

Bispectrum & finite-volume effects

in collaboration with:

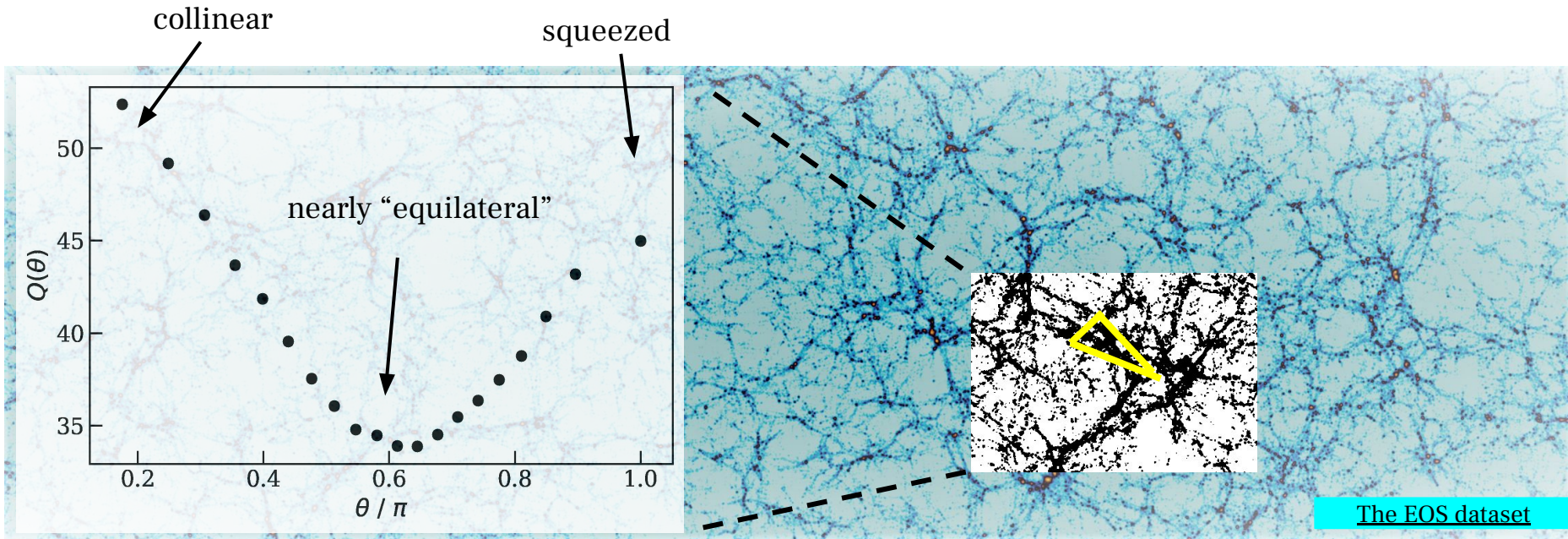
F.Rizzo, C.Moretti, M.Biagetti, E. Di Dio, E. Castorina, E. Sefusatti, P. Monaco, ...

New Physics from Galaxy Clustering II
6 Nov, IFPU (Trieste)

Kevin Pardede
INFN Parma



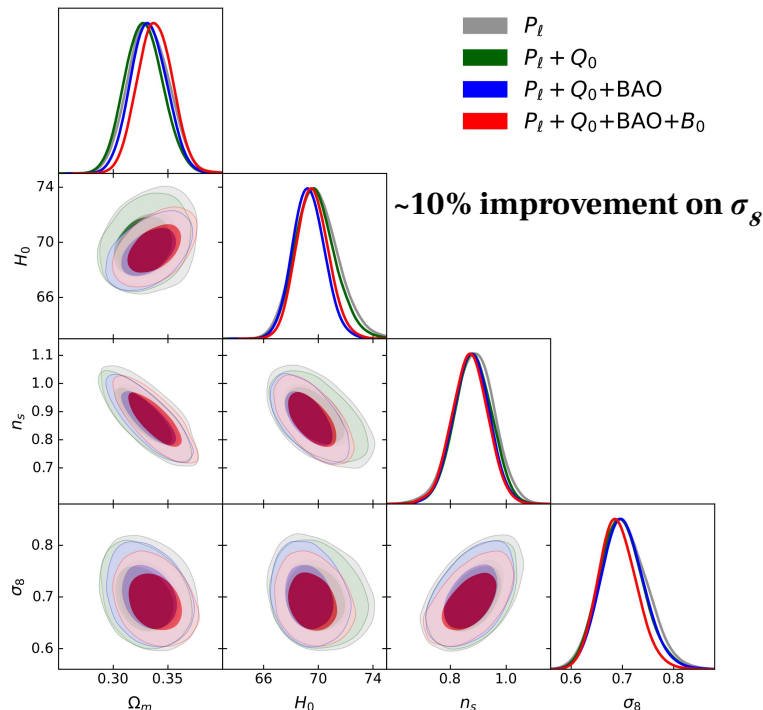
Bispectrum and the filamentary structure



