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Quasiperiodic oscillations from low mass X-ray binary systems

Kuantay Boshkayev

in collaboration with M. Muccino and H.Quevedo

Al-Farabi Kazakh National University, Almaty, Kazakhstan

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Motivation

Investigation of the motion of test particles in the field of rotating and deformed objects are relevant to several astrophysical phenomena:

- in particular to the observed high frequency, kilohertz Quasi Periodic Oscillations (kHz QPOs) in the X-ray luminosity from black hole and neutron star sources;
- it is believed that kHz QPO data may be used to test the strong field regime of Einstein's general relativity, and the physics of super-dense matter of which neutron stars are made of.

External Hartle-Thorne Solution

$$
ds^{2} = -\left(1 - \frac{2M}{r}\right)\left[1 + 2k_{1}P_{2}(\cos\theta) + 2\left(1 - \frac{2M}{r}\right)^{-1}\frac{J^{2}}{r^{4}}(2\cos^{2}\theta - 1)\right]dt^{2} + \left(1 - \frac{2M}{r}\right)^{-1}\left[1 - 2k_{2}P_{2}(\cos\theta) - 2\left(1 - \frac{2M}{r}\right)^{-1}\frac{J^{2}}{r^{4}}\right]dr^{2} + r^{2}[1 - 2k_{3}P_{2}(\cos\theta)](d\theta^{2} + \sin^{2}\theta d\phi^{2}) - 4\frac{J}{r}\sin^{2}\theta dt d\phi
$$

$$
k_1 = \frac{J^2}{Mr^3} \left(1 + \frac{M}{r} \right) - \frac{5}{8} \frac{Q - J^2/M}{M^3} Q_2^2 \left(\frac{r}{M} - 1 \right), \qquad k_2 = k_1 - \frac{6J^2}{r^4},
$$

\n
$$
k_3 = k_1 + \frac{J^2}{r^4} - \frac{5}{4} \frac{Q - J^2/M}{M^2 r} \left(1 - \frac{2M}{r} \right)^{-1/2} Q_2^1 \left(\frac{r}{M} - 1 \right), \qquad P_2(x) = \frac{1}{2} (3x^2 - 1),
$$

\n
$$
Q_2^1(x) = (x^2 - 1)^{1/2} \left[\frac{3x}{2} \ln \frac{x+1}{x-1} - \frac{3x^2 - 2}{x^2 - 1} \right], \qquad Q_2^2(x) = (x^2 - 1) \left[\frac{3}{2} \ln \frac{x+1}{x-1} - \frac{3x^3 - 5x}{(x^2 - 1)^2} \right].
$$

•*Hartle, J. B., ApJ 150, 1005 (1967)* •*Hartle, J. B. & Thorne, K. S., ApJ, 153, 807 (1968)*

Limiting cases

- Q=0, J=0, SCHW;
- Q=0, J \neq 0, neglecting terms \sim J \wedge 2, LT;
- Q≠0, J≠0, HT

or Kerr solution in the Boyer-Lindquist coordinates using the following substitution and coordinate transformations:

$$
J = -Ma, \quad Q = J^2/M,
$$

and

$$
t = t,
$$

\n
$$
r = R + \frac{a^2}{2R} \left[\left(1 + \frac{2M}{R} \right) \left(1 - \frac{M}{R} \right) - \cos^2 \Theta \left(1 - \frac{2M}{R} \right) \left(1 + \frac{3M}{R} \right) \right],
$$

\n
$$
\theta = \Theta + \frac{a^2}{2R^2} \left(1 + \frac{2M}{R} \right) \sin \Theta \cos \Theta,
$$

\n
$$
\phi = \phi.
$$

Figure 1 - Current observations of neutron star masses suggest the existence of stars with a mass of order 2 solar masses. Such stars require a **stiff equation of** state for the neutron matter that makes up most of the star to be able to balance the attraction of gravity and rules out the presence of **exotic forms of matter (pion-kaon condensates, quark matter or core solids)** in the core of any neutron star because at any mass its central density is less than the transition densities for these exotic phases.

Figure 2 – Mass-Radius relations of Neutron Stars. J.M. Lattimer, M. Prakash / Physics Reports 442 (2007) 109–165

Neutron Star Model

Belvedere, R.; Pugliese, D.; Rueda, J.A.; Ruffini, R; Xue, Sh.

