Pion Gravitational Form Factors in Holographic QCD

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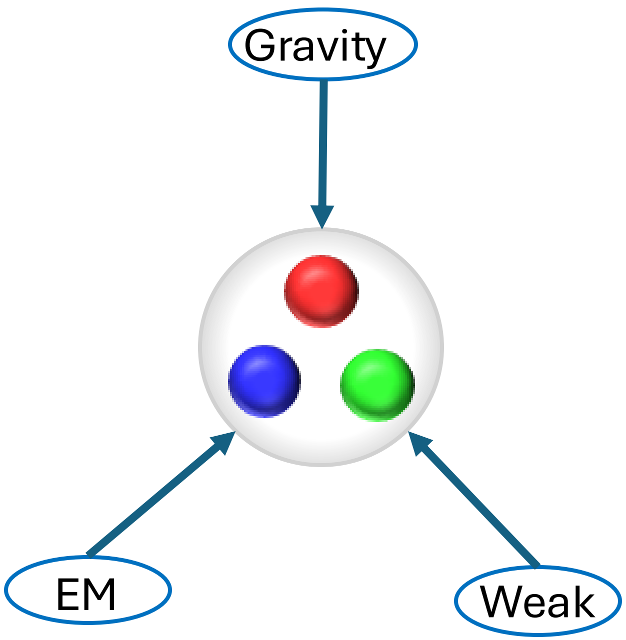
arXiv:2407.21113 [hep-ph] (2024).

Gravitational Form Factors(GFFs)

GFFs: Form factors defined through gravitational interaction



Probing through several forces

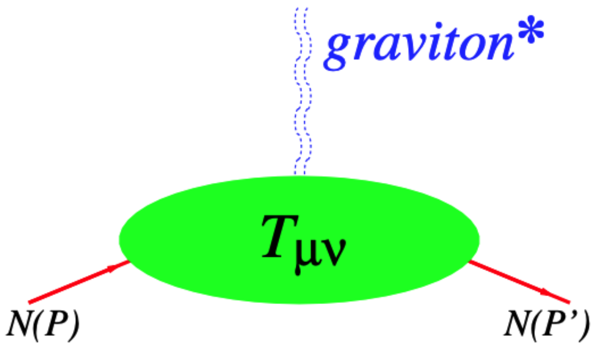
The source of gravitation is the energy momentum tensor

spin-0: $\langle p'|\hat{T}^{\mu\nu}(x)|p\rangle = \left[2P^\mu P^\nu A(t) + \frac{1}{2}(\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2)D(t)\right] e^{ix\cdot\Delta}$

GFFs

spin-$\frac{1}{2}$: $\langle p',s'|\hat{T}^{\mu\nu}(x)|p,s\rangle$

$$= \bar{u}'\left[\frac{P^\mu P^\nu}{m}A(t) + \frac{iP_{\{\mu}\sigma_{\nu\}\rho}\Delta^\rho}{2m}J(t) + \frac{\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2}{4m}D(t)\right]ue^{ix\cdot\Delta}$$



$P^\mu = (p'^\mu + p^\mu)/2$
$\Delta^\mu = p'^\mu - p^\mu,\ t = \Delta^2$

M. Polyakov and P. Schweiter, Int. J. Mod. Phys. A 33, 1830025 (2018)

Why we focus on GFFs?

GFFs may provide us insights into the non-perturbative phenomena of QCD
(confinement, chiral symmetry breaking etc.)

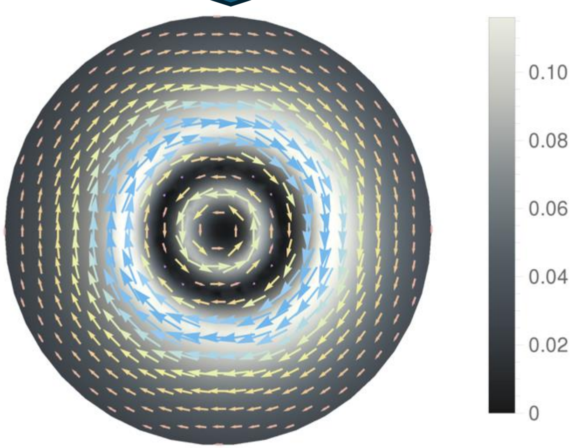
Remind:

