Flavour matters in Leptogenesis

claim: single-flavour approximation (usual Boltzmann Eqns for total lepton number) is unreliable

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1. review: thermal leptogenesis from $N_1$ decay, $M_1 \ll M_{2,3}$, flavour by flavour? but Boltz. Eqns obtained depend on how sum lepton flavour?

2. can not be: 1) Yukawas $h_e, h_\mu, h_\tau$ are pertubatively irrelevant correction 2) physics is basis independent

3. solution: matrix Boltz. Eqns in flavour space; trace $\to$ eqn for total $L$ number but terms in equation depends on basis

4. $\Rightarrow$ additional terms, numerical factors ($1 \rightarrow 1/3$) in the Boltzmann eqns
   • phenomenology no leptogenesis bound on neutrino mass scale lower bound on $M_1$ decreased(? $\sim 3$)
   • models: baryon asym can change by orders of magnitude, in specific models
The See-Saw

• in the charged lepton ("flavour") and $\mathcal{N}(=\nu_R)$ mass bases, at large energy scale $\gg M_i$:

$$\mathcal{L} = \mathcal{L}_{SM} + \lambda_{jk} \bar{N}_j \ell_k \cdot H_u - \frac{1}{2} \bar{N}_j M_j N^c_j$$

$\nu_{Li} \rightarrow m^{Ai}_{PD} \rightarrow M_A \rightarrow m^{Aj}_{PD} \rightarrow \nu_{Lj}$

$N_A$

• for $M_1 \ll M_{2,3}$, effective theory of SM + $N_1$ at leptogenesis scale $\sim M_1$:

$$\mathcal{L} = \mathcal{L}_{SM} + \lambda_{1j} \bar{N}_1 \ell_j \cdot H_u - \frac{1}{2} \bar{N}_1 M_1 N^c_1 - \kappa_{kn} \ell_k \cdot H_u \ell_n \cdot H_u$$

where $\kappa_{kn} = \sum_J \lambda_{jk} M_j^{-1} \lambda_{jn}$, $J = 2$ (2RHN model) or $J = 2, 3$

• at the weak scale, get effective light neutrino mass matrix

$$[m_\nu] \propto \lambda^T M^{-1} \lambda \langle H_u^0 \rangle^2$$
Making the Baryon Asymmetry by Thermal Leptogenesis

Suppose that $M_1 \lesssim 10^9$ GeV $\ll M_2, M_3$, consider lepton flavours $\alpha$ one at a time

- the lightest $N_1$ produced in the thermal soup after inflation by scatterings ($qt^c \rightarrow N\ell_\alpha$) and inverse decays ($H\ell_\alpha \rightarrow N$). At $T \lesssim M_1$, $N_1$ decays.
- generate lepton asymmetry in flavour $\alpha$ due to $CP$ in $N_1$ interactions: (including scattering)

\[
-\epsilon_{\alpha\alpha} = \frac{\Gamma(N_1 \rightarrow H\bar{\ell}_\alpha) - \Gamma(N_1 \rightarrow H\ell_\alpha)}{\Gamma(N \rightarrow H\ell) + \Gamma(N_1 \rightarrow \bar{H}\ell)}
\]

- non-perturbative SM processes: lepton asymmetry $\rightarrow$ baryon asymmetry
Making the Baryon Asymmetry by Thermal Leptogenesis

- the lightest $N_1$ is produced in the thermal soup after inflation by scatterings ($qt^c \rightarrow N\ell_\alpha$) and inverse decays ($H\ell_\alpha \rightarrow N$). At $T \lesssim M_1$, $N_1$ decays.

