RUB

European Research Council

Bundesministerium für Bildung und Forschung

DFG Deutsche Forschungsgemeinschaft

Ministerium für Kultur und Wissenschaft des Landes Nordrhein-Westfalen

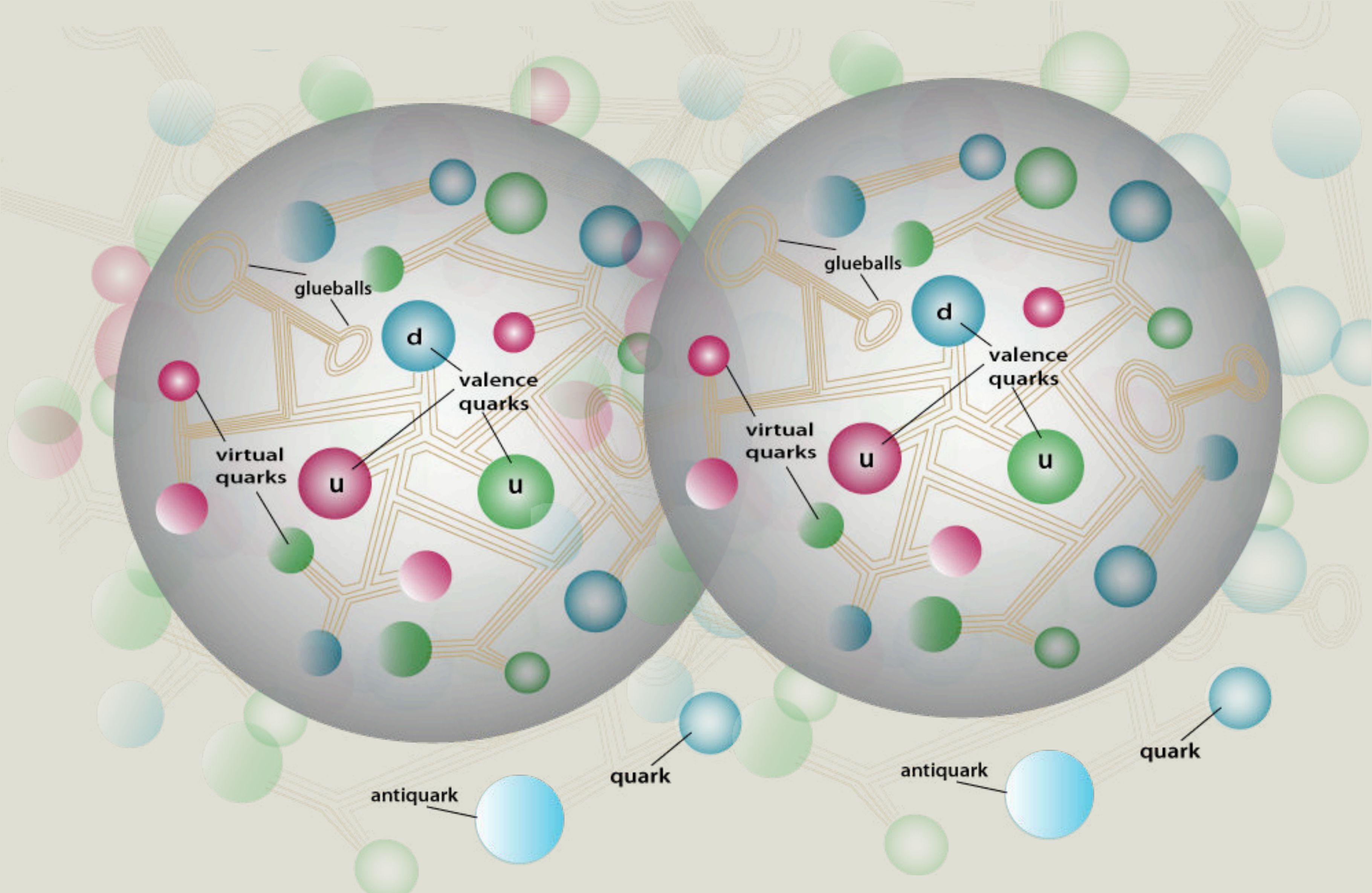
ERATO Exploratory Research for Advanced Technology

Chiral Dynamics of Nuclear Systems

Evgeny Epelbaum, Ruhr University Bochum, Germany

- Introduction
- ChPT and ChEFT for nuclear systems
- Chiral symmetry and the long-range two-nucleon force
- Precision two-nucleon physics with chiral EFT
- The three-nucleon force challenge
- Chiral EFT using gradient flow
- Summary

Deuteron as a bound state of quarks and gluons



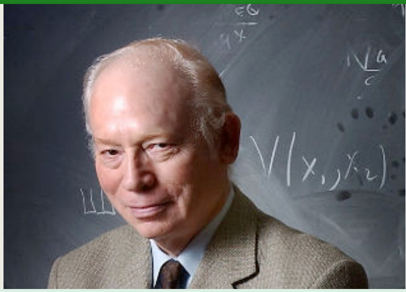
Is there a way to simplify the picture (without losing connection to QCD)?

Degrees of freedom

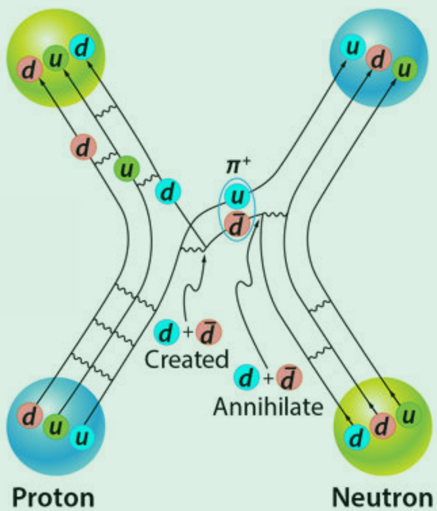
Weinberg's 3rd law of progress in theoretical physics:

You may use any degrees of freedom you like to describe a physical system, but if you use the wrong ones, you will be sorry…

in Asymptotic Realms of Physics, MIT Press, Cambridge, 1983



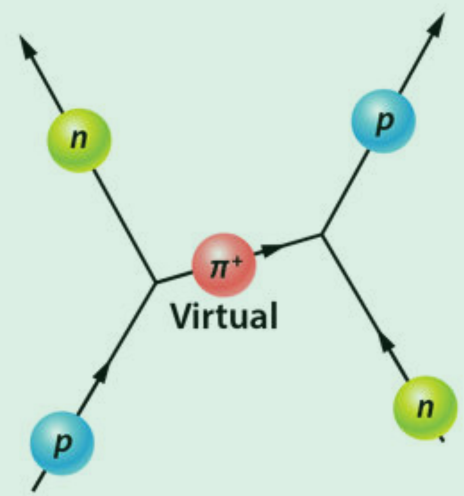
Fundamental degrees of freedom



(from physics.aps.org)

