Bootstrapping the Large Scale Structure of the Universe

Marco Marinucci
Technion Institute
marinucci@campus.technion.ac.il

Bias parameters & Cosmology

Approach: model independent, just assumes Equivalence Principle

Consistency relations: **MM,** Nishimichi T., Pietroni M., Phys. Rev. D100 (2019); **MM**, Nishimichi T., Pietroni M., JCAP 07 (2020)

LSS bootstrap & beyond: D'Amico G., **MM,** Pietroni M., Vernizzi F., JCAP 10 (2021); Amendola L., **MM**, Pietroni M., Quartin M., arXiv:2307.02117

Equivalence principle

Consistency relations

Exact equalities among correlation functions of different order
(Peloso M., Pietroni M., JCAP 2013,
Kehagias A., Riotto A., Nucl.Phys. 2013,
Creminelli P., Noreña J., Simonović M., Vernizzi F., JCAP 2013)

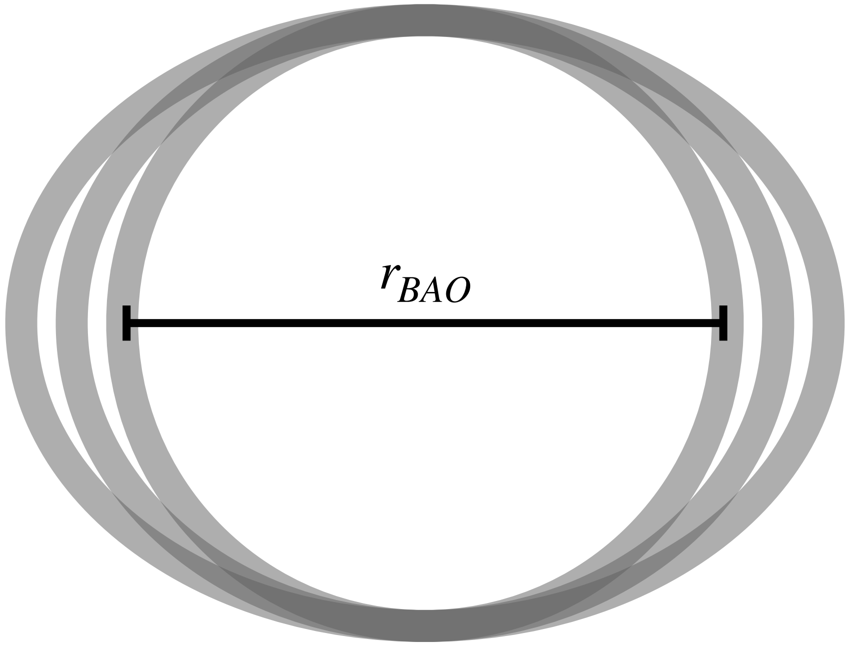
$$\langle \delta(\mathbf{x}_1, \tau_1) \dots \delta(\mathbf{x}_n, \tau_n) \,|\, \Phi_L \rangle = \langle \delta(\tilde{\mathbf{x}}_1, \tilde{\tau}_1) \dots \delta(\tilde{\mathbf{x}}_n, \tilde{\tau}_n) \rangle \, .$$

Effect of long modes on linear scales
$\longrightarrow$ **Equivalence Principle** (or Galilean Invariance)

$$\langle \delta_m(\mathbf{q}, \tau) \delta_g(\mathbf{k}_1, \tau_1) \dots \delta_g(\mathbf{k}_n, \tau_n) \rangle'_{\mathbf{q}\to 0} = - P_L(q, \tau) \sum_{i=1}^{n} \frac{D_+(\tau_i)}{D(\tau)} \frac{\mathbf{q} \cdot \mathbf{k}_i}{q^2} \langle \delta_g(\mathbf{k}_1, \tau_1) \dots \delta_g(\mathbf{k}_n, \tau_n) \rangle'$$

