NNLO QCD Calculation of $\bar{B} \to X_s \gamma$

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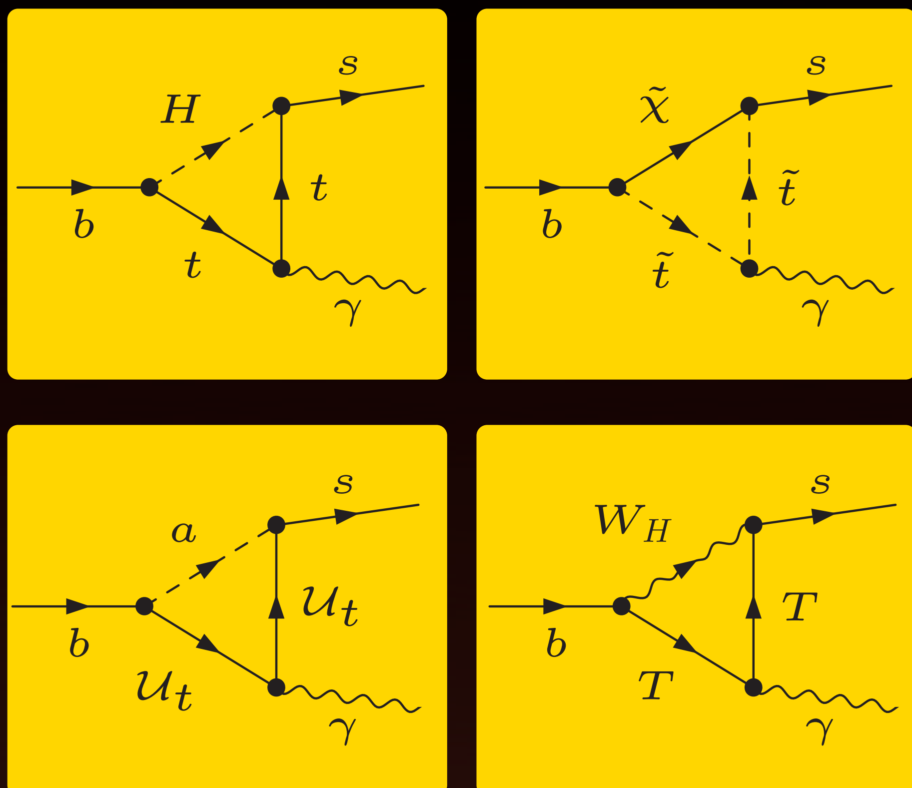
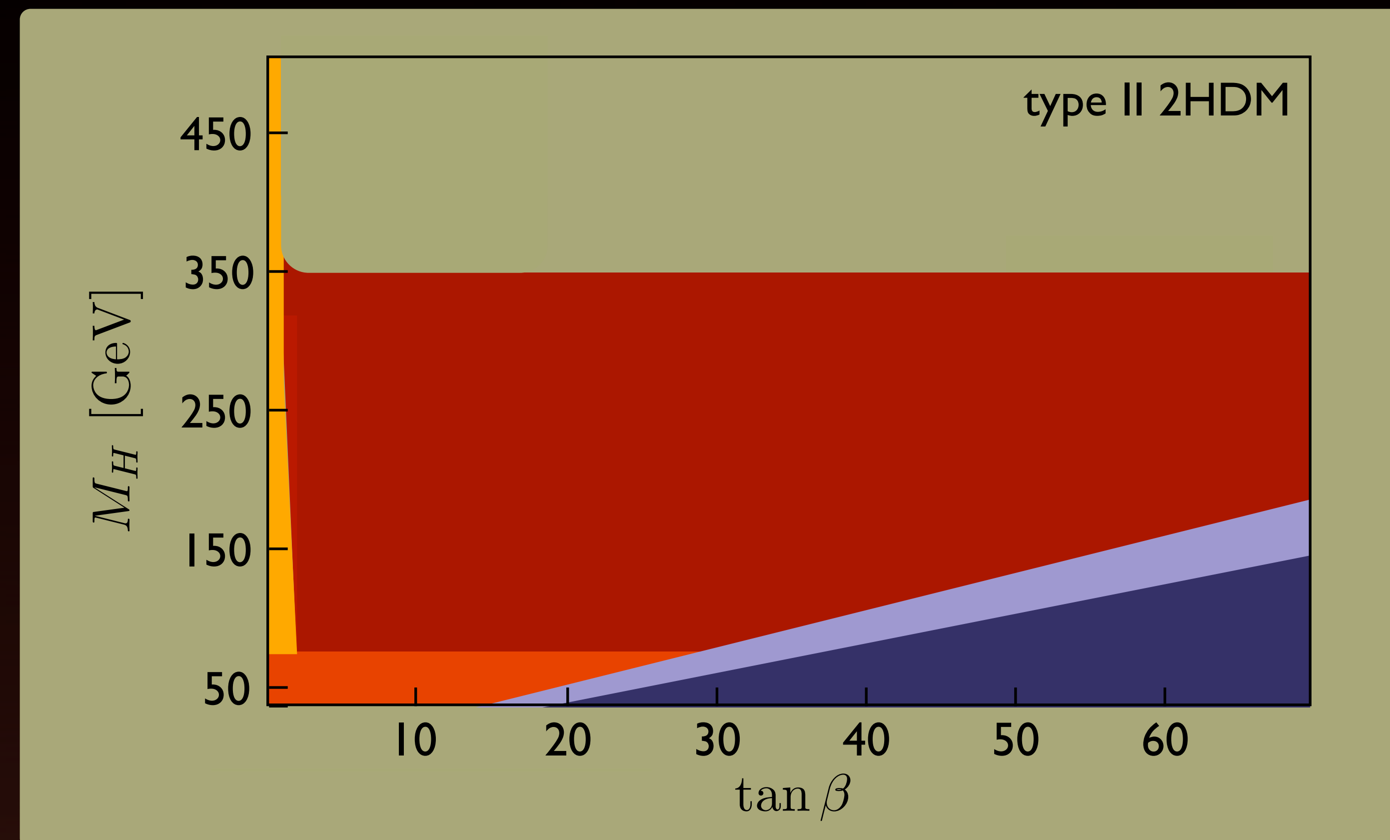
"Flavour in the era of the LHC" workshop,
3rd meeting, 15-17 May, 2006, CERN

Next 20 Minutes

- Introduction and Motivation
- Status of NNLO QCD Calculation
- Conclusions and Outlook

Introduction





Gambino & Misiak '01

- inclusive radiative $b \to s\gamma$ decay offers important precision tests of flavour sector in and beyond SM

- strong constraints on NP crucially depend on theoretical uncertainty of SM prediction

$\bar{B} \to X_s\gamma$

LEP2

R_b

$B \to \tau\nu$

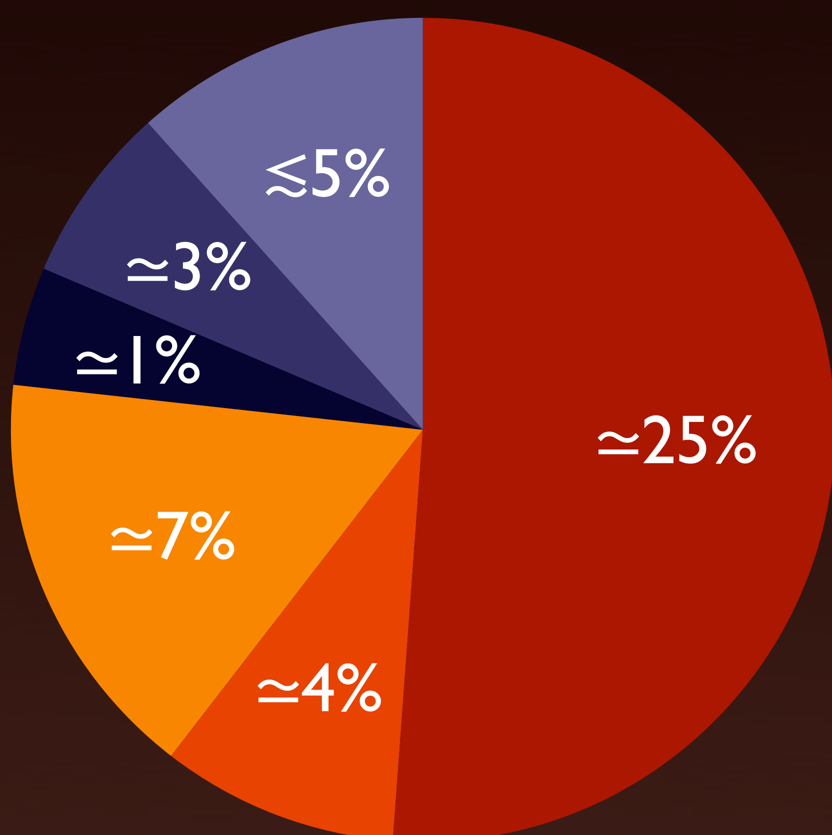
$B \to X\tau\nu$

General Structure

$$\mathcal{B}(\bar{B} \to X_s\gamma)^{E_\gamma > 1.6\,\mathrm{GeV}}_{\mathrm{SM}} = \mathcal{B}(\bar{B} \to X_c e \bar{\nu})_{\mathrm{exp}} \left[\frac{\Gamma(b \to s\gamma)}{\Gamma(b \to c e \bar{\nu})} \right]_{\mathrm{LO}} f\left(\frac{\alpha_s(M_W)}{\alpha_s(m_b)} \right)$$

$$\times \left\{ 1 + \mathcal{O}(\alpha_s) + \mathcal{O}(\alpha) + \mathcal{O}(\alpha_s^2) + \mathcal{O}\left(\frac{\Lambda^2}{m_b^2}\right) + \mathcal{O}\left(\frac{\Lambda^2}{m_c^2}\right) + \mathcal{O}\left(\alpha_s \frac{\Lambda}{m_b}\right) \right\}$$

✓ ✓ ✓ ? ✓ ✓ ?



