# $B \to K^*\ell^+\ell^-$ decay in soft-collinear effective theory

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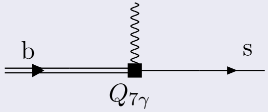
based on hep-ph/0601034, with A. Ali and G. Kramer

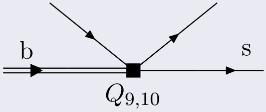
# Outline

# Why studies $B \to K^*(X_s)\ell^+\ell^-$

- $B \to K^*\ell^+\ell^-$, $X_s\ell^+\ell^-$ depend mainly on $Q_{7\gamma}$, $Q_{9,10}$.




$$Q_{7\gamma} = -\frac{g_{em}\overline{m}_b}{8\pi^2}\bar{s}\sigma^{\mu\nu}(1+\gamma_5)bF_{\mu\nu}$$




$$Q_{9,10} = \frac{\alpha_{em}}{2\pi}(\bar{s}b)_{V-A}\bar{\ell}\gamma^\mu\gamma_5\ell$$

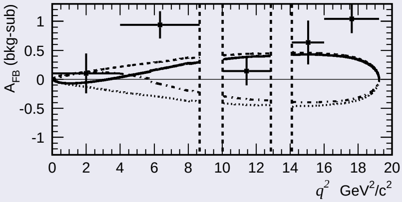
- Forward-backward Asymmetry

$$\frac{dA_{FB}}{dq^2} = \frac{1}{d\Gamma/dq^2}\left(\int_0^1 d\cos\theta \frac{d^2\Gamma}{dq^2 d\cos\theta} - \int_{-1}^0 d\cos\theta \frac{d^2\Gamma}{dq^2 d\cos\theta}\right)$$

- Determining the sign of $C_9/C_7 \Longrightarrow$ SM or NP
- Zero-point of the FB asymmetry: $A_{FB}(q_0^2) = 0$, almost free of hadronic uncertainties [Burdman1998, Ali et al., 2000]
- Combining with $B \to K^*\gamma$, $X_s\gamma \Longrightarrow$ information on the magnitude of $C_i \Longrightarrow$ SM or NP

## Exp. status

$$\mathcal{B}(B \to K^*\ell^+\ell^-) = (16.5^{+2.3}_{-2.2} \pm 0.9 \pm 0.4) \times 10^{-7} \quad (Belle)$$
$$= (7.3^{+2.0}_{-1.8} \pm 1.1) \times 10^{-7} \quad (BaBar)$$




FB asymmetry (Belle06)

- Solid: C7,C9<0 C10>0
- Dashed: C7,C9>0 C10<0
- Dotted: ...
- Dot-dashed: ...

## Theory

- Exclusive decays: complicated hadronic dynamics
- Large recoil $\Longrightarrow$ SCET $[E_{K^*} >> m_{K^*}, q^2 < m^2_{J/\psi}]$
- SCET only valid in the region $q^2 < 8$ GeV$^2$

# Why needs effective theory

- Uncover the hidden symmetry
  - Heavy quark spin symmetry in HQET
    4 $B \to D^{(*)}$ form factors $\Longrightarrow$ Isgur-Wise function
  - SCET: 10 $B \to P, V$ form factors $\Longrightarrow \zeta_P, \zeta_\parallel, \zeta_\perp$.
- Simplify the factorization proof, to all orders of $\alpha_s$.
- Power corrections can be treated in a systematic way.
- Sudakov double logs may be resummed by solving RGE in effective theory.

