PHOTOS and NLO -- for non-leptonic B Meson Decays
Work Report

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- Introduction
- Scalar QED for B decays
- PHOTOS standard and improved
- Numerical results
- Conclusion
- Grain of Salt

Purpose of my presentation

- Compute first order QED radiative corrections from the given Lagrangian and install them into PHOTOS. PHOTOS, if combined with other segments of MC simulations can be used for comparisons theory – raw experimental data.

- Thanks to program organization the single emission kernel can be used for runs at first and exponentiated mode.

- Such a method was shown to work at 0.1 % precision level in case of Z decay (see recent paper by P. Golonka and Z. W.

The following B Meson Decay will be presented today

$$B^0 \to \pi^- \pi^+ \gamma \qquad B^0 \to K^- \pi^+ \gamma$$

$$B^- \to \pi^- \pi^0 \gamma \qquad B^- \to K^- \pi^0 \gamma$$

- Technically easy (lagrangian exist),
- Of experimental interest, good step toward precision (if improvements due to fits to the data are added)
- Of technical interest: several processes with substantially massive final states.
- On the last meeting there was controversy on PHOTOS reliability for this channels, thus there is a need for clarification

SANC

- SANC is a network Client-Server System for a semi-automatic calculation of Electroweak, QCD and QED radiative corrections at a one-loop precision level for various processes (-decays) of elementary particle interactions

- The Present level of the system is realized in the version 1.0 (*“SANCscope – v.1.0”*, *hep-ph*/0411186)

- Application – LHC, Linear Colliders

- More information can be found on web page http://pcphsanc.cern.ch

Scalar QED Lagrangian

- To calculate QED radiative corrections we used the easiest Lagrangian for charged pions and electromagnetic field

$$L = -D^{*}_{\mu}\pi^{+}D_{\mu}\pi^{-} - m^{2}_{\pi}\pi^{+}\pi^{-} + G_{weak}B^{0}\pi^{+}\pi^{-}$$

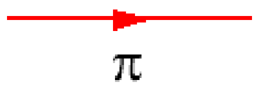
$B^0, \pi^{+,-}$ – The fields of B^0 and pions

G_{weak} – The effective weak coupling constant

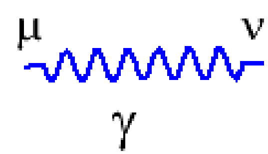
m_π – The pion mass.

$D_\mu = \partial_\mu - ieA_\mu \quad \rightarrow$ The covariant derivative

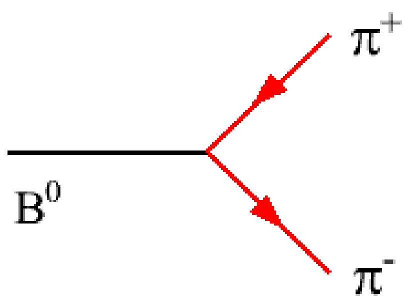
The Feynman rules



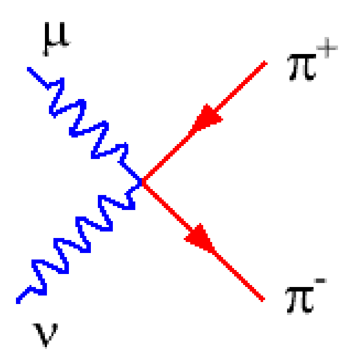
$$\frac{1}{p^2 + m_\pi^2}$$



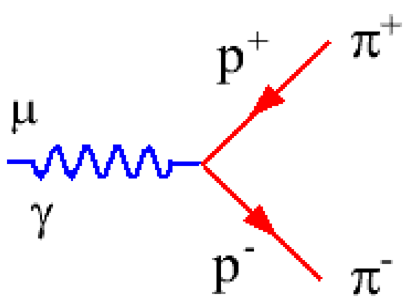
$$\frac{\delta_{\mu\nu}}{p^2 - i\in}$$



G_{weak}



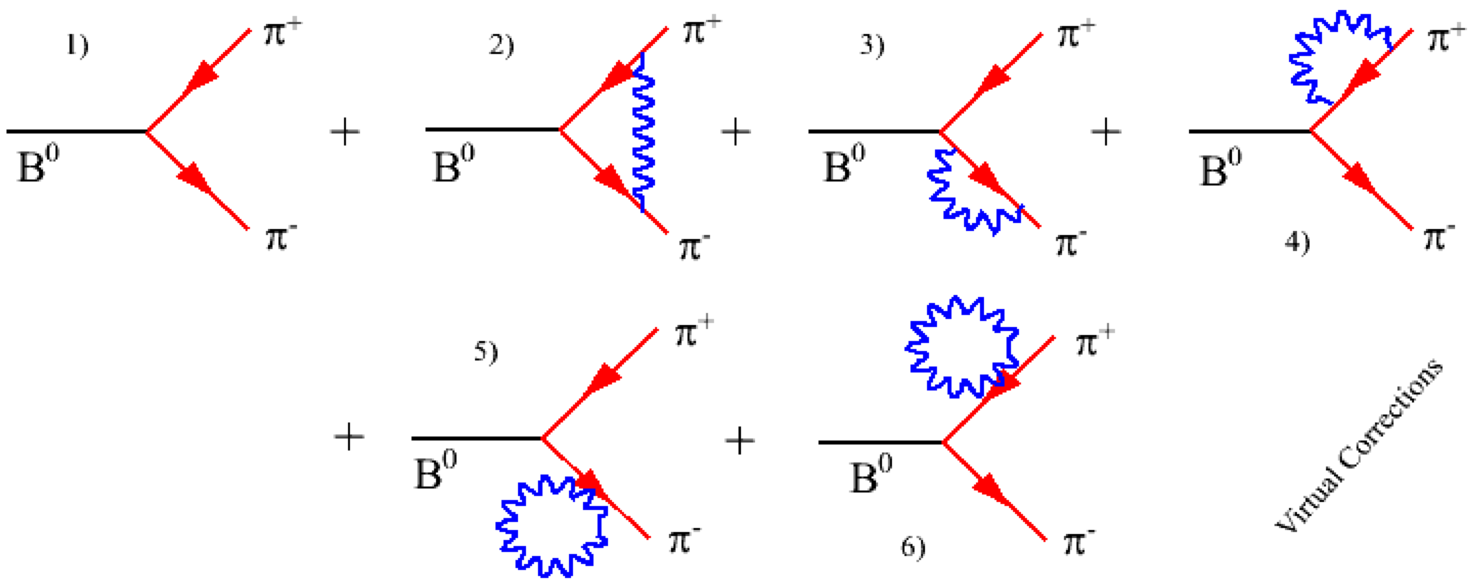
$-e^2 \delta_{\mu\nu}$

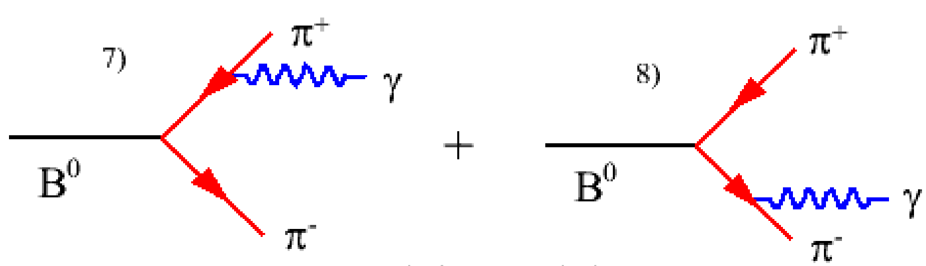


$e\,(p^-_\mu - p^+_\mu)$

Radiative Corrections

Diagrams of order $O(\alpha)$ contributing to $B^0 \rightarrow \pi^- \pi^+ \gamma$





Real Photon Emission

The QED radiative corrections modify the lowest order decay rate

$$d\Gamma^{\text{One-loop}}_{B^0\to\pi^-\pi^+\gamma}(\Lambda_{\text{QED}}) = d\Gamma^{\text{Born}}_{B^0\to\pi^-\pi^+}\left[1+\delta^{\text{Virt+Soft}}(\Lambda_{\text{QED}},\omega)\right]$$

$$+\ d\Gamma^{\text{Hard}}_{B^0\to\pi^-\pi^+\gamma}(\omega)$$

$$\delta^{\text{Virt+Soft}}(\Lambda_{\text{QED}},\omega) = 2\,\Re[\delta^{\text{Virt}}(\Lambda_{\text{QED}},m_\gamma)] + \delta^{\text{Soft}}(m_\gamma,\omega)$$

$$\Gamma^{\text{One-loop}}_{B^0\to\pi^-\pi^+\gamma}(\Lambda_{\text{QED}}) = d\Gamma^{\text{Born}}_{B^0\to\pi^-\pi^+}; \qquad \Lambda_{\text{QED}}=\mu_{\text{UV}}$$

- IR pole, which is represented here by the photon mass, cancels in the sum
- However, dependence on the **Ultraviolet Renormalization Scale** remains, we request that Kinoshita-Lee-Nauenberg theorem holds and QED will not affect total rate.
- SANC Monte Carlo for radiative B Meson decays is based on the above formula

– Hard photon contribution

$$d\Gamma^{\rm Hard} = |A^{\rm Born}|^2 4\pi\alpha \left(q_1 \frac{k_1.\epsilon}{k_1.k_\gamma} - q \frac{P.\epsilon}{P.k_\gamma} \right)^2 dLips_3(P \to k_1, k_2, k_\gamma)$$

– Total width (massless limit of the final mesons $m_1, m_2 \equiv m \to 0$)

$$\Gamma^{\rm Total} = \Gamma^{\rm Born} \left[1 + \frac{\alpha}{\pi} \left(\frac{3}{2} \ln \frac{\mu_{UV}^2}{M^2} - \frac{\pi^2}{3} + \frac{11}{2} \right) \right]$$

We have identified the parts of the weight in PHOTOS corresponding to phase space jacobiam and the term responsible for collinear times infrared double logarithmic distributions:
1/k 1/(1- β cosθ)

Once it is done, implementation of any matrix element is straightforward.