energy density momentum density

$$T^{\mu\nu} = \begin{pmatrix} T^{00} & T^{01} & T^{02} & T^{03} \\ T^{10} & T^{11} & T^{12} & T^{13} \\ T^{20} & T^{21} & T^{22} & T^{23} \\ T^{30} & T^{31} & T^{32} & T^{33} \end{pmatrix}$$

pressure

shear stress



The tangental force distribution in the nucleon from χQSM

M. Polyakov and P. Schweiter, Int. J. Mod. Phys. A 33, 1830025 (2018)

There are some relations

GFFs $\leftrightarrow$ GPD, GDA

In the case of the nucleon,

$$\int_{-1}^{1} dx\, x H^a(x,\xi,t) = A^a(t) + \xi^2 D^a(t)$$

$$\int_{-1}^{1} dx\, x E^a(x,\xi,t) = B^a(t) - \xi^2 D^a(t)$$

GFFs lead us to tackle issues such as

- Origin of mass
- Proton spin crisis

⋮

D-term

D-term: the value of $D(t)$ in the forward limit $t \to 0$

The EMT of a spin-0 particle

$$\langle p' | \hat{T}^{\mu\nu}(x) | p \rangle = \left[2P^\mu P^\nu A(t) + \frac{1}{2}(\Delta^\mu \Delta^\nu - g^{\mu\nu}\Delta^2) D(t) \right] e^{ix\cdot\Delta}$$

$A(0)$ must be 1 from general constraint

NO general constraint on $D(0)$ and should be determined experimentally

em: $\partial_\mu J^\mu_{\mathrm{em}} = 0$	$\langle N' \| J^\mu_{\mathbf{em}} \| N \rangle$	$\longrightarrow$	Q	$= 1.602176487(40) \times 10^{-19}\mathrm{C}$
			μ	$= 2.792847356(23)\mu_N$
weak: PCAC	$\langle N' \| J^\mu_{\mathbf{weak}} \| N \rangle$	$\longrightarrow$	g_A	$= 1.2694(28)$
			g_p	$= 8.06(55)$
gravity: $\partial_\mu T^{\mu\nu}_{\mathbf{grav}} = 0$	$\langle N' \| T^{\mu\nu}_{\mathbf{grav}} \| N \rangle$	$\longrightarrow$	m	$= 938.272013(23)\ \mathrm{MeV}/c^2$
			J	$= \frac{1}{2}$
			$\boldsymbol{D}$	$= \boldsymbol{?}$

The global properties of the proton

D-term is sometimes called "The last unknown property of hadron"

D-term

Interesting property of the D-term of a pion

- In free Klein-Gordon theory, D-term takes the value -1
- Introducing the interactions, it deviates from -1
- However, for Nambu-Goldstone (NG) bosons, the D-term remains -1 due to the restriction by chiral symmetry

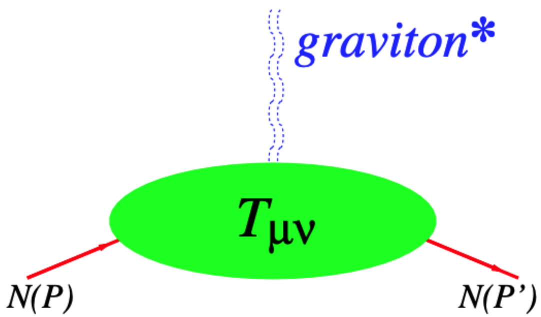
↓

D-term might reveal the role of chiral symmetry

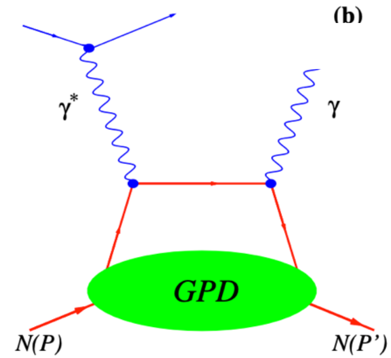
Experiment

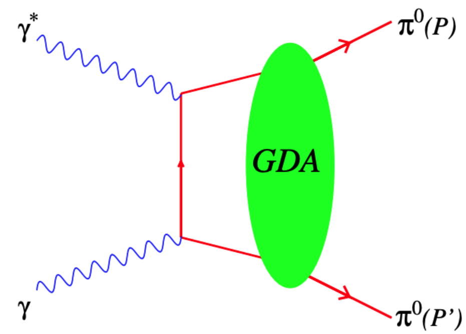
The experimental progress is also remarkable.