# Basics of SCET

- Applicable to any QCD processes which contain collinear meson or jet, i.e. $P^2 \ll Q^2$, in the final states.
- For $B \to K^* \ell^+ \ell^-$ decay, in the region $1\ \text{GeV}^2 \le q^2 \le 8\ \text{GeV}^2$
$$P^{\mu}_{K^*} = (2.34, 0, 0, 2.16)\ \text{GeV} \qquad [q^2 = 4\ \text{GeV}^2]$$
- Light-cone vectors $n^{\mu} = (1, 0, 0, 1)$, $\bar{n}^{\mu} = (1, 0, 0, -1)$, satisfying $n^2 = \bar{n}^2 = 0$ and $n \cdot \bar{n} = 2$.

$$P^{\mu} = n \cdot P \frac{\bar{n}^{\mu}}{2} + \bar{n} \cdot P \frac{n^{\mu}}{2} + P^{\mu}_{\perp} = (P_+, P_-, P_{\perp}) \sim E(\lambda^2, 1, \lambda)$$
$$[P_+ = 0.18\ \text{GeV},\ P_- = 4.5\ \text{GeV},\ \lambda \sim 0.2]$$

- power counting and expansion in $\lambda$, $\lambda \sim \frac{\Lambda_{QCD}}{E}$ .

# Effective fields of SCET

## Hard mode (h)

$P \sim E(1,1,1)$, integrated out in QCD $\to$ SCET$_I$

## Hard-collinear mode (hc)

$P \sim E(\lambda, 1, \sqrt{\lambda})$, integrated out in SCET$_I$ $\to$ SCET$_{II}$,

$\xi_{hc} \sim \sqrt{\lambda} \quad A_{hc} \sim (\lambda, 1, \sqrt{\lambda})$

## collinear mode (c)

$P \sim E(\lambda^2, 1, \lambda)$, long-distance mode, $\quad \xi_c \sim \lambda \quad A_c \sim (\lambda^2, 1, \lambda)$

## soft mode (s)

$P \sim E(\lambda, \lambda, \lambda)$, long-distance mode, $\quad q_s \sim \lambda^{3/2} \quad A_s \sim (\lambda, \lambda, \lambda)$

# The factorization formula in SCET

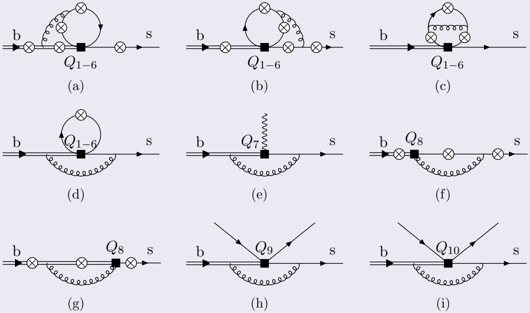
$$\langle K_a^* \ell^+ \ell^- | H_{eff} | B \rangle = T_a^I(q^2)\zeta_a(q^2) + \sum_{\pm} \int_0^\infty \frac{d\omega}{\omega} \phi_\pm^B(\omega) \int_0^1 du\ \phi_{K^*}^a(u) T_{a,\pm}^{II}(\omega, u, q^2)$$

where $a = \parallel, \perp$ denotes the polarization of the $K^*$ meson

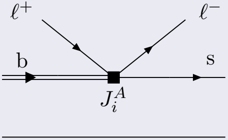
- formally coincide with the formula in BBNS [Beneke/Feldmann/Seidel2001], but valid to all orders of $\alpha_s$
- for $T^{II}$,the logarithms are summed from $\mu = m_b$ to $\sqrt{m_b \Lambda_h}$
- Compared with BBNS, the definition of $\zeta_{\perp,\parallel}$ is also different here

# SCET operators $J_i^A$

## Full QCD




## SCET$_I$




$\propto \zeta_{\perp,\parallel}(q^2)$

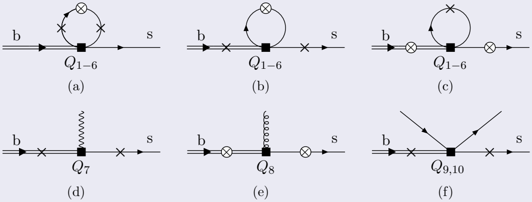
Crossed circles: the locations where the virtual photon is emitted

(a)-(d): calculated in $B \to X_s \ell^+ \ell^-$ [Asatryan/Asatryan/Greub/Walker,2001]

(e)-(i): Form factor analysis [Bauer/Fleming/Pirjol/Stewart,2001, Beneke/Kiyo/Yang,2004]