$$Q(k_1, k_2, k_3) \equiv \frac{B(k_1, k_2, k_3)}{P(k_1)P(k_2) + P(k_2)P(k_3) + P(k_3)P(k_1)}$$

Bispectrum analyses

Constraint on cosmological params:

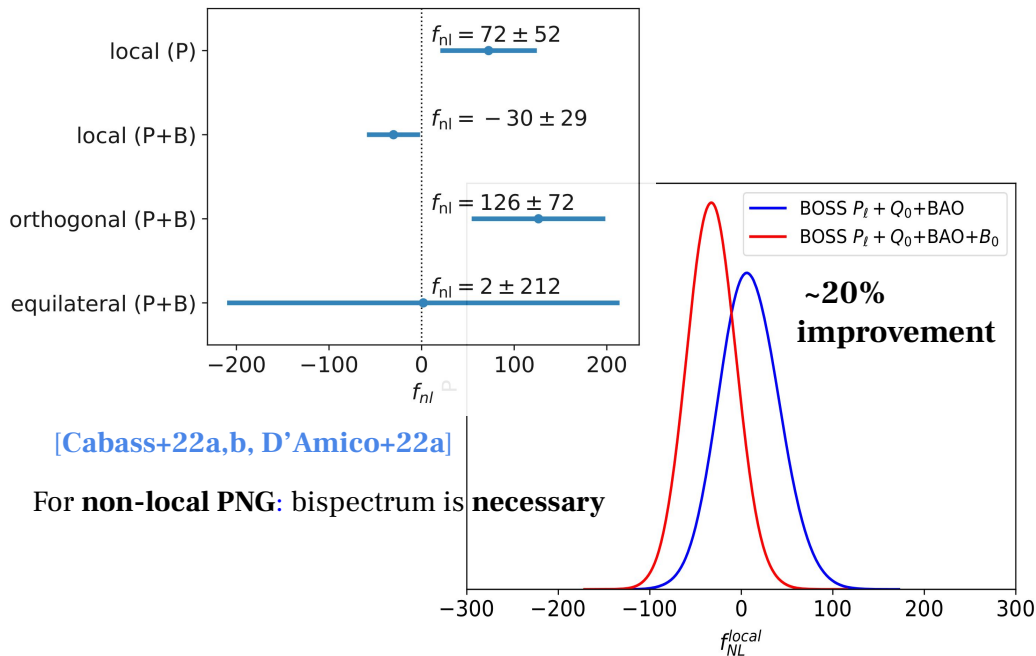


[Philcox&Ivanov21, D'Amico+19]

also one-loop bispectrum: [Philcox+22, D'Amico+22b]

also including bispectrum multipoles: [D'Amico+22b, Ivanov+23]

Constraint on primordial non-Gaussianity:



also recently: interacting DE [Tsedrik+22], w/ bootstrap [Amendola+23], w/ SBI [Tucci+23], nonlinear galaxy bispectrum [Hahn+23], ...

The bispectrum estimator

The Scoccimarro estimator [\[Scoccimarro15\]](#)

FFT-based

$$\hat{B}_L(k_1, k_2, k_3) = \frac{2L+1}{V_B V} \prod_{i=1}^3 \left[\int_{k_i} d^3 q_i \int_V d^3 x_i e^{-i \mathbf{q}_i \cdot \mathbf{x}_i} \right] \delta_D(\mathbf{q}_{123}) \tilde{\delta}(\mathbf{x}_1) \tilde{\delta}(\mathbf{x}_2) \tilde{\delta}(\mathbf{x}_3) \mathcal{L}_L(\hat{\mathbf{q}}_1 \cdot \hat{\mathbf{x}}_3)$$

$$\int_{k_1} d^3 q_1 \equiv \int_{|k_1 - \Delta k/2| \leq k_1 \leq |k_1 + \Delta k/2|} d^3 q_1 \quad V_B \equiv \int_{k_1} d^3 q_1 \int_{k_2} d^3 q_2 \int_{k_3} d^3 q_3 \delta_D(\mathbf{q}_{123})$$

