$B_s \to \mu^+\mu^-$ in Supersymmetry

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CERN, May 15, Flavour in the era of the LHC

Outline

- Why $B_s \to \mu^+\mu^-$ is interesting ?
- Higgs mediated contributions
  - Feynman Diagrammatic Approach – Results
  - Effective Lagrangian Approach – Results
- Conclusions

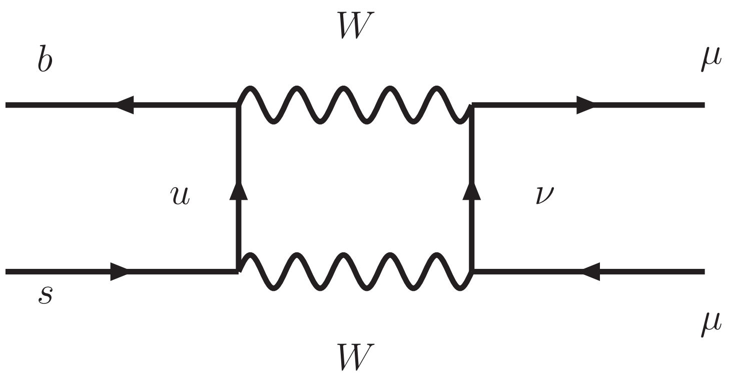
For a Higgs penguin review see, A. D., hep-ph/0309233

Why $B_s \to \mu^+\mu^-$ is interesting ?

SM predicts that $B_s \to \mu^+\mu^-$ is a very rare decay with

$$\mathcal{B}(B_s \to \mu^+\mu^-) = (3.8 \pm 1.0) \times 10^{-9} \; .$$

Main uncertainty from $[f_{B_s} = 230 \pm 30 \text{ MeV}]$[a]. Additional small uncertainty $\pm 0.3 \times 10^{-9}$ from $m_t = 175 \pm 5$ GeV. It originates from Z-penguin and box diagrams[b]



[a]Taken from Lattice, D. Becirevic 2003.

[b]T. Inami, C.S. Lim (1981); NLO QCD corrections by G. Buchalla, A. J. Buras (1993) and later by M. Misiak, and J. Urban (1999).

Other subdominant contributions :

- Higgs penguin (suppressed by m_b/M_W)
- The photon penguin does not contribute.

Current Experimental bound from CDF :

$$\mathcal{B}(B_s \to \mu^+\mu^-) \ < \ 1.0 \times 10^{-7} \quad \text{at } 95\% \text{ CL}$$

A possible evidence of $B_s \to \mu^+\mu^-$ at Tevatron will be a clear signal for new flavour physics.

Higgs mediated contributions

In SUSY, $\mathcal{B}(B_s \to \mu^+\mu^-)$ is enhanced by $\tan^6\beta$ making this decay interesting for Tevatron and LHC.

There are two approaches to calculate the dominant contributions :

- Feynman Diagrammatic Approach + Ressumation[a]
- Effective Lagrangian Approach

Both have advantages and disadvantages

[a] Choudhury and Gaur 1999; Huang, Liao, Yan, Zhu, 2001; Chankowski and Slawianowska 2001; Bobeth, Ewerth, Krüger, Urban, 2001,2002; Dedes, Dreiner, Nierste, 2001.

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[a] Babu and Kolda 2000; Isidori and Retico, 2001; Dedes and Pilaftsis 2003; Buras, Chankowski, Rosiek, Slawianowska, 2003; Foster, Okumura, Roszkowski, 2005; Carena, Menon, Noriega-Papaqui, Szynkman, Wagner, 2006.

Diagrammatic Approach

Step 1: Calculate the diagrams in the mass eigenbasis

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Step 2: Make everywhere the replacement[a]

$$h_b \to h_b = \frac{g}{\sqrt{2} M_W \cos\beta} \frac{m_b(Q)}{1 + \Delta m_b^{\mathrm{SQCD}}}$$

[a] Same method used in $b \to s\gamma$ calculation, by Carena,Garcia,Nierste, Wagner, 2000