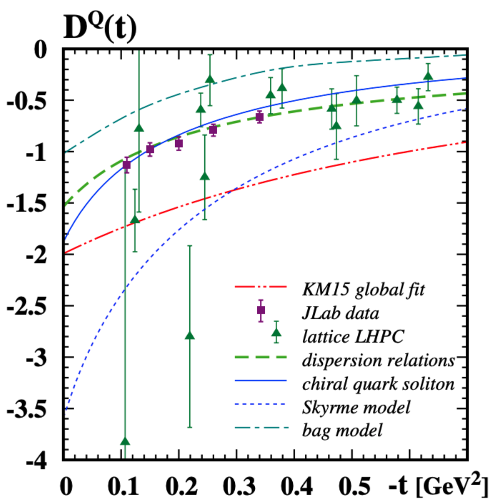
- Natural way, but not practical



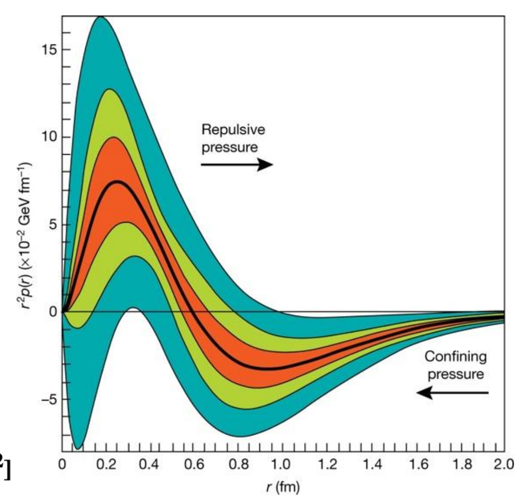
- Practical way: GFFs ↔ GPD, GDA



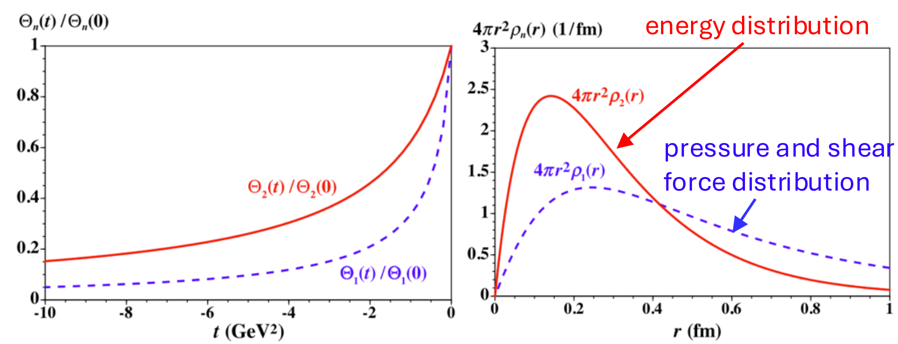




M. Polyakov and P. Schweiter,
Int. J. Mod. Phys. A 33, 1830025 (2018)



Burkert, Elouadrhiri, Girod,
Nature Vol.557, 396 (2018)



Kumano et al., Phys. Rev. D 97 (2018) 014020

$$\Theta_2(t) = 4A^{\pi}(t),\ \Theta_1(t) = -D^{\pi}(t)$$

Top-down Holographic QCD Approach to GFFs

We calculate the t-dependence of the pion's gravitational form factors from top-down holographic QCD (Sakai-Sugimoto Model).

