

# **Littlest Higgs Models Confronting the Data on $\Delta M_s$ , CP-Violation and $B \rightarrow X_s \gamma$**

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**Flavour in the Era of the LHC**  
**Third Workshop on the Interplay of Flavour and Collider Physics**  
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## Our Goals for the Next 25 Min

1. Overture:  
**Looking Desperately for Deviations** from the **SM** and **MFV**
2. **Signals of non-MFV** Interactions?
3. **LH Models** without and with T-Parity
4. **Problems of LH without T-Parity**
5. **LH with T-Parity**: the Rescue from Mirror Fermions
6. **Conclusions and Outlook**

The recent measurements of  $\Delta M$ s by CDF and D0 offer an important model-independent test of Minimal Flavour Violation

## Definition of **Constrained-MFV** and **(General) MFV**

**CMFV**: [AJB, Gambino, Jager, Silvestrini]

1. Flavour and CP-violation exclusively governed by the CKM matrix
2. The same operators as in the SM

**(General) MFV**: [D'Ambrosio, Giudice, Isidori, Strumia]

1. Only CKM matrix as in CMFV
2. New operators admitted

**MFV**, especially **CMFV**, implies strong correlations between observables in K,  $B_d$  and  $B_s$  systems

# Universal vs. Reference Unitarity Triangle

Two possible sets of parameters to construct the CKM matrix

Set 1: from tree-level decays only  $\longrightarrow$  Reference-UT

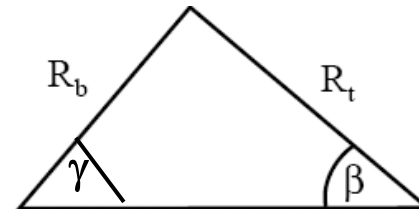
$$|V_{us}| \equiv \lambda, \quad |V_{cb}|, \quad R_b, \quad \gamma \longrightarrow \text{from } B \rightarrow D^{(*)} K$$

from semileptonic B-decays

Set 2: from loop-induced processes  $\longrightarrow$  Universal-UT

$$|V_{us}| \equiv \lambda, \quad |V_{cb}|, \quad R_t, \quad \beta \longrightarrow \sin 2\beta = S_{\psi K_S} \text{ valid in MFV}$$

$$R_t = \frac{\xi}{\lambda} \sqrt{\frac{\Delta M_d}{\Delta M_s}} \sqrt{\frac{m_{B_s}}{m_{B_d}}} \text{ valid in CMFV only}$$



Thanks to the recent first measurements

$$\Delta M_s = (17.33^{+0.42}_{-0.21} \pm 0.07)/\text{ps} \quad \text{CDF}$$

$$17/\text{ps} \leq \Delta M_s \leq 21/\text{ps} \text{ (90\%C.L.)} \quad \text{D0}$$

...other model independent relations:

$$\frac{Br(B_s \rightarrow \mu^+ \mu^-)}{Br(B_d \rightarrow \mu^+ \mu^-)} = \frac{\hat{B}_{B_d} \tau(B_s) \Delta M_s}{\hat{B}_{B_s} \tau(B_d) \Delta M_d} \quad (\text{CMFV})$$

$$\frac{Br(B \rightarrow X_s \nu \bar{\nu})}{Br(B \rightarrow X_d \nu \bar{\nu})} = \frac{|V_{ts}|^2}{|V_{td}|^2} = \frac{m_{B_d}}{m_{B_s}} \frac{1}{\xi^2} \frac{\Delta M_s}{\Delta M_d} \quad (\text{CMFV})$$

$$(\sin 2\beta)_{B \rightarrow \psi K_S} = (\sin 2\beta)_{K \rightarrow \pi \nu \bar{\nu}} \quad (\text{MFV})$$

The **violation** of these model independent **MFV (CMFV)** relations would signal **new flavour and CP-violating interactions (and/or new operators)**

# First direct tests and surprises

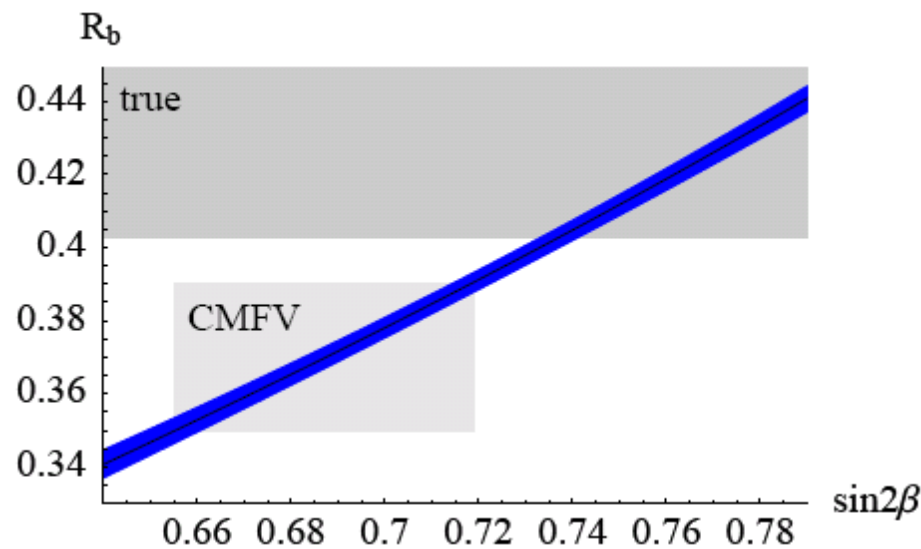
[M. Blanke, AJB, D. Guadagnoli and CT, hep-ph/0604057]

From CKM unitarity:  $R_b = \sqrt{1 + R_t^2 - 2R_t \cos \beta}$ ,  $\cot \gamma = \frac{1 - R_t \cos \beta}{R_t \sin \beta}$ ,

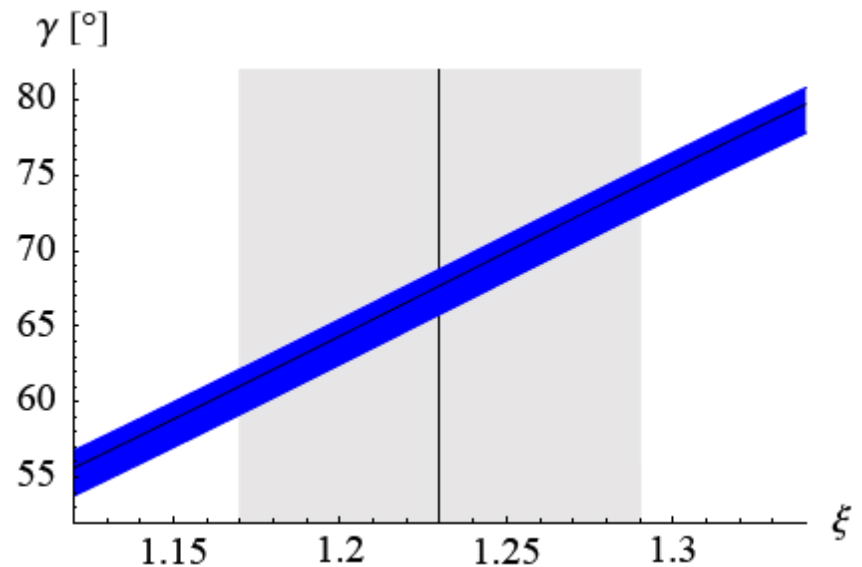
## Comparison between RUT and UUT

$$(R_b)_{\text{true}} = 0.440 \pm 0.037, \quad \gamma_{\text{true}} = (71 \pm 16)^\circ$$

