Bounds on contributions to B_s oscillations in general SUSY models to Next-to-Leading Order

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Outline

- **Introduction:** theoretical framework (OPE + RGE), the MSSM and the Mass Insertion Approximation (MIA)
- **Motivation** and status of the calculations for the $\Delta F = 2$ systems
- **NLO Calculation of the $\Delta F = 2$ Hamiltonian:** schemes considered, checks of the calculation
- **Phenomenological analysis:** B_s system in the light of ΔM_s by CDF-DØ

✔ Theoretical input

Experimental observables: $\Delta M_s\ ,\ \beta_s$ **depend on**

$$\underbrace{\langle \bar{B}_s^0 | H_{full}^{SM} | B_s^0 \rangle}_{\text{meson-antimeson oscillation amplitude}} \longrightarrow \langle \bar{B}_s^0 | H_{full}^{SM} + H_{full}^{SUSY} | B_s^0 \rangle$$

SUSY contributions:
Minimal Supersymmetric Standard Model (MSSM)

The full amplitude is expanded as:

$$\langle B_s^0 | H_{full}^{MSSM} | \bar{B}_s^0 \rangle = \sum_i^{\text{dim } 6} C_i(\mu) \langle B_s^0 | Q_i(\mu) | \bar{B}_s^0 \rangle + \left(\begin{array}{c} \text{co... from} \\ \text{dim} > 6 \text{ op...} \end{array} \right)$$

NEGLIGIBLE

Coefficient Functions (CFs):
encode physics beyond μ

Renormalization scale:
$\mu \sim$ few GeV

Operator Matrix Elements:
encode physics below μ

Step ❶
<u>Perturbative calculation</u>
of the CFs
at some "high" energy scale,
*O(*SUSY masses*)*.
Thus avoid large logs.

Step ❷
<u>RGE evolution</u> of the CFs
(and of the matrix elem's)
down to the scale μ.
ADM for the op. basis
needed (perturbatively).

Step ❸
<u>NON-Perturbative calculation</u>
of the op. matrix elem's
between the *physical* meson
states.
Lattice QCD mandatory.

On the Mass Insertion Approx (MIA)

☞ Redefinition of the squark fields: flavor × chirality basis vs. mass basis

I = flavor

"Left" squarks

$$\tilde{u}_L^I = \left(Z_U^\dagger\right)^{Ii} U_i \qquad \tilde{d}_L^I = \left(Z_D^\dagger\right)^{Ii} D_i$$

flavor × chirality basis

mass eigenstates basis

$$\tilde{u}_R^I = \left(Z_U^\dagger\right)^{(I+3)i} U_i \qquad \tilde{d}_R^I = \left(Z_D^\dagger\right)^{(I+3)i} D_i$$

"Right" squarks

i = 1,…,6

The 'MIA'

Hyp #1: diag terms ~ degenerate

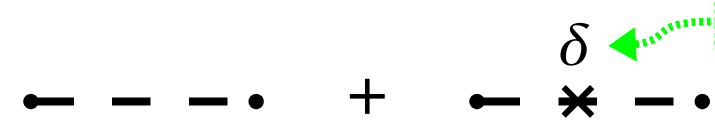
Hyp #2: non-diag small

'rotation'

$$Z_D^\dagger \underbrace{\begin{pmatrix} M_{1\,L}^2 & 0 & \cdots \\ 0 & M_{2\,L}^2 & \cdots \\ \vdots & \vdots & \ddots \end{pmatrix}}_{\textbf{mass basis}} Z_D = \underbrace{\begin{pmatrix} m_{11}^2 & \Delta_{12}^{LL} & \cdots \\ \Delta_{21}^{LL} & m_{22}^2 & \cdots \\ \vdots & \vdots & \ddots \end{pmatrix}}_{\textbf{flavor basis}} \simeq \bar{m}^2\,\mathbf{1} + \begin{pmatrix} 0 & \delta_{12}^{LL} & \cdots \\ \delta_{21}^{LL} & 0 & \cdots \\ \vdots & \vdots & \ddots \end{pmatrix}$$

squark propagator (flavor basis + MIA)

$\simeq$



SUSY source of FCNC's

diagonal **NON-diag.: Mass Insertion**

SUSY contributions to ΔM_s

$$\Delta M_s^{MSSM} = \Delta M_s^{SM} + \Delta M_s^{\tilde{g}} + \Delta M_s^{H-\text{box}} + \Delta M_s^{H-\text{DP}} + \Delta M_s^{\tilde{\chi}^+} +$$

neutralinos: negligible

gluinos

Dominant contribs, when NOT assuming any symmetry in the squark matrix

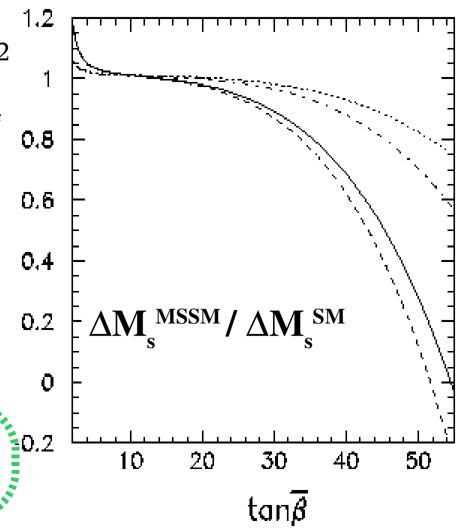
For low/moderate tan β, SUSY contrib's = gluinos

Higgses

$$\Delta M_s^{H-\text{box}} \propto -\tan\beta^2$$
$$\Delta M_s^{H-\text{DP}} \propto -\tan\beta^4$$

Dominant contribs (*negative*), for large tan β

Buras et al.
NPB 619 (2001)



charginos

One has in general

$$\frac{\Delta M_s^{\tilde{\chi}^+}}{\Delta M_s^{\tilde{g}}} \leq 10^{-2} \frac{\delta_u \times \delta_u}{\delta_d \times \delta_d}$$

Ball et al.
PRD 69 (2004)

Naïve ranking of gluino contributions

Assuming $M_{gluino} / M_{squark} \approx 1$, one finds

$$\frac{\Delta M_s^{\tilde{g}}}{\Delta M_s^{SM}} \simeq \left(\frac{500\,\text{GeV}}{m_{\tilde{q}}}\right)^2 \Big[\, O(1)\left(\delta_{LL}^2 + \delta_{RR}^2\right) + O(10)\left\{\left(\delta_{LR}^2 + \delta_{RL}^2\right) \;\&\; \left(\delta_{LR}\delta_{RL}\right)\right\} + O(100)\left(\delta_{LL}\delta_{RR}\right) \Big]$$

