

Equivariant Neural Networks for Robust CP Observables

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ICNFP2024, Kolumbari, Crete

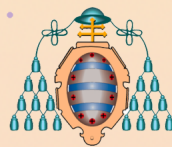
Dr. Pietro Vischia

work with Sergio Sánchez Cruz, Marina Kolosova, Clara Ramón Álvarez, and Giovanni Petrucciani

pietro.vischia@cern.ch

[@pietrovischia](https://www.instagram.com/pietrovischia)

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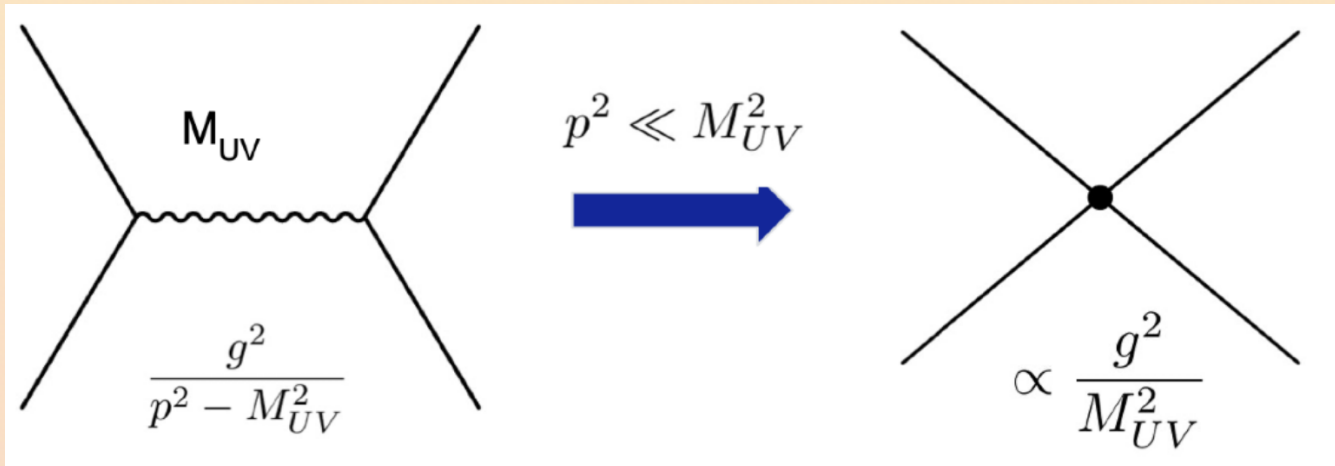
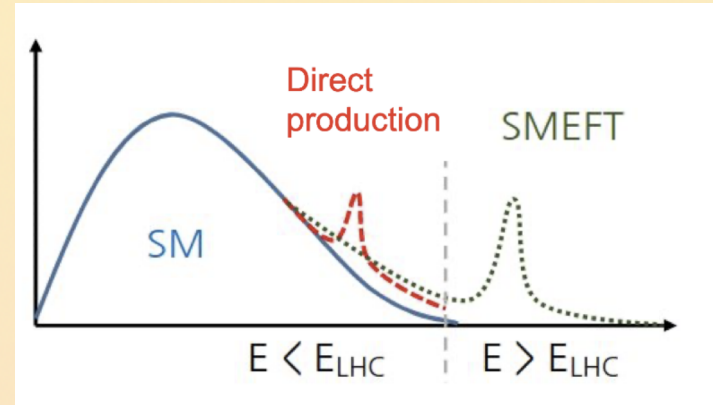
https://www.hep.uniovi.es/vischia/persistent/2024-09-04_EquivariantForCPAtICNFP2024_vischia.html