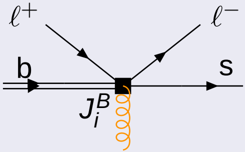
# SCET operators $J_i^B$

## Full QCD



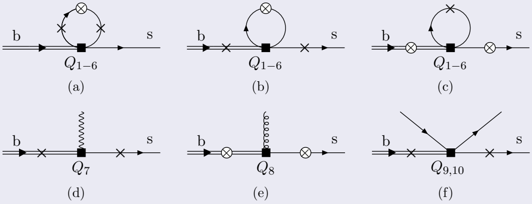

Crossed circles: the locations where the virtual photon is emitted. The crosses mark the possible places where a gluon line may be attached.

## SCET$_I$



# SCET operators $J_i^B$

## Full QCD

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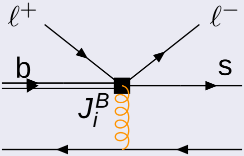
## SCET$_I$



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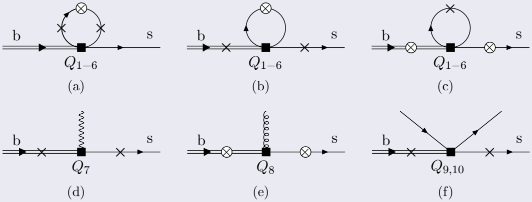


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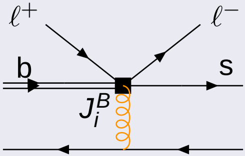
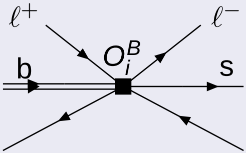
## SCET$_I$

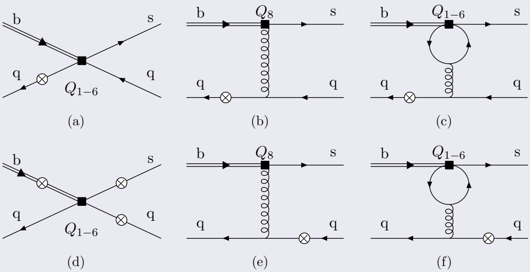



## SCET$_{II}$



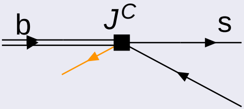
# SCET operators $J^C$

## Full QCD: photon from spectator quark




Crossed circles: the locations where the virtual photon is emitted. The last 3 diagrams are $1/m_b$ suppressed.

## SCET$_I$



# SCET operators $J^C$

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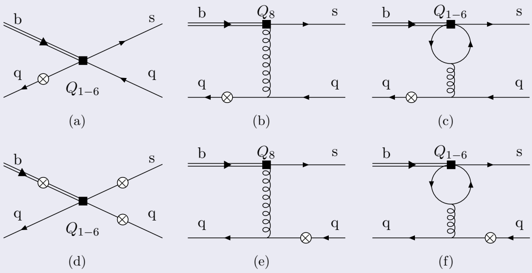


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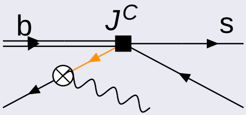
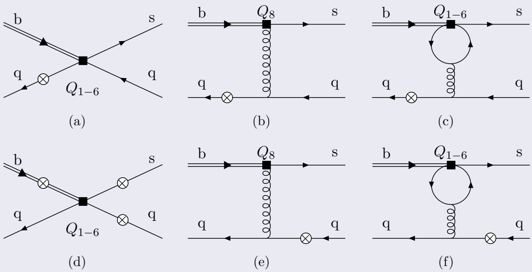
## $\text{SCET}_I$



# SCET operators $J^C$

## Full QCD: photon from spectator quark

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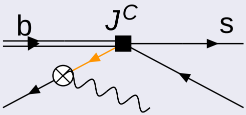
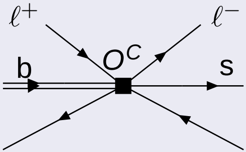
## SCET$_I$