Measurement vs. theory: the finite-volume effects

$$\hat{B}_L(k_1, k_2, k_3) = \frac{2L+1}{V_B V} \prod_{i=1}^3 \left[\int_{k_i} d^3 q_i \int_V d^3 x_i e^{-i\mathbf{q}_i \cdot \mathbf{x}_i} \right] \delta_D(\mathbf{q}_{123}) \tilde{\delta}(\mathbf{x}_1) \tilde{\delta}(\mathbf{x}_2) \tilde{\delta}(\mathbf{x}_3) \mathcal{L}_L(\hat{\mathbf{q}}_1 \cdot \hat{\mathbf{x}}_3)$$

1. binning effects $\frac{1}{V_B} \prod_{i=1}^3 \left[\int_{k_i} d^3 q_i \right] \delta_D(\mathbf{q}_{123})$

2. window function $\tilde{\delta}(\mathbf{x}) = W(\mathbf{x})\delta(\mathbf{x})$

3. LOS choice $\tilde{\delta}_L(\mathbf{q}) \equiv \int d^3 x \tilde{\delta}(\mathbf{x}) \mathcal{L}_L(\mathbf{q} \cdot \mathbf{x}) e^{-i\mathbf{q} \cdot \mathbf{x}}$

**... but, the estimator is
biased on large scale**

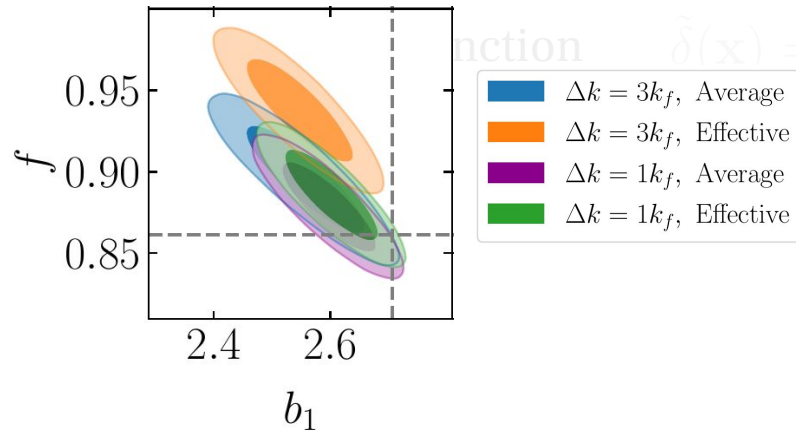
The binning effects

$$\hat{B}_L(k_1, k_2, k_3) = \frac{2L+1}{V_B V} \prod_{i=1}^3 \left[\int_{k_i} d^3 q_i \int_V d^3 x_i e^{-i\mathbf{q}_i \cdot \mathbf{x}_i} \right] \delta_D(\mathbf{q}_{123}) \tilde{\delta}(\mathbf{x}_1) \tilde{\delta}(\mathbf{x}_2) \tilde{\delta}(\mathbf{x}_3) \mathcal{L}_L(\hat{\mathbf{q}}_1 \cdot \hat{\mathbf{x}}_3)$$

1. binning effects

$$\frac{1}{V_B} \prod_{i=1}^3 \left[\int_{k_i} d^3 q_i \right] \delta_D(\mathbf{q}_{123})$$

on simulation boxes ...



[Rizzo, Moretti, Pardede+22]

The window function

$$\hat{B}_L(k_1, k_2, k_3) = \frac{2L+1}{V_B V} \prod_{i=1}^3 \left[\int_{k_i} d^3 q_i \int_V d^3 x_i e^{-i \mathbf{q}_i \cdot \mathbf{x}_i} \right] \delta_D(\mathbf{q}_{123}) \tilde{\delta}(\mathbf{x}_1) \tilde{\delta}(\mathbf{x}_2) \tilde{\delta}(\mathbf{x}_3) \mathcal{L}_L(\hat{\mathbf{q}}_1 \cdot \hat{\mathbf{x}}_3)$$

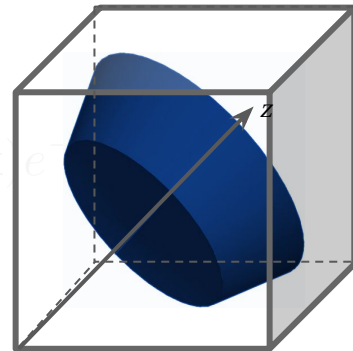
1. binning effects

$$\frac{1}{V_B} \prod_{i=1}^3 \left[\int_{k_i} d^3 q_i \right] \delta_D(\mathbf{q}_{123})$$

on real surveys ...

