γ extraction with 4-body D decays in $B^{\pm} \to DK^{\pm}$.

- Intro: γ from $B_u \to D^oK$ with 2 and 3 body D decays
- New: 4-body D decays like: $D^0 \to K^+K^-\pi^+\pi^-$
- Some Results from Toy MC studies.

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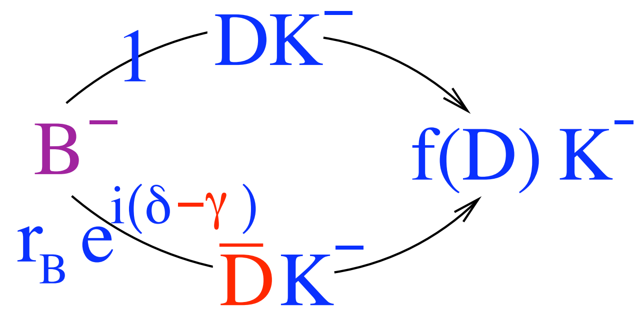
Many thanks to David Asner and Alberto Correa dos Reis for invaluable help.

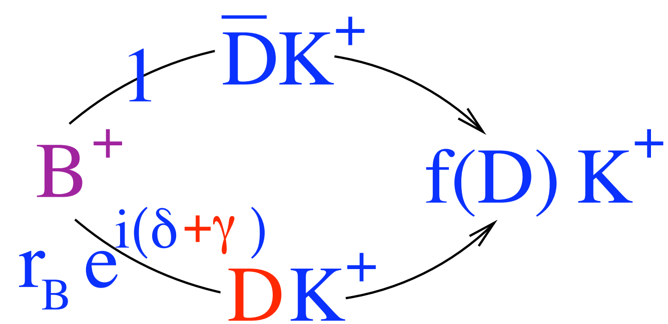
Flavour in the era of the LHC, WG2, 17 May 2006

γ from $\mathrm{B}^{\pm} \to \mathrm{DK}^{\pm}$

- Use interference between $\mathrm{B}^{\pm} \to \mathrm{D}^0\mathrm{K}^{\pm}$ and $\mathrm{B}^{\pm} \to \bar{\mathrm{D}}^0$ where D^0 and $\bar{\mathrm{D}}^0$ decay to the same final state f_D.

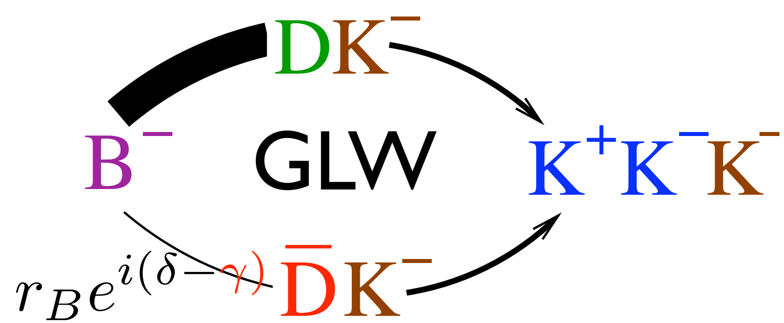


- No time dependence - simple.
- No tagging - exploit full statistics.
- $\frac{\langle B^- \to \bar{D}^0 K^- \rangle}{\langle B^- \to D^0 K^- \rangle} = r_B e^{i(\delta - \gamma)}$

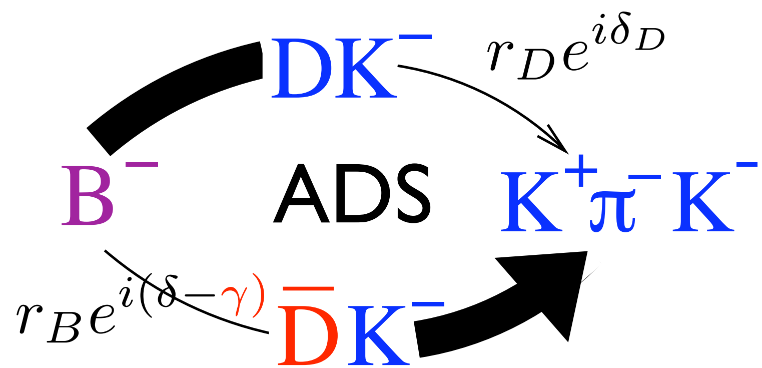


γ from $B^{\pm} \to DK^{\pm}$

- f(D) can be a CP-eigenstate (GLW) - advantage:
$$\frac{\langle D^0 \to f_{CP}\rangle}{\langle \bar{D}^0 \to f_{CP}\rangle} = 1$$



- ADS: Favoured B decay goes with the Cabbibo suppressed D decay. Interfering Amplitudes of similar size lead to larger interference effects.



2 body K^+K^-, $\pi^+\pi^-$, $K\pi$

3 body $K_S\pi^+\pi^-$ $K_S K^+K^-$

New, 4 body: $K^+K^-\pi^+\pi^-$ new
$K^+\pi^-\pi^+\pi^-$

2 body D decays, γ from counting decay rates.

- Extract γ simply by counting event numbers.
- Asymmetries can be very large with ADS, e.g:

$$\frac{\Gamma\left(\mathrm{B}^- \to \mathrm{D}(\mathrm{K}^-\pi^+)\mathrm{K}^-\right) - \Gamma\left(\mathrm{B}^+ \to \mathrm{D}(\mathrm{K}^+\pi^-)\mathrm{K}^+\right)}{\Gamma\left(\mathrm{B}^- \to \mathrm{D}(\mathrm{K}^-\pi^+)\mathrm{K}^-\right) + \Gamma\left(\mathrm{B}^+ \to \mathrm{D}(\mathrm{K}^+\pi^-)\mathrm{K}^+\right)} \approx 0.4$$

- Charged B's good for hadron machines where tagging is harder. But need to know production asymmetry.
- LHCb: ~60k $\mathrm{B}^\pm \to \mathrm{D}(\mathrm{K}\pi)\mathrm{K}$, 8k $\mathrm{B}^\pm \to \mathrm{D}(\mathrm{KK}/\pi\pi)\mathrm{K}$ p.a. Expected $\sigma(\gamma)$ from ADS+GLW at LHCb ~5deg.
- See Mithesh Patel's talk in prev. workshop (Mo 06-Feb-06, WG2) and Guy Wilkinson's note LHCb-2005-066.

