hep-ph/0601083
(hep-ph/0605...)

Flavour matters in Leptogenesis

claim: single-flavour approximation (usual Boltzmann Eqns for total lepton number) is unreliable

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1. review: thermal leptogenesis from N_1 decay, $M_1 \ll M_{2,3}$, flavour by flavour
   ? but Boltz. Eqns obtained depend on how sum lepton flavour ?

2. can not be: 1) Yukawas h_e, h_μ, h_τ are pertubatively irrelevant correction
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3. solution: matrix Boltz. Eqns in flavour space; trace $\rightarrow$ eqn for total L number
   but *terms in equation* depends on basis

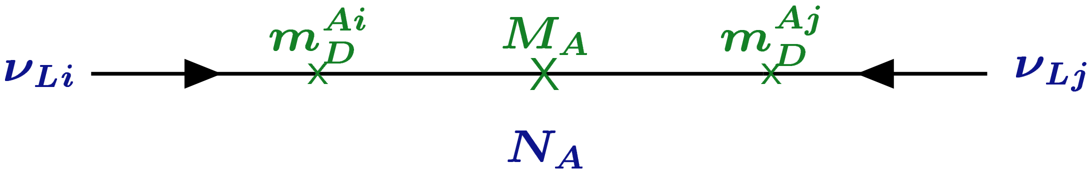
4. $\Rightarrow$ additional terms, numerical factors ($1 \rightarrow 1/3$) in the Boltzmann eqns

- phenomenology $\begin{array}{c}\text{no leptogenesis bound on neutrino mass scale}\\ \text{lower bound on } M_1 \text{ decreased}(? \sim 3)\end{array}$
- models: baryon asym can change by orders of magnitude, in specific models

The See-Saw

- in the charged lepton ("flavour") and $N(=\nu_R)$ mass bases, at large energy scale $\gg M_i$:

$$\mathcal{L} \quad = \quad \mathcal{L}_{SM} + \lambda_{Jk}\overline{N}_J \ell_k \cdot H_u - \frac{1}{2}\overline{N_J} M_J N_J^c$$



- for $M_1 \ll M_{2,3}$, effective theory of SM + N_1 at leptogenesis scale $\sim M_1$:

$$\mathcal{L} \quad = \mathcal{L}_{SM} + \lambda_{1j}\overline{N}_1 \ell_j \cdot H_u - \frac{1}{2}\overline{N_1} M_1 N_1^c - \kappa_{kn}\ell_k \cdot H_u \ell_n \cdot H_u$$

where $\kappa_{kn} = \sum_J \lambda_{Jk} {M_J}^{-1} \lambda_{Jn}$, $\quad J = 2$ (2RHN model) or $J = 2, 3$

- at the weak scale, get effective light neutrino mass matrix

$$[m_\nu] \quad \propto \quad \lambda^T M^{-1} \lambda \langle H_u^0 \rangle^2$$

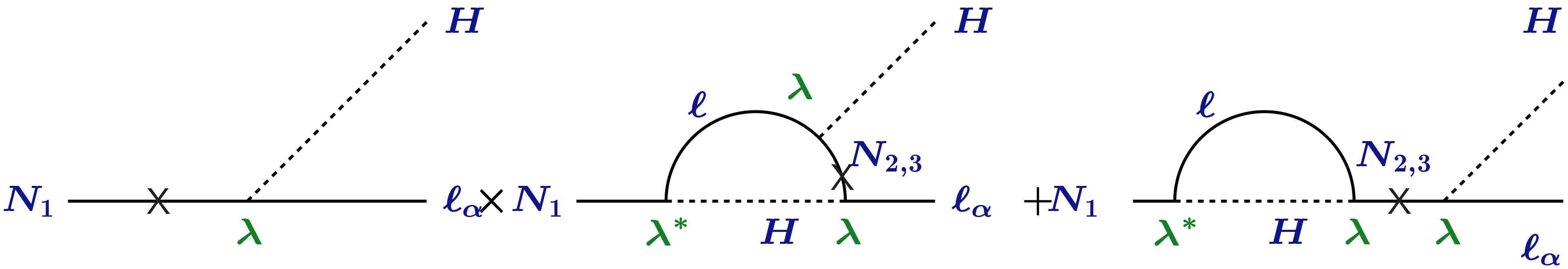
unflavoured: Fukugita Yanagida
Buchmuller et al
Covi et al
Branco et al
Giudice et al
...

