Entropy of Unruh radiation from a spherical source with an exponential energy spectrum

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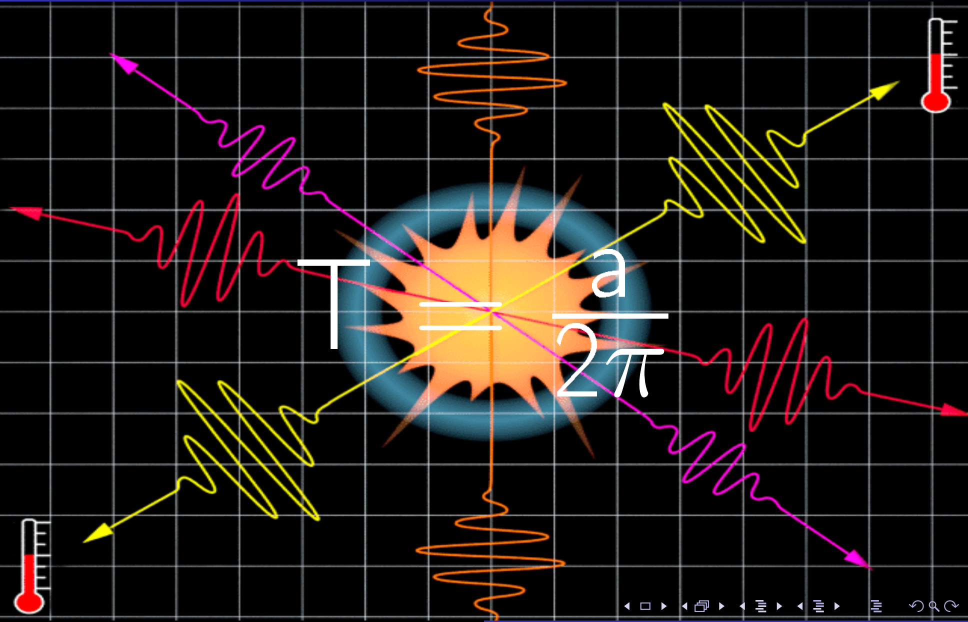

$$T = \frac{a}{2\pi}$$

Macro: astrophysics

[https://agilefacil.wordpress.com/] [https://pressbooks.online.ucf.edu/]

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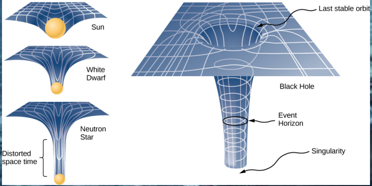

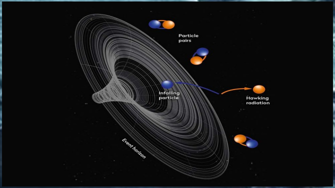

Black hole is black

Black hole is not black

[https://agilefacil.wordpress.com/]

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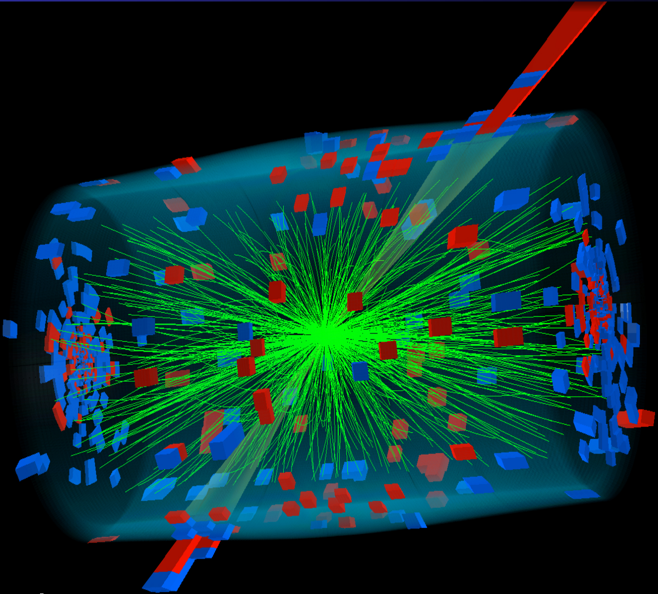
Micro: particle production