The dictionary with holographic renormalization

$$\int_{x_{11},\tau} \delta g_{\mu\nu} \sim \frac{2\kappa_7^2 L^5}{6} \frac{\langle T_{\mu\nu} \rangle}{r^4} + \cdots$$

S. Haro et al., Commun. Math. Phys., 217:595–622, 2001.

The metric fluctuations caused by matter fields

propagate to the boundary

The expectation values of EMT on the boundary theory

Basically, if we determine $\delta g_{\mu\nu}$, we can extract $\langle T_{\mu\nu} \rangle$ of QCD from it.

The Einstein equation that $\delta g_{\mu\nu}$ follows

$$\mathcal{H}_{MN} - \frac{g_{MN}}{2} \mathcal{H}_P^P = -2\kappa_7^2 \mathcal{T}_{MN}$$

$$\mathcal{H}_{MN} \equiv \nabla^2 \delta g_{MN} + \nabla_M \nabla_N \delta g_P^P - \nabla^P (\nabla_M \delta g_{NP} + \nabla_N \delta g_{MP}) - \frac{12}{L^2} \nabla_M \delta g_{MN}$$

$\mathcal{T}_{MN}$: the EMT in the bulk

Fujita et al., PTEP 2022, 093B06 (2022)

Our analysis: The EMT in the bulk

We consider the case where the pion field is the matter field as source of $\mathcal{T}_{MN}$

➡ matter fields: $A_\mu(x^\nu, z) = 0, \quad A_z(x^\nu, z) = \pi(x^\nu)\phi_0(z)$

Action:

$$S_{D8} = -\frac{C}{2\pi R_{11} g_s} \int \left(\delta(\tau) + \delta(\tau - \pi/M_{KK})\right) \frac{\sqrt{-G_{(11)}}}{\sqrt{G_{\tau\tau}}} \frac{1}{4} G^{MN} G^{PQ} \mathrm{tr}[F_{MP} F_{NQ}] + \cdots$$

The metric in the SS model

$$ds^2 = \frac{r^2}{L^2}\left[f(r)d\tau^2 - dx_0^2 + dx_1^2 + dx_2^2 + dx_3^2 + dx_{11}^2\right] + \frac{L^2}{r^2}\frac{dr^2}{f(r)} + \frac{L^2}{4}d\Omega_4^2,$$

$$f(r) = 1 - \frac{R^6}{r^6}, \quad R = \frac{L^2 M_{KK}}{3}, \quad L^3 = 8\pi g_s N_c l_s^3,$$

$$R_{11} = \frac{\lambda}{2\pi N_c M_{KK}}, \quad \lambda N_c = \frac{L^6 M_{KK}}{32\pi g_s l_s^5} = 216\pi^3\kappa,$$

$M_{KK} = 949$ MeV, $\kappa = 0.00745$ in this study

The definition of EMT $\mathcal{T}_{MN}$

$$\mathcal{T}_{MN} = -\frac{2}{\sqrt{-G_{(11)}}} \frac{\delta S_{D8}}{\delta G^{MN}_{(11)}} \mathrm{Vol}(S^4)$$

Fujita et al., PTEP 2022, 093B06 (2022)

Results

$$A(t) = \frac{6}{L^7}\int_R^\infty dr' \sum_{n=1}^{\infty} \frac{\alpha_n^{\rm T}\Psi_n^{\rm T}(r')}{(m_n^{\rm T})^2+\vec{\Delta}^2}\frac{r'^3}{L^3}P_{\rm t}^{\mu\nu}\mathcal{T}_{\mu\nu}(\vec{\Delta},r'),$$

$$D(t) =$$
$$-\frac{6}{L^7}\int_R^\infty dr' \sum_{n=1}^{\infty} \frac{\alpha_n^{\rm T}\Psi_n^{\rm T}(r')}{(m_n^{\rm T})^2+\vec{\Delta}^2}\frac{r'^3}{L^3}P_{\rm s}^{\mu\nu}(\mathcal{T}_{\mu\nu}+\mathcal{T}_{\mu\nu}^{\rm S}+\mathcal{T}_{\mu\nu}^{\rm other})$$
$$+\frac{6}{L^7}\int_R^\infty dr' \sum_{n=1}^{\infty} \frac{\alpha_n^{\rm S}\Psi_n^{\rm S}(r')}{(m_n^{\rm S})^2+\vec{\Delta}^2}\frac{r'^3}{L^3}P_{\rm s}^{\mu\nu}\mathcal{T}_{\mu\nu}^{\rm S},$$