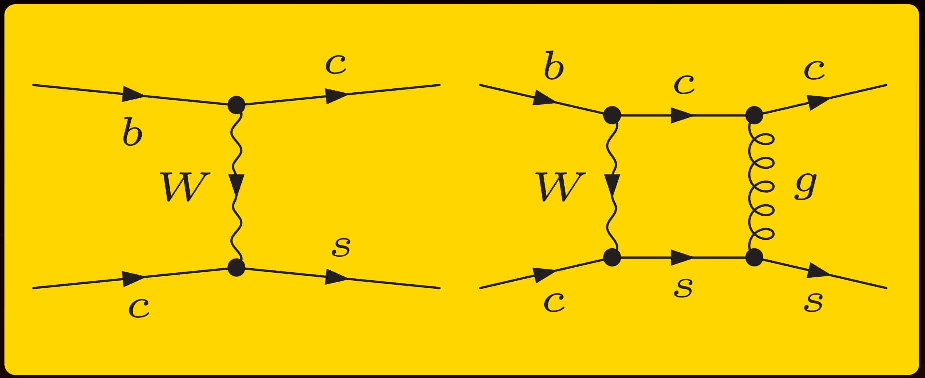
- NLO QCD
- NLO EW
- NNLO QCD

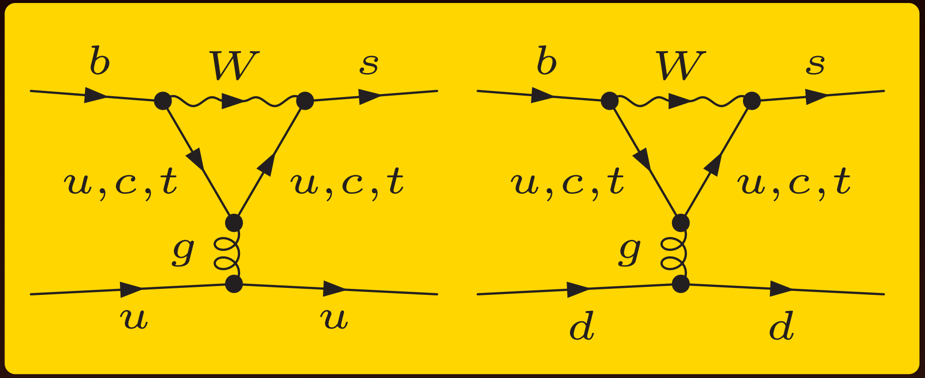
perturbative

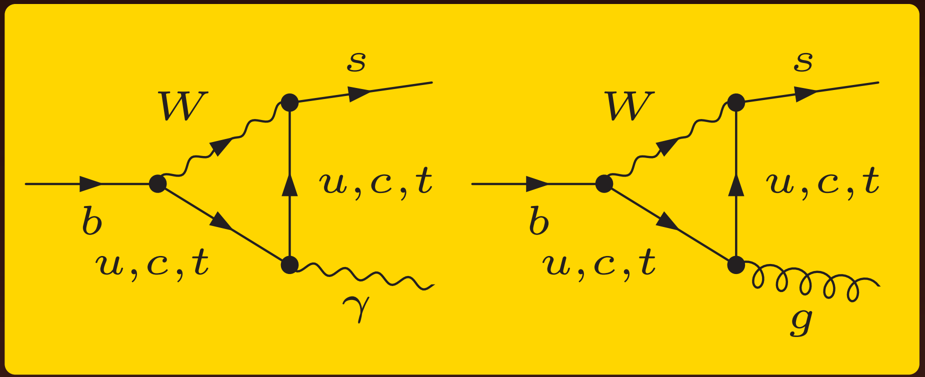
- LO QCD + NLO m_b
- LO QCD + NLO m_c
- NLO QCD + LO m_b

non-perturbative

Effective Theory



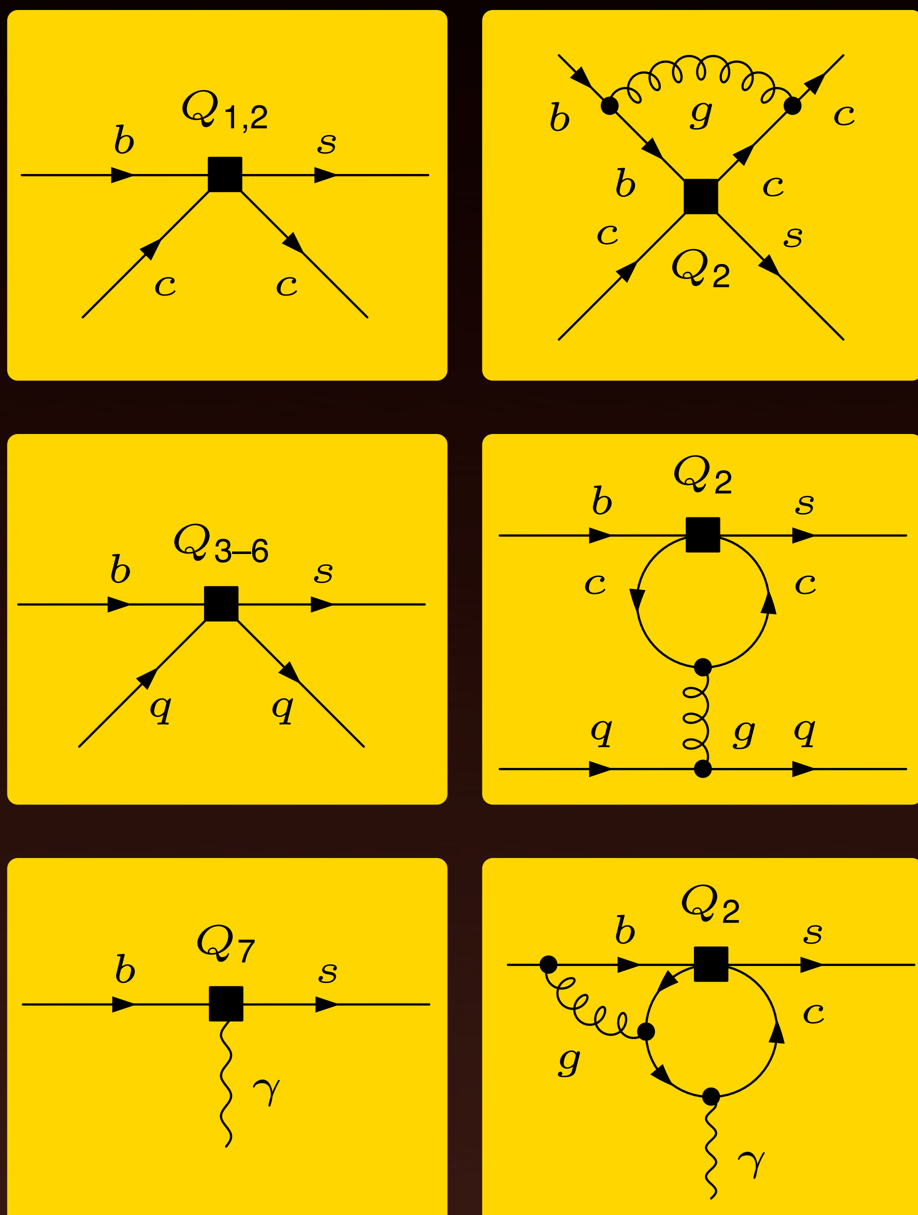




$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\rm QCD \times QED} + \frac{4 G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_{i=1}^{8} C_i(\mu) Q_i + \ldots$$

$Q_{1,2} = (\bar{s}\Gamma_i c)(\bar{c}\Gamma'_i b)$	$\lvert C_i(m_b)\rvert \sim 1$
$Q_{3-6} = (\bar{s}\Gamma_i b) \sum_q (\bar{q}\Gamma'_i q)$	$\lvert C_i(m_b)\rvert < 0.07$
$Q_7 = \frac{e m_b}{16\pi^2} (\bar{s}_L \sigma^{\mu\nu} T^a b_R) F_{\mu\nu}$	$C_7(m_b) \simeq -0.3$
$Q_8 = \frac{g m_b}{16\pi^2} (\bar{s}_L \sigma^{\mu\nu} T^a b_R) G^a_{\mu\nu}$	$C_8(m_b) \simeq -0.15$

Effective Theory



$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{QCD}\times\text{QED}} + \frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_{i=1}^{8} C_i(\mu) Q_i + \ldots$$

