$B \to \pi\pi$ decays: branching ratios and CP asymmetries

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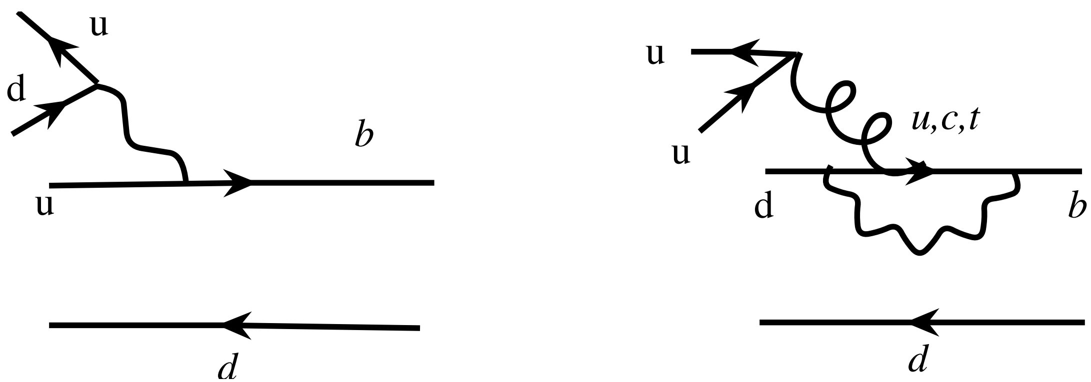
PLAN

- P/T - small parameter
- $B \to \pi\pi$ in tree approximation: $A_0, A_2, \delta \quad \alpha$
- factorisation: pro and contro
- $K \to \pi\pi, \quad D \to \pi\pi, \quad B \to D\pi$
- FSI in $B \to \pi\pi$ and $\pi\pi$ - $\rho\rho$ problem
- penguin corrections: C_{+-}, C_{00}, α
- Conclusions

$b \to ud\bar{u}$



$B \to \pi\pi,\ B \to \rho\rho,\ B \to \rho\pi$ decays

How large is penguin?

$$c_1 = 1.09, c_2 = -0.21;$$
$$c_3 = 0.013, c_4 = -0.032, c_5 = 0.009, c_6 = -0.037$$

P/T - small

$P/T = 0 \Longrightarrow \sin 2\alpha^T = S_{+-}$
$B \to \pi^+\pi^- : \alpha^T_{BABAR} = (99 \pm 5)^o$
$B \to \rho^+\rho^- : \alpha^T = (96 \pm 7)^o$
$B \to \pi^\pm\rho^\mp : \alpha^T = (94 \pm 4)^o$, or $(86 \pm 4)^o$
values of α from all 3 decays agree with each other and with global CKM fit result.
R.Aleksan, F.Buccella, A.Le Yaouanc, L.Oliver, O.Pene, J.-C.Raynal (1995):
$\Delta\alpha^{\pi\pi} > \Delta\alpha^{\rho\rho} > \Delta\alpha^{\pi\rho}$
from smallness of $B \to \rho^0\rho^0 / B \to \rho^+\rho^-$ one can PROVE smallness of penguins in $B \to \rho\rho$ decays; however from relatively large value of $B \to \pi^0\pi^0 / B \to \pi^+\pi^-$ it DOES NOT follow that penguins in $B \to \pi\pi$ decays are large.

Perturbation theory over P/T

$B \to \pi\pi$ exp data

	BABAR	Belle	Heavy Flavor Averaging Group
B_{+-}	5.5 ± 0.5	4.4 ± 0.7	5.0 ± 0.4
B_{00}	1.17 ± 0.33	2.3 ± 0.5	1.45 ± 0.29
B_{+0}	5.8 ± 0.7	5.0 ± 1.3	5.5 ± 0.6
S_{+-}	-0.30 ± 0.17	-0.67 ± 0.16	-0.50 ± 0.12
C_{+-}	-0.09 ± 0.15	-0.56 ± 0.13	-0.37 ± 0.10
C_{00}	-0.12 ± 0.56	-0.44 ± 0.56	-0.28 ± 0.39

B_{ik} in units 10^{-6}

C_{+-}: BABAR - little penguin; Belle - big penguin. Average?