Diagrammatic Approach

Step 1: Calculate the diagrams in the mass eigenbasis

Step 2: Make everywhere the replacement

Step 3: Use RGEs to connect with the Unification scale

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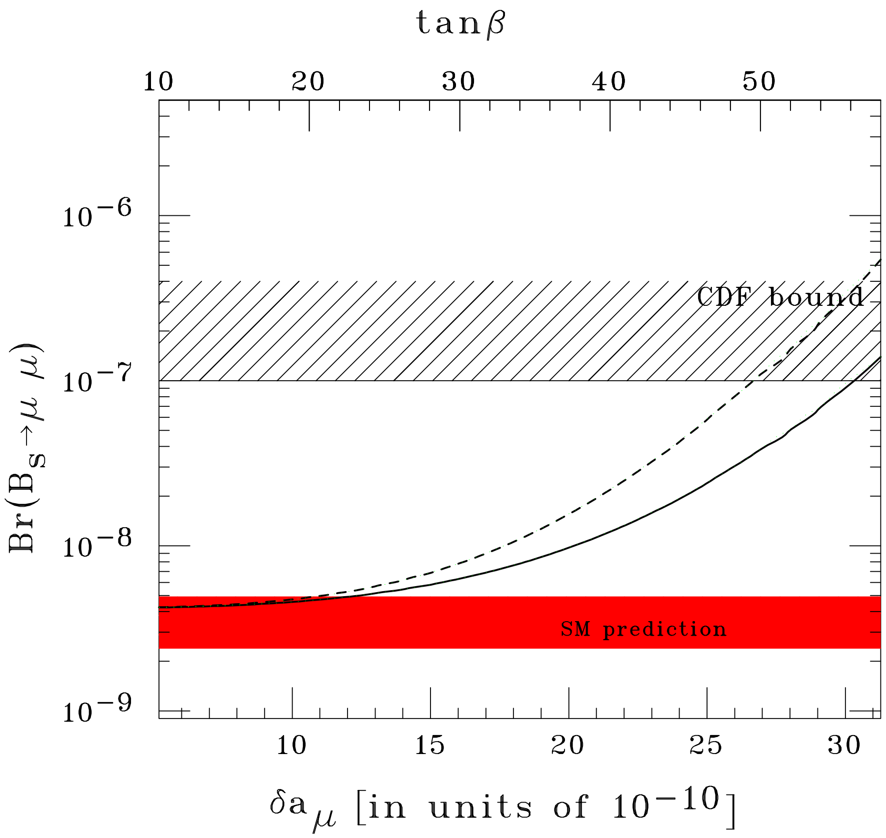
- $M_{1/2} = 450$ GeV
- $M_0 = 350$ GeV
- $A_0 = 0$ GeV
- $\mu > 0$ GeV,
- $m_t = 175$ GeV

Dedes, Dreiner, Nierste, 2001



Effective Lagrangian Approach

Step 1: Start out with the symmetric theory[a]

[a] Hempfling 1994; Hall, Rattazzi, Sarid, 1994

Effective Lagrangian Approach

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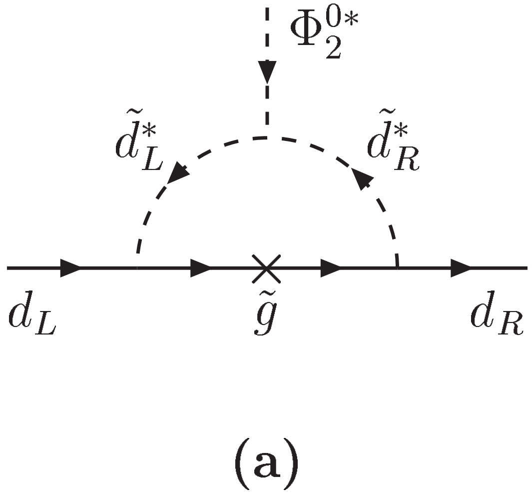
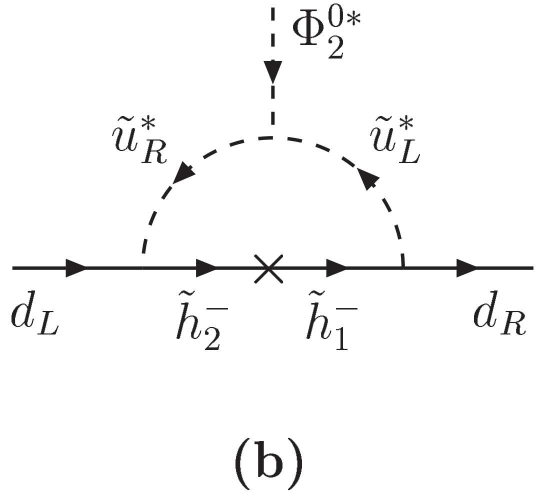
$$-\mathcal{L}_Y = \bar{d}_R^0\, \mathbf{h_d} \Big[\Phi_1^{0*}\,(\mathbf{1} + \ldots) + \Phi_2^{0*} \Big(\hat{\mathbf{E}}_\mathbf{g} + \hat{\mathbf{E}}_\mathbf{u}\, \mathbf{h}_\mathbf{u}^\dagger \mathbf{h_u} + \ldots \Big) \Big]\, d_L^0$$