CR and BAO

- **MM**, Nishimichi T., Pietroni M., Phys. Rev. D100 (2019)
- **MM**, Nishimichi T., Pietroni M., JCAP 07 (2020)



CR and BAO

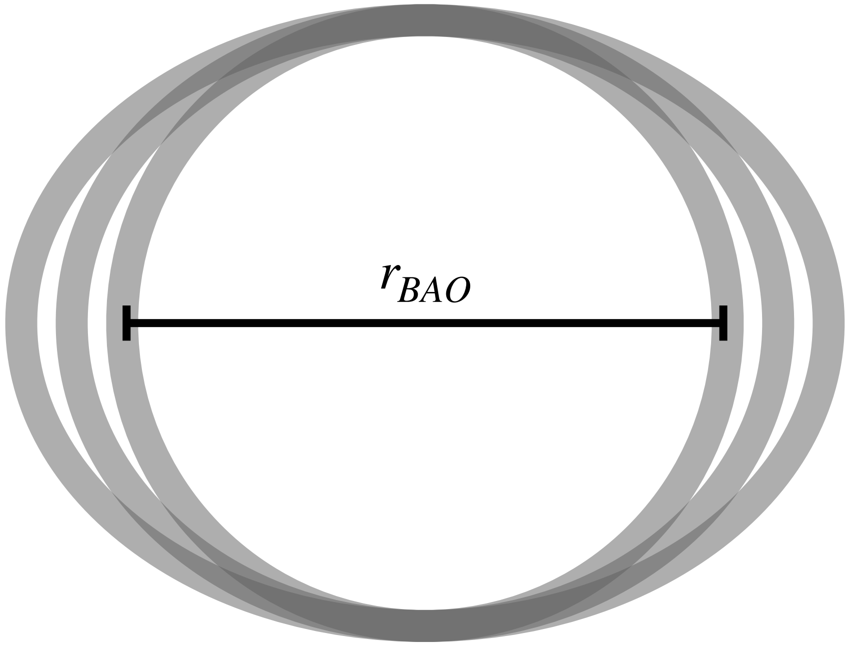
- **MM**, Nishimichi T., Pietroni M., Phys. Rev. D100 (2019)
- **MM**, Nishimichi T., Pietroni M., JCAP 07 (2020)

Equal-time squeezed limit

$$\lim_{q/k\to 0} \frac{B_{\alpha\alpha\alpha}(q, k_+ . k_-)}{P_{\alpha\alpha}(q) P_{\alpha\alpha}(k)} = -\frac{\mu^2}{b_\alpha(q)} \frac{d \log P_{\alpha\alpha}(k)}{d \log k} + O\left(\left(\frac{q}{k}\right)^0\right)$$

Unchanged by nonlinearities!

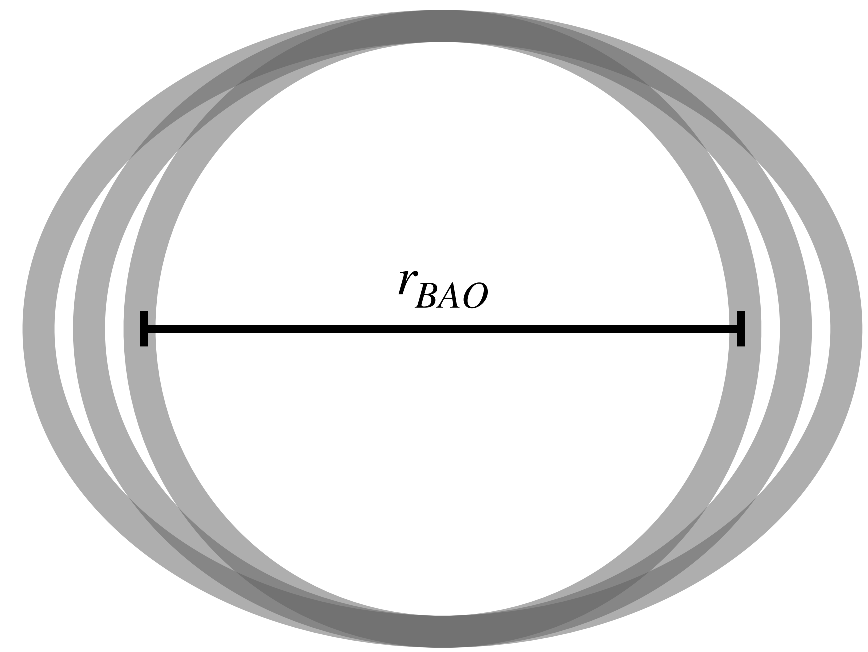
Peloso M. and Pietroni M., JCAP (2013)
Kehagias A. and Riotto A., Nucl. Phys. (2013)
Baldauf T. et al., Phys.Rev.D (2015)