$B \to \pi\pi$ phenomenology

$$M_{\bar{B}_d \to \pi^+\pi^-} = \frac{G_F}{\sqrt{2}} |V_{ub} V_{ud}^*| m_B^2 f_\pi f_+(0) \left\{ e^{-i\gamma} \frac{1}{2\sqrt{3}} A_2 e^{i\delta} + \right.$$
$$\left. + \quad e^{-i\gamma} \frac{1}{\sqrt{6}} A_0 + \left| \frac{V_{td}^* V_{tb}}{V_{ub} V_{ud}^*} \right| e^{i\beta} P e^{i\delta_p} \right\} , \qquad (1)$$

$$M_{\bar{B}_d \to \pi^0\pi^0} = \frac{G_F}{\sqrt{2}} |V_{ub} V_{ud}^*| m_B^2 f_\pi f_+(0) \left\{ -e^{-i\gamma} \frac{1}{\sqrt{3}} A_2 e^{i\delta} + \right.$$
$$\left. + \quad e^{-i\gamma} \frac{1}{\sqrt{6}} A_0 + \left| \frac{V_{td}^* V_{tb}}{V_{ub} V_{ud}^*} \right| e^{i\beta} P e^{i\delta_p} \right\} , \qquad (2)$$

$$M_{\bar{B}_u \to \pi^-\pi^0} = \frac{G_F}{\sqrt{2}} |V_{ub} V_{ud}^*| m_B^2 f_\pi f_+(0) \left\{ \frac{\sqrt{3}}{2\sqrt{2}} e^{-i\gamma} A_2 e^{i\delta} \right\} \qquad (3)$$

tree approximation: A_0, A_2, δ, α

3 parameters from 3 equations (B_{+-}, B_{00}, B_{+0}):

$A_0 = 1.53 \pm 0.23, A_2 = 1.60 \pm 0.20, \delta = \pm(53^o \pm 7^o)$

$A_0^f = 1.54,$ $A_2^f = 1.35$ but δ ...

$$\sin 2\alpha^{\mathrm{T}} = S_{+-} \quad ,$$

$$\alpha^{\mathrm{T}}_{\mathrm{BABAR}} = 99^o \pm 5^o \ , \ \ \alpha^{\mathrm{T}}_{\mathrm{Belle}} = 111^o \pm 6^o \ , \ \ \alpha^{\mathrm{T}}_{\mathrm{average}} = 105^o \pm 4^o \ \ .$$

FSI in $K \to \pi\pi, D \to \pi\pi, B \to D\pi$

K decays: 3 decay probabilities, or Watson theorem:
$\delta_0^K = 35^o \pm 3^o, \delta_2^K = -7^o \pm 0.2^o, \delta^K = 42^o \pm 4^o$

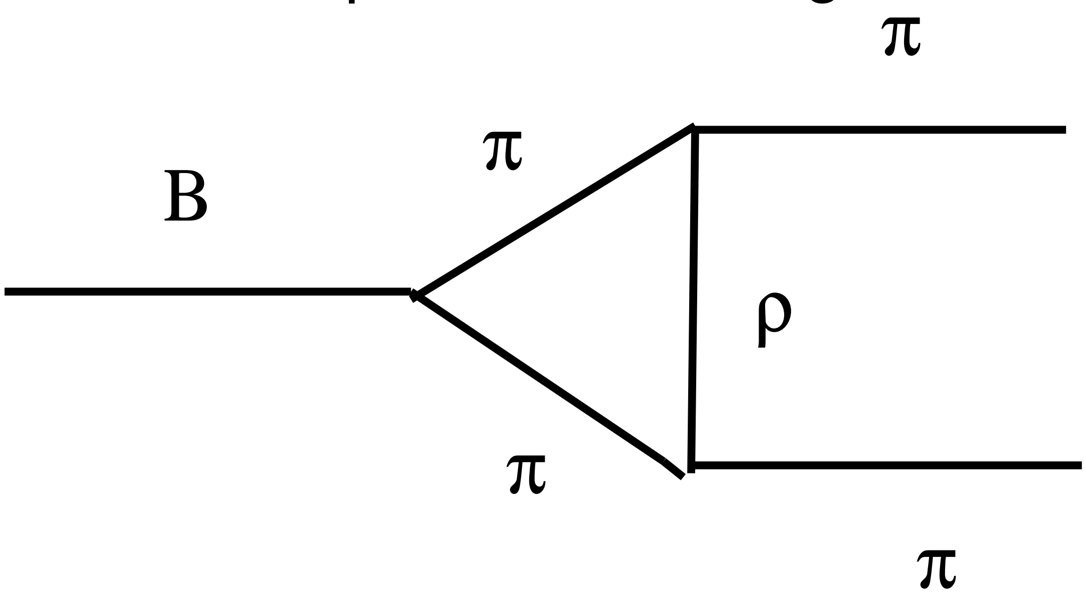
D decays: 3 decay probabilities (Watson theorem is not applicable): factorisation also good for moduli of decay amplitudes, while $\delta_2^D - \delta_0^D = 86^o \pm 4^o (!?)$
$1/M$ scaling of FSI phases?