Making the Baryon Asymmetry by Thermal Leptogenesis

Suppose that $M_1 \lesssim 10^9$ GeV $\ll M_2, M_3$, consider lepton flavours α one at a time

- the lightest N_1 produced in the thermal soup after inflation by scatterings $(qt^c \to N\ell_\alpha)$ and inverse decays $(H\ell_\alpha \to N)$. At $T \lesssim M_1$, N_1 decays.
- generate lepton asymmetry in flavour α due to $C\not{P}$ in N_1 interactions: (including scattering)

$$-\epsilon^{\alpha\alpha} = \frac{\Gamma(\bar{N}_1 \to \bar{H}\bar{\ell}_\alpha) - \Gamma(N_1 \to H\ell_\alpha)}{\Gamma(N \to H\ell) + \Gamma(\bar{N}_1 \to \bar{H}\bar{\ell})}$$



- non-perturbative SM processes: lepton asymmetry $\to$ baryon asymmetry

with flavour: Barbieri etal
Endoh etal
Pilaftsis Underwood
Vives
Abada etal
Nardi etal

Making the Baryon Asymmetry by Thermal Leptogenesis

- the lightest N_1 is produced in the thermal soup after inflation by scatterings ($qt^c \to N\ell_\alpha$) and inverse decays ($H\ell_\alpha \to N$). At $T \lesssim M_1$ N_1 decays.
- CP violation in N_1 interactions : $-\epsilon = \frac{\Gamma(\bar{N}_1 \to \bar{H}\bar{\ell_\alpha}) - \Gamma(N_1 \to H\ell_\alpha)}{\Gamma(N \to H\ell) + \Gamma(\bar{N}_1 \to \bar{H}\bar{\ell})} \to$ asym. in $n_{\ell_\alpha} - n_{\bar{\ell}_\alpha}$
- non-perturbative SM processes: lepton asymmetry $\to$ baryon asymmetry

described by Boltzmann Eqns for N_1 and lepton asymmetry number densities ($z = \frac{M_1}{T}$ is time var.):

$$\frac{dY_N}{dz} = -\frac{z}{sH}\left(\frac{Y_N}{Y_N^{eq}} - 1\right)\gamma$$

$Y_N = n_N/s$ (s = entropy density) pushed to equilibrium by $\gamma(z)$, which describes thermally averaged N decays (D), scatterings (S), inverse decays(ID). parametrise strength of rates by $\tilde{m}^{\alpha\alpha} \propto |\lambda_{1\alpha}|^2$

$$\frac{d}{dz}\frac{n_{\ell_\alpha} - n_{\bar{\ell}_\alpha}}{s} \equiv \frac{dY_L^{\alpha\alpha}}{dz} = \frac{z}{sH(M)}\left[\left(\frac{Y_N}{Y_N^{eq}} - 1\right)\epsilon^{\alpha\alpha}\gamma - \gamma^{\alpha\alpha}\frac{Y_L^{\alpha\alpha}}{Y_\ell^{eq}}\right]$$

asym $Y_L^{\alpha\alpha}$ produced by out-of-equilibrium Ns, washed out by same processes D, ID, S.
NB tension: $\gamma \gg$ to ensure $Y_N \sim Y_N^{EQ}$ at $T \gtrsim M_1$. But $\gamma \ll$ to minimise washout of $Y_L^{\alpha\alpha}$.

(for afictionados: calculate matrix elements in $T = 0$ field theory. include $\not{C}$P in N scattering.)

So Boltzmann Eqns for total lepton number are...