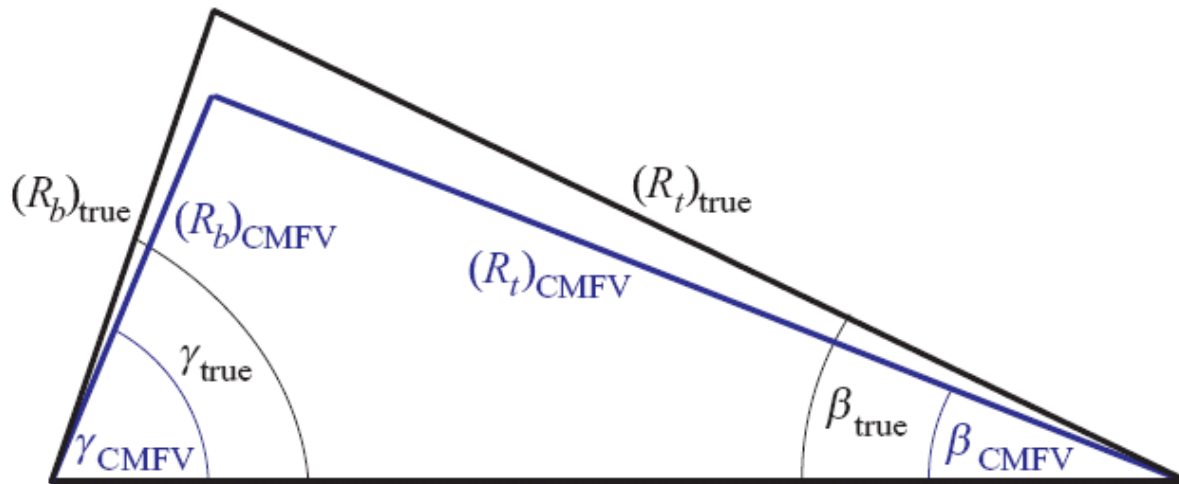
$$(R_b)_{\text{CMFV}} = 0.370 \pm 0.020, \quad \gamma_{\text{CMFV}} = (67.4 \pm 6.8)^\circ$$



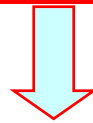
Tension between  $R_b$  and  $\sin 2\beta$  at  $1.7\sigma$



Main uncertainty from  $\xi$  in  $\Delta M_s / \Delta M_d$



The differences between RUT and UUT are within present uncertainties



Importance of:

- reducing theoretical non-perturbative uncertainties
- new experimental measurements to test the CMFV “magic numbers”

$$\frac{Br(B \rightarrow X_s \nu \bar{\nu})}{Br(B \rightarrow X_d \nu \bar{\nu})} = \frac{|V_{ts}|^2}{|V_{td}|^2} = \frac{m_{B_d}}{m_{B_s}} \frac{1}{\xi^2} \frac{\Delta M_s}{\Delta M_d} = 22.3 \pm 2.2 \quad \Rightarrow \quad \frac{|V_{td}|}{|V_{ts}|} = 0.212 \pm 0.011$$

$$\frac{Br(B_s \rightarrow \mu^+ \mu^-)}{Br(B_d \rightarrow \mu^+ \mu^-)} = \frac{\hat{B}_{B_d} \tau(B_s) \Delta M_s}{\hat{B}_{B_s} \tau(B_d) \Delta M_d} = 32.4 \pm 1.9$$

(consistent with  $B \rightarrow V_\gamma$   
[Ball, Zwicky],  
UTfit and CKMfitters)

Further consequence of the recent measurements:  
 $\Delta M_s$  is surprisingly small

CDF

$$\Delta M_s = (17.33_{-0.21}^{+0.42} \pm 0.07)/\text{ps}$$

D0

$$17/\text{ps} \leq \Delta M_s \leq 21/\text{ps} \text{ (90\% C.L.)}$$

### SM predictions from other constraints

$$(\Delta M_s)_{\text{UTfit}}^{\text{SM}} = (21.5 \pm 2.6)/\text{ps}$$

$$(\Delta M_s)_{\text{CKMfitter}}^{\text{SM}} = (21.7_{-4.2}^{+5.9})/\text{ps}$$

- $\Delta M_s$  seems smaller than the SM prediction
- Most MFV models predict higher values than the SM one (MSSM with MFV and large  $\tan\beta$  is an exception)  
[AJB, Chankowski, Rosiek, Slawianowska]



## From direct calculation in the SM

$$(\Delta M_s)_{\text{direct}}^{\text{SM}} = \frac{G_F^2}{6\pi^2} \eta_B m_{B_s} \left( \hat{B}_{B_s} F_{B_s}^2 \right) M_W^2 S(x_t) |V_{ts}|^2 = (17.8 \pm 4.8) / \text{ps}$$

•with

$$F_{B_s} \sqrt{\hat{B}_{B_s}} = 262(35) \text{ MeV}$$

from Wilson(JLQCD)+staggered(HPQCD) lattice fermions

A more accurate comparison is certainly desired

Lattice uncertainties need to be **reduced** (5% in 2010)

**New possible routes** to answer this question

## (Very general) Model Independent Parametrization

$$M_{12}^i = (M_{12}^i)_{SM} + (M_{12}^i)_{new} = (M_{12}^i)_{SM} C_i e^{2i\varphi_i}$$

Such modifications can be studied introducing effective one-loop functions

**In CMFV:**

$$(S_i)^{eff} = S_0 C_i e^{2i\varphi_i}, \quad i = K, B_d, B_s$$

$$C_K = C_{B_d} = C_{B_s}, \\ \varphi_K = \varphi_{B_d} = \varphi_{B_s} = 0$$

## Mass differences and CP-asymmetries in B-systems

$$\Delta M_q = 2 |M_{12}^q| = (\Delta M_q)_{SM} C_{B_q}$$

**Our aim:**

**determining  $C_{B_s}$  in a theoretically clean way!**

$$S_{\psi K_s} = \sin(2\beta + 2\varphi_{B_d}),$$

$$S_{\psi\phi} = \sin(2|\beta_s| - 2\varphi_{B_s})$$

• **Coefficients of  $\sin(\Delta M_q t)$  in**  
 $A_{CP}^{mix}(B_d \rightarrow \psi K_s), A_{CP}^{mix}(B_s \rightarrow \psi\phi),$

• **Attention to the relative sign!**

**New Idea:** let us consider the **semileptonic CP-asymmetry**  
**[B.B.G.T.]**

$$A_{\text{SL}}^s = \frac{\Gamma(\bar{B}_s^0 \rightarrow l^+ X) - \Gamma(B_s^0 \rightarrow l^- X)}{\Gamma(\bar{B}_s^0 \rightarrow l^+ X) + \Gamma(B_s^0 \rightarrow l^- X)} = \text{Im} \left( \frac{\Gamma_{12}^s}{M_{12}^s} \right)$$

$$A_{\text{SL}}^s = \text{Im} \left( \frac{\Gamma_{12}^s}{M_{12}^s} \right)^{\text{SM}} \frac{\cos 2\varphi_{B_s}}{C_{B_s}} - \text{Re} \left( \frac{\Gamma_{12}^s}{M_{12}^s} \right)^{\text{SM}} \frac{\sin 2\varphi_{B_s}}{C_{B_s}}$$

$$\approx -\text{Re} \left( \frac{\Gamma_{12}^s}{M_{12}^s} \right)^{\text{SM}} \frac{\sin 2\varphi_{B_s}}{C_{B_s}} \approx - \left| \text{Re} \left( \frac{\Gamma_{12}^s}{M_{12}^s} \right)^{\text{SM}} \right| \frac{1}{C_{B_s}} S_{\psi\phi}$$

$$\left| \text{Re}(\Gamma_{12}^s/M_{12}^s)^{\text{SM}} \right| = (2.6 \pm 1.0) \cdot 10^{-3}$$

$$\frac{\Delta M_s}{(\Delta M_s)^{\text{SM}}} = - \left| \text{Re} \left( \frac{\Gamma_{12}^s}{M_{12}^s} \right)^{\text{SM}} \right| \frac{S_{\psi\phi}}{A_{\text{SL}}^s}$$