2. window function* $\tilde{\delta}(\mathbf{x}) = W(\mathbf{x})\delta(\mathbf{x})$

$$\Rightarrow \tilde{B}(\vec{k}_1, \vec{k}_2) = \int \frac{d^3 p_1}{(2\pi)^3} \int \frac{d^3 p_2}{(2\pi)^3} B_W(\vec{k}_1 - \vec{p}_1, \vec{k}_2 - \vec{p}_2) B(\vec{p}_1, \vec{p}_2)$$



[Baumgart, Fry 1991]

*window-free estimator [Philcox20, Philcox21, Ivanov+23]
 see also Tri-poSH based bispectrum estimator [Sugiyama+18]
 see also [Alkhanishvili+23] for NN-based approach

Survey window effects in the bispectrum

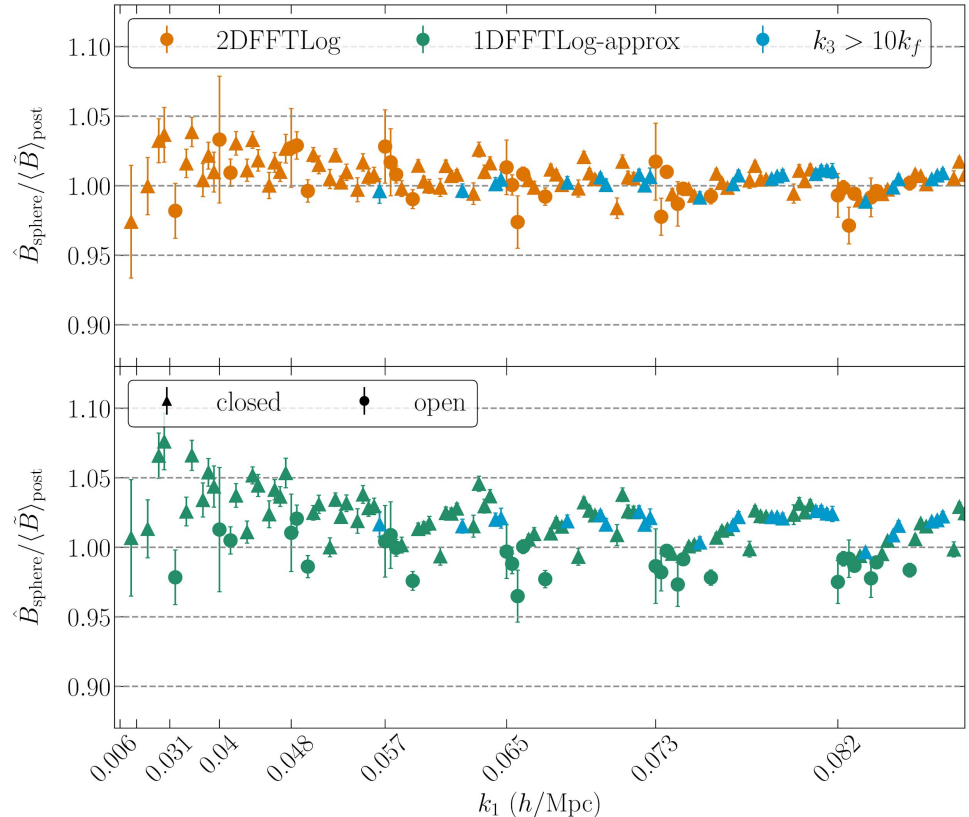
K. Pardede, F. Rizzo, M. Biagetti, E. Castorina, E. Sefusatti and P. Monaco, JCAP 10 (2022) 066 [2203.04174]

“**Mixing matrix** via (2D) FFTLog
of the window 3PCF”

$$\tilde{B}_\ell[T_i] = \sum_{j,\ell'} \mathcal{M}_{\ell\ell'}[T_i, T'_j] B_{\ell'}[T'_j]$$

Real-space spherical window tests

- Pinocchio mocks [Monaco+02]
- Total volume $\approx 3500 \text{ [Gpc}/h]^3$
- Monopole-monopole computation takes ~ 2 seconds



