$B_s \rightarrow \mu^+ \mu^-$ in Supersymmetry

Athanasios Dedes

IPPP, University of Durham

CERN, May 15, Flavour in the era of the LHC
Outline

- Why $B_s \rightarrow \mu^+ \mu^-$ is interesting?
- Higgs mediated contributions
  - Feynman Diagrammatic Approach – Results
  - Effective Lagrangian Approach – Results
- Conclusions

For a Higgs penguin review see, A. D., hep-ph/0309233
Why $B_s \rightarrow \mu^+ \mu^-$ is interesting?

SM predicts that $B_s \rightarrow \mu^+ \mu^-$ is a very rare decay with

$$\mathcal{B}(B_s \rightarrow \mu^+ \mu^-) = (3.8 \pm 1.0) \times 10^{-9}.$$ 

Main uncertainty from $[f_{B_s} = 230 \pm 30 \text{ MeV}]^a$. Additional small uncertainty $\pm 0.3 \times 10^{-9}$ from $m_t = 175 \pm 5 \text{ GeV}$. It originates from Z-penguin and box diagrams$^b$

$^a$Taken from Lattice, D. Becirevic 2003.

Other subdominant contributions:

- Higgs penguin (suppressed by $m_b/M_W$)
- The photon penguin does not contribute.

Current Experimental bound from CDF:

$$\mathcal{B}(B_s \rightarrow \mu^+ \mu^-) < 1.0 \times 10^{-7} \text{ at 95\% CL}$$

A possible evidence of $B_s \rightarrow \mu^+ \mu^-$ at Tevatron will be a clear signal for new flavour physics.
Higgs mediated contributions

In SUSY, $\mathcal{B}(B_s \to \mu^+ \mu^-)$ is enhanced by $\tan^6 \beta$ making this decay interesting for Tevatron and LHC.

There are two approaches to calculate the dominant contributions:

- Feynman Diagrammatic Approach + Ressumation
- Effective Lagrangian Approach

Both have advantages and disadvantages

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Diagrammatic Approach

Step 1: Calculate the diagrams in the mass eigenbasis
Diagrammatic Approach

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Step 2: Make everywhere the replacement\(^a\)

\[
h_b \rightarrow h_b = \frac{g}{\sqrt{2}M_W \cos \beta} \frac{m_b(Q)}{1 + \Delta m_b^{\text{SQCD}}}
\]

\(^a\)Same method used in \(b \rightarrow s\gamma\) calculation, by Carena, Garcia, Nierste, Wagner, 2000
Diagrammatic Approach

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Step 3: Use RGEs to connect with the Unification scale
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- $M_{1/2} = 450$ GeV
- $M_0 = 350$ GeV
- $A_0 = 0$ GeV
- $\mu > 0$ GeV,
- $m_t = 175$ GeV

Dedes, Dreiner, Nierste, 2001
Effective Lagrangian Approach

Step 1: Start out with the symmetric theory$^a$

$^a$Hempfling 1994; Hall, Rattazzi, Sarid, 1994

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Effective Lagrangian Approach

Step 1: Start out with the symmetric theory

\[- \mathcal{L}_Y = \bar{d}^0_R h_d \left[ \Phi^0_1 (1 + \ldots) + \Phi^0_2 \left( \hat{E}_g + \hat{E}_u h_u^\dagger h_u + \ldots \right) \right] d^0_L \]

\[+ \Phi^0_2 \bar{u}^0_R h_u (1 + \ldots) u^0_L + \text{H.c.} \]

with the finite non-holomorphic radiative corrections, $\hat{E}_g, \hat{E}_u$
Effective Lagrangian Approach

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with the finite non-holomorphic radiative corrections, \( \hat{E}_g, \hat{E}_u \)

(a): \( \hat{E}_g = \frac{1}{3\pi} \frac{2\alpha_s}{\bar{m}_g \mu^*} I(m_{d_L}^2, m_{d_R}^2, |m_{\tilde{g}}|^2) \)

(b): \( \hat{E}_u = \frac{1}{16\pi^2} \frac{1}{\mu^* A_U^*} I(m_{u_L}^2, m_{u_R}^2, |\mu|^2) \)
Effective Lagrangian Approach

Step 2: Find the quark mass terms

\[-L_{\text{mass}} = \frac{v_1}{\sqrt{2}} \bar{d}_R^0 h_d \left[ 1 + \tan \beta \left( \hat{E}_g + \hat{E}_u h_u^\dagger h_u \right) \right] d_L^0 + \frac{v_2}{\sqrt{2}} \bar{u}_R^0 h_u u_L^0 + \text{H.c.}\]
Effective Lagrangian Approach

Step 3: Redefine the quark fields-Diagonalize

\[
\begin{align*}
    u_0^L &= U_L^Q u_L, & d_0^L &= U_L^Q V d_L, \\
    u_0^R &= U_R^u u_R, & d_0^R &= U_R^d d_R,
\end{align*}
\]

\[
\downarrow
\]

\[
\hat{M}_u = \frac{v_2}{\sqrt{2}} \hat{h}_u,
\]

\[
U_R^{d\dagger} h_d U_L^{Q} = \frac{\sqrt{2}}{v_1} \hat{M}_d V^\dagger \hat{R}^{-1}, \tag{1}
\]

with \( \hat{R} = 1 + \hat{E}_g \tan \beta + \hat{E}_u \tan \beta |\hat{h}_u|^2 \)
Effective Lagrangian Approach