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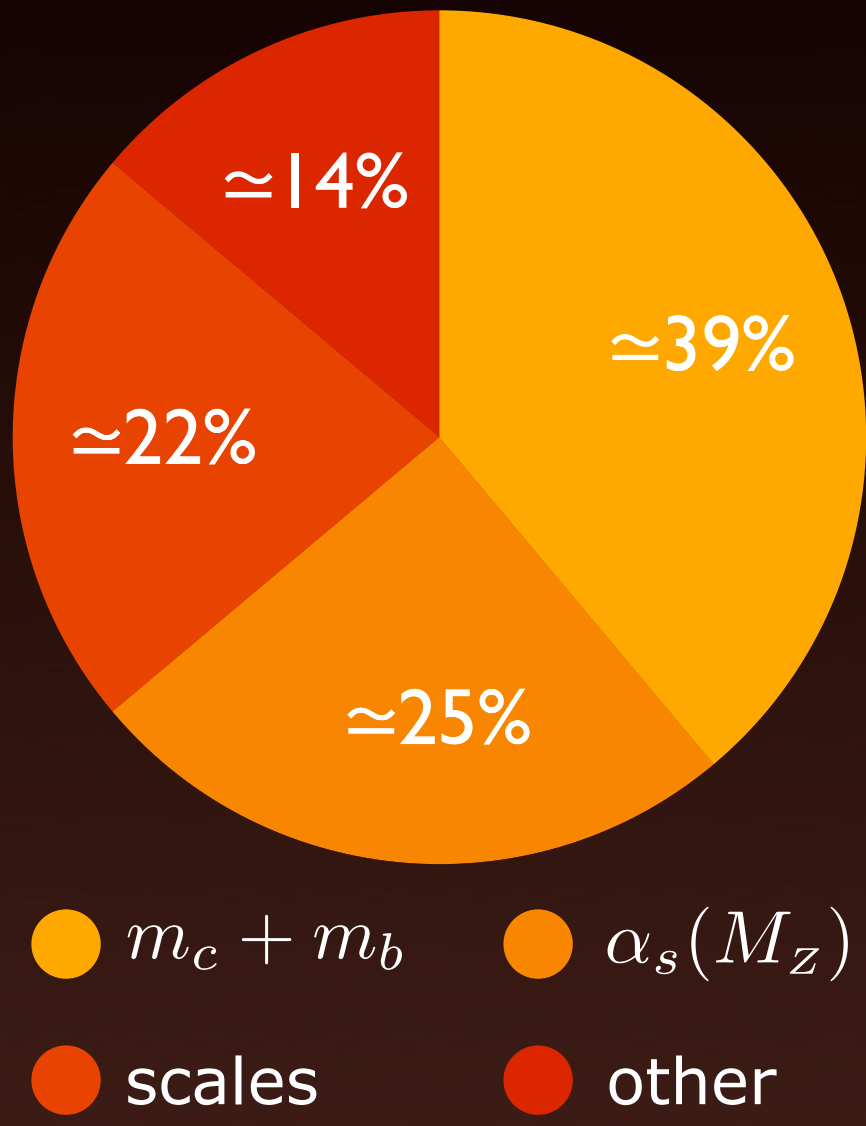
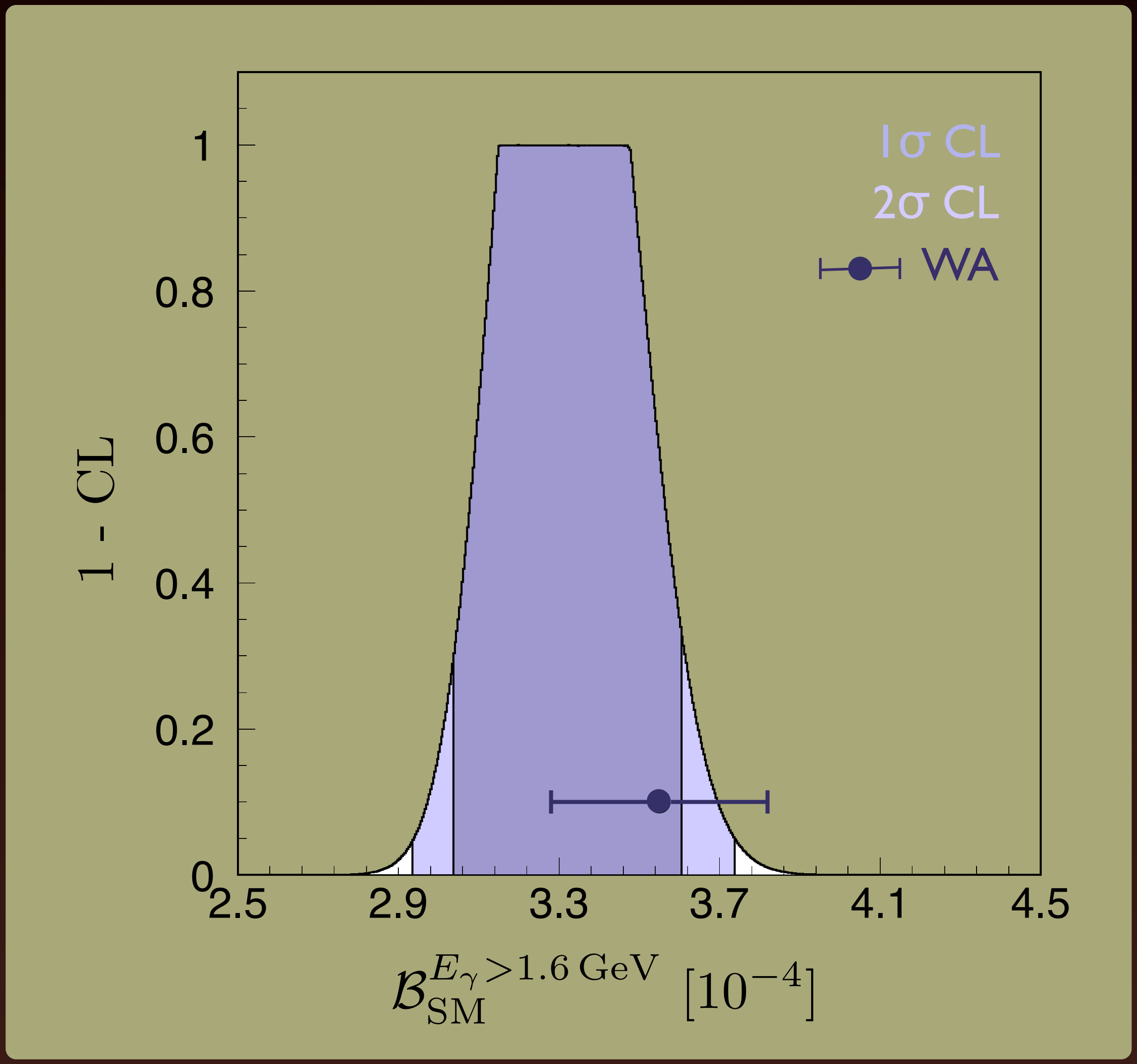
Experiment vs. Theory

$$\mathcal{B}_{\rm exp}^{E_\gamma > 1.6\,{\rm GeV}} = (3.55 \pm 0.24^{+9}_{-10} \pm 3) \times 10^{-4}$$