Note that until now, parts of matrix element and phase space were intermixed.

I will present details of the construction of my friday talk in theory division.

Case of the decay of neutral B is analogous.

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In calculations we do not use the approximated expression, but the following exact ones:

The one-loop QED correction to the decay width can be represented as the sum of the Born contribution with the contributions due to virtual loop diagrams and soft and hard photon emissions.

$$d\Gamma^{\rm Total} = d\Gamma^{\rm Born}\left\{1+\frac{\alpha}{\pi}\left[\delta^{\rm Soft}(m_\gamma,\omega)+\delta^{\rm Virt}(m_\gamma,\mu_{UV})\right]\right\} + d\Gamma^{\rm Hard}(\omega)$$

Neutral meson decay channels

– Virtual photon contribution $\Lambda=\lambda^{1/2}(M^2,m_1^2,m_2^2)$

$$\begin{aligned}\delta^{\rm Virt}(m_\gamma,\mu_{UV}) &= \left[1+\frac{M^2-m_1^2-m_2^2}{\Lambda}\ln\frac{2m_1m_2}{M^2-m_1^2-m_2^2+\Lambda}\right]\ln\frac{M^2}{m_\gamma{}^2}+\frac{3}{2}\ln\frac{\mu_{UV}^2}{M^2}\\ &+ \frac{M^2-m_1^2-m_2^2}{2\Lambda}\left[{\rm Li}_2\left(\frac{M^2+m_1^2-m_2^2+\Lambda}{2\Lambda}\right)-{\rm Li}_2\left(\frac{-M^2+m_2^2-m_1^2+\Lambda}{2\Lambda}\right)\right.\\ &\left.\qquad + 2\ln\frac{2Mm_1}{M^2+m_1^2-m_2^2+\Lambda}\ln\frac{m_1\Lambda}{M^3}+(1\leftrightarrow 2)+\pi^2\right]\\ &- \frac{\Lambda}{2M^2}\ln\frac{2m_1m_2}{M^2-m_1^2-m_2^2+\Lambda}+\frac{m_2^2-m_1^2}{4M^2}\ln\frac{m_2^2}{m_1^2}-\frac{1}{2}\ln\frac{m_1m_2}{M^2}+1\end{aligned}$$

– Soft photon contribution

$$\delta^{\rm Soft}(m_\gamma,\omega) = \left[1+\frac{M^2-m_1^2-m_2^2}{\Lambda}\ln\frac{2m_1m_2}{M^2-m_1^2-m_2^2+\Lambda}\right]\ln\frac{{m_\gamma}^2}{4\omega^2}$$
$$+\ \frac{M^2-m_1^2-m_2^2}{2\Lambda}\left[{\rm Li}_2\left(\frac{-2\Lambda}{M^2+m_1^2-m_2^2-\Lambda}\right)-{\rm Li}_2\left(\frac{2\Lambda}{M^2+m_1^2-m_2^2+\Lambda}\right)+(1\leftrightarrow2)\right]$$
$$-\ \frac{M^2+m_1^2-m_2^2}{\Lambda}\ln\frac{2Mm_1}{M^2+m_1^2-m_2^2+\Lambda}-(1\leftrightarrow2)$$

– Hard photon contribution

Case of two charged decay products

$$d\Gamma^{\rm Hard} = |A^{\rm Born}|^2 4\pi\alpha\left(q_1\frac{k_1.\epsilon}{k_1.k_\gamma}-q_2\frac{k_2.\epsilon}{k_2.k_\gamma}\right)^2 dLips_3(P\rightarrow k_1,k_2,k_\gamma)$$

– Total width (massless limit of the final mesons $m_1, m_2 \equiv m \rightarrow 0$)

$$\Gamma^{\rm Total} = \Gamma^{\rm Born}\left[1+\frac{\alpha}{\pi}\left(\frac{3}{2}\ln\frac{\mu_{UV}^2}{M^2}+5\right)\right]$$

- The infrared divergency, regularized by m_γ, cancels in the sum of virtul and soft contributions
- The virtual correction depends on ultraviolet scale μ_{UV}
- The total width is free of ω and of the final meson mass singularity (KLN theorem)

Charged meson decay channels

– Virtual photon contribution

$$\begin{aligned}
\delta^{virt}(m_\gamma, \mu_{UV}) &= \left[1 + \frac{M^2+m_1^2-m_2^2}{\Lambda} \ln \frac{2Mm_1}{M^2+m_1^2-m_2^2+\Lambda}\right] \ln \frac{Mm_1}{m_\gamma{}^2} + \frac{3}{2} \ln \frac{\mu_{UV}^2}{Mm_1} \\
&+ \frac{M^2+m_1^2-m_2^2}{2\Lambda} \left[\mathrm{Li}_2\left(\frac{M^2-m_1^2-m_2^2+\Lambda}{2\Lambda}\right) - \mathrm{Li}_2\left(\frac{M^2-m_1^2-m_2^2-\Lambda}{-2\Lambda}\right)\right. \\
&\qquad + \mathrm{Li}_2\left(\frac{M^2+m_2^2-m_1^2-\Lambda}{-2\Lambda}\right) - \mathrm{Li}_2\left(\frac{M^2+m_2^2-m_1^2+\Lambda}{2\Lambda}\right) \\
&\qquad \left. + 2\ln \frac{2Mm_1}{M^2+m_1^2-m_2^2+\Lambda} \ln \frac{\Lambda}{Mm_2} - \ln \frac{2Mm_2}{M^2+m_2^2-m_1^2+\Lambda} \ln \frac{M^2}{m_1^2}\right] \\
&+ \frac{\Lambda}{2m_2^2} \ln \frac{2Mm_1}{M^2+m_1^2-m_2^2+\Lambda} - \frac{M^2-m_1^2}{4m_2^2} \ln \frac{m_1^2}{M^2} + 1;
\end{aligned}$$

– Soft photon contribution

$$\begin{aligned}
\delta^{soft}(m_\gamma, \omega) &= \left[1 + \frac{M^2+m_1^2-m_2^2}{\Lambda} \ln \frac{2Mm_1}{M^2+m_1^2-m_2^2+\Lambda}\right] \ln \frac{m_\gamma{}^2}{4\omega^2} \\
&+ \frac{M^2+m_1^2-m_2^2}{2\Lambda} \left[\mathrm{Li}_2\left(\frac{-2\Lambda}{M^2+m_1^2-m_2^2-\Lambda}\right) - \mathrm{Li}_2\left(\frac{2\Lambda}{M^2+m_1^2-m_2^2+\Lambda}\right)\right] \\
&- \frac{M^2+m_1^2-m_2^2}{2\Lambda} \ln \frac{2Mm_1}{M^2+m_1^2-m_2^2+\Lambda}
\end{aligned}$$

Comparison With PHOTOS

- We used the same methodology as in the case of “ W decay” (*Acta Phys. Pol.* B34, (2003) 4561-4569; *hep-ph*/0303260)
- To visualize the usually small differences between SANC and PHOTOS, we plot the ratios of the predictions from the two programs for the certain class of (pseudo-)observables
- These observables are defined in the next transparency

