency ( $T = 0$ ).

The  $ep \rightarrow e'p'$  cross section will have the same angular and  $Q^2$  dependence as scattering of the electron on an unphysical proton which has no Pauli form factor.

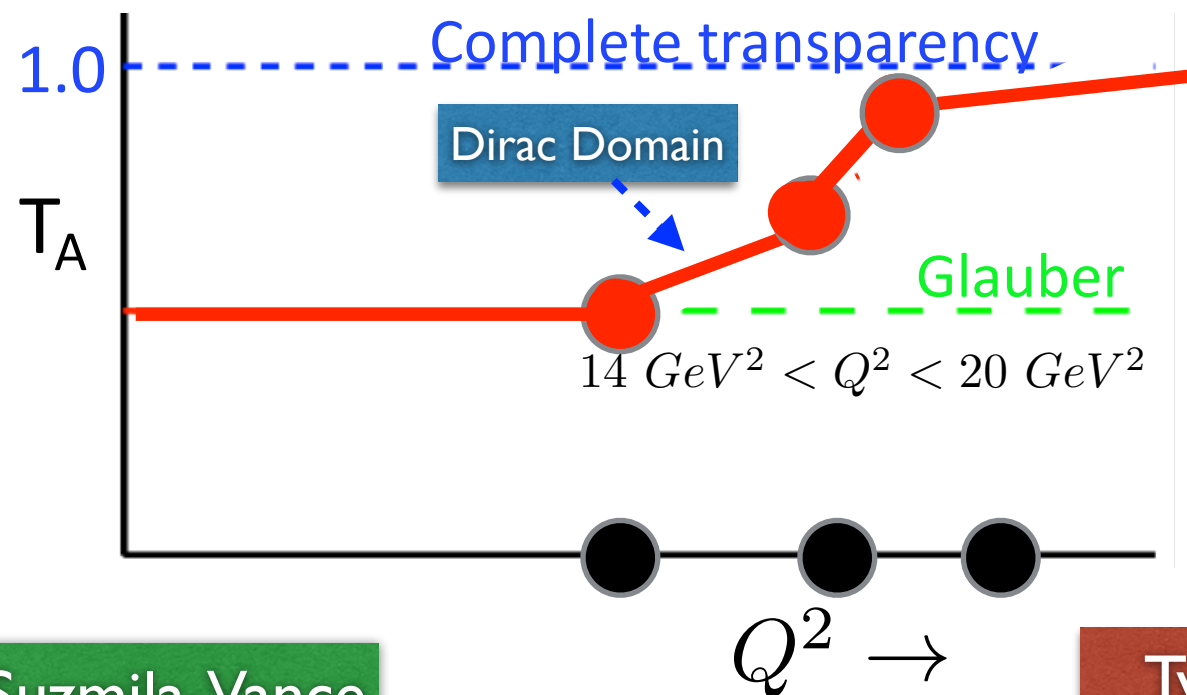
$$Q^2 > 20 \text{ GeV}^2$$

However, if the momentum transfer is increased to  $Q^2 > 20 \text{ GeV}^2$ , all events will have full color transparency, and the  $ep \rightarrow e'p'$  cross section will have the same angular and  $Q^2$  dependence as scattering of the electron on a physical proton eigenstate, with both Dirac and Pauli form factor components.

# Color transparency fundamental prediction of QCD



- Not predicted by strongly interacting hadronic picture  $\rightarrow$  arises in picture of quark-gluon interactions
- QCD: color field of singlet objects vanishes as size is reduced
- Signature is a rise in nuclear transparency,  $T_A$ , as a function of the momentum transfer,  $Q^2$



$$T_A = \frac{\sigma_A}{A \sigma_N} \quad \begin{array}{l} \text{(nuclear cross section)} \\ \text{(free nucleon} \\ \text{cross section)} \end{array}$$



