QCD effects in non-QCD theories

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Outline:

- Motivation
- QCD effects in non-QCD theories:
  - Mass gap in SU(2) theory Yang-Mills theory.
  - Flux tubes and particlelike configurations in Proca theories.
- Outline of nonperturbative quantization and possible appearance of:
  - a nonlinear spinor field (for a mass gap).
  - a mass term (for FT's and particlelike solutions).

Motivation:

In quantum chromodynamics (QCD), there are many effects caused by the strong nonlinearity of QCD. These effects are, for instance, (i) the appearance of a mass gap, (ii) the existence of tubes connecting quarks and filled with a longitudinal electric field, (iii) the presence of a contribution to the proton spin coming from gluon fields, etc.

Here we wish to discuss the possibility that, in some non-QCD theories, there may exist effects shared by QCD. The appearance of such QCD effects in non-QCD theories enables one to assume that such non-QCD theories may have some relevance to QCD. For example, it is possible that such non-QCD theories may serve as some *approximate* way to describe nonperturbative effects in QCD.

Mass gap:

The mass gap problem is an unsolved problem in physics. In the case of G=SU(3)—the strong nuclear interaction—it is necessary to prove that nucleons have a lower mass bound, and thus cannot be arbitrarily light. Mass/energy spectrum should have a global minimum.

Mass gap:

Now we want to demonstrate that the energy spectrum of a "particle-like" solution in SU(2) Yang-Mills theory with a source in the form of a nonlinear spinor field has a global minimum corresponding to a mass gap.
The Lagrangian describing a system consisting of a non-Abelian SU(2) field A^a_μ interacting with nonlinear spinor field ψ can be taken in the form

$$\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} + i\hbar c \bar{\psi} \gamma^\mu D_\mu \psi - m_f c^2 \bar{\psi}\psi + \frac{\Lambda}{2} \hbar c \left(\bar{\psi}\psi\right)^2$$

$[\Lambda] = \mathrm{sm}^2$

Mass gap

In order to find "monopole" solutions and their energy spectrum, we choose the following *Ansätze* for the Yang-Mills and spinor fields:

$$A^a_t = 0, \quad A^a_i = \frac{1}{g}(1-f)\begin{pmatrix} 0 & \sin\varphi & \sin\theta\cos\theta\cos\varphi \\ 0 & -\cos\varphi & \sin\theta\cos\theta\sin\varphi \\ 0 & 0 & -\sin^2\theta \end{pmatrix}$$

$$\psi^T = \frac{e^{-i\frac{Et}{\hbar}}}{gr\sqrt{2}}\left\{\begin{pmatrix} 0 \\ -u \end{pmatrix}, \begin{pmatrix} u \\ 0 \end{pmatrix}, \begin{pmatrix} iv\sin\theta e^{-i\varphi} \\ -iv\cos\theta \end{pmatrix}, \begin{pmatrix} -iv\cos\theta \\ -iv\sin\theta e^{i\varphi} \end{pmatrix}\right\},$$

Mass gap:

$$-f'' + \frac{f\,(f^2-1)}{x^2} + \tilde{g}^2\,\frac{\tilde{u}\,\tilde{v}}{x} = 0, \tag{1}$$

$$\tilde{v}' + \frac{f\,\tilde{v}}{x} = \tilde{u}\left(-\tilde{m}_f + \tilde{E} + \frac{\tilde{u}^2 - \tilde{v}^2}{x^2}\right), \tag{2}$$

$$\tilde{u}' - \frac{f\,\tilde{u}}{x} = \tilde{v}\left(-\tilde{m}_f - \tilde{E} + \frac{\tilde{u}^2 - \tilde{v}^2}{x^2}\right). \tag{3}$$

Equations for $f, \tilde{u}, \tilde{v}$. We assign boundary conditions near the origin $x = 0$ where solutions are sought in the form of the Taylor series

$$f = 1 + \frac{f_2}{2}x^2 + \ldots, \quad \tilde{u} = \tilde{u}_1 x + \frac{\tilde{u}_3}{3!}x^3 + \ldots, \quad \tilde{v} = \frac{\tilde{v}_2}{2}x^2 + \frac{\tilde{v}_4}{4!}x^4 + \ldots, \tag{4}$$