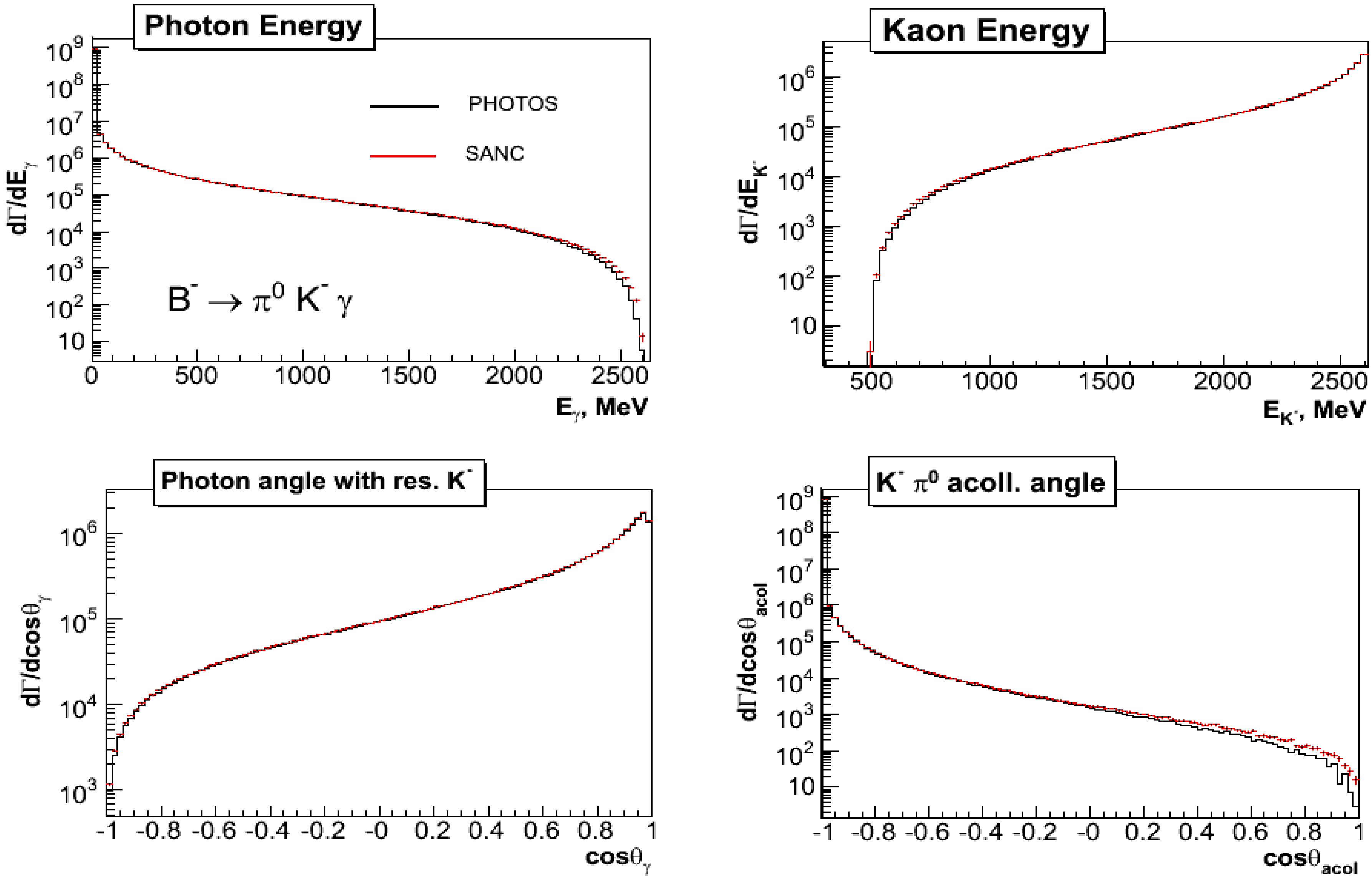
List of Observables

- Photon energy in the decaying particle rest frame – collinear configurations contribute to all bins

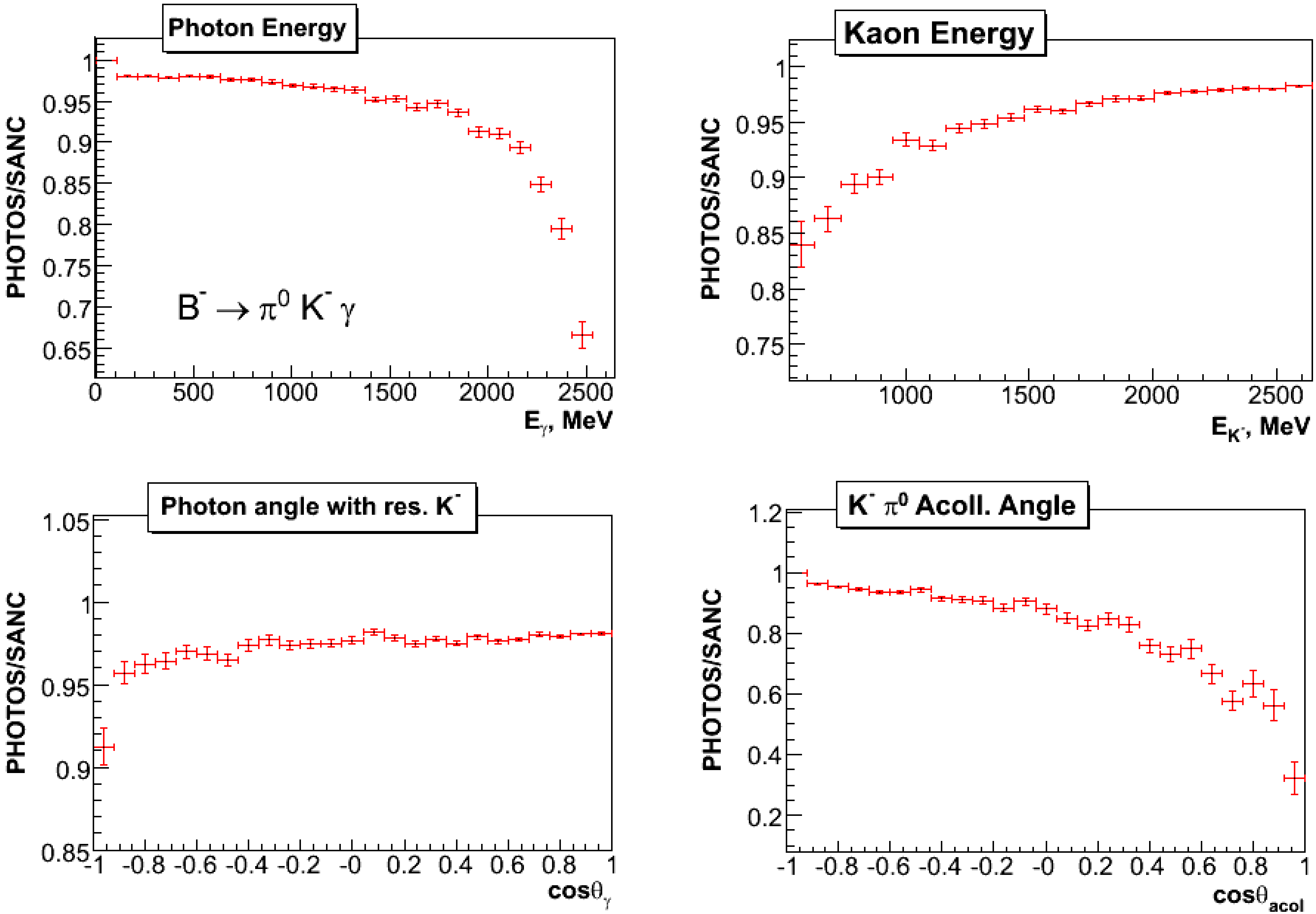
- Energy of final state charged particle – same as in the previous one

- Angle of photon with final-state charged particle – soft photons contribute to all bins.

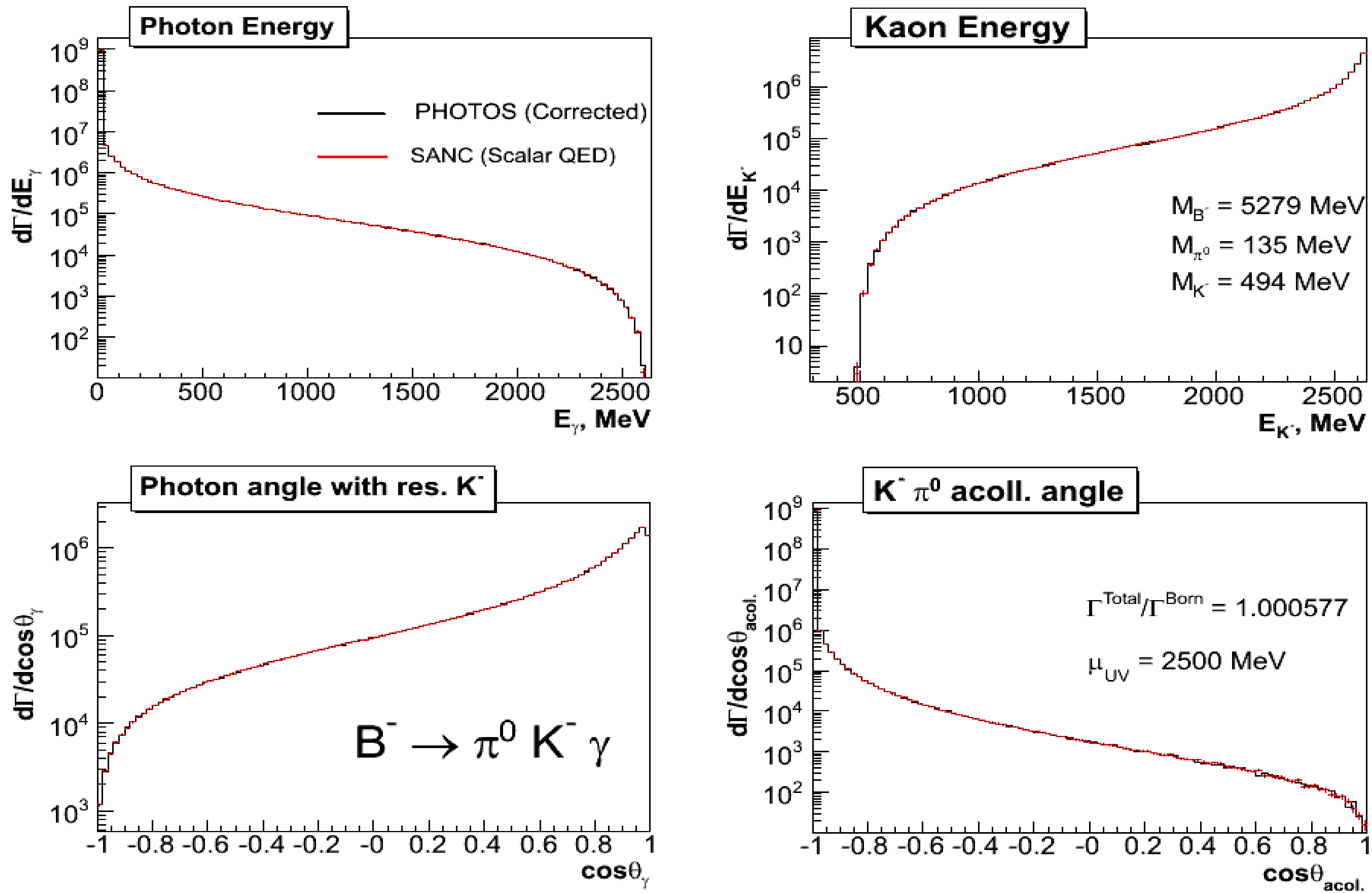
- Acollinearity angle of the final-state particles - visualizes non-soft and non-collinear, i.e non-leading componet of the distributions. Soft-collinear contributions are placed in the first bin of the histograms.



