Preliminary Note

1 Non-Minimal Flavour Violation in FeynArts and FormCalc

1.1 Introduction

FeynArts [1] and FormCalc [2] provide a high level of automation for perturbative calculations up to one loop. This is particularly important for models with a large particle content such as the MSSM [3].

This note describes the recent extension of the implementation of the MSSM in these packages to include non-minimal flavour violation (NMFV). This involves firstly the modification of the squark couplings in the existing FeynArts model file for the MSSM and secondly the corresponding initialization routines for the squark masses and mixings, i.e. the 6×6 diagonalization of the mass matrix in FormCalc.

1.2 Extended Squark Mixing

In the presence of NMFV the 2×2 mixing of the squark within each family is enlarged to a full 6×6 mixing among all three generations, such that the mixed states are

$$\tilde{u}_i = (R_u)_{ij} \begin{pmatrix} \tilde{u}_L \\ \tilde{c}_L \\ \tilde{t}_L \\ \tilde{u}_R \\ \tilde{c}_R \\ \tilde{t}_R \end{pmatrix}_j , \quad \tilde{d}_i = (R_d)_{ij} \begin{pmatrix} \tilde{d}_L \\ \tilde{s}_L \\ \tilde{b}_L \\ \tilde{d}_R \\ \tilde{s}_R \\ \tilde{b}_R \end{pmatrix}_j . \tag{1}$$

The mixing matrices R diagonalize the mass matrices

$$M^2_{u,d} = \left(\begin{array}{ccc|ccc} M^2_{\tilde{L},i} & 0 & 0 & m_i X_i & 0 & 0 \\ 0 & M^2_{\tilde{L},j} & 0 & 0 & m_j X_j & 0 \\ 0 & 0 & M^2_{\tilde{L},k} & 0 & 0 & m_k X_k \\ \hline m_i X_i^* & 0 & 0 & M^2_{\tilde{R},i} & 0 & 0 \\ 0 & m_j X_j^* & 0 & 0 & M^2_{\tilde{R},j} & 0 \\ 0 & 0 & m_k X_k^* & 0 & 0 & M^2_{\tilde{R},k} \end{array}\right) + \Delta_{u,d} \, , \tag{2}$$

where $\{i,j,k\} = \{\mathrm{u,c,t}\}$ for the up- and $\{\mathrm{d,s,b}\}$ for the down-squark mass matrix and

$$\begin{aligned}
M^2_{\tilde L,q} &= M^2_{\tilde Q,q} + m_q^2 + \cos 2\beta\,(T_3^q - Q_q s_W^2) m_Z^2\,, \\
M^2_{\tilde R,\{\mathrm{u,c,t}\}} &= M^2_{\tilde U,\{\mathrm{u,c,t}\}} + m^2_{\{\mathrm{u,c,t}\}} + \cos 2\beta\, Q_\mathrm{u} s_W^2 m_Z^2\,, \\
M^2_{\tilde R,\{\mathrm{d,s,b}\}} &= M^2_{\tilde D,\{\mathrm{d,s,b}\}} + m^2_{\{\mathrm{d,s,b}\}} + \cos 2\beta\, Q_\mathrm{d} s_W^2 m_Z^2\,, \\
X_{\{\mathrm{u,c,t}\}} &= A_{\{\mathrm{u,c,t}\}} - \mu\cot\beta\,, \\
X_{\{\mathrm{d,s,b}\}} &= A_{\{\mathrm{d,s,b}\}} - \mu\tan\beta\,.
\end{aligned} \tag{3}$$

The actual dimensionless input quantities δ are defined by

$$\Delta_{i_X j_Y} := M_{\tilde X,i} M_{\tilde Y,j} \delta_{i_X j_Y} \quad \text{where} \quad x_L = x,\ x_R = x + 3\,, \tag{4}$$

i.e. the 3×3 matrix $M_{\tilde X,i} M_{\tilde Y,j}$ is divided out of each block $XY = \{\mathrm{LL, LR, RL, RR}\}$ of Δ.

1.3 FeynArts Model File

The new model file `FVMSSM.mod` generalizes the squark couplings in `MSSM.mod` to the NMFV case. From the technical perspective it indeed reads in `MSSM.mod` and applies algebraic substitutions to yield the new couplings. A human-readable list of couplings has already been given in Ref. [4] and can also be found in `FVMSSM.ps.gz`, located in the `Models` subdirectory of FeynArts.