Mass gap:

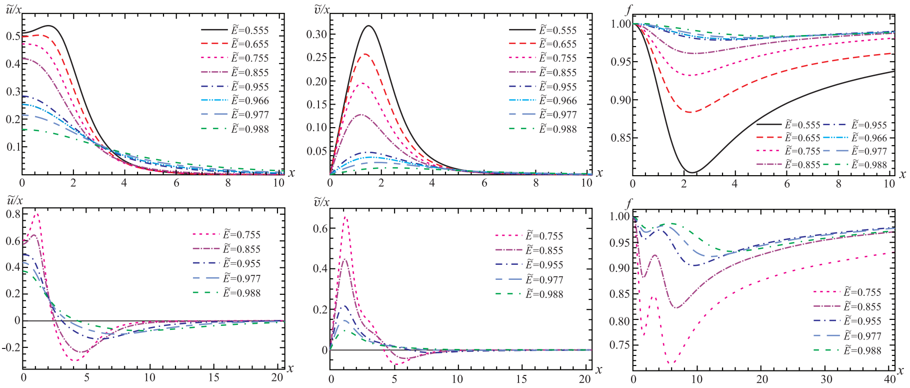


Рис.: Functions u, v, f

Mass gap:

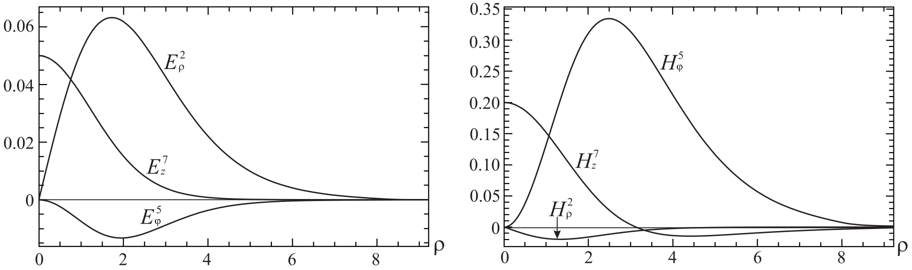

0.06
0.04
0.02
0
E_ρ^2
E_z^7
E_φ^5
0
2
4
6
8
ρ
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0
H_φ^5
H_z^7
H_ρ^2
0
2
4
6
8
ρ

Рис.:

Mass gap:

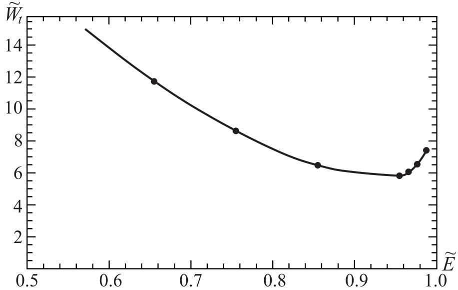


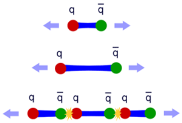
Рис.: A sketch of the energy spectrum of the total energy as a function of the spinor frequency $\tilde{E}$.

The total energy density of the monopole-plus-spinor-fields system under consideration is

$$\tilde{\epsilon} = \tilde{\epsilon}_m + \tilde{\epsilon}_s = \frac{1}{\tilde{g}^2}\left[\frac{f'^2}{x^2} + \frac{\left(f^2 - 1\right)^2}{2x^4}\right] + \left[\tilde{E}\frac{\tilde{u}^2 + \tilde{v}^2}{x^2} + \frac{1}{2}\frac{\left(\tilde{u}^2 - \tilde{v}^2\right)^2}{x^4}\right],$$

Flux tubes and particlelike configurations in Proca theories

A tube filled with a longitudinal non-Abelian electric field. Such a tube is a necessary ingredient of the confinement phenomenon, since it provides a constant force between quarks preventing their separation.