## SCET$_{II}$



# RGE of SCET$_I$

$$\begin{array}{ccccccc} & \text{hard} & & & \text{hard collinear} & \\ \text{QCD} & \Longrightarrow & \text{SCET}_I & \dashrightarrow & \text{SCET}_I & \Longrightarrow & \text{SCET}_{II} \\ & m_b & & \text{RGE} & & \sqrt{m_b \Lambda_{QCD}} & \end{array}$$

**$J_i^A$**

$$\langle K^*(p)|\bar{\mathfrak{X}}_{hc}\Gamma h|B(v)\rangle = -2E\zeta_{\perp,\parallel}(E)tr[\overline{\mathcal{M}}_{K^*}(n)\Gamma\mathcal{M}_B(v)]$$

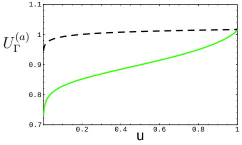
**$J^C$**

RGE unknown, but numerically $J^C$ contribution is very small, irrelevant for $A_{FB}$ $\Longrightarrow$ minor impact on phenomenology

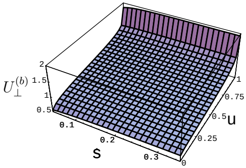
# RGE of SCET$_I$

$$\underset{\text{hard}}{\text{QCD} \Longrightarrow \text{SCET}_I} \dashrightarrow \underset{\text{hard collinear}}{\text{SCET}_I \Longrightarrow \text{SCET}_{II}}$$

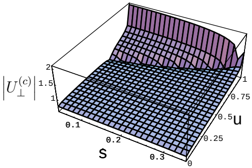
$m_b$ RGE $\sqrt{m_b \Lambda_{QCD}}$



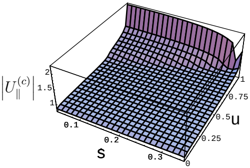

(a)




(b)




(c)




(d)

$\mu_h = 4.8$ GeV $\to 1.5$ GeV

$\widetilde{U}_\perp^a \; Q_{7,9,10} \to J_{1,3}^B$

$\widetilde{U}_\parallel^a \; Q_{7,9,10} \to J_{2,4}^B$

$\widetilde{U}_\perp^b \; Q_{8g} \to J_1^B$

$\widetilde{U}_{\perp,\parallel}^c \; Q_{1-6} \to J_{1,2}^B$

$$\frac{d^2\Gamma}{dq^2 d\cos\theta} = \frac{G_F^2 |V_{ts}^* V_{tb}|^2}{128\pi^3} \left(\frac{\alpha_{em}}{4\pi}\right)^2 m_B^3 \lambda_{K^*} (1 - \frac{q^2}{m_B^2})^2 \times$$
$$\left\{ 2\zeta_\perp^2 (1+\cos^2\theta) \frac{q^2}{m_B^2} (|\mathcal{C}_9^\perp|^2 + (\mathcal{C}_{10}^\perp)^2) - \right.$$
$$\left. \underline{8\zeta_\perp^2 \cos\theta \frac{q^2}{m_B^2} Re(\mathcal{C}_9^\perp)\mathcal{C}_{10}^\perp} + \zeta_\parallel^2 (1-\cos^2\theta)(|\mathcal{C}_9^\parallel|^2 + (\mathcal{C}_{10}^\parallel)^2) \right\}$$

- $Re(\mathcal{C}_9^\perp(q_0^2)) = 0 \Longrightarrow A_{FB}(q_0^2) = 0$
- $\zeta_{\perp,\parallel}(q^2)$ and $|V_{ts}^* V_{tb}|$ dominant sources of uncertainties

$$\mathcal{C}_9^\perp = \frac{2\pi}{\alpha_{em}} \left( C_1^A + \frac{m_B}{4} \frac{f_B \phi_+^B \otimes f_{K^*}^\perp \phi_{K^*}^\perp \otimes \mathcal{J}_\perp \otimes C_1^B}{\zeta_\perp} \right),$$
$$\mathcal{C}_{10}^\perp = \frac{2\pi}{\alpha_{em}} C_3^A$$