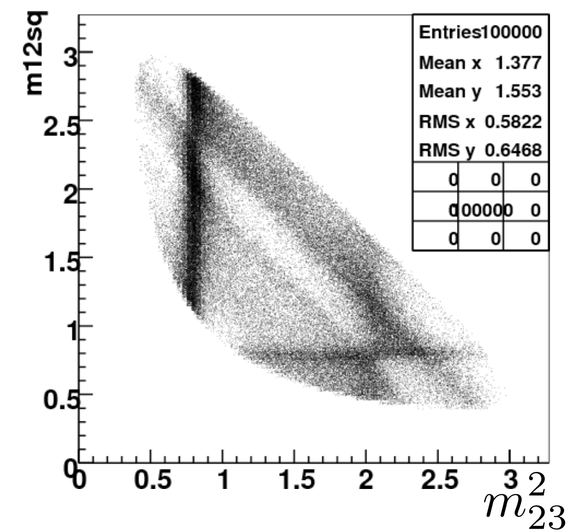
3-body D decays and Dalitz Plots

There are many paths from $\mathrm{D^O}$ to $\mathrm{K_S}\pi\pi$

$$\mathrm{D}^0 \rightarrow \quad \rightarrow \mathrm{K_S}\pi\pi$$

Intermediate state	Amplitude $|c_j|$	Phase δ_j (°)
$K^*(892)^+\pi^-$	1.656 ± 0.012	137.6 ± 0.6
$K^*(892)^-\pi^+$	$(14.9 \pm 0.7) \times 10^{-2}$	325.2 ± 2.2
$K_0^*(1430)^+\pi^-$	1.96 ± 0.04	357.3 ± 1.5
$K_0^*(1430)^-\pi^+$	0.30 ± 0.05	128 ± 8
$K_2^*(1430)^+\pi^-$	1.32 ± 0.03	313.5 ± 1.8
$K_2^*(1430)^-\pi^+$	0.21 ± 0.03	281 ± 9
$K^*(1680)^+\pi^-$	2.56 ± 0.22	70 ± 6
$K^*(1680)^-\pi^+$	1.02 ± 0.2	103 ± 11
$K_S\rho^0$	1.0 (fixed)	0 (fixed)
$K_S\omega$	$(33.0 \pm 1.3) \times 10^{-3}$	114.3 ± 2.3
$K_S f_0(980)$	0.405 ± 0.008	212.9 ± 2.3
$K_S f_0(1370)$	0.82 ± 0.10	308 ± 8
$K_S f_2(1270)$	1.35 ± 0.06	352 ± 3
$K_S\sigma_1$	1.66 ± 0.11	218 ± 4
$K_S\sigma_2$	0.31 ± 0.05	236 ± 11
non-resonant	6.1 ± 0.3	146 ± 3

3-body decay fully parametrised by 2 parameters:



- Every point in Dalitz plot ~ different decay channel, with different strong phase δ . Counting effectively averages over the entire plot.

- Should get much better precision than simple counting. The ***shape*** of the Dalitz plot is fitted, not the absolute numbers of B+, B-, no need to know prod. ratio.

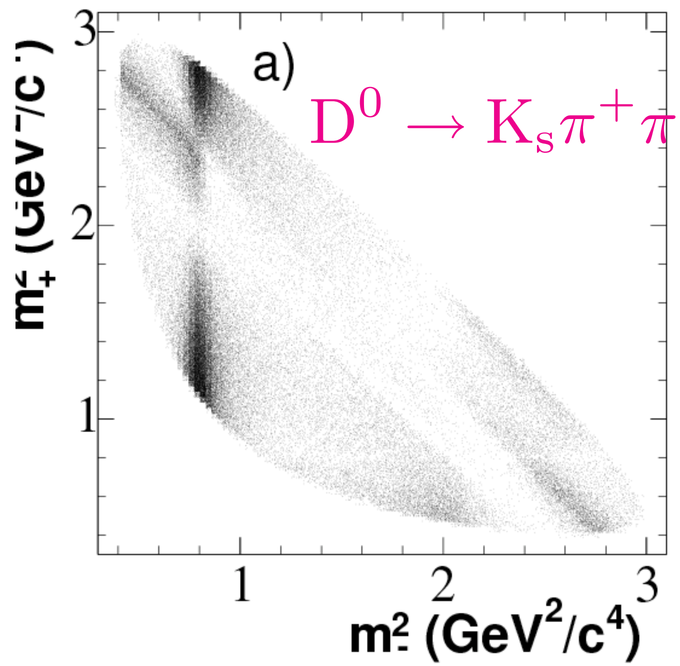
- Same argument applies to $\mathrm{B_u} \rightarrow \mathrm{D(KK\pi\pi)K}$ and $\mathrm{B_u} \rightarrow \mathrm{D(K\pi\pi\pi)K}$.

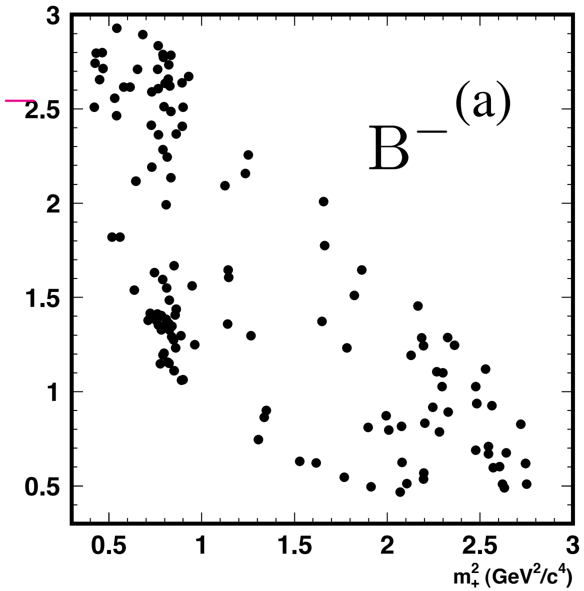
Dalitz Plot Analyses at Belle&BaBar

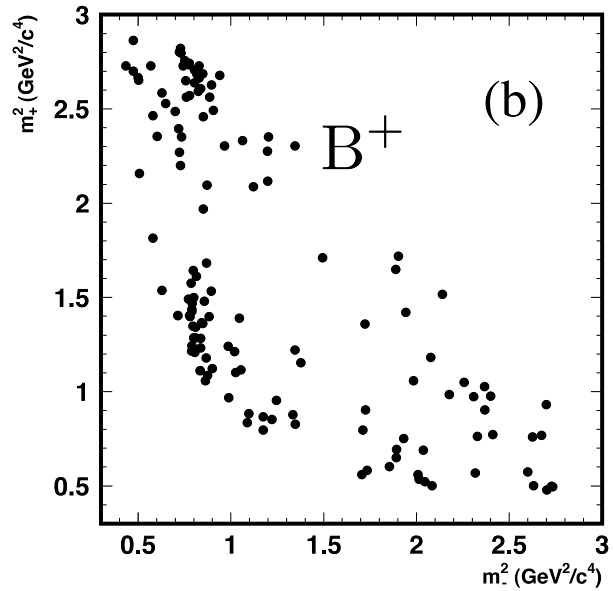
(Plots from BELLE)