Flux tubes and particlelike configurations in Proca theories

The Lagrangian describing such a system consisting of a non-Abelian SU(3) Proca field A^a_μ interacting with nonlinear scalar field ϕ can be taken in the form

$$\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} - \frac{\left(\mu^2\right)^{ab,\mu}_{\quad\ \nu}}{2} A^a_\mu A^{b\nu} + \frac{1}{2}\partial_\mu \phi \partial^\mu \phi + \frac{\lambda}{2}\phi^2 A^a_\mu A^{a\mu} - \frac{\Lambda}{4}\left(\phi^2 - M^2\right)$$

$[\lambda] = [g^2]$, $[\phi^2] = [A^2]$, $[\mu^2] = \text{sm}^{-2}$. With the following electric and magnetic field intensities:

$$E^2_\rho = -\frac{h'}{g}, \quad E^5_\varphi = -\frac{\rho h w}{2g}, \quad E^7_z = \frac{hv}{2g},$$

$$H^2_\rho = -\frac{vw}{2g}, \quad H^5_\varphi = -\frac{\rho v'}{g}, \quad H^7_z = \frac{1}{g}\left(w' + \frac{w}{\rho}\right)$$

Flux tubes and particlelike configurations in Proca theories

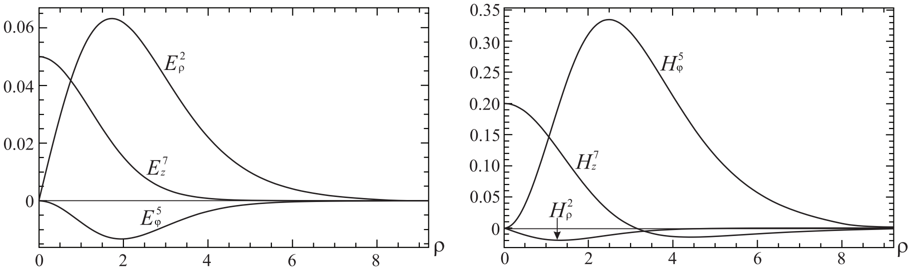


Рис.: A sketch of the distributions of color longitudinal electric fields E_z^7.

Important: it's not Nielsen – Olesen FT !

Tube with the energy flux/momentum density

Here we have the following components of the electric and magnetic field intensities:

$$E^2_\varphi = \frac{\rho f w}{2g}, \quad E^5_\rho = -\frac{f'}{g},$$

$$H^2_\rho = -\frac{v w}{2g}, \quad H^5_\varphi = -\frac{\rho v'}{g}, \quad H^7_z = \frac{1}{g}\left(w' + \frac{w}{\rho}\right),$$

In this case the Poynting vector S^i is already nonzero,

$$S^i = \frac{\epsilon^{ijk}}{\sqrt{\gamma}} E^a_j H^a_k \neq 0, \quad S^z = \frac{1}{g^2}\left(\frac{df}{d\rho}\frac{dv}{d\rho} + \frac{1}{4} f v w^2\right).$$

Tube with the energy flux/momentum density

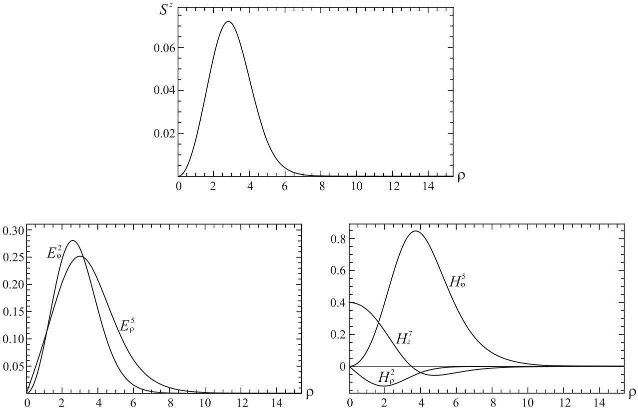


Рис.: Sketches of the z-component of the Poynting vector S^z.

