

# EFTofLSS meets simulation-based inference: $\sigma_8$ from biased tracers

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In collaboration with: Fabian Schmidt

Based on arXiv:2310.03741



**MAX-PLANCK-INSTITUT**  
FÜR ASTROPHYSIK

Ivana Babić talk today:  
Field-level BAO inference

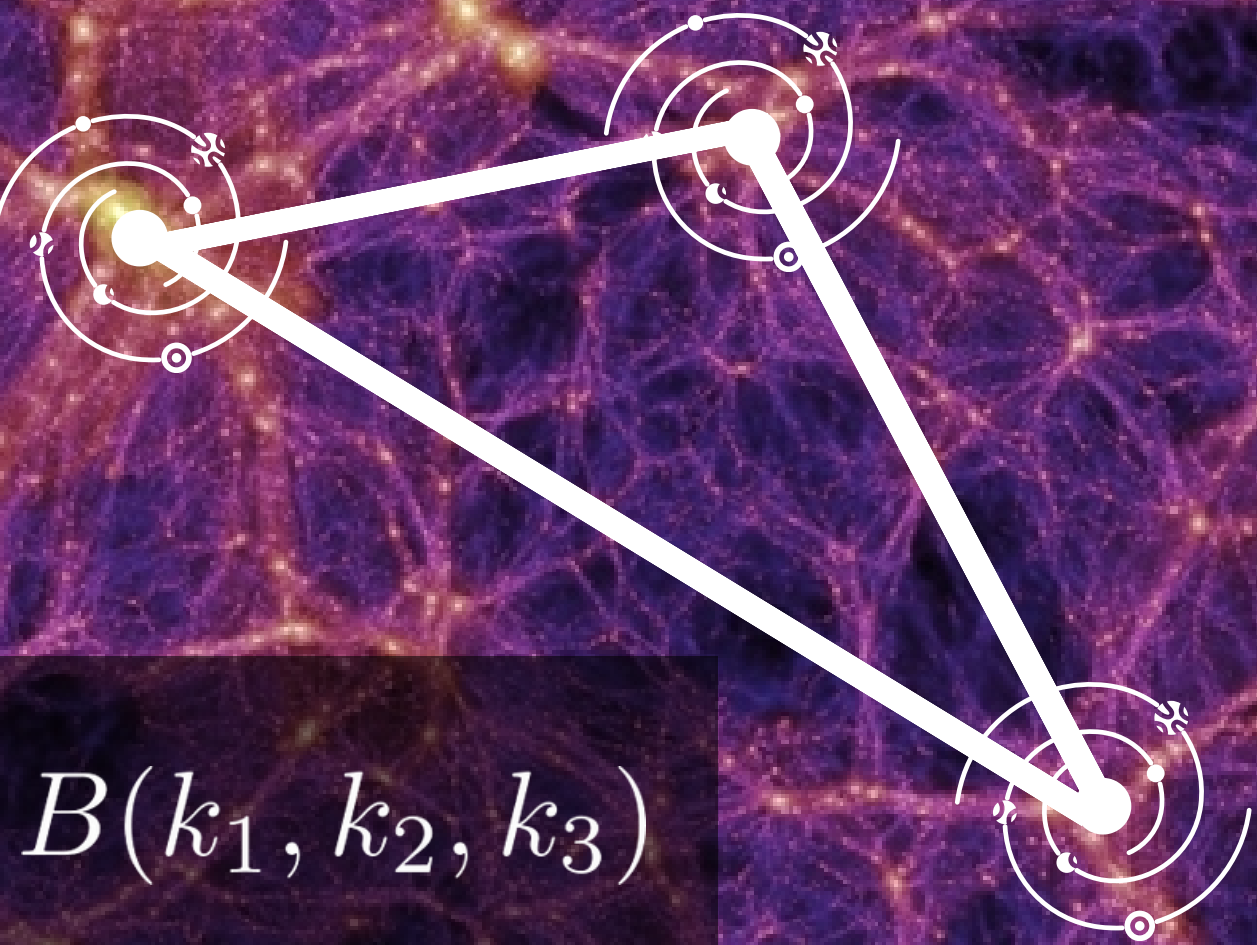
Field level



Summary  
Statistics





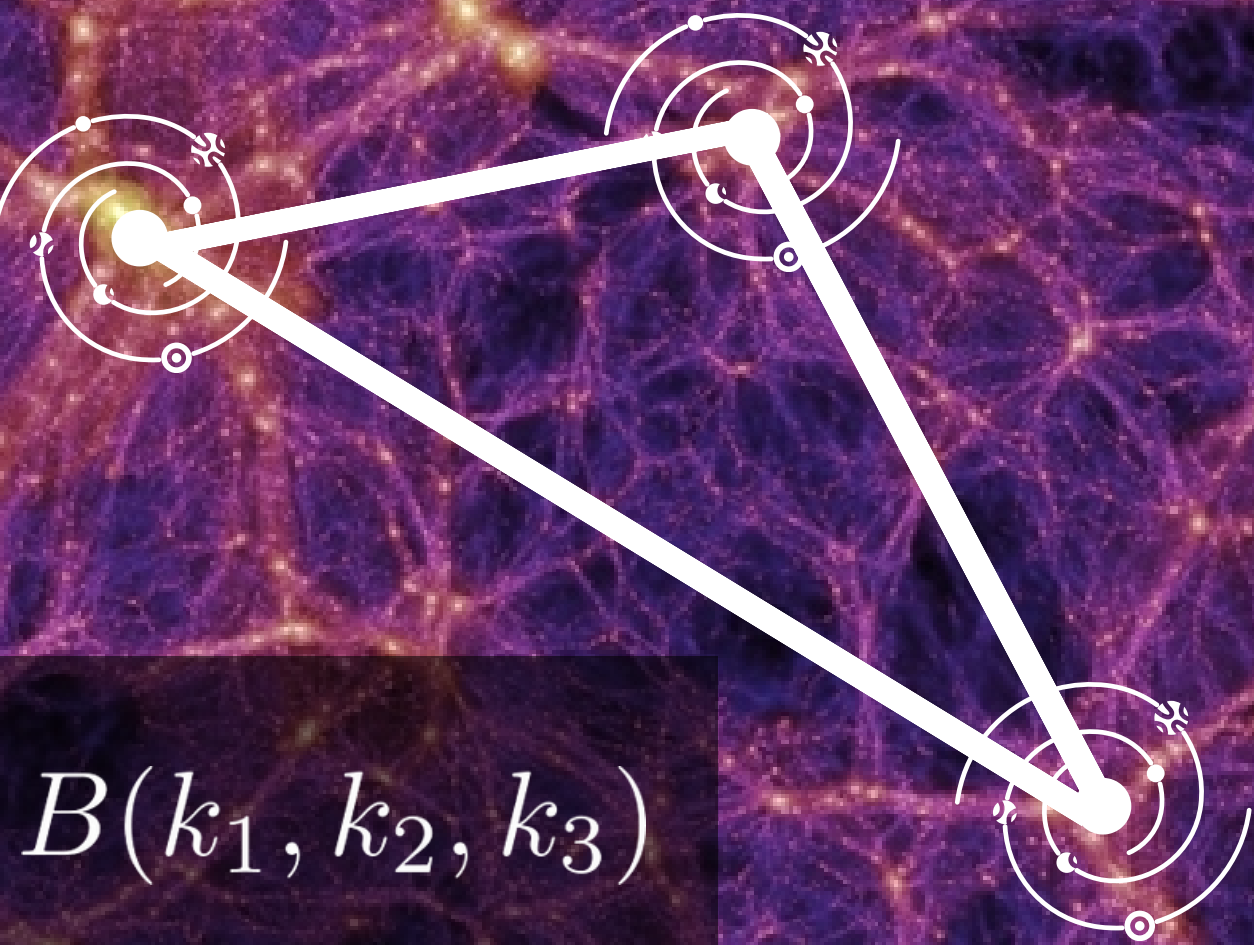


$B(k_1, k_2, k_3)$   
bispectrum

$P(k)$   
power spectrum

Summary  
Statistics





$B(k_1, k_2, k_3)$   
bispectrum



$P(k)$   
power spectrum



Gaussian likelihood?



# Standard inference in cosmology

Parameters  
posterior

Likelihood

Prior over the  
parameters

$$p(\boldsymbol{\theta}|\boldsymbol{x}) \propto \mathcal{L}(\boldsymbol{x}|\boldsymbol{\theta})\mathcal{P}(\boldsymbol{\theta})$$

# Standard inference in cosmology

Parameters posterior

Likelihood

Prior over the parameters

$$p(\boldsymbol{\theta} | \boldsymbol{x}) \propto \mathcal{L}(\boldsymbol{x} | \boldsymbol{\theta}) \mathcal{P}(\boldsymbol{\theta})$$

## Possible problems:

- Need of analytical approximations
- Cumbersome covariance estimations
- Binning effects



# Simulation-based inference

Parameters posterior

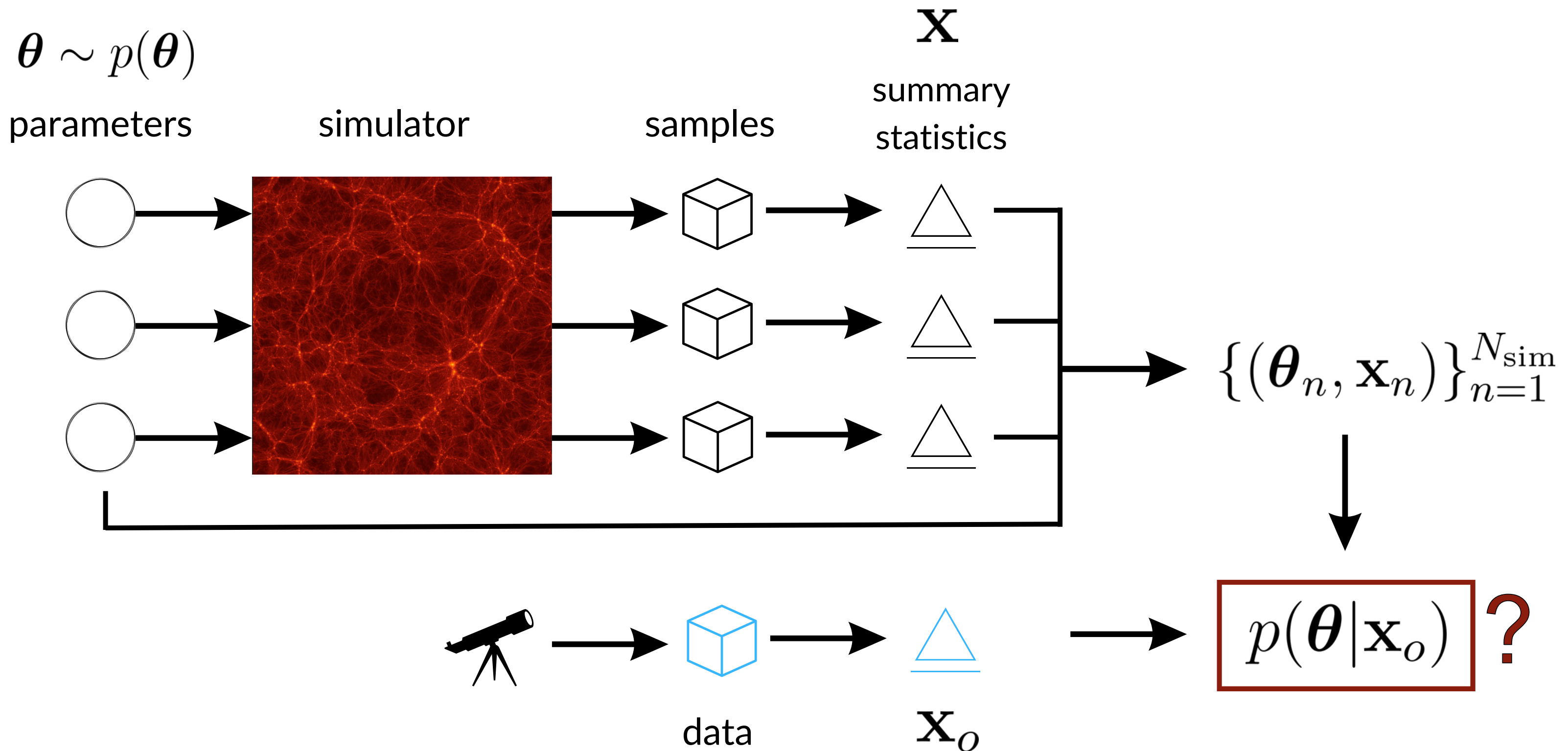
~~likelihood~~

Prior over the parameters

$$p(\boldsymbol{\theta} | \boldsymbol{x}) \propto \mathcal{L}(\boldsymbol{x} | \boldsymbol{\theta}) \mathcal{P}(\boldsymbol{\theta})$$