"Neutron star equilibrium configurations within a fully relativistic theory with strong, weak, electromagnetic, and gravitational interactions" Nuclear Physics A, Volume 883, p. 1-24. 2012

Belvedere, R.; Boshkayev, K.; Rueda, Jorge A.; Ruffini, R.

"Uniformly rotating neutron stars in the global and local charge neutrality cases"

Nuclear Physics A, Volume 921, p. 33-59. 2014

Figure 3 – Mass-Radius relations for Neutron Stars

Belvedere et. al. Nuclear Physics A, Volume 883, p. 1-24. (2012) Belvedere et. al. Nuclear Physics A, Volume 921, p. 33-59. (2014)

Constraints on the mass-radius relation given by J. E. Trumper in and the theoretical mass-radius in our work **the solid line is the upper limit of the surface gravity of XTE J1814-338**, **the dotteddashed curve corresponds to the lower limit to the radius of RX J1856-3754, the dashed line is the constraint imposed by the fastest spinning pulsar PSR J1748-2246ad, and the dotted curves are the 90% confidence level contours of constant R∞ of the neutron star in the lowmass X-ray binary X7**. Any mass-radius relation should pass through the area delimited by the solid, the dashed and the dotted lines and, in addition, it must have a maximum mass larger than the mass of PSR J1614-2230, M = 1.97 \pm 0.04M_o.

QPOs

In X-ray astronomy, **quasi-periodic oscillation** (**QPO**) is the manner in which the X-ray light from an astronomical object flickers about certain frequencies. In these situations, the X-rays are emitted near the inner edge of an accretion disk in which gas swirls onto a compact object such as a white dwarf, neutron star, or black hole.

QPOs were first identified in white dwarf systems and then in neutron star systems.

van der Klis et al. 1985, Nature, 316, 225 Middleditch and Priedhorsky 1986, Astrophysical Journal 306, 230

Figure 4 - Artist view of a LMXB system (http.//astro.virginia.edu)

In the context of X-ray binary systems, which often involve compact objects like neutron stars and black holes, QPOs are observed in the Xray emissions. The frequency of QPOs in these systems typically falls within the range of approximately 0.1 Hz (hertz) to several hundred hertz. The exact frequency range can vary from one binary system to another and may depend on the properties of the compact object and the accretion disk.

Origin of QPOs: Quasiperiodic oscillations, on the other hand, originate from different processes within the X-ray binary system. QPOs are often associated with variations in the X-ray emission and are thought to be linked to dynamic and complex interactions in the system, including the accretion disk and the compact object.

Modulation Frequency: The frequency of QPOs represents the characteristic timescales of the physical processes driving the variations in the X-ray emissions. These timescales can be significantly longer than the timescales associated with the production of X-rays. As a result, QPO frequencies are typically much lower, falling in the range of about 0.1 Hz to several hundred hertz.

The Relativistic Precession Model (RPM)

The RPM has been proposed in a series of papers by Stella and Vietri. *It explains the kHz QPOs as a direct manifestation of modes of relativistic epicyclic motion of blobs arising at various radii r in the inner parts of the accretion disk. The model identifies the lower and upper kHz QPOs with the periastron precession fper and Keplerian f^K frequency.*

$$
f_L = f_{per} = f_{\phi} - f_r \qquad f_U = f_{\phi} = f_K
$$

$$
\Delta f = f_U - f_L = f_r
$$

Stella, L., Vietri, M., 1999, Phys. Rev. Lett. , 82, 17.

Let us consider a generic stationary, axisymmetric, and asymptotically flat spacetime. We write the line element as

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 $\overline{1}$ model-independent analysis is reported in Fig.

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modes of ^a scal ar field (which onl y depend on the back -

ground metr ic) are not ver y differ ent from those of the

gravitational waves (which instead can only be der ived by

the field equati ons of the gravity theory) . The finding of

ref. [22] is that the observation of GW150914 cannot rule

 $\mathcal{I} \subset \mathcal{I}$. In ref . (23), one of us has discussed the constant raining the constant raining of \mathcal{I}

p ower of the iron line method by employing the same met-

r ic as in ref. [22], in or der to compare the gravitational

wave and the iron line approaches. While both the stud-

i es i n [22] and [23] are onl y prelimi nary anal yses to get an

idea of the potentialities of the two techniques, one can ar -

r ive at some inter esting conclusions. The iron line method

can potentially be quite competitive and pr ov ide str ingent

const raints. The reason is that one has to fit the whole

shape of the i ron line (actually the whol e reflected spec-

$$
ds^2 = g_{tt}dt^2 + g_{rr}dr^2 + g_{\theta\theta}d\theta^2 + 2g_{t\varphi}dtd\varphi + g_{\varphi\varphi}d\varphi^2.
$$