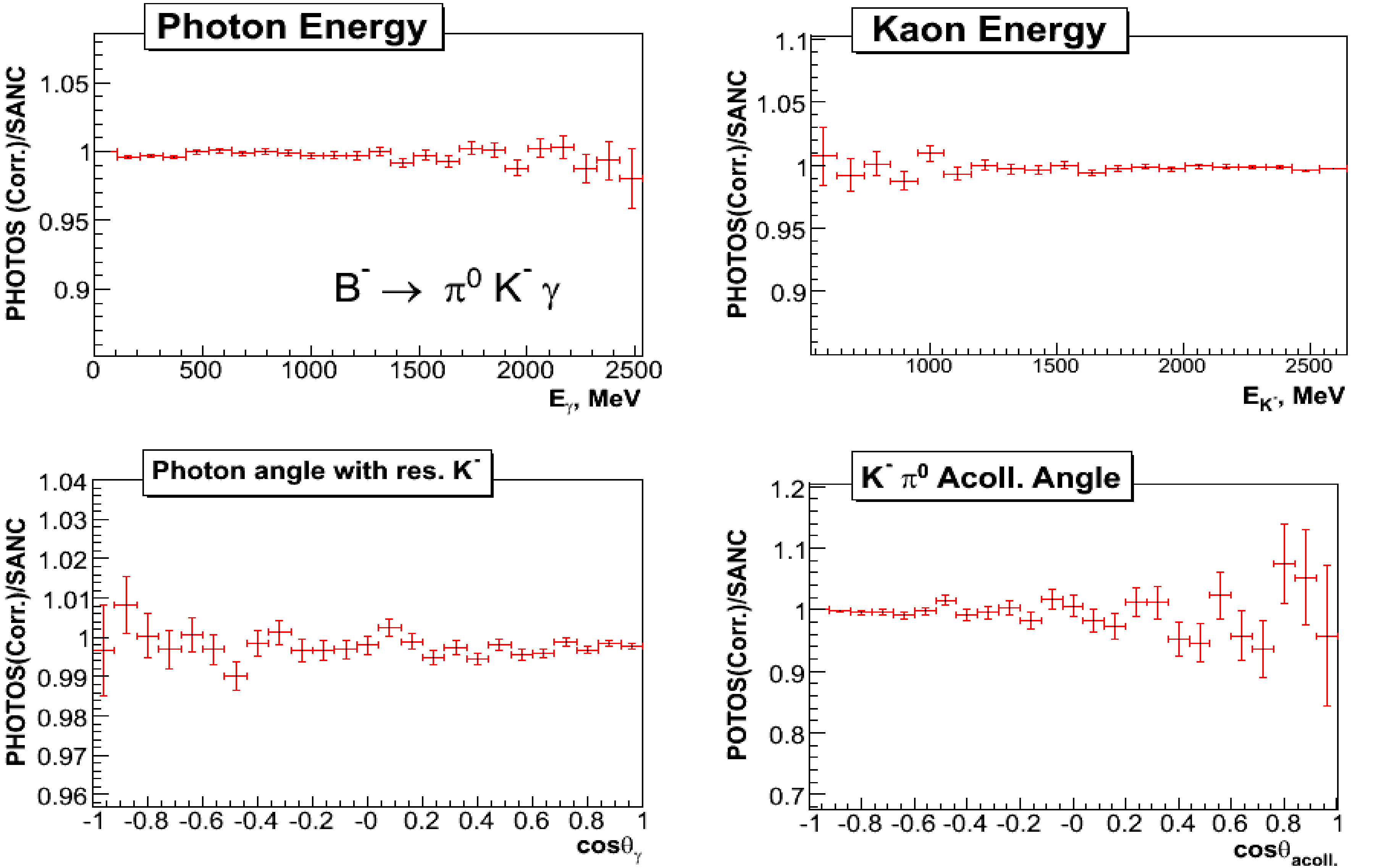
Differential distributions of charged B meson decay from *not corrected* PHOTOS and SANC (Scalar QED). Red color represents the results of SANC, black of PHOTOS.



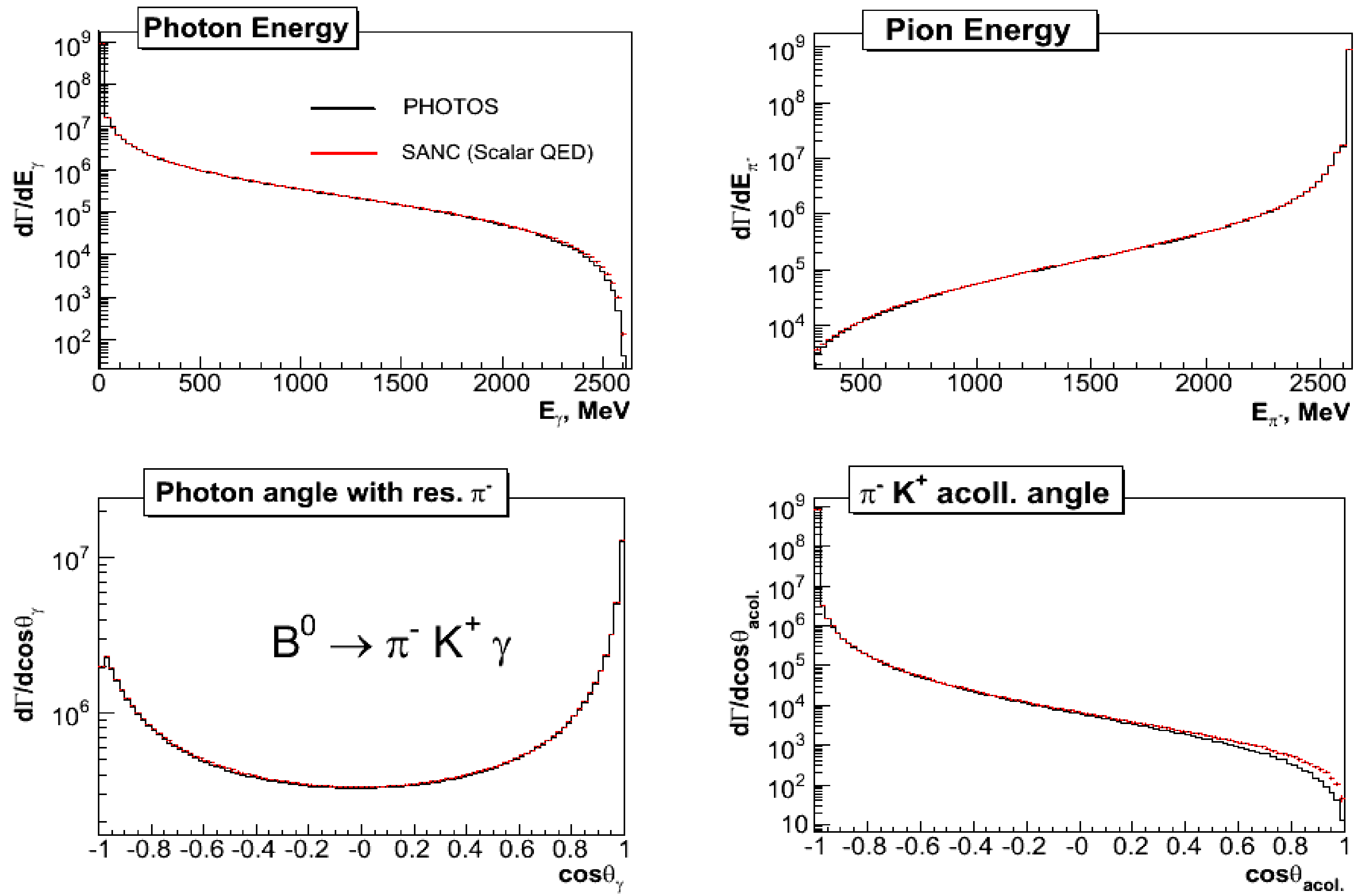
Ratio of distributions from *not corrected* PHOTOS and SANC (Scalar QED)