Recall that for $Y_L^{\alpha\alpha} = (n_{\ell_\alpha} - n_{\overline{\ell}_\alpha})/s$

$$\frac{d}{dz}Y_L^{ee} = \frac{z}{sH}\left[\gamma\left(\frac{Y_{N_1}}{Y_{N_1}^{eq}} - 1\right)\epsilon^{ee} - \frac{Y_L^{ee}}{Y_\ell^{eq}}\gamma^{ee}\right], \quad \text{with } \epsilon^{\alpha\alpha} = \frac{\Gamma_{N\to H\ell_\alpha} - \Gamma_{N\to\overline{H\ell_\alpha}}}{\Gamma_{N\to H\ell} + \Gamma_{N\to\overline{H\ell}}}$$

Sum up the flavoured BEs:

$$\sum_\alpha \frac{d}{dz}Y_L^{\alpha\alpha} = \frac{z}{sH}\left[\gamma\left(\frac{Y_{N_1}}{Y_{N_1}^{eq}} - 1\right)\underbrace{(\epsilon^{ee} + \epsilon^{\mu\mu} + \epsilon^{\tau\tau})}_{\epsilon} - \frac{1}{Y_\ell^{eq}}\underbrace{(\gamma^{ee}Y_L^{ee} + \gamma^{\mu\mu}Y_L^{\mu\mu} + \gamma^{\tau\tau}Y_L^{\tau\tau})}_{\neq\gamma Y_L}\right]$$

$$= \frac{z}{sH}\left[\left(\frac{Y_{N_1}}{Y_{N_1}^{eq}} - 1\right)\epsilon\gamma - \frac{\gamma^{ee}Y_L^{ee} + \gamma^{\mu\mu}Y_L^{\mu\mu} + \gamma^{\tau\tau}Y_L^{\tau\tau}}{Y_\ell^{eq}}\right]$$

Compare to the usual "single flavour" approx= consider lepton number, neglect flavour

$$\frac{dY_L}{dz} = \frac{z}{sH(M)}\left[\left(\frac{Y_N}{Y_N^{eq}} - 1\right)\epsilon\gamma - (\gamma^{ee} + \gamma^{\mu\mu} + \gamma^{\tau\tau})\frac{Y_L^{ee} + Y_L^{\mu\mu} + Y_L^{\tau\tau}}{Y_\ell^{eq}}\right]$$

NOT the same.

Abada etal
Nardi etal

where $z = \frac{M_1}{T}$ is a time var, $\gamma^{\alpha\alpha} = \gamma_D^{\alpha\alpha} + \gamma_{\Delta L=1}^{\alpha\alpha}$ is $N \leftrightarrow H\ell_\alpha$ and $qt^c \leftrightarrow N\ell_\alpha$ etc rates

Flavour is irrelevant—its obvious. Or not?

1. Production, decay of N_1 are controlled by λ;
   charged lepton Yukawas h_e, h_μ, h_τ are $\ll 1$, a small correction in perturbation theory (true)
   (ex: consider the far out-of-equilibrium decay of primordial thermal N abundance: make ϵ leptons per N_1, and h_α do not contribute to ϵ (true))
2. physics is basis-independent, can calculate lepton asym in the flavour combination into which N_1 decays (umm. not quite.)

So how can flavour matter? ?? ...its not a perturbative correction...

Flavour defines distinguishable mass eigenstates, if the h_τ, h_μ are in equilibrium

$$\Gamma_\tau \simeq 10^{-2} h_\tau^2 T > H \text{ for } T < 10^{12} \text{ GeV}, \qquad \Gamma_\mu > H \text{ for } T < 10^9 \text{ GeV}$$

So in "quantum mechanics", must work in mass basis, ($\leftrightarrow$ sum probabilities not amplitudes).

Will see: in any basis *other* than the flavour basis, h_α contribute additional fast terms to the Boltzmann Eqns.

Eqns for $Y_L^{\alpha\beta}$ in field theory?

Should be able to do Field Theory in any basis we like. Consider eqns of motion for

$$Y_L^{\alpha\beta} \propto \frac{\int d^3k \langle a^\dagger_{\ell_\alpha} a_{\ell_\beta} - a^\dagger_{\bar{\ell}_\beta} a_{\bar{\ell}_\alpha}\rangle}{s}$$

in flavour basis, diagonal elements are $(n_{\ell_\alpha} - n_{\bar{\ell}_\alpha})/s$, off-diagonals encode quantum correlations, and are driven rapidly to zero by charged lepton yukawas when these are in equilibrium.
So at $T \lesssim 10^9$ GeV, in flavour basis, and *only* in flavour basis, the Boltzmann Eqns are:

$$\frac{sH(M_1)}{z}\frac{d}{dz}\begin{bmatrix} Y_L^{ee} & \cdot & \cdot \\ \cdot & Y_L^{\mu\mu} & \cdot \\ \cdot & \cdot & Y_L^{\tau\tau} \end{bmatrix} = \gamma\left(\frac{Y_{N_1}}{Y_{N_1}^{eq}} - 1\right)\begin{bmatrix} \epsilon^{ee} & \cdot & \cdot \\ \cdot & \epsilon^{\mu\mu} & \cdot \\ \cdot & \cdot & \epsilon^{\tau\tau} \end{bmatrix}$$
$$-\frac{1}{Y_L^{eq}}\begin{bmatrix} \gamma^{ee} & \cdot & \cdot \\ \cdot & \gamma^{\mu\mu} & \cdot \\ \cdot & \cdot & \gamma^{\tau\tau} \end{bmatrix}\begin{bmatrix} Y_L^{ee} & \cdot & \cdot \\ \cdot & Y_L^{\mu\mu} & \cdot \\ \cdot & \cdot & Y_L^{\tau\tau} \end{bmatrix}$$

Same as previous flavoured. Take Trace to get eqn for total Y_L—take it in any basis? (*eg* direction into which N_1 decays? This give the usual BEs) **NO**. Dropped from BE $\sim -|h_\alpha|^2 T Y_L^{\alpha\beta}$ terms that drive off-diagonals to zero.

outline (again)

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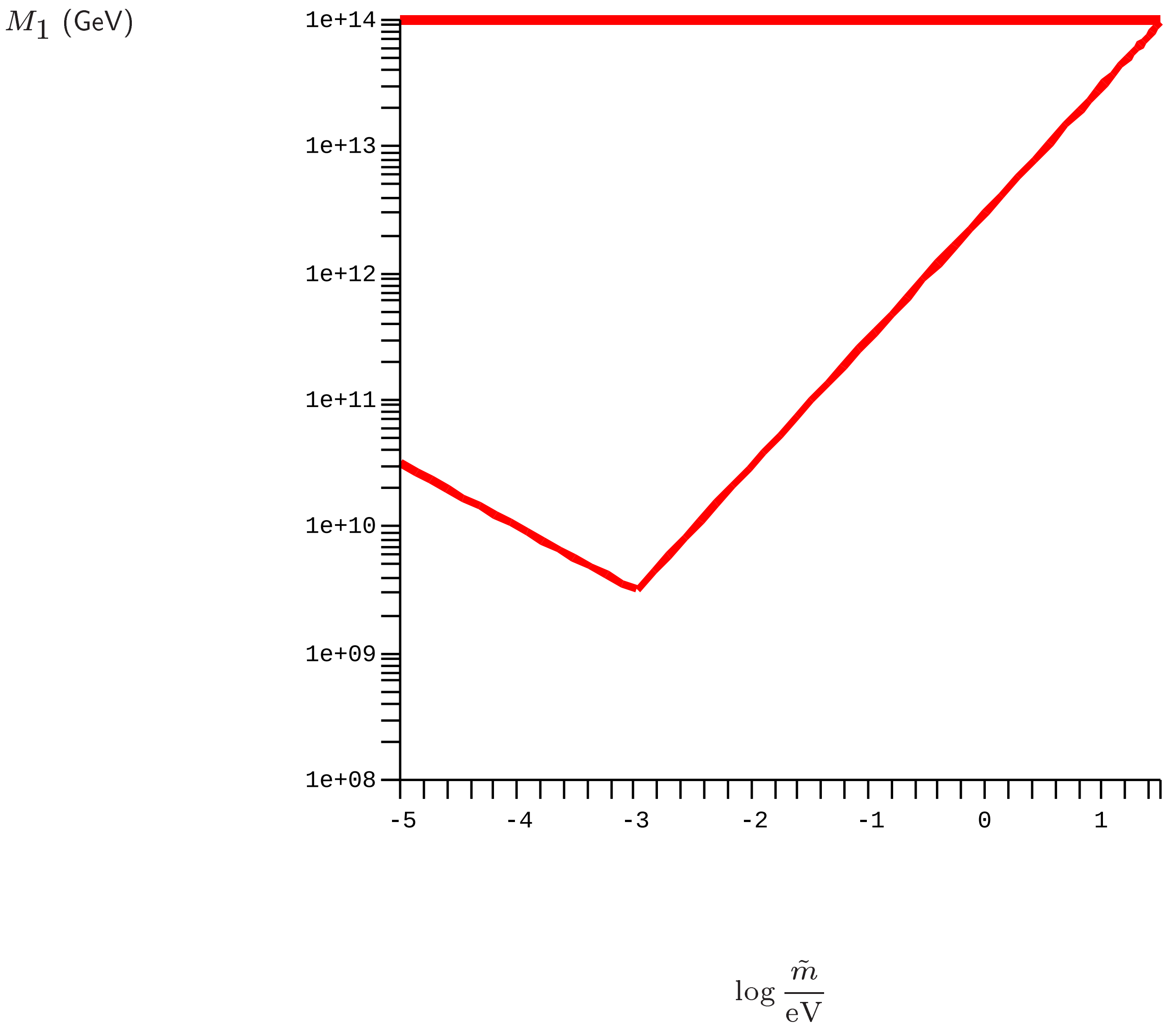
phenomenological consequences