low resolution →

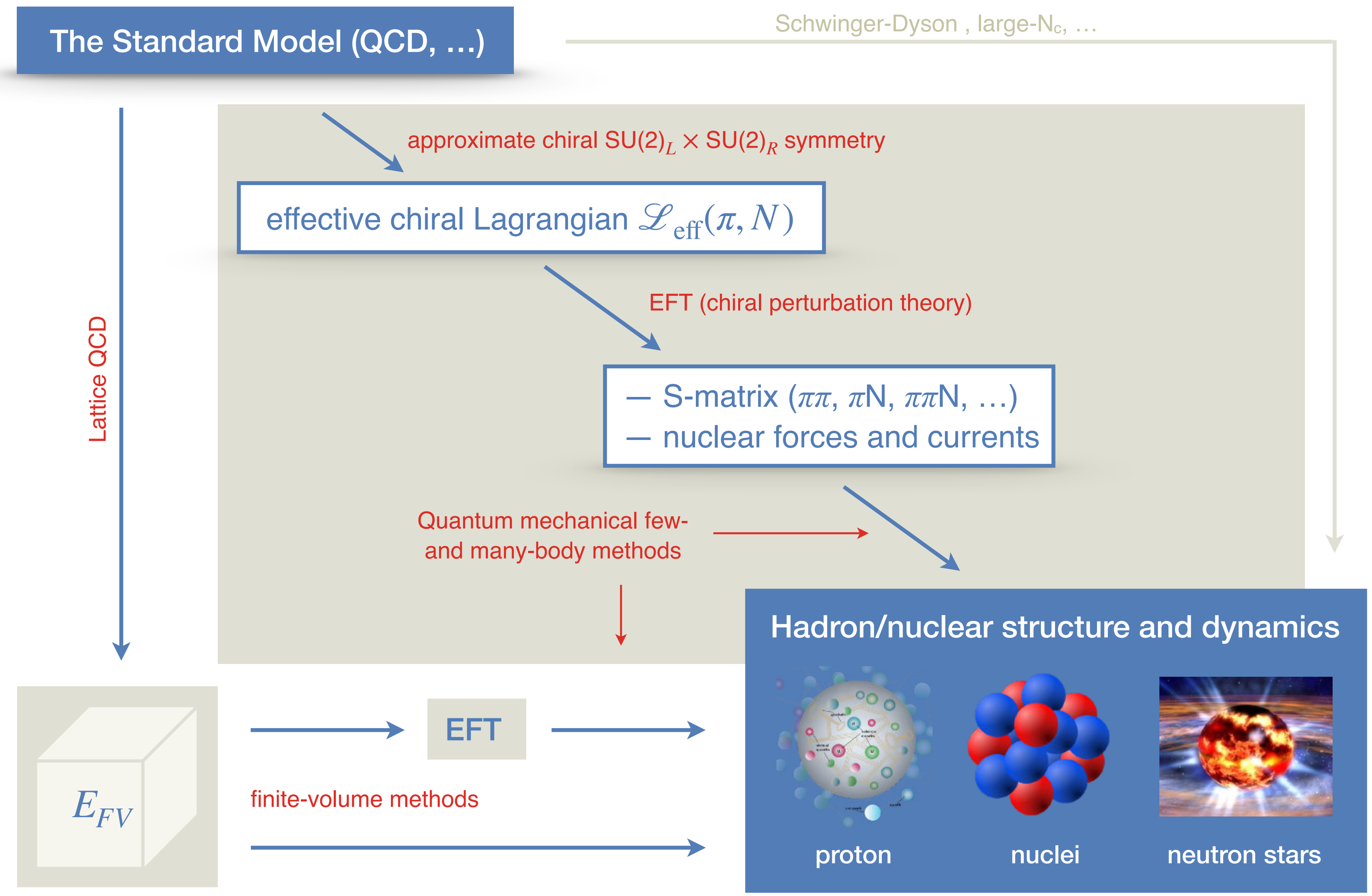
Effective degrees of freedom



⇒ non-relativistic description in the framework of the A-body Schrödinger equation:

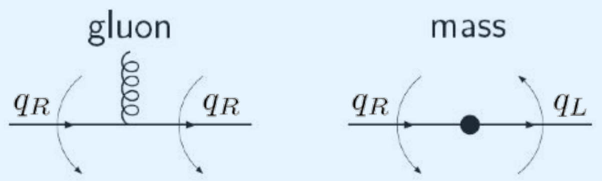
$$\left[\left(\sum_{i=1}^{N} \frac{-\vec{\nabla}^2}{2m} + \mathcal{O}(m^{-3})\right) + \underbrace{V_{2N} + V_{3N} + V_{4N} + \ldots}_{\text{derived in ChPT}}\right] |\Psi\rangle = E|\Psi\rangle$$

From QCD to nuclear physics



Chiral perturbation theory

QCD and the chiral symmetry



$$\mathcal{L}_{\rm QCD} = -\frac{1}{4}G_a^{\mu\nu}G_{a,\mu\nu} + \bar{q}(i\gamma^\mu D_\mu - \mathcal{M})q$$

$$= -\frac{1}{4}G_a^{\mu\nu}G_{a,\mu\nu} + \underbrace{\bar{q}_L i\not{D} q_L + \bar{q}_R i\not{D} q_R}_{\text{SU(N}_f)_L \times \text{SU(N}_f)_R \text{ invariant}} - \underbrace{q_L\mathcal{M}q_R - q_R\mathcal{M}q_L}_{\text{small for } N_f = 2,(3)}$$

SSB to $\mathrm{SU(N_f)_V} \le \mathrm{SU(N_f)_L \times SU(N_f)_R} \Rightarrow N_f^2 - 1$ GBs

Low-energy QCD dynamics can be described in terms of $\mathscr{L}_{\rm eff}$[GBs + matter fields $(N, \Delta\ldots)$]

Chiral perturbation theory

Weinberg, Physica A96 (79) 327; Gasser, Leutwyler, NPB 250 (85) 465; Leutwyler, Annals Phys. 235 (94) 165

QCD in the presence of external sources: $\mathcal{L} = \mathcal{L}^0_{\rm QCD} + \bar{q}(\gamma^\mu v_\mu + \gamma_5\gamma^\mu a_\mu - s - ip)q$

$$\langle 0,\, \text{out}|0,\, \text{in}\rangle_{\rm v,a,s,p} = Z[v,a,s,p] = \int [DG_\mu][Dq][D\bar{q}]\, e^{i\int d^4x \mathcal{L}(q,\bar{q},G_{\mu\nu};v,a,s,p)}\Big|_{\text{low energy}}$$

$$= \int \underbrace{[DU]}_{\text{pion fields}}\, e^{i\int d^4x \mathcal{L}_{\rm eff}(U;v,a,s,p)}\Big|_{\text{low energy}} \xrightarrow{\text{loop expansion}} \text{S-matrix}$$

(perturbation theory)

ChPT for pion-nucleon scattering

Effective chiral Lagrangian:

low-energy constants

$$\mathcal{L}_\pi = \mathcal{L}_\pi^{(2)} + \mathcal{L}_\pi^{(4)} + \ldots$$

$$\mathcal{L}_{\pi N} = \underbrace{\bar N \left(i\gamma^\mu D_\mu[\pi] - m + \frac{g_A}{2}\gamma^\mu \gamma_5 u_\mu[\pi] \right) N}_{\mathcal{L}_{\pi N}^{(1)}} + \underbrace{\sum_i \mathbf{c}_i \bar N \hat O_i^{(2)}[\pi] N}_{\mathcal{L}_{\pi N}^{(2)}} + \underbrace{\sum_i \mathbf{d}_i \bar N \hat O_i^{(3)}[\pi] N}_{\mathcal{L}_{\pi N}^{(3)}} + \underbrace{\sum_i \mathbf{e}_i \bar N \hat O_i^{(4)}[\pi] N}_{\mathcal{L}_{\pi N}^{(4)}} + \ldots$$