- CP violation in $N_1$ interactions: $-\varepsilon = \frac{\Gamma(N_1\rightarrow H\ell_\alpha) - \Gamma(N_1\rightarrow H\ell\bar{\alpha})}{\Gamma(N\rightarrow H\ell) + \Gamma(N_1\rightarrow H\ell)} \rightarrow$ asym. in $n_{\ell\alpha} - n_{\ell\bar{\alpha}}$

- non-perturbative SM processes: lepton asymmetry $\rightarrow$ baryon asymmetry

described by Boltzmann Eqns for $N_1$ and lepton asymmetry number densities ($z = \frac{M_1}{T}$ is time var.):

$$\frac{dY_N}{dz} = -\frac{z}{sH} \left( \frac{Y_N}{Y_N^{eq}} - 1 \right) \gamma$$

$Y_N = n_N / s$ ($s =$ entropy density) pushed to equilibrium by $\gamma(z)$, which describes thermally averaged $N$ decays (D), scatterings (S), inverse decays(ID). parametrise strength of rates by $\tilde{m}^{\alpha\alpha} \propto |\lambda_{1\alpha}|^2$

$$\frac{d}{dz} \left( \frac{n_{\ell\alpha} - n_{\ell\bar{\alpha}}}{s} \right) = \frac{dY_{L}^{\alpha\alpha}}{dz} = \frac{z}{sH(M)} \left[ \left( \frac{Y_N}{Y_N^{eq}} - 1 \right) \varepsilon^{\alpha\alpha} \gamma - \gamma^{\alpha\alpha} \frac{Y_L^{\alpha\alpha}}{Y_{\ell}^{eq}} \right]$$

asym $Y_{L}^{\alpha\alpha}$ produced by out-of-equilibrium $N$s, washed out by same processes D, ID, S.

NB tension: $\gamma \gg$ to ensure $Y_N \sim Y_N^{EQ}$ at $T \gtrsim M_1$. But $\gamma \ll$ to minimise washout of $Y_L^{\alpha\alpha}$.

(for aficionados: calculate matrix elements in $T = 0$ field theory. include CP in $N$ scattering.)
So Boltzmann Eqns for total lepton number are...

Recall that for $Y_{\ell\alpha} = (n_{\ell\alpha} - n_{\ell\alpha}\bar{\alpha}) / s$

$$\frac{d}{dz} Y_{\ell\alpha}^{ee} = \frac{z}{sH} \left[ \gamma \left( \frac{Y_{N_1}}{Y_{eq}} - 1 \right) \epsilon^{ee} - \frac{Y_{\ell\alpha}^{ee}}{Y_{eq}^{ee}} \gamma^{ee} \right], \quad \text{with } \epsilon^{\alpha\alpha} = \frac{\Gamma_{N \rightarrow H \ell_\alpha} - \Gamma_{N \rightarrow H \bar{\ell}_\alpha}}{\Gamma_{N \rightarrow H \ell} + \Gamma_{N \rightarrow H \bar{\ell}}}
$$

Sum up the flavoured BEs:

$$\sum_{\alpha} \frac{d}{dz} Y_{\ell\alpha}^{\alpha\alpha} = \frac{z}{sH} \left[ \gamma \left( \frac{Y_{N_1}}{Y_{eq}} - 1 \right) \left( \epsilon^{ee} + \epsilon^{\mu\mu} + \epsilon^{\tau\tau} \right) - \frac{1}{Y_{\ell\ell}^{eq}} \left( \gamma^{ee} Y_{\ell\ell}^{ee} + \gamma^{\mu\mu} Y_{\ell\ell}^{\mu\mu} + \gamma^{\tau\tau} Y_{\ell\ell}^{\tau\tau} \right) \right]$$

$$= \frac{z}{sH} \left[ \left( \frac{Y_{N_1}}{Y_{eq}} - 1 \right) \epsilon\gamma - \frac{\gamma^{ee} Y_{\ell\ell}^{ee} + \gamma^{\mu\mu} Y_{\ell\ell}^{\mu\mu} + \gamma^{\tau\tau} Y_{\ell\ell}^{\tau\tau}}{Y_{eq}^{\ell\ell}} \right]$$

Compare to the usual "single flavour" approx= consider lepton number, neglect flavour

$$\frac{dY_{\ell\alpha}}{dz} = \frac{z}{sH(M)} \left[ \left( \frac{Y_{N}}{Y_{eq}} - 1 \right) \epsilon\gamma - \left( \gamma^{ee} + \gamma^{\mu\mu} + \gamma^{\tau\tau} \right) \frac{Y_{\ell\ell}^{ee} + Y_{\ell\ell}^{\mu\mu} + Y_{\ell\ell}^{\tau\tau}}{Y_{eq}^{\ell\ell}} \right]$$

NOT the same.

where $z = \frac{M}{T}$ is a time var, $\gamma^{\alpha\alpha} = \gamma^D_{\alpha\alpha} + \gamma_{\Delta L=1}$ is $N \leftrightarrow H \ell_\alpha$ and $qt^c \leftrightarrow N \ell_\alpha$ etc rates

Abada etal
Nardi etal
Flavour is irrelevant—its obvious. Or not?