The wide-angle effects

$$\hat{B}_L(k_1, k_2, k_3) = \frac{2L+1}{V_B V} \prod_{i=1}^3 \left[\int_{k_i} d^3 q_i \int_V d^3 x_i e^{-i \mathbf{q}_i \cdot \mathbf{x}_i} \right] \delta_D(\mathbf{q}_{123}) \tilde{\delta}(\mathbf{x}_1) \tilde{\delta}(\mathbf{x}_2) \tilde{\delta}(\mathbf{x}_3) \mathcal{L}_L(\hat{\mathbf{q}}_1 \cdot \hat{\mathbf{x}}_3)$$

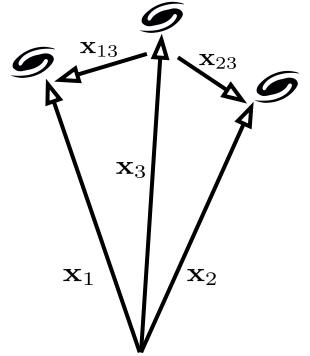
1. binning effects

not plane parallel ...

$$B(\mathbf{k}_1, \mathbf{k}_2, \mathbf{d}) = \sum_{n, i+j=n} B^{(i+j)}(\mathbf{k}_1, \mathbf{k}_2, \hat{\mathbf{d}}) (k_1 d)^{-i} (k_2 d)^{-j}$$

3. LOS choice*

$$\tilde{\delta}_L(\mathbf{q}) \equiv \int d^3 x \tilde{\delta}(\mathbf{x}) \mathcal{L}_L(\mathbf{q} \cdot \mathbf{x}) e^{-i \mathbf{q} \cdot \mathbf{x}}$$

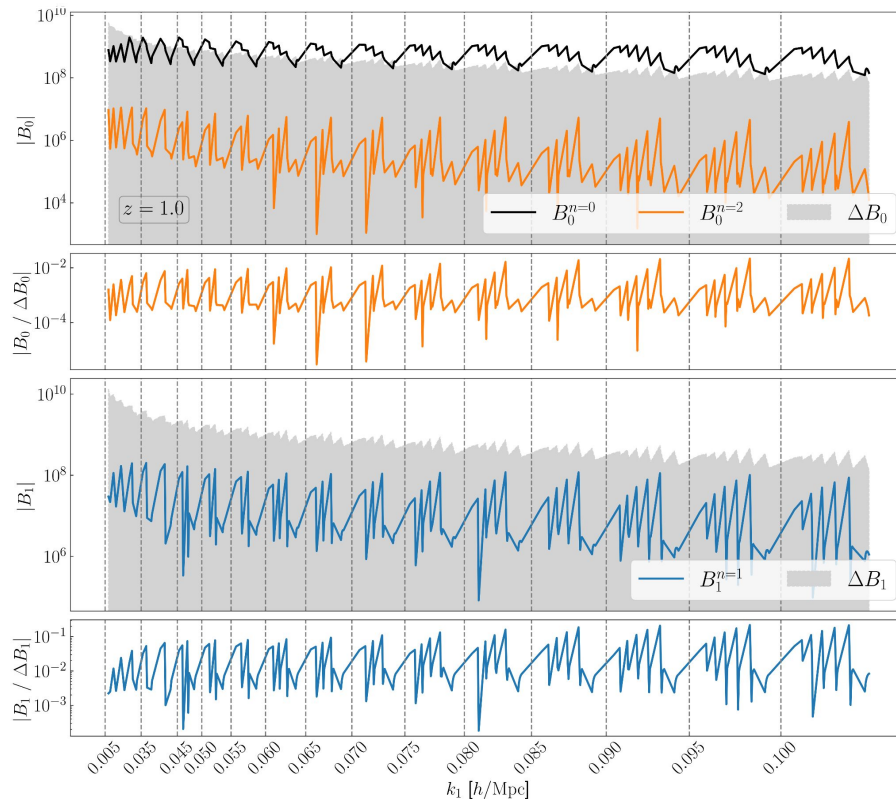


*see also [\[Milad Noorikuhani & Scoccimarro 2022\]](#)

The wide-angle effects in the bispectrum

K. Pardede, E. Di Dio, E. Castorina, JCAP 09 (2023) 030, [2302.12789]

- Monopole correction only generated at second order $\sim 0.1\%$
- Dipole correction $\sim 1\%$ of the flat-sky monopole
- S/N of dipole $\sim [V/(8 \text{ Gpc}^3 h^{-3})]^{1/2}$