- **Wilson coefficients at the NLO and  $\mathcal{O}(1/m_b^4)$**   
 [Ciuchini, Franco, Lubicz, Mescia, CT]  
 [Beneke, Buchalla, Greub, Lenz, Nierste]
- **B-parameters from LatticeQCD +VSA**  
 [SPQ<sub>cd</sub>R, JLQCD, UKQCD]

**Theoretically:**

- **free** from the  $F_{B_s}$  uncertainties
- but **dimension-seven operators**

**Experimentally:**

**$S_{\psi\phi}$  and  $A_{\text{SL}}^s$  have to be measured**

Advantage and Limit of the **model-independent analysis**:  
“New Physics is knocking on our door but  
we have no idea what it is”

We focus now on  
The **Littlest Higgs Model** without and with T-Parity

Main goals:

1. In the **MFV limit**: is  $(\Delta M_s)_{LH} < (\Delta M_s)_{SM}$  possible?
2. With **new flavour and CP-violating interactions** from mirror fermions:
  - Can  $(\Delta M_s)_{LH} < (\Delta M_s)_{SM}$  and  $\varphi_{B_d} < 0$  ?
  - Are large values of  $A_{CP}(B \rightarrow X_s \gamma)$ ,  $A_{CP}(B_s \rightarrow \psi \phi)$ ,  $A_{SL}^s$ , consistent with all other constraints, possible?

# Littlest Higgs Models

Non-linear  
Sigma-Models

valid up to  $(4\pi f) \equiv \Lambda$

Original model: Arkani-Hamed, Cohen, Katz, Nelson (2002)

$$f \approx O(1\text{TeV})$$

LH

Global:  $SU(5) \longrightarrow SO(5)$

Local:  $[SU(2) \otimes U(1)]_1 \otimes [SU(2) \otimes U(1)]_2 \longrightarrow SU(2)_L \otimes U(1)_Y$   
 $(g_1) \quad (g'_1) \quad (g_2) \quad (g'_2)$

Model with T-Parity: Cheng, Low (2003)

LHT

Theory symmetric under  $[SU(2) \otimes U(1)]_1 \longleftrightarrow [SU(2) \otimes U(1)]_2$   
 $\longleftrightarrow \quad g_1 = g_2 \quad g'_1 = g'_2$

# Littlest Higgs Models without and with T-Parity

New particles: (with  $O(f)$  masses)

LH

Gauge Bosons:  $W_{\text{H}}^{\pm}, Z_{\text{H}}^0, A_{\text{H}}^0$

Fermions: T

Scalars:  $\Phi^{\pm}, \dots$

LHT

T-even  
Sector

T-odd  
Sector

SM Particles +  $T_{+}$

Gauge Bosons:  $W_{\text{H}}^{\pm}, Z_{\text{H}}^0, A_{\text{H}}^0$

Fermions:  $T_{-}$ , Mirror Fermions

Scalars:  $\Phi^{\pm}, \dots$

# The World of Mirror Fermions

Required to cut-off large 4-fermion operators constrained by LEP

$$\begin{pmatrix} u_{1H} \\ d_{1H} \end{pmatrix} \quad \begin{pmatrix} u_{2H} \\ d_{2H} \end{pmatrix} \quad \begin{pmatrix} u_{3H} \\ d_{3H} \end{pmatrix}$$

Vectorial couplings under  $SU(2)_L$

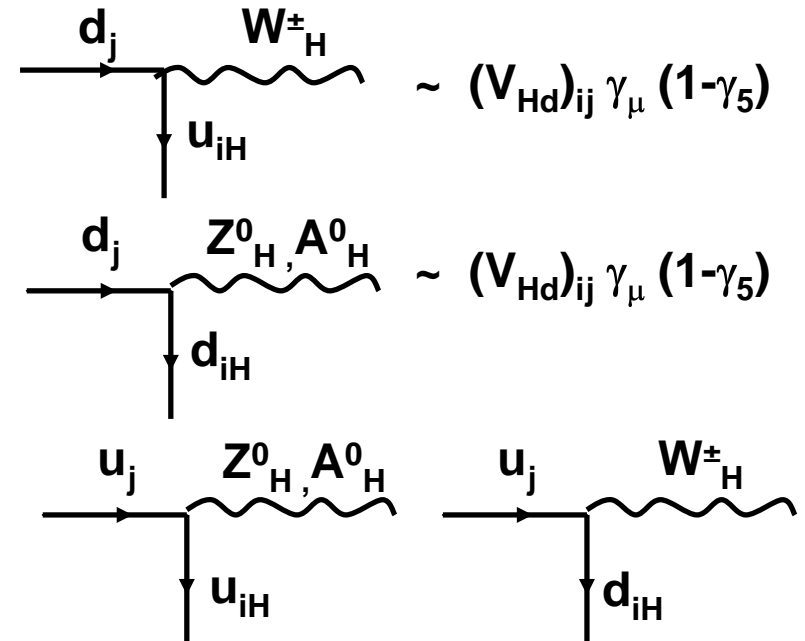
Similarly for Leptons

$$m_{Hi}^u = m_{Hi}^d \quad i=1,2,3 \text{ to first order in } v/f$$

**New Flavour Interactions** involving SM fermions, Mirror Fermions and  $W^\pm_H, Z^0_H, A^0_H$

$$V_{Hu}^\dagger V_{Hd} = V_{CKM}$$

$(V_{Hu})_{ij}$  for:



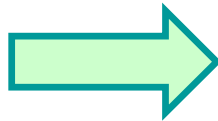
# Determination of the LHT Parameters at the LHC

Discovery of  
 $W_{\pm}^H, Z^0_H, A^0_H$



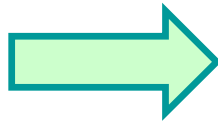
$f$

Discovery of  $T_{\pm}$



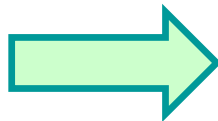
$x_L$

Discovery of  
Mirror Fermions



$m_{H1}$   
 $m_{H2}$   
 $m_{H3}$

•FCNC Processes  
•CP-Asymmetries



$\hat{s}_{12}, \hat{s}_{23}, \hat{s}_{13}$   
and  $\hat{\delta}$



# General Structure of the Amplitudes

## LH (CMFV Model)

Non-perturbative factors

Real functions

$$A(\text{Decay}) = \sum_i B_i^{SM} \eta_{QCD}^i V_{CKM}^i F_i(m_t, m_T, m_{W_H}, \dots)$$

## LHT

Real functions

$$A(\text{Decay}) = \sum_i B_i^{SM} \eta_{QCD}^i \left[ V_{CKM}^i F_i(m_t, m_T) + V_{Hd}^i G_i(m_H^u, m_H^d, W_H^\pm, Z_H^0, A_H^0) \right]$$

T-even contribution: CMFV

T-odd contribution: **New CP and Flavour violating Interactions**  
but only SM operators

# Problems of LH without T-Parity

1.

**Custodial SU(2)  
Symmetry violated  
at tree level**

$$f \geq 2 \text{ TeV}$$

$$\frac{M_W^2}{M_Z^2} = \cos^2 \theta_w \left( 1 + \mathcal{O} \left( \frac{v^2}{f^2} \right) \right)$$

**Effects in FCNC**

**small**

[AJB, Poschenrieder, Uhlig]  
(hep-ph/0410309)  
[Choudhury et al.]