[CMS, CERN]

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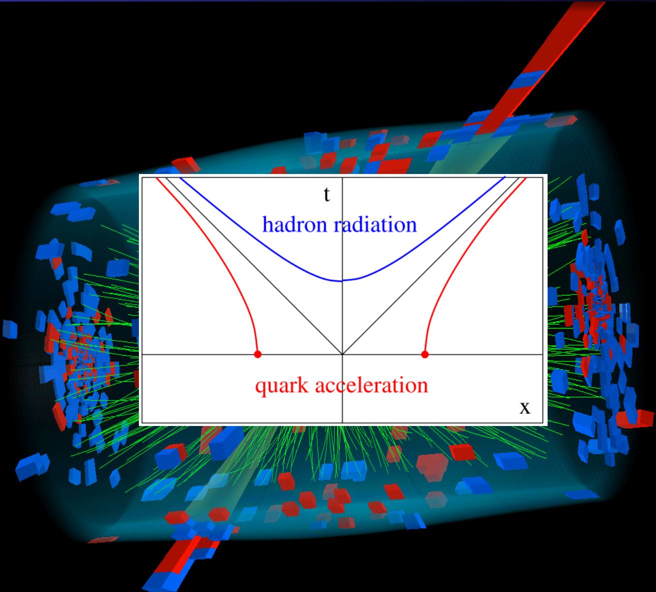
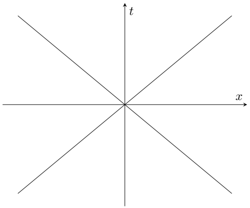


[CMS, CERN] [Castorina et al, EPJC 52 (2007) 187]

Reference frames

Inertial reference frame (IRF)

Minkowski vacuum $|0\rangle_M$



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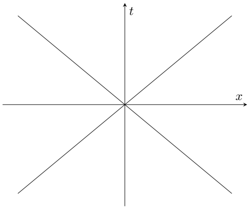

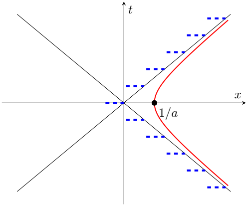

Non-inertial reference frame (NRF)

Acceleration a, horizon thermal radiation at temperature $T = a/2\pi$ [Unruh, 1976]

Another basis is required

$\Rightarrow$ Rindler modes $|n\rangle_{\rm in}\, |n\rangle_{\rm out}$

NRF density matrix

IRF

$|0\rangle_M$ is a pure state

NRF

Only $|n\rangle_{\mathrm{out}}$ modes are detectable $\Rightarrow$ mixed state

$$\begin{aligned}\rho_{\mathrm{out}} &= \mathrm{Tr}_{\mathrm{in}}\, |0\rangle_M \langle 0|_M \\ &= \frac{1-e^{-E/T}}{1-e^{-NE/T}} \sum_{n=0}^{N-1} e^{-nE/T}\, |n\rangle_{\mathrm{out}} \langle n|_{\mathrm{out}}\end{aligned}$$

Eigenvalues of ρ_{out}:
probability to find n particles at energy E, temperature T, and(?) maximal multiplicity $N-1$

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Eigenvalues of $\rho_{\rm out}$:
probability to find n particles at energy E, temperature T, and(?) maximal multiplicity $N-1$
$\equiv$ conditional distribution $\{n|E, T, N\}$

Distribution & Shannon entropy H

Distribution $\{X\}$

Find x with probability $p(x)$: $\sum_x p(x) = 1$

$H(X) = -\sum_x p(x) \ln p(x)$

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Joint distribution $\{X, Y\}$

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H is information we need to describe our system
$\equiv$ how much we do not know