$(D\pi) : I = 1/2$ or $3/2$; 3 decay probabilities: $\delta_{D\pi} = 30^o \pm 7^o$
Important: above the resonances

So: FSI phases can be L A R G E

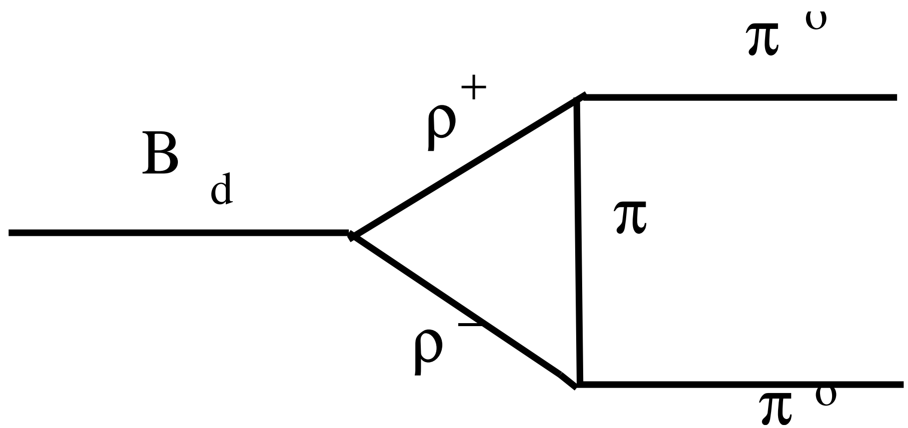
FSI in $B \to \pi\pi$

a lot of papers; one example: Cheng, Chua, Soni, P.R. D 71 (2005): "rescattering effects enhance color-suppressed neutral modes". FSI phase is extracted from the diagrams with vector particle exchanges in t-channel:



for elementary ρ meson partial wave amplitude does not decrease with energy. However all exchanges should be reggeized, $s^{\alpha(0)-1}, \alpha_\rho \approx 0.5$

phase from $\rho^+\rho^-$intermediate state



1. $Br(B_d \to \rho^+\rho^-) = 26 * 10^{-6}$

2. Will generate negligible phases in $B \to \rho\rho$ amplitudes, because $Br(B_d \to \pi^+\pi^-) = 5 * 10^{-6}$

$\delta \approx 30^o$ is generated by intermediate $\rho\rho$ and $\pi\pi$ states

penguin corrections

To avoid shifts of B_{+-} and B_{00} we should shift A_0 and δ :

$$A_0 \to A_0 + \tilde{A}_0 \; , \quad \delta \to \delta + \tilde{\delta} \quad ,$$

$$\tilde{A}_0 = \sqrt{6}\left|\frac{V_{td}}{V_{ub}}\right| \cos\alpha \cos\delta_p P \quad ,$$

$$\tilde{\delta} = -\sqrt{6}\left|\frac{V_{td}}{V_{ub}}\right| \cos\alpha \sin\delta_p P/A_0 \quad ,$$

where only the terms linear in P are taken into account

In the factorisation approach we have:

$$P^f = -a_4 - \frac{2m_\pi^2}{(m_u + m_d)m_b} a_6 = 0.06 \ \ ,$$

$a_4 = c_4 + c_3/3, a_6 = c_6 + c_5/3$ and shifts of A_0 and δ are small:

$$-0.12 < \tilde{A}_0 < 0.12 \ , \ \ -4^o < \tilde{\delta} < 4^o$$

for

$$A_0 = 1.5 \ , \ \ -1 < \cos\delta_p \ , \ \ \sin\delta_p < 1 \ \text{ and } \ 70^o < \alpha < 110^o$$