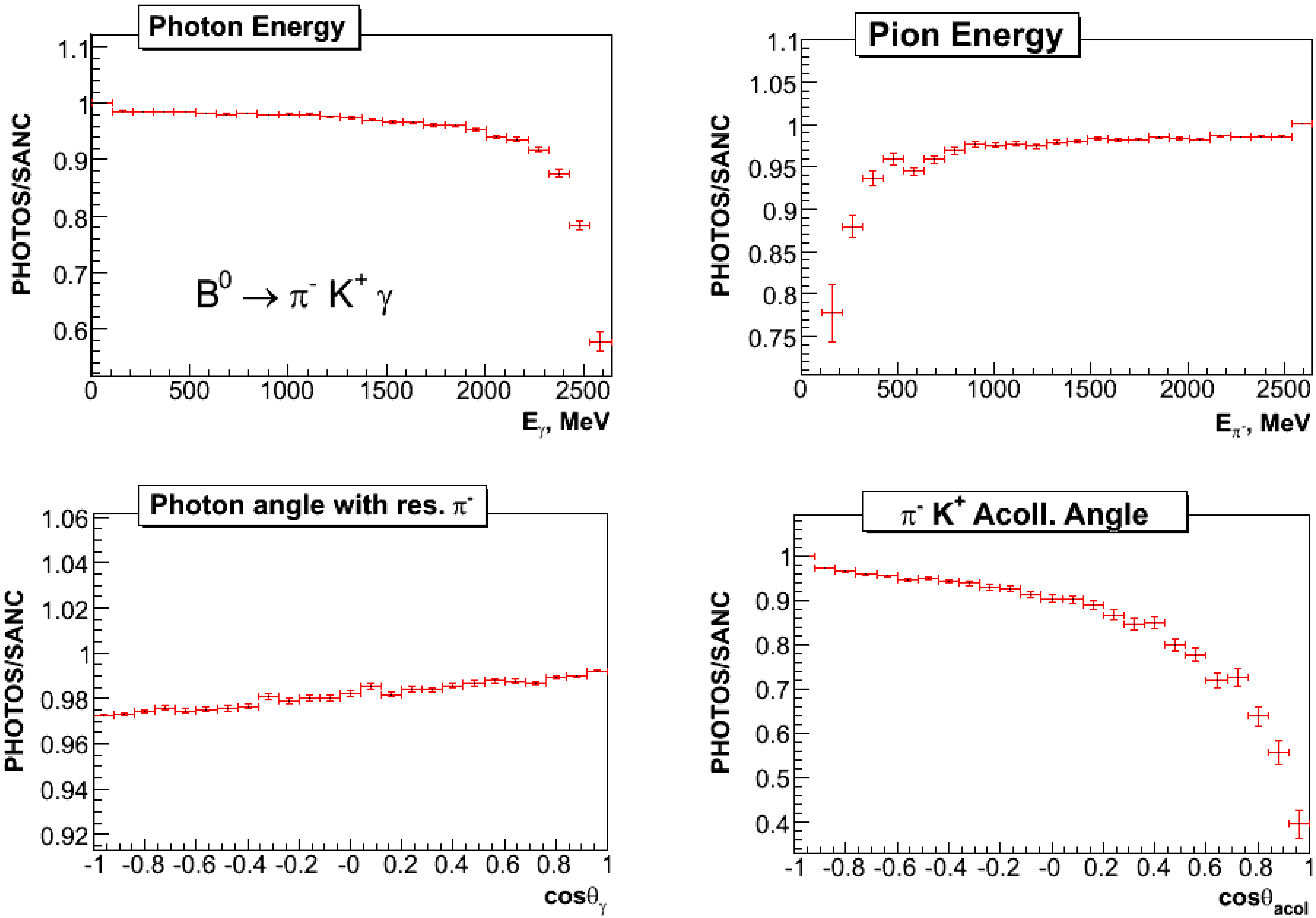
Differential distributions of charged B meson decay from *corrected* PHOTOS and SANC (Scalar QED). Red color represents the results of SANC, black of PHOTOS



Ratio of distributions from *corrected* PHOTOS and SANC (Scalar QED)

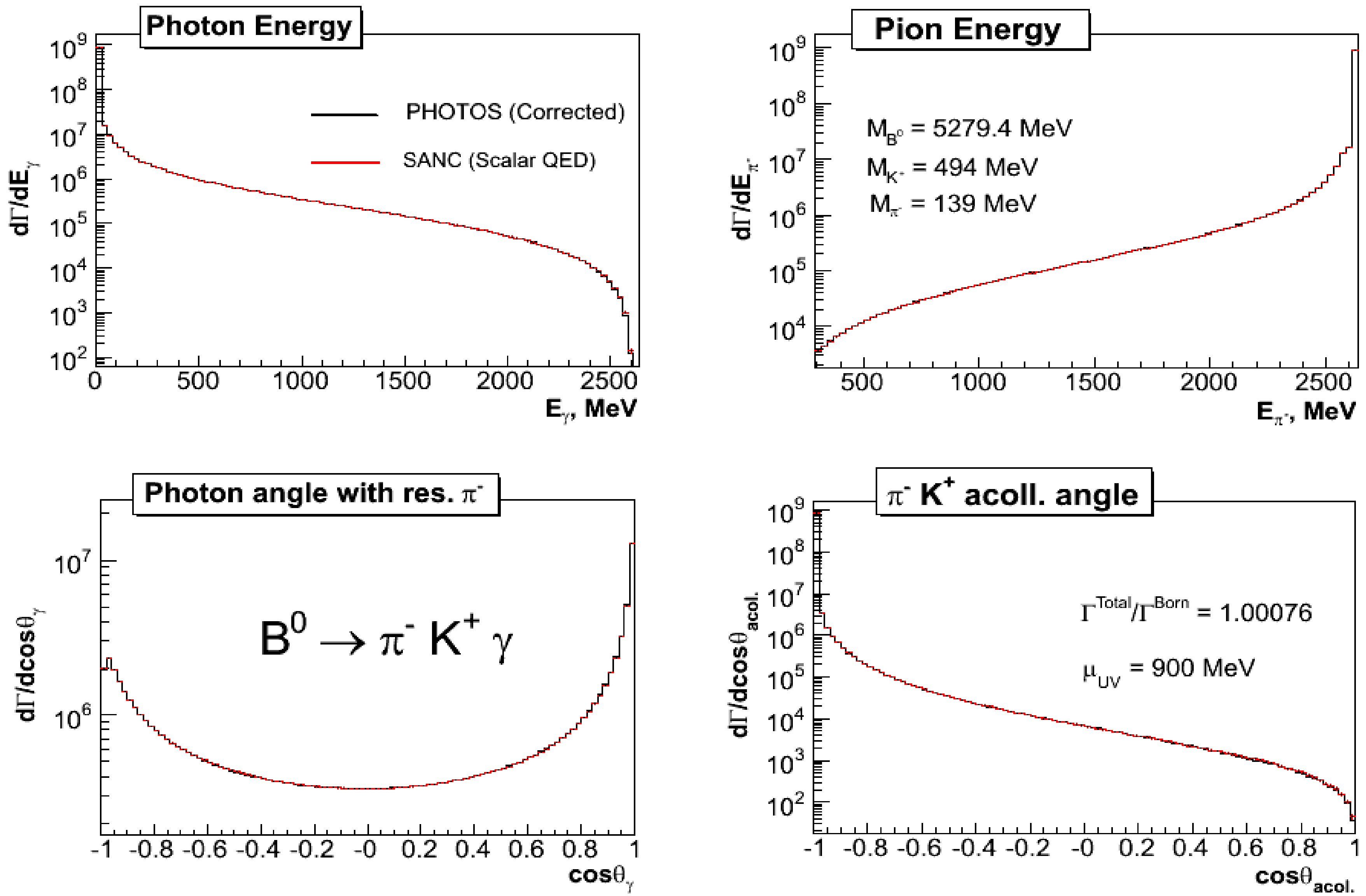


Differential distributions of neutral B meson decay from *not corrected* PHOTOS and SANC (Scalar QED). Red color represents the results of SANC, black of PHOTOS.