Step 4: Express $\mathcal{L}_Y$ in terms of the mass eigenstates

\[- \mathcal{L}_Y = \frac{\sqrt{2}}{v_2} \left( \tan \beta \Phi_1^0 - \Phi_2^0 \right) \bar{d}_R \hat{M}_d V^\dagger \hat{R}^{-1} V d_L \]

\[+ \frac{\sqrt{2}}{v_2} \Phi_2^0 \bar{d}_R \hat{M}_d d_L + \Phi_2^0 \bar{u}_R \hat{h}_u u_L + \text{H.c.} \]

with $\hat{R} = 1 + \hat{E}_g \tan \beta + \hat{E}_u \tan \beta |\hat{h}_u|^2$

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Effective Lagrangian Approach

Step 4: Express $\mathcal{L}_Y$ in terms of the mass eigenstates

Even if we had started with general matrices $\tilde{E}_u$, $\tilde{E}_g$ we end up with the same $\mathcal{L}_Y$ where

$$E_g = U_L^Q \tilde{E}_g U_L^Q, \quad E_u = U_L^Q \tilde{E}_u U_L^Q,$$

$$R = 1 + \tan \beta \left( E_g + E_u |\hat{h}_u|^2 + \ldots \right),$$
Effective Lagrangian Approach

Step 4: Express $\mathcal{L}_Y$ in terms of the mass eigenstates

RGE induced operators proportional to $U^L_{Q} h_d^\dagger h_d U^L_{Q}$

Take the hermitian square of the modified Eq.(1) and solve for $U^L_{Q} h_d^\dagger h_d U^L_{Q}$ and then iterate
Effective Lagrangian Approach

Step 5: Express the Higgs fields $\Phi_{1,2}^0$ in terms of their mass eigenstates $H_{1,2,3}$ in the presence of CP violation\(^a\)

$$
\Phi_1^0 \rightarrow \frac{1}{\sqrt{2}} \left[ O_{1i} H_i + i \left( \cos \beta G^0 - \sin \beta O_{3i} H_i \right) \right],
$$

$$
\Phi_2^0 \rightarrow \frac{1}{\sqrt{2}} \left[ O_{2i} H_i + i \left( \sin \beta G^0 + \cos \beta O_{3i} H_i \right) \right],
$$

\(^a\) Carena, Ellis, Pilaftsis, Wagner, 2000

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Effective Lagrangian Approach

Step 5: Combine all to construct the Higgs penguin

\[ H_1, H_2, H_3 \]

\[ \tan^2 \beta \]
Effective Lagrangian Approach

\[ \mathcal{L}_{H_i \tilde{d} d'} = - \frac{g_w}{2M_W} \sum_{i=1}^{3} H_i \tilde{d} \left( \hat{M}_d g^L_{H_i \tilde{d} d'} P_L + g^R_{H_i \tilde{d} d'} \hat{M}_d P_R \right) d' \]

\[ g^L_{H_i \tilde{d} d'} = V^\dagger R^{-1} V \frac{O_{1i}}{\cos \beta} + \left( 1 - V^\dagger R^{-1} V \right) \frac{O_{2i}}{\sin \beta} \]

\[ - i \left( 1 - \frac{1}{\cos^2 \beta} V^\dagger R^{-1} V \right) \frac{O_{3i}}{\tan \beta}, \]

where \( R = 1 + \tan \beta \left( E_g + E_u |\hat{h}_u|^2 + \ldots \right) \)

Dedes., Pilaftsis, 2003

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Effective Lagrangian Approach

- Even with flavour and mass squark universality we observe Higgs mediated FCNCs!
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- The method is limited to large $\tan \beta \gtrsim 40$ and $M_{\text{SUSY}} \gg M_Z$
Results

A benchmark scenario

\[ m_{\tilde{q}} \]

\[ \rho \times M_{SUSY} \]

\[ m_{\tilde{\tau}} = |\mu| = |A_U| = |m_{\tilde{\tau}}| = M_{SUSY} \]

\[ m_{\tilde{\chi}} \sim 2 m_t \]

\[ M_{H^+} \sim m_t \]

\[ M_W \]
GIM operative points

$M_{\text{SUSY}} = 1 \text{ TeV}, \ M_{H^0} = 0.2 \text{ TeV}, \ \tan \beta = 50, \ \text{arg}(m_g) = 180^\circ$

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Bounding the Higgs sector....

- $\tan \beta = 50$
- $\tilde{m} < 2.5$ TeV
- Random SUSY parameter scan assuming neutralino LSP
- Max Br formula

$$B(B_S \rightarrow \mu^+ \mu^-) = 5 \times 10^{-7} \left( \frac{\tan \beta}{50} \right)^6 \left( \frac{650 \text{ GeV}}{M_A} \right)^4 + 1.0 \times 10^{-8}$$

Dedes, Huffman, 2004
Conclusions

\( \mathcal{B}(B_s \rightarrow \mu^+\mu^-) \) is large in SUSY because of the \( \tan \beta \)-enhanced Higgs penguin

\(^a\) Numerical codes are available for both approaches, (send an e-mail to Athanasios.Dedes@durham.ac.uk)
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- Two approaches exist: Diagrammatic and Effective Lagrangian$^a$
- GIM operative points exist
- Indirect probe of the heavy Higgs sector

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