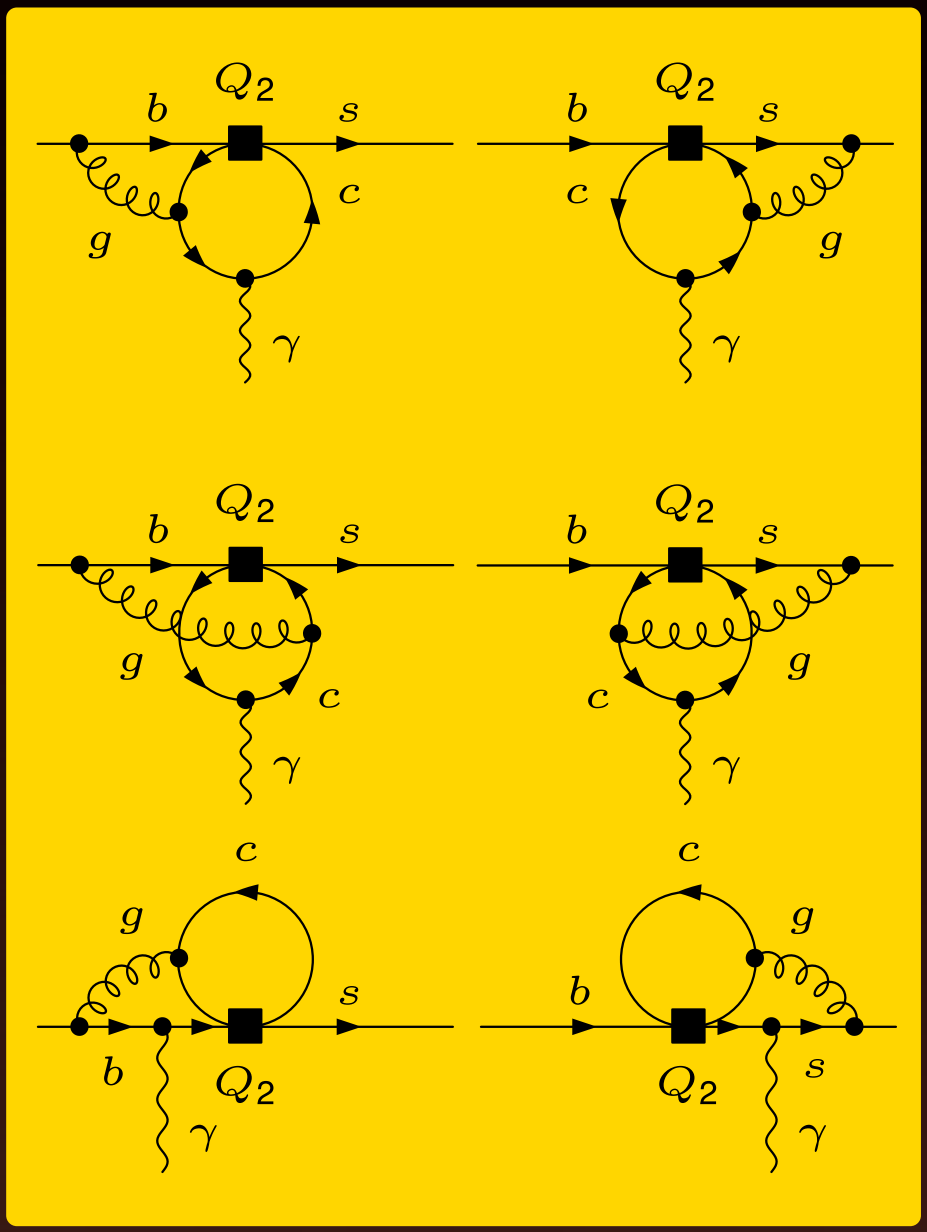
HFAG '06

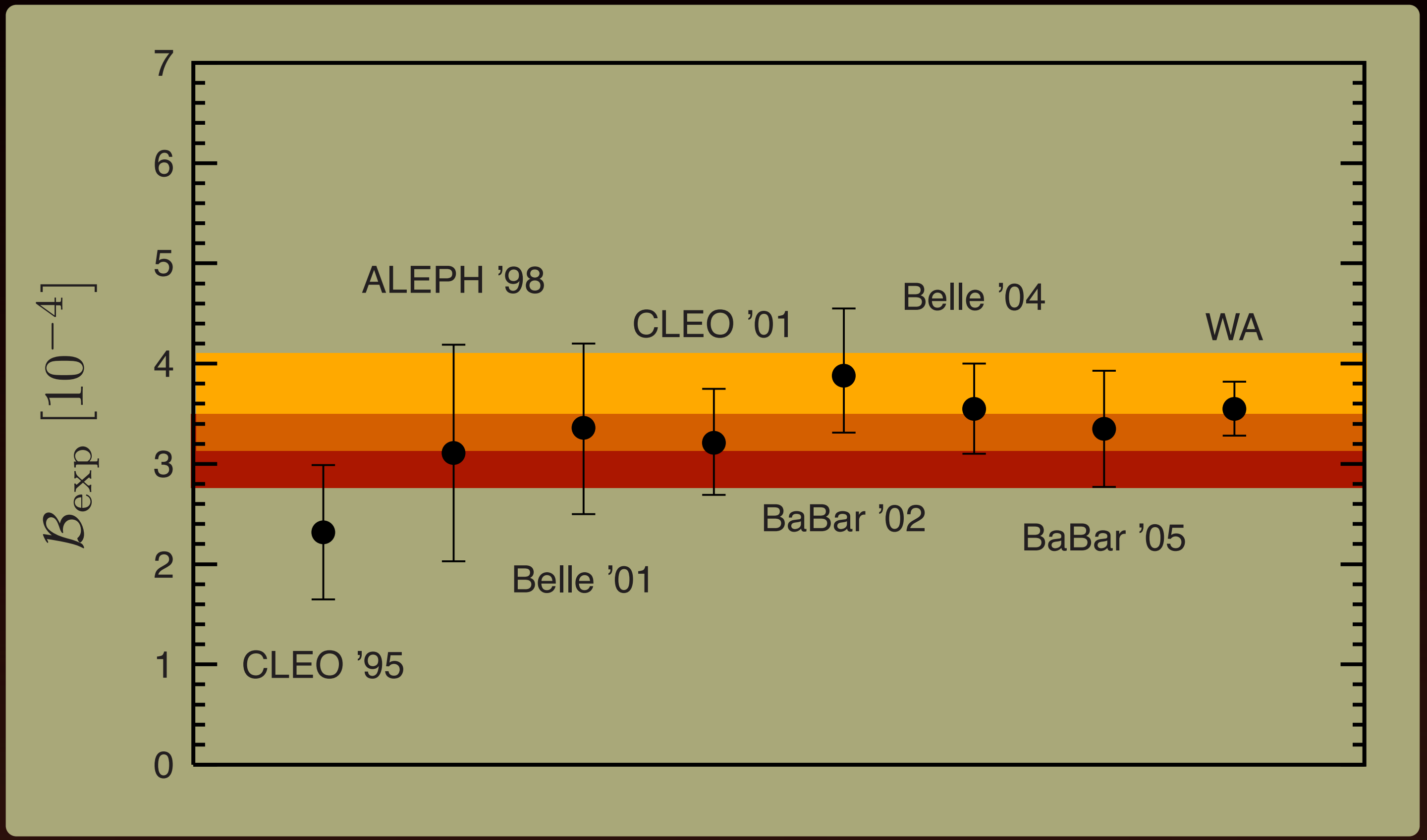
$$\mathcal{B}_{\rm SM}^{E_\gamma > 1.6\,{\rm GeV}} = (3.33 \pm 0.29) \times 10^{-4} \quad \text{(NLO)}$$





Charm Quark Mass





- $m_c/m_b = 0.29 \pm 0.02$ (on-shell)
- $m_c/m_b = 0.22 \pm 0.04$ ($\overline{\text{MS}}$)

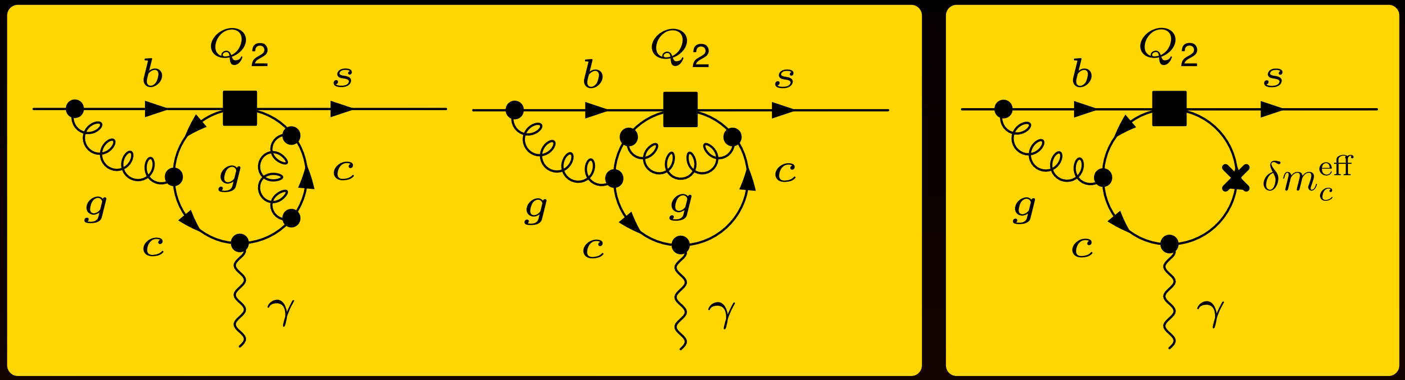
NNLO Error Estimate

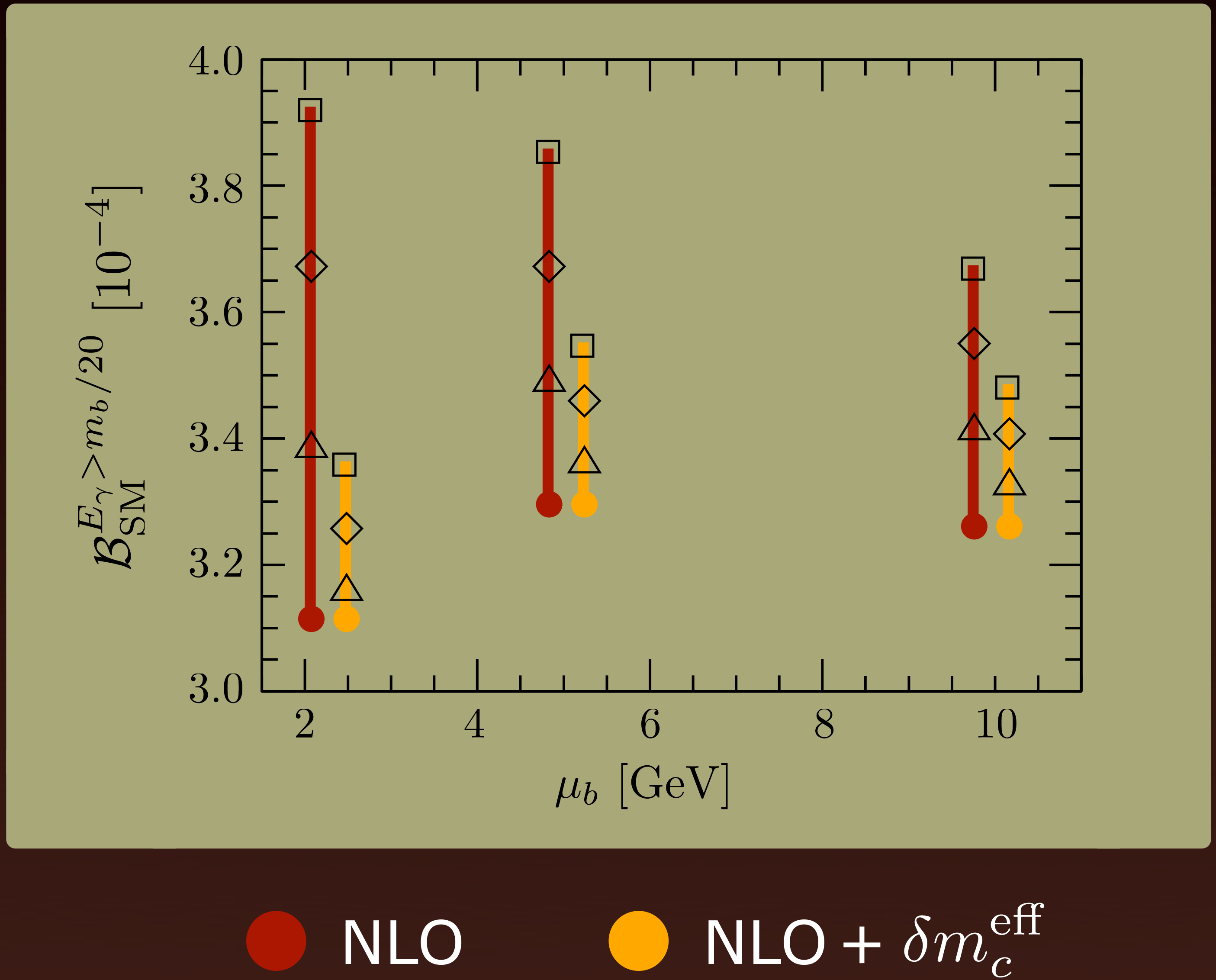
- unrenormalized quark self-energy $\Sigma(\not{p})$ is gauge-dependent
- consider gauge-independent mass insertion