Pion-nucleon scattering amplitude for $\pi^a(q_1) + N(p_1) \to \pi^b(q_2) + N(p_2)$:

$$T_{\pi N}^{ba} = \frac{E+m}{2m}\left(\delta^{ba}\left[g^+(\omega,t) + i\vec\sigma\cdot\vec q_2 \times \vec q_1\, h^+(\omega,t)\right] + i\epsilon^{bac}\tau^c\left[g^-(\omega,t) + i\vec\sigma\cdot\vec q_2\times\vec q_1\, h^-(\omega,t)\right]\right)$$

calculated by means of the chiral expansion

Pion-nucleon scattering up to Q⁴ in heavy-baryon ChPT

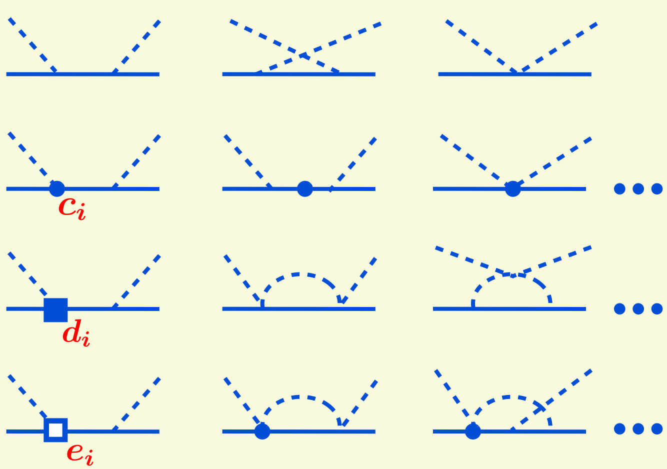
Fettes, Meißner ’00; Krebs, Gasparyan, EE ’12

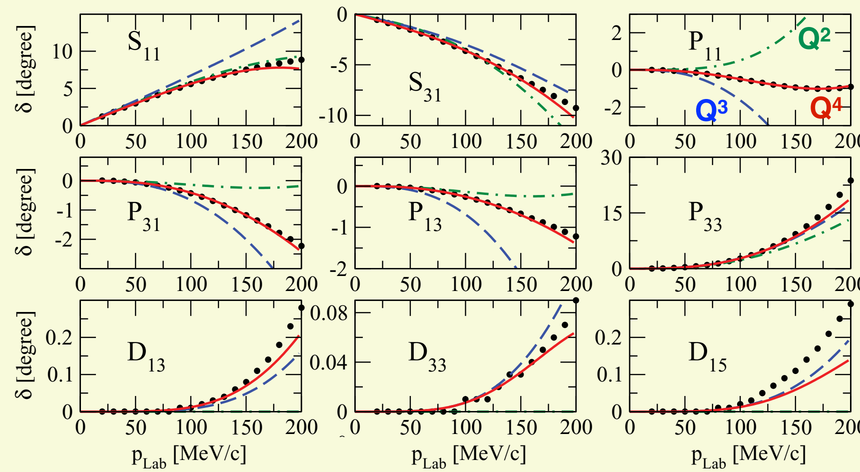
Order Q:

Order Q²:

Order Q³:

Order Q⁴:



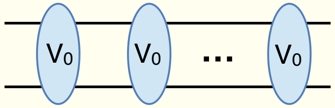


Chiral EFT for nuclear systems

For few-N, ladder diagrams are enhanced and must be re-summed Weinberg ’90, ’91

$$T_{\rm LO} = V_0 + V_0 G V_0 + V_0 G V_0 G V_0 + \ldots = \sum_{n=0}^{\infty} V_0 (G V_0)^n$$

↑ order in the ChEFT expansion



$$T_{\rm NLO} = T_{\rm LO} + \sum_{m,n=0}^{\infty} (V_0 G)^m V_2 (V_0 G)^n + \mathcal{O}\big((V_2)^2\big)$$

automatically included when solving $T_{\rm NLO} = (V_0 + V_2) + (V_0 + V_2) G T_{\rm NLO}$

(divergent integrals in the Lippmann-Schwinger equation are usually regularized with a cutoff Λ)

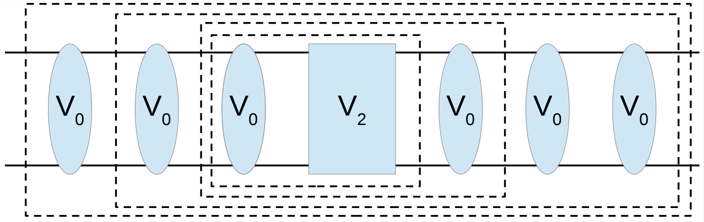
Finite-cutoff EFT ($\Lambda \sim \Lambda_b \sim 600$ MeV) Lepage, EE, Gegelia, Meißner, Reinert, Entem, Machleidt, …

— implicit renormalization (achieved by tuning bare LECs to data)

— approximate Λ-independence of calculated observables has to be verified a posteriori

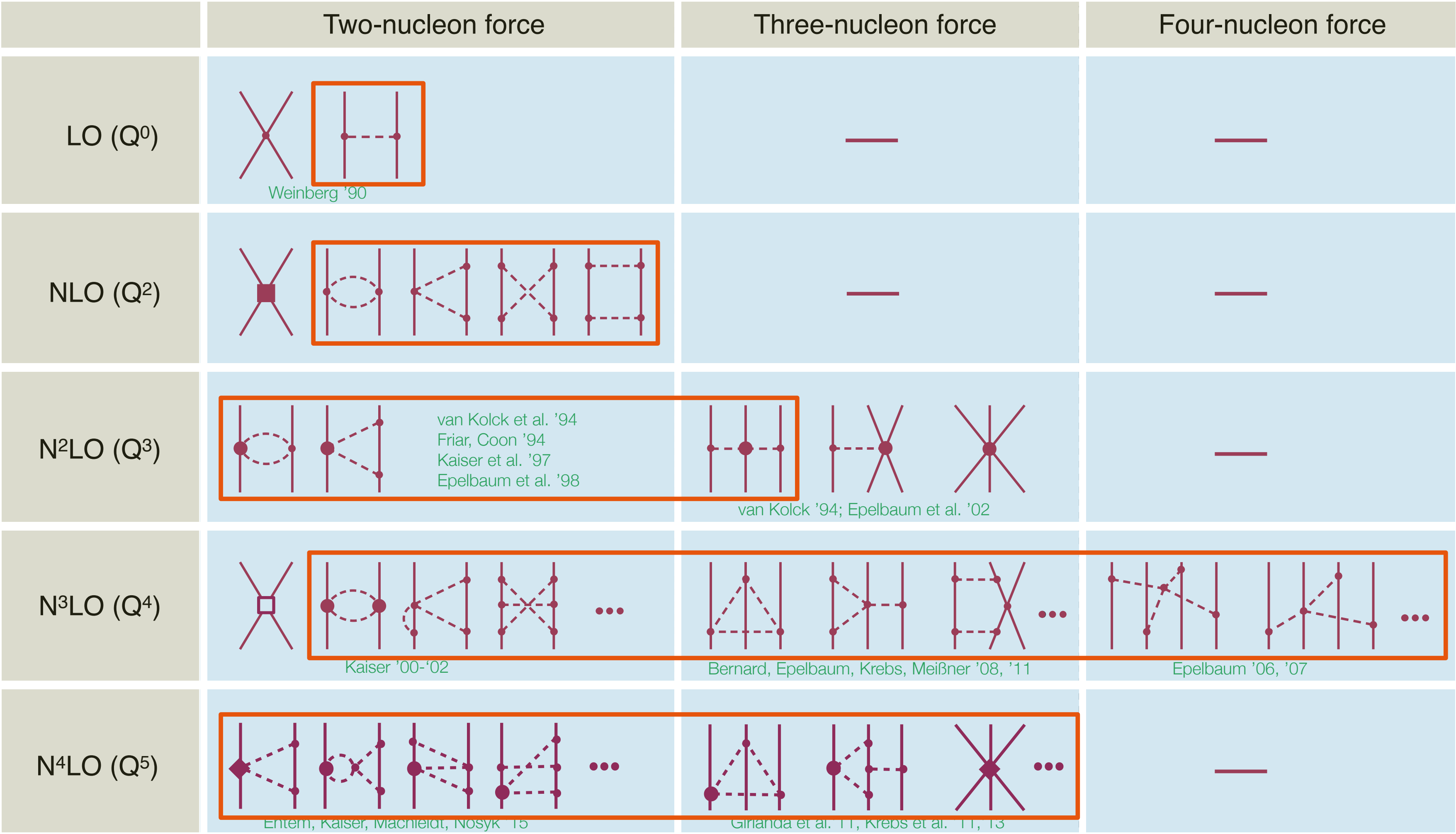
— explicit proof of renormalizability (in the EFT sense) has been given to NLO using the BPHZ formalism