$$\boldsymbol{x} \sim \text{simulator}(\boldsymbol{\theta})$$

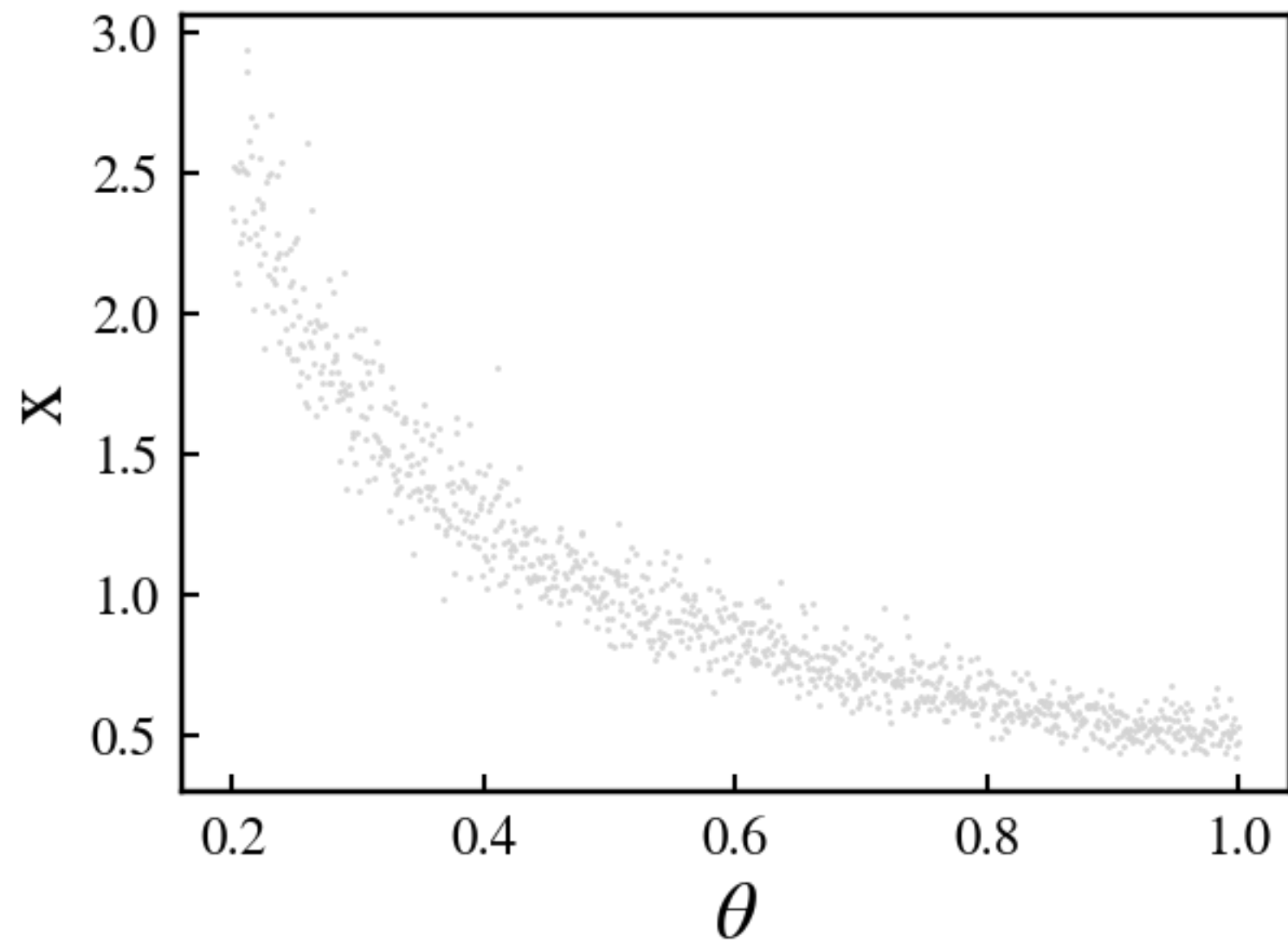
# Simulation-based inference





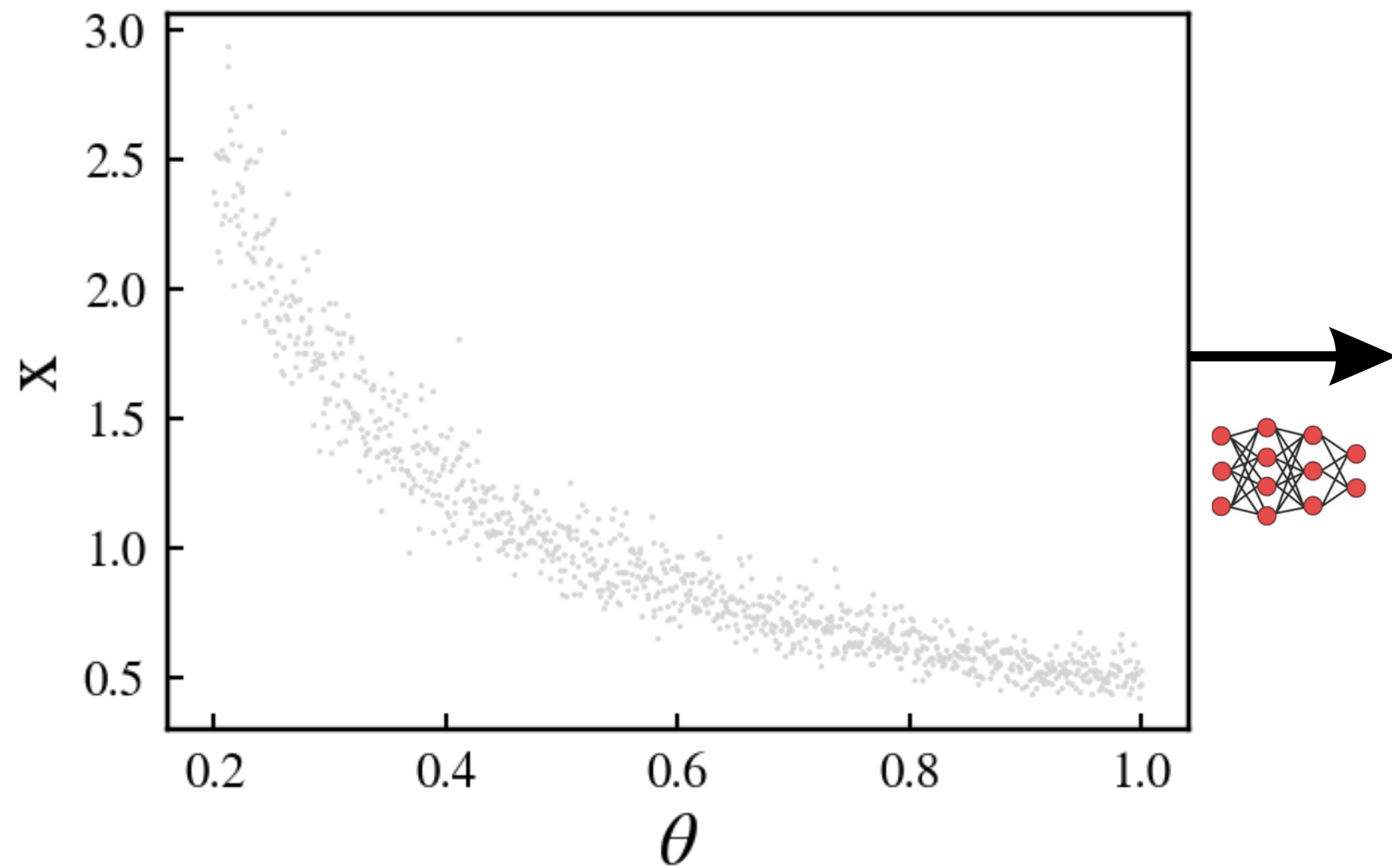
# Simulation-based inference

$$\{(\boldsymbol{\theta}_n, \mathbf{x}_n)\}_{n=1}^{N_{\text{sim}}}$$

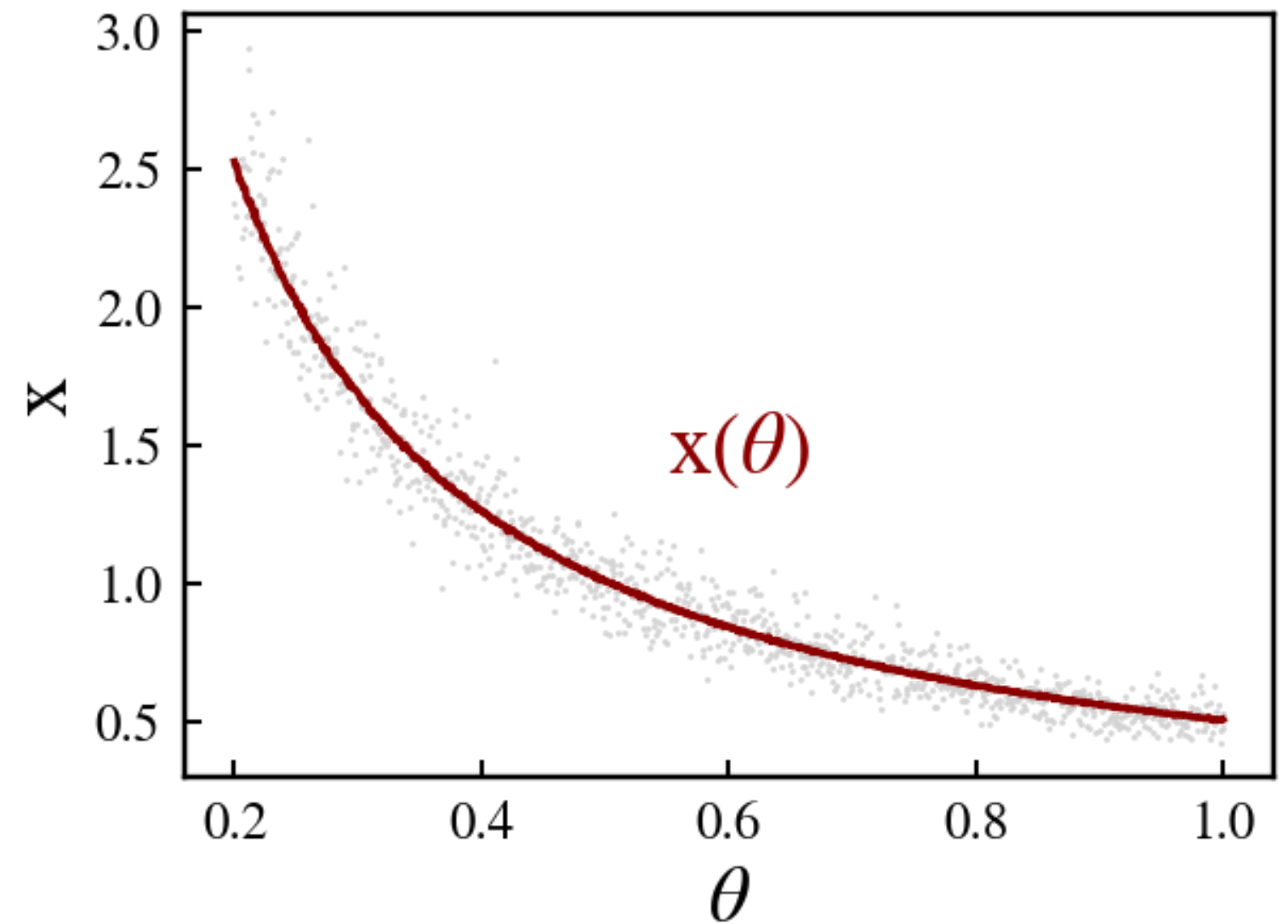


# Simulation-based inference

$$\{(\boldsymbol{\theta}_n, \mathbf{x}_n)\}_{n=1}^{N_{\text{sim}}}$$



Summary statistics  
emulators

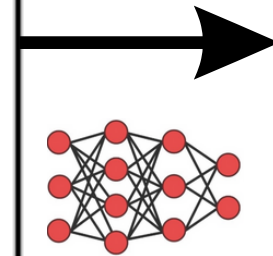
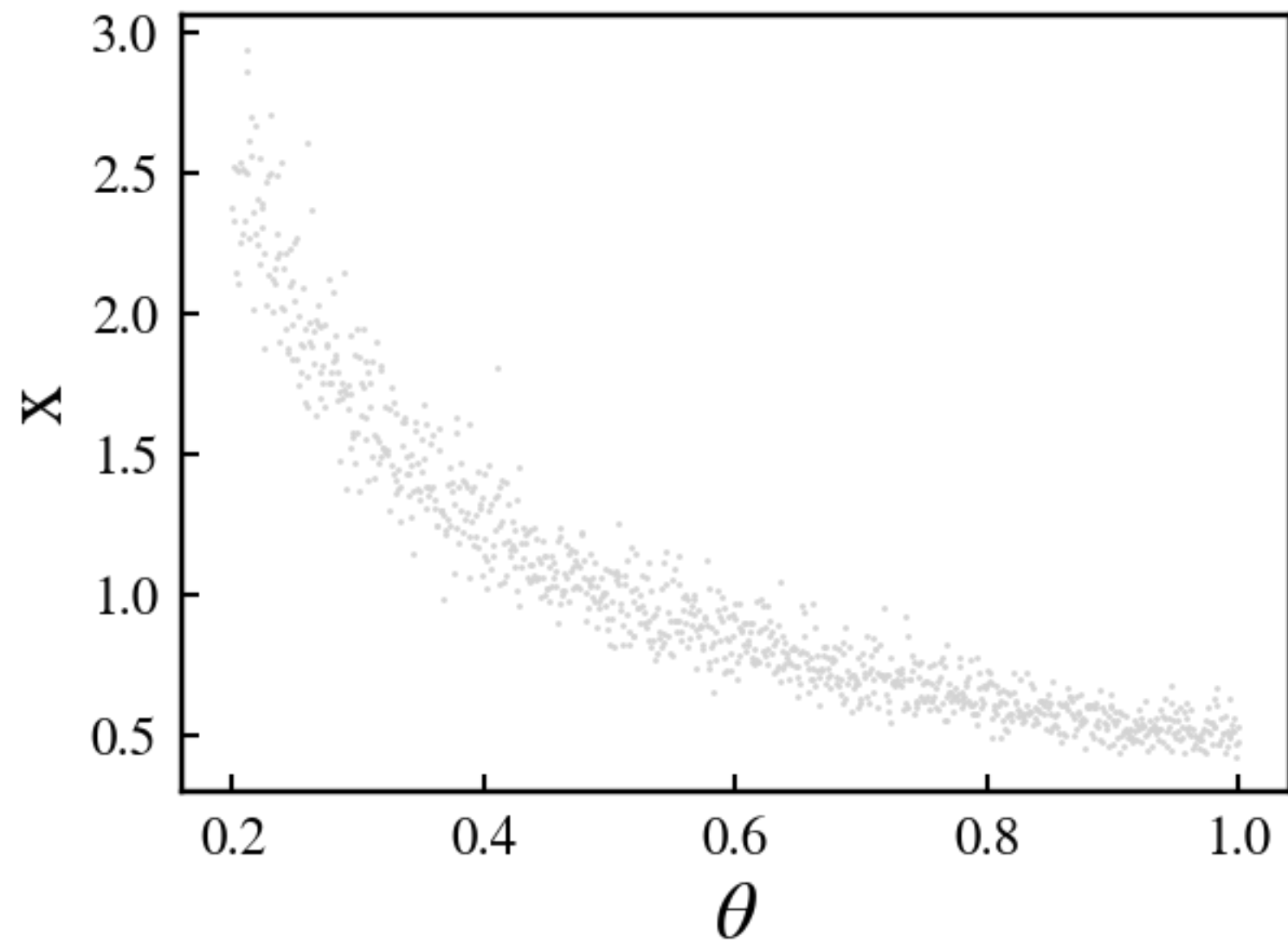


This is **not** what we are doing!