# Soft form factors in SCET

**Exp.** in principle $B \to \rho \ell \nu$ + flavor SU(3) symmetry

**Lattice** not applicable to the region $q^2 \ll m_b^2$

## QCD sum rules

$$\frac{m_B}{m_B + m_{K^*}} V^{B\to K^*}(q^2) = C_V^A(E,\mu)\zeta_\perp(E,\mu) + \mathcal{O}(\alpha_s^2, 1/m_b)$$

$$A_0^{B\to K^*}(q^2) = C_{A0}^A(E,\mu)\zeta_{||}(E,\mu) + \phi_B \otimes f_{K^*}^{||}\phi_{K^*}^{||} \otimes C_{A0}^B$$

$$\zeta_{||}(0) = 0.40 \pm 0.05 \qquad \zeta_\perp(0) = 0.40 \pm 0.04$$

# Soft form factors in SCET

**Exp.** in principle $B \to \rho\ell\nu$ + flavor SU(3) symmetry

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**QCD sum rules + $B \to K^*\gamma$**

$$\frac{m_B}{m_B + m_{K^*}} V^{B\to K^*}(q^2) = C_V^A(E,\mu)\zeta_\perp(E,\mu) + \mathcal{O}(\alpha_s^2, 1/m_b)$$

$$A_0^{B\to K^*}(q^2) = C_{A0}^A(E,\mu)\zeta_{||}(E,\mu) + \phi_B \otimes f_{K^*}^{||}\phi_{K^*}^{||} \otimes C_{A0}^B$$

$\zeta_{||}(0) = 0.40 \pm 0.05$ $\zeta_\perp(0) = 0.40 \pm 0.04$ $\zeta_\perp(0) = 0.32 \pm 0.02$

$B \to \rho\gamma$, $\rho\ell\nu$ also implies the overestimation of LCSRs

# Soft form factors in SCET

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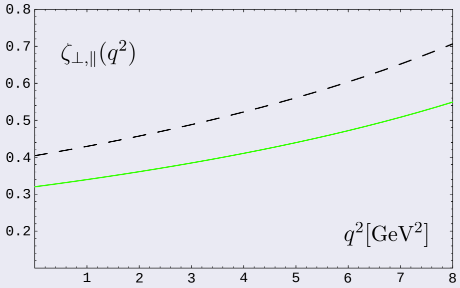

# Input parameters

## Branching ratios

| | | | |
|---|---|---|---|
| $M_W$ | 80.425 GeV | $\sin^2\theta_W$ | 0.2312 |
| $m_t^{pole}$ | $(172.7 \pm 2.9)$ GeV | $\Lambda^{(5)}_{\overline{MS}}$ | $(217^{+25}_{-23})$ MeV |
| $\lvert V_{ts}V_{tb}^*\rvert$ | $(40.3 \pm 2.0) \times 10^{-3}$ | $\alpha_{em}(m_b)$ | $1/133$ |
| $m_B$ | 5.279 GeV | $m_b^{pole}$ | 4.8 GeV |
| $\tau_{B^+}$ | 1.643 ps | $\tau_{B^0}$ | 1.528 ps |
| $m_c/m_b$ | $0.29 \pm 0.02$ | $\mu_I$ | 1.5 GeV |
| $\lambda_{B,+}^{-1}$ | $(1.86 \pm 0.34)$ GeV$^{-1}$ | $f_B$ | $(200 \pm 30)$ MeV |
| $\zeta_\perp(0)$ | $0.32 \pm 0.02$ | $\zeta_\parallel(0)$ | $0.40 \pm 0.05$ |
| $f_{K^*}^\perp(1\text{ GeV})$ | $(185 \pm 10)$ MeV | $f_{K^*}^\parallel$ | $(217 \pm 5)$ MeV |
| $a_1^{\perp,\parallel}$ | $0.1 \pm 0.1$ | $a_2^{\perp,\parallel}$ | $0.1 \pm 0.1$ |