$$+ \;\; \Phi_2^0\, \bar{u}_R^0\, \mathbf{h_u}\,(\mathbf{1} + \ldots)\, u_L^0 \; + \; \text{H.c.}$$

with the finite non-holomorphic radiative corrections, $\hat{\mathbf{E}}_\mathbf{g}, \hat{\mathbf{E}}_\mathbf{u}$





Effective Lagrangian Approach

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$$-\mathcal{L}_Y \;=\; \bar{d}^0_R\,\mathbf{h_d}\Big[\,\Phi_1^{0*}\,(\mathbf{1}+\ldots)\;+\;\Phi_2^{0*}\Big(\hat{\mathbf{E}}_\mathbf{g}\;+\;\hat{\mathbf{E}}_\mathbf{u}\,\mathbf{h_u^\dagger}\mathbf{h_u}+\;\ldots\Big)\Big]\,d^0_L$$

$$+\;\;\Phi_2^0\,\bar{u}^0_R\,\mathbf{h_u}\,(\mathbf{1}+\;\ldots)\,u^0_L\;+\;\mathrm{H.c.}$$

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$$(\mathbf{a}):\quad \hat{\mathbf{E}}_\mathbf{g} \;=\; \mathbf{1}\,\frac{2\alpha_s}{3\pi}\,m^*_{\tilde g}\mu^*\,I(m^2_{\tilde d_L}, m^2_{\tilde d_R}, |m_{\tilde g}|^2)$$

$$(\mathbf{b}):\quad \hat{\mathbf{E}}_\mathbf{u} \;=\; \mathbf{1}\,\frac{1}{16\pi^2}\,\mu^* A^*_U\,I(m^2_{\tilde u_L}, m^2_{\tilde u_R}, |\mu|^2)$$

Effective Lagrangian Approach

Step 2: Find the quark mass terms

$$-\mathcal{L}_{\rm mass} = \frac{v_1}{\sqrt{2}}\,\bar{d}^0_R\,\mathbf{h_d}\Big[\,\mathbf{1} + \tan\beta\,\Big(\,\hat{\mathbf{E}}_\mathbf{g} + \hat{\mathbf{E}}_\mathbf{u}\,\mathbf{h}^\dagger_\mathbf{u}\mathbf{h_u}\Big)\Big]\,d^0_L$$

$$+ \frac{v_2}{\sqrt{2}}\,\bar{u}^0_R\,\mathbf{h_u}\,u^0_L + {\rm H.c.}$$

Effective Lagrangian Approach

Step 3: Redefine the quark fields-Diagonalize

$$u_L^0 = \mathcal{U}_\mathbf{L}^\mathbf{Q}\, u_L\,, \qquad d_L^0 = \mathcal{U}_\mathbf{L}^\mathbf{Q}\,\mathbf{V}\, d_L\,,$$
$$u_R^0 = \mathcal{U}_\mathbf{R}^\mathbf{u}\, u_R\,, \qquad d_R^0 = \mathcal{U}_\mathbf{R}^\mathbf{d}\, d_R\,,$$

$$\Downarrow$$

$$\hat{\mathbf{M}}_\mathbf{u} = \frac{v_2}{\sqrt{2}}\,\hat{\mathbf{h}}_\mathbf{u}\,,$$
$$\mathcal{U}_\mathbf{R}^{\mathbf{d}\,\dagger}\mathbf{h}_\mathbf{d}\mathcal{U}_\mathbf{L}^\mathbf{Q} = \frac{\sqrt{2}}{v_1}\,\hat{\mathbf{M}}_\mathbf{d}\mathbf{V}^\dagger\,\hat{\mathbf{R}}^{-1}\,, \tag{1}$$

with $\hat{\mathbf{R}} = \mathbf{1} + \hat{\mathbf{E}}_\mathbf{g}\tan\beta + \hat{\mathbf{E}}_\mathbf{u}\tan\beta\,|\hat{\mathbf{h}}_\mathbf{u}|^2$