$A_D \equiv A(D^0 \to K_s\pi^+\pi^-)$ $\quad m_- \equiv M(K_s^0\pi^-), m_+ \equiv M(K_s^0\pi^+)$

$$A\left[\mathrm{B}^- \to (\mathrm{K_s}\pi^+\pi^-)_\mathrm{D}\mathrm{K}^-\right](m_-^2, m_+^2) = A_D(m_-^2, m_+^2) + r_B e^{i(\delta_B - \gamma)} A_D(m_+^2, m_-^2)$$







- BELLE's result: $\gamma = 64^o \pm 19^o \pm 13^o \pm 11^o$ for 209 DK events (w/o D*)
- BaBar's result: $\gamma = 67^o \pm 28^o \pm 13^o \pm 11^o$ for 282 DK, 134D*K evts

BRs, Yields of 3 and 4 body Dalitz channels

$D^0 \to \ldots$	B.R.	
$K_S\pi^+\pi^-$	2.9%	BaBar/Belle
$K_S K^+ K^-$	0.51%	new gamma channel
$K^+K^-\pi^+\pi^-$	0.25%	new gamma channel, this talk
$K^+\pi^-\pi^+\pi^-$	7.5%	new Dalitz channel

4 body channels promising for an experiment like LHCb:

o Only charged particles in final state, no Ks.

o Kaons can be identified by RICH.

LHCb yield p.a.: ~1.5k KKππ, 60k Kπππ (~1-2k doubly Cabbibo supr.)

(Yields are <u>*guesstimates*</u> based on yields in LHCb reopt TDR (CERN/LHCC 2003-040) for topologically similar channels, and B.R.s)

4-body Dalitz: What's different?

- Need 5 variables instead of 2 to describe decay. With the following numbering scheme $D^o_0 \rightarrow K^+_1 K^-_2 \pi^+_3 \pi^+_4$, define:

$$
\begin{aligned}
t_{01} &= (p_0 - p_1)^2 = (p_2 + p_3 + p_4)^2 = s_{234} \\
\underline{s_{12}} &= (p_1 + p_2)^2 \\
\underline{s_{23}} &= (p_2 + p_3)^2 \\
s_{34} &= (p_3 + p_4)^2 \\
t_{40} &= (p_4 - p_0)^2 = (p_1 + p_2 + p_3)^2 = s_{123}
\end{aligned}
$$

- From this, get all 4 momenta, and remaining 2, 3 body masses: $\underline{s_{13}}, s_{14}, s_{24}, s_{124}, s_{134}$

- Resonance structure is a bit more complex, including decay-chains with intermediate states, etc.

- For 3 body decays, phase space is flat in s_{12}, s_{23} . It is not for any sensible set of 5 variables describing 4 body decays.

- And, of course, it hasn't been done before to extract γ.

Fit Model

$$B^{\pm} \rightarrow D(K_1^{\pm} K_2^{\mp} \pi_3^{\pm} \pi_4^{\mp}) K^{\pm}$$

Unbinned likelihood fit.

PDF = $|A_B(s_{234}, s_{12}, s_{23}, s_{34}, s_{123})|^2 \cdot (\text{phase space})_4$

with $A_B(s_{234}, s_{12}, s_{23}, s_{34}, s_{123}) =$

$$A_D(s_{234}, s_{12}, s_{23}, s_{34}, s_{123})$$

$$r_B\, e^{i(\delta-\gamma)} \quad A_D(s_{134}, s_{12}, s_{14}, s_{34}, s_{124})$$

and $A_D = \sum |c_j| e^{i\phi_j} A_j$

$$A_j = F_D \cdot s_J \cdot F_r \cdot BW$$

$$A_j = F_D \cdot s_J \cdot F_{rA} \cdot BW_A \cdot F_{rB} \cdot BW_B$$

Form factors

(Blatt-Weisskopf Penetration Factors)

and $BW = \dfrac{1}{m^2 - s - im\Gamma(s)}$

spin factor

Strategy

- Assume that the coefficients c_j and the phases ϕ_j in $\mathrm{A(D \to KK\pi\pi).= \sum |c_j| e^{i\phi_j} A_j}$ are well known (i.e. don't fit them, keep them as constants). Here we take them FOCUS Phys.Lett. B610 (2005) 225-234 (hep-ex/0411031).

- Only fit the bit that's B specific, i.e. $\mathrm{r_B}$, δ and γ in

$$\mathrm{A_B(s_{234}, s_{12}, s_{23}, s_{34}, s_{123})} =$$
$$\mathrm{A_D(s_{234}, s_{12}, s_{23}, s_{34}, s_{123})}$$
$$\mathrm{r_B\ e^{i(\delta-\gamma)}} \quad \mathrm{A_D(s_{134}, s_{12}, s_{14}, s_{34}, s_{124})}$$

- No backg. Ignore all detector effects/efficiencies.

