Decay rate difference in the neutral B_s-system: $\Delta\Gamma(B_s)$

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May 16th 2006

Flavour in the era of the LHC, CERN

in collaboration with Uli Nierste, Uni Karlsruhe

Contents:

- Introduction
  - Mixing
  - Experimental status
- Theoretical Tools
  - **H**eavy **Q**uark **E**xpansion
  - Matrix elements of local four quark operators
  - State of the art
- Results for the Lifetime difference: $\Delta\Gamma(B_s)$
  - LO calculation
  - NLO QCD calculation
  - Non-perturbative parameters
  - Subleading 1/m and $1/m^2$ corrections
  - Normalization
  - new operator basis
- Summary and wish-list

Introduction I: Mixing

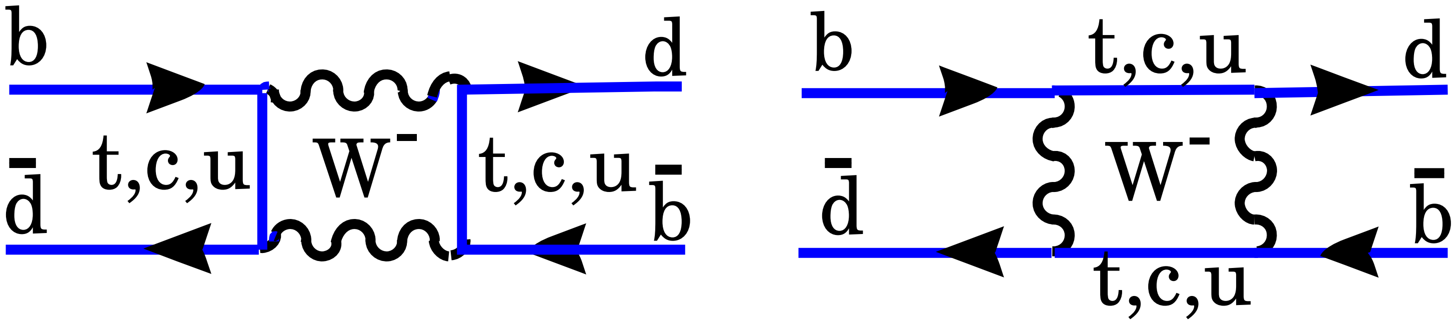
Time evolution of a decaying particle:

$$B(t) = \exp\left[-i m_B t - \Gamma_B/2 t\right]$$

This can be written as

$$i\frac{d}{dt}\begin{pmatrix} |B(t)\rangle \\ |\bar{B}(t)\rangle \end{pmatrix} = \left(\hat{M} - \frac{i}{2}\hat{\Gamma}\right)\begin{pmatrix} |B(t)\rangle \\ |\bar{B}(t)\rangle \end{pmatrix}$$

BUT: In the neutral B-system transitions like $B_{d,s} \to \bar{B}_{d,s}$ are possible due to weak interaction: **Boxdiagrams**



This leads to **off-diagonal elements** in the matrices $\hat{M}$, $\hat{\Gamma}$

Diagonalization of $\hat{M}$, $\hat{\Gamma}$ gives the physical eigenstates B_H and B_L with the masses M_H, M_L and the deacy rates Γ_H, Γ_L

Introduction II: Mixing

$$
\begin{aligned}
B_H &:= p\, B + q\, \bar{B} \\
B_L &:= p\, B - q\, \bar{B}
\end{aligned}
$$

with $|p|^2 + |q|^2 = 1$,

$$
\begin{aligned}
\Delta M &:= M_H - M_L \\
\Delta \Gamma &:= \Gamma_L - \Gamma_H
\end{aligned}
$$

These quantities can be related to the off-diagonal elements of $\hat{M}$ and $\hat{\Gamma}$: $|M_{12}|$, $|\Gamma_{12}|$ and $\Phi = arg(-M_{12}/\Gamma_{12})$:

$$
\begin{aligned}
\Delta M &= 2|M_{12}| \left(1 + \frac{1}{8}\frac{|\Gamma_{12}|^2}{|M_{12}|^2}\sin^2\Phi + ...\right) \\
&\quad |M_{12}| : \text{heavy particles: } t, \text{SUSY}, ... \\
\Delta \Gamma &= -2|\Gamma_{12}|\cos\Phi \left(1 - \frac{1}{8}\frac{|\Gamma_{12}|^2}{|M_{12}|^2}\sin^2\Phi + ...\right) \\
&\quad |\Gamma_{12}| : \text{light particles:} u, c, ... \text{ no NP!!!} \\
a_{sl} &\equiv a_{fs} = -2\left(\left|\frac{q}{p}\right| - 1\right) = \text{Im}\frac{\Gamma_{12}}{M_{12}} = \frac{\Delta\Gamma}{\Delta M}\tan\Phi
\end{aligned}
$$