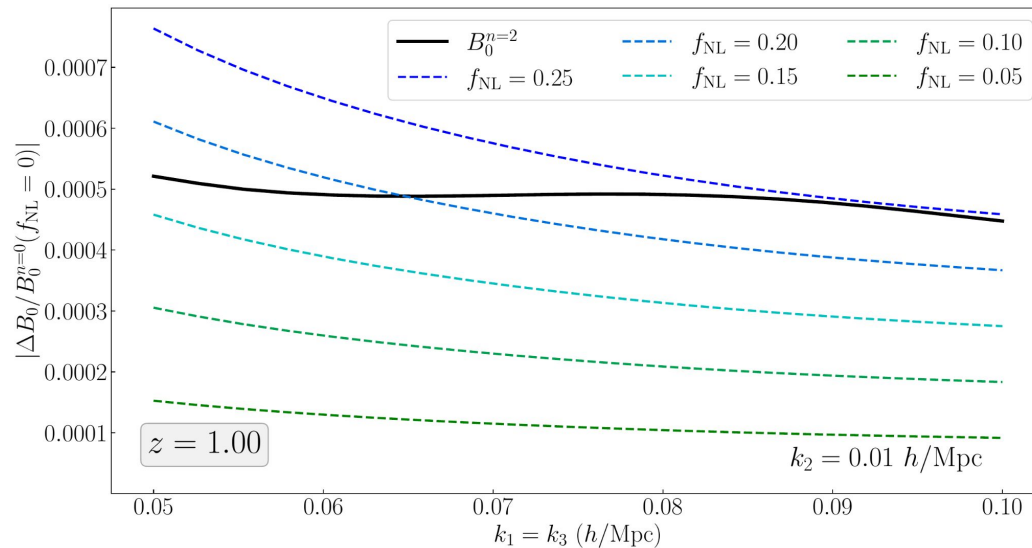


comparison with a Gaussian variance with $V = 8 (\text{Gpc}/h)^3$, $n_g \sim 6.10^{-4} (h/\text{Mpc})^3$ @ $z = 1$

Monopole correction \sim small local f_{NL}

K. Pardede, E. Di Dio, E. Castorina, JCAP 09 (2023) 030, [2302.12789]

- Monopole correction scales as k^{-2}
- Can mimic a local PNG signal



Conclusions & Outlook

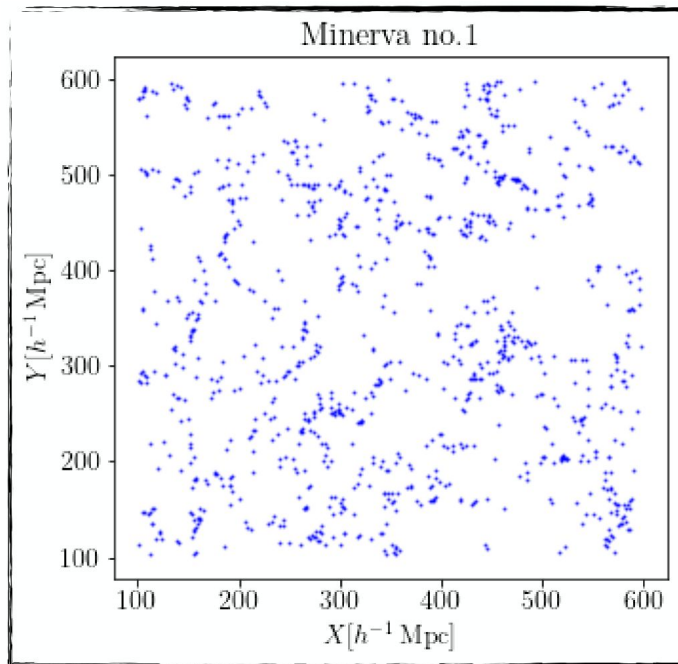
- Finite-volume effects affect the bispectrum on large-scales
- We gave a formulation for bispectrum-window convolution and bispectrum beyond plane-parallel approximation
- **Next:** application on real data, + GR effects, signal in bispectrum dipole, squeezed bispectrum, ...

THANK YOU!