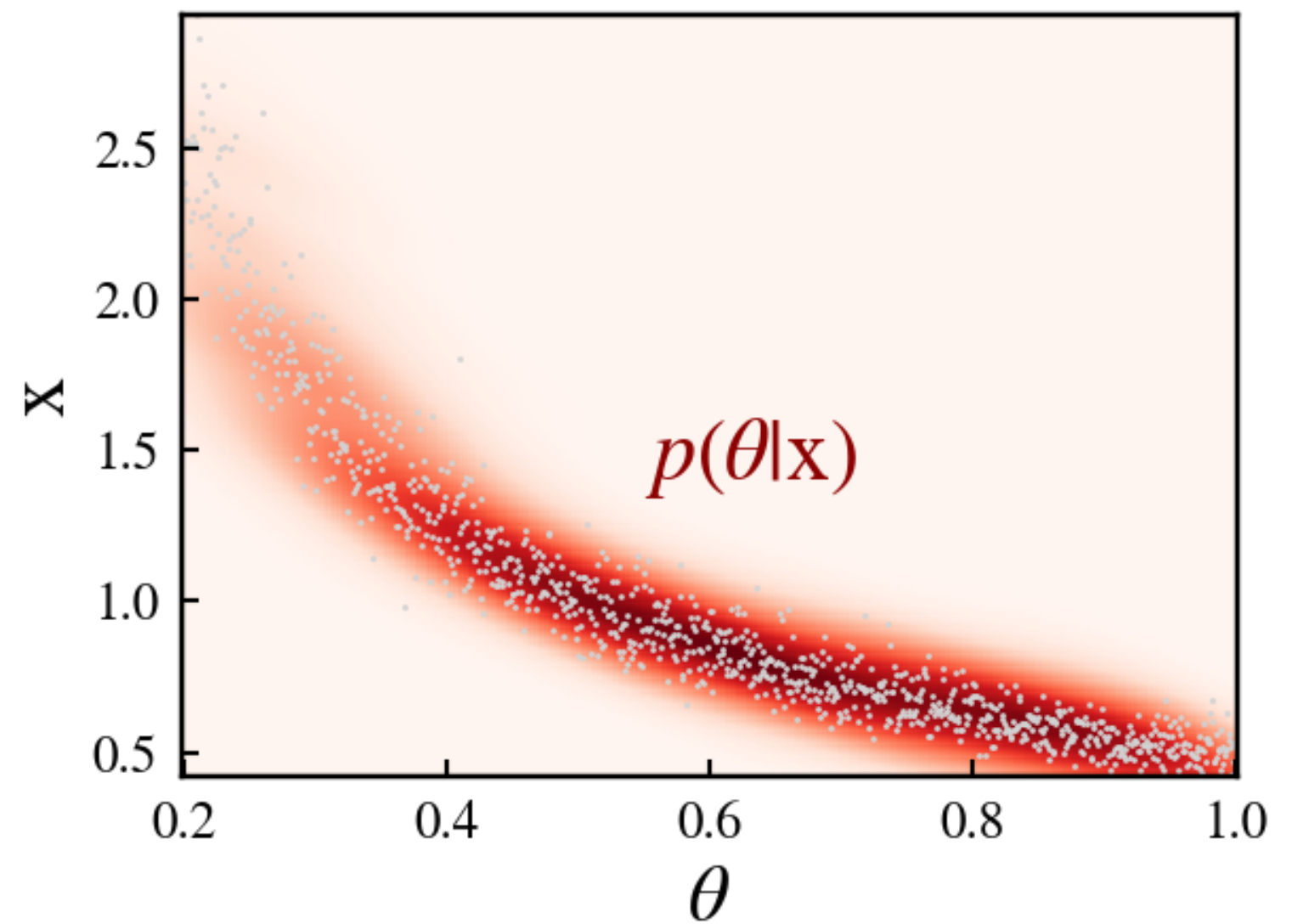


# Simulation-based inference

$$\{(\boldsymbol{\theta}_n, \mathbf{x}_n)\}_{n=1}^{N_{\text{sim}}}$$

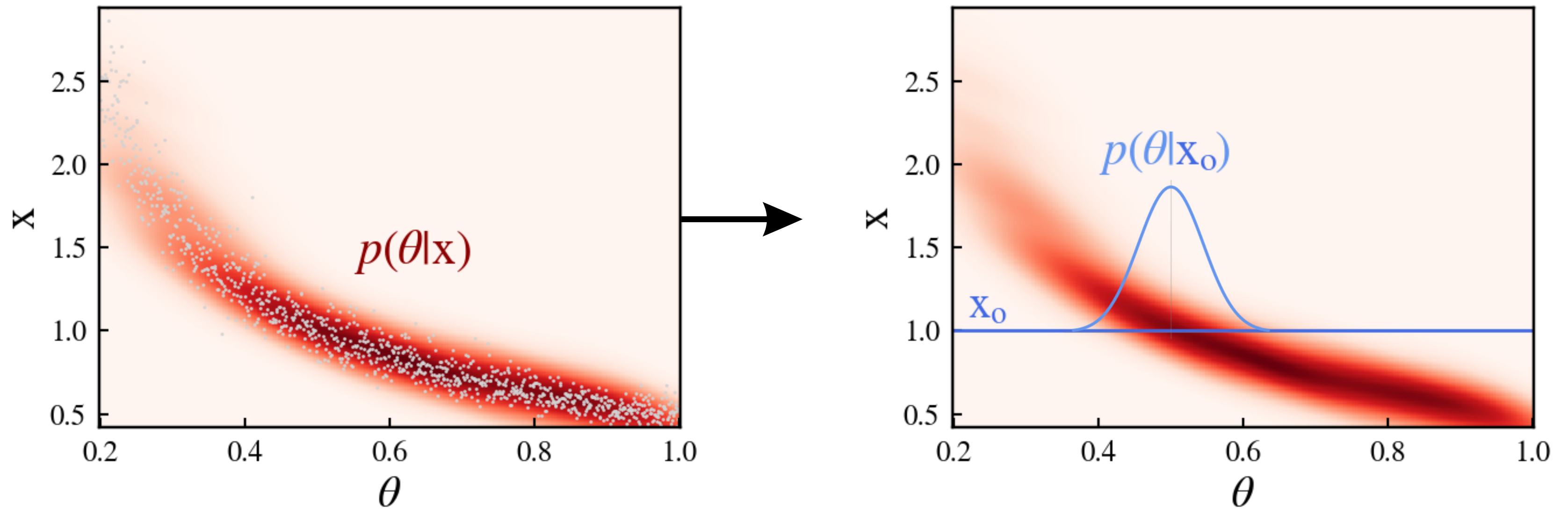


## Neural Posterior Estimation (NPE)



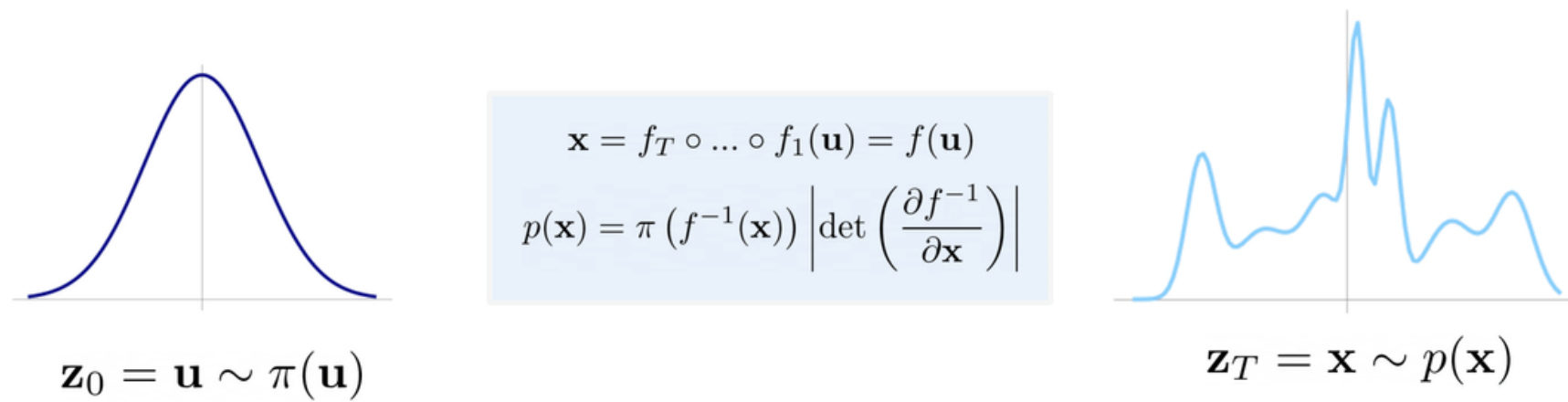
# Simulation-based inference

## Posterior





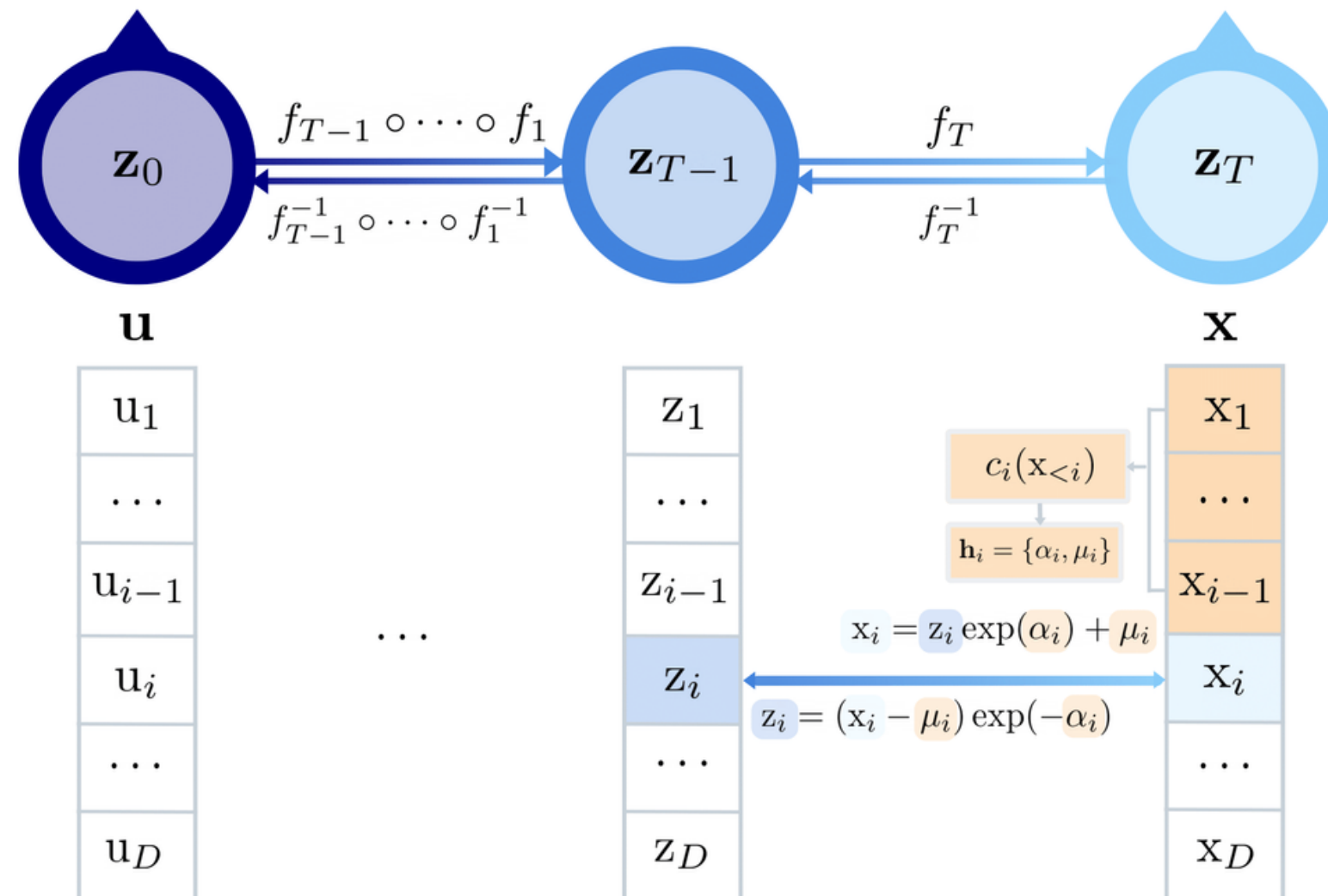
# Normalizing Flows



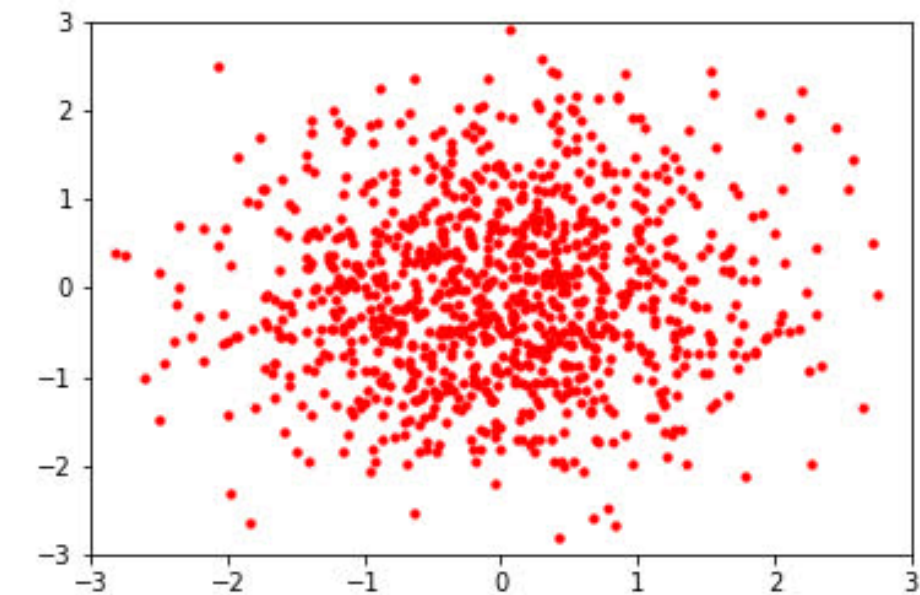
## Neural Density Estimators (NDEs):

- Neural Posterior Estimation (NPE)
- Neural Likelihood Estimation (NLE)

*sbi: A toolkit for simulation-based inference*  
 Tejero-Cantero et al. (2020)



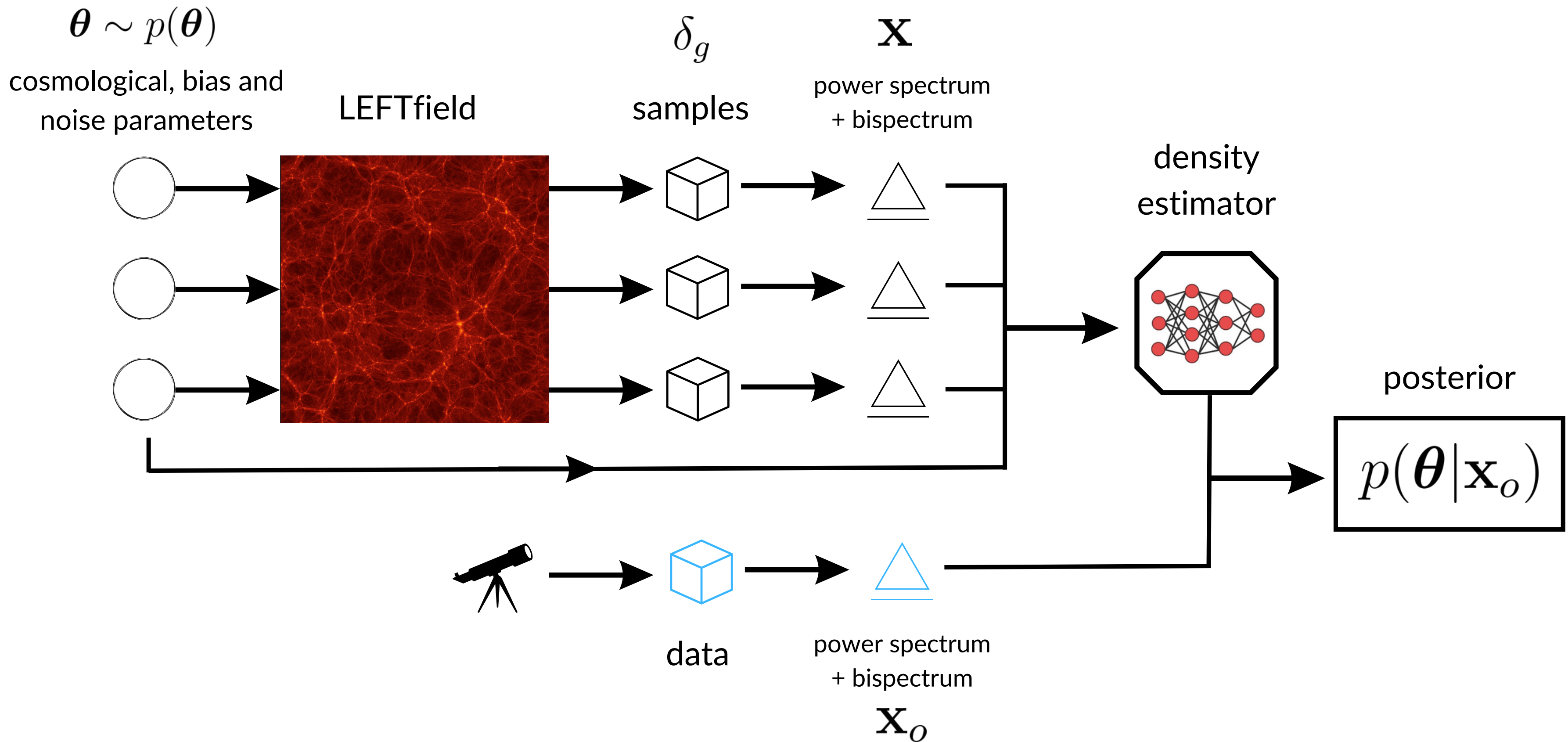
Tucci, Schmidt (2023)



Credits: Miles Cranmer

Learns an **invertible dynamical model**  
for samples of the distribution

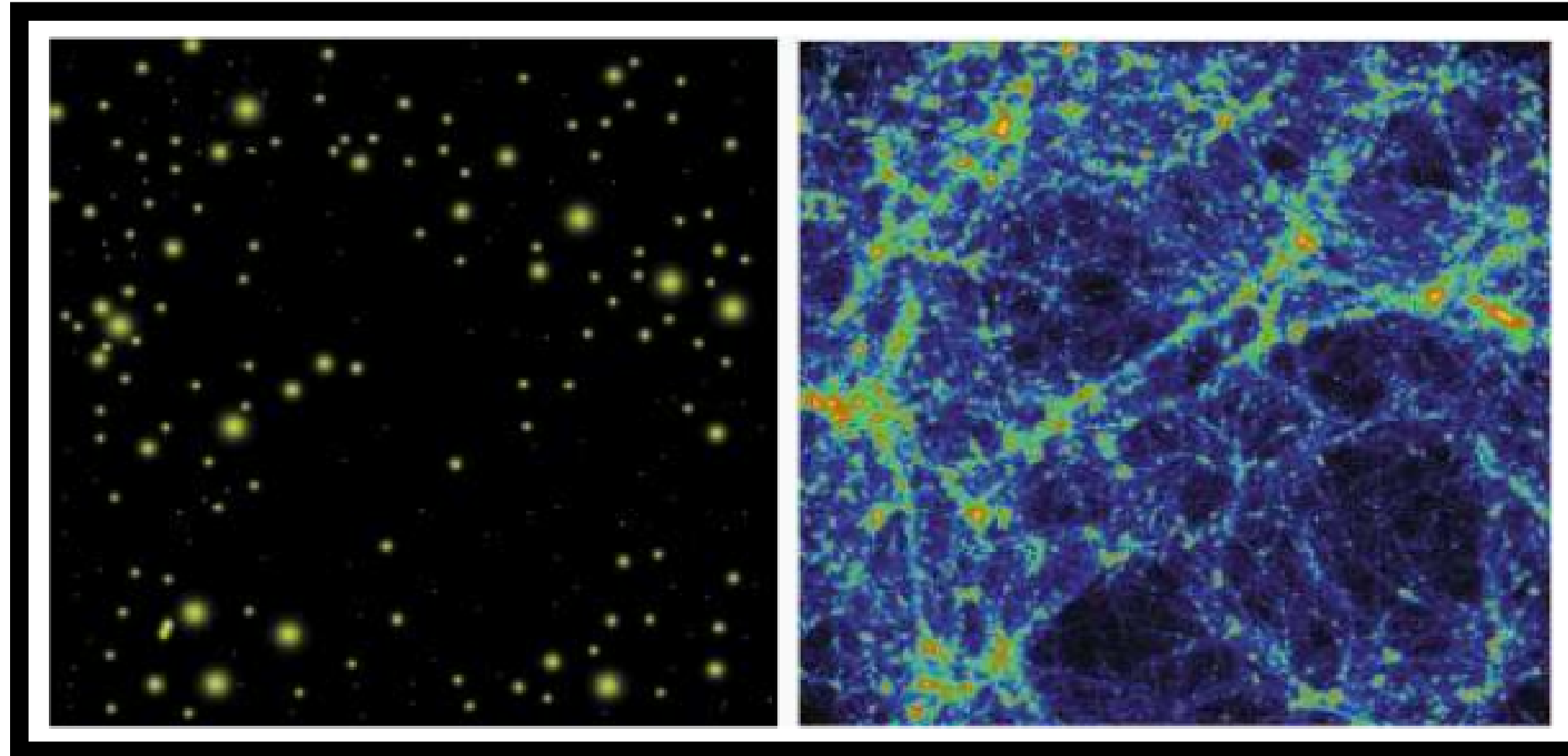
# Simulation-based inference



# The bias expansion

Cooray & Sheth (2002)