to get the version with working animations

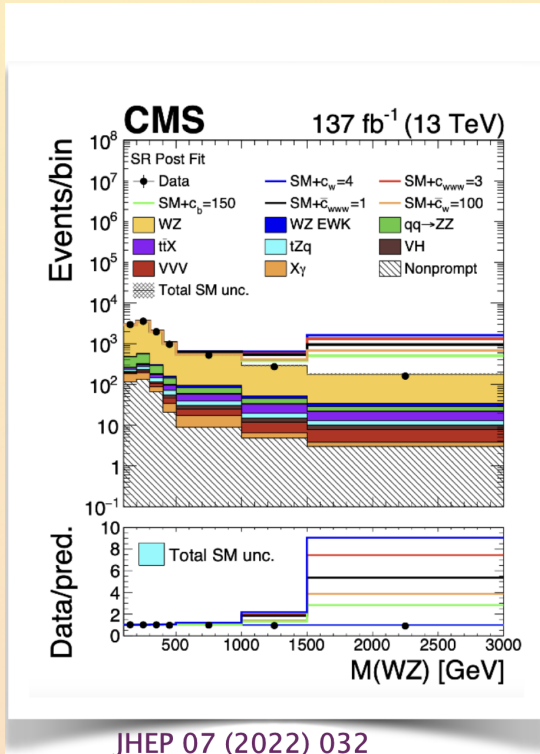
SMEFT and CP Violation

- SMEFT: standard model extended by postulating high-mass BSM particles
- 1350 CP-even operators, 1149 CP-odd operators

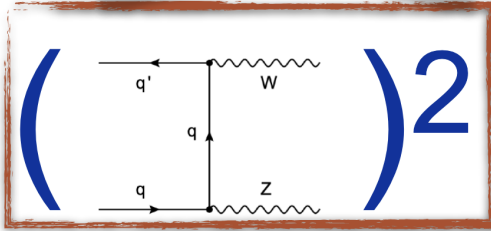
$$\mathcal{L} = \mathcal{L}_{SM} + \sum_i \frac{C_i}{\Lambda^2} \mathcal{O}_i^{(6)}$$



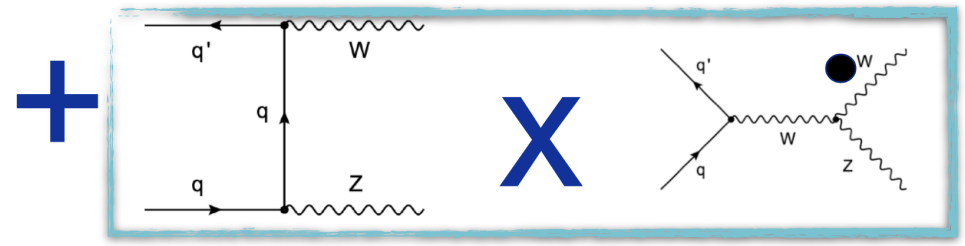
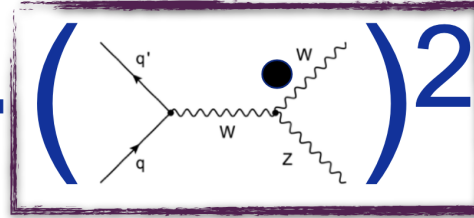
EFT Observables



SM contribution



Pure BSM contribution



SM-BSM interference

CP-violating operators

- **SM contribution**: mostly CP-invariant
- **Pure BSM contribution**: CP-invariant e.g. in top/Higgs sectors
- **SM-BSM** interference: odd under CP transformations
- Sensitivity to the interference given only by CP-odd observables. LHC cross section program insensitive.
- CP-odd observables are robust against signal mismodelling/background

Our Algorithm

- Build observables that are *equivariant with respect to CP symmetry
 - CP-invariant observables
 - discriminate between different SM backgrounds
 - discriminate between SM and quadratic terms or CP-even contribution
 - CP-odd observables
 - discriminate between signal-like and interference-like contributions
 - discriminate between interference-like and other SM backgrounds
- We fix $n_1 = 0$ and $n_2 = 1$, obtaining a single CP-odd observable
 - Can generalize to n_1 CP-invariant and n_2 CP-odd components

Our algorithm (reprise)

- A function $f : D \rightarrow R$ is odd under CP transformations if
$$f(CP(event)) = -f(event)$$
 - Most general function satisfying this is $f(event) = g(event) - g(CP(event))$
 - We parameterize g using a neural network, training f to minimize a loss function
- Parameterizations of g : can be any parametric function, you don't strictly need a neural network
- Space of input features is fully general
 - Kinematics of set of particles, low- or high-level variables, particle set, graph network
 - Can also add features for background discrimination