Extras

Dark matter halo catalogs

1. **298 Minerva** (N-body) [Grieb+16](#)
2. 10000 **Pinocchio** (3LPT) [Monaco+02](#)



credit: A.Veropalumbo

@ $z = 1$

Λ CDM cosmology

$L_{\text{box}} = 1500 \text{ Mpc}/h$

$V_{\text{eff}} \approx 1000 \text{ (Gpc}/h)^3$

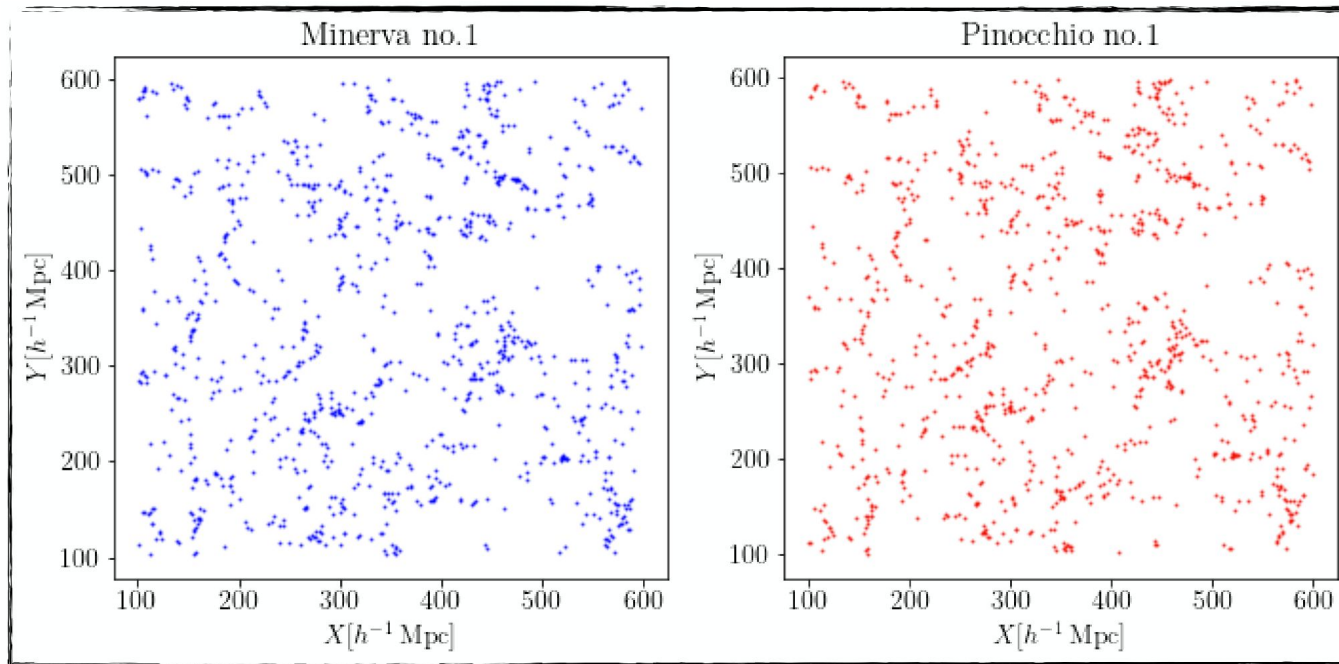
$\approx 2x$ volume in [Nishimichi+20](#)

Dark matter halo catalogs + numerical covariance

1. 298 **Minerva** (N-body) Grieb+16
2. 10000 **Pinocchio** (3LPT) [Monaco+02](#)

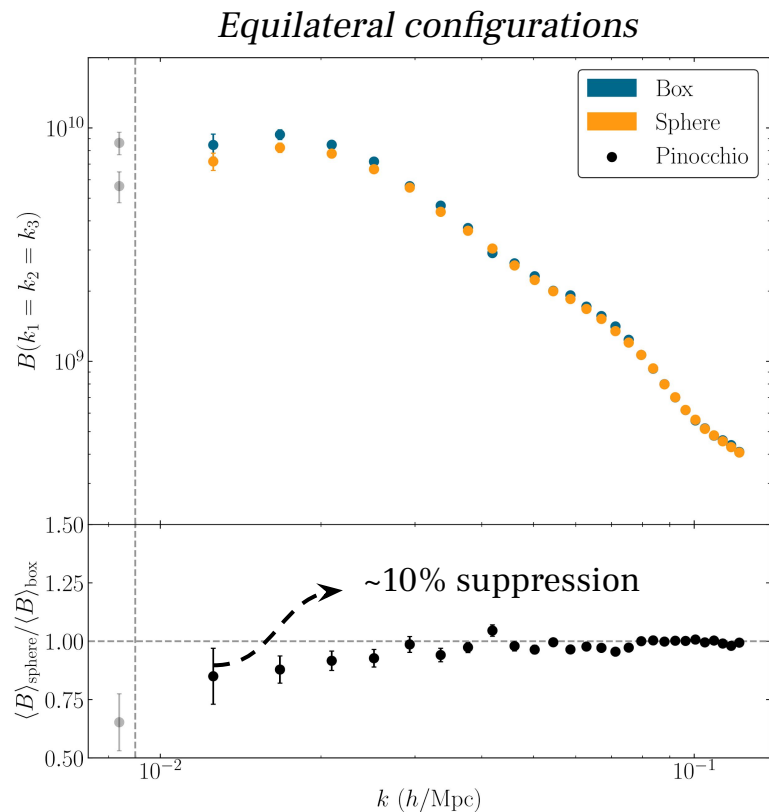
- approx. based on Lagrangian perturbation theory
- relatively fast and accurate