Effective Lagrangian Approach

Step 4: Express $\mathcal{L}_Y$ in terms of the mass eigenstates

$$- \mathcal{L}_Y = \frac{\sqrt{2}}{v_2} \left(\tan\beta\, \Phi_1^{0*} - \Phi_2^{0*} \right) \bar{d}_R \, \hat{\mathbf{M}}_{\mathbf{d}} \mathbf{V}^\dagger \hat{\mathbf{R}}^{-1} \mathbf{V} \, d_L$$

$$+ \frac{\sqrt{2}}{v_2} \Phi_2^{0*} \, \bar{d}_R \, \hat{\mathbf{M}}_{\mathbf{d}} \, d_L + \Phi_2^0 \, \bar{u}_R \hat{\mathbf{h}}_{\mathbf{u}} u_L + \mathrm{H.c.}$$

with $\hat{\mathbf{R}} = \mathbf{1} + \hat{\mathbf{E}}_{\mathbf{g}} \tan\beta + \hat{\mathbf{E}}_{\mathbf{u}} \tan\beta \, |\hat{\mathbf{h}}_{\mathbf{u}}|^2$

Effective Lagrangian Approach

Step 4: Express $\mathcal{L}_Y$ in terms of the mass eigenstates

Even if we had started with general matrices $\tilde{\mathbf{E}}_\mathbf{u}$, $\tilde{\mathbf{E}}_\mathbf{g}$ we end up with the same $\mathcal{L}_Y$ where

$$\mathbf{E_g} \;=\; \mathcal{U}_\mathbf{L}^{\mathbf{Q}\dagger}\,\tilde{\mathbf{E}}_\mathbf{g}\,\mathcal{U}_\mathbf{L}^{\mathbf{Q}}\,, \qquad \mathbf{E_u} \;=\; \mathcal{U}_\mathbf{L}^{\mathbf{Q}\dagger}\,\tilde{\mathbf{E}}_\mathbf{u}\,\mathcal{U}_\mathbf{L}^{\mathbf{Q}}\,,$$

$$\mathbf{R} \;=\; \mathbf{1} + \tan\beta\left(\mathbf{E_g} + \mathbf{E_u}\,|\hat{\mathbf{h}}_\mathbf{u}|^2 + \ldots\right),$$

Effective Lagrangian Approach

Step 4: Express $\mathcal{L}_Y$ in terms of the mass eigenstates

RGE induced operators proportional to $\mathcal{U}_{\mathbf{L}}^{\mathbf{Q}\dagger}\mathbf{h_d}^\dagger\mathbf{h_d}\mathcal{U}_{\mathbf{L}}^{\mathbf{Q}}$

Take the hermitian square of the modified Eq.(1) and solve for $\mathcal{U}_{\mathbf{L}}^{\mathbf{Q}\dagger}\mathbf{h_d}^\dagger\mathbf{h_d}\mathcal{U}_{\mathbf{L}}^{\mathbf{Q}}$ and then iterate

Effective Lagrangian Approach

Step 5: Express the Higgs fields $\Phi^0_{1,2}$ in terms of their mass eigenstates $H_{1,2,3}$ in the presence of CP violation[a]

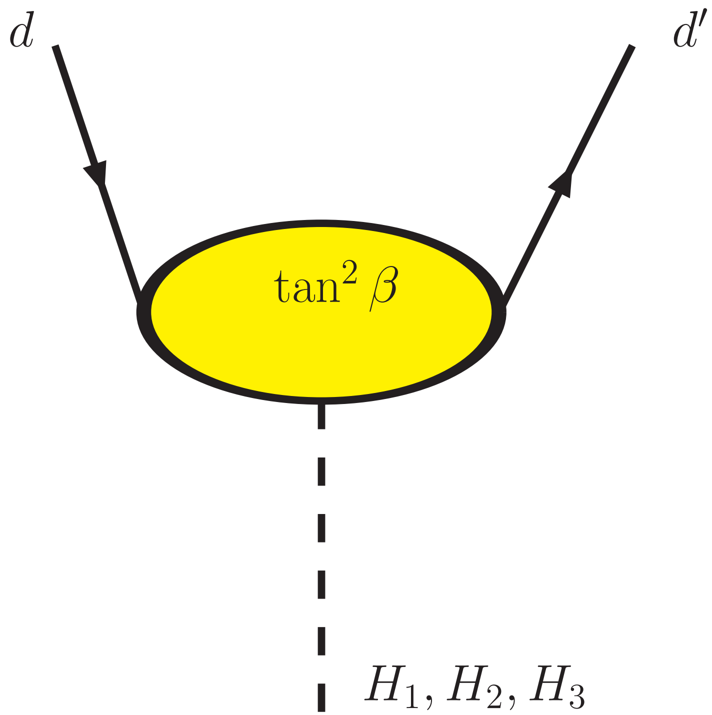
$$\Phi^0_1 \;\to\; \frac{1}{\sqrt{2}}\left[\, O_{1i}\, H_i \;+\; i\left(\cos\beta\, G^0 \;-\; \sin\beta\, O_{3i}\, H_i \right)\right],$$