Experimental status: ΔM, $\Delta\Gamma_d$ and a_{sl}

- ΔM, see talks by G. Borissov and S. De Cecco
- $\Delta\Gamma_d$: HFAG, 2006 "These averages are not ready or finalized yet, and are still evolving."

$$(\Delta\Gamma/\Gamma)_d = (9 \pm 37) \cdot 10^{-3}$$

SM prediction including NLO-QCD and 1/m corrections

$$(\Delta\Gamma/\Gamma)_d = (3.0 \pm 1.2) \cdot 10^{-3}$$

Beneke, Buchalla, A.L, Nierste;
Ciuchini, Franco, Lubicz, Mescia, Tarantino (2003)

- a_{sl}^d: HFAG, 2006 vs. Grossmann, Nir, Raz

$$a_{sl}^d = -(3.0 \pm 7.8) \cdot 10^{-3} \quad \text{vs.} \quad a_{sl}^d = -(0.4 \pm 5.5) \cdot 10^{-3}$$

$$a_{sl}^d = -(5.0 \pm 1.1) \cdot 10^{-4}$$

Expect new physics: $\mathcal{O}(5 \cdot 10^{-3})$!!!

- a_{sl}^s: Grossmann, Nir, Raz using Casey, D0 at FPCP 06

$$a_{sl}^s = -(1.3 \pm 1.5) \cdot 10^{-2} \quad \text{vs.} \quad a_{sl}^s = +(2.1 \pm 0.4) \cdot 10^{-5}$$

Experimental status: $\Delta\Gamma_s$

Rick van Kooten, FPCP 2006

Unofficial World average:

$$\Delta\Gamma_s = 0.097^{+0.041}_{-0.042}\,\text{ps}^{-1}$$

which corresponds to

$$\left(\frac{\Delta\Gamma}{\Gamma}\right)_{B_s} \approx 0.15 \pm 0.06$$

for $\tau_{B_s} \approx \tau_{B_d} = 1.527 \pm 0.008$ps and not $\tau_{B_s} = 1.461 \pm 0.030$ps

Wait for more results from TeVatron, Belle? and LHC

Theoretical Status - 2006

$$\Delta\Gamma = \left(\frac{\Lambda}{m_b}\right)^3 \left(\Gamma_3^{(0)} + \frac{\alpha_s}{4\pi}\Gamma_3^{(1)} + \ldots\right) + \left(\frac{\Lambda}{m_b}\right)^4 \left(\Gamma_4^{(0)} + \ldots\right) + \ldots$$

$\Gamma_3^{(0)}$: Hagelin; Buras, Slominski, Steger; Datta, Paschos, Türke,Wu; Voloshin, Uraltsev, Khoze, Shifman; Chau; Franco,Lusignoli, Pugliese; (1981 . . .)

$\Gamma_3^{(1)}$: Beneke, Buchalla, Greub, A.L., Nierste (1998) Ciuchini, Franco, Lubicz, Mescia, Tarantino (2003)

$\langle || \rangle$: JLQCD, Becirevic et al.; Gimenez, Reyes; Jamin, Lange Huang, Zhang, Zhou (1999 . . .)

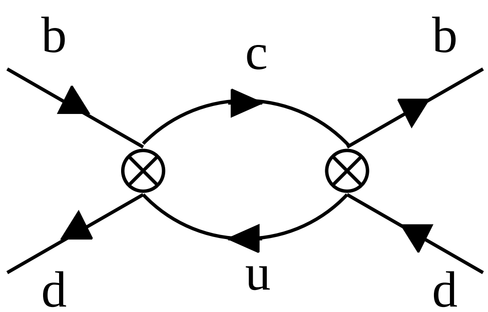
$\Gamma_4^{(0)}$: Beneke, Buchalla, Dunietz (1996); Ciuchini, Franco, Lubicz, Mescia, Tarantino (2003)

$\Gamma_5^{(0)}$: A.L., Nierste preliminary

$\langle || \rangle$: part of the appearing operators of dim 7 and 8 Becirevic, Gimenez, Martinelli, Papinutto, Reyes (2001)

Getting started

- <u>To get an idea:</u>
  Calculate the leading term and determine the matrix elements in vacuum insertion approximation (VIA)



$$\langle \bar{B}_s | (\bar{b}_i s_i)_{V-A} (\bar{b}_j s_j)_{V-A} | B_s \rangle \approx f_{B_s}^2 M_{B_s}^2 \cdot color$$

To get a number:

* NLO QCD:
Dress the above diagram with one gluon in all possible ways

* Non-perturbative evaluation of the matrix elements

$$\langle \bar{B}_s | (\bar{b}_i s_i)_{V-A} (\bar{b}_j s_j)_{V-A} | B_s \rangle \stackrel{!}{=} f_{B_s}^2 M_{B_s}^2 B \cdot color$$