Looking at **single** δ's, bounds are hierarchical

δ_{LL} (or δ_{RR}) only ☞ $/\delta/ \leq 0.4$

$(\delta_{LR}, \delta_{RL})$ ☞ $b \to s\,\gamma$ does better

$\delta_{LL} \times \delta_{RR}$ only ☞ $/\delta/ \leq 0.02 \div 0.05$

‘State of the art’ of the calculations for $\Delta F = 2$ Hamiltonians

Step ❶
CFs(M_{SUSY})

SM: complete to NLO

MSSM: complete to LO

{ NLO available only for the 2HDM }

Herrlich + Nierste 1994, ‘95 & ‘96

Buras et al. 1990

Gabbiani et al. 1996

Step ❷
CFs evolution:
CFs(M_{SUSY}) → CFs(μ)

SM + MSSM: complete to NLO

{ calculation to 2 loops of the *anomalous dimension matrix* for the basis Q_i *(dim = 6)* }

Ciuchini et al. 1998

Step ❸
Hadronic Matrix Elements

SM + MSSM: *quenched* lattice estimate

{ for the basis Q_i *(dim = 6)*
[unquenched (= full QCD) estimate **highly desirable**] }

Bécirévic et al. 2001

☞ **NLO corrections to the CFs in the MSSM** are still missing (... and important)

Motivations

0) They permit (by definition) to **verify the perturbativeness** of the expansion in α_s

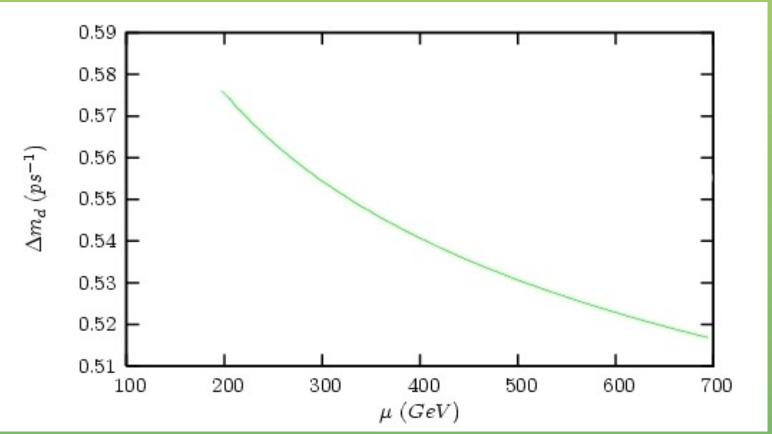
1) A *complete-to*-NLO analysis requires both the NLO ADM for the operator basis (already present) **and** of the NLO corrections to the CFs. If one or both the ingredients are missing, the error in the analyses is a NLO one.

D.Bécirévic et al. **NPB 634 (2002)**

M.Ciuchini *et al.* **JHEP 10 (1998)**

2) Existing phenomenological analyses are affected by a **residual scheme and μ dependence** corresponding to an $O(\alpha_s(M_{SUSY}))$ error.

$2\,Abs\{\langle \bar{B}_d | H_{eff}^{\Delta B=2} | B_d \rangle\}$
(LL & RR contribs.)
vs. scale $\boldsymbol{\mu}$



3) They allow to reach within the MSSM the **same precision** as that in the SM. (then the error is all in the hands of the non-pert. parts)

NLO calculation of the ΔF=2 Hamiltonian

CFs calculation

CFs are obtained enforcing (at the scale M_{SUSY}) the following equality ('matching' full-eff)

Full *Eff*

$$\langle \bar{b}\, d | H_{full}^{MSSM} | b\, \bar{d} \rangle = \sum_{i}^{dim\ 6} C_i \langle \bar{b}\, d | Q_i | b\, \bar{d} \rangle_R = \sum_{i\,j}^{dim\ 6} C_i\, r_{ij} \langle \bar{b}\, d | Q_j | b\, \bar{d} \rangle_{tree}$$

2 loop amplitude in the full theory

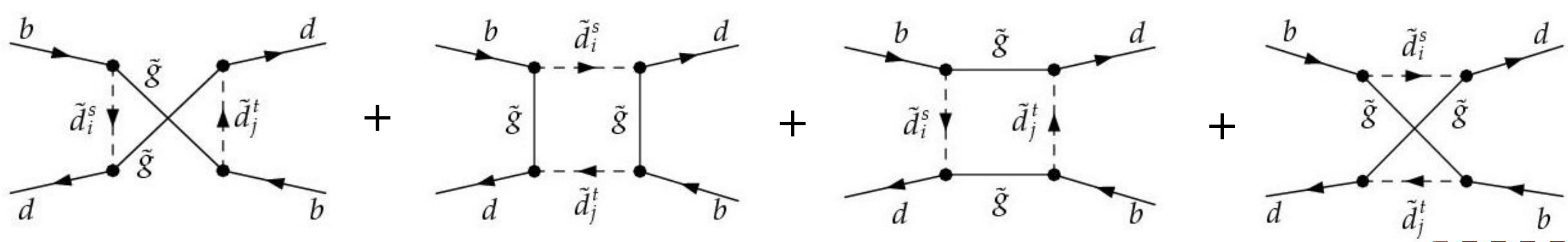
external states with arbitrary kinematics

same external states as in the full theory

1 loop corrections in the effective theory

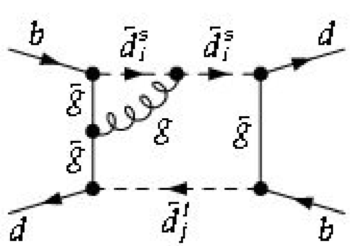
Amplitude in the full theory

$LO =$



F.Gabbiani *et al.* **NPB 477 (1996)**

$NLO =$



+ *another **70** 1PI, "non-equivalent" diagrams*

It becomes important
- ✔ to **automatize** as much of the calculation as possible
- ✔ to be sure **not to forget** any diagram

NLO calculation: general strategy

The whole calculation was performed within the program Mathematica

Generation of all the **2 loop** Wick contractions in the MSSM (strong)

Basis: package FeynArts

- ✔ it **generates all the 2 loop topologies** with insertion of vertices in a given model
- ✔ it writes the single diagram in the form of a Feynman Amplitude

Modification of the implemented model (MSSM) for allowing the most general squark mixing

Creation of the **Feynman Amplitude** for the single diagrams

Manipulations on the single diagram. In particular: **gamma matrix reduction**

Basis: package TRACER

- ✔ it allows to calculate traces of gamma structures in arbitrary dimensions
- ✔ it allows the use of different regularization schemes. In particular **DRED e NDR**

Use of both schemes (def. of the **evanescents** mandatory)

Integration over the loop momenta

- ✔ adopting the Mass Insertion Approximation (MIA), only ≤ 3 loop masses ('feasible')
- ✔ integrals diverge borh UV and IR. **Use of 2 IR regs. as a check:** gluon mass and IR-dimensional