Gutting the algo: the cost function

- Inductive bias (see the [Machine Learning course!!](#)) by learning the likelihood ratio
 - Method inspired by the SALLY procedure ([Brehmer et al.](#))
 - Other loss functions can encode different properties (see [recent example](#))
- Weighted simulations: $w(z) = w_{SM}(z) + cw_{int}(z) + c^2w_{quad}(z)$
 - Weights are functions of parton level kinematics
- Intractable likelihood ratio:

$$\frac{p(d, z|c_1)}{p(d, z|c = 0)} = \frac{w_{SM} + cw_{int} + c^2w_{quad}}{w_{SM}}$$

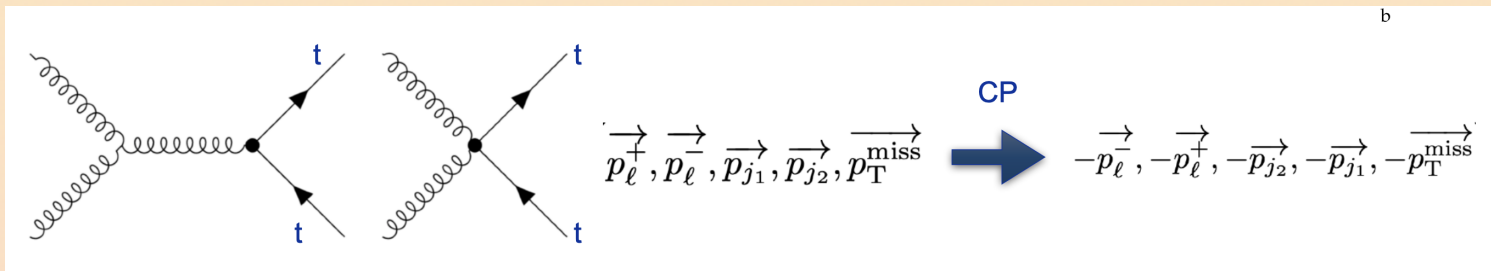
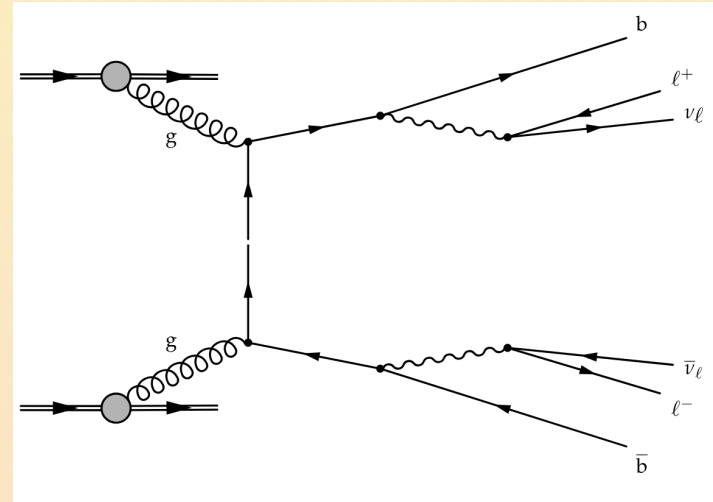
- The likelihood score at the SM point will be a sufficient statistic for small values of c
 - In the small- c regime, the linear component, describing the interference, is dominant
- Learn a surrogate model of the score