$\mu = \hat{\mathbf{k}} \cdot \hat{\mathbf{q}}$

CR and BAO

- **MM**, Nishimichi T., Pietroni M., Phys. Rev. D100 (2019)
- **MM**, Nishimichi T., Pietroni M., JCAP 07 (2020)



$$\mu = \hat{\mathbf{k}} \cdot \hat{\mathbf{q}}$$

Equal-time squeezed limit

$$\lim_{q/k \to 0} \frac{B_{\alpha\alpha\alpha}(q, k_+ . k_-)}{P_{\alpha\alpha}(q) P_{\alpha\alpha}(k)} = -\frac{\mu^2}{b_\alpha(q)} \frac{d \log P_{\alpha\alpha}(k)}{d \log k} + O\left(\left(\frac{q}{k}\right)^0\right)$$

Unchanged by nonlinearities!

Peloso M. and Pietroni M., JCAP (2013)
Kehagias A. and Riotto A., Nucl. Phys. (2013)
Baldauf T. et al., Phys.Rev.D (2015)

In presence of a scale like the BAO, the oscillating part of the derivative is enhanced by a $\sim k\, r_{BAO}$ factor, we can isolate it to verify CR and measure bias

CR and BAO

N-body simulations: **redshift space w/ biased tracers,** MM+ (2020)

Multipoles + Kaiser

$$\lim_{q/k\to 0} \frac{B_t^{(0)}(q,k)}{P_t^{(0)}(q)P_t^{(0)}(k)} = -\left[\frac{1}{3b_t} + \frac{b_t-1}{9b_t}\beta_t \frac{1+\frac{3}{5}\beta_t}{1+\frac{2}{3}\beta_t+\frac{1}{5}\beta_t^2}\right]\frac{\partial \log P_t^{(0)}(k)}{\partial \log k} + ..$$

$$\lim_{q/k\to 0} \frac{B_t^{(l_k=2)}(q,k)}{P_t^{(0)}(q)P_t^{(0)}(k)} = -\frac{2\beta_t}{45b_t}\frac{2+b_t(5+3\beta_t)}{1+\frac{2}{3}\beta_t+\frac{1}{5}\beta_t^2}\frac{\partial \log P_t^{(0)}(k)}{\partial \log k} + \ldots$$

$$\lim_{k\to 0} \frac{P^{(2)}(k)}{P_t^{(0)}(k)} = \frac{4\beta_t}{21}\frac{7+3\beta_t}{1+\frac{2}{3}\beta_t+\frac{1}{5}\beta_t^2}$$