Tube with the energy flux/momentum density

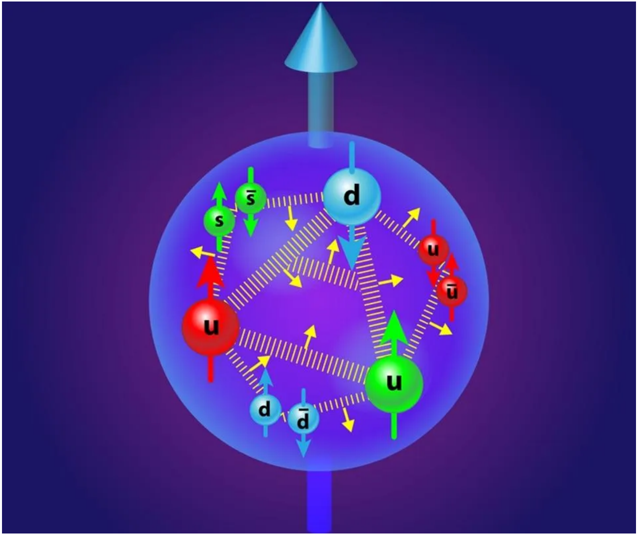


Рис.:

Proca balls with angular momentum

The Lagrangian describing a system consisting of a non-Abelian SU(2) Proca field A^a_μ coupled to a triplet of real Higgs scalar fields ϕ^a can be taken in the form

$$\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} - \frac{1}{2} \left(\mu^2\right)^{ab,\mu}_{\quad\ \nu} A^a_\mu A^{b\nu} + \frac{1}{2} D_\mu \phi^a D^\mu \phi^a - \frac{\Lambda}{4} \left(\phi^a \phi^a - v^2\right)^2 .$$

Proca balls with angular momentum

For such a choice, there are the following nonvanishing color electric and magnetic fields (physical components):

$$E^1_z = -\frac{f_{,z}}{g}, \quad E^3_z = -\frac{h_{,z}}{g}, \quad E^1_\rho = -\frac{f_{,\rho}}{g}, \quad E^3_\rho = -\frac{h_{,\rho}}{g}, \tag{6}$$

$$H^1_z = -\frac{\rho\, k_{,\rho} + k}{g\rho}, \quad H^3_z = -\frac{\rho\, w_{,\rho} + w}{g\rho}, \quad H^1_\rho = \frac{k_{,z}}{g}, \quad H^3_\rho = \frac{w_{,z}}{g}, \tag{7}$$

Proca balls with angular momentum

In turn, for the above mentioned strengths the expression of the Poynting vector is:

$$S_\varphi = \frac{2}{g^2}\left[f_{,\rho}\left(w_{,\rho} + \frac{w}{\rho}\right) + f_{,z}w_{,z}\right].$$

Making use of this expression, one can obtain the expression for a linear angular momentum density,

$$\mathcal{P}_z = 2\pi \int\limits_0^\infty S_\varphi \rho^2 d\rho,$$

Proca balls with angular momentum

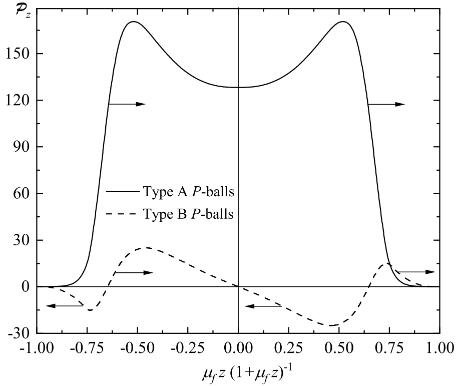


Рис.: Schematic sketch of the linear angular momentum density $\mathcal{P}_z$ along the tube axis for the P-balls under consideration. The arrows show the directions of the angular momentum density.

Outline of nonperturbative quantization and possible appearance of a nonlinear spinor field

As we see from Fig. 3, the energy of the "dipole-like" supported by the nonlinear spinor field has a minimum which corresponds to a mass gap. In this connection the natural question arises: whether there exists a relation between the mass gap obtained and a mass gap in QCD? In other words, whether the appearance of the mass gap for such a "dipole-like" solution is coincidental or this is related somehow to nonperturbative quantization of essentially nonlinear non-Abelian Yang-Mills fields? If one starts from the assumption that such a relation does really exist, it may consist in the fact that the nonlinear spinor field occurs as a consequence of some approximate description of quantum nonpertubative effects in QCD.