Cosmological  
tracers



Matter  
distribution

$$\delta_g(\mathbf{x}, \tau) = \sum_{\mathcal{O}} b_{\mathcal{O}}(\tau) \mathcal{O}(\mathbf{x}, \tau) + \varepsilon(\mathbf{x}, \tau) + \sum_{\mathcal{O}} \varepsilon_{\mathcal{O}}(\mathbf{x}, \tau) \mathcal{O}(\mathbf{x}, \tau)$$

For a review, see:  
Desjacques, Jeong  
& Schmidt (2016)



# Forward model



$$\alpha \equiv \sigma_8 / \sigma_8^{\text{fid}} \quad \hat{\mathbf{s}}(\mathbf{x}) \sim \mathcal{N}(0, 1)$$

$$\delta_{\Lambda}^{(1)}(\mathbf{k}, z) = W_{\Lambda}(k) \sqrt{\alpha^2 P_L(k, z)} \hat{\mathbf{s}}(\mathbf{k})$$

# Forward model



$$\alpha \equiv \sigma_8 / \sigma_8^{\text{fid}} \quad \hat{\mathbf{s}}(\mathbf{x}) \sim \mathcal{N}(0, 1)$$

$$\delta_{\Lambda}^{(1)}(\mathbf{k}, z) = W_{\Lambda}(k) \sqrt{\alpha^2 P_L(k, z)} \hat{\mathbf{s}}(\mathbf{k})$$