Some checks of the calculation

The CFs were calculated
- ☑ in two regularization schemes (DRED & NDR)
- ☑ with two IR regulators: 'gluon mass' & dimensional

Case A: DRED, with IR-reg = gluon mass

1st check

Case B:
DRED, with IR-reg = dim

2nd check

Case C:
NDR, with
IR-reg = gluon mass

Note
residual *($1/\epsilon_{IR}$)* divergences after renormalization

Necessary to include LO contributions from **DRED evanescents**

Martin + Vaughn
PLB **318**, 331 (93)

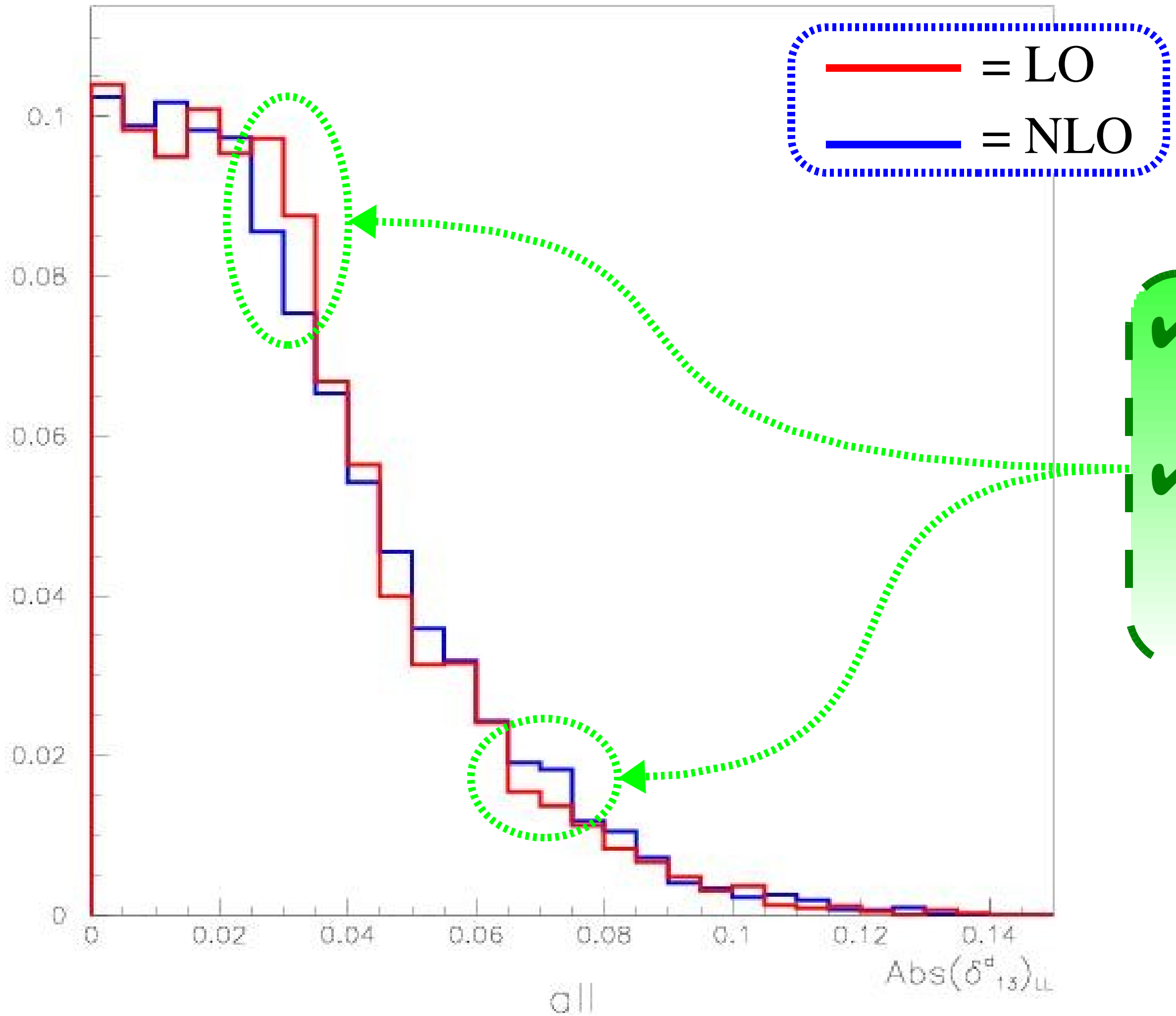
Notes for the check A vs. C

- ✔ Consider the *ΔZ* of **scheme changing** (DRED – NDR) in the effective theory
- ✔ **NDR breaks SUSY:** add suitable *finite corrections* to masses and *SU(3)* couplings

Impact of NLO CFs

Previous phenomenological analyses used LO values for the CFs.
What is the effect of going to the NLO ?

Example: constraints on *Abs[δ]* , case "*LL* only"



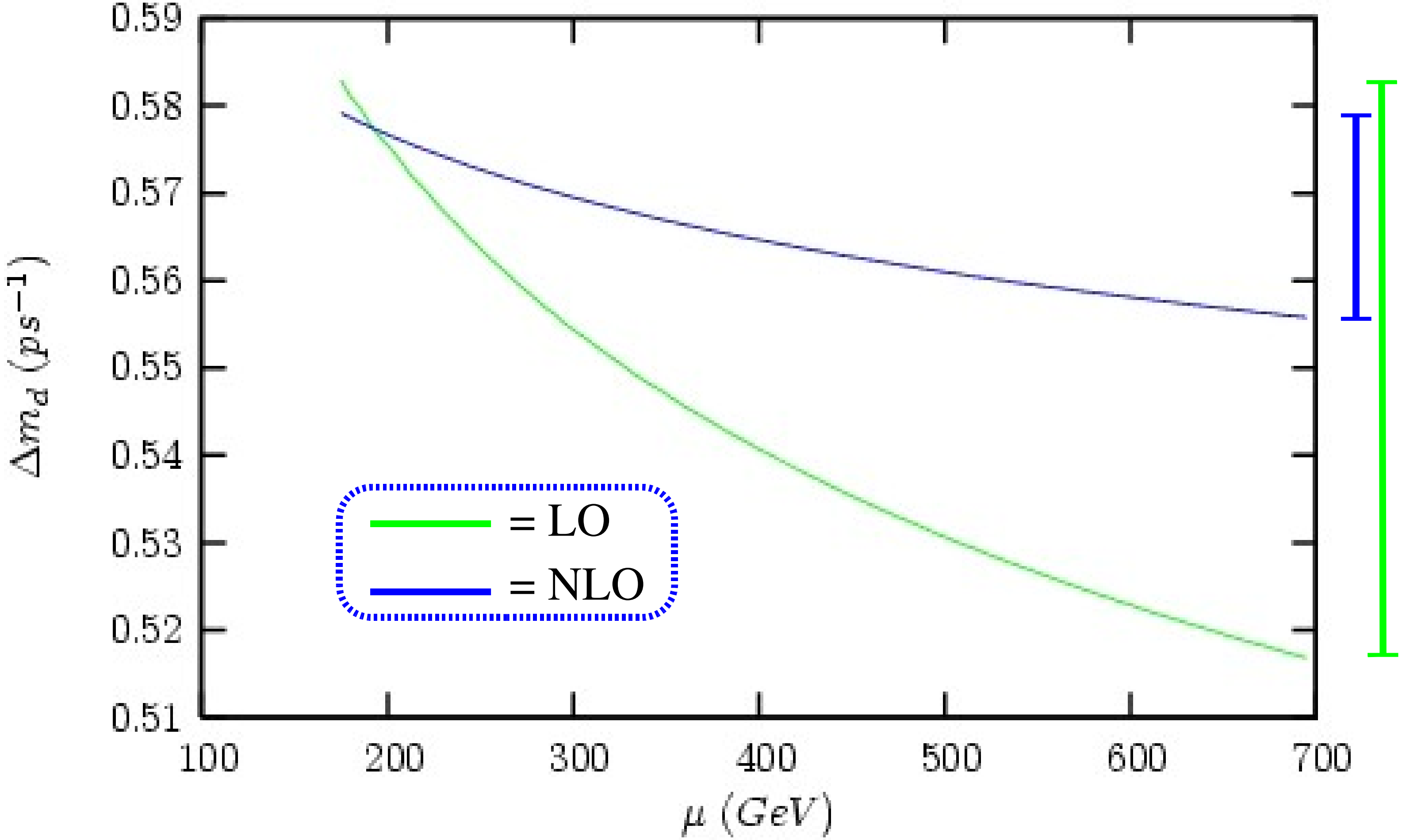
✔ NLO tends to slightly **relax** limits on δ's