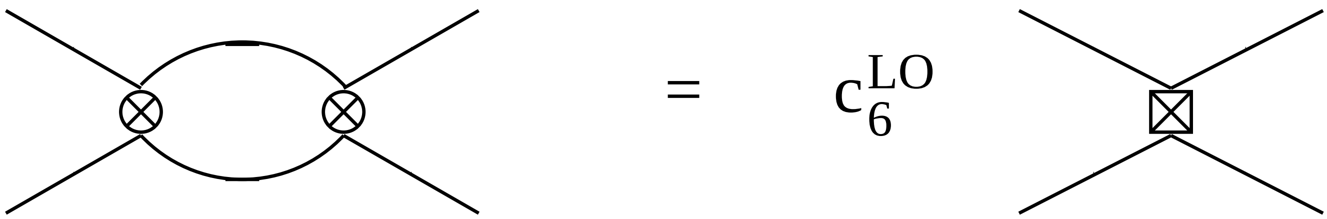
Evaluation of decay constants and bag parameters

* $1/m_b^4$:
Take the momentum of the external d-quark into account:
New operators like $\frac{1}{m_b^2} \bar{b} \overleftarrow{D} \Gamma D s \cdot \bar{b} \Gamma s$ arise

LO Calculation I

$$\Delta\Gamma = -\frac{1}{m_{B_s}} \langle \overline{B}_s | \mathcal{T} | B_s \rangle$$

Matching of $\Delta B = 1$ double insertion to $\Delta B = 2$ insertion



!!! four $\Delta B = 2$-operators arise – eliminate two!!!

$$Q = (\bar{b}_i s_i)_{V-A} \cdot (\bar{b}_j s_j)_{V-A}$$

$$Q_s = (\bar{b}_i s_i)_{S-P} \cdot (\bar{b}_j s_j)_{S-P}$$

$$\tilde{Q} = Q \quad ; \quad \tilde{Q}_S = -Q_s - \frac{1}{2} Q$$

$$\langle \bar{B}_s | Q | B_s \rangle = \frac{8}{3} f_{B_S}^2 M_{B_S}^2 B$$

$$\langle \bar{B}_s | Q_S | B_s \rangle = -\frac{5}{3} f_{B_S}^2 M_{B_S}^2 B_S'$$

$$B_S' = \frac{M_{B_S}^2}{(\bar{m}_b + \bar{m}_s)^2} B_S$$

LO Calculation II

$$\mathcal{T} = -\frac{G_F^2 m_b^2}{12\pi}(V_{cb}^* V_{cs})^2 \left[F(z)Q(\mu_2) + F_S(z)Q_S(\mu_2)\right]$$

with

$$F(z) = F_{11}(z)C_1^2(\mu_1) + F_{12}(z)C_1(\mu_1)C_2(\mu_1) + F_{22}(z)C_2^2(\mu_1),$$

$$F_{ij}(z) = F_{ij}^{(0)}(z) + \frac{\alpha_s(\mu_1)}{4\pi}F_{ij}^{(1)}(z)$$

LO:

$$F_{11}^{(0)}(z) = 3\sqrt{1-4z}\,(1-z) \qquad F_{S,11}^{(0)}(z) = 3\sqrt{1-4z}\,(1+2z),$$

$$F_{12}^{(0)}(z) = 2\sqrt{1-4z}\,(1-z) \qquad F_{S,12}^{(0)}(z) = 2\sqrt{1-4z}\,(1+2z),$$

$$F_{22}^{(0)}(z) = \frac{1}{2}(1-4z)^{3/2} \qquad F_{S,22}^{(0)}(z) = -\sqrt{1-4z}\,(1+2z).$$