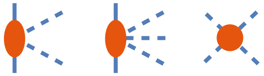
Ashot Gasparyan, EE, PRC 105 (22); PRC 107 (23)



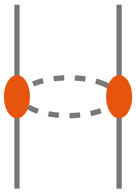
Chiral expansion of the nuclear forces

(NDA, chiral EFT with pions and nucleons as the only DoF)



Chiral dynamics: Long-range interactions are predicted in terms of on-shell amplitudes 

Two-pion exchange and the πN amplitude



← exchanged pions can become on-shell for $\mathbf{q}^2 \leq -(2M_\pi)^2$

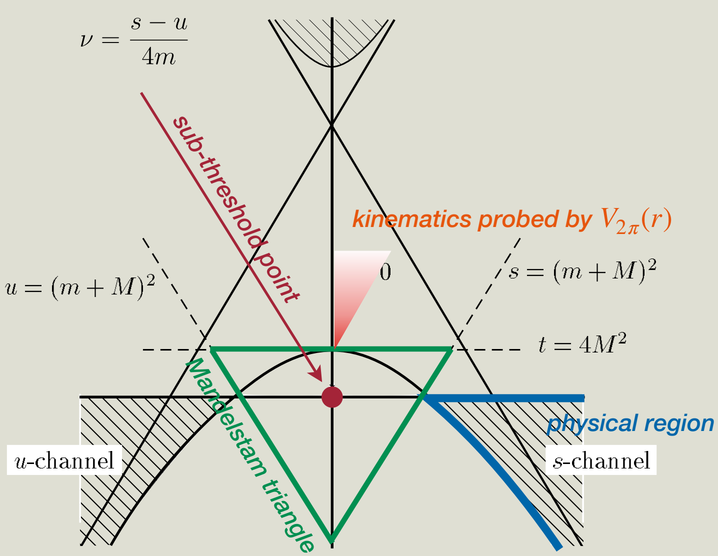
⇒ $V_{2\pi}(\mathbf{q}^2)$ are analytic functions except for the branch cut $\mathbf{q}^2 \in (-\infty, -4M_\pi^2]$

⇒ $V_{2\pi}^{c}(\mathbf{q}^2) = \frac{2}{\pi}\int_{2M_\pi}^{\infty} d\mu \mu \frac{\rho(\mu)}{\mathbf{q}^2+\mu^2}$ where $\rho(\mu) = \mathrm{Im}\big[V_{2\pi}^{c}(q=0^+ - i\mu)\big]$

in practice, subtractions are needed to make the integral convergent

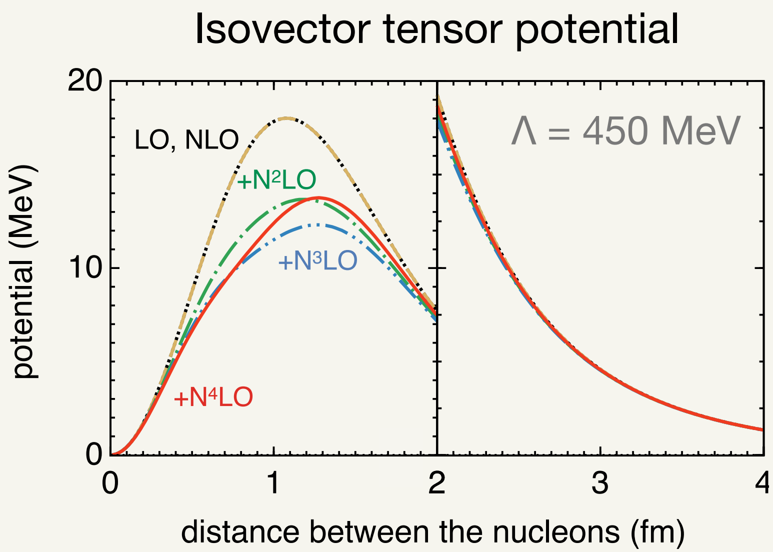
The spectral functions $\rho(\mu)$ determine the long-range behavior of $V_{2\pi}(r)$ and can be calculated from the on-shell πN amplitude (ChPT) using Cutkosky cutting rules (Kaiser, 1999)

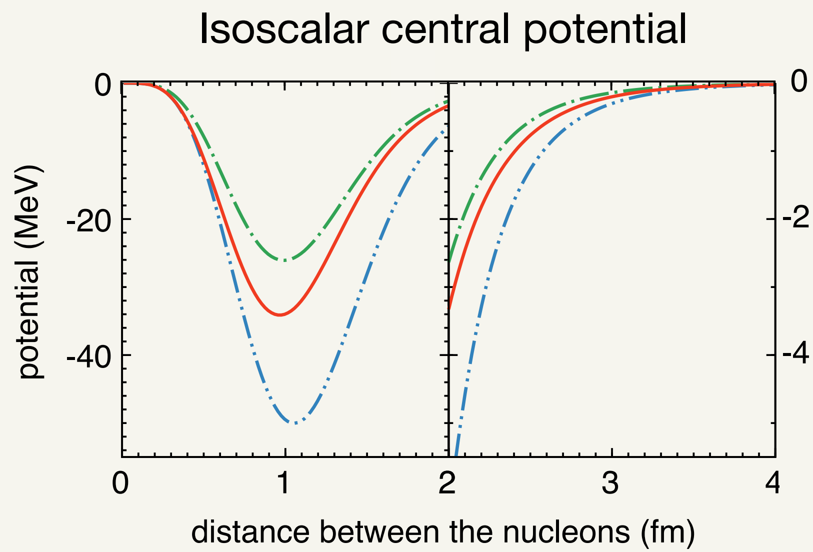
Kinematic regions for πN scattering



- πN amplitude from the numerical solution of the dispersive Roy-Steiner equations Ditsche et al., 2012
- Extract LECs from matching ChPT at the *sub-threshold point* Hoferichter et al., 2015
- Closer to the *kinematics probed by $V_{2\pi}(r)$* than the physical region
- Beyond HBChPT: Δ, 1/m Siemens et al., 2017

Chiral expansion of the multi-pion exchange





— Long-distance behavior of the NN force is **a parameter-free prediction of chiral EFT**

— Agrees with phenomenology (strong intermediate-range attraction from TPEP)

— Reasonable convergence of the chiral expansion (at large r)

— Short-range interactions parametrized by contacts

Regularization

EE, Krebs, Meißner EPJA 51 (15); Phys. Rev. Lett. 115 (15); Reinert, Krebs, EE, EPJA 54 (18)

$$V_{1\pi}(q) = \frac{\alpha}{\vec{q}^2 + M_\pi^2} \; e^{-\frac{\vec{q}^2 + M_\pi^2}{\Lambda^2}} + \text{subtraction},$$

$$V_{2\pi}(q) = \frac{2}{\pi}\int_{2M_\pi}^{\infty} d\mu\,\mu\,\frac{\rho(\mu)}{\vec{q}^2+\mu^2} \; e^{-\frac{\vec{q}^2+\mu^2}{2\Lambda^2}} + \text{subtractions}$$