The metric coefficients are independent of the *t* and *φ* coordinates, leading to the existence of the conserved specific energy at infinity, *E*, and the conserved *^z*-component of the specific angul ar momentum at infinity, *L^z* . The *t*- and *φ*-component of the 4-velocity of ^a test-particle can thus be written as

$$
\dot{t} = \frac{E g_{\varphi\varphi} + L_z g_{t\varphi}}{g_{t\varphi}^2 - g_{tt}g_{\varphi\varphi}} \qquad \varphi = -\frac{E g_{t\varphi} + L_z g_{tt}}{g_{t\varphi}^2 - g_{tt}g_{\varphi\varphi}} \ . \qquad (1)
$$

From the conservation of the rest-mass, *gµν x*˙ *µ x*˙ *ν* = *[−]* 1, we have

$$
g_{rr}r^2 + g_{\theta\theta}\dot{\theta}^2 = V_{\text{eff}}(r, \theta, E, L_z), \qquad (2)
$$

where the effective potential *V*_{eff} is

r ic as in ref. [22], in or der to compare the gravitational

wave and the incident studies.

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idea of the potentialities of the two techniques, one can are \mathcal{I}

r ive at some inter esting conclusions. The iron line method

can potentially be quite competitive and pr ov ide str ingent

const raints. The reason is that one has to fit the whole

shape of the i ron line (actually the whol e reflected spec-

t rum, but most of t he informat ion on t he st rong-gravity

field i s encoded i n the i ron line), while i n the case of the

gravitational waves one has just two number s (in the case

of GW150914) associated to the frequency of the observed

quasi -normal mode. The weak poi nt of the i ron line i s the

astrophysical model, and there is not ^a common consen-

sus that the i ron line can really be used t o get precise

The ai m of this work i s t o further i nvestigate the con-

st raining capability of different techniques. We consider

the quasi-periodic oscillations ($\mathcal{O}(\mathcal{P}(\mathcal{C}))$

black hole in GRO J1655-40 and we constrain possible de-

v iations from the Kerr metr ic by employ ing the same met-

r ic as in [22,23]. Interestingly, current constraints would

b e at least comparable, but mayb e even b etter, than the

$$
V_{\text{eff}} = \frac{E^2 g_{\varphi\varphi} + 2EL_z g_{t\varphi} + L_z^2 g_{tt}}{g_{t\varphi}^2 - g_{tt} g_{\varphi\varphi}} - 1.
$$
 (3)

Circular orbits in the equatorial plane have $r = \dot{\theta} = \ddot{r} = 0.$ We write the geodesic equations as

$$
\frac{\mathrm{d}}{\mathrm{d}\lambda}\left(g_{\mu\nu}\dot{x}^{\nu}\right)=\frac{1}{2}\left(\partial_{\mu}g_{\nu\rho}\right)\dot{x}^{\nu}\dot{x}^{\rho},\tag{4}
$$

and we consider the radial component (namely $\mu = r$)

$$
(\partial_r g_{tt}) t^2 + 2(\partial_r g_{t\varphi}) t\varphi + (\partial_r g_{\varphi\varphi}) \varphi^2 = 0.
$$
 (5)

From eq. (5) we obtain the orbital angular velocity $\Omega_{\varphi} = \varphi / t$

st raining capability of different techniques. We consider

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 $\mathbb{E}[Y]$ in Group

 $\sqrt{2\pi}$

r ic as in [22,23]. Interestingly, current constraints would

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constraints from gr av itational waves and iron line. How-

ever, there i s ^a st rong parameter degeneracy. Even if we

assume to have much better measur ements in the futur e,

i t i s difficul t to break the parameter degeneracy wi thout

T est ing t he K err metric wit h QPOs. – QPOs are

a common feature in the X-ray power density spectrum

of black-hole binar ies [24,25]. Ther e are several types of

QPOs. Low-frequency QPOs are in the range 0.1–30Hz

and are divided into ty pe-A , type-B, and type-C accord-

ing to their properties. Hi gh-f requencies QPOs are i n the

range [∼] 100–500Hz and some sources show an upper and

a lower high-frequency QPO with the ratio 3:2. At the

moment, there i s no common consensus on the mecha-

nism responsible for these QPOs. H owever, recent studies

$$
\Omega_{\varphi} = \frac{-\partial_r g_{t\varphi} \pm \sqrt{(\partial_r g_{t\varphi})^2 - (\partial_r g_{t\bar{t}}) (\partial_r g_{\varphi\varphi})}}{\partial_r g_{\varphi\varphi}}, \qquad (6)
$$