with $z = (m_c/m_b)^2$

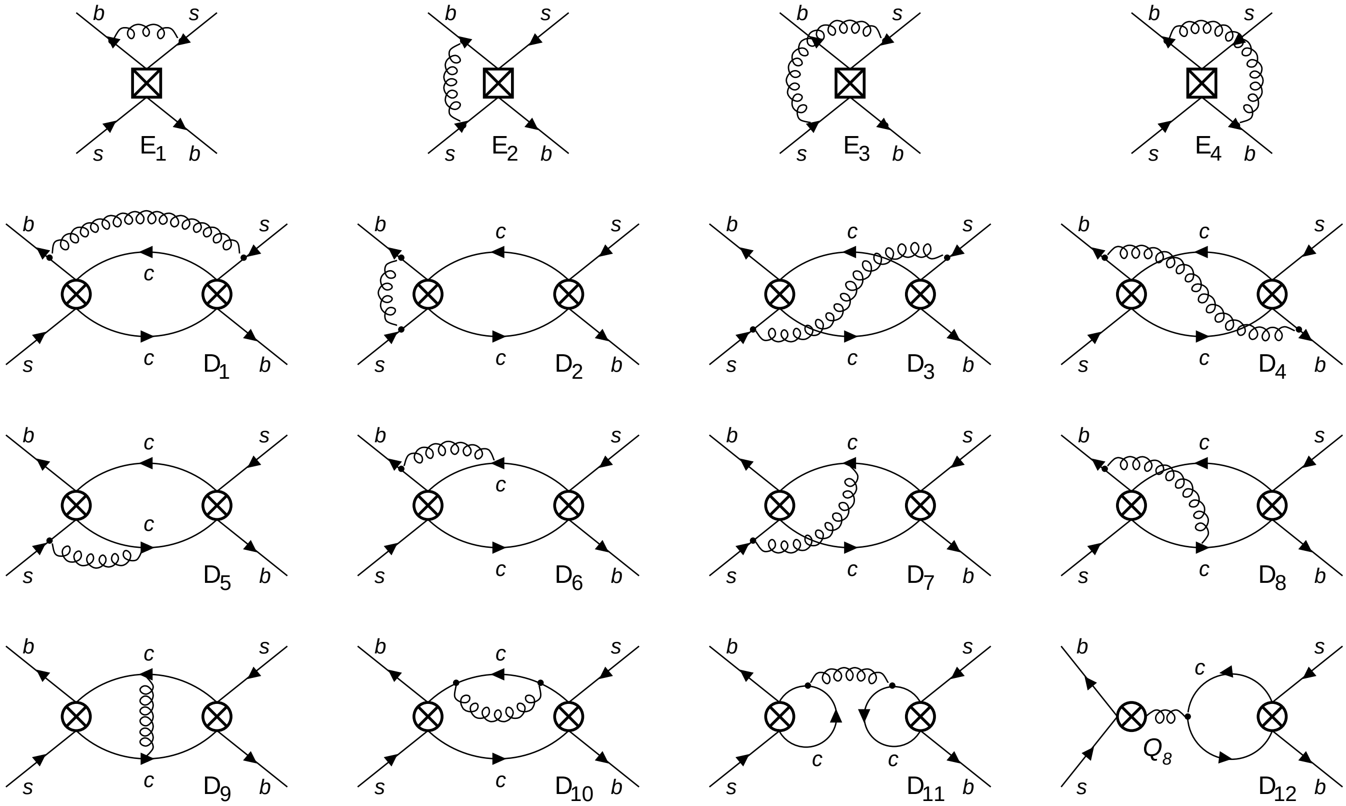
$$\Longrightarrow \quad \left(\frac{\Delta\Gamma}{\Gamma}\right)_{B_s} = \mathcal{O}(28...42)\%$$

Why should we calculate up to NLO?

To extract parameters and to check the theory
(search for new physics)
a precise theoretical result is needed!

- Test of expansion in α_s and m_b^{-1}
- Reduction of theoretical error
  e.g. μ-dependence in QCD
- **Renormalization scheme dependence**
  (!!! matching to lattice calculations !!!)
- Proper use of $\Lambda_{\overline{MS}}$
- Definition of quark masses $m_{pole} \leftrightarrow m_{\overline{MS}}$
- up to 30 % correction expected
- LO coefficients for lifetime predictions are anomalously small
- Verify IR-safetey of QCD-corrections to Weak Annihilation and Pauli interference diagrams

NLO-QCD calculation



Beneke, Buchalla, Greub, A.L., Nierste (1998)
Ciuchini, Franco, Lubicz, Mescia, Tarantino (2003)

$$\mathcal{T} = -\frac{G_F^2 m_b^2}{12\pi}(V_{cb}^* V_{cs})^2 \left[F(z)Q(\mu_2) + F_S(z)Q_S(\mu_2)\right]$$

with

$$F(z) = F_{11}(z)C_1^2(\mu_1) + F_{12}(z)C_1(\mu_1)C_2(\mu_1) + F_{22}(z)C_2^2(\mu_1),$$