There is an envelope, in space of parameters leptogenesis depends on : M_1, $\tilde{m}$, ϵ, where leptogenesis *can* work.



real plot in
Buchmuller, diBari,Plumacher
Giudice et al

$\tilde{m} \propto \Gamma(N \to HL)$

pheno consequences 1 - $C\not{P}$ and the bound on m_ν

$C\not{P}$ delicate — more symmetries, fewer phases. $\Rightarrow$?? with flavour, more $C\not{P}$??

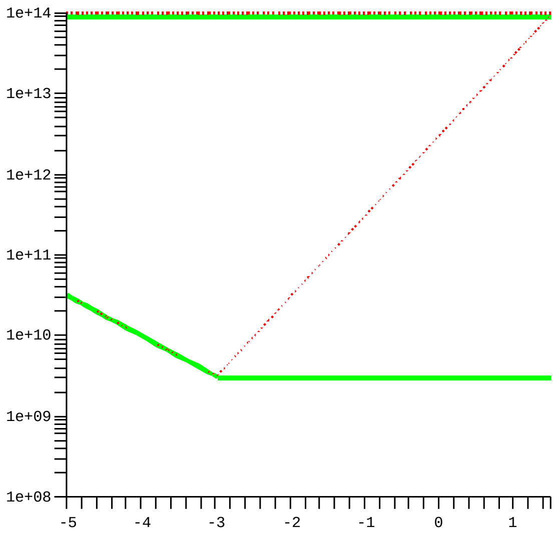
known that total ϵ decreases for degenerate light m_ν:

$$\sum_\alpha \epsilon^{\alpha\alpha} \leq \frac{3M_1 \Delta m^2_{atm}}{8\pi v^2 m_{max}}$$

but individual flavour asyms can grow with light ν mass scale m_{max}:

$$\epsilon^{\alpha\alpha} \leq \frac{3M_1 m_{max}}{8\pi v^2}$$

$\Rightarrow$ no leptogenesis bound $m_{max} \lesssim .1$ eV
(leptogenesis can work between green lines
compensate strong washout (large $\tilde{m}^{\alpha\alpha}$) by larger CP asyms)



where did the bound go?

in "single flavour" approx, successful thermal leptogenesis $\Rightarrow m_\nu \leq .1$ because:

In strong washout (generic situation), can approximate

$$Y_B = \frac{\text{baryon excess}}{\text{entropy}} \simeq 10^{-3}\epsilon \frac{m_* = 3 \times 10^{-3} eV}{\tilde{m}}, \qquad \tilde{m} = \frac{8\pi\Gamma(N \to H\ell)v^2}{M_1^2}$$

$$\epsilon \leq \frac{3M_1 \Delta m^2_{atm}}{8\pi v^2 m_{max}} \qquad m_{max} = \text{heaviest } \nu_L \text{ mass}$$

if $m_{max} > m_{atm}$ grows, raise M_1 (or decrease $\tilde{m} \geq m_{max}$) to keep Y_B constant.

but M_1 up, $\Rightarrow$ leptogenesis temperature up, eventually $\Gamma(\Delta L = 2) > H$

with flavour

$$\epsilon^{\alpha\alpha} \leq \frac{3M_1 m_{max}}{8\pi v^2} \qquad Y_B \sim 10^{-3} \sum_\alpha \epsilon^{\alpha\alpha} \frac{m_*}{\widetilde{m}^{\alpha\alpha}}$$

m_{max} grows, $\Rightarrow \epsilon^{\alpha\alpha}$ grows, $\tilde{m}^{\alpha\alpha}$ grows, $\Rightarrow Y_B$ constant

pheno consequences 2 - mild decrease in min M_1

There is an envelope, in space of parameters leptogenesis depends on $(M_1,\ \tilde{m},\ \epsilon ...)$ where leptogenesis *can* work. Flavour effects change this envelope.