Wavefunctions of n's glueball of T_4/S_4 modes

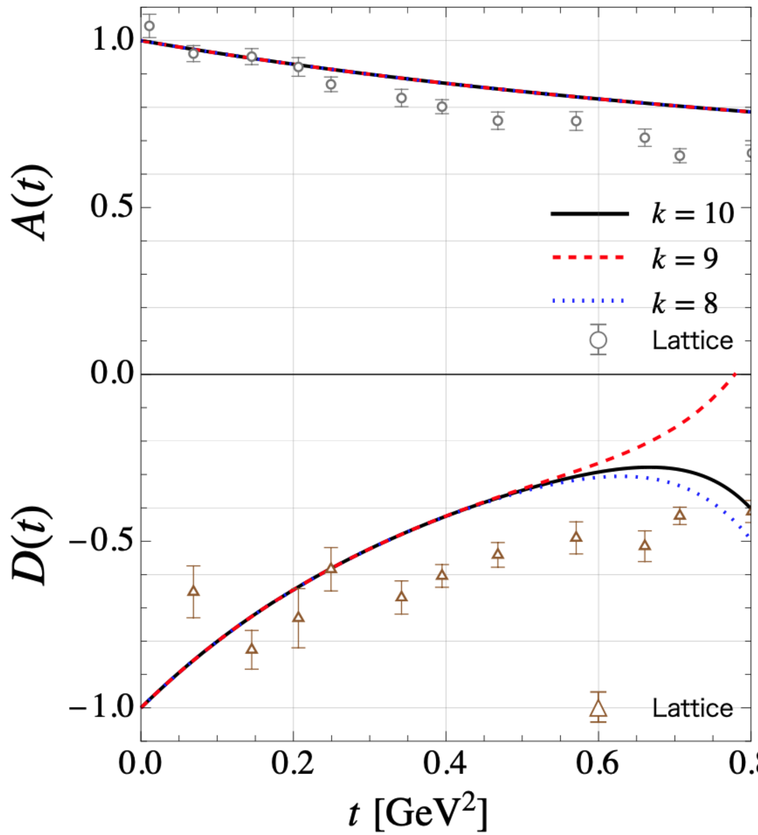
$$\alpha^{\rm T/S} = \frac{(m_n^{\rm T/S})^2 L^4}{6}\int_R^\infty dr r^3 \Psi_n^{\rm T/S}(r),$$

Masses of n's glueball of T_4/S_4 modes

Decay constants of n's glueball of T_4/S_4 modes

To calculate infinite sums, we expand it in terms of momentum

$$\frac{6}{L^7}\int_R^\infty dr' \sum_{n=1}^{\infty} \frac{\alpha_n^{\rm T/S}\Psi_n^{\rm T/S}(r')}{(m_n^{\rm T/S})^2+\vec{\Delta}^2} = \sum_{k=0}^{\infty} F_k^{\rm T/S}(r')(-\vec{\Delta}^2)^k,$$