Outline of nonperturbative quantization and possible appearance of a nonlinear spinor field

In a more rigorous mathematical language, this looks like

$$
\begin{aligned}
\left\langle i\gamma^\mu \hat{\psi}_{;\mu} - m\hat{\psi} \right\rangle =& 0, \\
\frac{1}{\sqrt{-g}} \frac{\partial}{\partial x^\nu} \left(\sqrt{-g} \left\langle \hat{F}^{a\mu\nu} \right\rangle \right) =& -4\pi \left\langle \hat{j}^{a\mu} \right\rangle , \\
\left\langle \hat{\bar{\psi}} \left(i\gamma^\mu \hat{\psi}_{;\mu} - m\hat{\psi} \right) \right\rangle =& 0, \\
\left\langle \hat{A}_\nu \left(i\gamma^\mu \hat{\psi}_{;\mu} - m\hat{\psi} \right) \right\rangle =& 0, \\
\frac{1}{\sqrt{-g}} \frac{\partial}{\partial x^\nu} \left(\sqrt{-g} \left\langle \hat{F}^{a\mu\nu} \hat{\psi} \right\rangle \right) =& -4\pi \left\langle \hat{j}^{a\mu} \hat{\psi} \right\rangle , \qquad (8) \\
& \ldots
\end{aligned}
$$

Outline of nonperturbative quantization and possible appearance of a nonlinear spinor field

It is of great importance to note that these equations greatly simplify in the stationary case.
It is evident that such an infinite set of equations can be solved neither analytically nor numerically. Therefore, it is necessary to have some approximate methods of solving Eq's. One of such methods consists in using the procedure that enables one to cut off an infinite set of equations to get a finite one. In this case it is necessary to have a hypothesis about the behavior of higher-order Green's function.

Outline of nonperturbative quantization and possible appearance of a nonlinear spinor field

If we wish to cut off the infinite set of Dyson-Schwinger equations at just the point where one may derive separate equations for the gauge field $\left\langle A^a_\mu \right\rangle$ and for the spinor field $\langle \psi \rangle$, we have to have a hypothesis about an approximate description of the interaction $\left\langle \hat{\bar{\psi}} \hat{A}^a_\mu \hat{\psi} \right\rangle$. For this purpose, one can suggest different approximations, one of which is

$$\left\langle \hat{\bar{\psi}} \gamma^\mu \hat{A}^a_\mu \hat{\psi} \right\rangle = \left\langle \hat{\bar{\psi}} \gamma^\mu \left(\left\langle \hat{A}^a_\mu \right\rangle + \widehat{\delta A^a_\mu} \right) \hat{\psi} \right\rangle \approx \left\langle \hat{\bar{\psi}} \gamma^\mu \hat{\psi} \right\rangle \left\langle A^a_\mu \right\rangle + \lambda \left\langle \bar{\hat{\psi}} \hat{\psi} \right\rangle^2$$

where the closure constant λ appears.

Nonperturbative quantization and the hypothetical appearance of a mass term

Here we would like to discuss the question of whether, in solving infinite eq's system for all Green's functions approximately, there can occur additional terms (in particular, a mass-like term) which might play the role of an effective mass in Proca theory.

The only way to solve these infinite set of equations approximately is the use of the cut-off method for obtaining a finite set of equations. According to this approach, it is necessary to keep only the first n equations. In this case, the last equation will contain higher-order Green's functions (G_{n+1}), equations for which are already ruled out from a consideration. In order that the first n equations form a closed system, it is necessary to propose a hypothesis about the last Green's function G_{n+1}. For instance, one can assume that such Green's function is some polylinear combination of lower-order Green's functions. For example, for a fourth-order Green's function, this can be schematically represented as

$$G_4 \approx (G_2)^2 + \alpha G_2 + \beta,$$

Analogy between nonperturbative quantization and the turbulence modeling

We may notice here a remarkable analogy between such procedure of nonperturbative quantization and what happens in turbulence modeling when analysing the Navier-Stokes equation

$$\rho \left(\frac{\partial v_i}{\partial t} + v_j \frac{\partial v_i}{\partial x_j} \right) = - \frac{\partial p}{\partial x_i} + \frac{\partial t_{ij}}{\partial x_j},$$