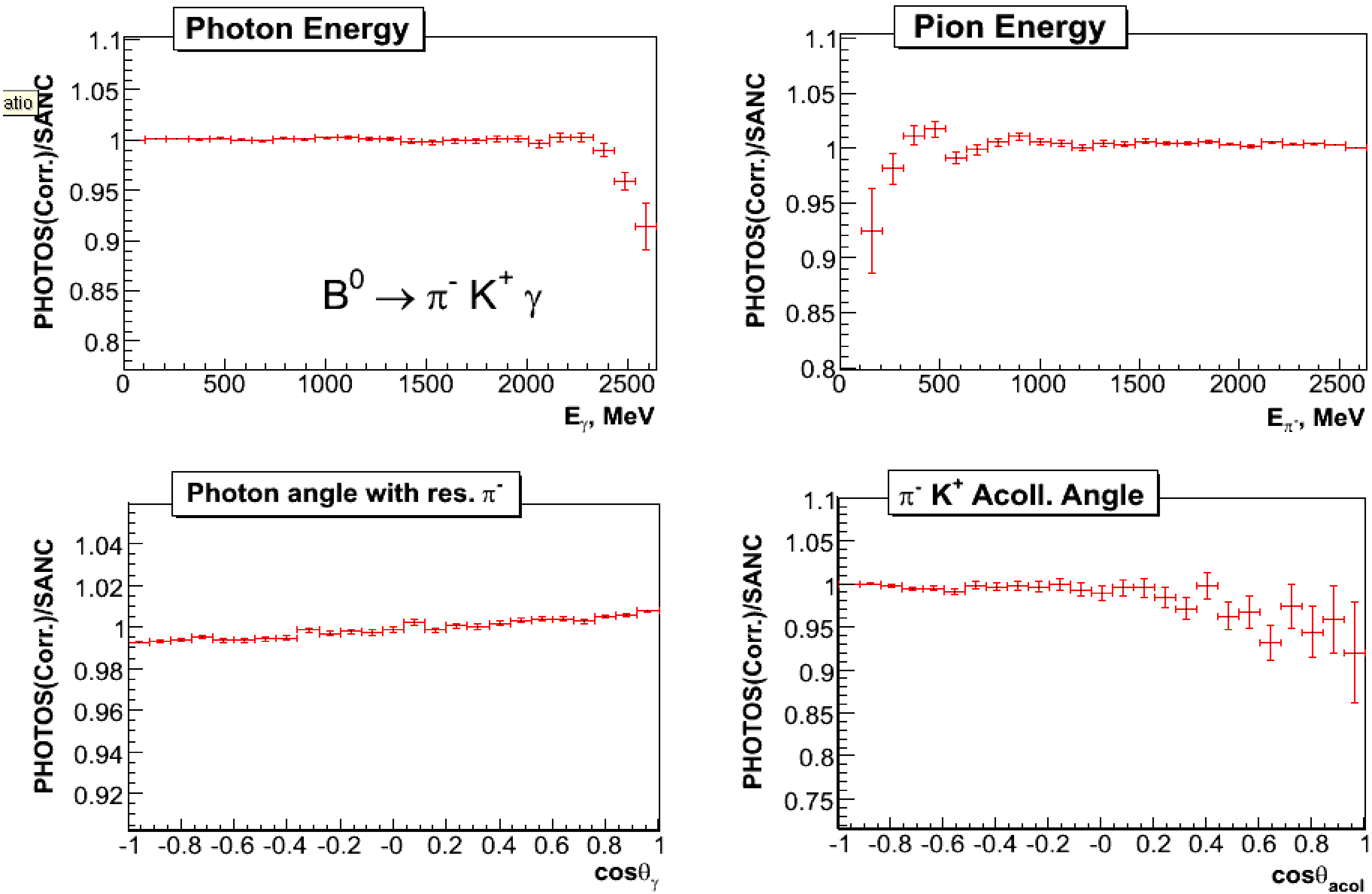


Ratio of distributions from *not corrected* PHOTOS and SANC (Scalar QED)

Looks perfect again, but is not: see the next transparency.



Differential distributions of neutral B meson decay from *corrected* PHOTOS and SANC (Scalar QED). Red color represents the results of SANC, black of PHOTOS.



Ratio of distributions from *corrected* PHOTOS and SANC (Scalar QED)

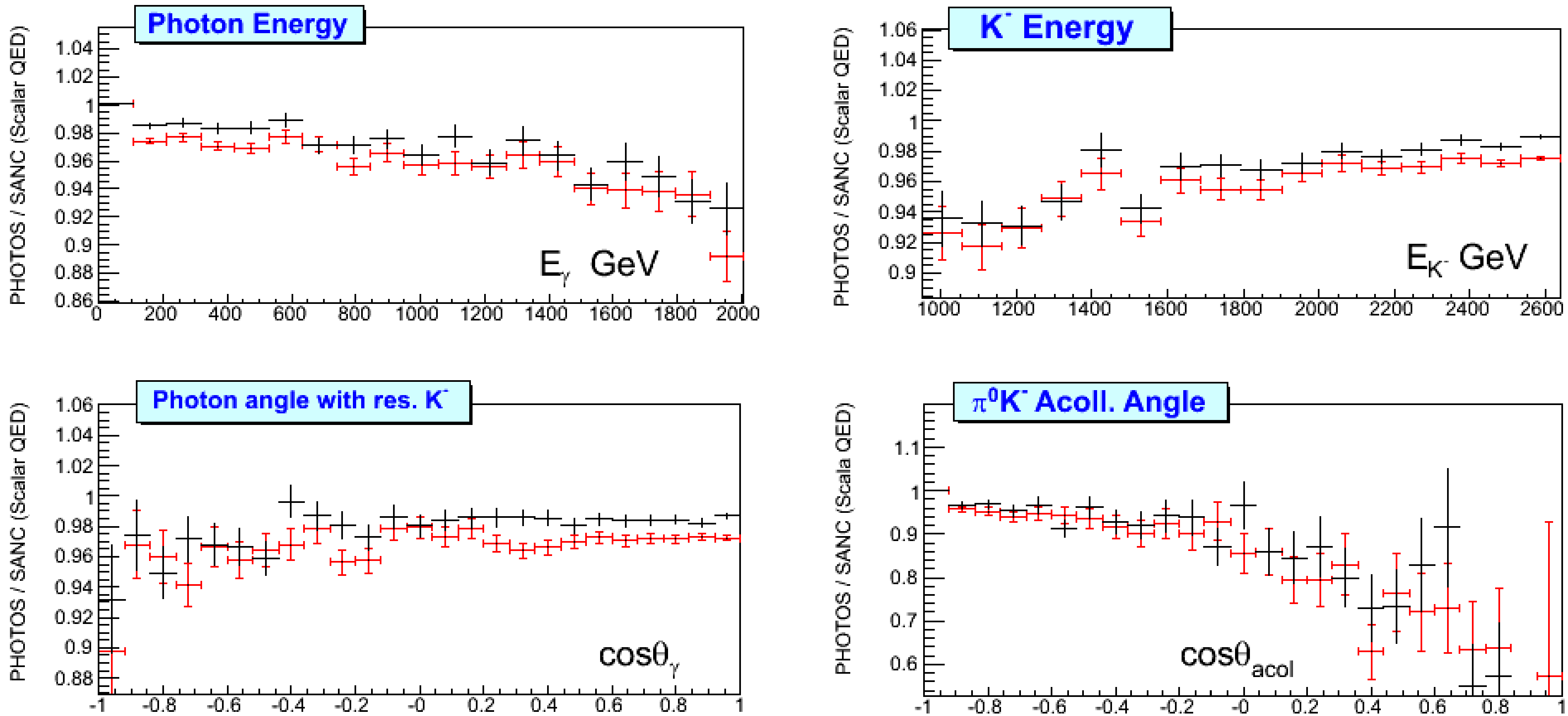
Residual problem related to still inappropriate matching of tangent space is visible, but the effect is at the 0.01 % level !! Exellent test bed for the geometry of the algorithm.

Conclusion

- Comparisons of SANC and PHOTOS for non-leptonic B Meson decays within the Scalar QED Lagrangian were presented.
- Complete first order matrix element was pre-installed into PHOTOS. Contrary to Z decay, it does not enforce changes in interface to event record (simple ME)
- Both standard and new version of PHOTOS agrees for practical applications, sufficiently well with the reference calculation (SANC).
- This is however not the end of the necessary work as measured formfactors will be probably needed.
- NLO effects could be quantified with samples of 1 Gevt each. Good benchmark study completed.
- However ...

Grain of salt

- In the presentation we could see that PHOTOS reproduces with permille level distribution of the density varying by 8 orders of magnitude.
- Having in mind excellent performance of the iteration algorithm as tested in case of Z decay we could conclude that the algorithm will work perfectly for the decay of any particle.
- **This does not need to be always the case. It depends on quality of input !!**
- In our tests, Born level events were double precision: energy momentum conservation, and on shell momenta. These were decomposed into variables for bremssthahlung phase space parametrization.
- This is not always the case !!! PHOTOS searches event record to find elementary decay branches and then sometimes changes it content.
- It is useful. Provide automated answers for the size of bremsstrahlung corrections in decay of any particle. However if event record content is of low quality, so will be the results from PHOTOS.
- We would like to stress importance to check the benchmark results before using PHOTOS.
- That is the price one needs to keep in mind.