2.

**$\sin(2\beta) - R_b$  Problem not solved**

3.

$$(\Delta M_s)_{\text{LH}} > (\Delta M_s)_{\text{SM}}$$

**Go to  
LHT!**

## Virtues of LHT

1.

Custodial **SU(2)**  
Symmetry **unbroken**  
at tree level

$f \geq 500 \text{ GeV}$

Effects in **FCNC**  
can be larger

[Blanke,AJB,Poschenrieder,  
CT,Uhlig,Weiler]  
[Hubisz et al.]

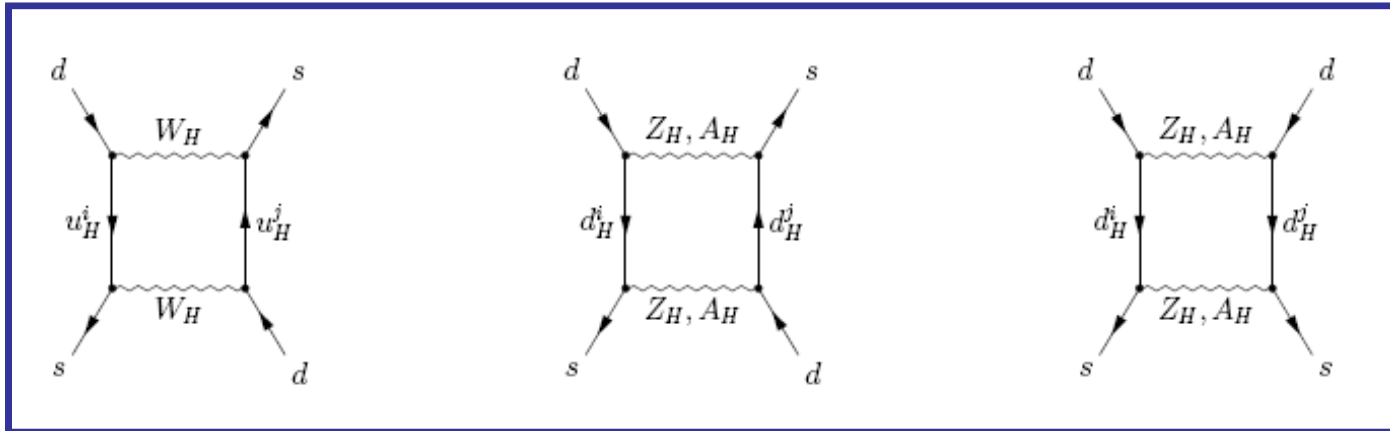
2. **New Flavour Violating Interactions**  
in the Mirror fermion Sector

3. **All existing  $\Delta F=2$  Problems**  
can be **solved**

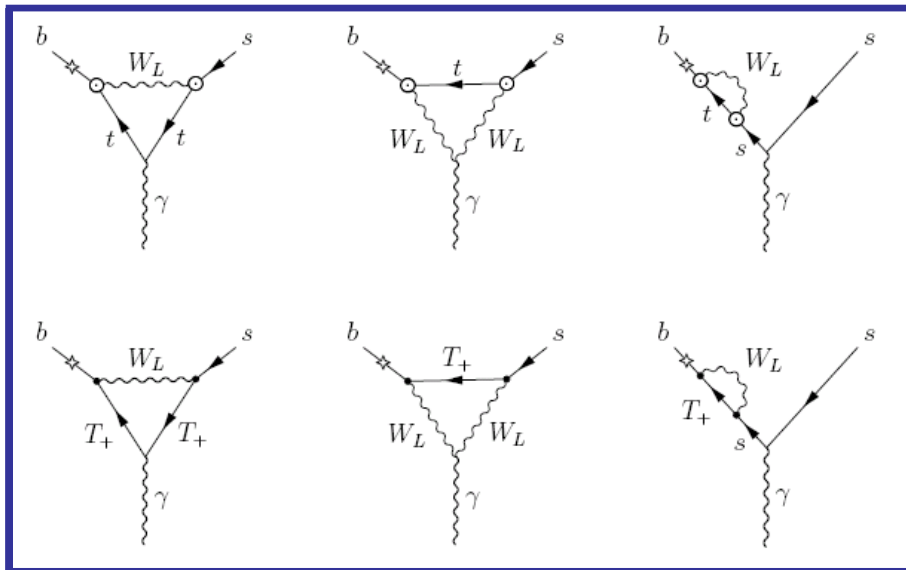
$A_{SL}^s, S_{\psi\phi}$  can be  
by an order of magnitude  
**larger** than in the SM

# Feynman Diagrams for:

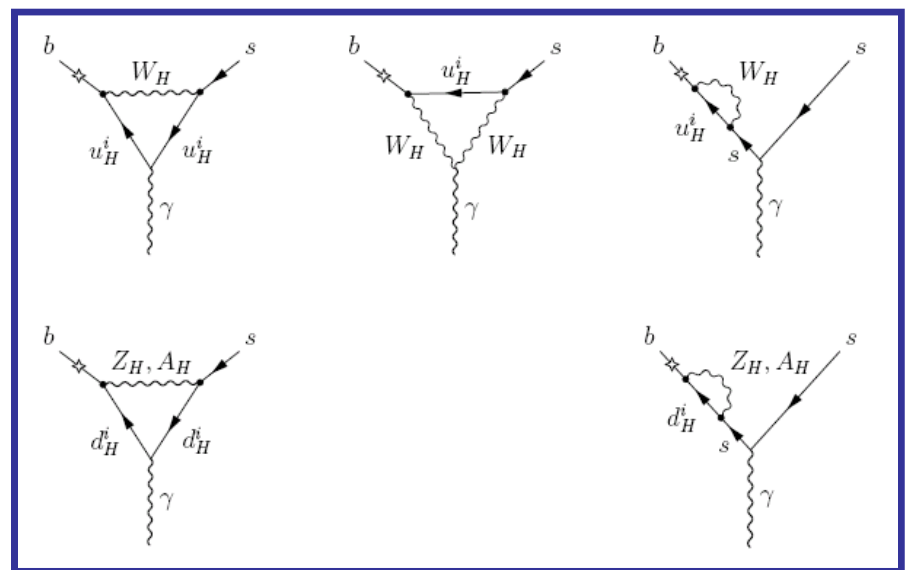
# Particle-Antiparticle Mixing



## $B \rightarrow X_s \gamma$



T-even contribution



T-odd contribution

# Benchmark Scenarios in LHT

$f \equiv 1000 \text{ TeV}$

1. { Degenerate Mirror Fermion Spectrum }  $\rightarrow$  CMFV
2. { Non-degenerate Mirror Fermion Spectrum } +  $V_{\text{Hd}} = V_{\text{CKM}}$   $\rightarrow$  Approximate CMFV
3. {  $m_{\text{H}1} = 350 \text{ GeV}, m_{\text{H}2} = 400 \text{ GeV}$   
 $m_{\text{H}3} = 450 \text{ GeV}$  } +  $V_{\text{Hd}} \neq V_{\text{CKM}}$
4. {  $m_{\text{H}1} \approx m_{\text{H}2} = 500 \text{ GeV}$   
 $m_{\text{H}3} = 1000 \text{ GeV}$  } +  $V_{\text{Hd}}$  hierarchy different from the  $V_{\text{CKM}}$  one  
Favourite scenario (large effects in  $B_s$ !)
5. {  $\gamma_{\text{tree}} = - (109 \pm 16)^\circ$   
(forbidden in the SM, MFV) } Can Mirror Fermions rescue it?  
(large effects in  $B_d$ !)