Conditional distribution

Conditional distribution $\{X|Y\}$

Probability to find x being given y

$$p(x|y) = \frac{p(x,y)}{p(y)}, \quad p(y) = \sum_x p(x,y)$$

Entropy

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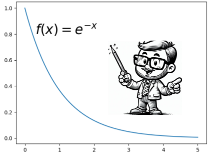
Entropy

$$H(X|y) = -\sum_x p(x|y) \ln p(x|y)$$

$$H(X,Y) = H(Y) + \sum_y p(y) H(X|y)$$

$$= H(Y) + \langle H(X|y) \rangle_y$$

Model assumptions



1. Particle spectrum

Energy of the particles: $m \leq E \leq M$

Exponential distribution: $p(E) = \frac{e^{-E/T}}{D}$

Other DOF contribute independently

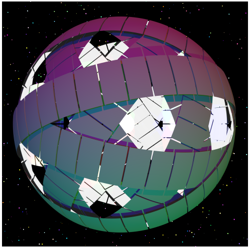
2. $1D \to 3D$

Unruh effect is D=1+1

For D=3+1 one needs to take angular DOF into account

$$H \to \sum_{l=0}^{l=L} \sum_{-l}^{l} H = (L+1)^2 H$$

$$\sqrt{L(L+1)} = r\sqrt{E^2 - m^2}$$



[Copilot, Wikipedia]

Final expression

$$H = H(E|T) + \langle H(\rho, Q|E, T, N)\rangle_E$$
$$= (h_E + h_\rho + h_Q)\, H_{BH},$$

where h_E is the contribution of exponential energy spectrum, h_ρ describes the Unruh part, and h_Q stands for any other intrinsic DOF

Entropy of a Schwarzschild black hole $H_{BH} = \frac{1}{16\pi T^2}$ is used for scaling reasons

Notations

$$f(x)\Big|_{x=a}^{x=b} = f(b) - f(a)$$

Binomial coefficients:

$$(1+x)^{\alpha} = \sum_{n=0}^{\infty} \binom{\alpha}{n} x^n, \qquad |x| < 1.$$

Lower incomplete gamma function $\gamma(\nu, x)$:

$$\gamma(\nu, x) = \int_0^x t^{\nu-1} e^{-t} \mathrm{d}t = (\nu-1)! \left(1 - e^{-x} \sum_{j=0}^{\nu-1} \frac{x^j}{j!}\right)$$

Notations

$$A_{nk} = \frac{4\,(2\pi)^{1-2n}\,T^2}{e^{-m/T} - e^{-M/T}} \binom{1/2}{n} \binom{n}{k} (-1)^{n-k} \left(\frac{m}{T}\right)^{2(n-k)}$$

$$B_{nk} = \frac{4\,(2\pi)^{2n}\,T^2}{e^{-m/T} - e^{-M/T}} \binom{1/2}{n} \binom{1/2-n}{k} (-1)^{k} \left(\frac{m}{T}\right)^{2k}$$

$$\mu = \begin{cases} \sqrt{4\pi^2 T^2 + m^2}, & M > 2\pi T \\ M, & M < 2\pi T. \end{cases}$$

Term h_E

$$h_E = 16\pi T^2 \Bigg[-\langle \ln \mathrm{d}E \rangle_E + 1 + \ln T + \ln \left(e^{-m/T} - e^{-M/T} \right) + \frac{1}{T} \frac{m e^{-m/T} - M e^{-M/T}}{e^{-m/T} - e^{-M/T}} \Bigg]$$

Term h_ρ

$$h_\rho = \sum_{n=0}^{\infty} \Bigg\{ \sum_{k=0}^{n} A_{nk} \sum_{r=1}^{\infty} \frac{1}{(1+\beta r)^{1+2k}} \times \left[\frac{\beta}{1+\beta r} \gamma(2+2k, x) + \frac{1}{r} \gamma(1+2k, x) \right] \Bigg|_{x=(1+\beta r)m/T}^{x=(1+\beta r)\mu/T}$$