$$\Phi^0_2 \;\to\; \frac{1}{\sqrt{2}}\left[\, O_{2i}\, H_i \;+\; i\left(\sin\beta\, G^0 \;+\; \cos\beta\, O_{3i}\, H_i \right)\right],$$

[a] Carena,Ellis,Pilaftsis,Wagner, 2000

Effective Lagrangian Approach

Step 5: Combine all to construct the Higgs penguin



Effective Lagrangian Approach

$$\mathcal{L}_{H_i\bar{d}d'} = -\frac{g_w}{2M_W}\sum_{i=1}^{3} H_i\,\bar{d}\left(\hat{\mathbf{M}}_{\mathbf{d}}\,\mathbf{g}^L_{H_i\bar{d}d'}\,P_L + \mathbf{g}^R_{H_i\bar{d}d'}\,\hat{\mathbf{M}}_{\mathbf{d'}}\,P_R\right)d'$$

$$\begin{aligned}\mathbf{g}^L_{H_i\bar{d}d'} &= \mathbf{V}^\dagger\mathbf{R}^{-1}\mathbf{V}\,\frac{O_{1i}}{\cos\beta} + \left(\mathbf{1} - \mathbf{V}^\dagger\mathbf{R}^{-1}\mathbf{V}\right)\frac{O_{2i}}{\sin\beta}\\ &- \quad i\left(\mathbf{1} - \frac{1}{\cos^2\beta}\mathbf{V}^\dagger\mathbf{R}^{-1}\mathbf{V}\right)\frac{O_{3i}}{\tan\beta}\,,\end{aligned} \tag{2}$$

where $\mathbf{R} = \mathbf{1} + \tan\beta\left(\mathbf{E_g} + \mathbf{E_u}\,|\hat{\mathbf{h}}_\mathbf{u}|^2 + \ldots\right)$