4 Body Amplitudes (FOCUS)

Phys.Lett. B610 (2005) 225-234 hep-ex/0411031

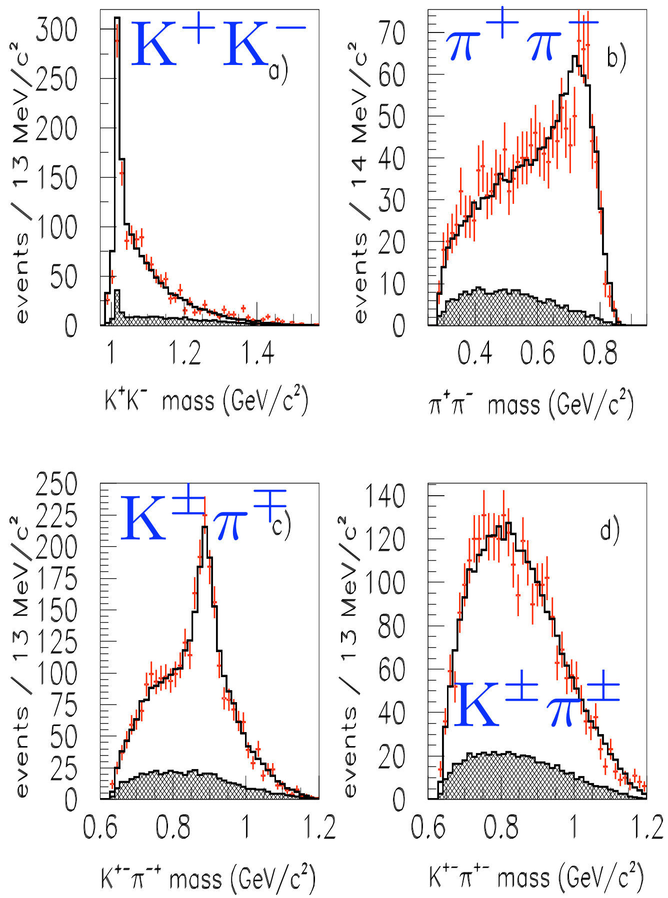
Mode	Magnitude	Phase	Fraction (%)
$K_1(1270)^+K^-,\ K_1 \to \rho(770)^0K^+$	1 (fixed)	0 (fixed)	$18 \pm 6 \pm 3$
$K_1(1270)^+K^-,\ K_1 \to K_0^*(1430)\pi^+$	$0.27 \pm 0.08 \pm 0.06$	$354 \pm 19 \pm 19$	$2 \pm 1 \pm 0$
$K_1(1270)^+K^-,\ K_1 \to K^*(892)^0\pi^+$	$0.94 \pm 0.16 \pm 0.13$	$12 \pm 12 \pm 15$	$16 \pm 4 \pm 5$
$K_1(1270)^+K^-$, (all modes)	—	—	$33 \pm 6 \pm 4$
$K_1(1400)^+K^-$	$1.18 \pm 0.19 \pm 0.09$	$259 \pm 11 \pm 13$	$22 \pm 3 \pm 4$
$K^*(892)0\bar{K}^*(892)^0$	$0.39 \pm 0.09 \pm 0.11$	$28 \pm 13 \pm 10$	$3 \pm 2 \pm 1$
$\phi(1020)\rho(770)^0$	$1.30 \pm 0.11 \pm 0.07$	$49 \pm 11 \pm 12$	$29 \pm 2 \pm 1$
$\rho(770)^0K^+K^-$	$0.33 \pm 0.12 \pm 0.16$	$278 \pm 26 \pm 20$	$2 \pm 2 \pm 2$
$\phi(1020)\pi^+\pi^-$	$0.30 \pm 0.06 \pm 0.06$	$163 \pm 16 \pm 15$	$1 \pm 1 \pm 0$
$K^*(892)^0K^+\pi^-$	$0.83 \pm 0.09 \pm 0.10$	$234 \pm 10 \pm 11$	$11 \pm 2 \pm 1$
$f_0(980)\pi^+\pi^-$	$0.91 \pm 0.13 \pm 0.05$	$240 \pm 11 \pm 17$	$15 \pm 3 \pm 2$

The sign of each spin factor sJ is purely a matter of convention which can vary from amplitude to amplitude - so the phases in this table are only defined up to +/- 180 deg.

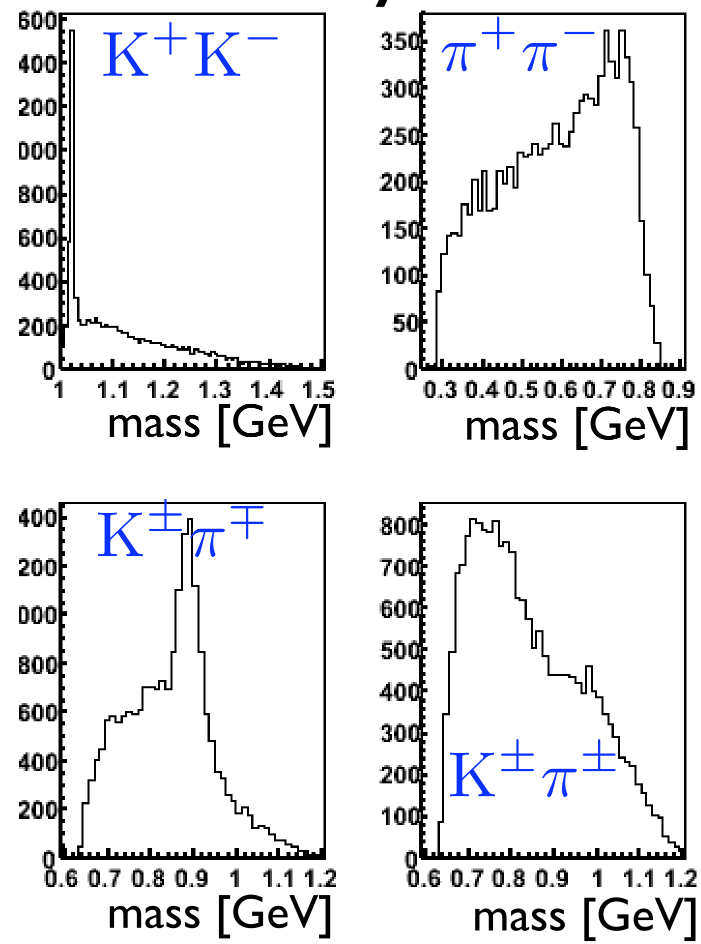
Mass distributions

$D \rightarrow K^+K^-\pi^+\pi^-$

FOCUS



Our Toy MC

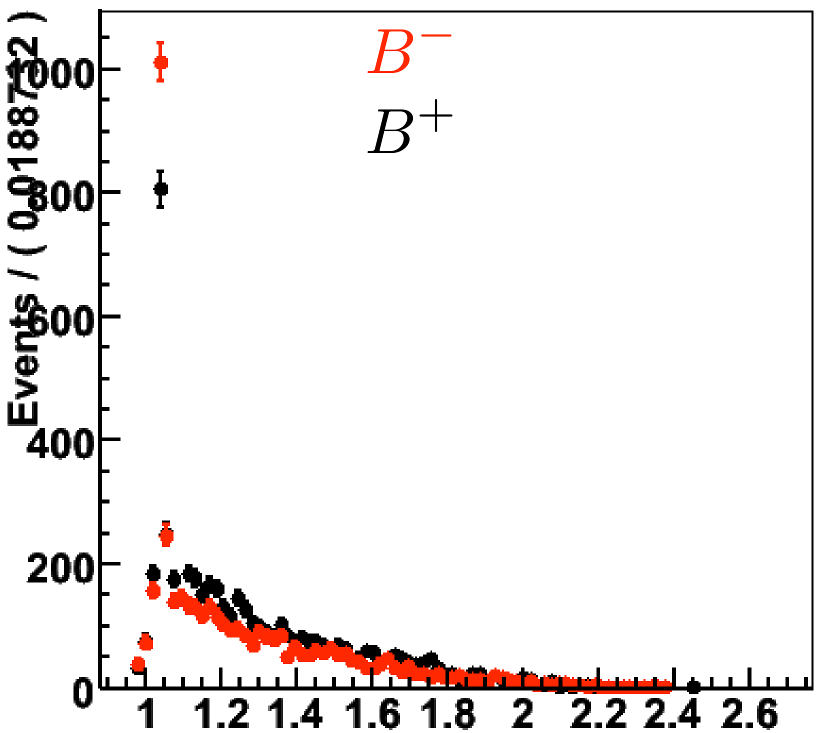


Basic features reproduced - better agreement expected once phase-ambiguities are resolved.