1. Production, decay of $N_1$ are controlled by $\lambda$; charged lepton Yukawas $h_e, h_\mu, h_\tau$ are $\ll 1$, a small correction in perturbation theory \textcolor{red}{(true)} (ex: consider the far out-of-equilibrium decay of primordial thermal $N$ abundance: make $\epsilon$ leptons per $N_1$, and $h_\alpha$ do not contribute to $\epsilon$ \textcolor{red}{(true)})

2. physics is basis-independent, can calculate lepton asym in the flavour combination into which $N_1$ decays \textcolor{red}{(untrue, not quite.)}

So how can flavour matter? ?? ...its not a perturbative correction...

Flavour defines distinguishable mass eigenstates, if the $h_\tau, h_\mu$ are in equilibrium

$$\Gamma_\tau \simeq 10^{-2} h_\tau^2 T > H \text{ for } T < 10^{12} \text{ GeV}, \quad \Gamma_\mu > H \text{ for } T < 10^9 \text{ GeV}$$

So in "quantum mechanics", must work in mass basis, (↔ sum probabilities not amplitudes).

Will see: in any basis other than the flavour basis, $h_\alpha$ contribute additional fast terms to the Boltzmann Eqns.
Eqns for $Y^\alpha_{L\beta}$ in field theory?

Should be able to do Field Theory in any basis we like. Consider eqns of motion for

$$Y^\alpha_{L\beta} \propto \frac{\int d^3k \langle a_{\ell\alpha}^\dagger a_{\ell'\beta} - a_{\ell'\beta}^\dagger a_{\ell\alpha} \rangle}{s}$$

in flavour basis, diagonal elements are $(n_{\ell\alpha} - n_{\ell'\alpha})/s$, off-diagonals encode quantum correlations, and are driven rapidly to zero by charged lepton yukawas when these are in equilibrium.

So at $T \lesssim 10^9$ GeV, in flavour basis, and only in flavour basis, the Boltzmann Eqns are:

$$\frac{s H(M_1)}{z} \frac{d}{dz} \begin{bmatrix} Y^{ee}_{L} & \cdot & \cdot \\ \cdot & Y^{\mu\mu}_{L} & \cdot \\ \cdot & \cdot & Y^{\tau\tau}_{L} \end{bmatrix} = \gamma \left( \frac{Y_{N_1}}{Y_{eq}} - 1 \right) \begin{bmatrix} \epsilon^{ee} \\ \cdot \\ \cdot \end{bmatrix} + \begin{bmatrix} \gamma^{ee} \\ \cdot \\ \cdot \end{bmatrix} \begin{bmatrix} Y^{ee}_{L} \\ \cdot \\ \cdot \end{bmatrix}$$

Same as previous flavoured. Take Trace to get eqn for total $Y_L$—take it in any basis? (eg direction into which $N_1$ decays? This give the usual BEs) NO. Dropped from BE $\sim -|h_\alpha|^2 T Y^{\alpha\beta}_{L}$ terms that drive off-diagonals to zero.
**outline (again)**

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phenomenological consequences

There is an envelope, in space of parameters leptogenesis depends on: $M_1$, $\tilde{m}$, $\epsilon$, where leptogenesis can work.

\[ M_1 \text{ (GeV)} \]

\[ \log \frac{\tilde{m}}{\text{eV}} \]

$\tilde{m} \propto \Gamma(N \rightarrow H L)$
Pheno consequences 1: $C_P$ and the bound on $m_\nu$

$C_P$ delicate — more symmetries, fewer phases. $\Rightarrow$ ?? with flavor, more $C_P$??