The lower bound on M_1 decreases ($\leftrightarrow$ leptogenesis works at lower reheat temp):

ex: $\epsilon^{\alpha\alpha} = \epsilon/3$, $\gamma^{\alpha\alpha} = \gamma/3$, for all flavours. Strong washout, non-degen m_ν

Recall:summed flavour $\neq$ usual "single flavour" eqn

$$\frac{dY_L}{dz} = \frac{z}{sH}\left[\left(\frac{Y_{N_1}}{Y^{eq}_{N_1}} - 1\right)\epsilon\gamma - \frac{\gamma^{ee}Y^{ee}_L + \gamma^{\mu\mu}Y^{\mu\mu}_L + \gamma^{\tau\tau}Y^{\tau\tau}_L}{Y^{eq}_\ell}\right] \quad \neq \quad \frac{z}{sH(M)}\left[\left(\frac{Y_N}{Y^{eq}_N} - 1\right)\epsilon\gamma - \gamma\frac{Y_L}{Y^{eq}_\ell}\right]$$

$$= \quad \frac{z}{sH(M)}\left[\left(\frac{Y_N}{Y^{eq}_N} - 1\right)\epsilon\gamma - \frac{\gamma}{3}\frac{Y_L}{Y^{eq}_\ell}\right]$$

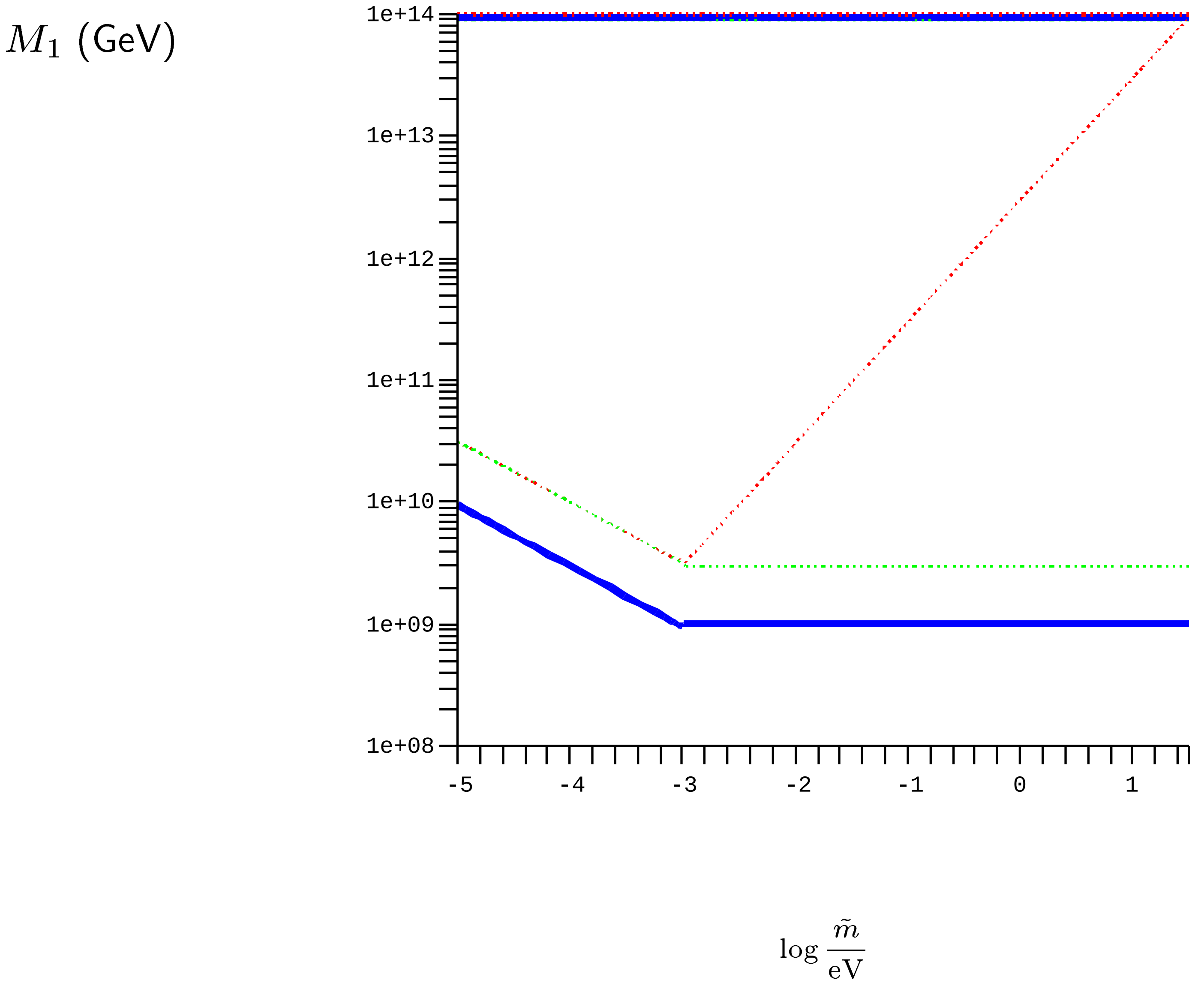
Analytic approximations to final baryon asym (strong washout $\Gamma^{\alpha\alpha} \gg H$):