Averaging this equation, one can obtain

$$\rho \frac{\partial V_i}{\partial t} + \rho V_j \frac{\partial V_i}{\partial x_j} = - \frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j} \left(2\mu S_{ji} - \overline{\rho v_j' v_i'} \right),$$

Analogy between nonperturbative quantization and the turbulence modeling

The quantity $\overline{\rho v'_j v'_i} = -\overline{\rho v'_i v'_j}$. is known as the Reynolds-stress tensor. One can obtain the following equation for the Reynolds stress tensor:

$$\frac{\partial \tau_{ij}}{\partial t} + V_k \frac{\partial \tau_{ij}}{\partial x_k} = -\tau_{ik}\frac{\partial V_j}{\partial x_k} - \tau_{jk}\frac{\partial V_i}{\partial x_k} + 2\mu \overline{\frac{\partial v'_i}{\partial x_k}\frac{\partial v'_j}{\partial x_k}} - \overline{p'\left(\frac{\partial v'_i}{\partial x_j} + \frac{\partial v'_j}{\partial x_i}\right)} + \frac{\partial}{\partial x_k}\left(\nu \frac{\partial \tau_{ij}}{\partial x_k} + C_{ijk}\right),$$

Thus, we see that the approximate approach to solving the infinite set of equations of nonperturbative quantization leads to the appearance of the closure constants, one of which may be a mass of a Yang-Mills field. It is quite conceivable that in the limit $n \to \infty$ some of such constants will remain. This means that the nonperturbative quantization leads to the appearance of dimensional constants which are absent in the initial classical theory; this phenomenon is called dimensional transmutation.

Discussion and conclusions

In this study, we have shown that some QCD effects can occur in non-QCD theories which differ from QCD by the presence, for example, a nonlinear spinor field or a massive Yang-Mills field (a Proca field).
We have considered two possible ways of solving this problem:

- Green's function $\left\langle \hat{\bar{\psi}} \gamma^\mu \hat{A}^a_\mu \hat{\psi} \right\rangle$) is approximately described as the sum of two terms. The first term is $\left\langle \hat{\bar{\psi}} \gamma^\mu \hat{\psi} \right\rangle \left\langle A^a_\mu \right\rangle$. The second term is the product of closure constant λ, and a nonlinear term $\left\langle \hat{\bar{\psi}} \hat{\psi} \right\rangle^2$.
- Green's function of the product of Yang-Mills field potentials is a poly-linear combination of lower-order Green's functions of the gauge field, where closure constants may appear.

Discussion and conclusions

As concrete examples, we have demonstrated that there exist the following QCD effects in non-QCD theories:

- In SU(2) Yang-Mills theory with a source in the form of a nonlinear spinor field, a mass gap is present.
- In non-Abelian Proca theories, there are:
  - Infinite tubes containing a longitudinal electric field.
  - Infinite tubes with the momentum density created by crossed electric and magnetic fields and directed along the tube axis.
  - Particlelike solutions possessing a nonzero total angular momentum created by crossed electric and magnetic fields.
  - For all the above solutions, the Meissner-like effect is observed: the Proca fields are expelled by the Higgs fields.

Questions and assumptions

Question: In what and how does nonperturbative quantization manifest itself ?

Assumption: $\hbar$, new dimensional constants (dimensional transmutation) ?

Questions

In connection with the assumptions made here, there arise the following questions requiring more detailed investigations:

- What happens with the closure constants in the limit $n \to \infty$? If the closure constants survive in this limit, then this means that there will take place dimensional transmutation for nonperturbative quantization.
- Whether there exists any generalization of the procedure of renormalization for the procedure of nonperturbative quantization ?
- How deep is the analogy between nonperturbative quantization and turbulence modeling ?

Movie created by Nikolay Kotlyarevsky

Thanks for your attention !