Lagrangian Bias Operators

$$\begin{array}{ll} 1^{\text{st}} & \text{tr}[\mathbf{M}_{\Lambda}^{(1)}] \\ 2^{\text{nd}} & \text{tr}[\mathbf{M}_{\Lambda}^{(1)} \mathbf{M}_{\Lambda}^{(1)}], (\text{tr}[\mathbf{M}_{\Lambda}^{(1)}])^2 \end{array}$$

$$1 + \delta(\mathbf{x}, \tau) = |\mathbf{1} + \mathbf{M}(\mathbf{q}, \tau)|^{-1} \quad M_{ij} \equiv \partial_i s_j$$

$$\text{tr}[\mathbf{M}_{\Lambda}^{(1)}] = -\delta_{\Lambda}^{(1)}$$

LPT recursion relations

$$\mathbf{s}^{(n)}$$

$$\delta_{g, \text{det}}^L(\mathbf{q}, \tau) = \sum_{\mathcal{O}^L} b_{\mathcal{O}^L}(\tau) \mathcal{O}^L(\mathbf{q}, \tau)$$

# Forward model



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Lagrangian Bias Operators

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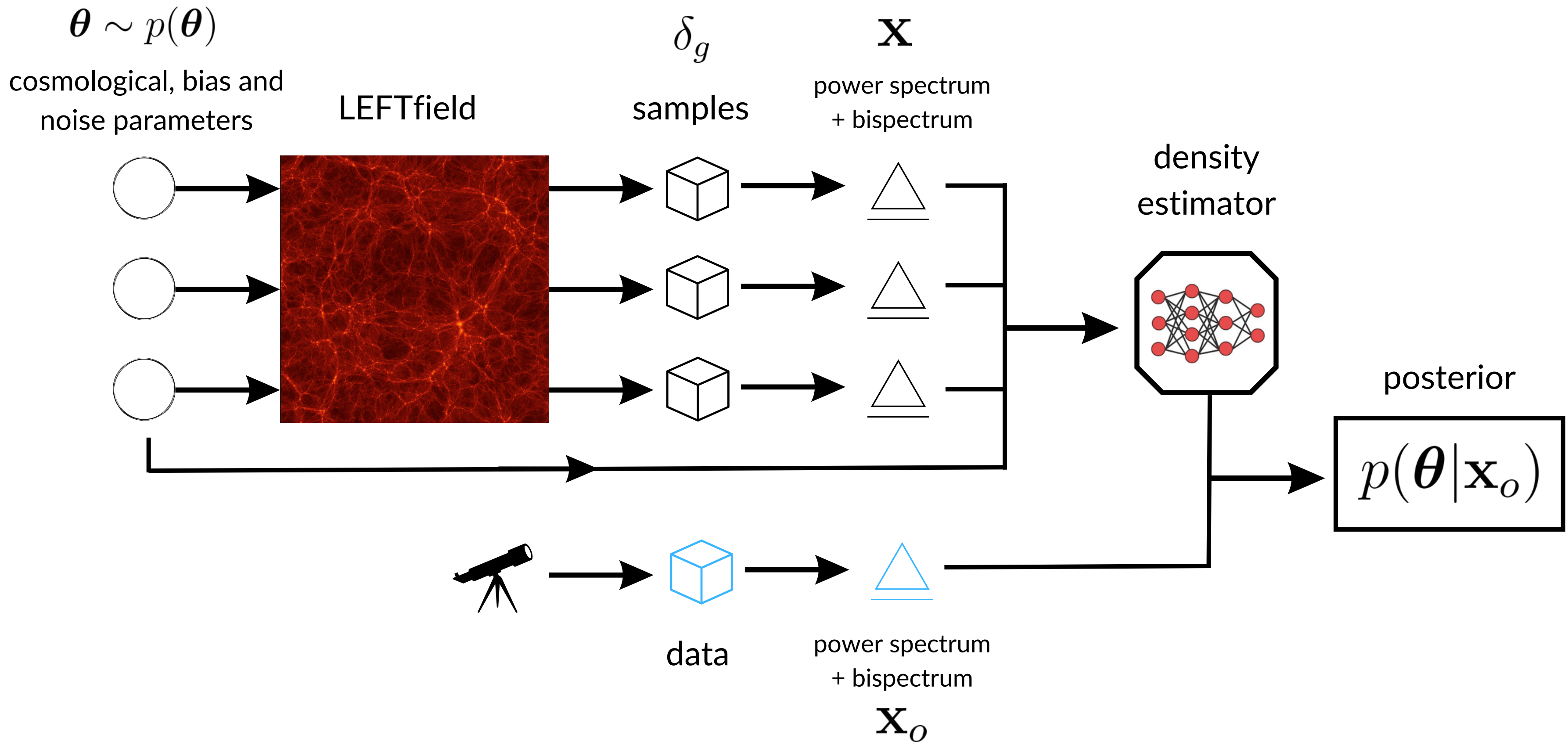
$$\delta_{g,\text{det}}^L(\mathbf{q}, \tau) = \sum_{\mathcal{O}^L} b_{\mathcal{O}^L}(\tau) \mathcal{O}^L(\mathbf{q}, \tau)$$

$$\mathbf{x}(\mathbf{q}, \tau) = \mathbf{q} + \mathbf{s}(\mathbf{q}, \tau)$$

$$\varepsilon \sim \mathcal{N}(0, P_{\varepsilon})$$

$$\delta_g(\mathbf{x}, \tau) = \delta_{g,\text{det}}(\mathbf{x}, \tau) + \varepsilon(\mathbf{x}, \tau) + c_{\varepsilon\delta}(\tau) \varepsilon(\mathbf{x}, \tau) \delta(\mathbf{x}, \tau) + c_{\varepsilon^2}(\tau) \varepsilon^2(\mathbf{x}, \tau)$$

# Simulation-based inference





# Our main goals

Considering an Euclid-like mock survey, we want to answer:

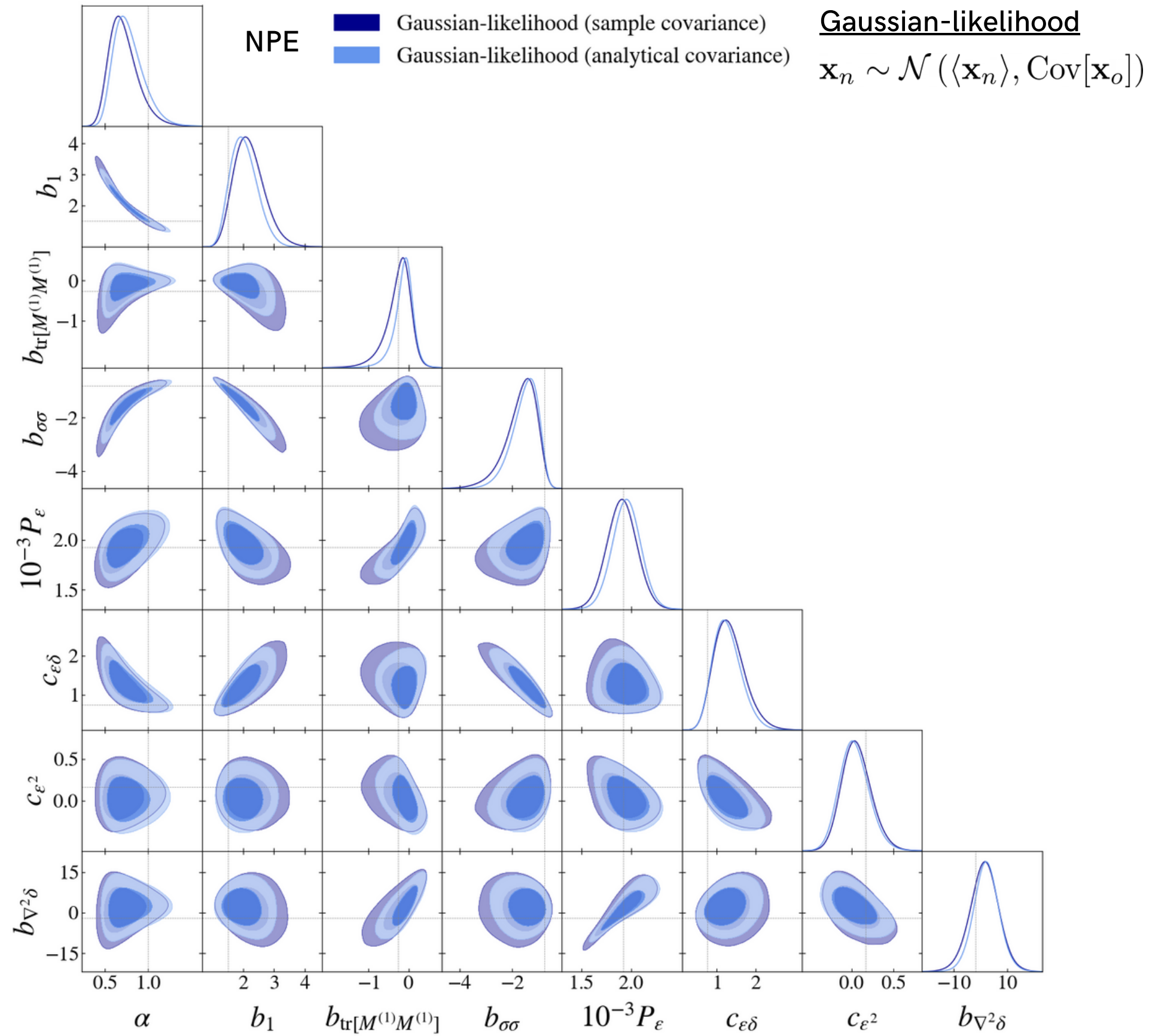
- Does the *non-Gaussianity* of the power-spectrum and bispectrum distributions at *low- $k$*  affect cosmological inference?
- *How many simulations* are needed for posterior estimation?

# Cosmological inference | Euclid configuration

$$N_{\text{sim}} = 10^5$$

$$k_{\text{max}} = \Lambda = 0.1 h\text{Mpc}^{-1}$$

$$D = N_{\text{bin}} + N_{\text{tri}} = 33$$



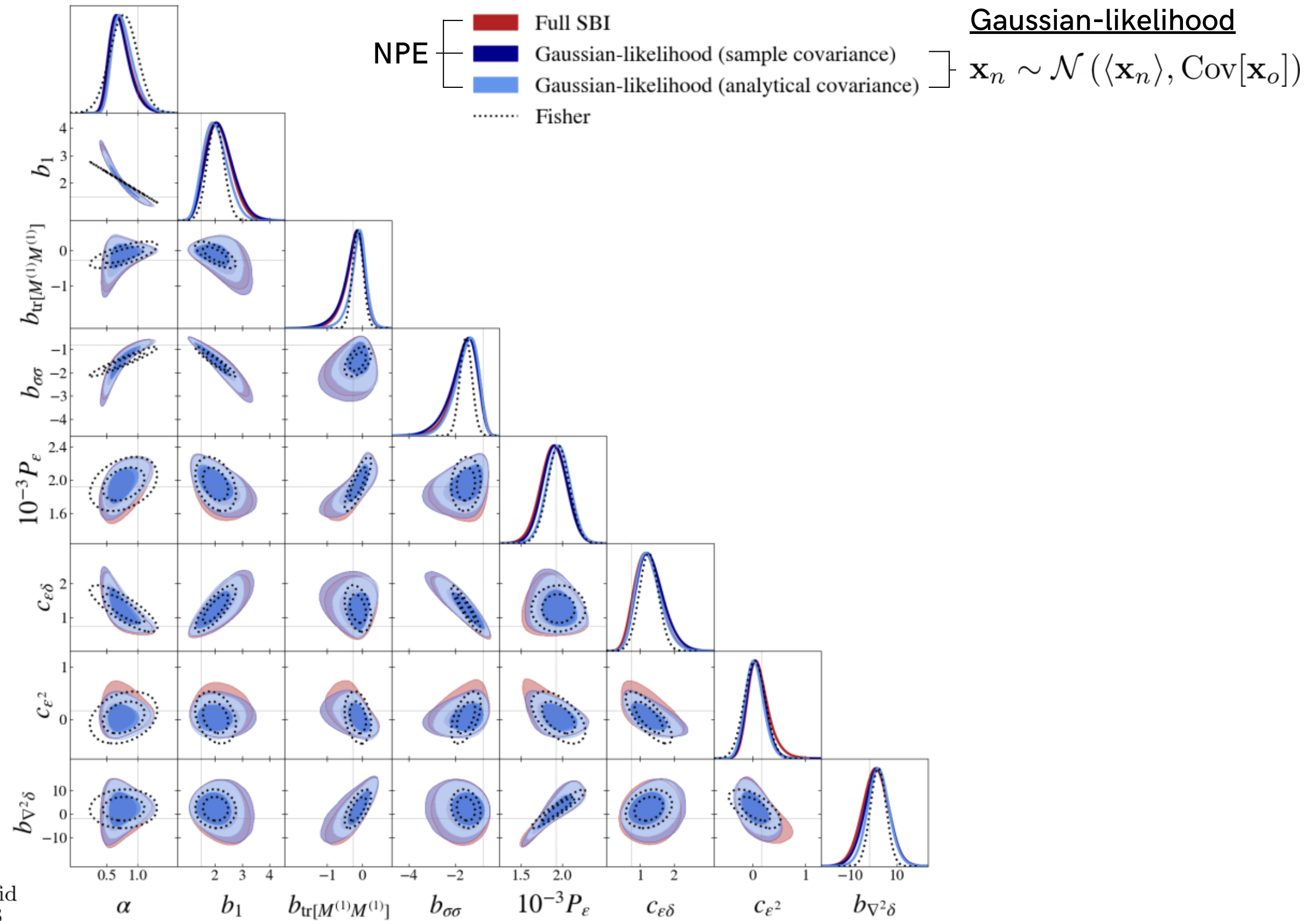
$$\alpha \equiv \sigma_8 / \sigma_8^{\text{fid}}$$