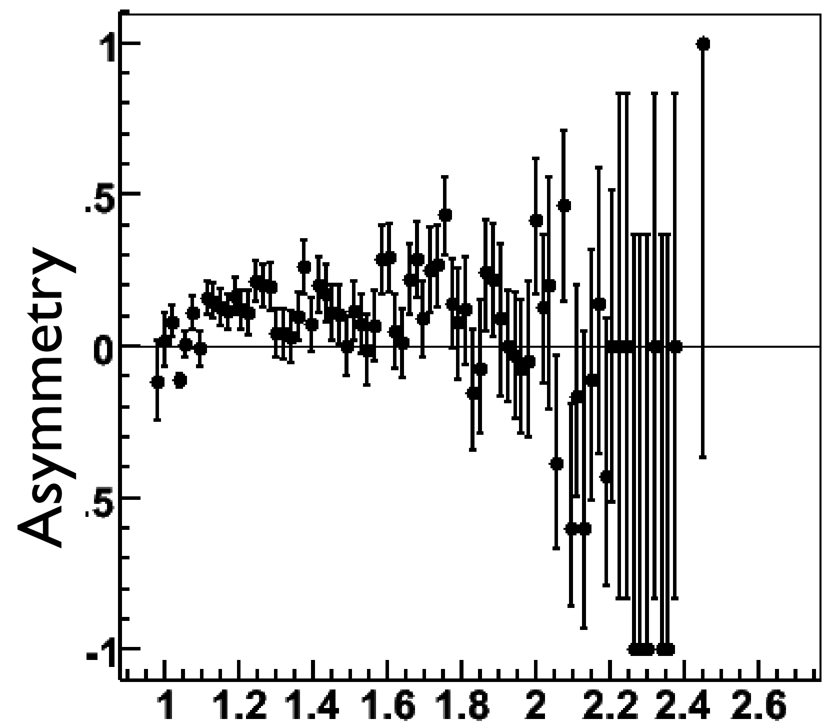
~10k events

5k B+, 4.4k B-

$$r_B = 0.15, \delta = 130^o, \gamma = 60^o$$



$$s_{12} = M(K^+K^-)^2$$



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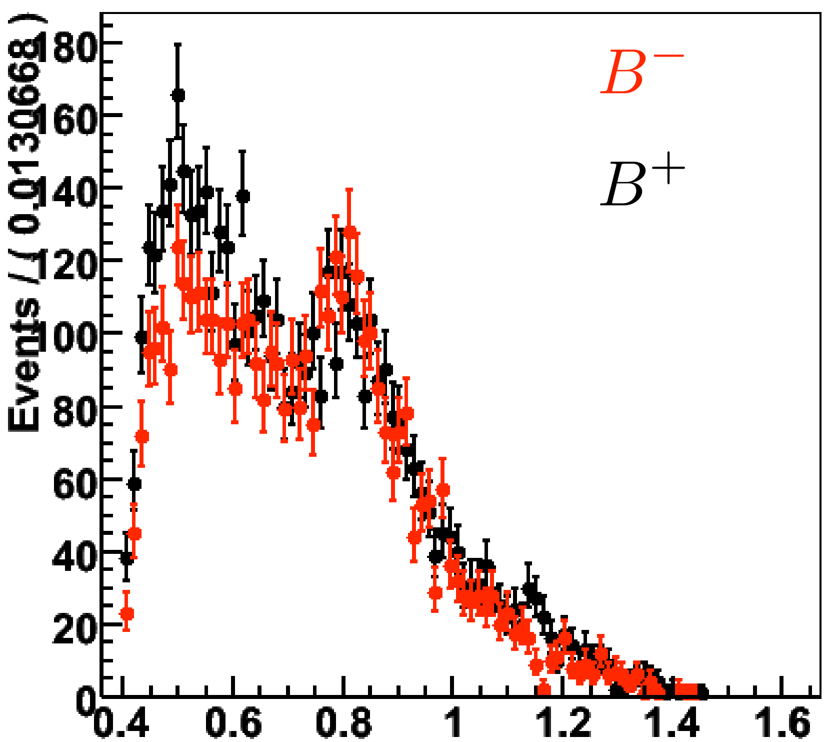
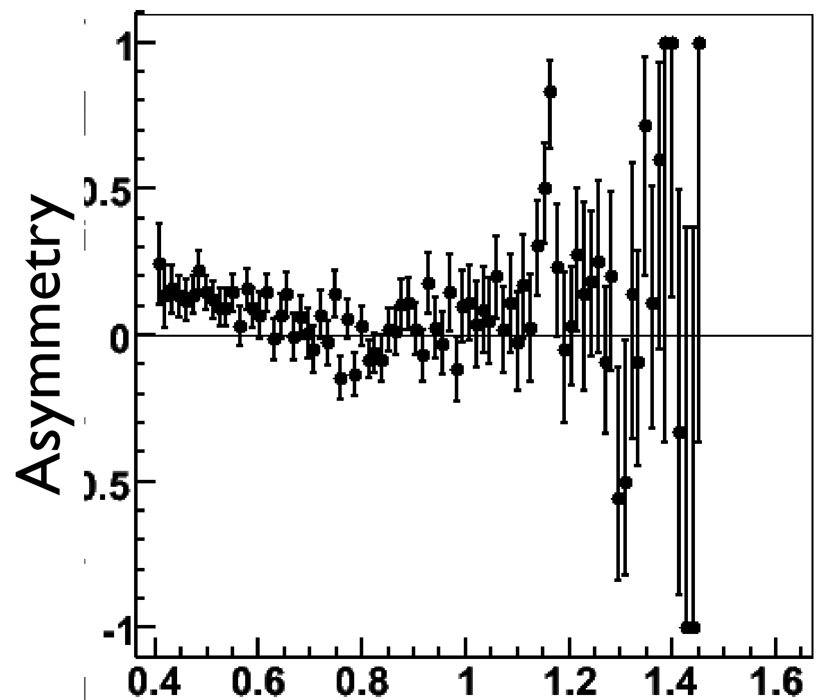