# *Color Transparency and Light-Front Holography*

- Essential prediction of QCD
- LF Holography: Spectroscopy, dynamics, structure
- Transverse size predicted by LF Holography as a function of Q
- Q scale for CT increases with twist, number of constituents
- Two-Stage Proton Transparency: Equal probability L=0,1
- No contradiction with present experiments

$Q_0^2(p) \simeq 18 \text{ GeV}^2$  vs.  $Q_0^2(\pi) \simeq 4 \text{ GeV}^2$  for onset of color transparency in  $^{12}\text{C}$

## Other Consequences of $[ud]_{\bar{3}_C, I=0, J=0}$ diquark cluster

### QCD Hidden-Color Hexadiquark in the Core of Nuclei

J. Rittenhouse West, G. de Teramond, A. S. Goldhaber, I. Schmidt, sjb

$$|\Psi_{HDQ}\rangle = |[ud][ud][ud][ud][ud][ud]\rangle$$

mixes with

$${}^4He|npnp\rangle$$

**Increases alpha binding energy, EMC effects**

### Diquarks Can Dominate Five-Quark Fock State of Proton

$$|p\rangle = \alpha|[ud]u\rangle + \beta|[ud][ud]\bar{d}\rangle$$

Natural explanation why  $\bar{d}(x) \gg \bar{u}(x)$  in proton

**Excitations and Decay of HdQ in Alpha-Nuclei  
may explain ATOMKI X17 signal**

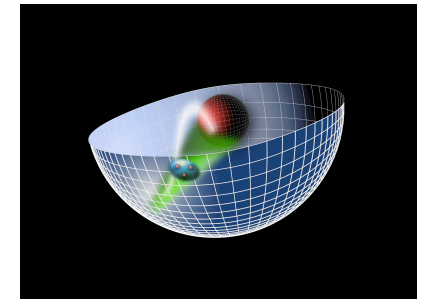
# Underlying Principles

- **Polncarè Invariance: Independent of the observer's Lorentz frame: Quantization at Fixed Light-Front Time  $\tau$**

- **Causality: Information within causal horizon: Light-Front**

- **Light-Front Holography:  $AdS_5 = LF (3+1)$**

$$z \leftrightarrow \zeta \text{ where } \zeta^2 = b_{\perp}^2 x(1-x)$$



- **Introduce mass scale  $\kappa$  while retaining conformal invariance of the Action (dAFF)**

*“Emergent Mass”*

- **Unique Dilaton in  $AdS_5$ :  $e^{+\kappa^2 z^2}$**

- **Unique color-confining LF Potential  $U(\zeta^2) = \kappa^4 \zeta^2$**

- **Superconformal Algebra: Mass Degenerate 4-Plet:**

Meson  $q\bar{q} \leftrightarrow$  Baryon  $q[qq] \leftrightarrow$  Tetraquark  $[qq][\bar{q}\bar{q}]$

# *Light-Front Holography: First Approximation to QCD*

- **Color Confinement, Analytic form of confinement potential**
- **Retains underlying conformal properties of QCD despite mass scale (DeAlfaro-Fubini-Furlan Principle)**
- **Massless quark-antiquark pion bound state in chiral limit, GMOR**
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*Supersymmetric Features of Hadron Physics  
from Superconformal Algebra  
and Light-Front Holography*

## Holographic light-front QCD (HLFQCD)

Present analytic approach follows from a semiclassical approximation to light-front QCD and its holographic embedding in AdS space: It leads to relativistic wave equations similar to the Schrödinger equation in atomic physics

Further constraints from a superconformal algebraic structure introduce a mass scale and fix the effective confinement potential: It is not SUSY QCD

The zero energy eigenmode is identified with the pion and it is massless in the chiral limit

The new framework leads to relations between the Regge trajectories of mesons, baryons, and tetraquarks

Holographic QCD also incorporates features of the Veneziano model as emerging properties

Further extensions incorporate the exclusive-inclusive connection in QCD and provide nontrivial relations between hadron form factors and quark and gluon distributions