The two-nucleon system

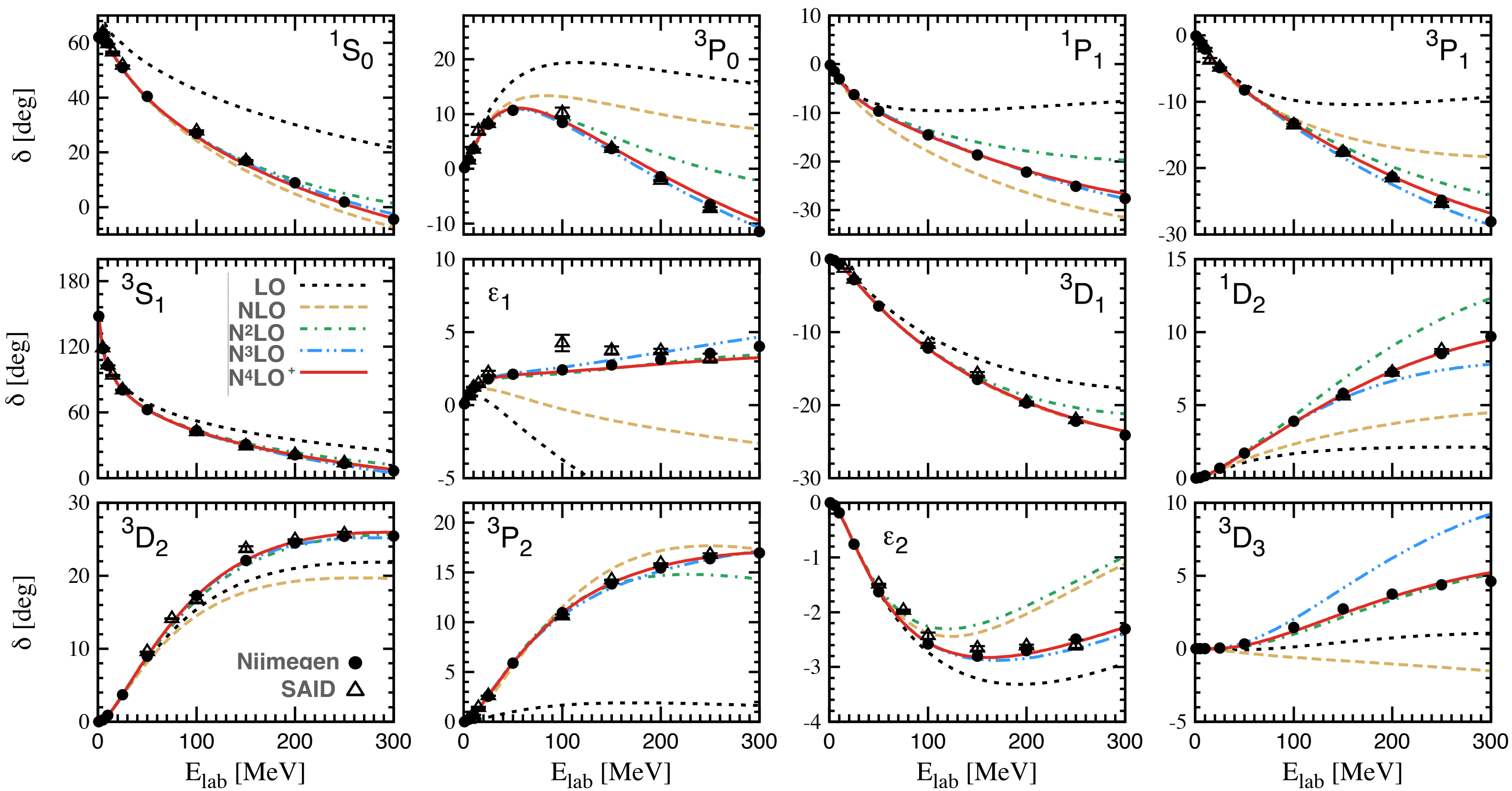
How far in the EFT expansion does one need to go to precisely describe low-energy NN data?

Results for $\Lambda = 450$ MeV

from: P. Reinert, H. Krebs, EE, EPJA 54 (2018) 88

	LO (Q^0)	NLO (Q^2)		N^{2}LO (Q^3)	N^{3}LO (Q^4)		N^{4}LO (Q^5)	N^{4}LO$^+$
χ^2/datum (np, 0 – 300 MeV)	75	14	no new LECs →	4.1	2.01	no new LECs →	1.16	1.06
χ^2/datum (pp, 0 – 300 MeV)	1380	91		41	3.43		1.67	1.00
	2 LECs	+ 7 + 1 IB LECs			+ 12 LECs		+ 1 LEC (np)	+ 4 LECs

Chiral expansion of the neutron-proton phase shifts [Λ = 450 MeV]



Truncation uncertainty

How large is truncation uncertainty stemming from neglected higher-order terms?

Consider an arbitrary observable X(p):

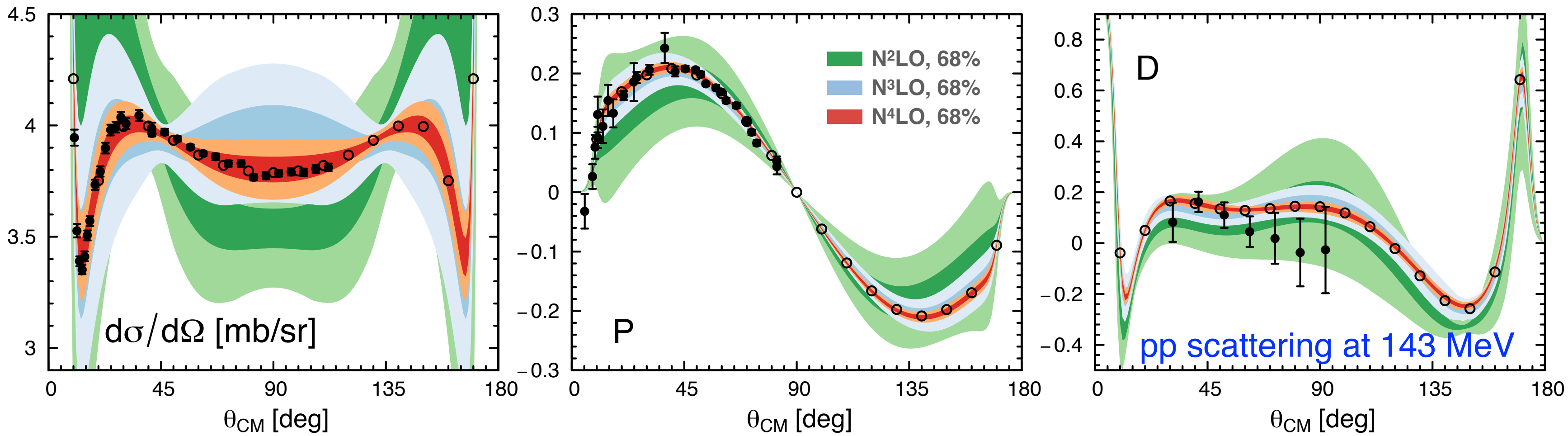
$$X(p) = \underbrace{c_0 + c_2 Q^2 + c_3 Q^3 + \ldots + c_i Q^i}_{\text{known from explicit calculations}} + \underbrace{c_{i+1} Q^{i+1} + \ldots}_{\text{truncation error to be estimated}} \quad \text{where} \quad Q = \max\left(\frac{p}{\Lambda_b}, \frac{M_\pi^{\text{eff}}}{\Lambda_b}\right)$$