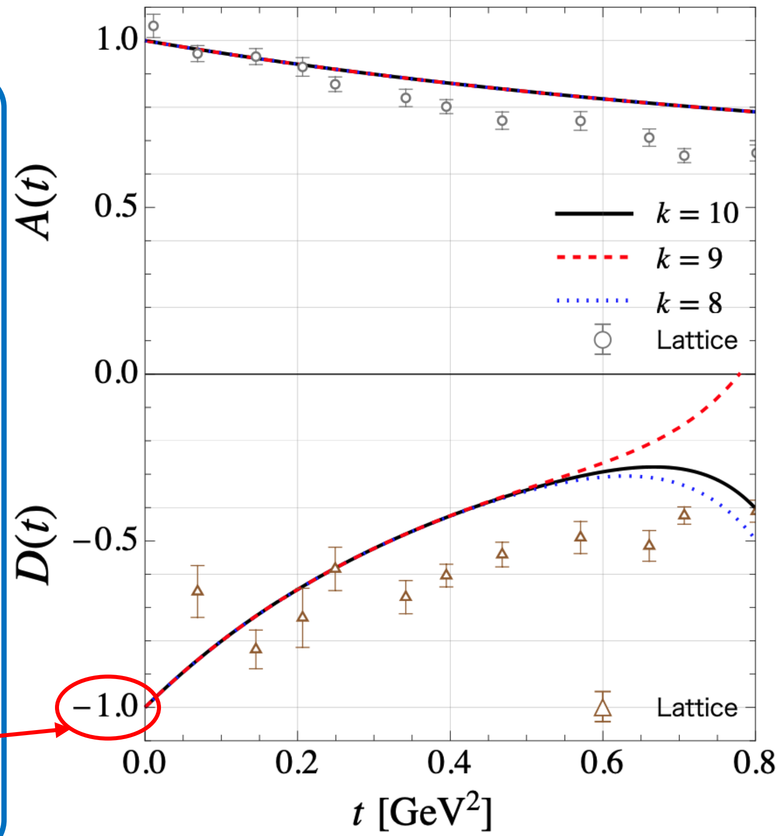


The pion GFFs $A(t)$ and $D(t)$ with the lattice data

Tanaka et al., arXiv:2407.21113 [hep-ph] (2024).
D. C. Hackett et al., Phys. Rev. D 108, 114504 (2023)

Results

- The absolute values of $D(t)$ dumps more rapidly than that of $A(t)$
  ✓ qualitatively consistent with the lattice QCD
- The convergence of $D(t)$ becomes poor around $t \sim 0.5\ \mathrm{GeV}^2$
- $A(t)$ converges well in the range $0 < t < 0.8\ \mathrm{GeV}^2$
- These differences between $A(t)$ and $D(t)$ are due to the contribution to $D(t)$ from S_4 modes
- D-term is -1



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Results

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$$D(t) =$$
$$-\frac{6}{L^7}\int_R^\infty dr' \sum_{n=1}^{\infty} \frac{\alpha_n^{\rm T}\Psi_n^{\rm T}(r')}{(m_n^{\rm T})^2 + \vec{\Delta}^2}\frac{r'^3}{L^3} P_{\rm s}^{\mu\nu}(\mathcal{T}_{\mu\nu} + \mathcal{T}_{\mu\nu}^{\rm S} + \mathcal{T}_{\mu\nu}^{\rm other})$$
$$+\frac{6}{L^7}\int_R^\infty dr' \sum_{n=1}^{\infty} \frac{\alpha_n^{\rm S}\Psi_n^{\rm S}(r')}{(m_n^{\rm S})^2 + \vec{\Delta}^2}\frac{r'^3}{L^3} P_{\rm s}^{\mu\nu}\mathcal{T}_{\mu\nu}^{\rm S},$$

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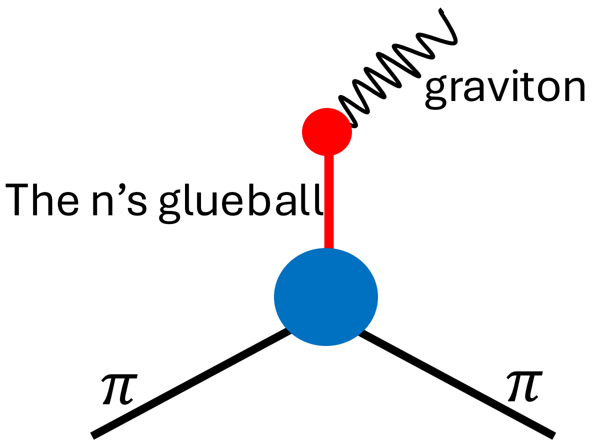
Glueball dominance



pions have gravitational interactions via infinite glueball spectra

Summary

✓ We have calculated for the first time the t-dependence of the pion's gravitational form factor from top-down holographic QCD.

✓ We have successfully reproduce the feature where the behaviors of $A(t)$ and $D(t)$ are different .

✓ We obtained the D-term as $D(0) = -1$.