$$Loss = w_{SM} \left| f(d) - \frac{w_{int}(z)}{w_{SM}(d)} \right|_2$$

Use case: $t\bar{t}$ production

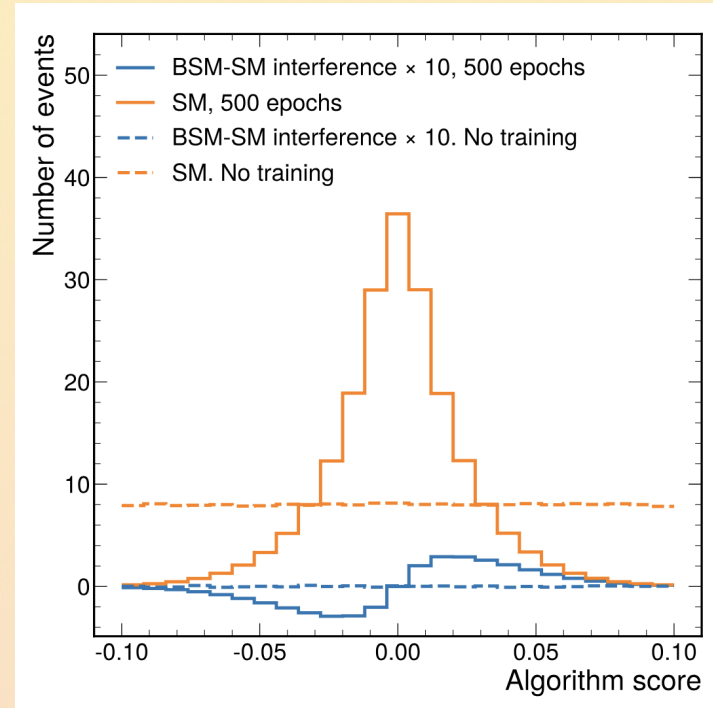
- Dileptonic final state
 - Semileptonic difficult, need to estimate jet charge (BSc thesis of Santiago Vila Domínguez)
- CP-violating chromoelectric dipole moment operator

$$g_s \frac{v}{\sqrt{2}} (\bar{t} \sigma^{\mu\nu} \gamma_5 T^A t) G_{\mu\nu}^A$$



Use case: ttbar production

- The score after the training is CP-odd!
 - Symmetric for SM
 - Any SM-like mismodelling/background will be symmetric by construction!
 - Constructive/destructive interference pattern for positive/negative values
- Equivariance respected at all stages of training

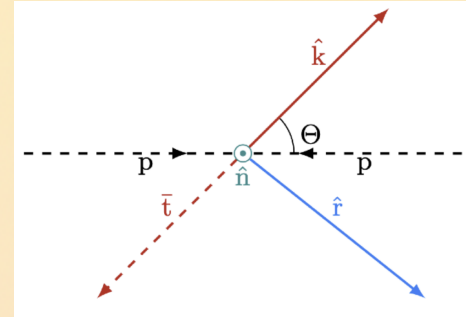


- The observable is robust even before training convergence

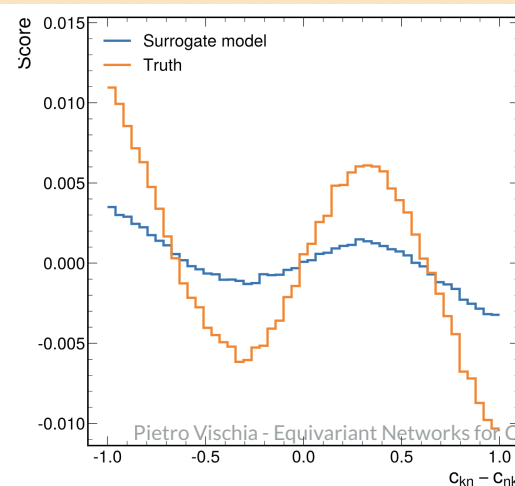
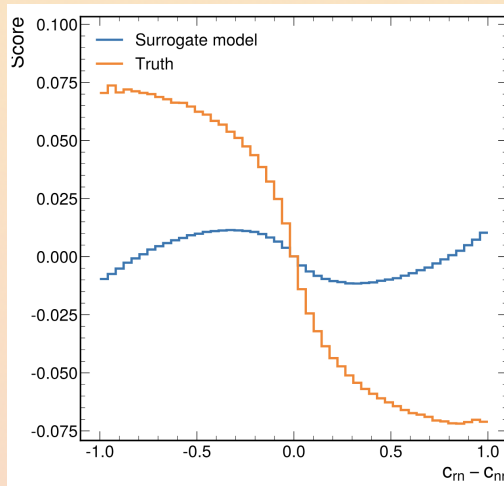
Use case: ttbar production

- Reweight events by the score, compare with parton-level CP-odd observables
- Reconstruct the ttbar system based on angles

$$\begin{aligned} c_{nr} - c_{nr} &= \cos(\theta_r^+) \cos(\theta_r^-) - \\ &\cos(\theta_n^+) \cos(\theta_n^-) \\ c_{nk} &= \cos(\theta_k^+) \cos(\theta_n^-) - \\ &\cos(\theta_n^+) \cos(\theta_k^-) \end{aligned}$$