## Results in Scenarios: 1 - 3

1. and 2.  
(uninteresting)

No Problems  
solved

$$(\Delta M_s) > (\Delta M_s)_{SM}$$

3.

$\sin(2\beta) - R_b$  Problem  
solved

but

$$(\Delta M_s) > (\Delta M_s)_{SM}$$

and

~~CP~~ effects in  $B_s^0 - \bar{B}_s^0$  remain tiny

Non-degenerate  $m_{H1}$  and  $m_{H2}$  force through  $\Delta M_K$  and  $\varepsilon_K$   
 $\hat{V}_{Hd}^{32}$  to be almost real

## 4. Favourite "B<sub>s</sub>" Scenario

$V_{\text{CKM}}$

$V_{\text{Hd}}$  +

Degeneracy of  
Mirror Fermions  
in the first two  
generations

$$\begin{pmatrix} c_{12} & s_{12} & s_{13}e^{-iy} \\ -s_{12} & c_{12} & s_{23} \\ s_{12}s_{23} - s_{13}e^{iy} & -s_{23} & 1 \end{pmatrix}$$

$$\begin{pmatrix} \hat{c}_{12} & \hat{s}_{12} & \hat{s}_{13}e^{-i\delta} \\ -\hat{s}_{12} & \hat{c}_{12} & \hat{s}_{23} \\ -\hat{c}_{12}\hat{s}_{13}e^{i\delta} & -\hat{s}_{12}\hat{s}_{13}e^{i\delta} & 1 \end{pmatrix}$$

Large  $\cancel{CP}$  in  $B_d^0 - \bar{B}_d^0$   
Tiny  $\cancel{CP}$  in  $B_s^0 - \bar{B}_s^0$

Large  $\cancel{CP}$  in  $B_s^0 - \bar{B}_s^0$   
Small  $\cancel{CP}$  in  $B_d^0 - \bar{B}_d^0$

$$s_{13} \ll s_{23} \ll s_{12}$$

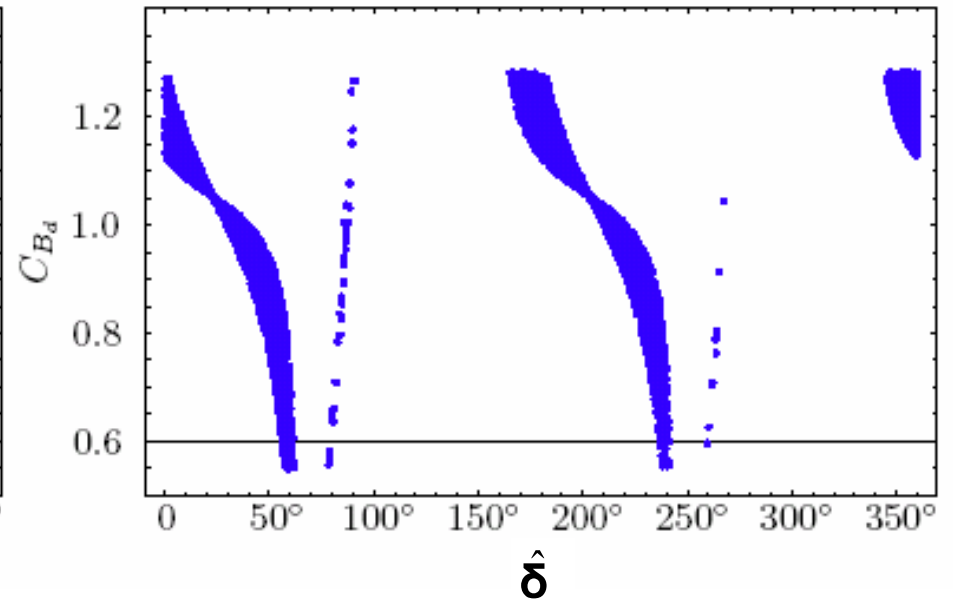
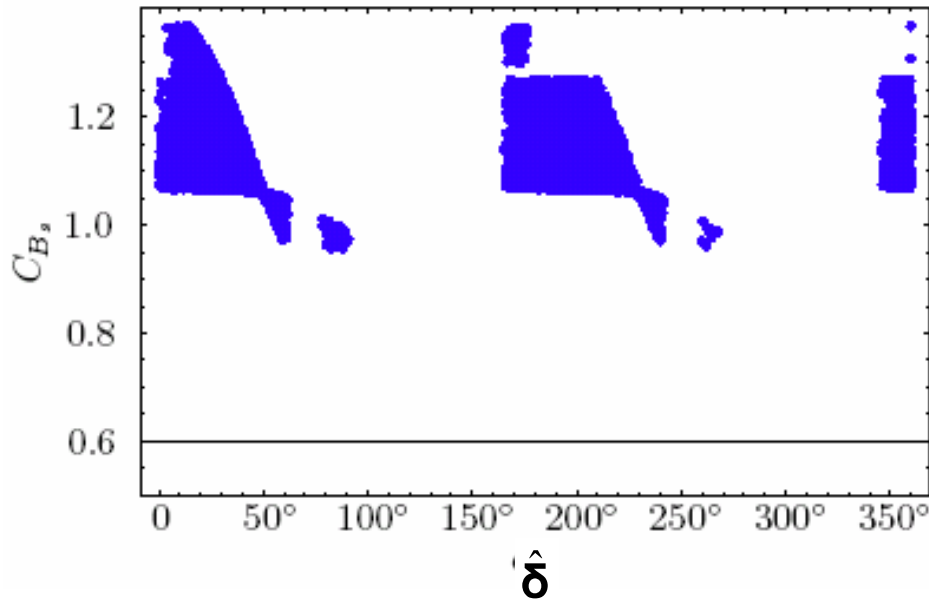
$$(4 \cdot 10^{-3}) \quad (4 \cdot 10^{-2}) \quad (0.2)$$

$$\hat{s}_{23} \ll \hat{s}_{13} \ll \hat{s}_{12}$$

$$(4 \cdot 10^{-4}) \quad (8 \cdot 10^{-2}) \quad (0.90)$$

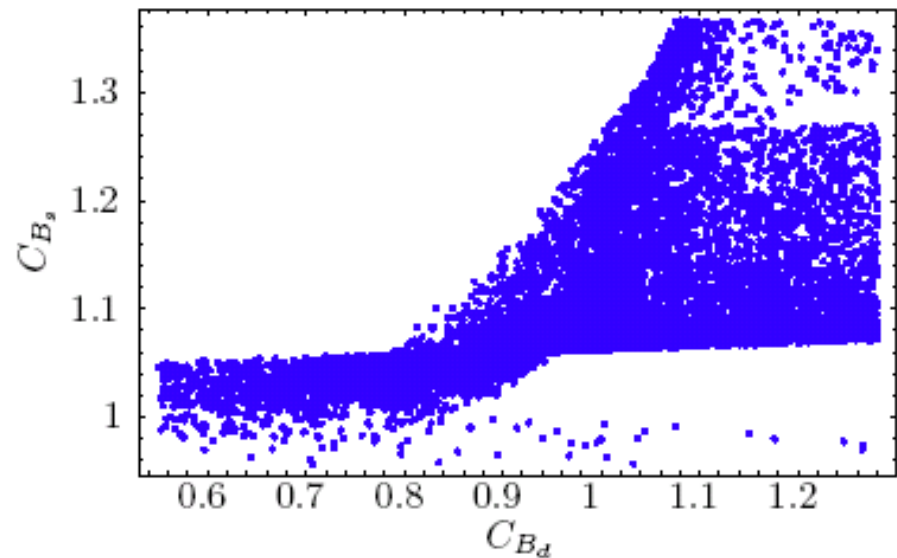
## Results in Scenario 4

“favourite!”