$$\delta m_c^{\text{eff}} = \Sigma(\not{p} = m_c) + \delta m_c$$

- including δm_c^{eff} insertions reduces uncertainty due to m_c from ±6% to ±3%

Asatrian et al. '05



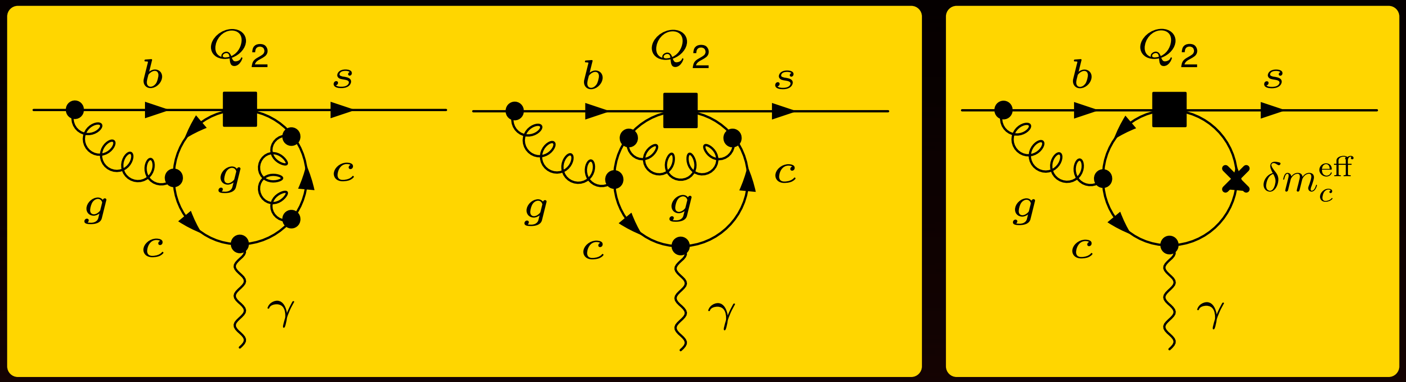


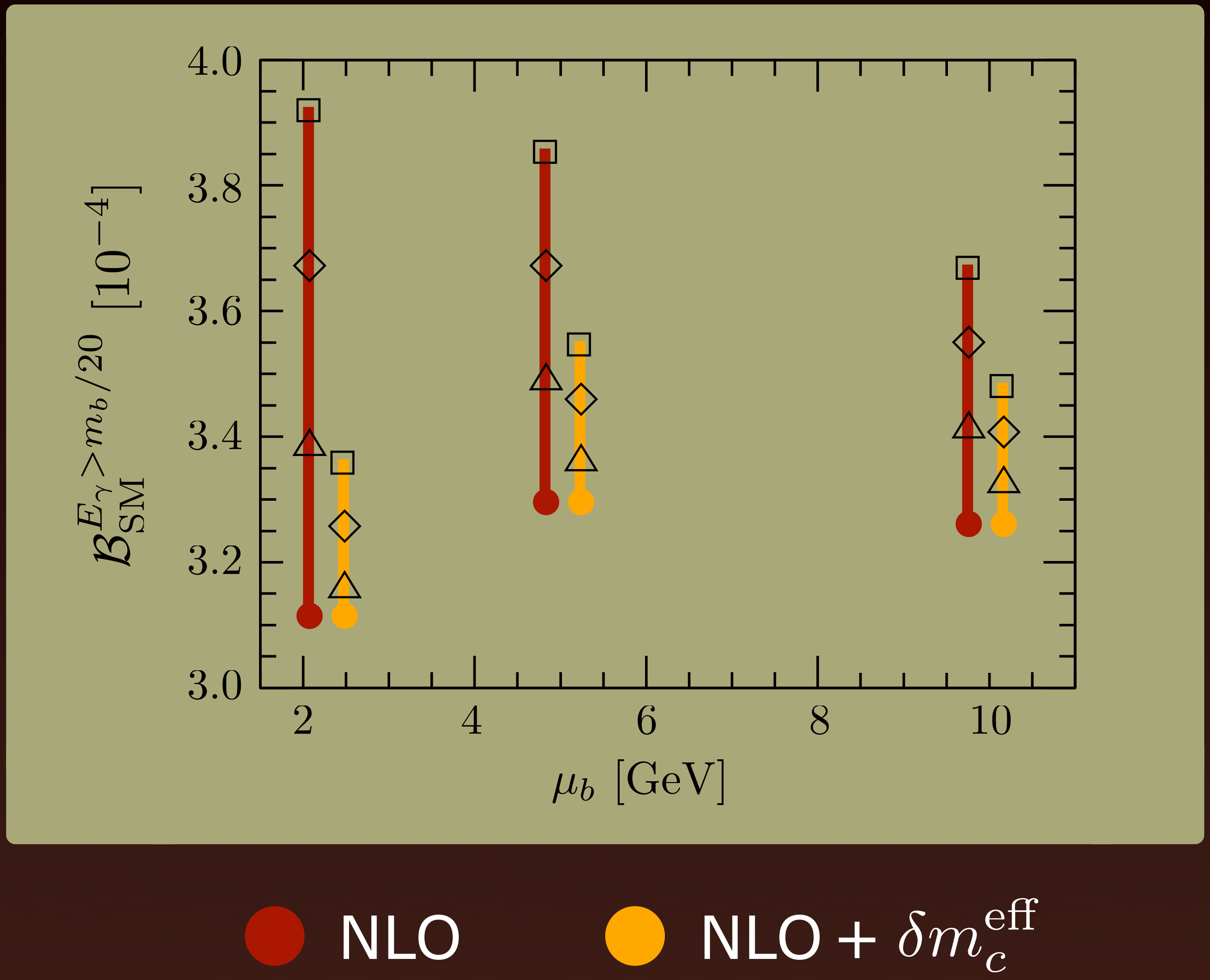
NNLO Error Estimate

- no loop calculation needed since RG gives us answer

$$\delta \mathcal{A}_{\rm NNLO} \propto \gamma_m^{(0)} \frac{d\mathcal{A}_{\rm NLO}}{d \ln m_c} \ln \frac{m_b}{m_c}$$

- NNLO corrections expected to drastically reduce theoretical due to definition of m_c
- to determine shift of central value of $\mathcal{B}_{\rm SM}$ going beyond NLO unavoidable