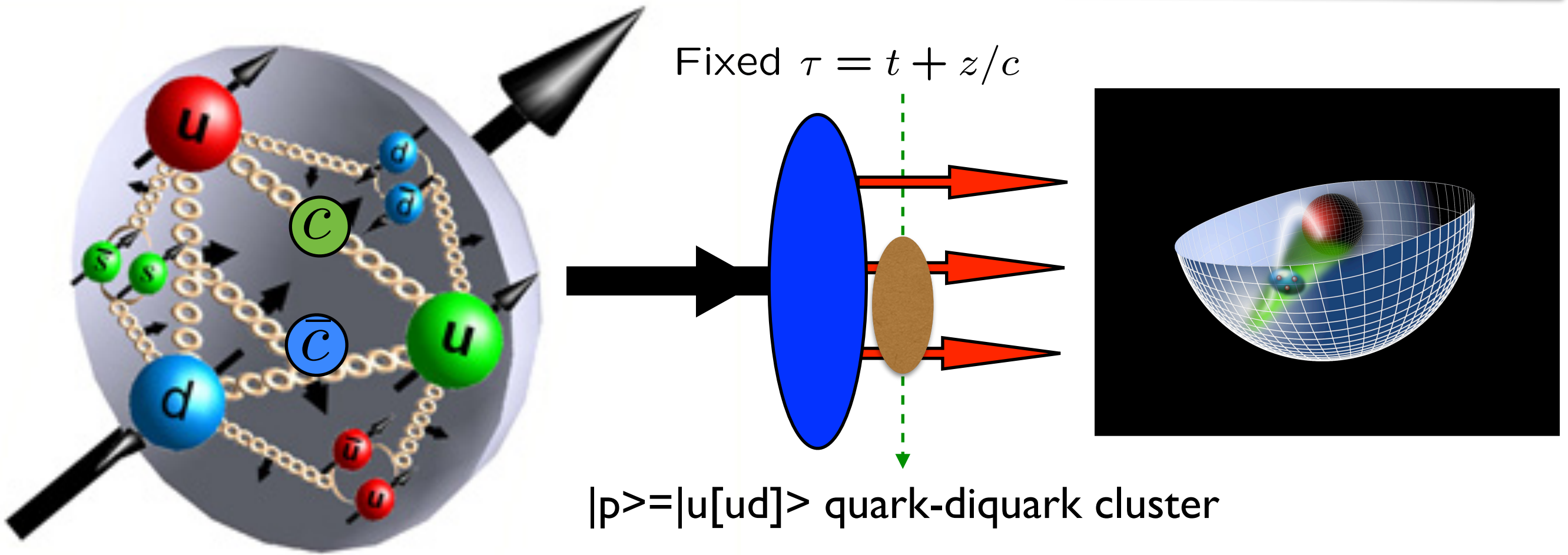
# Future Directions for AdS/QCD

- **Hadronization at the Amplitude Level**
- **Diffraction dissociation of pion and proton to jets**
- **Factorization Scale for ERBL, DGLAP evolution:  $Q_0$**
- **Calculate Sivers Effect including FSI and ISI**
- **Compute Tetraquark Spectroscopy: Sequential Clusters**
- **Update SU(6) spin-flavor symmetry**
- **Heavy Quark States: Supersymmetry, not conformal**
- **Compute higher Fock states; e.g. Intrinsic Heavy Quarks**
- **Nuclear States — Hidden Color**
- **Basis LF Quantization**

*Basis light-front quantization: A new approach to non-perturbative scattering and time-dependent production processes*

*James P. Vary, Xingbo Zhao, Anton Ilderton, Heli Honkanen, Pieter Maris, Stanley J. Brodsky*

# New Perspectives for Hadron Spectroscopy and Dynamics and the QCD Coupling from Color-Confining Holographic Light-Front QCD



with Guy de Tèramond, Hans Günter Dosch, Cèdric Lorcè, Alexandre Deur, and Joshua Erlich

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