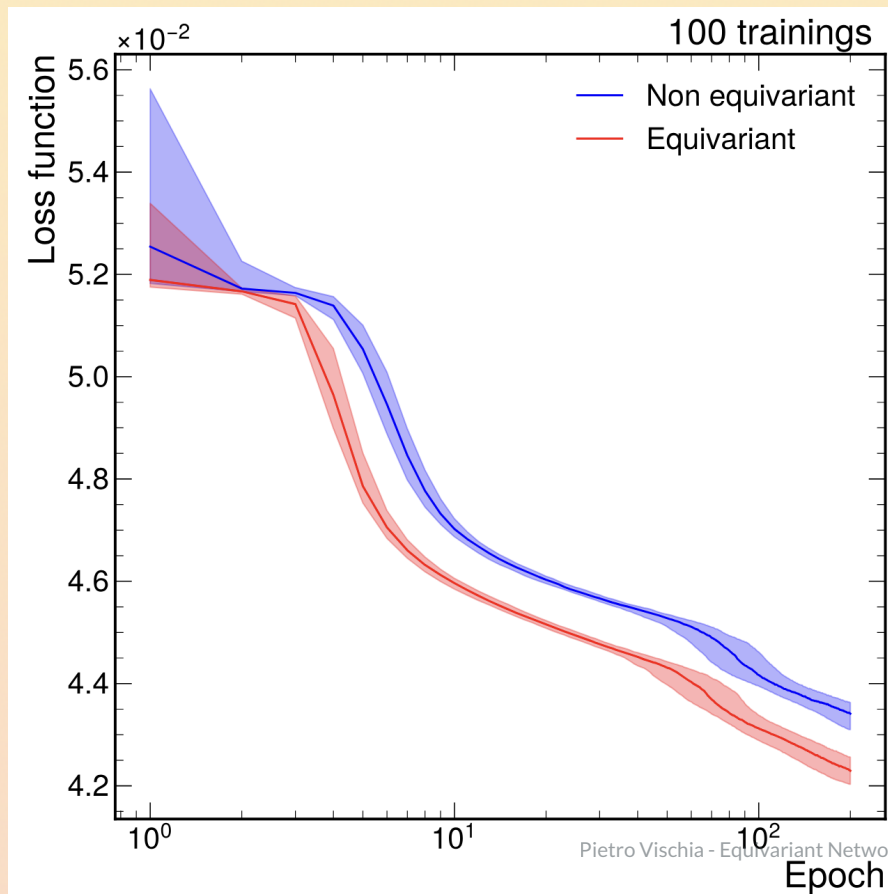


- Limitation is the reconstruction of the ttbar system



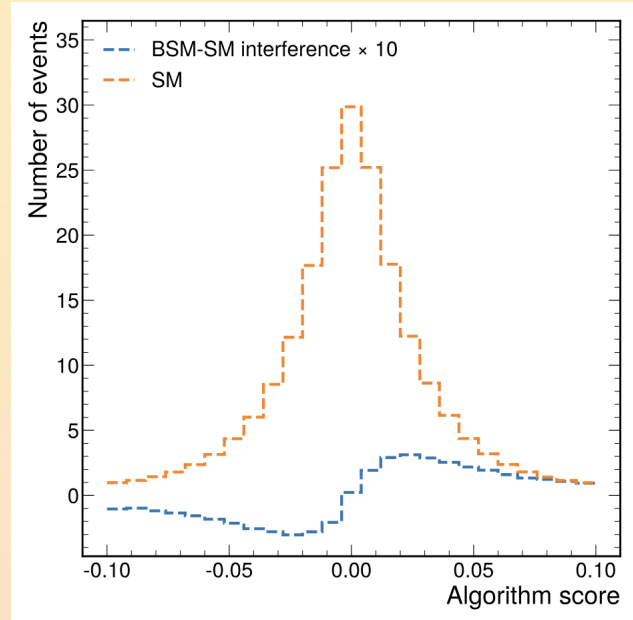
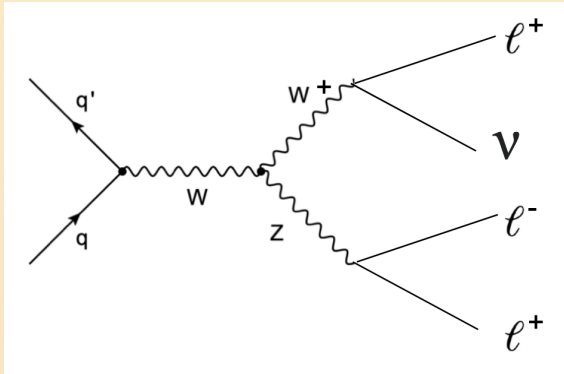
Use case: ttbar production

- Equivariance as inductive bias speeds up convergence
- Between **40% and 300% less iterations** needed to achieve the same loss value!!!