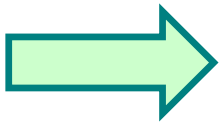
$$C_{B_s} \geq 0.93$$

$(\Delta M_s) < (\Delta M_s)_{SM}$   
possible,  
but more difficult than  
 $(\Delta M_d) < (\Delta M_d)_{SM}$



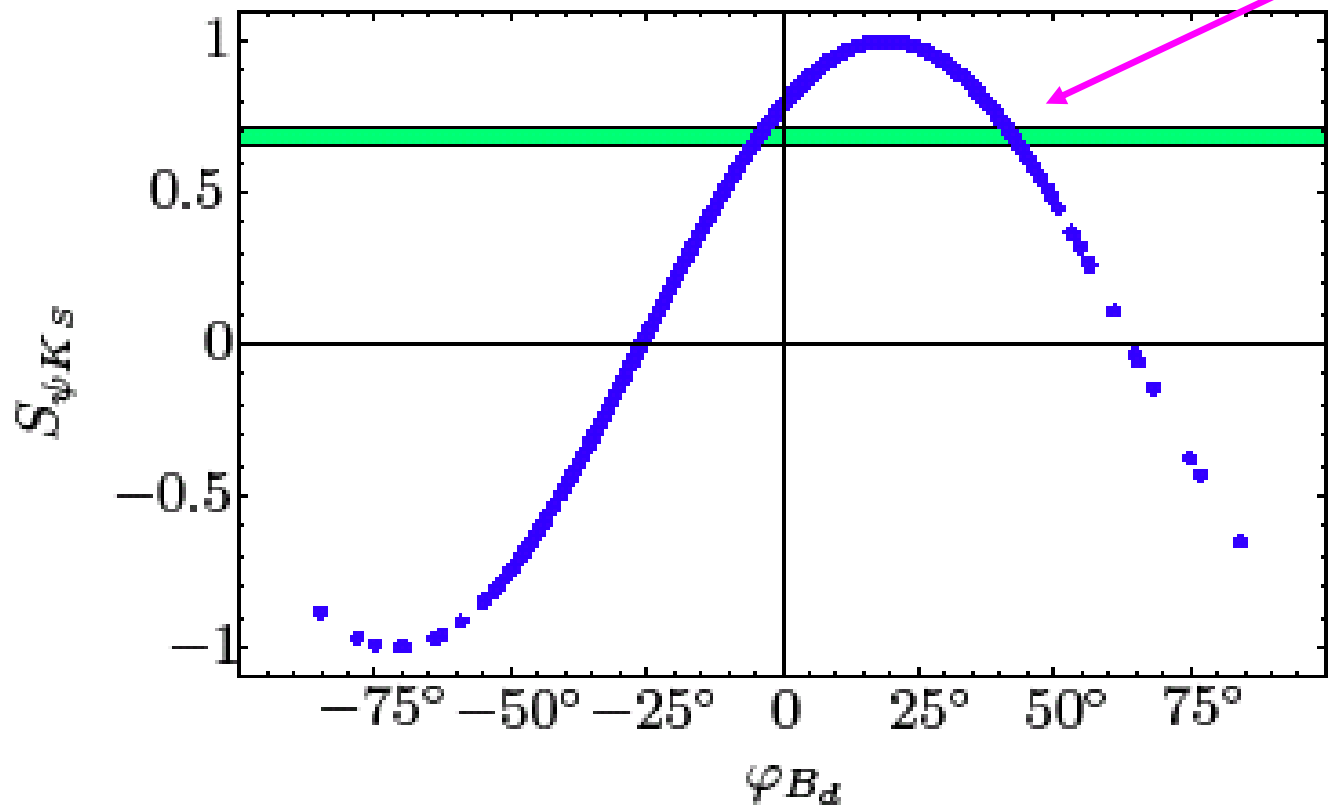


$\sin(2\beta) - R_b$  Problem solved

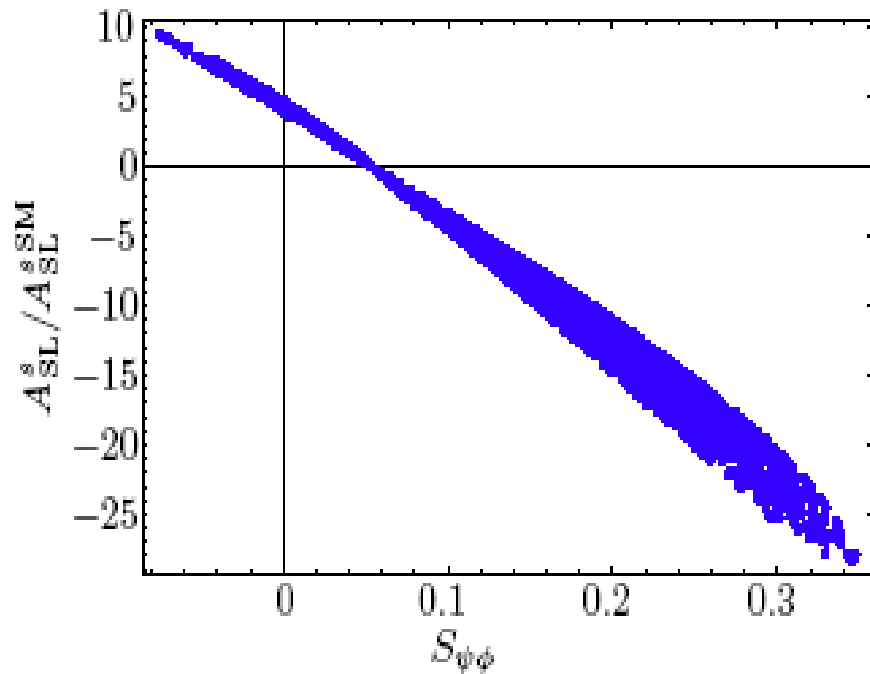


$\varphi_{Bd} \approx -3^\circ, +43^\circ$

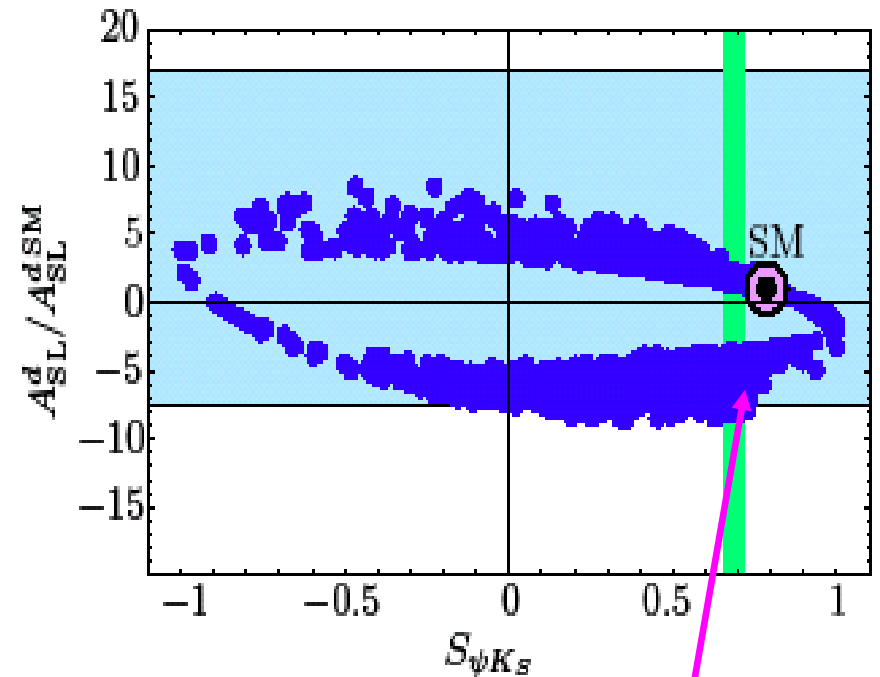
strongly disfavoured by  $\cos(2\beta + 2\varphi_{Bd})^{\text{exp}}$