C_{+-}, C_{00}

In linear in P approximation for direct CP asymmetries we obtain:

$$C_{+-} = -2P\sin\alpha\left|\frac{V_{td}}{V_{ub}}\right|\frac{\cos(\delta_p+\varphi)}{\sqrt{\frac{1}{12}A_2^2+\frac{1}{6}A_0^2+\frac{1}{3\sqrt{2}}A_0A_2\cos\delta}} =$$

$$-4.7P\cos(\delta_p+68^o)$$

$$C_{00} = -2P\sin\alpha\left|\frac{V_{td}}{V_{ub}}\right|\frac{\cos(\delta_p+\psi)}{\sqrt{\frac{1}{3}A_2^2+\frac{1}{6}A_0^2-\frac{\sqrt{2}}{3}\cos\delta A_0A_2}} = 6.2P\cos\delta_p$$

where

$$\varphi = \arccos \frac{\frac{1}{\sqrt{3}} A_2 \sin\delta}{\sqrt{\frac{1}{3}A_2^2 + \frac{2}{3}A_0^2 + \frac{2\sqrt{2}}{3}A_0 A_2 \cos\delta}} \approx 68^o \ ,$$

$$\psi = \arccos \frac{-\frac{2}{\sqrt{3}} A_2 \sin\delta}{\sqrt{\frac{4}{3}A_2^2 + \frac{2}{3}A_0^2 - \frac{4\sqrt{2}}{3}A_0 A_2 \cos\delta}} \approx 175^o$$

From experimental values of C_{ik} we can determine P and δ_p - Gronau-London pass;

since experimental uncertainty in C_{00} is big while Belle and BABAR contradicts each other in C_{+-} this pass is (temporary) closed

Let us look which values of direct asymmetries follow from our formulas. With $P = P_f$ we get:

$$C_{+-} = -0.28 \cos(\delta_p + 68^o) \quad ,$$

and for the theoretically motivated value $\delta_p \leq 30^o$ we obtain:

$$0 > C_{+-} > -0.10 \quad ,$$

which is close to BABAR result.
For direct CP asymmetry in $B_d \to \pi^0 \pi^0$ decay we get:

$$C_{00} = 0.4 \cos \delta_p \approx 0.4 \quad ,$$

which differs in sign from C_{+-} and is rather big. It is very interesting to check these predictions experimentally.

α

S_{+-} is not changed when penguins are taken into account:

$$\alpha = \alpha^T + \tilde{\alpha} \quad ,$$

$$\tilde{\alpha} = -\left|\frac{V_{td}}{V_{ub}}\right| P(1 + C_{+-}) \sin\alpha \frac{\cos(\delta_p - \kappa)}{\sqrt{\frac{1}{12}A_2^2 + \frac{1}{6}A_0^2 + \frac{1}{3\sqrt{2}}A_0 A_2 \cos\delta}} \quad ,$$

$$\kappa = \frac{\pi}{2} - \varphi \quad .$$

$$\tilde{\alpha} = -2.4(1+C_{+-})P\cos(\delta_p-\kappa) = -0.14(1+C_{+-})\cos(\delta_p-22^o) \quad ,$$

$$\tilde{\alpha}_{\mathrm{BABAR}} \approx -7^o \,, \quad \alpha_{\mathrm{BABAR}} = \alpha^T_{\mathrm{BABAR}} + \tilde{\alpha}_{\mathrm{BABAR}} = 92^o \pm 5^o \quad .$$

$$\tilde{\alpha}_{\rm average} = -5^o \ , \ \ \alpha_{\rm average} = \alpha^T_{\rm average} + \tilde{\alpha}_{\rm average} = 100^o \pm 4^o \ \ .$$

Theoretical uncertainty of the value of α can be estimated in the following way. Let us suppose that the accuracy of the factorisation calculation of the penguin amplitude is 100%:

$$\tilde{\alpha}_{\rm average} = -5^o \pm 5^o_{\rm theor} \ \ ,$$

$$\alpha_{\rm average} = 100^o \pm 4^o_{\rm exp} \pm 5^o_{\rm theor}$$

$B \to \rho\rho$

Better theor accuracy of α follows from $B \to \rho\rho$ decays, where penguin contribution is 2 times smaller, and FSI phases are small:

$$\alpha^{\rho\rho} = 92^o \pm 7^o_{exp} \pm 4^o_{theor}$$

The model independent isospin analysis of $B \to \rho\rho$ decays performed by BABAR gives:

$$\alpha^{\rho\rho}_{BABAR} = 100^o \pm 13^o$$

The global CKM fit results are:

$$\alpha_{CKMFitter} = 98^o \pm 8^o \quad , \quad \alpha_{UTfit} = 95^o \pm 5^o$$

Conclusions

- The moduli of the amplitudes A_0 and A_2 are given with good accuracy by factorisation; FSI phase shift is very large, $\delta \approx 50^o$.
- Theoretical uncertainty of the value of α extracted from $B \to \pi\pi$ data on S_{+-} is at the level of few degrees.
- Resolution of the contradiction of Belle and BABAR data on CP asymmetries are very important both for checking the correctness of our approach (C) and determination of angle α (S).