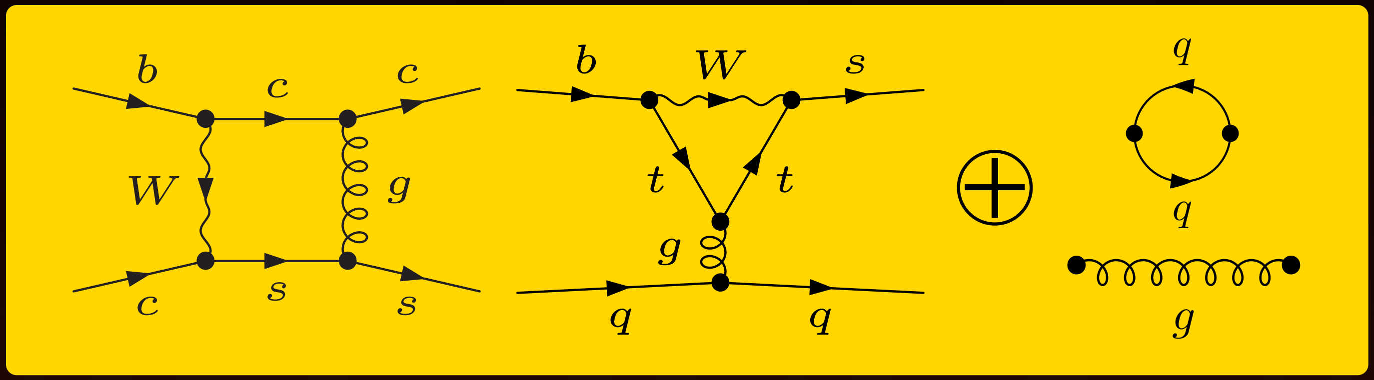




NNLO Matching Conditions

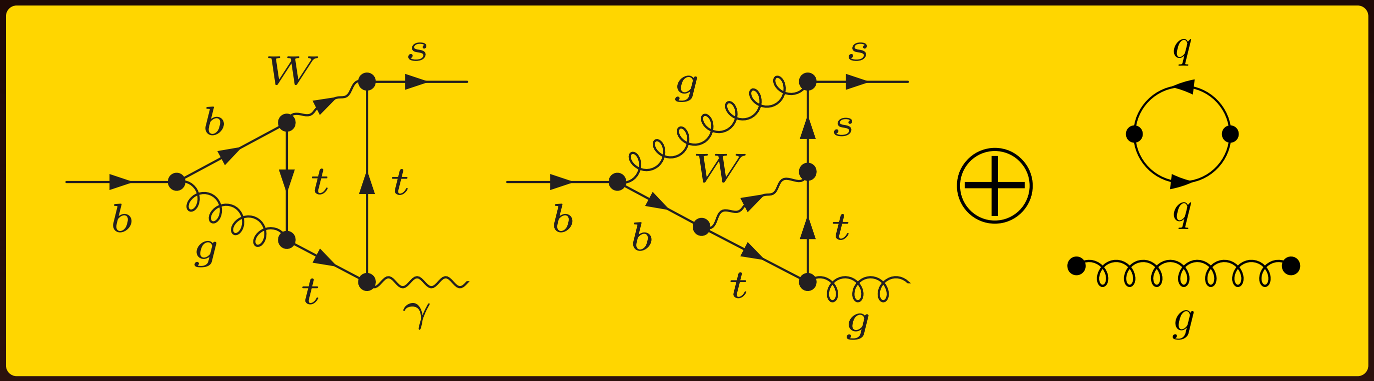
- analytic expressions of 2-loop $\mathcal{O}(\alpha_s^2)$ corrections to $C_{1\text{–}6}(M_W)$ known

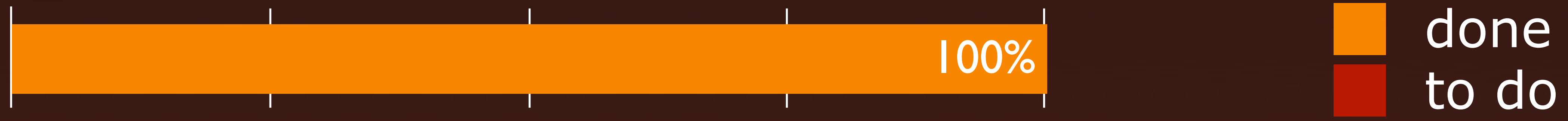
  Bobeth et al. '00, Buras et al. '06



- asymptotic expansions of 3-loop $\mathcal{O}(\alpha_s^2)$ corrections to $C_{7,8}(M_W)$ available

  Misiak & Steinhauser '04

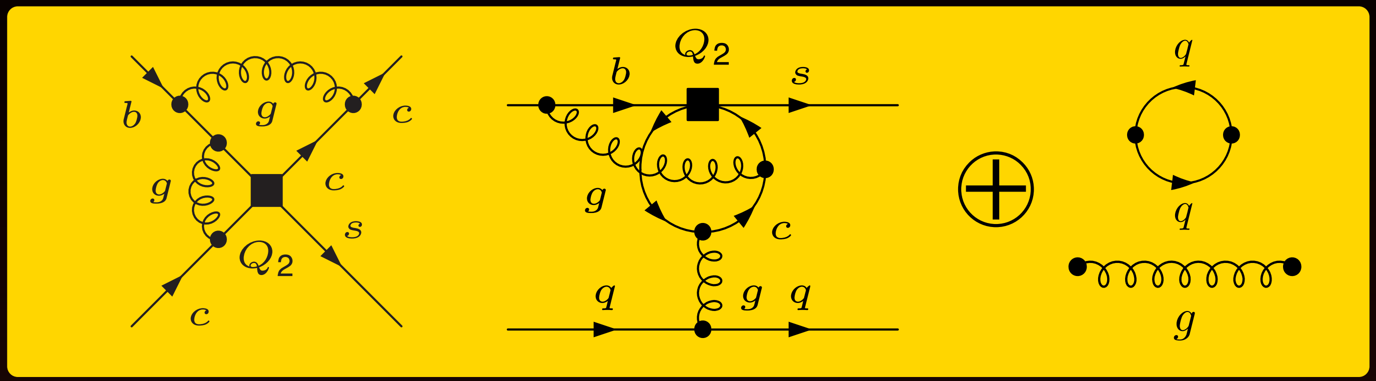




NNLO Anomalous Dimensions

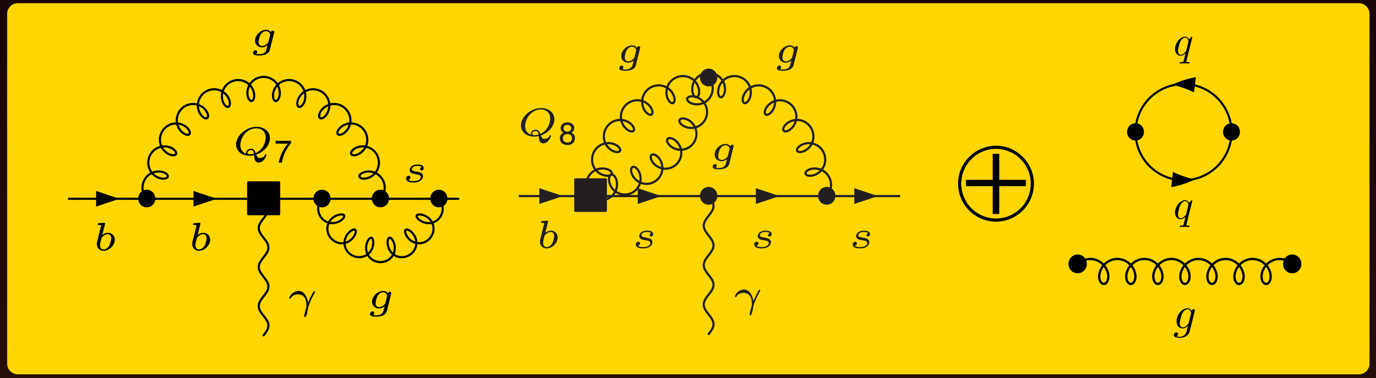
- 3-loop $\mathcal{O}(\alpha_s^3)$ mixing in Q_{1-6} sector calculated

  Gorbahn & UH ‘04



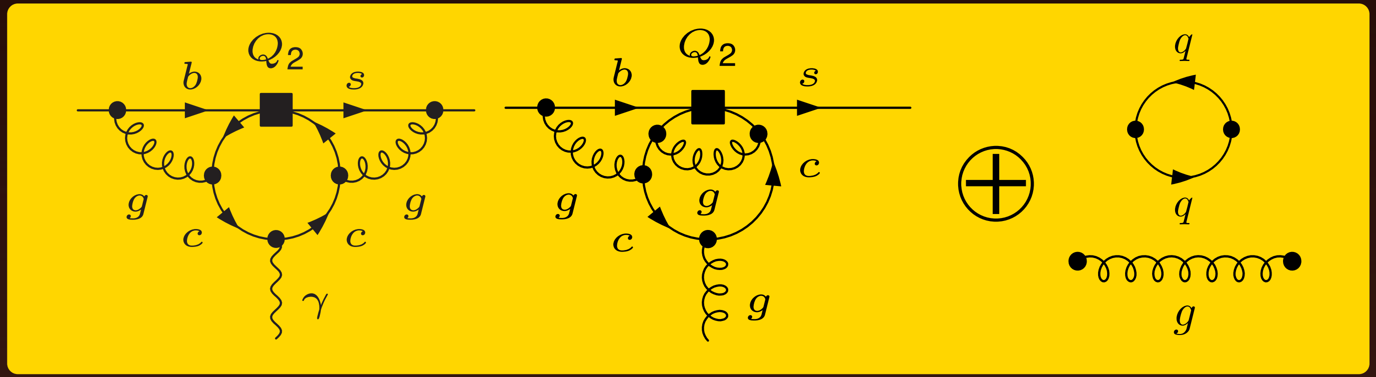
- 3-loop $\mathcal{O}(\alpha_s^3)$ mixing in $Q_{7,8}$ sector found

  Gorbahn et al. ‘05

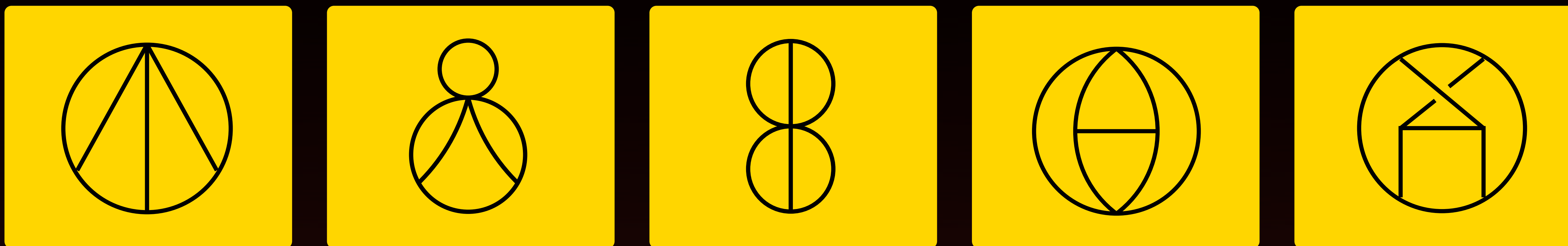


- 4-loop $\mathcal{O}(\alpha_s^3)$ mixing of Q_{1-6} into Q_7 already computed and Q_8 almost finished

  Czakon et al.