# Cosmological inference | Euclid configuration

$$N_{\text{sim}} = 10^5$$

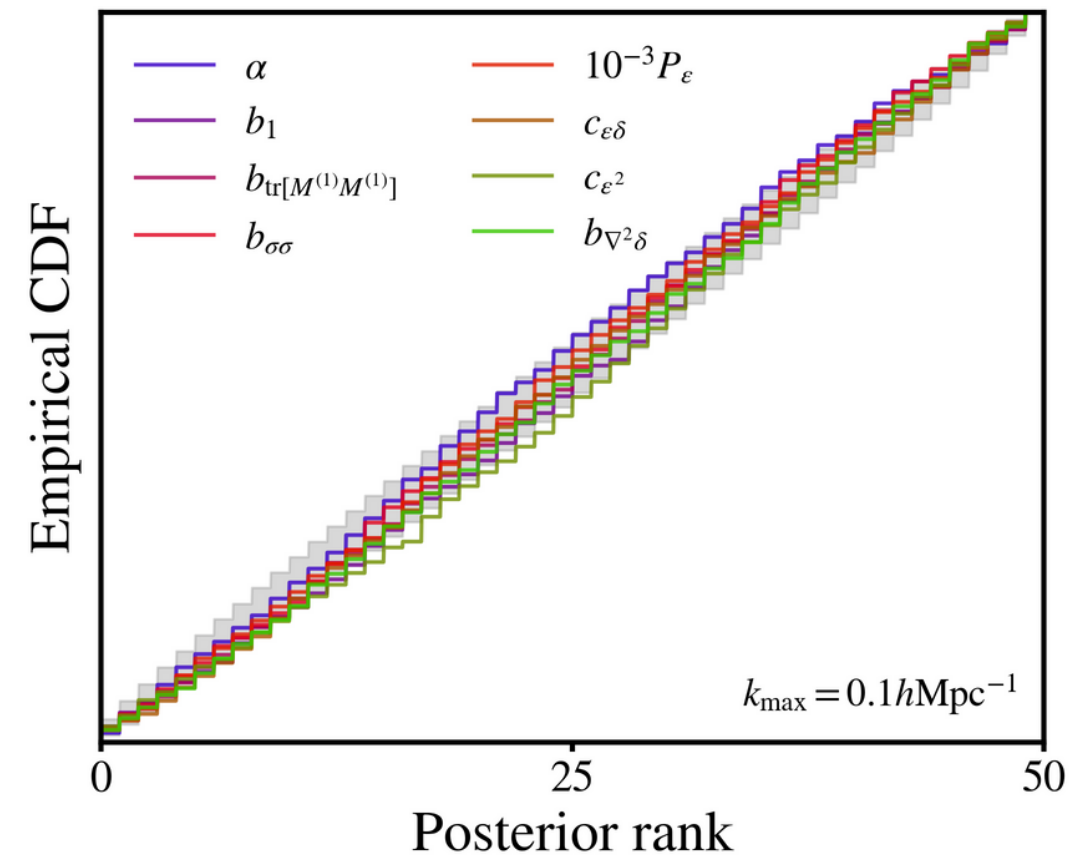
$$k_{\text{max}} = \Lambda = 0.1 h\text{Mpc}^{-1}$$

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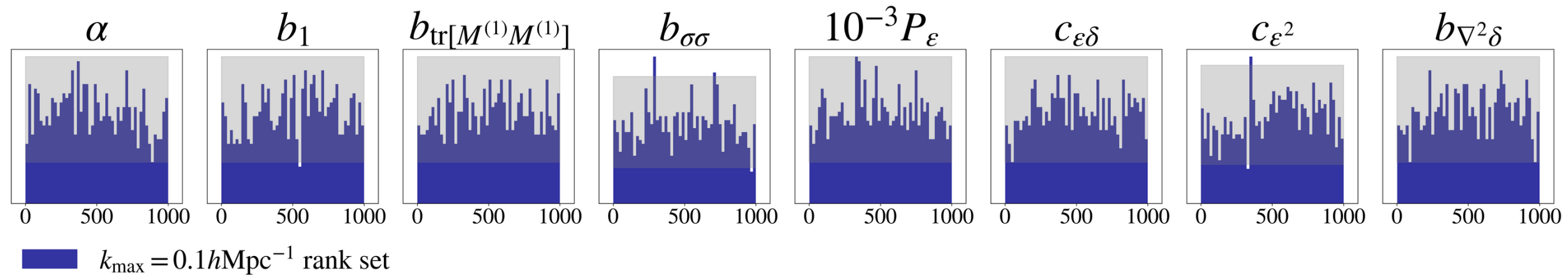
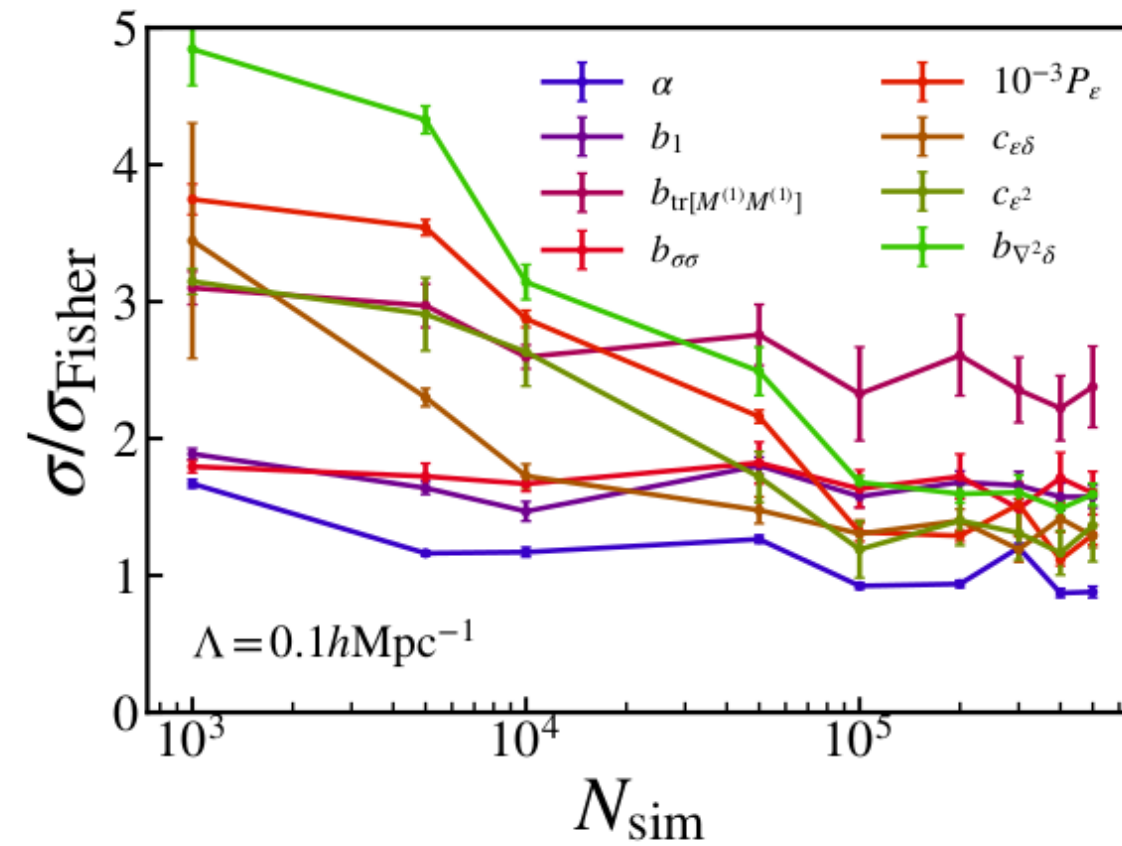


# Posterior diagnostics

Simulation-based calibration



Convergence





# Conclusion & Next Steps

- Simulation-based inference has proven to be a **powerful tool for galaxy clustering** analysis and offers several **advantages** over the likelihood-based approach;
- In the future, we plan to sample more **cosmological parameters**, add more **summary statistics** and improve **observational aspects** of the forward model (masks, systematic effects, etc);
- Comparison of field-level inference with HMC and SBI with summary statistics (P+B).

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**Thank you!**

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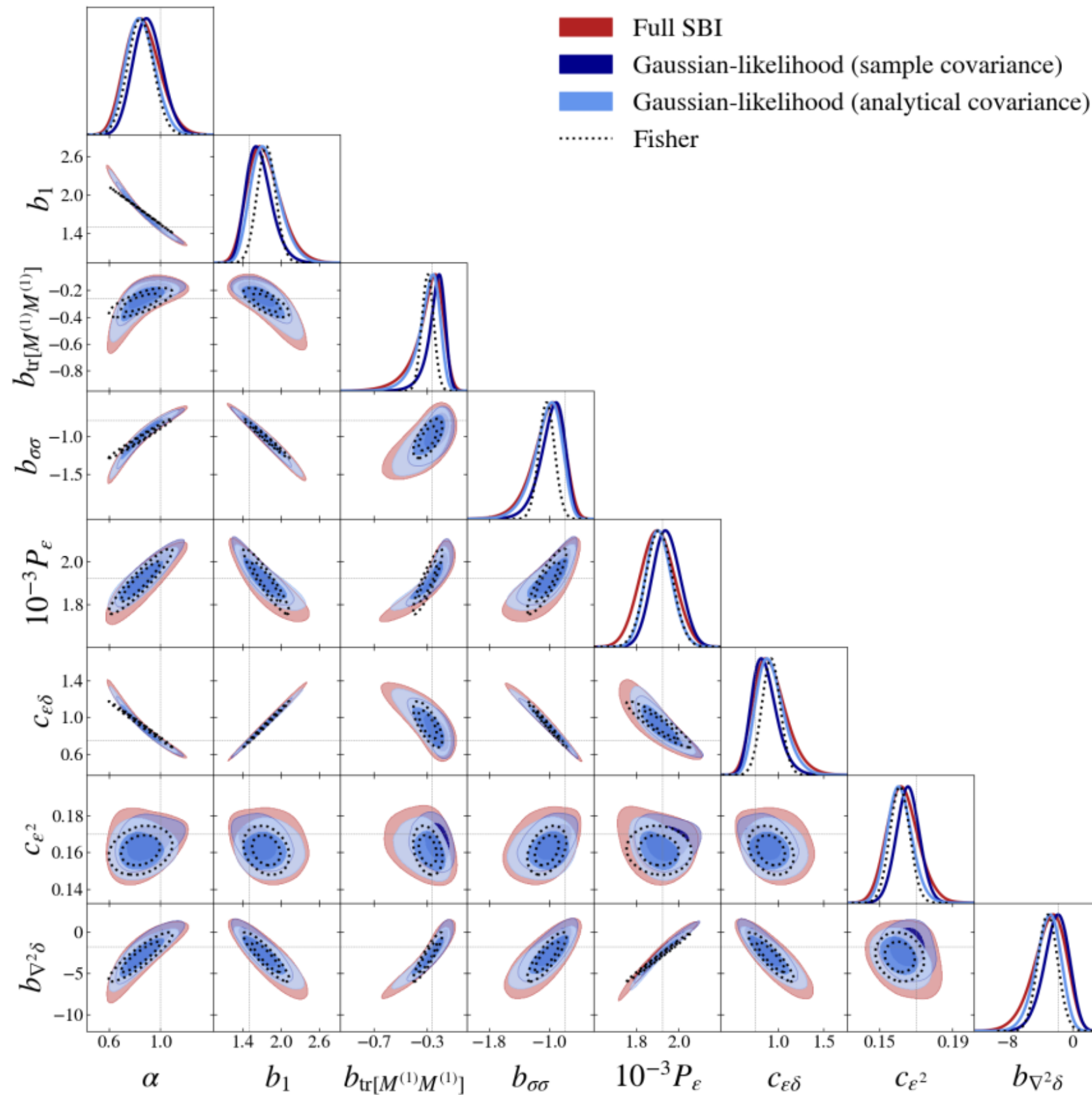
# Cosmological inference | Euclid configuration

Tucci, Schmidt (in prep.)