$$\text{single flavour}: \ Y_B \quad \propto \quad \frac{\epsilon}{\Gamma_D} \simeq 10^{-3}\epsilon\frac{m_*}{\tilde{m}}$$

$$\text{summed flavours}: \ Y_B \quad \propto \quad \left(\frac{\epsilon^{ee}}{\Gamma^{ee}_D} + \frac{\epsilon^{\mu\mu}}{\Gamma^{\mu\mu}_D} + \frac{\epsilon^{\tau\tau}}{\Gamma^{\tau\tau}_D}\right) \simeq 3 \times 10^{-3}\epsilon\frac{m_*}{\tilde{m}}$$

$\Rightarrow$ leptogenesis can work for $\epsilon \leq 3M_1\Delta m^2_{atm}/(8\pi v^2 m_{max})$ a factor of 3 *smaller*, so M_1 a factor of 3 smaller.

pheno allowed parameter space (for maximal $\epsilon^{\alpha\alpha}$)



consequences for models...to estimate Y_B

$$\text{“Usual'' analytic approximations :} \quad Y_B \simeq \begin{cases} 10^{-3}\epsilon\frac{m_*}{\tilde{m}} & \tilde{m} > m_* \simeq 3 \times 10^{-3} eV \\ \\ 10^{-3}\epsilon\frac{\tilde{m}}{m_*} & \tilde{m} < m_* \Leftrightarrow \Gamma < H \end{cases}$$

$$\Rightarrow \text{ calculate} \qquad \epsilon = \frac{\Gamma(N_1 \to H\ell) - \Gamma(\bar{N}_1 \to \bar{H}\bar{\ell})}{\Gamma(N \to H\ell) + \Gamma(\bar{N}_1 \to \bar{H}\bar{\ell})} \qquad \tilde{m} = \frac{8\pi\Gamma(N \to H\ell)v^2}{M_1^2}$$

model predictions with flavour : ϵ^{ee}, $\epsilon^{\mu\mu}$, $\epsilon^{\tau\tau}$, $\widetilde{m}^{ee}$, $\widetilde{m}^{\mu\mu}$, $\widetilde{m}^{\tau\tau}$

1. strong washout all flavours: $\widetilde{m}^{\alpha\alpha} > m_*$

$$Y_B \simeq \quad 10^{-3} \sum_\alpha \epsilon^{\alpha\alpha} \frac{m_*}{\widetilde{m}^{\alpha\alpha}}$$