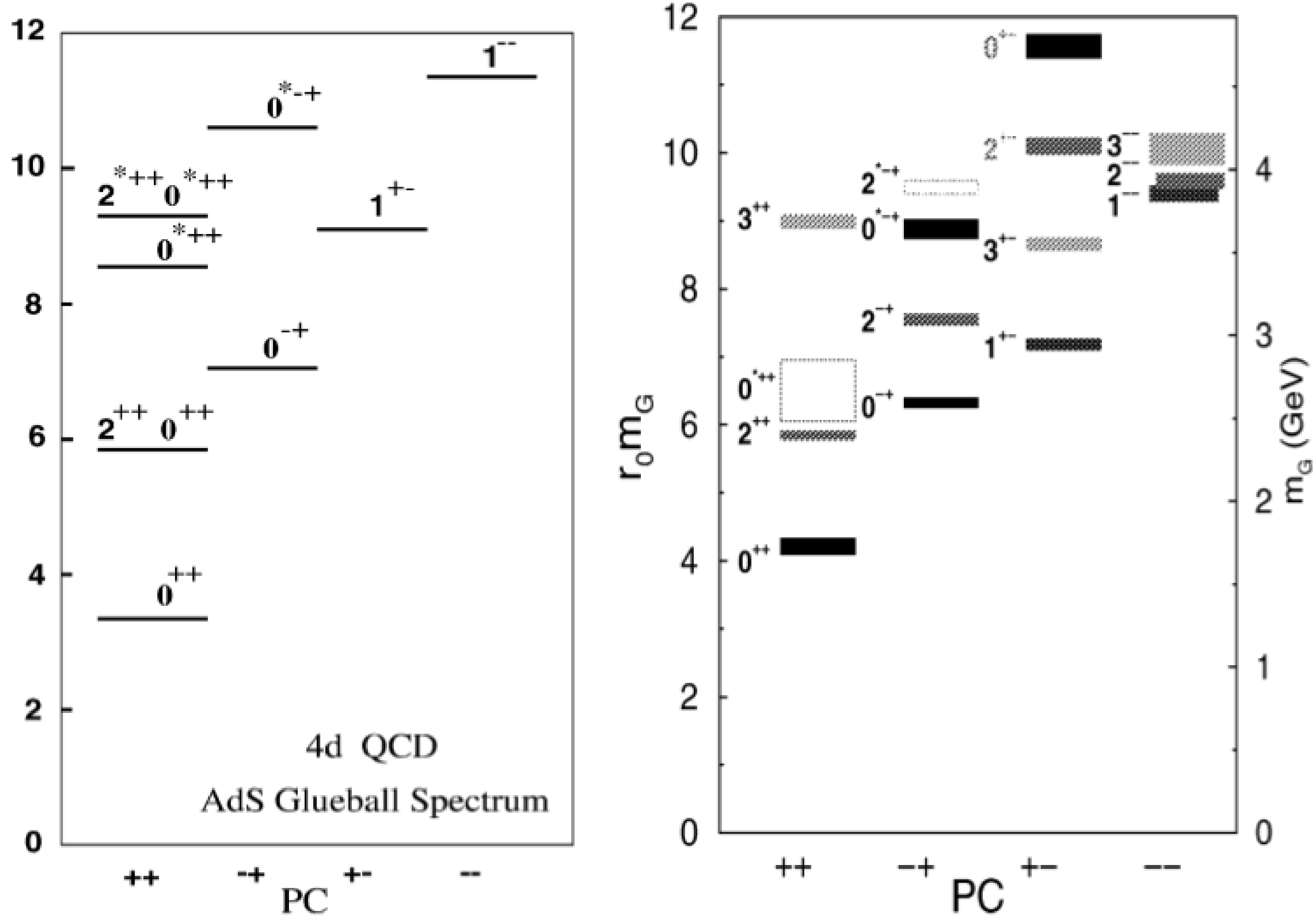
✓ We have confirmed the glueball dominance.

Future Works

- Extend to other hadrons
- Investigate beyond the chiral limit by introducing the quark mass

Back Up

The comparison with lattice QCD



Brower at al., arXiv:hep-th/000315