provide a robust estimate of the covariance



credit: A.Veropalumbo

Window effects in the equilateral bispectrum



window convolution will mix modes

10000 Pinocchio sphere catalogue

Note: this is a huge volume ≈ 3500 $[\text{Gpc}/h]^3$

... main effect is on the large scale

Recovering bias parameters

K. Pardede, F. Rizzo, M. Biagetti, E. Castorina, E. Sefusatti and P. Monaco, JCAP 10 (2022) 066 [2203.04174]

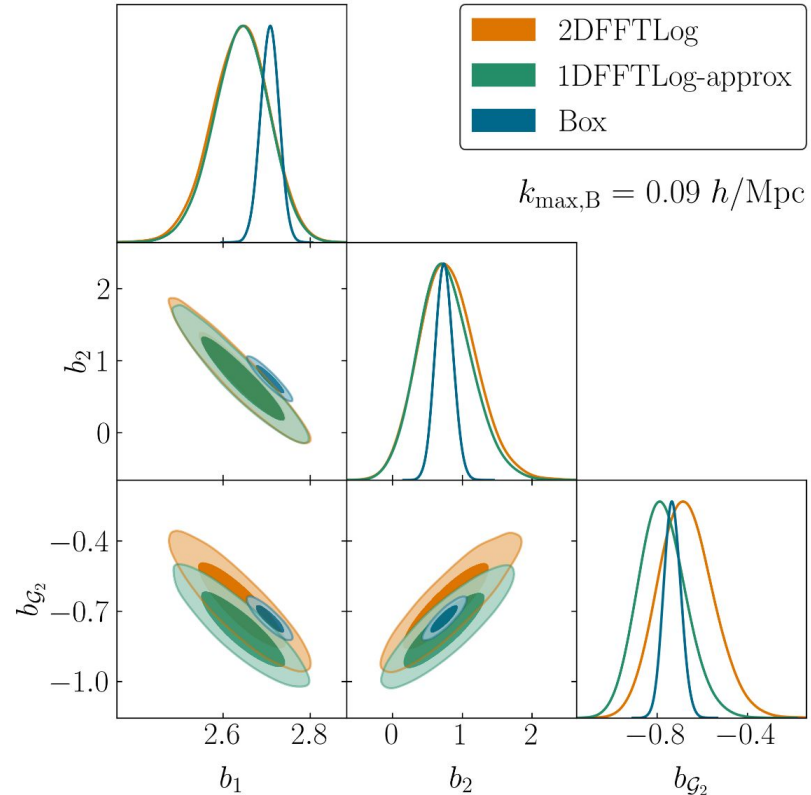
Analysis on **Minerva** data

[Grieb+16]

$\approx 1/6$ times volume in [Nishimichi+20]

≈ 6 times $z \in [1.5, 1.8]$

Euclid volume



Wide-angle effects in the bispectrum

K. Pardede, E. Di Dio, E. Castorina, JCAP 09 (2023) 030, [2302.12789]

$$B_L(k_1, k_2, k_3) = \sum_{i+j \leq n, \ell_1, \ell_2, \dots} B_{L, \ell_1, \ell_2, \dots}^{(n)}(\mathbf{k}_1, \mathbf{k}_2) \mathcal{M}_{L, \ell_1, \ell_2, \dots}^{(n)}(\mathbf{k}_1, \mathbf{k}_2) \left(\frac{1}{k_1 x_3} \right)^i \left(\frac{1}{k_2 x_3} \right)^j$$

- The “mixing-matrix” and the ($n > 0$) “bispectrum” has a high dimensionality (# ℓ s ~ 10)
- Evaluating the ($n > 0$) “bispectrum” terms takes ~ 10 minutes
- The “multiplication” can be done analytically in the case of no window