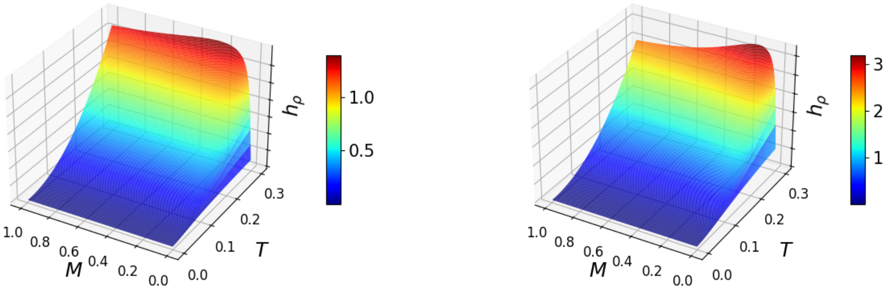
$$+ \sum_{k=0}^{\infty} B_{nk} \sum_{r=1}^{\infty} (1+\beta r)^{2(n+k-1)} \times \left[\frac{\beta}{1+\beta r} \gamma(3-2n-2k, x) + \frac{1}{r} \gamma(2-2n-2k, x) \right] \Bigg|_{x=(1+\beta r)\mu/T}^{x=(1+\beta r)M/T} \Bigg\} \Bigg|_{\beta=N}^{\beta=1}$$

Term h_Q

$$h_Q = \frac{H(Q)}{\pi}\left[-\frac{M^2 - m^2}{e^{(M-m)/T} - 1} + 2T\frac{me^{-m/T} - Me^{-M/T}}{e^{-m/T} - e^{-M/T}} + 2T^2\left(2\pi^2 + 1\right)\right]$$

$$+ H(Q)\sum_{n=0}^{\infty}\left[\sum_{k=0}^{n} A_{nk}\gamma(1 + 2k, x)\Big|_{x=m/T}^{x=\mu/T} + \sum_{k=0}^{\infty} B_{nk}\gamma(2 - 2n - 2k, x)\Big|_{x=\mu/T}^{x=M/T}\right]$$

Results



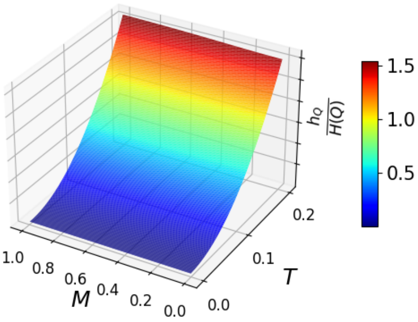
h_ρ as a function of upper energy bound M and temperature T for $m = 0$

Left: $N = 2$

Right: $N = 5$

[M.T. et al, Particles 7 (2024) 634]

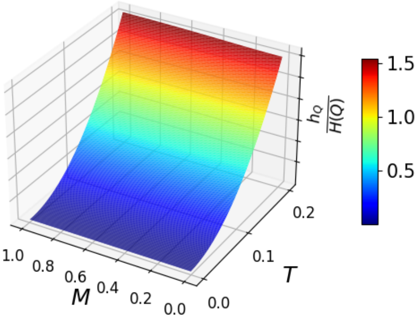
Results

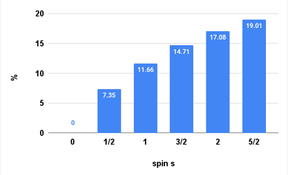


Quantity $\frac{h_Q}{H(Q)}$ as a function of M and T for $m = 0$

[M.T. et al, Particles 7 (2024) 634]

Results





[M.T. et al, Particles 6 (2023) 864]

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Conclusions

- The total entropy H of spherical Unruh source is estimated analytically for an exponential spectrum within arbitrary energy range.
- Particle multiplicity of the entropy allowed to analyze the contribution of both bosons and fermions.
- Additional degrees of freedom contribute equally likely to avoid perturbing the background metric.
- $H = 0$ for $T = 0$. The entropy gradually increases with temperature rising.
- The Unruh term h_ρ has a peak as a function of the upper energy bound M. The maximum becomes more pronounced for larger N.
- The results reveal strong dependence of the entropy on conditional distributions governing particle emission probability.