# Input parameters

## Forward-backward Asymmetry

| | | | |
|---|---|---|---|
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| $\tau_{B^+}$ | 1.643 ps | $\tau_{B^0}$ | 1.528 ps |
| $m_c/m_b$ | $0.29 \pm 0.02$ | $\mu_l$ | 1.5 GeV |
| $\lambda_{B,+}^{-1}$ | $(1.86 \pm 0.34)$ GeV$^{-1}$ | $f_B$ | $(200 \pm 30)$ MeV |
| $\zeta_\perp(0)$ | $0.32 \pm 0.02$ | $\zeta_\parallel(0)$ | $0.40 \pm 0.05$ |
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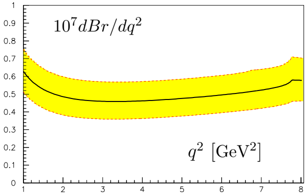
$2.4\text{ GeV} \le \mu_h \le 9.6\text{ GeV}$, $1.1\text{ GeV} \le \mu_l = \sqrt{\mu_h * 0.5} \le 2.2\text{ GeV}$

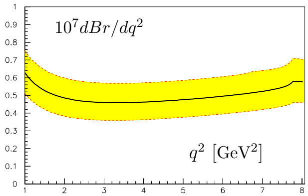
# Numerical results




**Total branching ratio ($q^2 \in [0.1, 20]$ GeV$^2$)**

$$\mathcal{B}(B \to K^* \ell^+ \ell^-) = (16.5^{+2.3}_{-2.2} \pm 0.9 \pm 0.4) \times 10^{-7} \quad (Belle)$$
$$= (7.3^{+2.0}_{-1.8} \pm 1.1) \times 10^{-7} \quad (BaBar)$$

# Numerical results




**Theor. vs. Belle ($q^2 \in [4, 8]$ GeV$^2$)**

$$10^7 Br = 1.94^{+0.44}_{-0.40}$$
$$= 4.8^{+1.4}_{-1.2}|_{\text{stat}} \pm 0.3|_{\text{syst}} \pm 0.3|_{\text{model}}$$

**Theor.( [1,8] GeV$^2$) vs. BaBar( [0.1,8.4] GeV$^2$)**

$$10^7 Br = 3.36^{+0.64}_{-0.57}|_{\zeta_\parallel}\ {}^{+0.34}_{-0.32}|_{\text{CKM}}\ {}^{+0.23}_{-0.21}$$
$$= 2.7^{+1.2}_{-1.0}|_{\text{stat}} \pm 0.5|_{\text{syst}}$$

**Total branching ratio ($q^2 \in [0.1, 20]$ GeV$^2$)**

$$\mathcal{B}(B \to K^* \ell^+ \ell^-) = (16.5^{+2.3}_{-2.2} \pm 0.9 \pm 0.4) \times 10^{-7} \quad (Belle)$$
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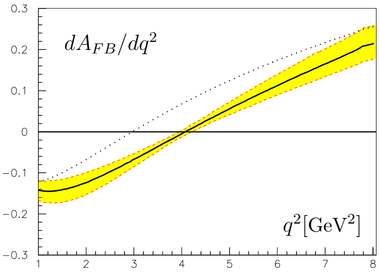
# Forward-backward asymmetry




- $A_{FB}(q_0^2) = 0$ free of hadronic uncertainties [Burdman, Ali et al.]
- $q_0^2 = (4.07^{+0.16}_{-0.13})$ GeV$^2$ with $\Delta(q_0^2)_{\text{scale}} = {}^{+0.08}_{-0.05}$ GeV$^2$
- Uncertainty due to power corrections is not included
- A model-dependent estimation: power corrections could be about $\pm 0.45$ GeV$^2$[Beneke/Feldmann/Seidel 2001]