$\Lambda_{QED} = M_{B^-}$ (black) $\Lambda_{QED} = 770$ MeV (red)

$\Lambda \rightarrow$ Ultraviolet Renormalization Scale

$M_{B^-} = 5279.57$ MeV

$M_{\pi 0} = 134.98$ Mev

$M_{K^-} = 493.677$ MeV

ω (soft/hard cut-off) = M_{B^-} / 200 MeV

$$B^- \rightarrow K^- \pi^0 \gamma$$

– Hard photon contribution

$$d\Gamma^{\text{Hard}} = |A^{\text{Born}}|^2 4\pi\alpha \left(q_1 \frac{k_1.\epsilon}{k_1.k_\gamma} - q \frac{P.\epsilon}{P.k_\gamma} \right)^2 dLips_3(P \rightarrow k_1, k_2, k_\gamma)$$

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$$\Gamma^{\text{Total}} = \Gamma^{\text{Born}} \left[1 + \frac{\alpha}{\pi} \left(\frac{3}{2} \ln \frac{\mu_{UV}^2}{M^2} - \frac{\pi^2}{3} + \frac{11}{2} \right) \right]$$

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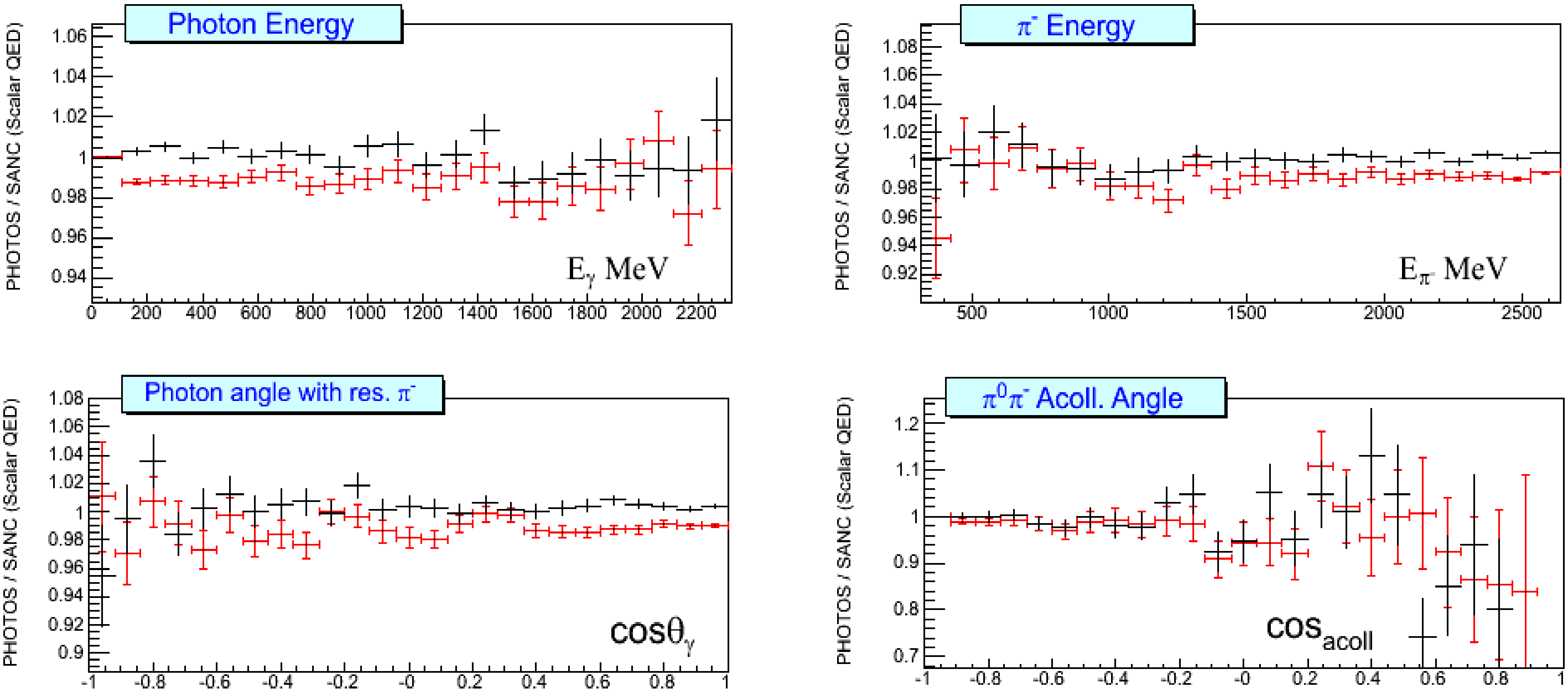

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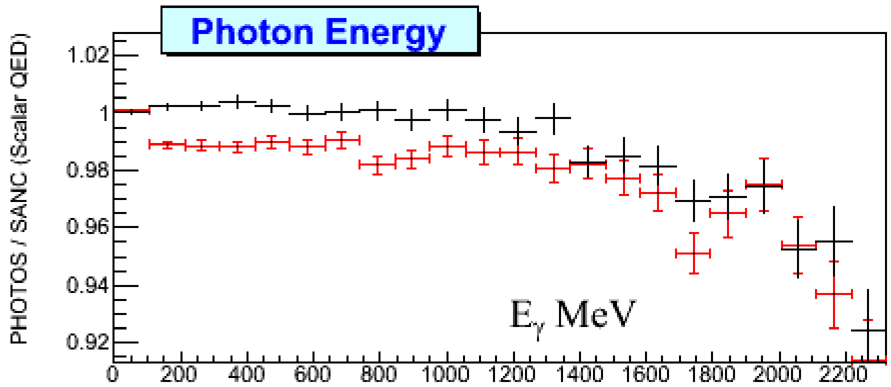
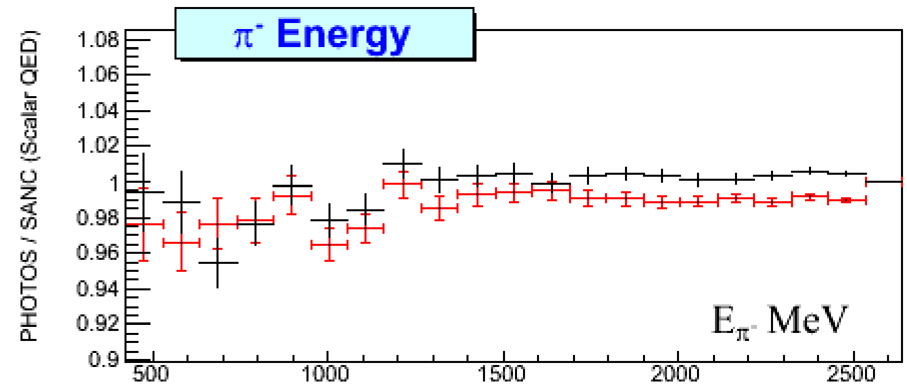
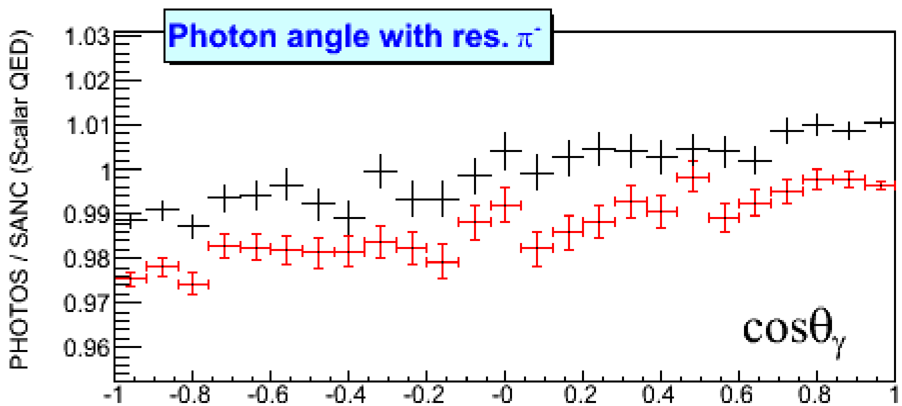
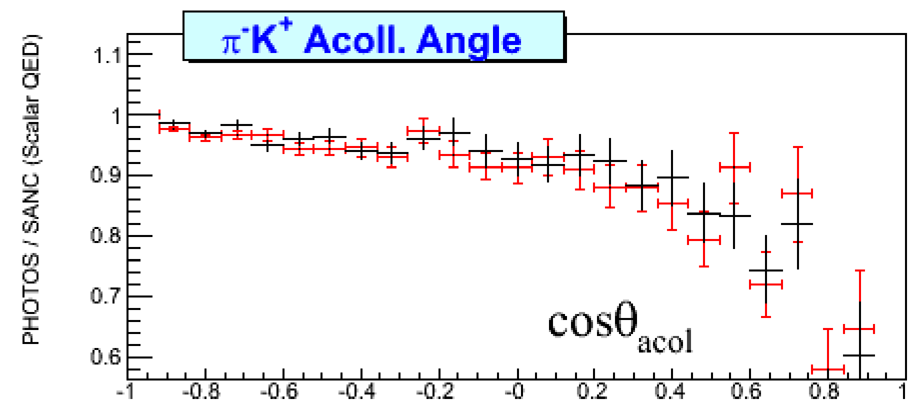