Use case: WZ production

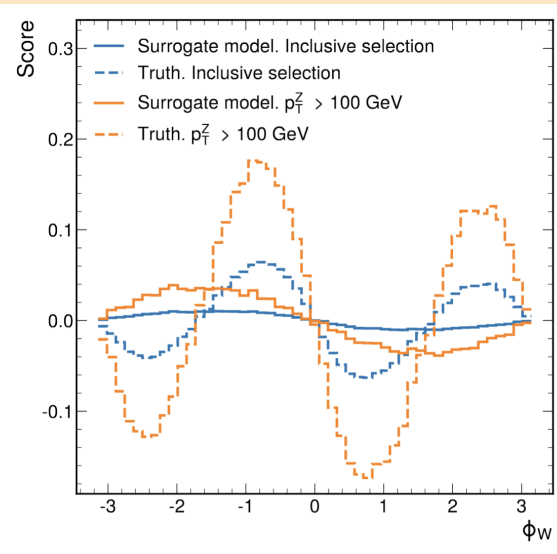
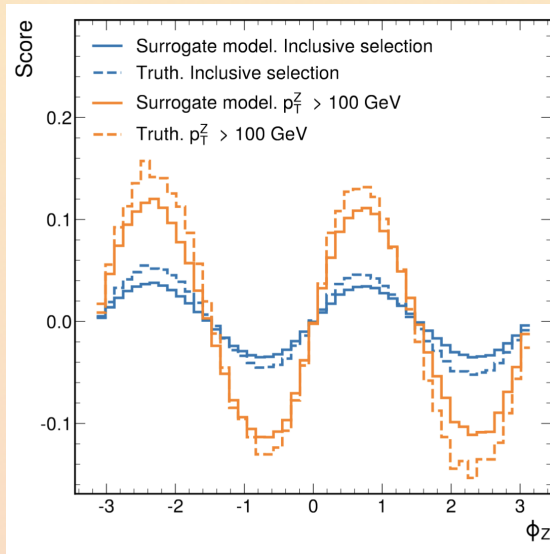
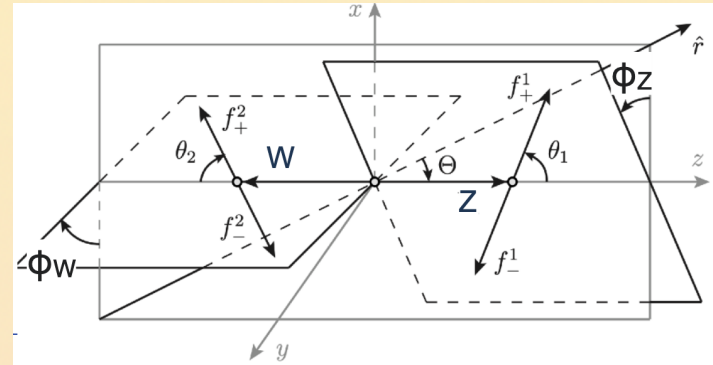
- Trilepton final state
- CP-odd operator: $c_{\tilde{W}}$



$$\begin{array}{c}
 \vec{p}_{\ell^+}^Z, \vec{p}_{\ell^-}^Z, \vec{p}_{\ell^+}^W, Q^W, p_{\text{miss}}^{\text{T}} \\
 \searrow \text{CP} \\
 -\vec{p}_{\ell^-}^Z, -\vec{p}_{\ell^+}^Z, -\vec{p}_{\ell^+}^W, -Q^W, -p_{\text{miss}}^{\text{T}}
 \end{array}$$