✔ The effect is a percent level one. Many percent for the "*LR* = *RL*" case.

Impact of NLO CFs

***μ*-dependence of the amplitude:** case "*LL* only"

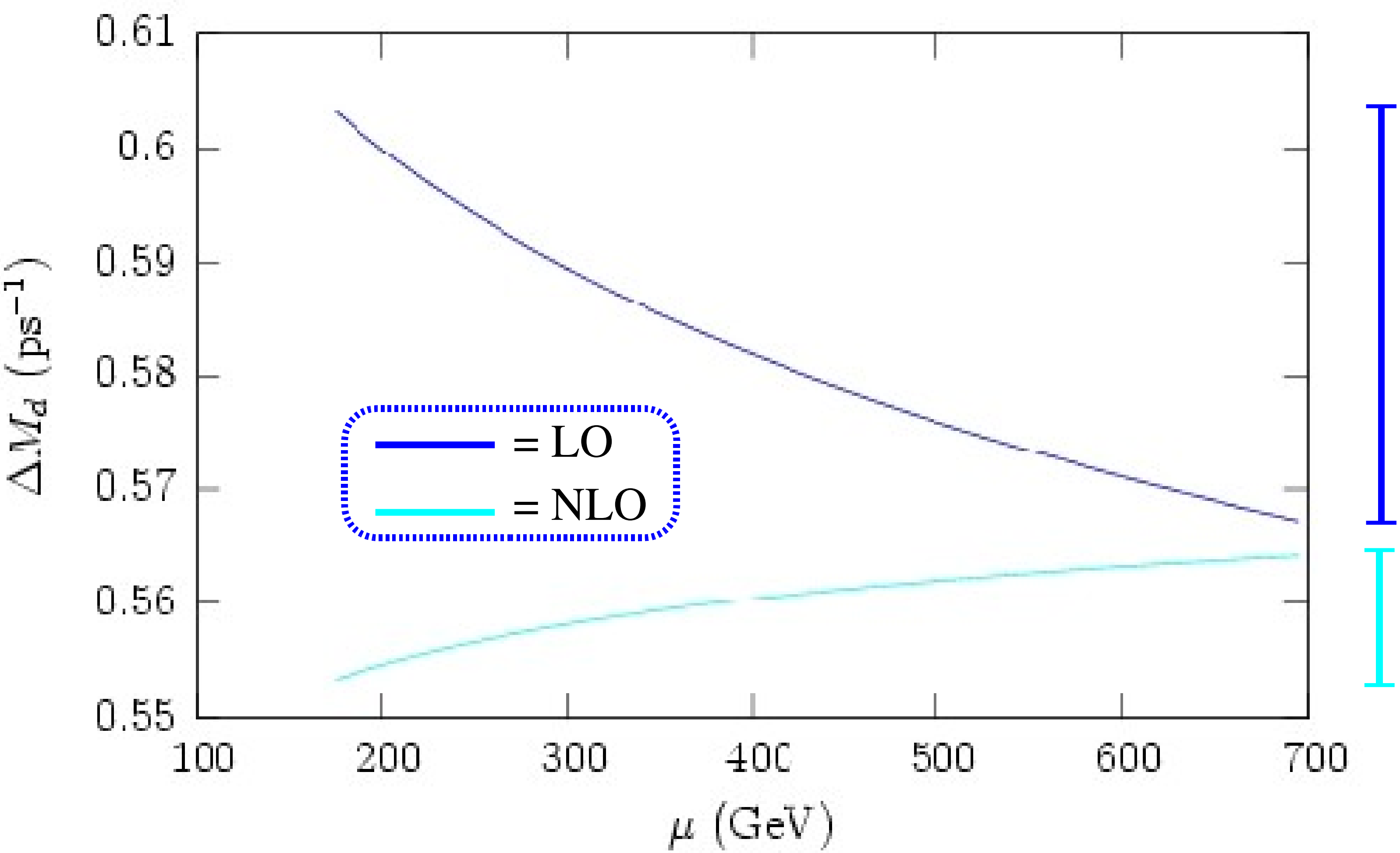
scale dependence
~10 % (LO) → ~ 3.5% (NLO)



Impact of NLO CFs: continued

***μ*-dependence of the amplitude:** case "*LR* only"

scale dependence
~6 % (LO) → ~ 2% (NLO)



Phenomenological analysis (B_s system)

Phenomenological analysis

With the $O(\alpha_s)$ corrections to the CFs it becomes possible a full NLO analysis

Exp quantities

$$\Delta m_s = 17.33^{+0.42}_{-0.21} \pm 0.07\ ps^{-1}$$

$\Delta F = 1$ decays: $b \to s\gamma$, $b \to s\, l^+ l^-$

Theory relations

$$\Delta m_s = 2\, Abs\{\langle \bar{B}_s | H_{eff}^{\Delta B, S=2} | B_s \rangle\}$$

and MSSM formulae for the decays **(1)**

Exp input

- ☑ SM couplings & masses
- ☑ ϱ, η from tree-level processes
- ☑ bag parameters from the lattice