Dedes., Pilaftsis, 2003

Effective Lagrangian Approach

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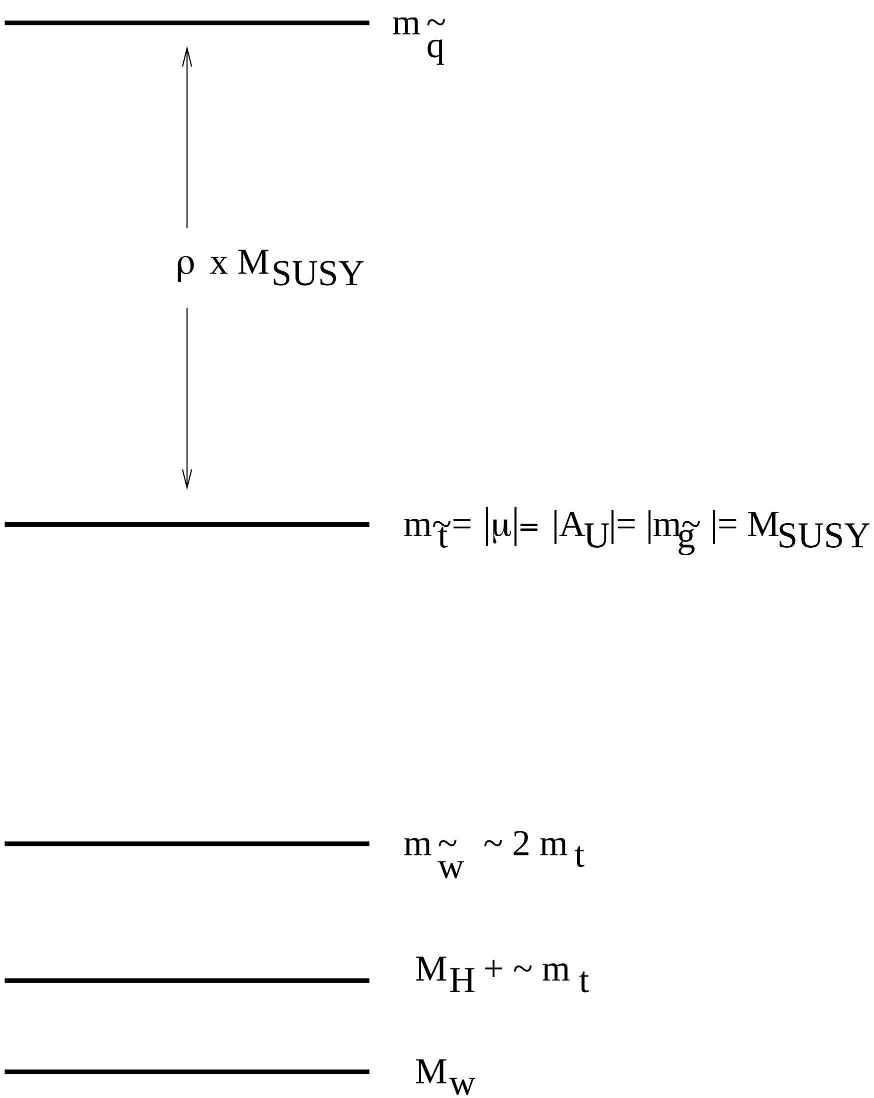
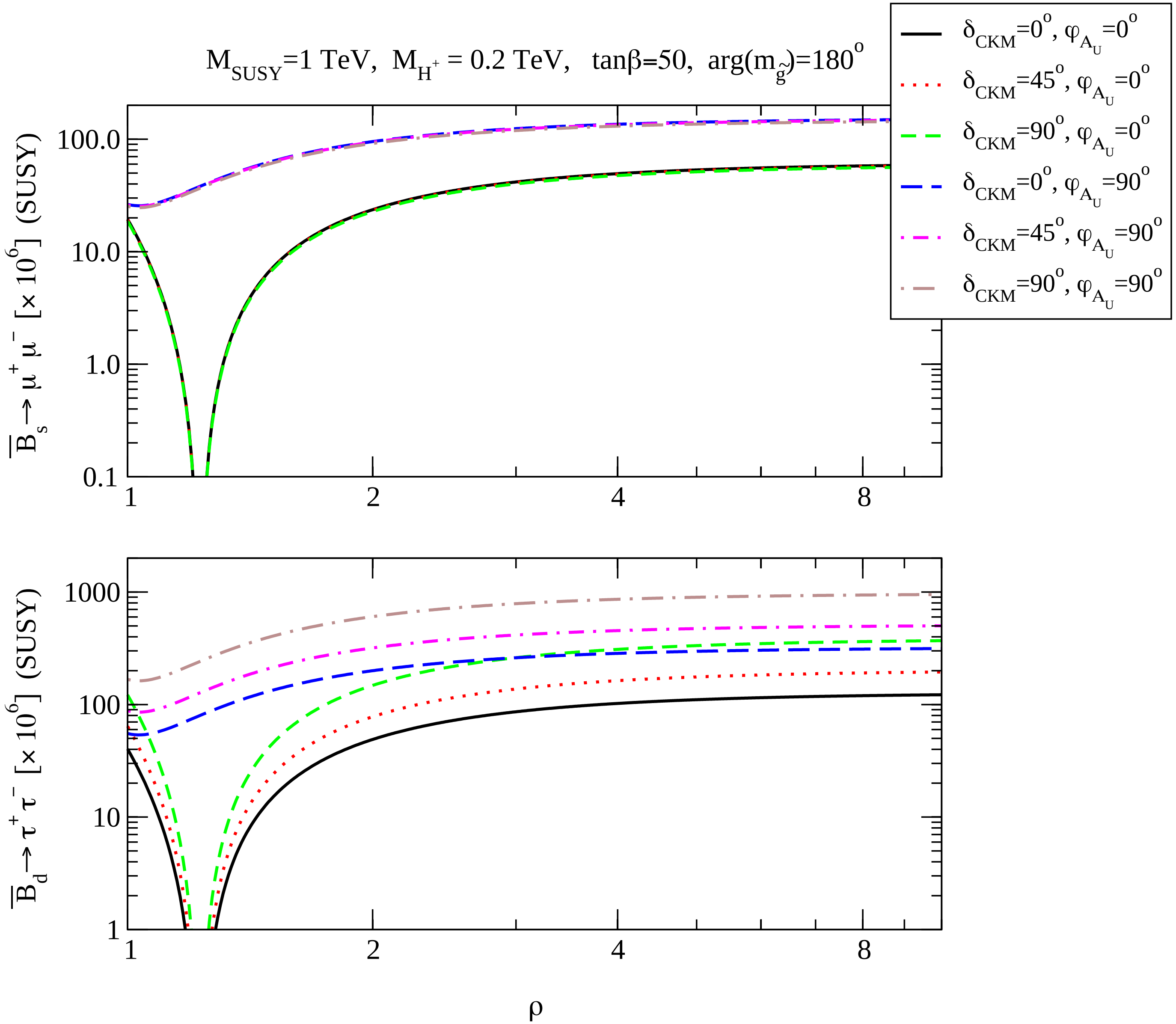
- The method is limited to large $\tan\beta \gtrsim 40$ and $M_{\rm SUSY} >> M_Z$