- $A_{\text{SL}}^s$  enhanced by 10-20,  $A_{\text{SL}}^d$  by  $\sim 3$
- $S_{\psi\phi}$  can be as high as  $+0.3$
- $B \rightarrow X_s \gamma$  and  $A_{\text{CP}}(B \rightarrow X_s \gamma)$  consistent with the data



Model-independent correlation [Ligeti et al.],  
more efficient in the specific LHT Model



$\varphi_{B_d} = +43^\circ$  disfavoured  
also by  $(A_{\text{SL}}^d)_{\text{exp}}$

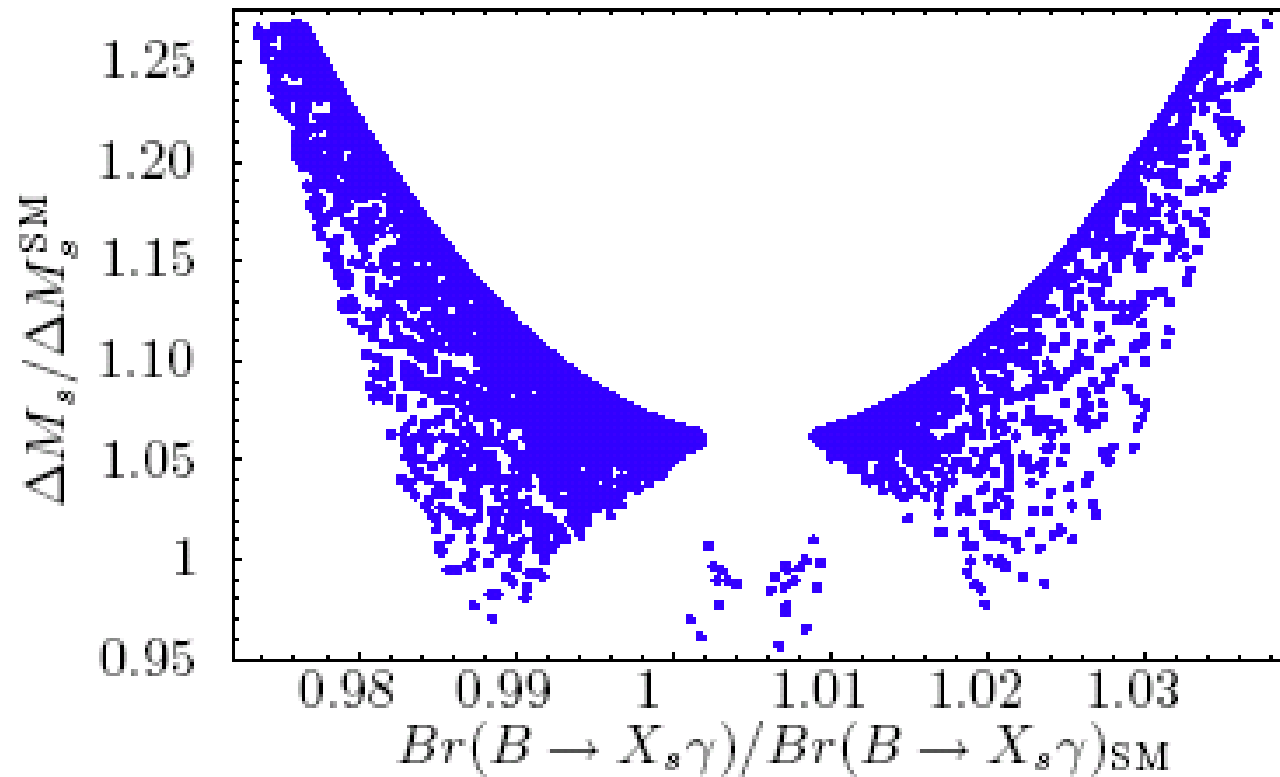
# $\text{Br}(B \rightarrow X_s \gamma)$

At most  $\pm 4\%$  effects in the LHT Model



Good agreement with data

Small effects also  
in  $A_{\text{CP}}(B \rightarrow X_s \gamma)$



5. **The  $\gamma_{\text{tree}} \approx -109^\circ$  ( $\hat{B}_d$ ) Scenario**

$m_{H1} = 500 \text{ GeV}, m_{H2} = 450 \text{ GeV}$   
 $m_{H3} = 1000 \text{ GeV}$