(2)

Evaluation of the matrix element

$$\langle \bar{B}_s | H_{eff}^{\Delta B, S=2} | B_s \rangle$$

...and the other input in **(1)**

Constraining δ's

a) Generate values for the matrix element by

✔ extracting the exp input **(2)** with normal distributions

✔ extracting Abs[δ's] and Arg[δ's] with flat distributions

MonteCarlo

b) Calculate observables O in **(1)**, with the values generated in ***a)***, and give them a 'gaussian weight', *i.e.* weigh with $\mathrm{Exp}[-(O_{th} - O_{exp})^2]/(2\sigma^2_{exp})$

Constraints on *Re[δ]* and *Im[δ]*

General procedure: Our matrix element is written as follows

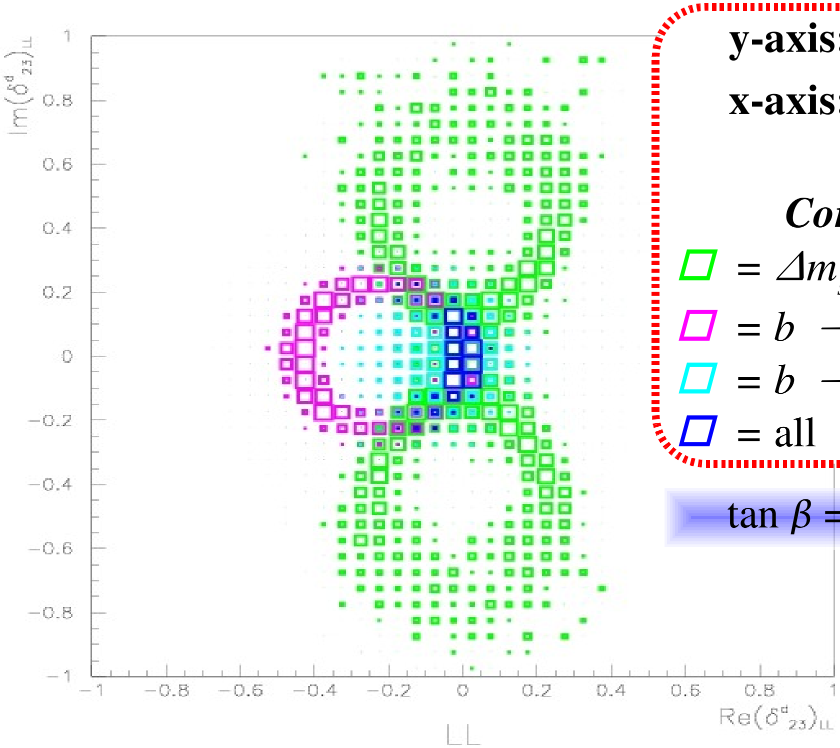
$$\langle \bar{B}_s | H_{eff}^{\Delta B,\, S=2} | B_s \rangle \;=\; \underbrace{Re\, A_{SM} + i\, Im\, A_{SM}}_{\textbf{SM part}} + A_{SUSY} \left(Re\, (\delta_{AB}^{(23)})^2 + i\, Im\, (\delta_{AB}^{(23)})^2 \right)$$

Always pairs of δ's appear in the ampliude

SUSY part proportional "to a given δ^2_{AB}"

We switch on one δ^2_{AB} at a time: *LL* only, *RR* only, *LL=RR*, *LR*, *RL* or *LR=RL*

Example 1: constraints on *Re[δ]* vs. *Im[δ]*, case "*LL* only"



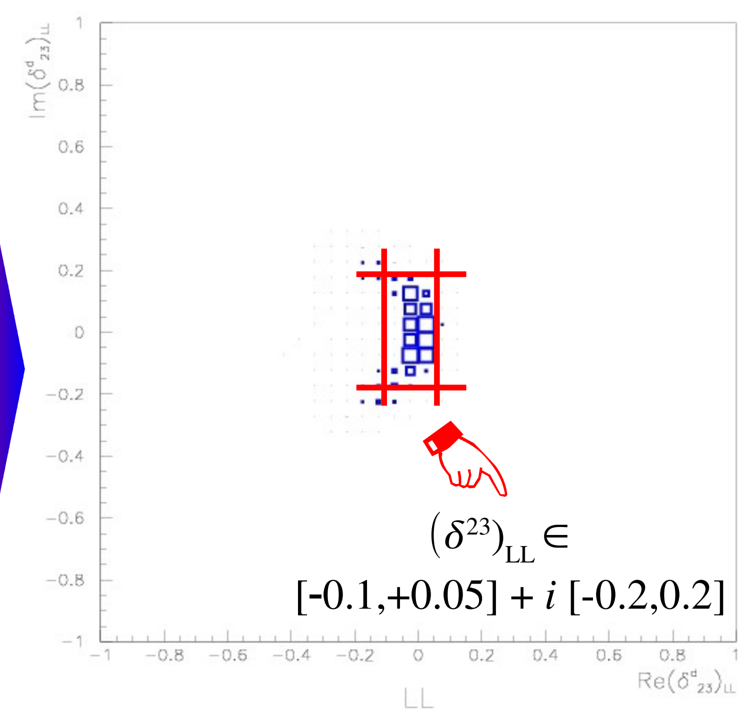
y-axis: $Im[(\delta^{23})_{LL}]$

x-axis: $Re[(\delta^{23})_{LL}]$

Constraints

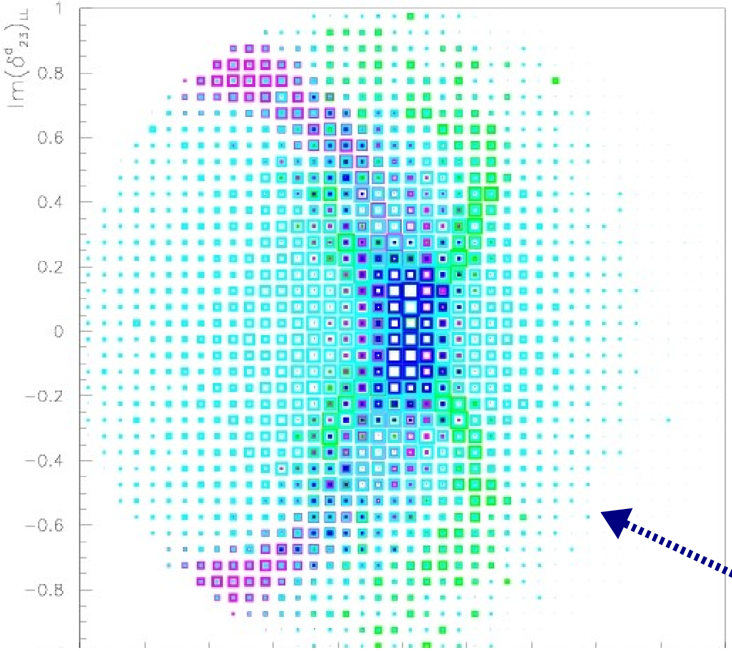
- $= \Delta m_s$
- $= b \to s\ \gamma$
- $= b \to s\, l^+ l^-$
- = all