where the sign is + (-) for corotating (counter-rotating) orbits. The orbital fr equency is thus *^ν^φ* = Ω*φ/* ²*^π* From $g_{\mu\nu}x^{\mu}x^{\nu} = -1$ with $r = \dot{\theta} = 0$ we have

$$
\dot{t} = \frac{1}{\sqrt{-g_{tt} - 2g_{t\varphi}\Omega_{\varphi} - g_{\varphi\varphi}\Omega_{\varphi}^2}}.\tag{7}
$$

The radial and vertical epicyclic frequencies can be obtained by studying small perturbations around circular equator ial orbits. If *δ^r* and *δ^θ* are the small displacements equator ial orbits. If *δ^r* and *δ^θ* are the small displacements around the mean orbit (*i.e.*, $r = r_0 + \delta_r$ and $\theta = \pi/2 + \delta_\theta$), they are governed by the following differential equations:

$$
\frac{d^2 \delta_r}{dt^2} + \Omega_r^2 \delta_r = 0, \quad \frac{d^2 \delta_\theta}{dt^2} + \Omega_\theta^2 \delta_\theta = 0, \quad (10)
$$

where $m_{\rm H}$

$$
\Omega_r^2 = -\frac{1}{2g_{rr}\dot{t}^2}\frac{\partial^2 V_{\text{eff}}}{\partial r^2}, \quad \Omega_\theta^2 = -\frac{1}{2g_{\theta\theta}\dot{t}^2}\frac{\partial^2 V_{\text{eff}}}{\partial \theta^2}.
$$
 (11)

 Γ he radial epicyclic frequency is $\nu = \Omega / 2\pi$ The vertical epicyclic f *requency* is *ν*_θ = Ω*/* 2*π* $P₁$ be *n* periodicity $P₀$ *γ*_p $P₂$ *γ*_p $P₃$ *ν*_p $P₄$ *γ*_p $P₅$ *γ*_p $P₆$ *γ*_p $P₇$ *γ*_p $P₈$ *γ*_p $P₉$ *γ*_p $P₉$ *γ*_p $P₉$ The radi al epicyclic f requency i ^s *^ν^r* = Ω*^r /* 2*π*. Thevertical epicyclic f requency i ^s *^ν^θ* = Ω*θ/* 2*π*.

 $f_L = f_{per} = f_{\phi} - f_r$ $f_U = f_{\phi} = f$ $\mu - yr$ $L = Jper = J\varphi$ $Jr = J\varphi = Jr$.

Figure 5 - QPOs from LMXBs

Figure 6 - QPOs from GX 5-1

Figure 7 – Contours plots of the best-fit parameters (black dots) and the associated 1–σ (dark gray) and 2–σ (light gray) confidence regions of the source GX 5-1.

Figure 8 – Contours plots of the best-fit parameters (black dots) and the associated 1–σ (dark gray) and 2–σ (light gray) confidence regions of the source Cir X1.

Figure 9 - The mass in solar masses (M/M⊙), the dimensionless angular momentum j and quadrupole moment q as a function of the central density of a maximally rotating neutron star. Solid curves indicate GCN and dashed curves indicate LCN cases.

Conclusion and prospects

- On the basis of the RPM using the QPOs data of GX 5-1 and Cir X1, we inferred the mass, angular momentum and quadrupole moment of the source with error bars (from observation).
- From the neutron star model of Belvedere et. al. (2012, 2014) we derived the rest parameters of the source such as radius, angular velocity (frequency) etc. of the neutron star (from theory).
- Different models for different sources.
- arXiv:2303.03248, arXiv:2212.10186.

Work in progress…

Thank you for your time and attention!

Figure 2. Power density spectrum showing quasi-periodic oscillations (QPOs) in the black hole X-ray binary XTE J1550-564. Credit: Motta et al. 2018.

Figure 2: A detailed view of the kilohertz QPO in Sco X-1.