# Overview of theoretical uncertainties

## Branching ratio

CKM factor: $|V_{ts}^* V_{tb}| = (40.3 \pm 2.0) \times 10^{-3} \Longrightarrow$ 10% uncertainty

$$|V_{ts}^* V_{tb}| \simeq |V_{cs}^* V_{cb}| \Longrightarrow B \to D^{(*)} \ell \bar{\nu},\ X_c \ell \bar{\nu}$$

Soft form factors $\Longrightarrow$ 20% level uncertainty

- Exp: in principle $B \to \rho \ell \nu$ + flavor SU(3) symmetry
- Lattice: not applicable to the region $q^2 \ll m_b^2$
- QCD sum rules: inherent systematic errors
- SCET: calculable in terms of $B$ and $K^*$ LCDAs [Manohar& Stewart,2006]

Power corrections: $1/m_b \sim$ 20% uncertainty
Perturbative uncertainty: $\mathcal{O}(\alpha_s^2) <$ 10% uncertainty

## Future improvement

- $1/m_b$ power-suppressed contributions
- better determination of soft form factors

# Overview of theoretical uncertainties

## Forward-backward asymmetry

- uncertainties from CKM factors and soft form factors are largely canceled
- scale uncertainty: few percents level
- power corrections:
  - model-dependent estimation: roughly 10% uncertainty [Beneke & Feldmann, 2001]
  - model-independent investigation in SCET is possible [Manohar& Stewart,2006]

## Future improvement

$1/m_b$ power suppressed contributions $\Longrightarrow$ 10% level precision

$$q_0^2 = (4.0 \pm 0.4)\ \mathrm{GeV}^2$$

# Summary

- SCET is only applicable to the large recoiled region $q^2 \ll m_b^2$.
- Using SCET, we get the factorization formula holds to all orders of $\alpha_s$ and leading order in $1/m_b$.
- The log are resummed between $\mu = m_b$ and $\mu = \sqrt{m_b \Lambda_h}$.
- Soft form factors $\zeta_\perp$, $\zeta_{||}$ are determined, by combining QCD sum rule and $B \to K^* \gamma$ decay.
- Our branching ratio estimations are consistent with BaBar, but twice smaller than that of Belle.
- $q_0^2 = 4.07^{+0.16}_{-0.13}$ GeV$^2$, power corrections are not included in the error estimation.

## Future

model-independent study of power corrections in SCET

# Scale dependence of $q_0^2$

- BBNS $\equiv$ Physical form factor scheme (PFF) of SCET [Beneke/Yang2006]
- Physical results are independent on the choice of scheme
- RGE in SCET$_I$ not included

- $\Delta(q_0^2)_{\text{scale}} = {}^{+0.08}_{-0.05}\ \text{GeV}^2 \xrightarrow{\text{no RGE}} {}^{+0.17}_{-0.07}$ (BBNS $\pm 0.25$)

- $\zeta_\perp^P \equiv \frac{m_B}{m_B+m_{K^*}} V = C_V^A \zeta_\perp$
- Our analytic formulae are consistent with those of PFF scheme, up to $\mathcal{O}(\alpha_s^2)$

# Scale dependence of $q_0^2$(continue)

- $\mathcal{O}(\alpha_s^2)$ has minor impact on the branching ratios
- Accidentally, $Re(\mathcal{C}_9^\perp) = 0 \longleftrightarrow Re(\mathcal{C}_9^{\perp P}) = 0$

**Our result**

$q_0^2 = 4.07^{+0.08}_{-0.05}$ GeV$^2$

$q_0^2 = 4.12^{+0.17}_{-0.07}$ GeV$^2$ (no RGE)

**PFF Scheme**

$q_0^2 = 3.98 \pm 0.18$ GeV$^2$

$q_0^2 = 4.03 \pm 0.22$ GeV$^2$ (no RGE)

- when $\mathcal{O}(\alpha_s^2)$ corrections are included, the difference should be much smaller
- Notice that $1/m_b \sim \alpha_s$, power corrections are not included in the error estimation
- model-dependent: $\pm 0.45$GeV$^2$ [Beneke et al., 2001] $\Longrightarrow$ Further investigation