$$F_{ij}(z) = F_{ij}^{(0)}(z) + \frac{\alpha_s(\mu_1)}{4\pi}F_{ij}^{(1)}(z)$$

$$
\begin{aligned}
F_{11}^{(1)}(z) &= 32(1-z)(1-2z)\left(\mathrm{Li}_2(\sigma^2)+\ln^2\sigma+\frac{1}{2}\ln\sigma\ln(1-4z)-\ln\sigma\ln z\right)\\
&+ 64(1-z)(1-2z)\left(\mathrm{Li}_2(\sigma)+\frac{1}{2}\ln(1-\sigma)\ln\sigma\right)\\
&- 4(13-26z-4z^2+14z^3)\ln\sigma\\
&+ \sqrt{1-4z}\Big[4(13-10z)\ln z-12(3-2z)\ln(1-4z)\\
&\quad +\frac{1}{6}(109-226z+168z^2)\Big]+2\sqrt{1-4z}(5-8z)\ln\frac{\mu_2}{m_b},\\
F_{S,11}^{(1)}(z) &= 32(1-4z^2)\left(\mathrm{Li}_2(\sigma^2)+\ln^2\sigma+\frac{1}{2}\ln\sigma\ln(1-4z)-\ln\sigma\ln z\right)\\
&+ 64(1-4z^2)\left(\mathrm{Li}_2(\sigma)+\frac{1}{2}\ln(1-\sigma)\ln\sigma\right)\\
&- 16(4-2z-7z^2+14z^3)\ln\sigma\\
&+ \sqrt{1-4z}\Big[64(1+2z)\ln z-48(1+2z)\ln(1-4z)\\
&\quad -\frac{8}{3}(1-6z)(5+7z)\Big]-32\sqrt{1-4z}(1+2z)\ln\frac{\mu_2}{m_b},\\
F_{12}^{(1)}(z) &= \frac{64}{3}(1-z)(1-2z)\left(\mathrm{Li}_2(\sigma^2)+\ln^2\sigma+\frac{1}{2}\ln\sigma\ln(1-4z)-\ln\sigma\ln z\right)\\
&+ \frac{128}{3}(1-z)(1-2z)\left(\mathrm{Li}_2(\sigma)+\frac{1}{2}\ln(1-\sigma)\ln\sigma\right)\\
&+ (2-259z+662z^2-76z^3-200z^4)\frac{\ln\sigma}{6z}\\
&- \sqrt{1-4z}\left[(2-255z+316z^2)\frac{\ln z}{6z}+8(3-2z)\ln(1-4z)\right.\\
&\quad \left.+\frac{2}{9}(127-199z-75z^2)\right]\\
&- 2\sqrt{1-4z}(17-26z)\ln\frac{\mu_1}{m_b}+\frac{4}{3}\sqrt{1-4z}(5-8z)\ln\frac{\mu_2}{m_b},\\
F_{S,12}^{(1)}(z) &= \frac{64}{3}(1-4z^2)\left(\mathrm{Li}_2(\sigma^2)+\ln^2\sigma+\frac{1}{2}\ln\sigma\ln(1-4z)-\ln\sigma\ln z\right)\\
&+ \frac{128}{3}(1-4z^2)\left(\mathrm{Li}_2(\sigma)+\frac{1}{2}\ln(1-\sigma)\ln\sigma\right)\\
&+ (1-35z+4z^2+76z^3-100z^4)\frac{4\ln\sigma}{3z}
\end{aligned}
$$

$$
\begin{aligned}
&- \quad \sqrt{1-4z}\left[(1-33z-76z^2)\frac{4\ln z}{3z} + 32(1+2z)\ln(1-4z)\right. \\
&\quad \left. +\frac{4}{9}(68+49z-150z^2)\right] \\
&- \quad 16\sqrt{1-4z}(1+2z)\ln\frac{\mu_1}{m_b} - \frac{64}{3}\sqrt{1-4z}(1+2z)\ln\frac{\mu_2}{m_b},
\end{aligned}
$$

$$
\begin{aligned}
F_{22}^{(1)}(z) \quad = \quad & \frac{4}{3}(4-21z+2z^2)\left(\mathrm{Li}_2(\sigma^2)+\ln^2\sigma+\frac{1}{2}\ln\sigma\ln(1-4z)-\ln\sigma\ln z\right) \\
+ \quad & \frac{4}{3}(1-2z)(5-2z)\left(\mathrm{Li}_2(\sigma)+\frac{1}{2}\ln(1-\sigma)\ln\sigma\right) \\
- \quad & (7+13z-194z^2+304z^3-64z^4)\frac{\ln\sigma}{6z} - \frac{\pi^2}{3}(1-10z) \\
+ \quad & \sqrt{1-4z}\left[(7+27z-250z^2)\frac{\ln z}{6z} - 4(1-6z)\ln(1-4z)\right. \\
& \left. -\frac{1}{18}(115+632z+96z^2)\right] \\
- \quad & 2\sqrt{1-4z}(5-2z)\ln\frac{\mu_1}{m_b} + \frac{4}{3}\sqrt{1-4z}(2-5z)\ln\frac{\mu_2}{m_b},
\end{aligned}
$$

$$
\begin{aligned}
F_{S,22}^{(1)}(z) \quad = \quad & -\frac{32}{3}(1+z)(1+2z)\left(\mathrm{Li}_2(\sigma^2)+\ln^2\sigma+\frac{1}{2}\ln\sigma\ln(1-4z)-\ln\sigma\ln z\right) \\
+ \quad & \frac{32}{3}(1-4z^2)\left(\mathrm{Li}_2(\sigma)+\frac{1}{2}\ln(1-\sigma)\ln\sigma\right) \\
+ \quad & (1+7z+10z^2-68z^3+32z^4)\frac{4\ln\sigma}{3z} + \frac{8\pi^2}{3}(1+2z) \\
- \quad & \sqrt{1-4z}\left[(1+9z+26z^2)\frac{4\ln z}{3z} - 16(1+2z)\ln(1-4z)\right. \\
& \left. +\frac{8}{9}(19+53z+24z^2)\right] \\
- \quad & 16\sqrt{1-4z}(1+2z)\ln\frac{\mu_1}{m_b} + \frac{32}{3}\sqrt{1-4z}(1+2z)\ln\frac{\mu_2}{m_b}.
\end{aligned}
$$

with: $\sigma = (1-\sqrt{1-4z})/(1+\sqrt{1-4z})$, $z = m_c^2/m_b^2$

NLO-QCD Result

Result of the NLO-QCD corrections:

- Overall reduction of the LO result by 38%
- Minor reduction of the μ-dependence
  $\longrightarrow$ α_s^2-corrections
- Renormalization scheme dependence is introduced
  $\longrightarrow$ Non-perturbative evaluation of the matrix elements

NLO-QCD corrections plus matrix elements in VIA give

$$\Longrightarrow \quad \boxed{\left(\frac{\Delta\Gamma}{\Gamma}\right)_{B_s} = \mathcal{O}(26)\%}$$

Matrix elements I

Typical 4-quark matrix element

$$\begin{aligned} & \langle \bar{B}_s | (\bar{b}_i s_i)_{V-A} (\bar{b}_j s_j)_{V-A} | B_s \rangle \\ \propto \quad & B \langle \bar{B}_s | (\bar{b}_i s_i)_{V-A} | 0 \rangle \langle 0 | (\bar{b}_j s_j)_{V-A} | B_s \rangle \\ \approx \quad & B \cdot f_{B_s} p^\mu_{B_s} \cdot f_{B_s} p_{B_s\,\mu} = B f^2_{B_s} M^2_{B_s} \end{aligned}$$

- Decay constant f_{B_d} in MeV
  - 176^{+28+20}_{-23-19} from $B \to \tau \nu_\tau$ at BELLE '06
  - $191 \pm 10^{+12}_{-22}$ from JLQCD '03 (N=2, Wilson)
  - 210 ± 19 from sum rules Jamin, Lange; Penin Steinhauser '01
  - $216 \pm 9 \pm 20$ from HPQCD '05 (N=2+1, Staggered)
- Decay constant f_{B_s} (15%-increase due to unquenching)
  - $215 \pm 9 \pm 13$ from JLQCD '03 (N=2, Wilson)
  - 244 ± 21 from sum rules Jamin, Lange '01
  - $260 \pm 7 \pm 28$ from HPQCD '05 (N=2+1, Staggered)

At LATTICE 2005 (Okamoto) and FPCP 2006 (Mackenzie)
the HPQCD values were chosen to be the "best ones"
We use $f_{B_s} = 250 \pm 30$ MeV

Matrix elements II

- Bag parameters for B_s-mixing
  in ΔM_B B appears
  in $\Delta\Gamma_B$ B, B_S and more dim 6,7,8 operators appear

  – JLQCD '03 (N=2, Wilson), small unquenching effect

  $$B = 0.84 \pm 0.06 \qquad B_S = 0.85 \pm 0.06$$

  – Becirevic, Gimenez, Martinelli, Papinutto and Reyes '01
  Complete set of $\Delta B = 2$ operators:
  B, B_S, $\tilde{B}_S$ and part of dim 7 and dim 8 operators
  But: quenched

  $$B = 0.87 \pm 0.06 \qquad B_S = 0.84 \pm 0.05$$

$$\Longrightarrow \quad \boxed{\left(\frac{\Delta\Gamma}{\Gamma}\right)_{B_s} = \mathcal{O}(22)\%}$$

$1/m$ and $1/m^2$-Corrections

Take the LO diagram and expand to first (Γ_4) or second (Γ_5) order in the light quark momentum p_s

$$F Q + F_S Q_S \to F Q + F_S Q_S + \delta_{1/m}$$

$$\begin{aligned}
\delta_{1/m} = \; & (1+2z) && \Big(K_1 \big[(S_1 - 2R_1) + (S_2 - 2R_2) \big] \\
& && + K_2 \big[R_0 + (\tilde{S}_1 - 2\tilde{R}_1) + (\tilde{S}_2 - 2\tilde{R}_2) \big] \Big) \\
& + \tfrac{6z^2}{1-4z} && \Big(K_1 \big[(S_2 - 2R_2) + 2(S_3 - 2R_3) \big] \\
& && + K_2 \big[(\tilde{S}_2 - 2\tilde{R}_2) + 2(\tilde{S}_3 - 2\tilde{R}_3) \big] \Big) \\
& - \tfrac{24z^2}{(1-4z)^2} && \Big(K_1 \big[3S_5 + 2S_6 + z(S_4 - 10S_5 - 8S_6) \big] \\
& && + K_2 \big[3\tilde{S}_5 + 2\tilde{S}_6 + z(\tilde{S}_4 - 10\tilde{S}_5 - 8\tilde{S}_6) \big] \Big) \\
& + \tfrac{6z}{\sqrt{1-4z}} && \left(K_1 G_1 + K_2 \tilde{G}_2 \right)
\end{aligned}$$