Angles

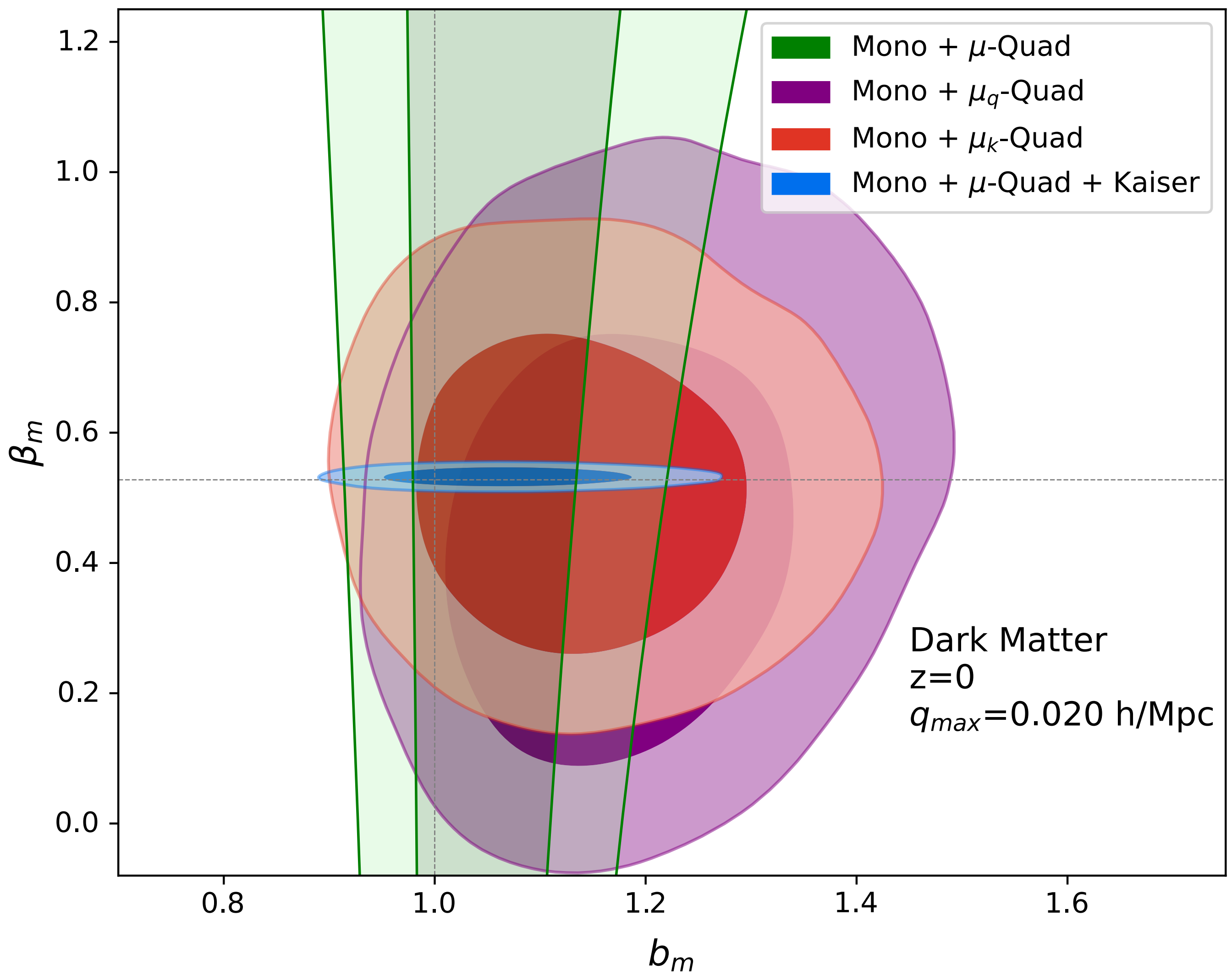
Growth rate $f \equiv \dfrac{d\log D}{d\log a}$