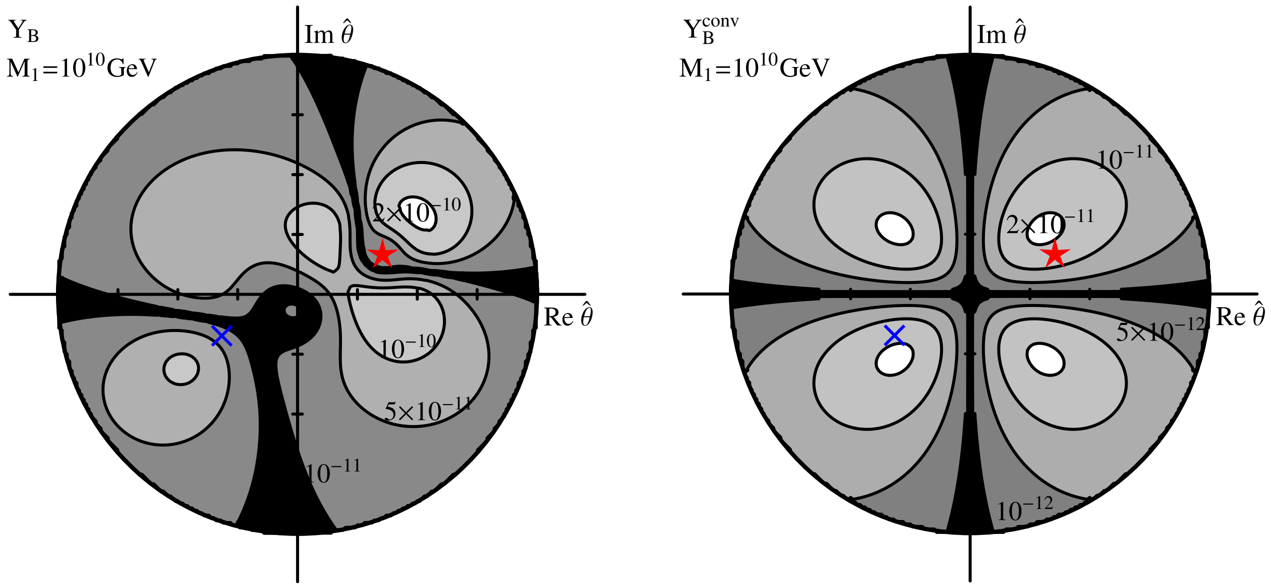
2. weak washout all flavours: $\widetilde{m}^{\alpha\alpha} < m_*$

$$Y_B \simeq 5 \times 10^{-3} \frac{\tilde{m}}{m_*} \sum_\alpha \epsilon^{\alpha\alpha} \frac{\tilde{m}^{\alpha\alpha}}{m_*} \qquad (Y_N \propto \tilde{m}; \quad C\not{P} \text{ in scatt.} \to \epsilon^{\alpha\alpha}\widetilde{m}^{\alpha\alpha})$$

3. weak washout in flavours: α , strong in β

$$Y_B \simeq \; 10^{-3} \frac{\tilde{m}^{\alpha\alpha}}{m_*} \epsilon^{\alpha\alpha} + 5 \times 10^{-3} \epsilon^{\beta\beta} \frac{m_*}{\widetilde{m}^{\beta\beta}}$$

useful factors of 10...



Y_B in the 2RHN model, hierarchical ν_L.
LHS flavoured leptogenesis, RHS "single flavour approx".
(1,2) texture zero   (1,3) texture zero
(in plane of complex angle of R matrix)

Summary

thermal leptogenesis is an attractive, minimal mechanism to make the Baryon asymmetry of the Universe. The baryon to entropy ratio Y_B can be calculated with Boltzmann Eqns (BE).

When the interaction rates of the charged lepton Yukawas are faster than the leptogenesis rates in the BE, the charged Yukawas should be "integrated out". $\Rightarrow$ work in flavour basis.

Resulting "flavoured" BE are different, and the solutions are different:

- allowed phenomenological parameter space is enlarged
  1) leptogenesis works for degenerate light neutrinos
  2) minimum T_{reheat} reduced by ~ 3(hier. ν_L) to ~ 10(degen ν_L)

- Y_B obtained in specific models can be (?is?) enhanced
  1) flavoured $\epsilon^{\alpha\alpha}$ can be bigger than ϵ
  2) cancellations reduced: $Y_B \propto \sum_\alpha \epsilon^{\alpha\alpha} f(\tilde{m}^{\alpha\alpha})$