$s_{23} = M(K^-\pi^+)^2$

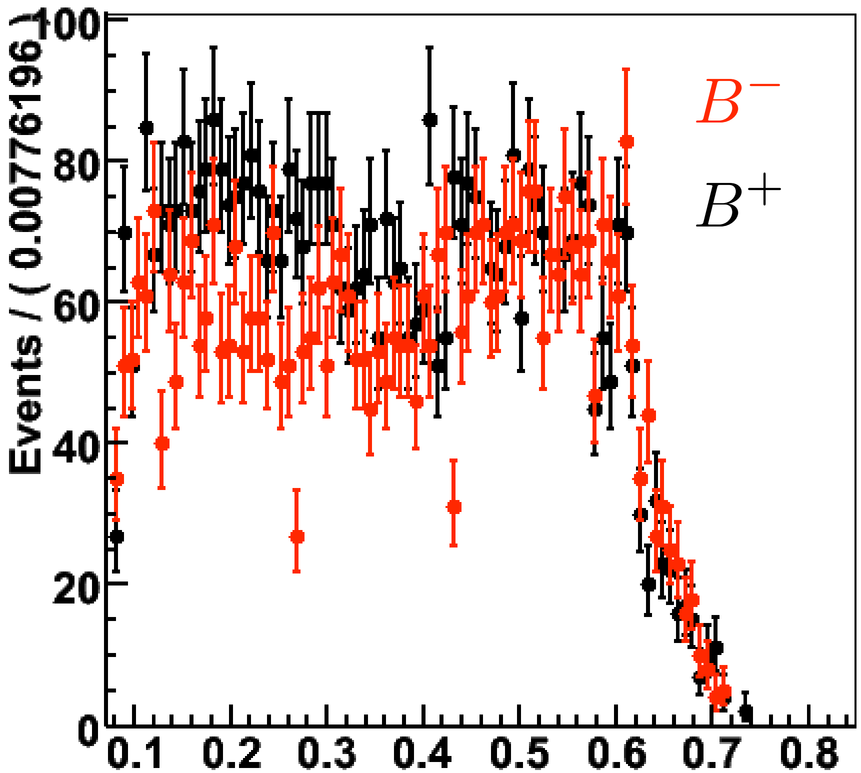


$s_{23} = M(K^-\pi^+)^2$

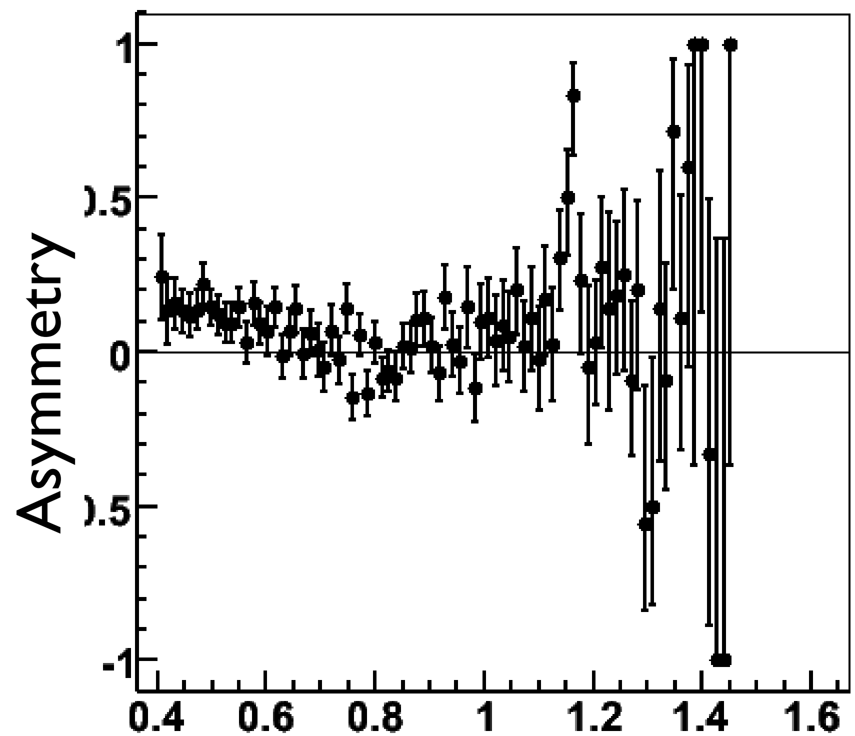
~10k events

5k B+, 4.4k B-

$$r_B = 0.15, \delta = 130^o, \gamma = 60^o$$



$s_{34} = M(\pi^+\pi^-)^2$

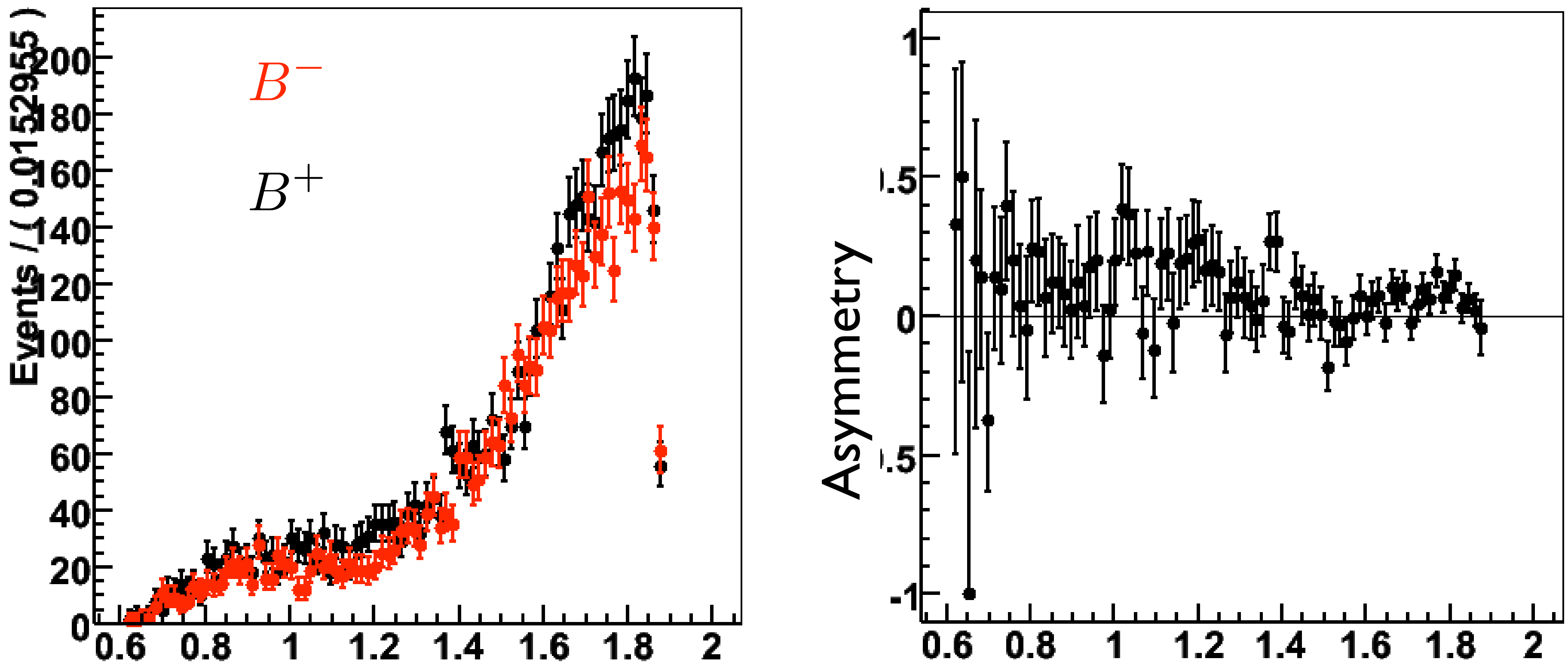


$s_{34} = M(\pi^+\pi^-)^2$

~10k events

5k B+, 4.4k B-

$$r_B = 0.15, \delta = 130^o, \gamma = 60^o$$