$$\beta_t \equiv \frac{f}{b_t}$$

$$\mu \equiv \hat{\mathbf{k}}\cdot\hat{\mathbf{q}}$$

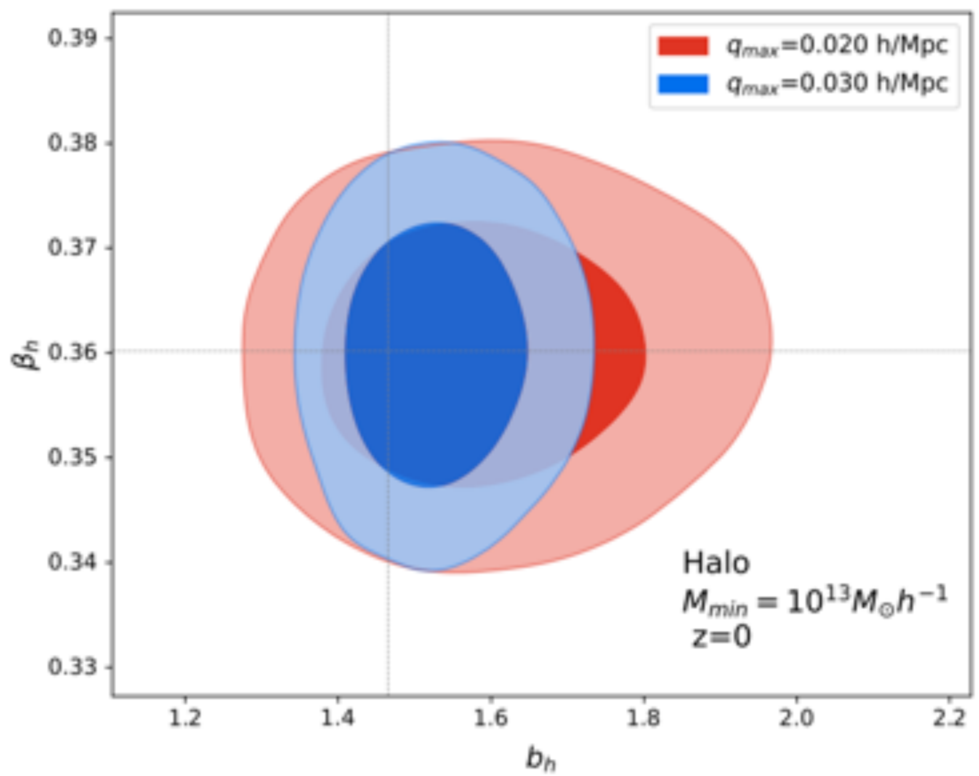
$$\mu_k \equiv \hat{\mathbf{k}}\cdot\hat{\mathbf{z}}$$

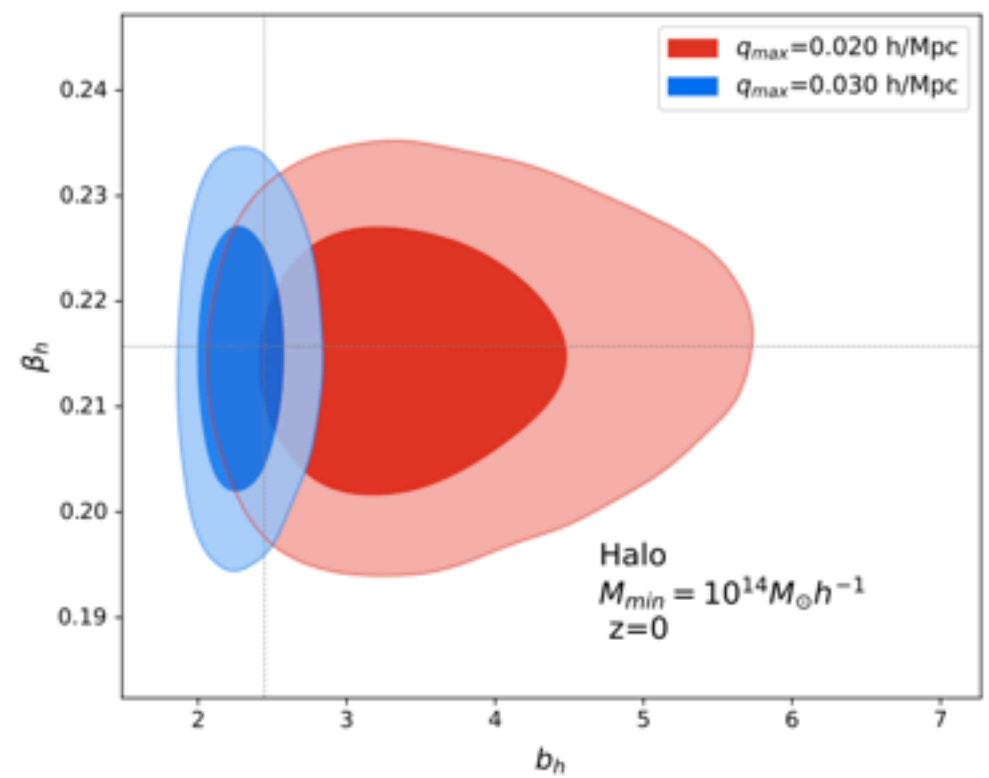
$$\mu_q \equiv \hat{\mathbf{q}}\cdot\hat{\mathbf{z}}$$