Beneke, Buchalla, Dunietz (1996);
Ciuchini, Franco, Lubicz, Mescia, Tarantino (2003);
A.L., Nierste !!!preliminary!!! - numerics not yet finished

$$K_1 = 3C_1^2 + 2C_1 C_2; \; K_2 = C_2^2$$

1/m-Corrections

$$
\begin{aligned}
R_0 &= Q_s + \tilde{Q}_S + \frac{1}{2} Q \\
R_1 &= \frac{m_s}{m_b} (\bar{b}_i s_i)_{S-P} (\bar{b}_j s_j)_{S+P} \\
R_2 &= \frac{1}{m_b^2} (\bar{b}_i \overleftarrow{D}_\rho D^\rho s_i)(\bar{b}_j \gamma_\mu (1-\gamma_5) s_j) \\
R_3 &= \frac{1}{m_b^2} (\bar{b}_i \overleftarrow{D}_\rho (1-\gamma_5) D^\rho s_i)(\bar{b}_j (1-\gamma_5) s_j) \\
\tilde{R}_i &= \tilde{R}_i(R_j)
\end{aligned}
$$

$\longrightarrow$ matrix elements of Q, Q_s, $\tilde{Q}_S$ and $m_b/m_s R_1$: Becirevic et al.

$\longrightarrow$ only R_2 and R_3 remain to be unknown $\Rightarrow$ VIA

$\longrightarrow$ R_2 and R_0 are dominant

$\longrightarrow$ all important 1/m contributions have the same sign

$$
m_b^{\text{pow}} = 4.8 \text{GeV} \Longrightarrow \quad \left(\frac{\Delta\Gamma}{\Gamma} \right)_{B_s} = \mathcal{O}(12)\%
$$

$\longrightarrow$ large dependence on m_b: $\mathcal{O}(8)\%$ for $m_b^{\text{pow}} = 4.6$ GeV

Final-Result I

NLO-result:

$$\left(\frac{\Delta\Gamma}{\Gamma}\right)_{B_s} = \left(\frac{f_{B_s}}{250\text{MeV}}\right)^2 [0.006B + 0.256B_S - 0.097]$$

QCD: Beneke, Buchalla, Greub, A.L., Nierste; Ciuchini et al.

$1/m_b$ corrections: Beneke, Buchalla, Dunietz; Ciuchini et al.

$$\Longrightarrow \quad \left(\frac{\Delta\Gamma}{\Gamma}\right)_{B_s} = 12 \pm 5\%$$

$f_{B_s} = 250$ MeV: JLQCD, HPQCD; Jamin, Lange;

Bag parameters: Becirevic et al.;

Remarks - Dominant errors:

- Coefficient of $B \ll$ coefficient of B_S
- Conspiracy: all corrections in the same direction
- $1/m_b$ corrections: They are 25% of LO-result (78% of final result) huge dependence on value of m_b
  Matrix elements (dim. 7) and α_s corrections
- Residual scale dependence α_s^2 corrections
- Non-perturbative parameters smaller error on f_{B_s}, B and B_S
  Unquenched values of all $\Delta B = 2$ operators

New operator basis I

Determine $\Delta\Gamma/\Delta M \rightarrow f_{B_s}^2$ cancels

$$\left(\frac{\Delta\Gamma}{\Gamma}\right)_{B_s} = \Delta\Gamma_{B_s}\tau_{B_s} = \frac{G_F^2 M_{B_s}(V_{cb}^* V_{cs})^2}{12\pi} m_b^2 f_{B_s}^2 K \tau_{B_s}$$

$$\frac{\Delta\Gamma_s}{\Delta M_s} = \frac{\pi}{2M_W^2}\frac{(V_{cb}^* V_{cs})^2}{(V_{tb}V_{ts})^2}\frac{m_b^2}{\hat{\eta}_B S_0(x_t)}\frac{K}{B}$$

$$3K = 8\,F\,B - 5\,F_S\,B_S + 3\tilde{\delta}_{1/m}\,; \quad \delta_{1/m} = f_{B_s}^2 M_{B_s}^2 \tilde{\delta}_{1/m}$$

- Unfortunateley: $(8F) \ll (-5F_s)$

$\Rightarrow$ try a different operator basis, i.e. eliminate Q_S instead of $\tilde{Q}_S$

$$R_0 = Q_S + \tilde{Q}_S + \frac{1}{2}Q = \mathcal{O}(1/m_b, \alpha_s)$$

$$\langle \bar{B}_s|\tilde{Q}_S|B_s\rangle = \frac{1}{3} f_{B_S}^2 M_{B_S}^2 \frac{M_{B_S}^2}{(\bar{m}_b + \bar{m}_s)^2}\tilde{B}_S$$

$$3K = 8\left[F - \frac{F_S}{2}(1 + \alpha_s(\ldots))\right]B - F_S\left[1 + \alpha_s(\ldots)\right]\tilde{B}_S + 3\left[\tilde{\delta}_{1/m} + \frac{F_S\langle R_0\rangle}{M_{B_s}^2 f_{B_s}^2}\right.$$