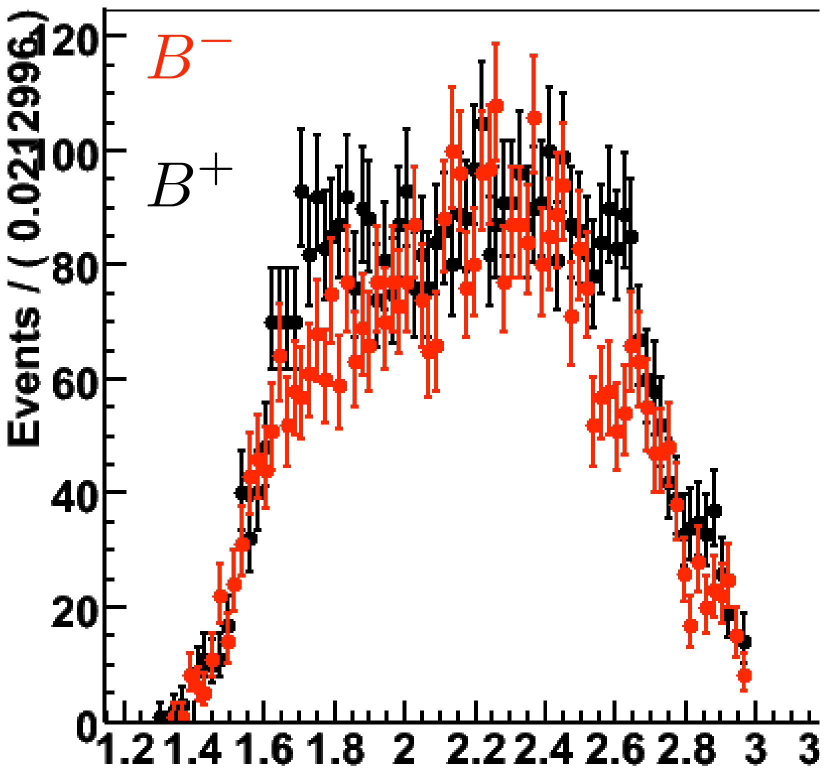


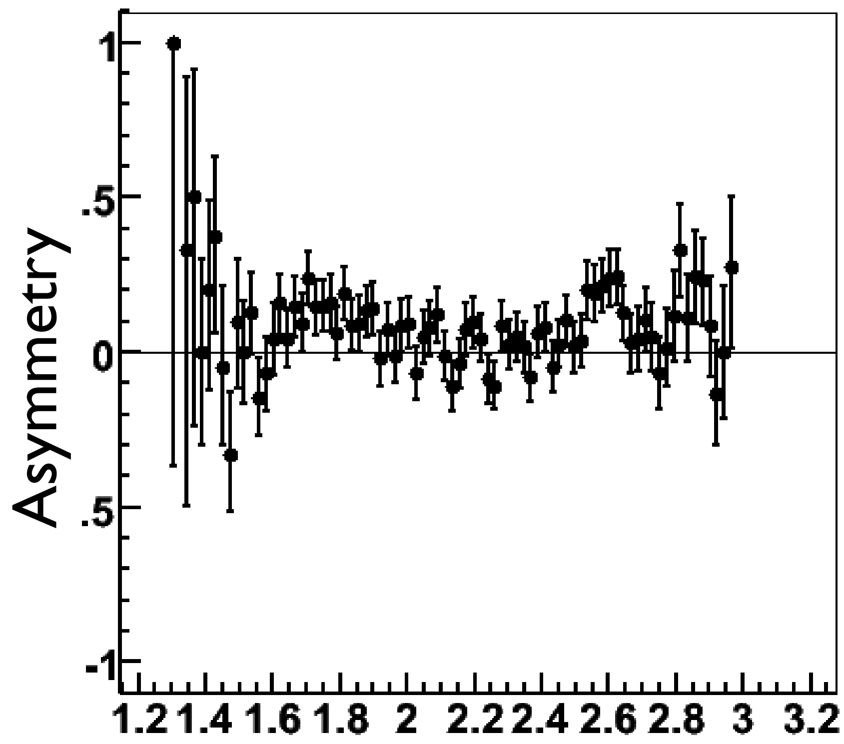
$$t_{01} = s_{234} = M(K^-\pi^+\pi^-)^2 \qquad t_{01} = s_{234} = M(K^-\pi^+\pi^-)^2$$

~10k events

5k B+, 4.4k B-

$$r_B = 0.15, \delta = 130^o, \gamma = 60^o$$





$$t_{40} = s_{123} = M(K^+K^-\pi^+)^2 \qquad t_{40} = s_{123} = M(K^+K^-\pi^+)^2$$

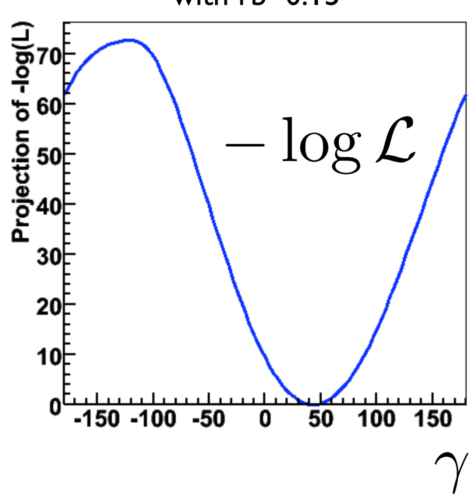
Fitting γ with $B_u \to D^0(KK\pi\pi)K$.