CR and BAO

N-body simulations: **redshift space w/ biased tracers,** MM+ (2020)





$M_{\rm min} = 10^{13} h^{-1} M_{\odot}$ $z=0$				
q_{max} (h/Mpc)	b_h	b_h^{fid}	$f = \beta_h b_h$	f^{fid}
0.020	$1.58^{+0.15}_{-0.13}$	1.47	$0.57^{+0.06}_{-0.06}$	0.528
0.030	$1.53^{+0.08}_{-0.08}$	1.47	$0.55^{+0.04}_{-0.04}$	0.538
$M_{\rm min} = 10^{13} h^{-1} M_{\odot}$ $z=1$				
q_{max} (h/Mpc)	b_h	b_h^{fid}	$f = \beta_h b_h$	f^{fid}
0.020	$2.85^{+0.39}_{-0.32}$	2.686	$0.93^{+0.14}_{-0.14}$	0.877
0.030	$2.58^{+0.17}_{-0.16}$	2.686	$0.82^{+0.07}_{-0.07}$	0.877
$M_{\rm min} = 10^{14} h^{-1} M_{\odot}$ $z=0$				
q_{max} (h/Mpc)	b_h	b_h^{fid}	$f = \beta_h b_h$	f^{fid}
0.020	$3.40^{+0.83}_{-0.61}$	2.446	$0.73^{+0.18}_{-0.18}$	0.528
0.030	$2.29^{+0.21}_{-0.18}$	2.446	$0.49^{+0.06}_{-0.06}$	0.528

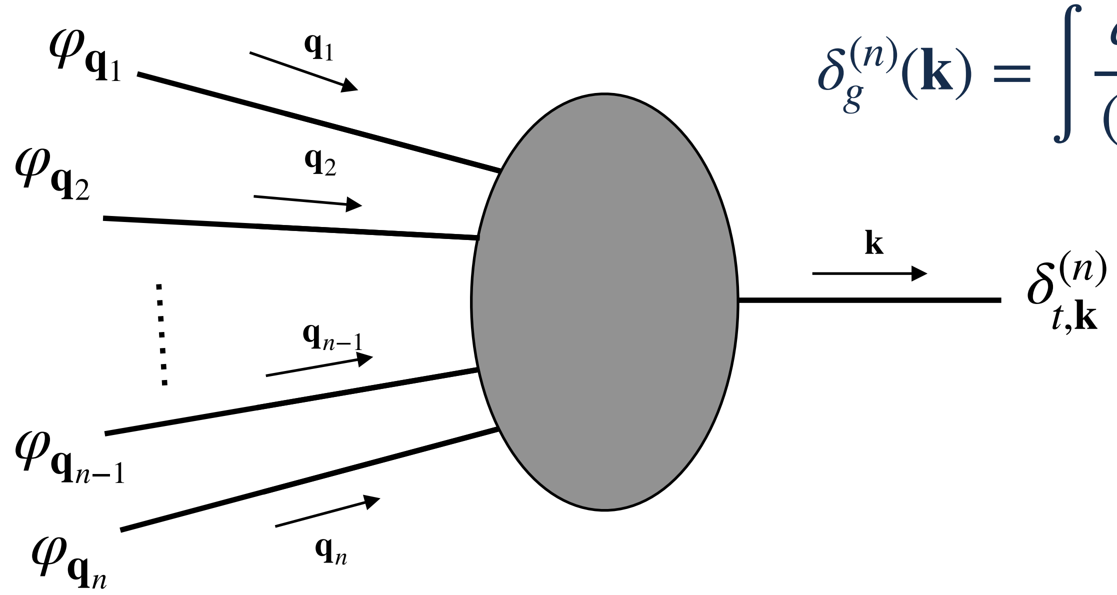
$5-7\,\%$ accuracy on b_1 and f with a model-independent approach!