$\tan\beta = 10$



$(\delta^{23})_{LL} \in$
$[-0.1, +0.05] + i\, [-0.2, 0.2]$

LL, RR insertions: the ΔM_s constraint is relevant / fundamental



y-axis: $Im[(\delta^{23})_{XY}]$

x-axis: $Re[(\delta^{23})_{XY}]$

Constraints

□ = Δm_s

□ = $b \rightarrow s\ \gamma$

□ = $b \rightarrow s\ l^+ l^-$

□ = all

LL only, tan β=3

⇩

[-0.15,+0.15] + *i* [-0.25,0.25]

RR only, tan β=3

⇩

[-0.4,+0.4] + *i* [-0.9,0.9]

LL=RR, tan β=3

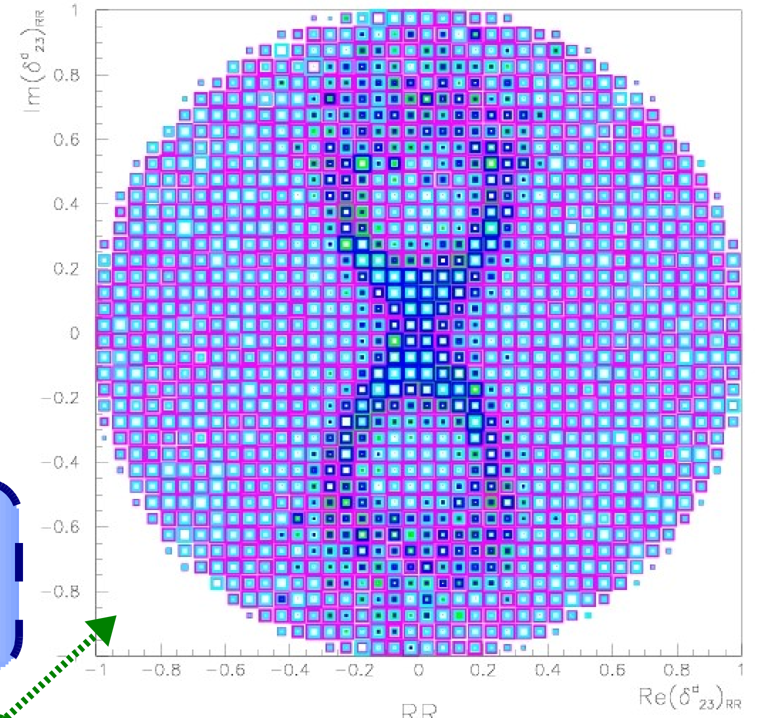
⇩

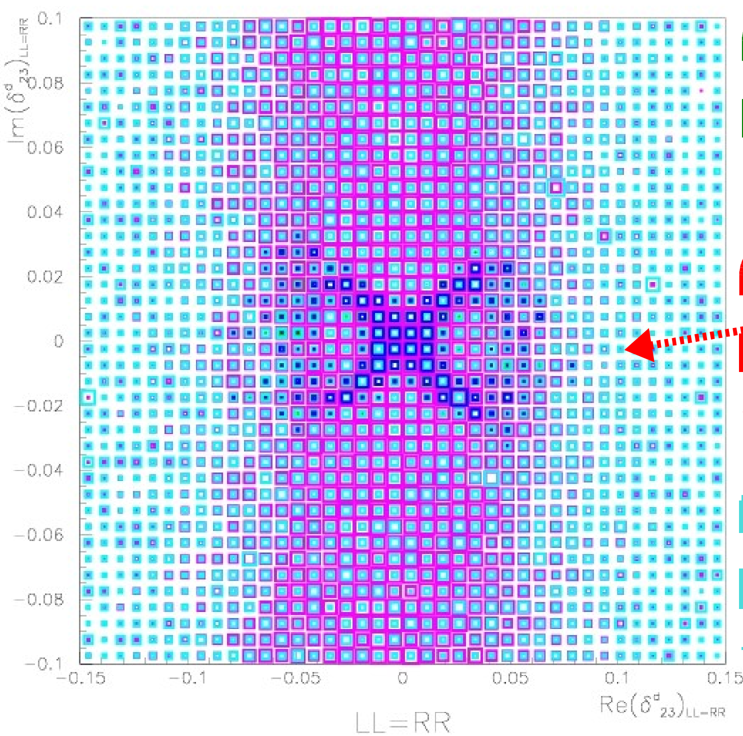
[-0.05,+0.05] + *i* [-0.03,0.03]