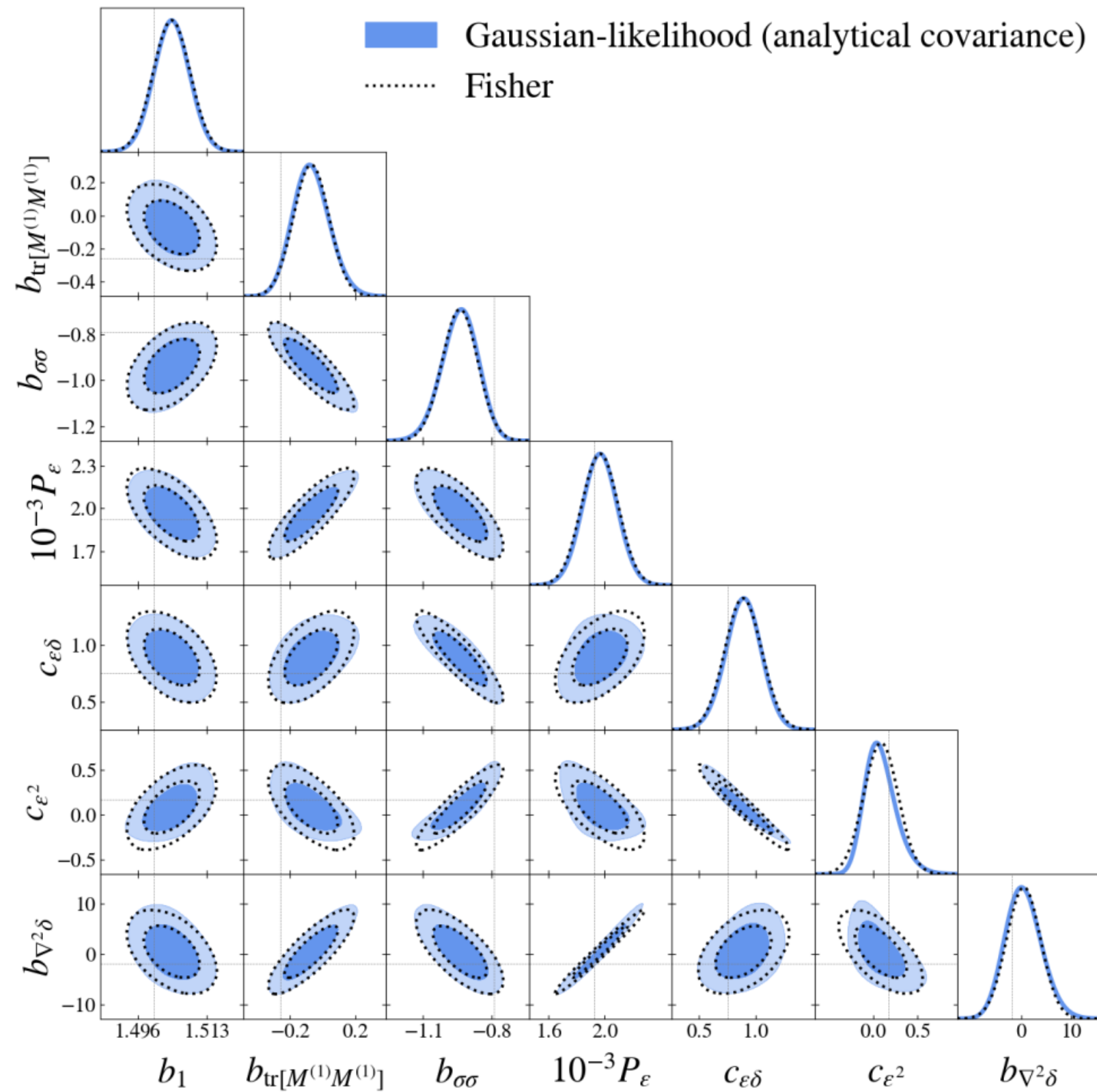
$$N_{\text{sim}} = 10^5$$

$$k_{\text{max}} = \Lambda = 0.2h\text{Mpc}^{-1}$$

$$D = 49$$



# Euclid | Gaussian-likelihood

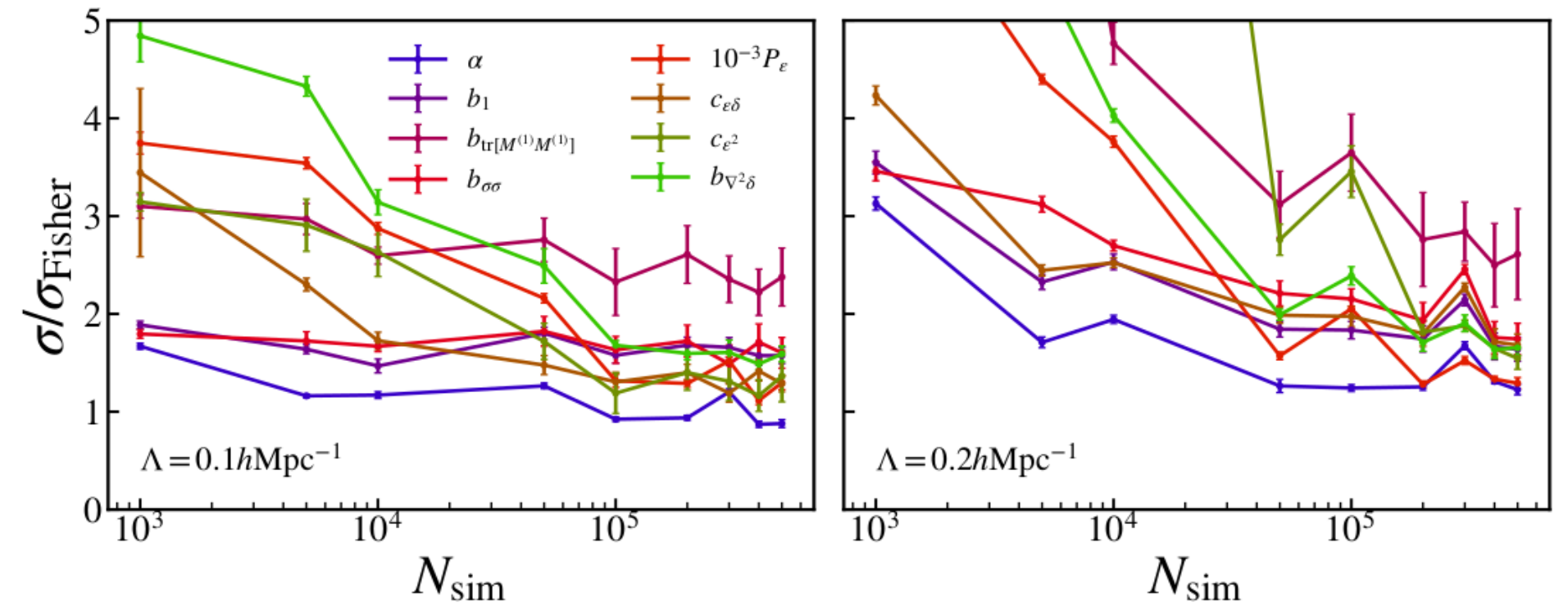
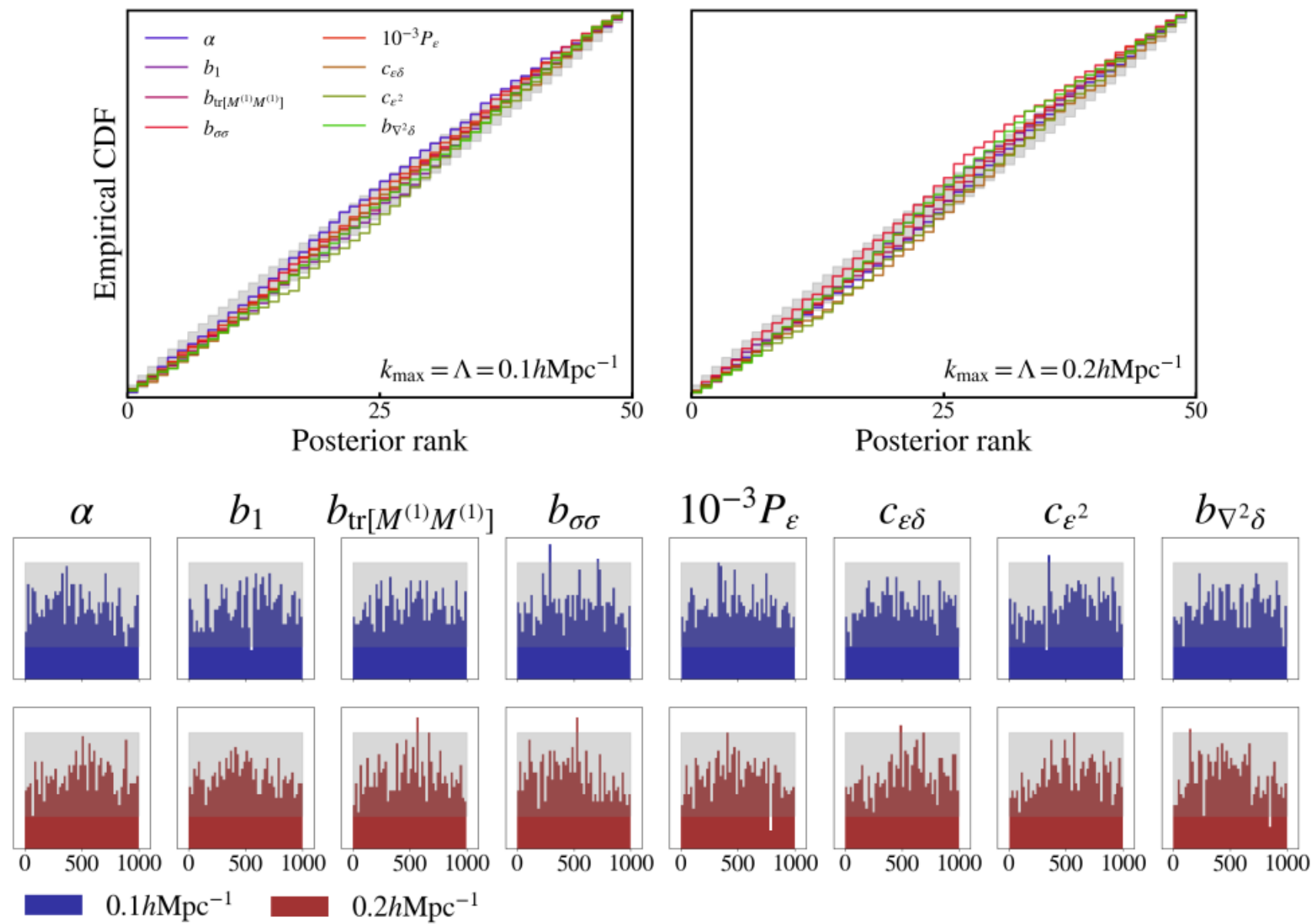


$$\mathbf{x}_n \sim \mathcal{N}(\langle \mathbf{x}_n \rangle, \text{Cov}[\mathbf{x}_o])$$