Bayesian approach to estimate the truncation error Furnstahl et al. (**BUQEYE**) '15…'23, EE et al. '20

— assume some common pdf for the coefficients c_i (prior)

— learn the convergence pattern from the known $c_0, \ldots, c_i$ to obtain the posterior pdf

— marginalize over the unknown $c_{i+1}, c_{i+2}, \ldots$



Precision physics in the 2N sector I: πN couplings

Reinert, Krebs, EE, Phys. Rev. Lett. 126 (2021) 9, 092501

Our $g_{\pi \mathrm{NN}}$ value corresponding to f_c^2 reads:

$$g_{\pi \mathrm{NN}} = 13.23 \pm 0.04$$

Pionic hydrogen exp. at PSI (GMO sum

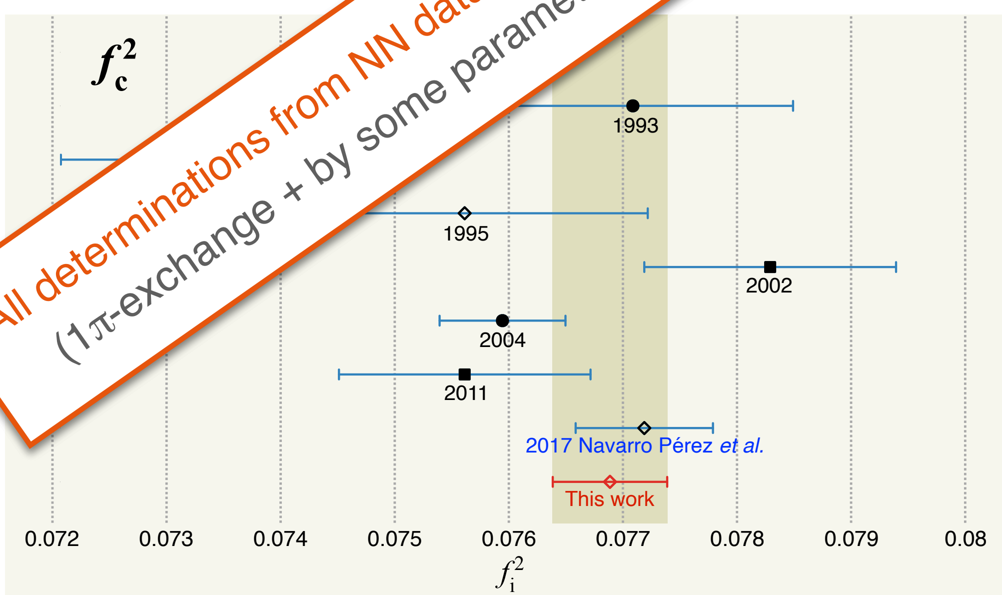
[Hirtl et al., Eur. Phys. J. A57 (2021) 2, 70]

$$\epsilon_{1s}^{\pi \mathrm{H}} + \epsilon_{1s}^{\pi \mathrm{D}}: \quad g_{\pi \mathrm{NN}} = 13.10 \pm$$

$$\Gamma_{1s}^{\pi \mathrm{H}}: \quad g_{\pi \mathrm{NN}} = 1$$

All determinations from NN data rely on **phenomenological models**

(1π-exchange + by some parametrizations of short-range terms)



Standard notation ($f_{\pi NN} = \frac{M_{\pi^\pm}}{2\sqrt{4\pi} m_N} g_{\pi NN}$):

$$\begin{aligned} f_0^2 &= -f_{\pi^0 \mathrm{nn}}\, f_{\pi^0 \mathrm{pp}} \\ f_\mathrm{p}^2 &= f_{\pi^0 \mathrm{pp}}\, f_{\pi^0 \mathrm{pp}} \\ 2 f_\mathrm{c}^2 &= f_{\pi^\pm \mathrm{pn}}\, f_{\pi^\pm \mathrm{pn}} \end{aligned}$$

2017 Granada PWA: claimed to find significant charge dependence of the coupling constants:

$$f_0^2 - f_\mathrm{p}^2 = 0.0029(10)$$

Navarro Perez et al., PRC 95 (2017) 6, 064001

Our result (χEFT at N⁴LO):

Bayesian determination; statistical **and systematic** uncertainties.

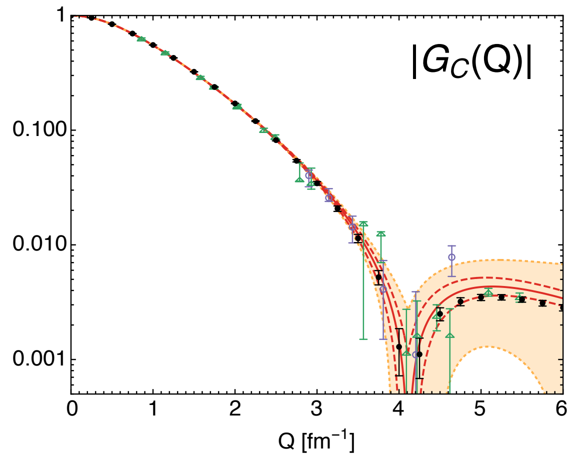
No evidence for charge dependence of the πN coupling constants

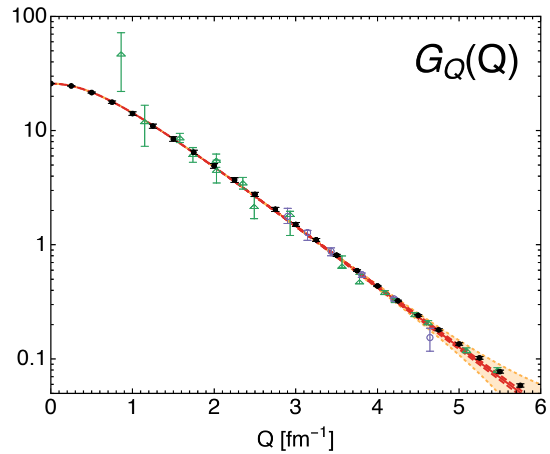
Reinert, Krebs, EE, Phys. Rev. Lett. 126 (2021) 092501

Precision physics in the 2N sector II: Deuteron FFs

Filin, Möller, Baru, EE, Krebs, Reinert, PRL 124 (2020) 082501; PRC 103 (2021) 024313

Charge and quadrupole form factors of the deuteron at N⁴LO





Extracted quadrupole moment:

$$Q_{\rm d} = 0.2854^{+0.0038}_{-0.0017}\ {\rm fm}^2$$

EFT truncation, choice of fitting range, NN, πN and γNN LECs

to be compared with experiment

$$Q_{\rm d}^{\rm exp} = 0.285\,699(15)(18)\ {\rm fm}^2$$

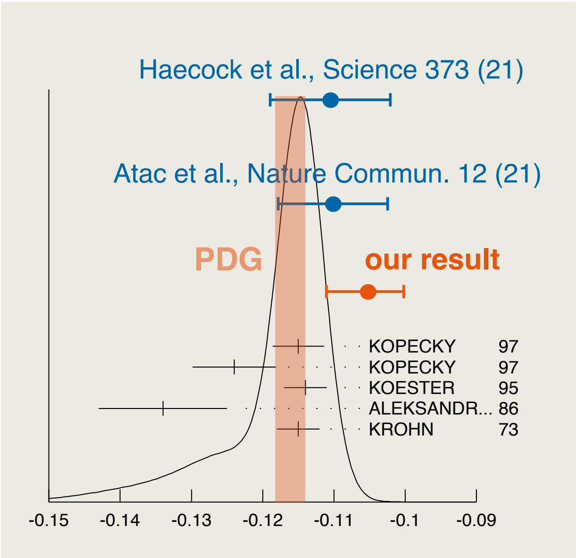
Puchalski et al., PRL 125 (2020)