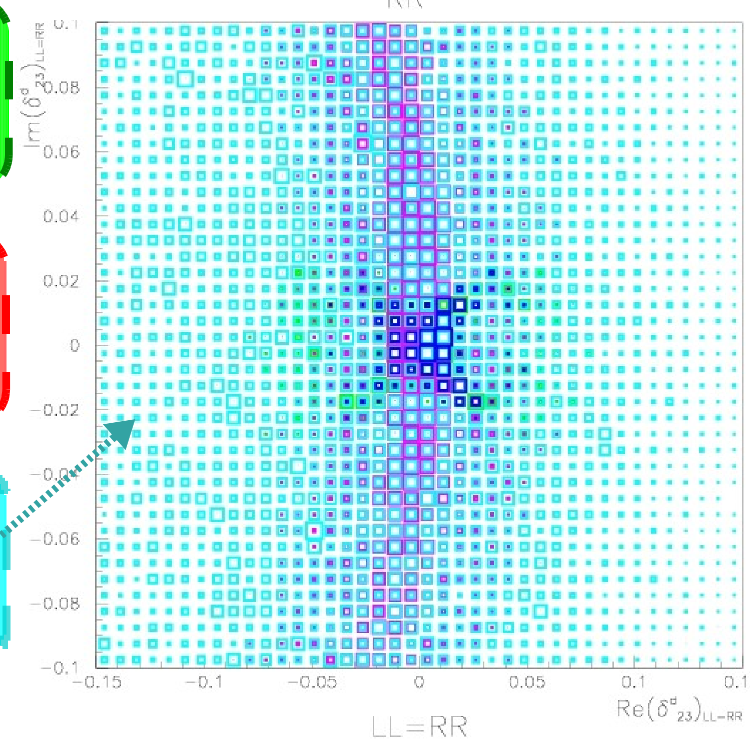
LL=RR, tan β=10

⇩

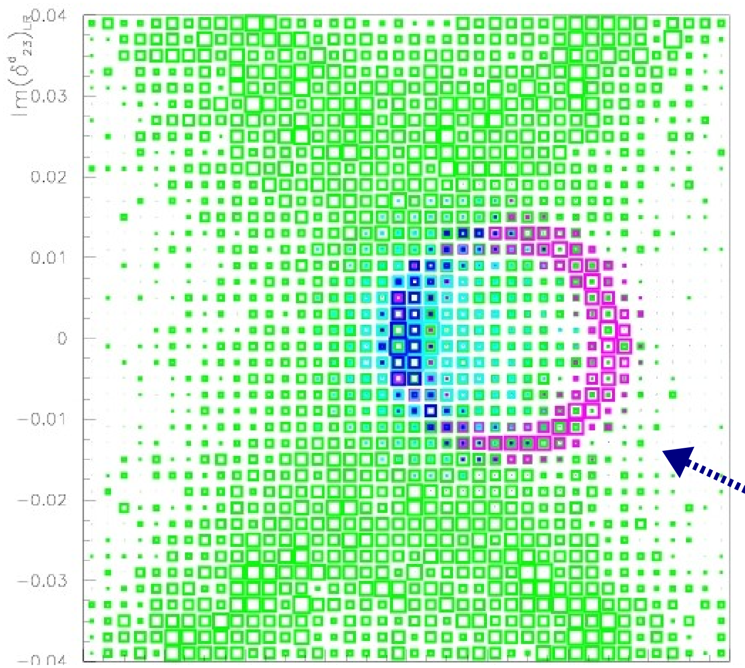
[-0.03,+0.03] + *i* [-0.02,0.02]

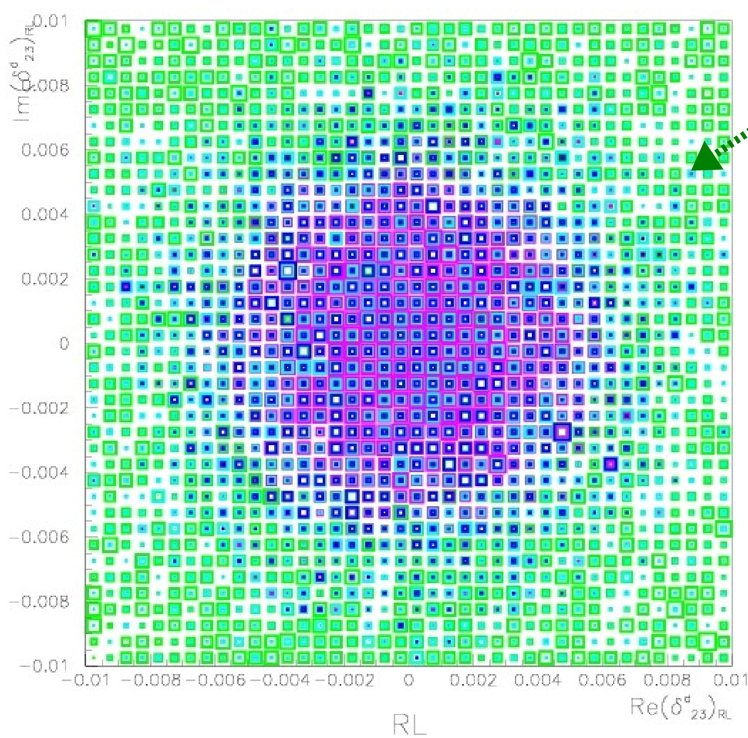






LR, RL insertions: $\Delta F=1$ constraints rule





y-axis: $Im[(\delta^{23})_{XY}]$

x-axis: $Re[(\delta^{23})_{XY}]$

Constraints

= Δm_s

= $b \to s\ \gamma$

= $b \to s\ l^+ l^-$

= all

LR only, tan β=3

[-0.0025,+0.01] + i [-0.015,0.015]

RL only, tan β=3

[-0.008,+0.008] + i [-0.008,0.008]

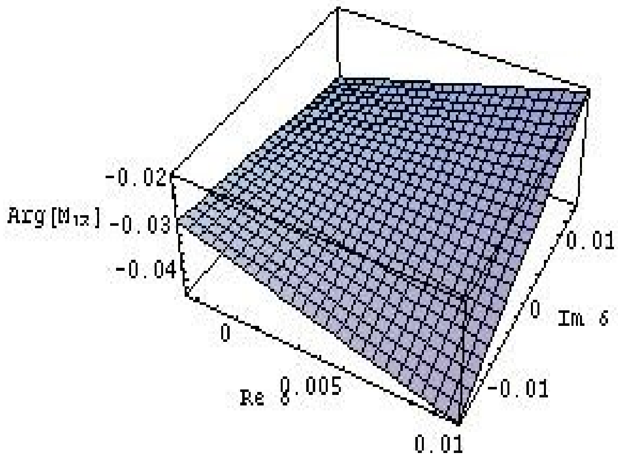
Remarks

- Constraints on LL and LL=RR mass insertions are severe
- Constraints on LR and RL mass insertions are *very* severe
- Do these constraints have a severe impact on the B_s – mixing ***phase*** ?

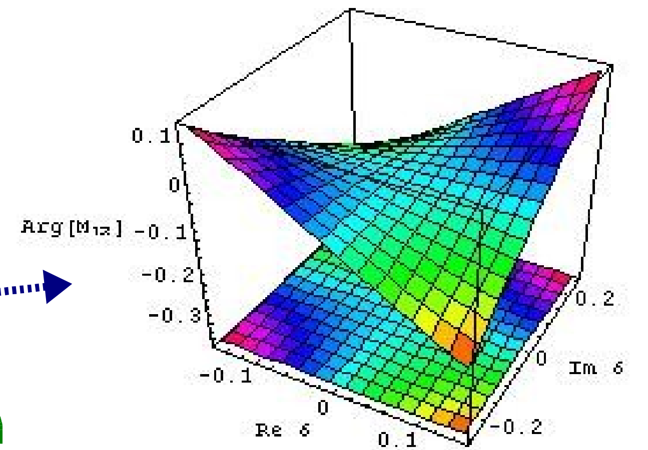
B_s – mixing phase

✔ In the SM one has $Arg\, M_{12}^{SM} \equiv Arg\{\langle \bar{B}_s | H_{eff,\,SM}^{\Delta B,\, S=2} | B_s \rangle\} = -2\lambda^2 \eta \simeq -0.03$

☞ What is the allowed range for $Arg\, M_{12}^{MSSM}$ with the previous limits on the δ's ?