$$N_{\text{sim}} = 10^5$$

$$k_{\text{max}} = \Lambda = 0.1 h \text{Mpc}^{-1}$$

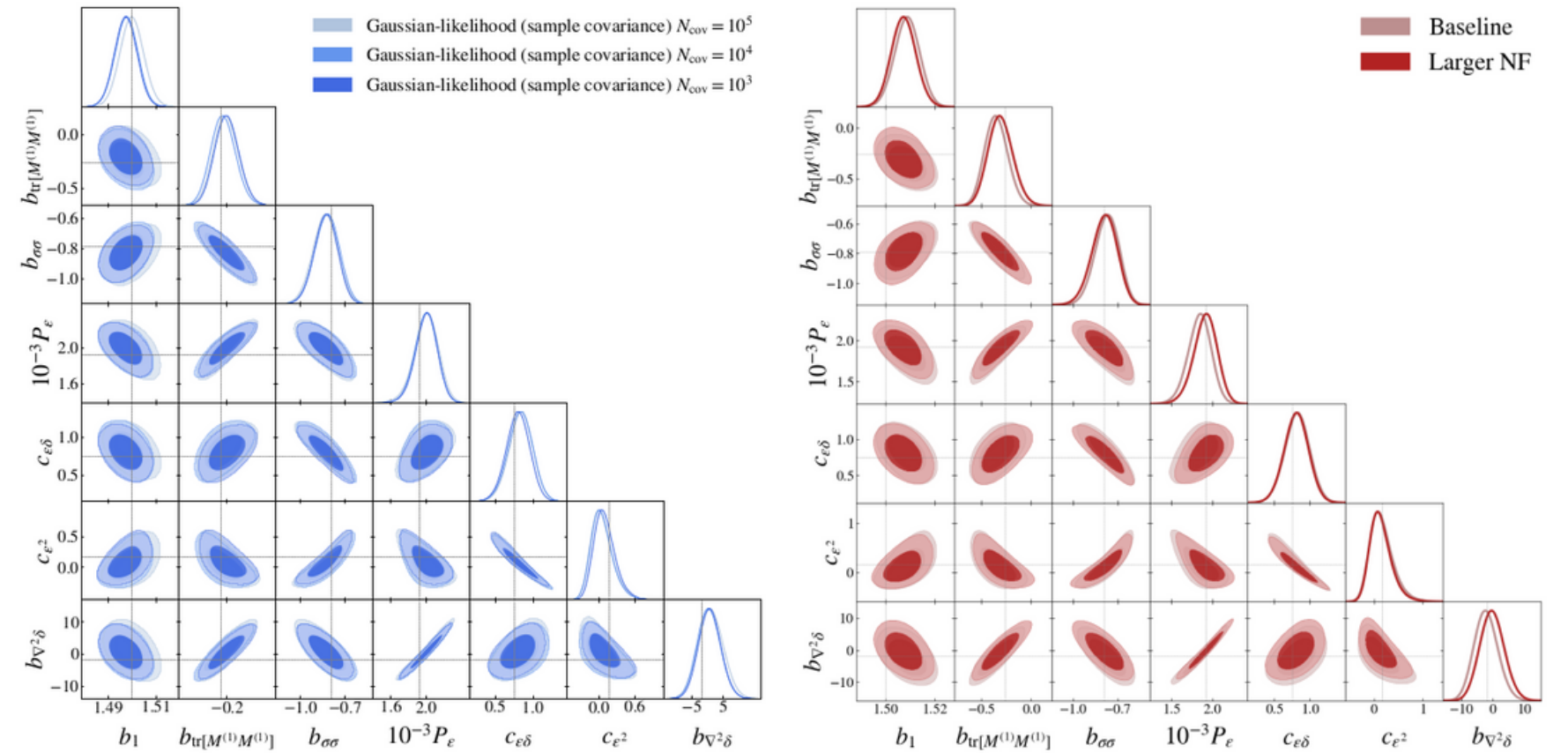
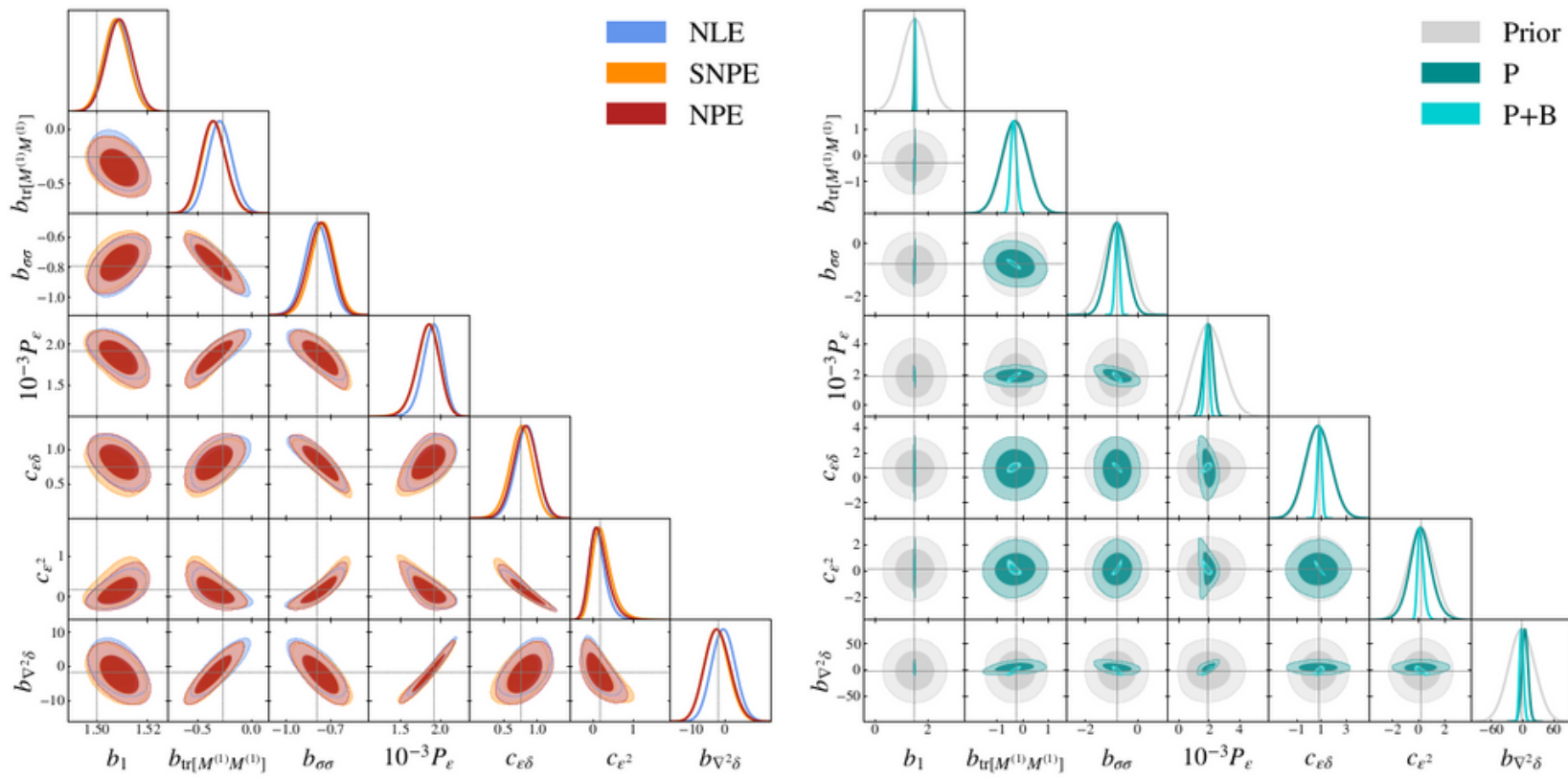
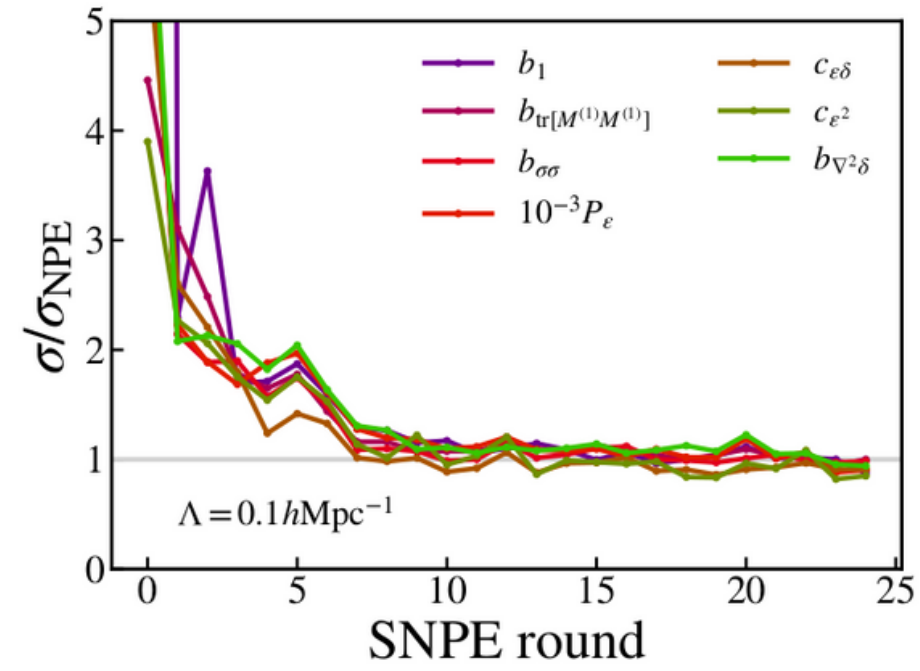
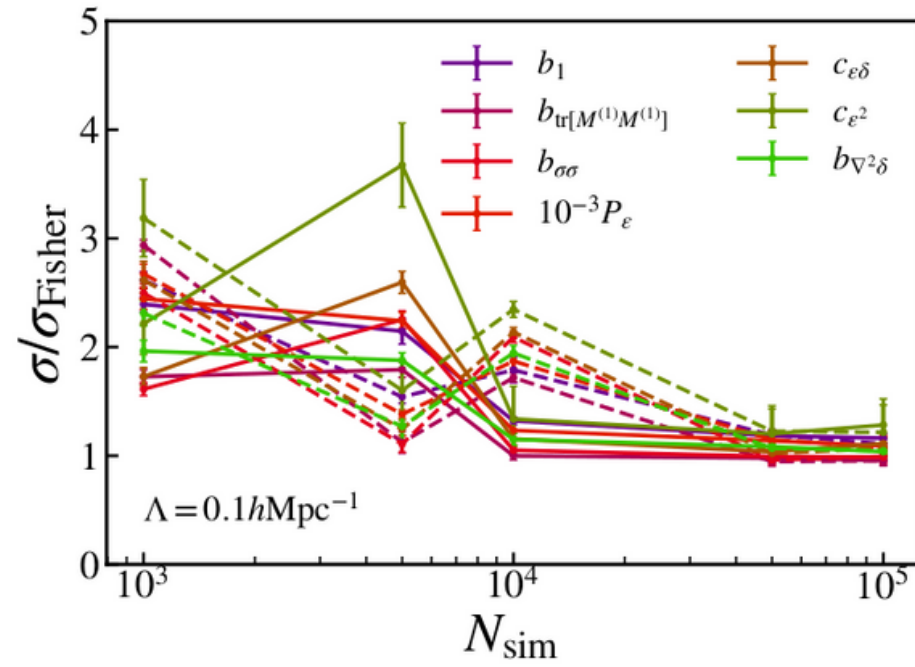
# Posterior diagnostics



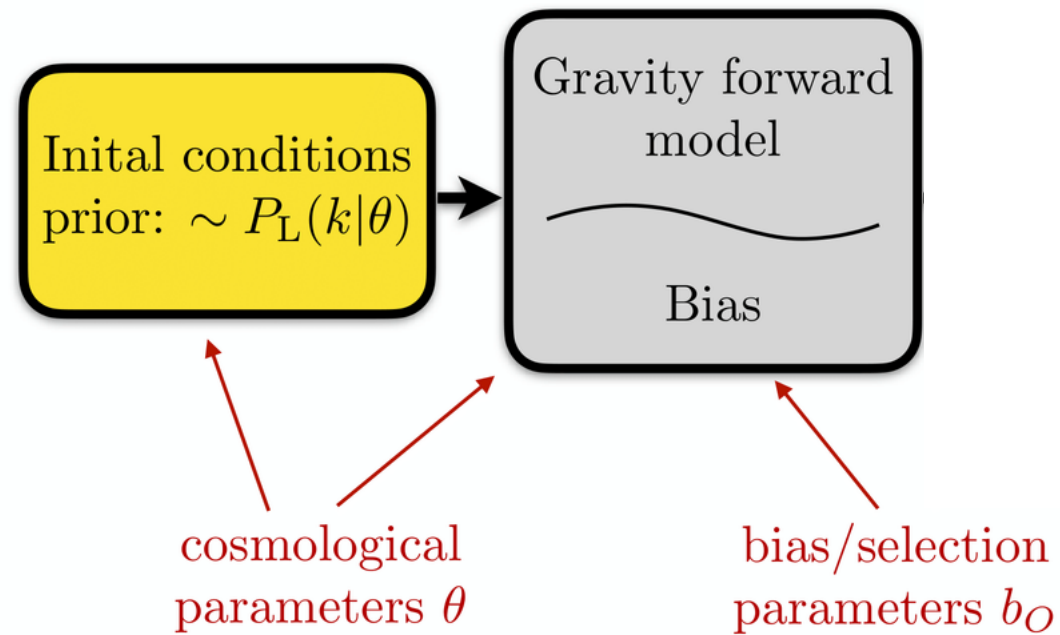


# Inference tests

PB normalization



## EFTofLSS based approach



$$\varepsilon \sim \mathcal{N}(0, \sigma_\varepsilon)$$

### Perturbation Theory

$$\langle \delta_g(k_1) \delta_g(k_2) \delta_g(k_3) \rangle'_{\text{stoch}}{}^{\text{LO}} = B_\varepsilon + 2b_1 P_{\varepsilon\varepsilon\delta} (P_m(k_1) + 2 \text{ perm.})$$

### Forward Model

$$\langle \delta_g(k_1) \delta_g(k_2) \delta_g(k_3) \rangle'_{\text{stoch}}{}^{\text{LO}} = 6c_\varepsilon^{\text{NG}} P_\varepsilon^2 + 2b_1 P_\varepsilon \sigma_{\varepsilon\delta} (P_m(k_1) + 2 \text{ perm.})$$

An n-th order Lagrangian Forward Model for Large-Scale Structure  
Fabian Schmidt (2021)

$$\delta_g(\mathbf{x}, \tau) = \delta_{g,\text{det}}(\mathbf{x}, \tau) + \varepsilon(\mathbf{x}, \tau) + \sigma_{\varepsilon\delta}(\tau) \varepsilon(\mathbf{x}, \tau) \delta(\mathbf{x}, \tau) + c_\varepsilon^{\text{NG}}(\tau) \varepsilon^2(\mathbf{x}, \tau)$$