Input values: $\gamma = 60^o \quad \delta = 130^o, r_B = 0.15$

Fitting 1k events

$$\gamma = 60.6^o \pm 9.7^o$$
$$\delta = 132.0^o \pm 9.8^o$$
$$r_B = 16.0\% \pm 2.2\%$$

Means = average of 5 toy experiments.
Errors = MINUIT errors for a typical result with rb=0.15



Fitting 10k events:

$$\gamma = 64.6^o \pm 3.1^o$$
$$\delta = 132.5^o \pm 3.1^o$$
$$r_B = 15.0\% \pm 0.7\%$$

Result from a single toy experiment.

With guessed LHCb yields (~1.5k/year), expect $\sigma(\gamma) \sim 10^o$ after 1 year.

Result for different r_B, for 1k events (full fit).

rB(fixed)	gamma fit, 4 body
0.10	$55.2^o \pm 14.2^o$
0.15	$53.4^o \pm 9.9^o$
0.20	$53.1^o \pm 7.4^o$
0.25	$53.2^o \pm 6.3^o$

Data samples for different values of rB are not independent.

DISCLAIMER: These results are from an earlier version of the Toy MC that did not reproduce the FOCUS data quite as well. Where we checked, the old and new version give the same precision on γ though.

Note: For the purpose of this study, r_B had to be fixed, because MINUIT calculates the error on γ for the value of r_B at the end of the fit. The actual precision on γ is not affected when r_B is a fitted simultaneously.

5D asymmetry

- Rather than extracting gamma from a fit to the absolute PDFs, fit for each set of $s_{234}, s_{12}, s_{23}, s_{34}, s_{123}$ the binomial probability to find a B+ event rather than a B- event (and vice versa):

$$P_+ = \frac{P(B^+)}{P(B^+) + f \cdot P(B^-)}$$

- Acceptance effects cancel.

- Additional fit parameter f for relative normalisation of P (B+), P(B-) makes fit independent of production rates - also, no need to get relative normalisation between P(B+), P(B-), saves a lot of time.

Fits with 5D asymmetry method

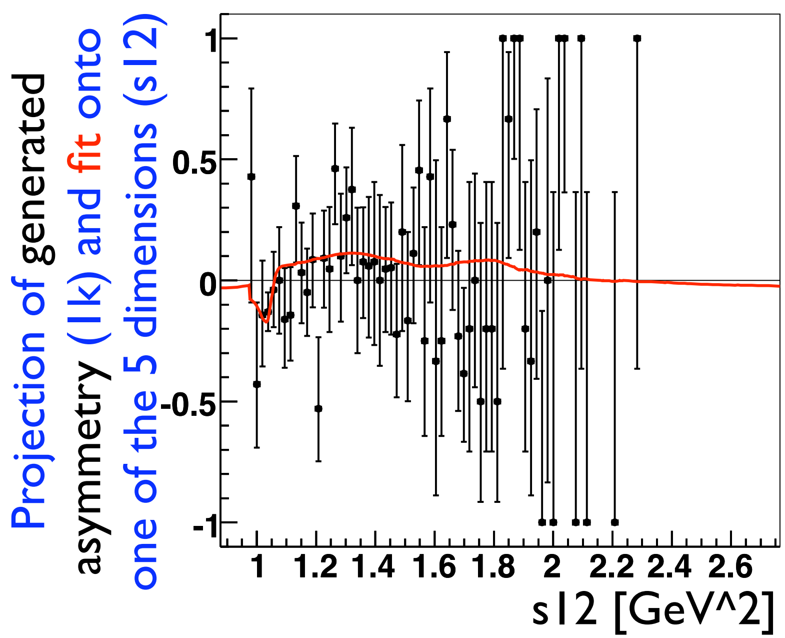
input values: $\gamma = 60^o, \delta = 130^o, r_B = 0.15$

1k events

$$\begin{aligned} \gamma &= 112^{o}{}^{+20^o}_{-30^o} \\ \delta &= 124^o \pm 11^o \\ r_B &= 14\% \pm 3\% \end{aligned}$$

10k events

$$\begin{aligned} \gamma &= 55.4^o \pm 8.9^o \\ \delta &= 126.3^o \pm 3.8^o \\ r_B &= 15.7\% \pm 1.8\% \end{aligned}$$



It works, but at a price: Need ~10 times as many events for the same error as full fit. It'll be a trade off between statistics and systematic.

Conclusion

- Introduced new 4-body "Dalitz" method to extract γ. Successfully fitted γ to Toy-MC generated $\mathrm{B}^{\pm} \rightarrow \mathrm{D}(\mathrm{KK}\pi\pi)\mathrm{K}^{\pm}$ events.

- Extracting γ with using $\mathrm{B}^{\pm} \rightarrow \mathrm{D}(\mathrm{KK}\pi\pi)\mathrm{K}^{\pm}$ particularly suited to "messy" hadron environment: No tagging, no neutrals.

- Precision: $\sigma(\gamma) \sim 10^{o}$ for 1k events (no Bg, detector effects). Complementary to other B->DK methods - combined fit possible. Full study will be pursued within the context of LHCb.

- Alternative method introduced: Unbinned Dalitz *Asymmetries*. Acceptance effects cancel, but it's costly in stat. precision

- Method should work for other modes, eg. $\mathrm{B}^{\pm} \rightarrow \mathrm{D}(\mathrm{K}\pi\pi\pi)\mathrm{K}^{\pm}$ with 30 times larger BR (Cabbibo-supressed channel on its own has similar BR, but larger asymmetry due to ADS effect).

Backup

Next: $B_u \to D(K_s\pi\pi\pi)K$.

- Expect 60k $B_u \to D(K\pi\pi\pi)K$ evts/year.

- For the ~1.4k Cabbibo supressed decays, interference effects should be stronger than for the ~1k $B_u \to D(KK\pi\pi)K$ (ADS vs GLW)

- Just counting total rates (~ averaging over the entire Dalitz Space) gives $\sigma(\gamma) \sim 7^o$ (see Guy's note).