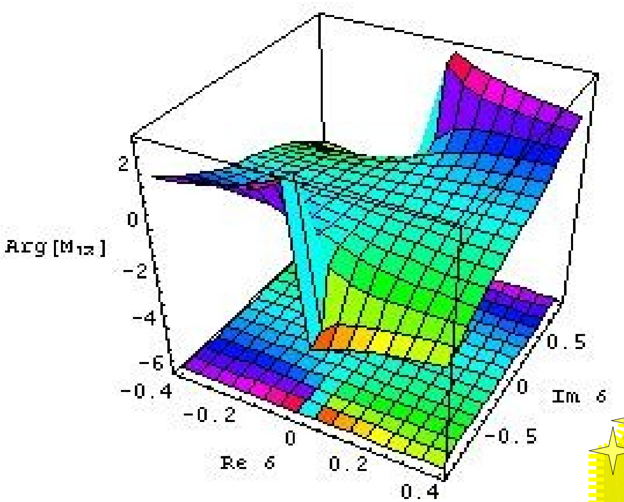


LR only, tan β=3
no sizable deviations
from the SM

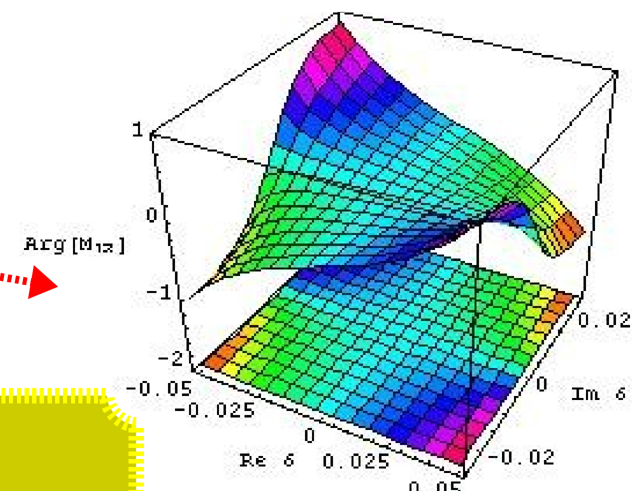


LL only, tan β=3
~ 10 × SM value are allowed

RR only, tan β=3
~ 100 × SM value are easy to get
(but RR is still mildly constrained…)



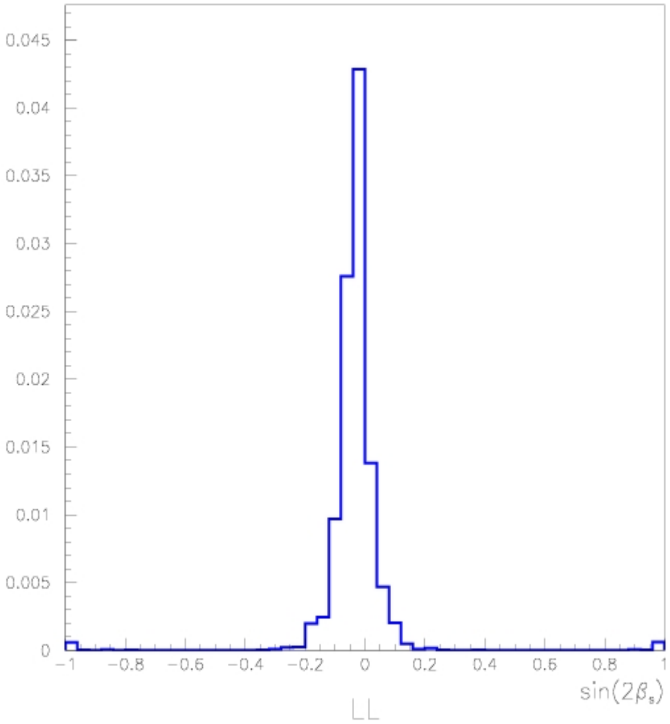
LL=RR, tan β=3
~ 100 × SM value are again easy
(yet LL=RR is severely constrained!)



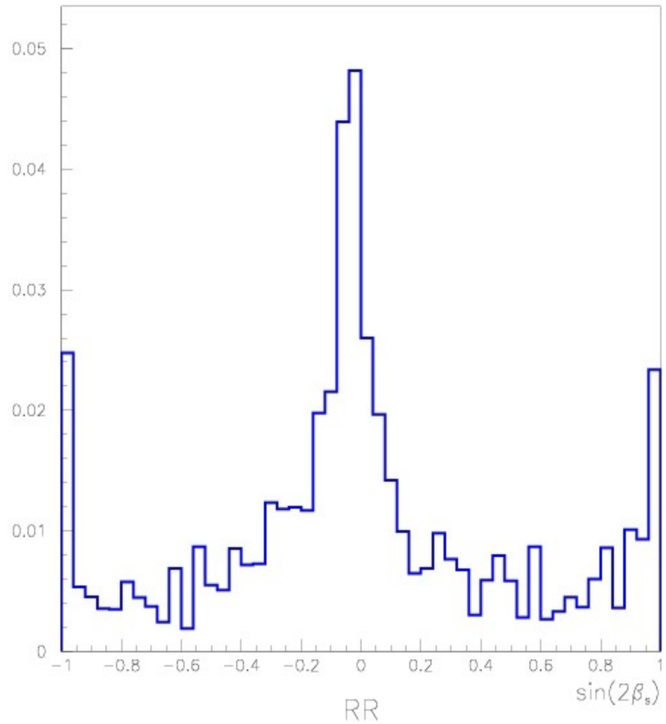
The CP asymmetry in $B_s \to \psi\phi$
will provide a truly fantastic probe!

B_s – mixing phase

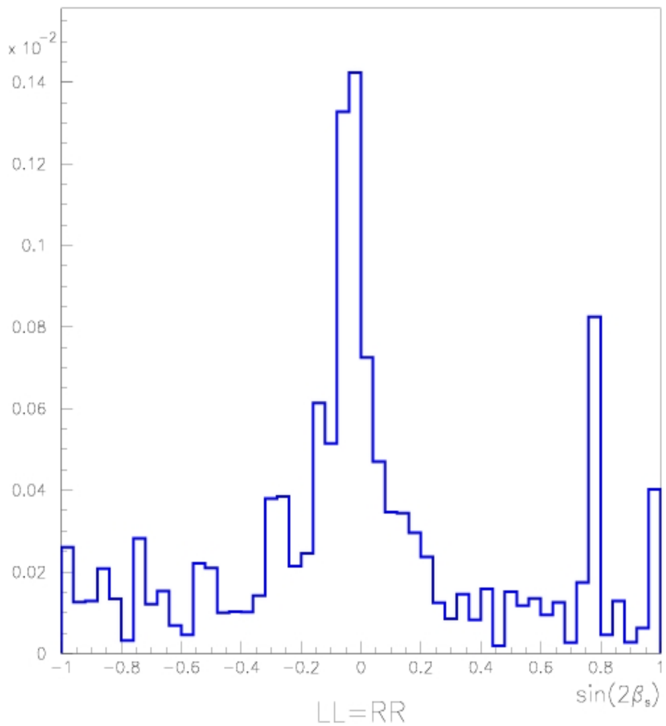
✔ PDF's for *sin 2* β_s in the LL, RR and LL=RR cases



LL only, tan β=3



RR only, tan β=3



LL=RR, tan β=3