Results of 4-Loop Mixing



- numerical effect of $\mathcal{O}(\alpha_s^3)$ mixing of Q_{1-6} into Q_7 on $\mathcal{B}_{\rm SM}$ is −2.4%

- impact of remaining $\mathcal{O}(\alpha_s^3)$ mixing of Q_{1-6} into Q_8 expected to be ≈10 times smaller

$$\gamma^{(2)}_{1-6,7} = \begin{pmatrix} \frac{150994745}{1062882} + \frac{1272596}{6561}\zeta_3 \\ \frac{138336202}{177147} - \frac{2713672}{2187}\zeta_3 \\ -\frac{58397866}{177147} + \frac{3236560}{2187}\zeta_3 \\ -\frac{5108749081}{2125764} + \frac{2007886}{6561}\zeta_3 \\ \frac{5824017302}{177147} + \frac{112180720}{2187}\zeta_3 \\ \frac{3603565835}{531441} + \frac{15361912}{6561}\zeta_3 \end{pmatrix}$$

NNLO Matrix Elements

- large β_0 limit of 2- and 3-loop $\mathcal{O}(\alpha_s^2)$ matrix elements of $Q_{7,8}$ and $Q_{1,2}$ worked out

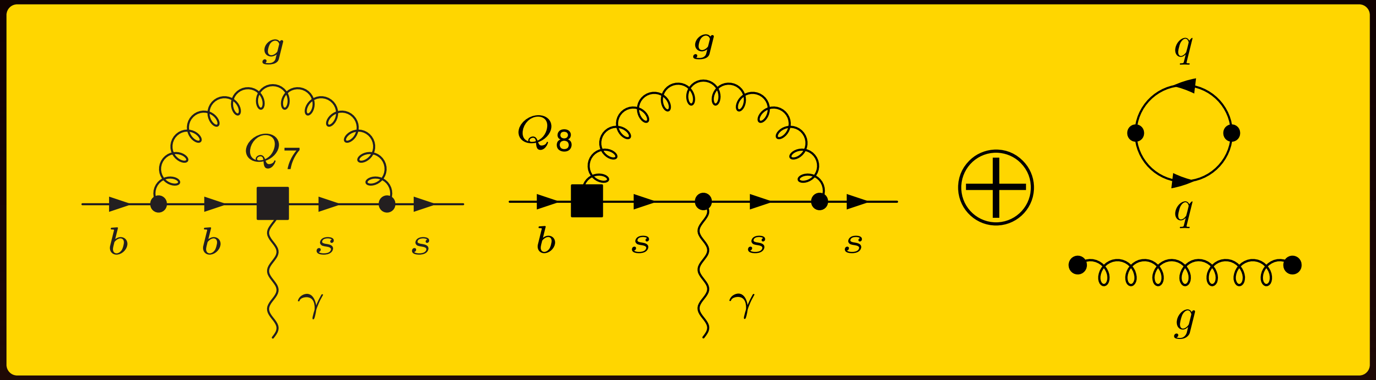
  Bieri et al. '03

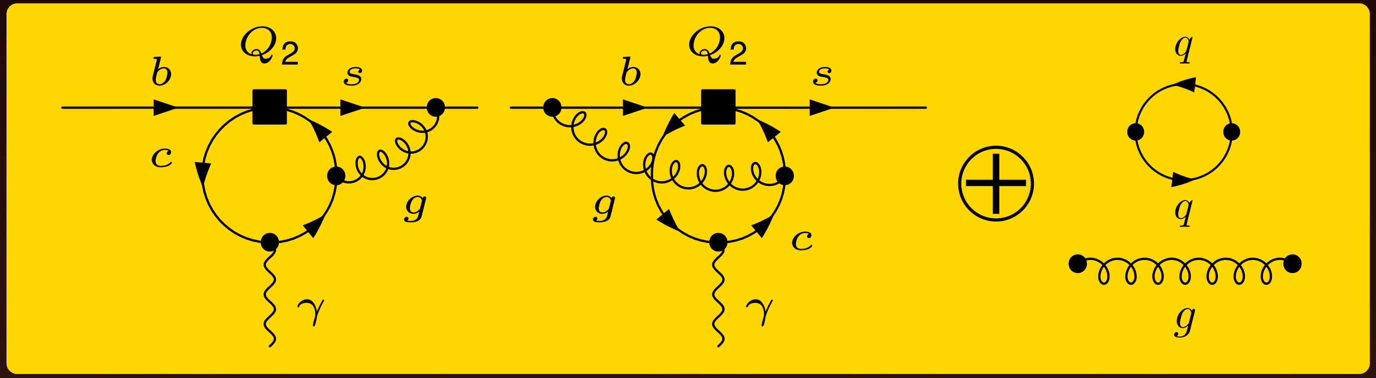
- analytic results for 2-loop $\mathcal{O}(\alpha_s^2)$ corrections to self-interference of Q_7 derived

  Blokland et al. '05, Asatrian et al. '06; Melnikov & Mitov '05, Asatrian et al., Ferroglia & Gambino

- interpolation in m_c of 3-loop $\mathcal{O}(\alpha_s^2)$ matrix elements of $Q_{1,2}$ very far advanced

  Misiak & Steinhauser





done

to do

Individual NNLO Contributions

$$C_i = \sum_{n=0}^{2} \left(\frac{\alpha_s}{4\pi}\right)^n C_i^{(n)}$$

$$\mathcal{B}_{\rm NNLO}^{E_\gamma > 1.6\,{\rm GeV}} \equiv \mathcal{B}_{\rm NLO}^{E_\gamma > 1.6\,{\rm GeV}} \big(1 + \delta_1 + \delta_2(r) + \delta_3(r)\big)$$

$$r \equiv \frac{m_c(m_c)}{m_b^{1S}}$$

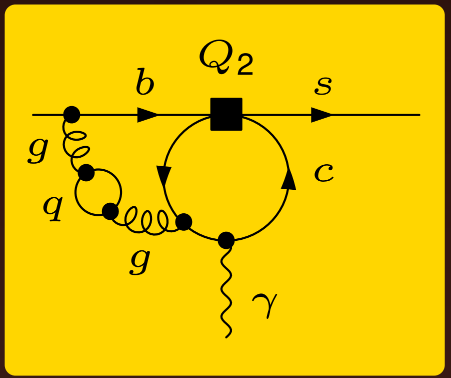
$$\delta_1 \propto C_i^{(0)} C_j^{(2)}, C_i^{(1)} C_j^{(1)}$$

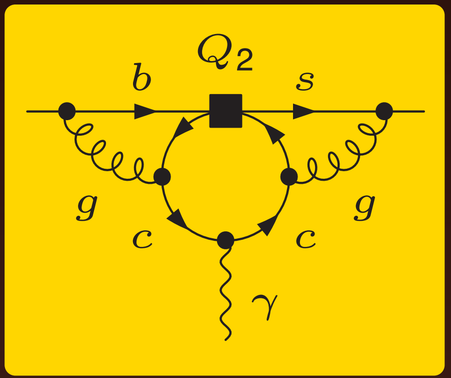
$$\delta_2(r) \propto C_i^{(0)} C_j^{(0)}$$

$$\delta_3(r) \propto C_i^{(0)} C_j^{(1)}$$

$$\delta_2(r) = A\, n_f + B = -\frac{3}{2}\left(11 - \frac{2}{3} n_f\right) A + \left(\frac{33}{2} A + B\right) \equiv \delta_2^{\beta_0}(r) + \delta_2^{\text{non-}\beta_0}(r)$$

- while $\delta_2^{\beta_0}(r)$ known for arbitrary r $\delta_2^{\text{non-}\beta_0}(r)$ has been computed so far only in limit $r \gg 1/2$