The charge and structure radius:

$$r_d^2 = (-6)\frac{\partial G_C(Q^2)}{\partial Q^2}\bigg|_{Q^2=0} = r_{str}^2 + r_p^2 + r_n^2 + \frac{3}{4m_p^2}$$

Combining our result $r_{\rm str} = 1.9729^{+0.0015}_{-0.0012}$ fm with very precise isotope-shift spectroscopy data for $r_d^2 - r_p^2$, we determine the neutron m.s. charge radius:

$$r_n^2 = -\,0.105^{+0.005}_{-0.006}\ {\rm fm}^2$$

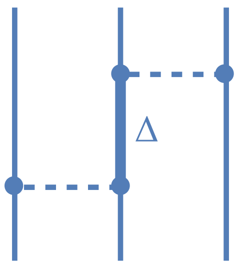


In progress: magnetic (Daniel Möller, PhD thesis) and gravitational (Julia Panteleeva, PhD thesis) FFs of ²H

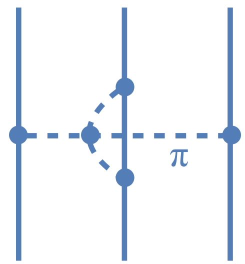
Three-body force: A frontier in nuclear physics

- Three-nucleon forces (3NF) are small but important corrections to the dominant NN forces

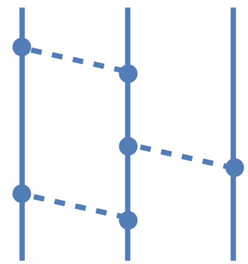
- 3NF mechanisms:



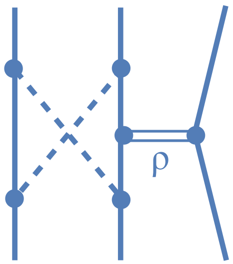
intermediate Δ-excitation
Fujita, Miyazawa '57



multi-pion interactions



off-shell behavior of the V_{NN}
$$V_{\rm ring} = \mathscr{A}_{3\pi} - V_\pi G_0 V_\pi G_0 V_\pi$$

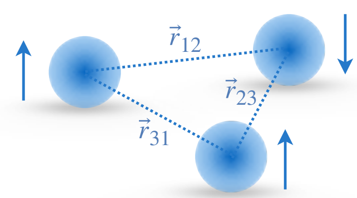


short-range

⇒ 3NF are not directly measurable and depend on the scheme (DoF, off-shell V_{NN}, …)

- 3NF have extremely rich and complex structure

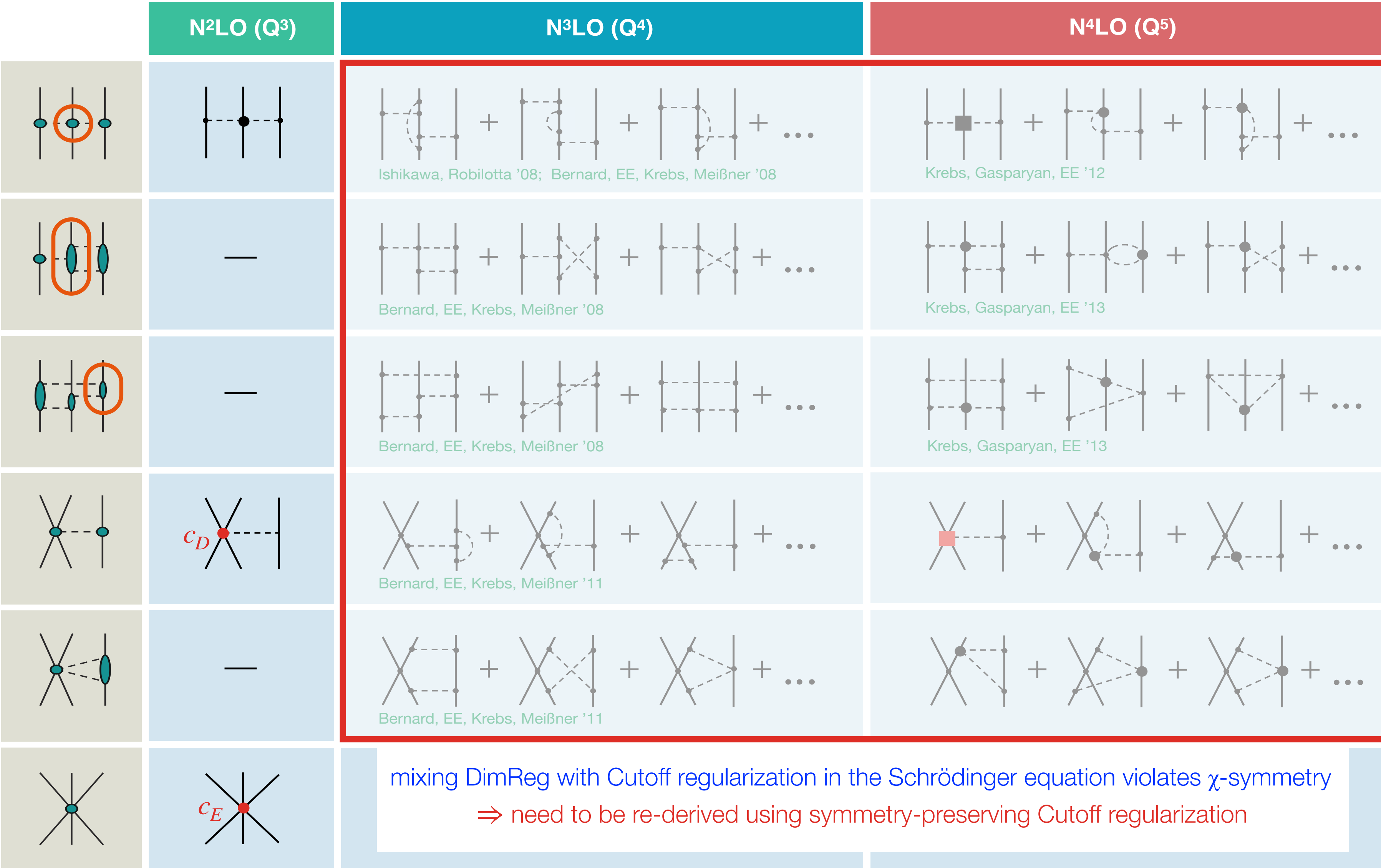
— most general ***local*** 3NF: $V_{3N} = \sum_{i=1}^{20} O_i\, f_i(r_{12}, r_{23}, r_{31}) + $ permutations
EE, Gasparyan, Krebs, Schat '15



— most general ***nonlocal*** 3NF: **320 (!)** operators Topolnicki '17

⇒ Guidance from theory indispensable — an opportunity for χEFT!