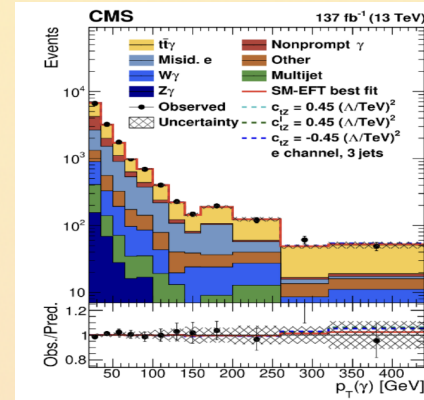
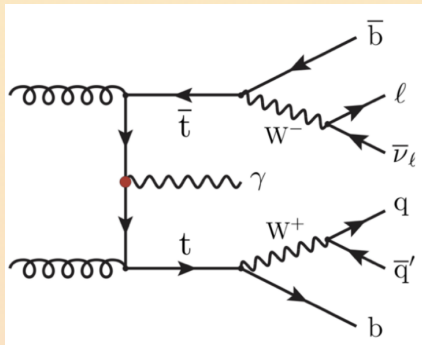
Use case: WZ production

- Performance on parton-level observables **even better than dedicated observables!!!**
 - Can capture energy growth
 - Insensitivity to ϕ_W due to ambiguity in W decay reconstruction



Use case: ttgamma production

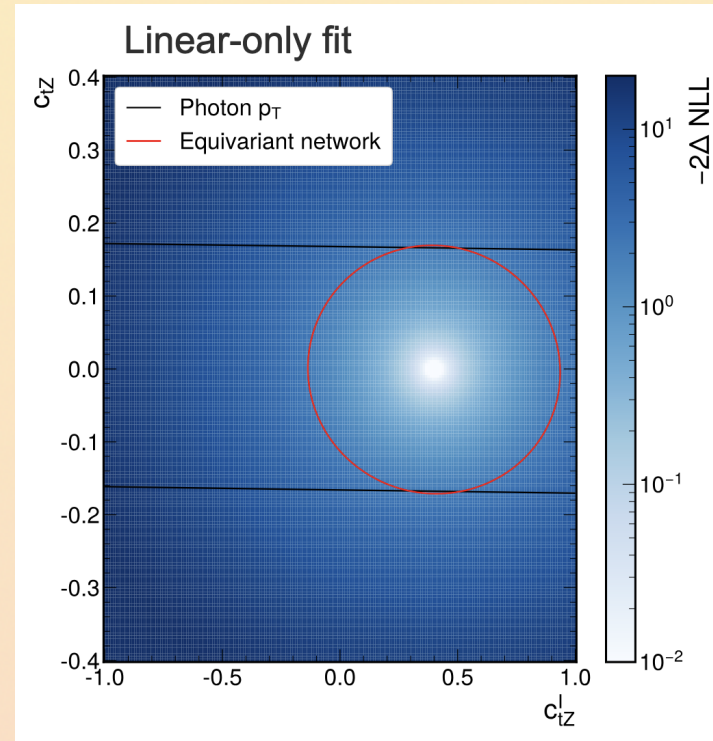
- Single lepton channel, CP-odd operator c_{tZl}
- Literature mostly checks photon p_T , which is CP-even



$$\begin{array}{c}
 \vec{p}_\gamma, \vec{p}_\ell, Q_\ell, \vec{p}_{b_1}, \vec{p}_{b_2}, \vec{p}_{j_1}, \vec{p}_{j_2} \\
 \downarrow \text{CP} \\
 -\vec{p}_\gamma, -\vec{p}_\ell, -Q_\ell, -\vec{p}_{b_2}, -\vec{p}_{b_1}, -\vec{p}_{j_2}, -\vec{p}_{j_1}
 \end{array}$$