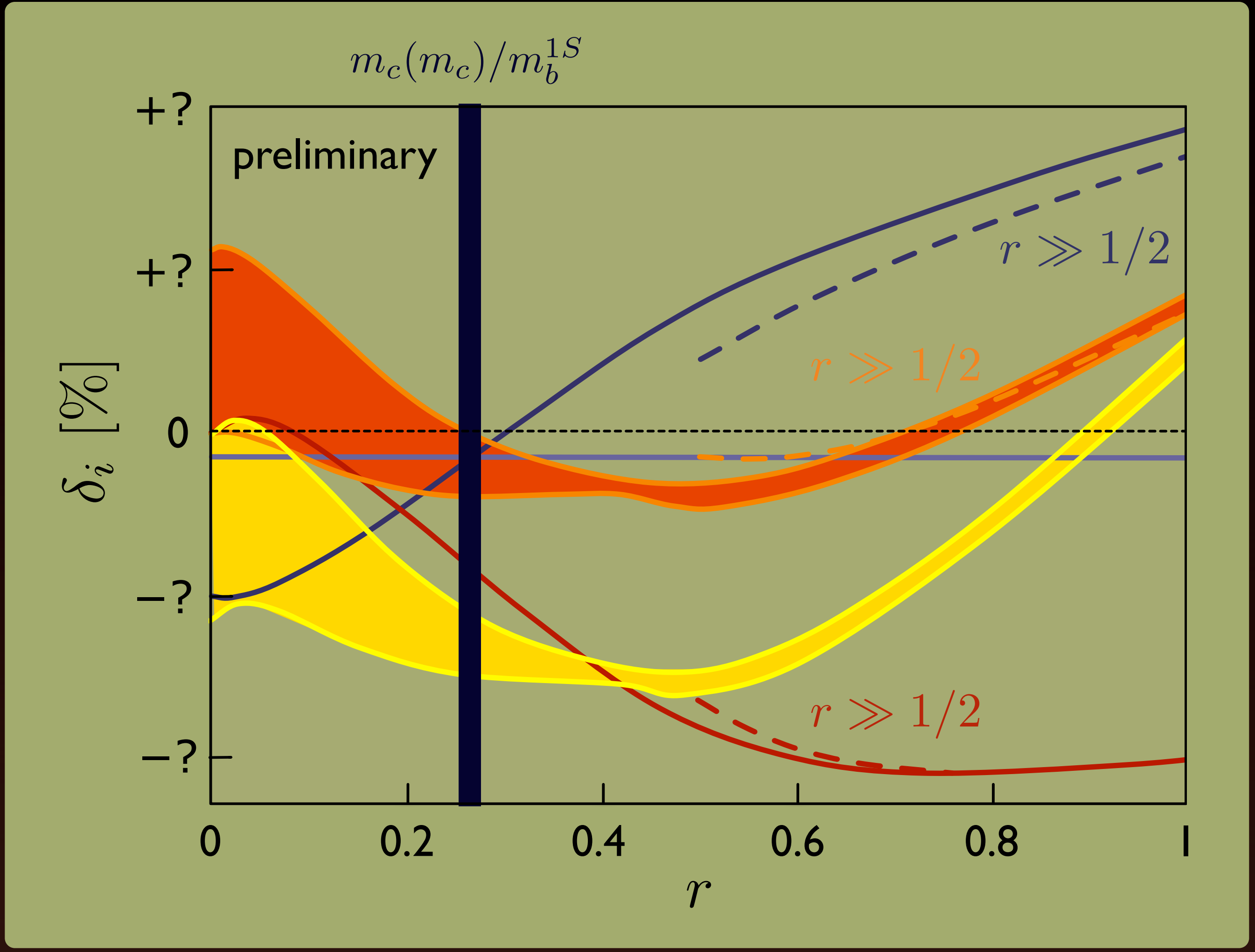


Interpolation in Charm Quark Mass

- interpolate beyond large β_0 correction $\delta_2^{\text{non-}\beta_0}(r)$ by

$$a\,\frac{d\mathcal{B}_{\rm NLO}}{d\ln r}+b\,\mathcal{B}_{\rm NLO}+c\,\delta_2^{\beta_0}(r)+d$$

- coefficients a, b, c and d are determined from behaviour at large r and requiring that correction vanishes at $r=0$
- lower curves correspond to $\delta_2^{\text{non-}\beta_0}(0)=0$ upper ones to $\delta_1+\delta_2^{\text{non-}\beta_0}(0)+\delta_3(0)=0$



Misiak et al.

- $\delta_1+\delta_2(r)+\delta_3(r)$
- $\delta_2^{\text{non-}\beta_0}(r)$
- δ_1
- $\delta_2^{\beta_0}(r)$
- $\delta_3(r)$

Summary and Outlook

- current experimental world average already slightly more accurate than SM prediction at NLO
- completion of NNLO enterprise will reverse situation and drastically increase sensitivity of $\bar{B} \to X_s \gamma$ to NP
- to resolve all doubts in interpolation in charm quark mass explicit calculation of 3-loop matrix elements for $m_c = 0$ should be attempted
- mandatory to perform a study of non-perturbative effects that scale like $\mathcal{O}(\alpha_s \Lambda / m_b)$ in limit of vanishing charm quark mass

Credits

Michal Czakon,
Paolo Gambino,
Martin Gorbahn,
Mikolaj Misiak &
Matthias Steinhauser

NNLO SCET Analysis

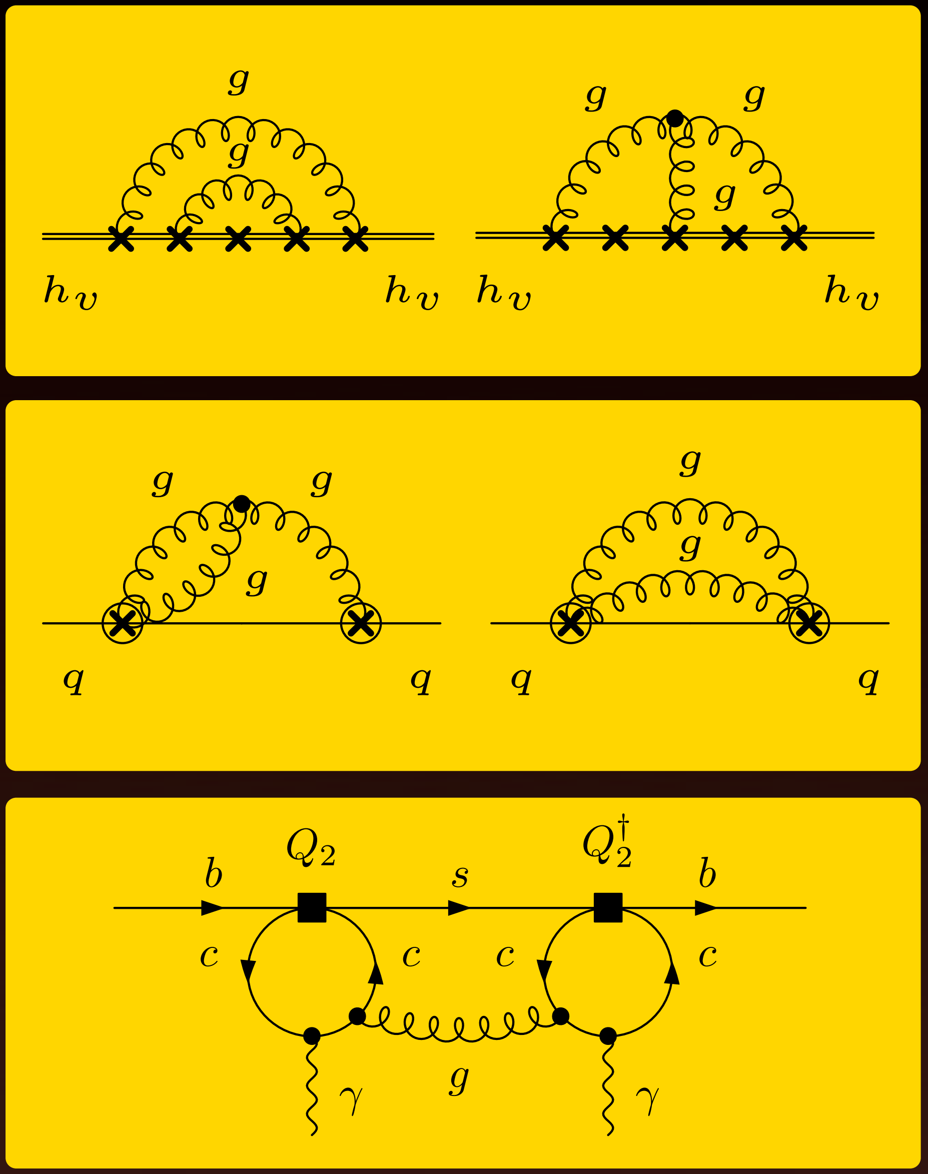
- 2-loop $\mathcal{O}(\alpha_s^2)$ corrections to soft function $\tilde{s}(L,\mu)$ available now

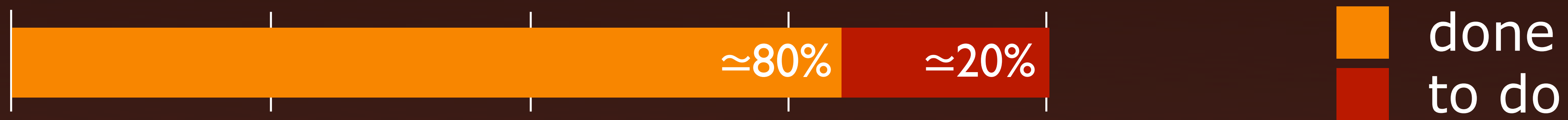
  Becher & Neubert '05

- 2-loop $\mathcal{O}(\alpha_s^2)$ corrections to jet function $\tilde{j}(L,\mu)$ calculated recently

  Becher & Neubert '06

- analysis of non-perturbative effects in matrix elements of $Q_{1,2}$ at $\mathcal{O}(\alpha_s)$ open issue





Klingon Military Branch Insignia