$\hat{S}_{12} \ll \hat{S}_{23} \ll \hat{S}_{13}$

**Within  
the SM:**

$(R_t)_{\text{true}} \approx 1.22$

$\beta_{\text{true}} = -20^\circ$

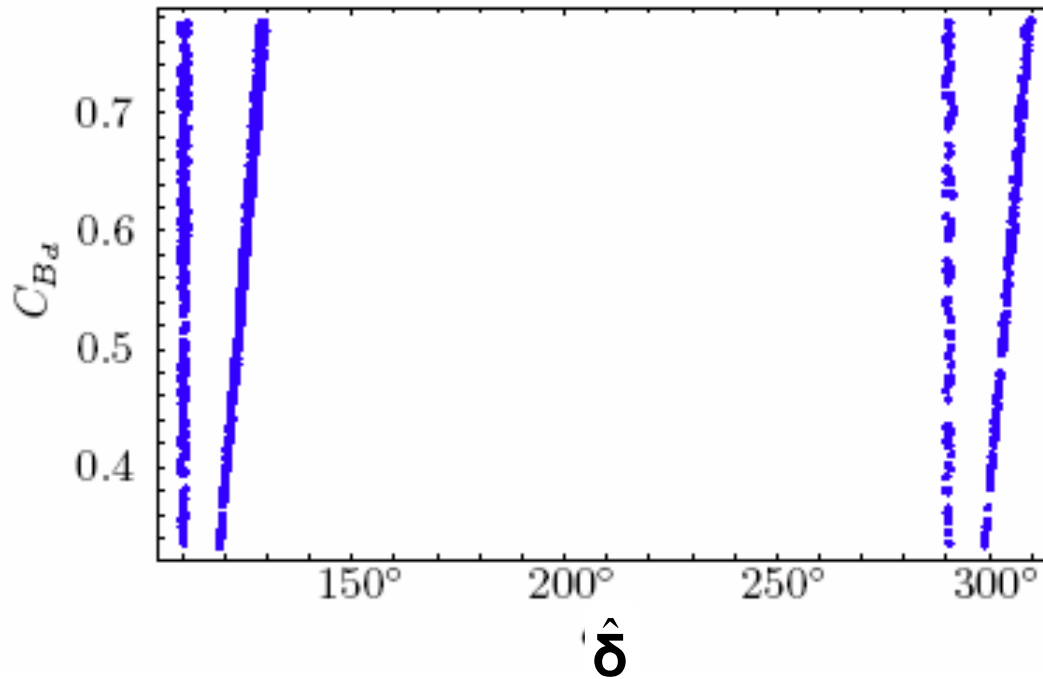
$S_{\psi K_S} = -0.64$

$\epsilon_K = -3.7 \cdot 10^{-3} \cdot e^{i\pi/4}$

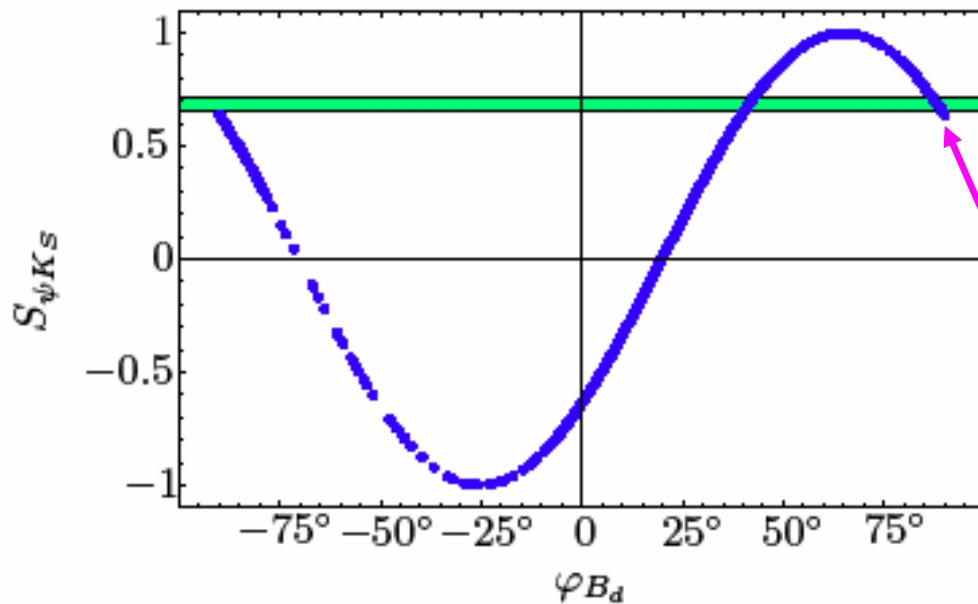
$\Delta M_d = 0.90/\text{ps}$

**Hard Work for  
Mirror Fermions:**

- Reversing signs of  $\epsilon_K$  and  $S_{\psi K_S} = \sin(2\beta_{\text{true}} + 2\varphi_{B_d})$  :  $\varphi_{B_d} \approx +42^\circ, +88^\circ!$
- Decreasing  $\Delta M_d$ :  $C_{B_d} < 1!$
- Can they still give large  $\mathcal{CP}$  effects in  $B^0_s - \bar{B}^0_s$  and  $\Delta M_s < (\Delta M_s)_{\text{SM}}$  ?



$C_{Bd} < 1$   
is indeed possible

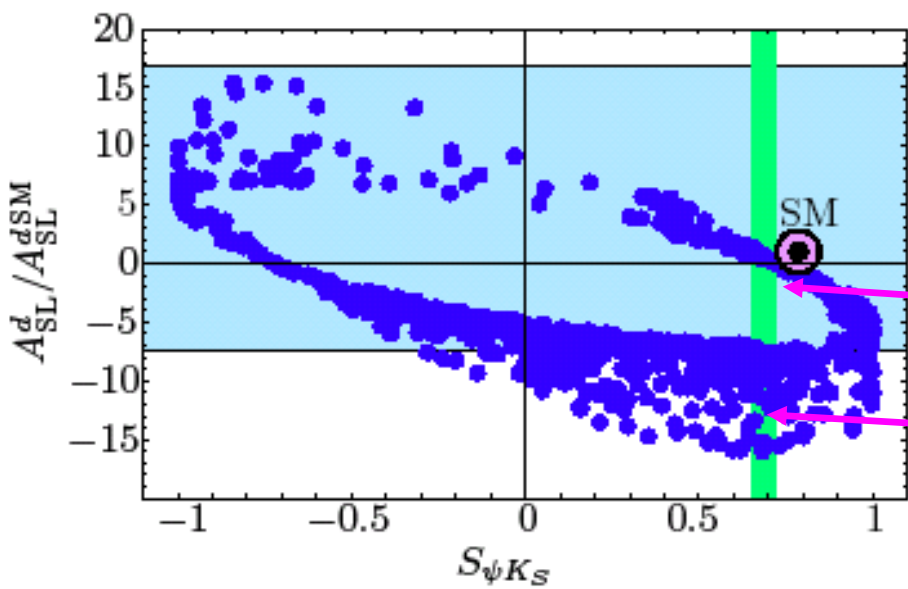


2 large values for  $\varphi_{Bd}$   
( $+42^\circ$ ,  $+88^\circ$ )  
are pointed out by data

strongly disfavoured  
by  $\cos(2\beta + 2\varphi_{Bd})^{\text{exp}}$

$$A_{SL}^d / (A_{SL}^d)_{SM}$$

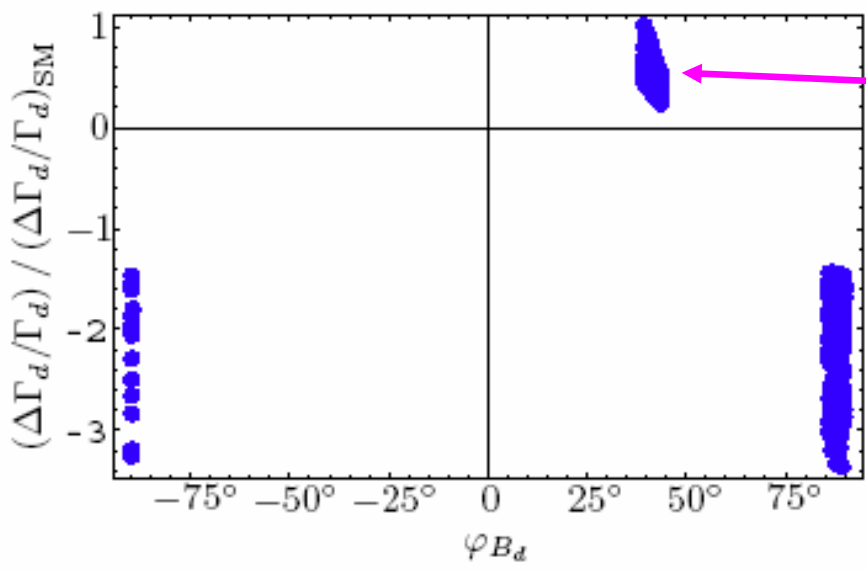
Reversed sign w.r.t. the  $\gamma=71^\circ$  case



$\varphi_{Bd} = +88^\circ$

$\varphi_{Bd} = +42^\circ$

$$\Delta\Gamma_d / (\Delta\Gamma_d)_{SM}$$



$\varphi_{Bd} = +42^\circ$

$\varphi_{Bd} = +88^\circ$

$\gamma = -109^\circ$  Scenario still survives  
 • Importance to reduce the experimental uncertainties

## Main Messages from Blanke, AJB, Poschenrieder, CT, Uhlig, Weiler

- The **LHT** Model offers a **useful playground** for studying **non-MFV** interactions
- All the existing **“Problems”** can be **solved**
- Large  **$\mathcal{CP}$** -effects in  **$B^0_s - \bar{B}^0_s$**  are allowed
- Mirror Fermions rescue** the  **$\gamma = -109^\circ$**  solution

### The analysis of

$$B_{s,d} \rightarrow \mu^+ \mu^-, B \rightarrow X_{s,d} \nu \bar{\nu}, B \rightarrow X_{s,d} l^+ l^-$$

$$K^+ \rightarrow \pi^+ \nu \bar{\nu}, K_L \rightarrow \pi^0 \nu \bar{\nu}, K_L \rightarrow \pi^0 l^+ l^-$$

is coming soon !

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