`FVMSSM.mod` contains the following new quantities:

`UASf[`s_1`,`s_2`,`t`]`	the squark mixing matrix $R_{u,d}$, where $s_1, s_2 = 1\dots 6$, $t = 3(u), 4(d)$,
`MASf[`s`,`t`]`	the squark masses, where $s = 1\dots 6$, $t = 3(u), 4(d)$.

1.4 Model Initialization in FormCalc

The initialization of the generalized squark-mixing parameters `MASf` and `UASf` is already built into FormCalc's regular MSSM model-initialization file `model_mssm.F` but not turned on by default. It must be enabled by defining a preprocessor flag in `run.F`:

```
#define FLAVOUR_VIOLATION
```

The NMFV parameters δ are represented in FormCalc by the `deltaSf` matrix:

`double complex deltaSf(`s_1`,`s_2`,`t`)`	the matrix $(\delta_t)_{s_1 s_2}$, where $s_1, s_2 = 1\dots 6$, $t = 3(u), 4(d)$.

Since δ is a Hermitian matrix, only the entries on and above the diagonal need to be filled. For convenience, the following abbreviations can be used for individual matrix elements:

$$\begin{array}{rlrl}
\texttt{deltaLLuc} = (\delta_u)_{12} & \texttt{deltaLRuc} = (\delta_u)_{15} & \texttt{deltaLLds} = (\delta_d)_{12} & \texttt{deltaLRds} = (\delta_d)_{15} \\
\texttt{deltaRLucC} = (\delta_u)_{24} & \texttt{deltaRRuc} = (\delta_u)_{45} & \texttt{deltaRLdsC} = (\delta_d)_{24} & \texttt{deltaRRds} = (\delta_d)_{45} \\[1ex]
\texttt{deltaLLct} = (\delta_u)_{23} & \texttt{deltaLRct} = (\delta_u)_{26} & \texttt{deltaLLsb} = (\delta_d)_{23} & \texttt{deltaLRsb} = (\delta_d)_{26} \\
\texttt{deltaRLctC} = (\delta_u)_{35} & \texttt{deltaRRct} = (\delta_u)_{56} & \texttt{deltaRLsbC} = (\delta_d)_{35} & \texttt{deltaRRsb} = (\delta_d)_{56} \\[1ex]
\texttt{deltaLLut} = (\delta_u)_{13} & \texttt{deltaLRut} = (\delta_u)_{16} & \texttt{deltaLLdb} = (\delta_d)_{13} & \texttt{deltaLRdb} = (\delta_d)_{16} \\
\texttt{deltaRLutC} = (\delta_u)_{34} & \texttt{deltaRRut} = (\delta_u)_{46} & \texttt{deltaRLdbC} = (\delta_d)_{34} & \texttt{deltaRRdb} = (\delta_d)_{46}
\end{array}$$

Note the somewhat special treatment of the RL elements: As indicated by the trailing `C` in the name, one has to provide the complex conjugate of the element. This is because the original lies below the diagonal and would be ignored by the eigenvalue routine.

Furthermore, the trilinear couplings A acquire non-zero off-diagonal entries in the presence of NMFV through the relations

$$m_{q,i}(A_q)_{ij} = (M_q^2)_{i,j+3}\,, \quad q = u, d\,, \quad i, j = 1 \dots 3\,. \tag{5}$$

These off-diagonal trilinear couplings appear in the Higgs–squark–squark couplings.

1.5 Summary

Non-minimal flavour violation effects are now available within the FeynArts and FormCalc packages through a new model file for FeynArts and an extended MSSM initialization file for FormCalc. Including NMFV effects requires only three minor changes compared to calculations with the MFV MSSM:

- choosing `FVMSSM.mod` instead of `MSSM.mod`,
- setting the `FLAVOUR_VIOLATION` preprocessor flag in `run.F`,
- providing values for the `deltaSf` matrix.

The described changes are contained in the FeynArts and FormCalc packages which are publicly available from www.feynarts.de.

References

[1] J. Küblbeck, M. Böhm, A. Denner, *Comput. Phys. Commun.* **60** (1990) 165;
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[3] T. Hahn, C. Schappacher, *Comput. Phys. Commun.* **143** (2002) 54 [hep-ph/0105349].

[4] T. Hahn, W. Hollik, J.I. Illana, S. Peñaranda, hep-ph/0512315.