New operator basis II

Changing the operator Basis

$$3K = 8F \qquad B - 5F_S \qquad B_S + 3\tilde{\delta}_{1/m}$$

$$3K = 8\left[F - \frac{F_S}{2}(1 + \alpha_s(\ldots))\right]B - F_S\left[1 + \alpha_s(\ldots)\right]\tilde{B}_S + 3\left[\tilde{\delta}_{1/m} + \frac{F_S\langle R_0\rangle}{M_{B_s}^2 f_{B_s}^2}\right.$$

we observe

- Coefficient of B becomes larger
- Coefficient of $\tilde{B}_S$ becomes a factor 5 smaller

$\Rightarrow$ Coefficient of $B \gg$ Coefficient of $\tilde{B}_S$

$\Rightarrow$ **Theoretical clean determination of $\Delta\Gamma/\Delta M$**

- new $1/m$-corrections that enhance $\Delta\Gamma$
- new, positive α_s-corrections

$\Rightarrow$ no conspiracy anymore

- α_s- and $1/m$-corrections have now contributions with different signs
- α_s- and $1/m$- corrections become smaller

Final-Result II

!!Preliminary!!

LO-result:

$$\left(\frac{\Delta\Gamma}{\Gamma}\right)_{B_s} = \left(\frac{f_{B_s}}{250\text{MeV}}\right)^2 \left[0.222B + 0.081\tilde{B}_S - 0.055\right]$$

NLO-result:

$$\left(\frac{\Delta\Gamma}{\Gamma}\right)_{B_s} = \left(\frac{f_{B_s}}{250\text{MeV}}\right)^2 \left[0.154B + 0.057\tilde{B}_S - 0.055\right]$$

$$\left(\frac{\Delta\Gamma}{\Delta M}\right)_{B_s} = \left[39.6 + 14.5\frac{\tilde{B}_S}{B} - 14.0\frac{1}{B}\right] 10^{-4}$$

with the Bag parameters from Becirevic et al. we get

$$\left(\frac{\Delta\Gamma}{\Gamma}\right)_{B_s} = 13{\pm}5\%, \qquad \Delta\Gamma_s = 0.086 \pm 0.031\text{ps}^{-1}$$

$$\left(\frac{\Delta\Gamma}{\Delta M}\right)_{B_s} = 0.0039 \rightarrow \Delta\Gamma_s = \frac{0.07}{\text{ps}} \equiv f_{B_s} \approx 221\text{MeV}$$

Summary

$$\Delta\Gamma_s \approx 0.09 \pm 0.03\text{ps}^{-1}$$

- Change of basis is theoretically advantageous
  - Coefficient of B becomes dominant
  - $1/m$-corrections become smaller
  - α_s-corrections become smaller

- Theoretical clean prediction of $\Delta\Gamma/\Delta M$

- Γ_5 in progress $\rightarrow$ test HQE

Wish-list

- Unquenched results for all $\Delta B = 2$-operators

- Precise value of the decay constant f_{B_s}

- Precise experimental number

Quark-Hadron-Duality:

—— Deviation from OPE based prediction ——

??What could go wrong??

- Terms in OPE do not decrease, actual expansion parameter is $\Lambda/Q \Rightarrow \Delta\Gamma(B_s)$ is an ideal testing ground
- OPE does not account for exponential terms

$$X = \sum_n x_n \left(\frac{\Lambda}{Q}\right)^n + \tilde{x} e^{-\frac{Q}{\Lambda}}$$

Nethertheless: This is QCD based, not like a hadronic model!!!

Why should we believe in duality?

- Poggio-Quinn-Weinberg (1976)
  $\sigma^{tot}(e^- e^+ \to h) \equiv \sigma^{tot}(e^- e^+ \to quarks)$
- Determination of V_{cb} from inclusive and exclusive semileptonic B-decays agree
  e.g. Neubert; Shifman, Uraltsev, Vainsthein
- Experiment and Theory agree in hadronic τ-decays
  Pich et al.; Altarelli et al.; Ball, Beneke, Braun; Neubert; Shifman
- t'Hooft model: QCD in $1+1$ dimensions and $N_c \to \infty$
  Bigi, Shifman, Uraltsev, Vainshtein Grinstein
- $(\Delta\Gamma/\Gamma)_{B_s} : \sum$ exklusive $=$ inclusive
  in the limit $N_c \to \infty$ and $\Lambda_{QCD} \ll m_b - 2m_c \ll m_b$
  Aleksan et al.