known that total $\epsilon$ decreases for degenerate light $m_\nu$.

but individual flavor asymms can grow with light $\nu$ mass scale $m_{\nu_{max}}$:

$$\sum_{\alpha} \epsilon^{\alpha} \leq \frac{3M_1 \Delta m_{atm}^2}{8 \pi^2 m_{\nu_{max}}^2}$$

$\epsilon^{\alpha} \leq \frac{3M_1 m_{\nu_{max}}}{8 \pi^2}$

$\Rightarrow$ no leptogenesis bound $m_{\nu_{max}} \lesssim 1$ eV

(leptogenesis can work between green lines to compensate strong washout by larger CP asyms)

$\sim$
where did the bound go?

in “single flavour” approx, successful thermal leptogenesis $\Rightarrow m_v \leq .1$ because:

In strong washout (generic situation), can approximate

$$Y_B = \frac{\text{baryon excess}}{\text{entropy}} \simeq 10^{-3} \epsilon \frac{m_*}{\tilde{m}} = 3 \times 10^{-3} eV,$$

$$\tilde{m} = \frac{8 \pi \Gamma(N \rightarrow H\ell)v^2}{M_1^2}$$

$$\epsilon \leq \frac{3M_1 \Delta m_{atm}^2}{8\pi v^2 m_{max}}$$

$m_{max} = \text{heaviest } \nu_L \text{ mass}$

if $m_{max} > m_{atm}$ grows, raise $M_1$ (or decrease $\tilde{m} \geq m_{max}$) to keep $Y_B$ constant.

but $M_1$ up, $\Rightarrow$ leptogenesis temperature up, eventually $\Gamma(\Delta L = 2) > H$

with flavour

$$\epsilon^{\alpha\alpha} \leq \frac{3M_1 m_{max}}{8\pi v^2} \quad Y_B \sim 10^{-3} \sum_{\alpha} \epsilon^{\alpha\alpha} \frac{m_*}{\tilde{m}^{\alpha\alpha}}$$

$m_{max}$ grows, $\Rightarrow \epsilon^{\alpha\alpha}$ grows, $\tilde{m}^{\alpha\alpha}$ grows, $\Rightarrow Y_B$ constant
pheno consequences 2 - mild decrease in min $M_1$

There is an envelope, in space of parameters leptogenesis depends on ($M_1$, $\tilde{m}$, $\epsilon$...) where leptogenesis can work. Flavour effects change this envelope.

The lower bound on $M_1$ decreases ($\leftrightarrow$ leptogenesis works at lower reheat temp):

ex: $\epsilon^{\alpha\alpha} = \epsilon / 3$, $\gamma^{\alpha\alpha} = \gamma / 3$, for all flavours. Strong washout, non-degen $m_\nu$

Recall: summed flavour $\neq$ usual "single flavour" eqn

$$\frac{dY_L}{dz} = \frac{z}{sH} \left[ \left( \frac{Y_{N_1}}{Y_{N_1}^{eq}} - 1 \right) \epsilon\gamma - \frac{\gamma^{ee}Y_{eeL} + \gamma^{\mu\mu}Y_{\mu\muL} + \gamma^{\tau\tau}Y_{\tau\tauL}}{Y_{\ell}^{eq}} \right] \neq \frac{z}{sH(M)} \left[ \left( \frac{Y_{N}}{Y_{N}^{eq}} - 1 \right) \epsilon\gamma - \frac{Y_{L}}{3Y_{\ell}^{eq}} \right]$$

$$= \frac{z}{sH(M)} \left[ \left( \frac{Y_{N}}{Y_{N}^{eq}} - 1 \right) \epsilon\gamma - \frac{\gamma Y_{L}}{3Y_{\ell}^{eq}} \right]$$

Analytic approximations to final baryon asym (strong washout $\Gamma^{\alpha\alpha} \gg H$):