Results

A benchmark scenario

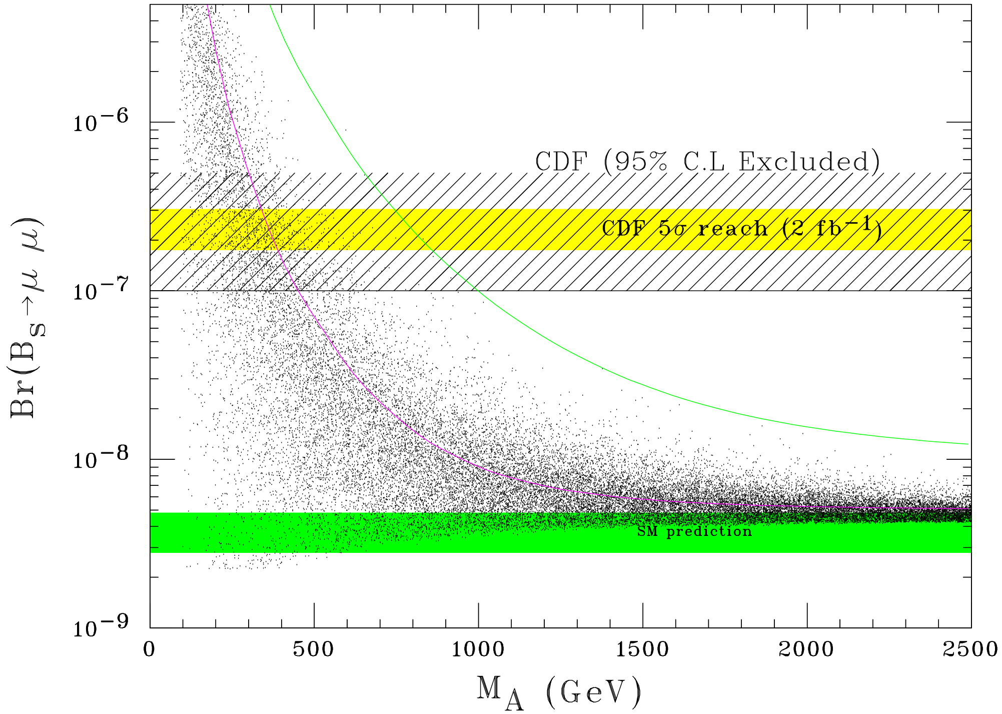


GIM operative points



Bounding the Higgs sector....

- $\tan\beta = 50$
- $\tilde{m} < 2.5$ TeV
- Random SUSY parameter scan assuming neutralino LSP
- Max Br formula



Dedes, Huffman, 2004

$$\mathcal{B}(B_s \to \mu^+\mu^-) \;=\; 5\times 10^{-7}\left(\frac{\tan\beta}{50}\right)^6\left(\frac{650\ \mathrm{GeV}}{M_A}\right)^4 + 1.0\times 10^{-8}$$

Conclusions

- $\mathcal{B}(B_s \to \mu^+\mu^-)$ is large in SUSY because of the $\tan\beta$-enhanced Higgs penguin

[a] Numerical codes are available for both approaches, (send an e-mail to Athanasios.Dedes@durham.ac.uk)

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