$\Lambda_{QED} = M_{B^0}$ (black) — $\Lambda = 770$ MeV (red)

$\lambda_{QED} \rightarrow$ Ultraviolet Renormalization Scale

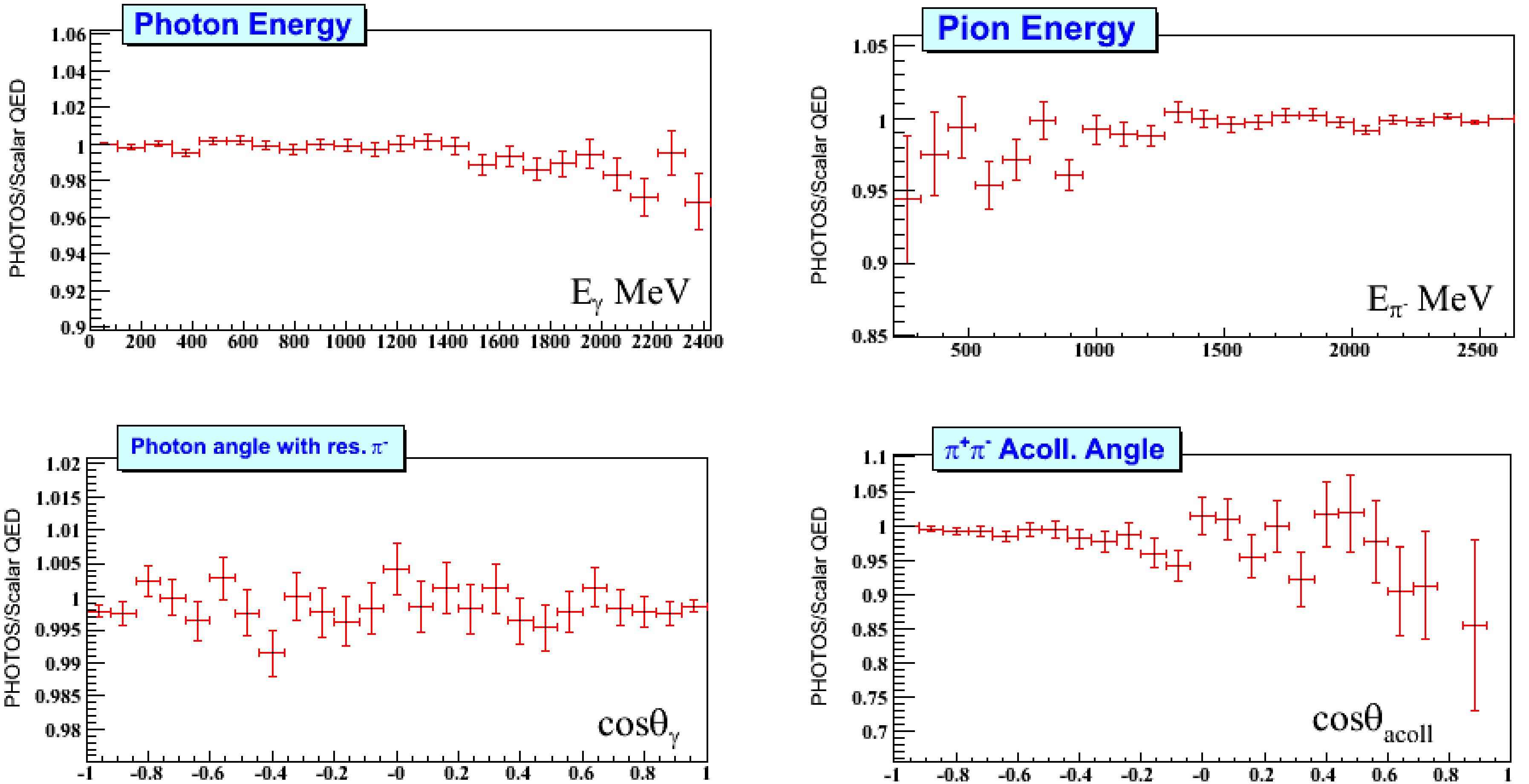
$M_{B^0} = 35279.4$ MeV

$M_{\pi^-} = 139.57$ MeV

$M_{K^+} = 493.677$ MeV

ω (soft/hard cut-off) $= M_{B^0} / 200$ MeV

$$B^0 \rightarrow \pi^- K^+ \gamma$$

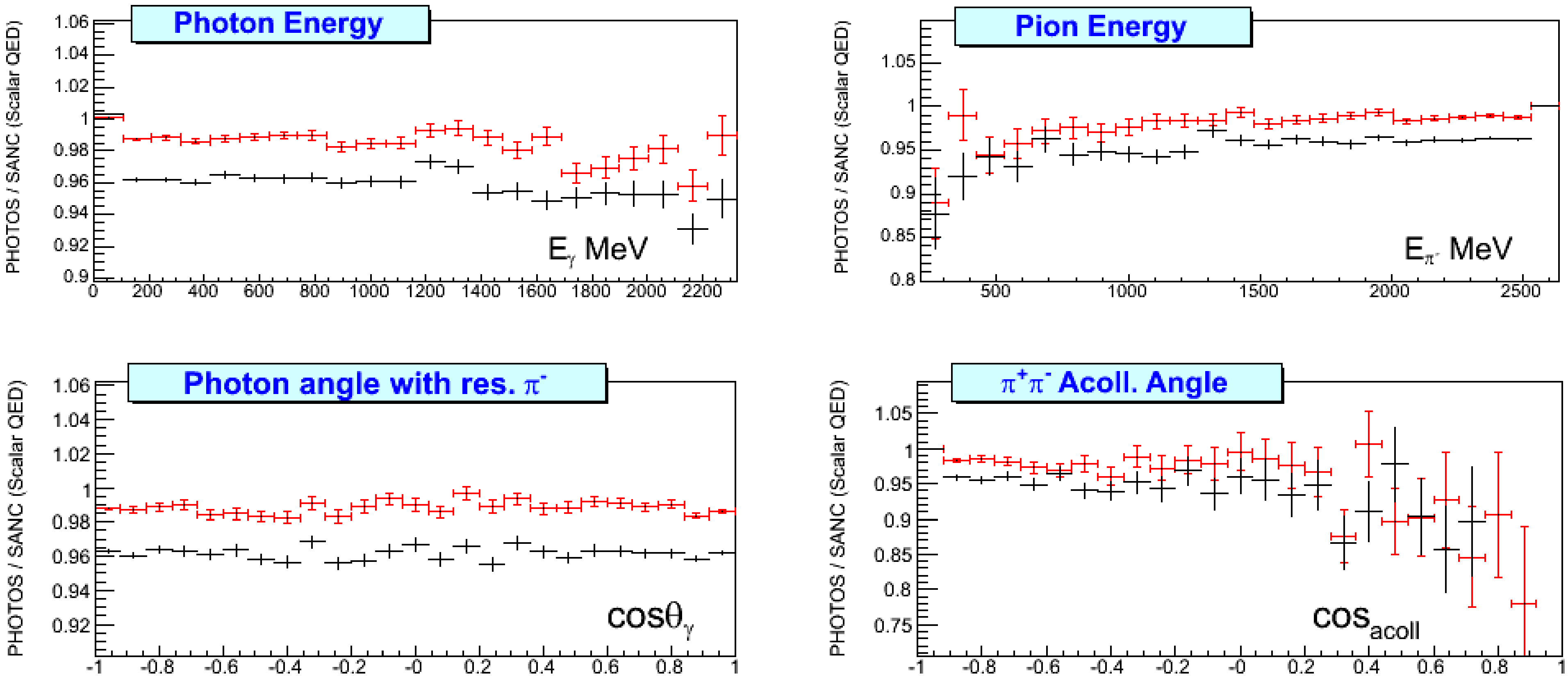


$M_\pi = 139.57$ MeV

$$B^0 \rightarrow \pi^+ \pi^- \gamma$$

$M_{B^0} = 5279.4$ Mev

ω (soft/hard cut-off) = $M_{B^0}/200$ MeV



$\Lambda_{QED} = M_{\pi}$ (black line) $\Lambda_{QED} = M_{B^0}$ (red line)

$\Lambda_{QED} \rightarrow$ Ultraviolet Renormalization Scale

M_{π} = 139.57 MeV

M_{B^0} = 5279.4 MeV

ω (soft/hard cut-off) = M_{B^0}/200 MeV

$$B^0 \rightarrow \pi^- \pi^+ \gamma$$