**single flavour**: $Y_B \propto \frac{\epsilon}{\Gamma_D} \approx 10^{-3} \frac{\epsilon m_*}{\tilde{m}}$

**summed flavours**: $Y_B \propto \left( \frac{\epsilon^{ee}}{\Gamma_D^{ee}} + \frac{\epsilon^{\mu\mu}}{\Gamma_D^{\mu\mu}} + \frac{\epsilon^{\tau\tau}}{\Gamma_D^{\tau\tau}} \right) \approx 3 \times 10^{-3} \frac{\epsilon m_*}{\tilde{m}}$

$\Rightarrow$ leptogenesis can work for $\epsilon \leq 3M_1 \Delta m_{atm}^2 / (8\pi v^2 m_{max})$ a factor of 3 smaller, so $M_1$ a factor of 3 smaller.
$M_1$ (GeV)
consequences for models...to estimate $Y_B$

"Usual" analytic approximations:

\[
Y_B \simeq \begin{cases} 
10^{-3} \epsilon \frac{m_*}{\tilde{m}} & \tilde{m} > m_* \simeq 3 \times 10^{-3} eV \\
10^{-3} \epsilon \frac{\tilde{m}}{m_*} & \tilde{m} < m_* \Rightarrow \Gamma < H 
\end{cases}
\]

\[\Rightarrow \text{calculate} \quad \epsilon = \frac{\Gamma(N_1 \to H\ell) - \Gamma(\bar{N}_1 \to \bar{H}\ell)}{\Gamma(N \to H\ell) + \Gamma(\bar{N}_1 \to \bar{H}\ell)} \quad \tilde{m} = \frac{8 \pi \Gamma(N \to H\ell)v^2}{M_1^2}\]

model predictions with flavour: $\epsilon^{ee}, \epsilon^{\mu\mu}, \epsilon^{\tau\tau}, \tilde{m}^{ee}, \tilde{m}^{\mu\mu}, \tilde{m}^{\tau\tau}$

1. strong washout all flavours: $\tilde{m}^{\alpha\alpha} > m_*$

\[Y_B \simeq 10^{-3} \sum_\alpha \epsilon^{\alpha\alpha} \frac{m_*}{\tilde{m}^{\alpha\alpha}}\]

2. weak washout all flavours: $\tilde{m}^{\alpha\alpha} < m_*$

\[Y_B \simeq 5 \times 10^{-3} \frac{\tilde{m}}{m_*} \sum_\alpha \epsilon^{\alpha\alpha} \frac{\tilde{m}^{\alpha\alpha}}{m_*} \quad (Y_N \propto \tilde{m}; \quad CP \text{ in scatt. } \rightarrow \epsilon^{\alpha\alpha} \tilde{m}^{\alpha\alpha})\]

3. weak washout in flavours: $\alpha$, strong in $\beta$

\[Y_B \simeq 10^{-3} \frac{\tilde{m}^{\alpha\alpha}}{m_*} \epsilon^{\alpha\alpha} + 5 \times 10^{-3} \epsilon^{\beta\beta} \frac{m_*}{\tilde{m}^{\beta\beta}}\]
useful factors of 10...

$Y_B$ in the 2RHN model, hierarchical $\nu_L$.
LHS flavoured leptogenesis, RHS "single flavour approx".
(1,2) texture zero    (1,3) texture zero
(in plane of complex angle of R matrix)
Summary

thermal leptogenesis is an attractive, minimal mechanism to make the Baryon asymmetry of the Universe. The baryon to entropy ratio $Y_B$ can be calculated with Boltzmann Eqns (BE).

When the interaction rates of the charged lepton Yukawas are faster than the leptogenesis rates in the BE, the charged Yukawas should be "integrated out". ⇒ work in flavour basis.

Resulting "flavoured" BE are different, and the solutions are different:

- allowed phenomenological parameter space is enlarged
  1) leptogenesis works for degenerate light neutrinos
  2) minimum $T_{reheat}$ reduced by $\sim 3$(hier. $\nu_L$) to $\sim 10$(degen $\nu_L$)

- $Y_B$ obtained in specific models can be (?is?) enhanced
  1) flavoured $\epsilon^{\alpha\alpha}$ can be bigger than $\epsilon$
  2) cancellations reduced: $Y_B \propto \sum_{\alpha} \epsilon^{\alpha\alpha} f(\tilde{m}^{\alpha\alpha})$