Three-nucleon force in chiral EFT



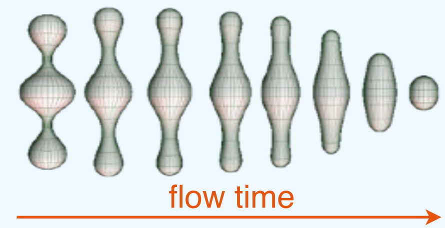
Gradient flow for chiral EFT

Hermann Krebs, EE, e-Print: 2311.10893, 2312.13932

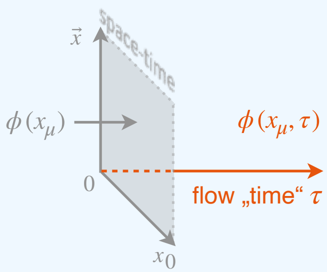
A rigorous approach to regularize nuclear interactions and currents
in harmony with the chiral and gauge symmetries

Gradient flow

Gradient flows: methods for smoothing manifolds
(e.g., Ricci flow used in the proof of the Poincaré conjecture)



Gradient flow as a regulator in field theory



Flow equation: $\dfrac{\partial}{\partial \tau}\phi(x,\tau) = -\dfrac{\delta S[\phi]}{\delta \phi(x)}\bigg|_{\phi(x)\to\phi(x,\tau)}$

subject to the boundary condition $\phi(x,0) = \phi(x)$

Free scalar field:

$$\left[\partial_\tau - (\partial^x_\mu \partial^x_\mu - M^2)\right]\phi(x,\tau) = 0 \quad \Rightarrow \quad \phi(x,\tau) = \underbrace{\int d^4y\, \overbrace{G(x-y,\tau)}^{G(x,\tau) = \frac{\theta(\tau)}{16\pi^2\tau^2} e^{-\frac{x^2+4M^2\tau^2}{4\tau}}}\phi(y)}_{\textit{heat kernel}} \quad \Rightarrow \quad \tilde{\phi}(q,\tau) = e^{-\tau(q^2+M^2)}\tilde{\phi}(q)$$

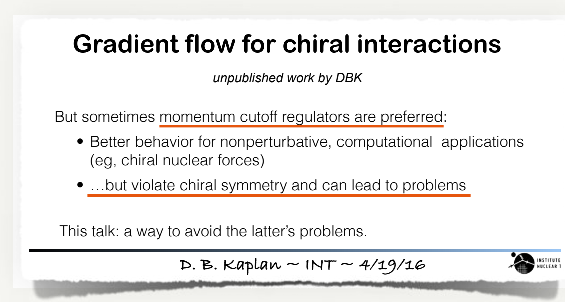
YM gradient flow Narayanan, Neuberger '06, Lüscher, Weisz '11: $\partial_\tau A_\mu(x,\tau) = D_\nu G_{\nu\mu}(x,\tau)$ ← extensively used in LQCD

Chiral gradient flow Krebs, EE, 2312.13932

Start with $U(\boldsymbol{\pi}(x)) \in \mathrm{SU}(2) \;\to\; RU(x)L^\dagger$

Generalize $U(x)$ to $W(x,\tau)$: $\partial_\tau W = -i\underbrace{w}_{\sqrt{W}}\,\overbrace{\mathrm{EOM}(\tau)}^{[D_\mu, w_\mu] + \frac{i}{2}\chi_-(\tau) - \frac{i}{4}\mathrm{Tr}\,\chi_-(\tau)}\,w, \quad W(x,0) = U(x)$

We have proven $\forall\tau$: $W(x,\tau) \in \mathrm{SU}(2), \;\; W(x,\tau) \;\to\; RW(x,\tau)L^\dagger$



Gradient flow for chiral interactions

unpublished work by DBK

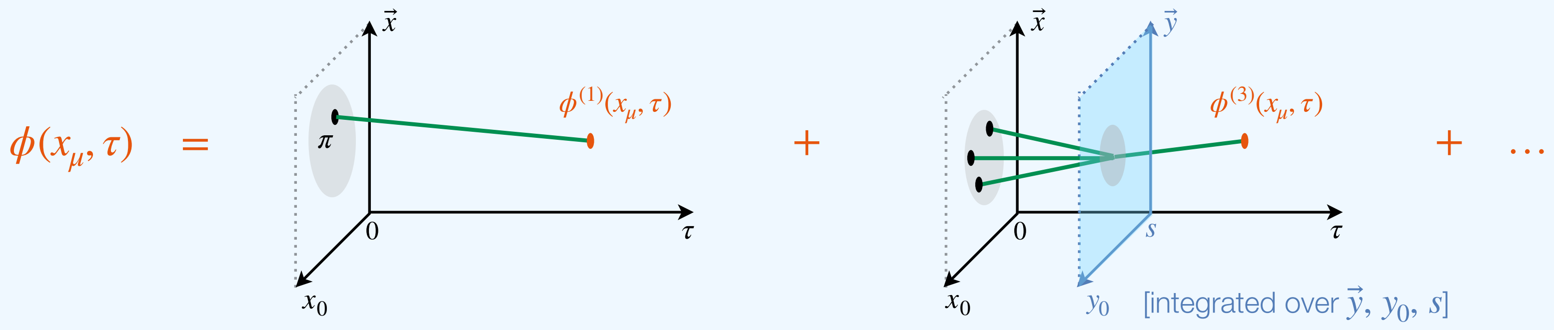
But sometimes momentum cutoff regulators are preferred:

- Better behavior for nonperturbative, computational applications (eg, chiral nuclear forces)
- ...but violate chiral symmetry and can lead to problems

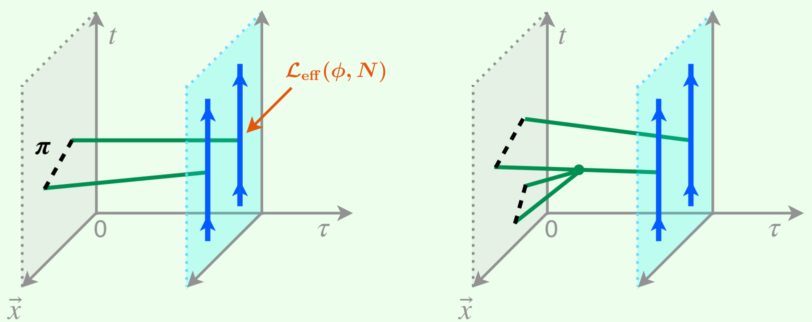
This talk: a way to avoid the latter's problems.

D. B. Kaplan ~ INT ~ 4/19/16

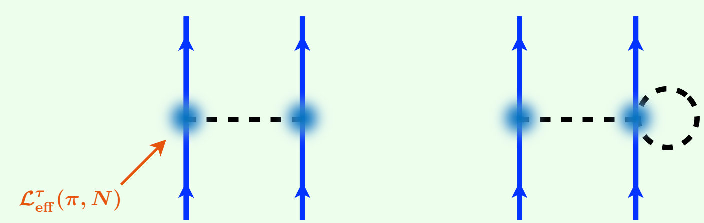
Gradient flow regularization



Local field theory in 5d



Smeared (non-local) theory in 4d



We now have the regularized Lagrangian, but cannot use the canonical-quantization-based UT method to derive nuclear forces ($\partial_0^n \pi$ with arbitrary n…). Path Integral approach [Krebs, EE, e-Print: 2311.10893]:

$$Z[\eta^\dagger,\eta] \;=\; A\int \mathcal{D}N^\dagger\,\mathcal{D}N\,\mathcal{D}\pi\,\exp\biggl(iS^\Lambda_{\rm eff} + i\int d^4x[\eta^\dagger N + N^\dagger \eta]\biggr)$$

nonlocal redefinitions of $N, N^\dagger$ → instantaneous

$$A\int \mathcal{D}\tilde{N}^\dagger\,\mathcal{D}\tilde{N}\exp\biggl(iS^\Lambda_{\rm eff,\,N} + i\int d^4x[\eta^\dagger \tilde{N} + \tilde{N}^\dagger \eta]\biggr)$$

The horizon is always three miles away

…but is becoming increasingly brighter

Thank you for your attention