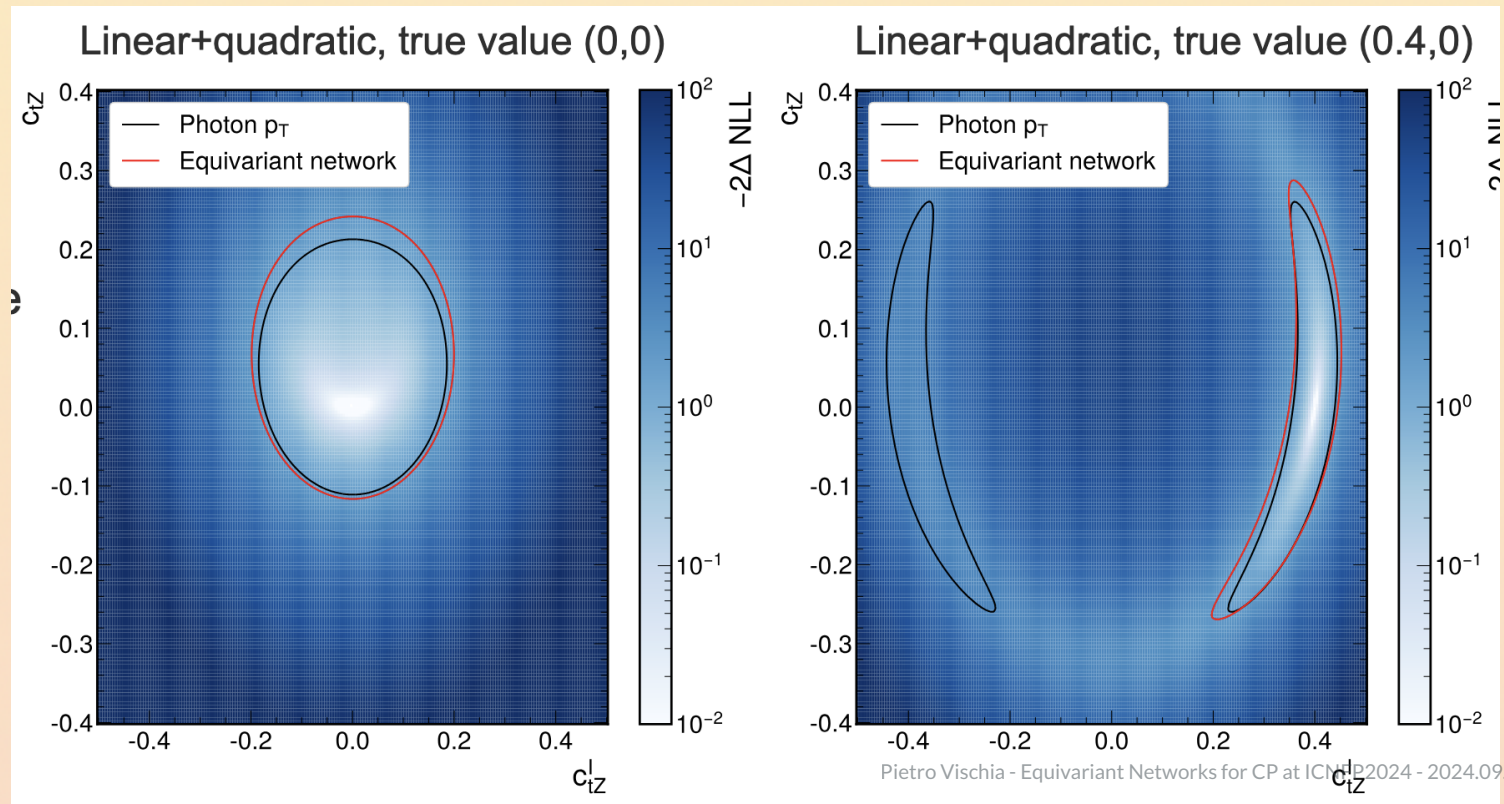
Use case: ttgamma production

- Linear contribution constrainable **only by our approach**
- c_{tZ}^l (CP-odd): Comparison with photon p_T is damning (for the photon p_T , which is CP even)
- c_{tZ} (CP-even): similar sensitivity



Use case: ttgamma production

- Assuming the SM: same sensitivity
 - Our approach retains performance in CP-even observables!
- BSM cases: our approach disentangles the sign of c_{tZ}^i !!!
 - Equivariant training is superior, even if not trained for quadratic components!



Conclusions

- Implemented equivariant networks to obtain robust observables for CP violation
- Inductive bias encoded in the network structure
 - Robust regardless of convergence status
 - Training is faster than regular network
- Benchmarks: $t\bar{t}$, WZ , $t\bar{t}\gamma$
 - Our approach is better than existing state-of-the-art observables
- Extensions under exploration
 - Maybe CP-invariant networks (to target CP-even observables)
- Already being employed for upcoming CMS analyses

Thank you!