LSS Bootstrap

- D'Amico G., **MM**, Pietroni M., Vernizzi F., JCAP 10 (2021)

LSS Bootstrap

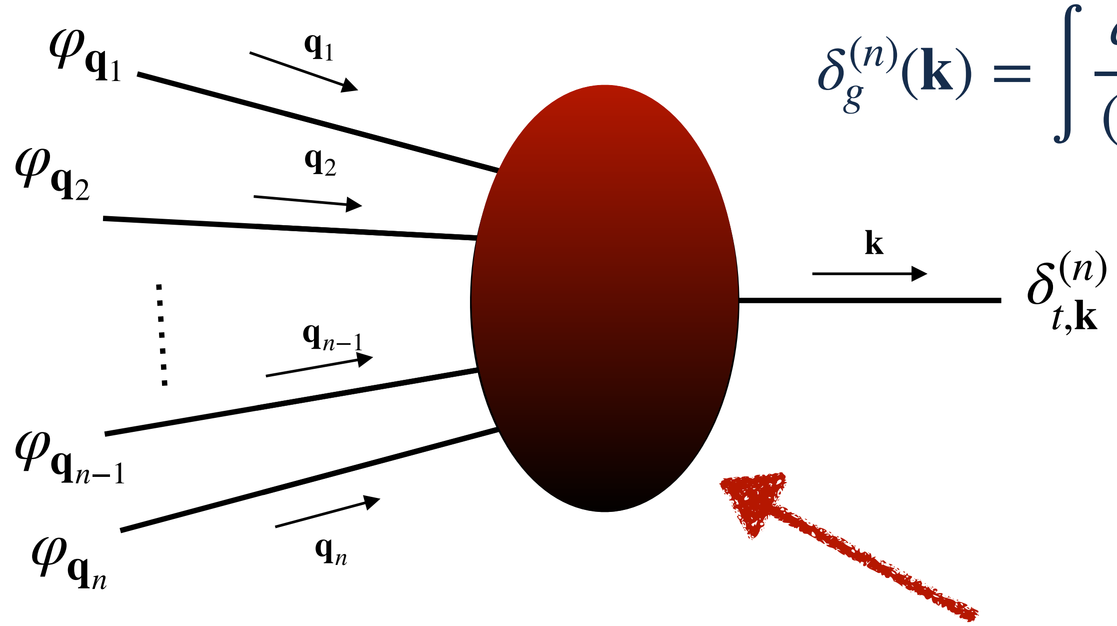
- D'Amico G., **MM**, Pietroni M., Vernizzi F., JCAP 10 (2021)



$$\delta_g^{(n)}(\mathbf{k}) = \int \frac{d^3q_1}{(2\pi)^3} \ldots \int \frac{d^3q_n}{(2\pi)^3} K_n\left(\mathbf{q}_1, \ldots, \mathbf{q}_n\right) \delta^{(1)}(\mathbf{q_1}) \ldots \delta^{(1)}(\mathbf{q_n})$$

LSS Bootstrap

- D'Amico G., **MM**, Pietroni M., Vernizzi F., JCAP 10 (2021)

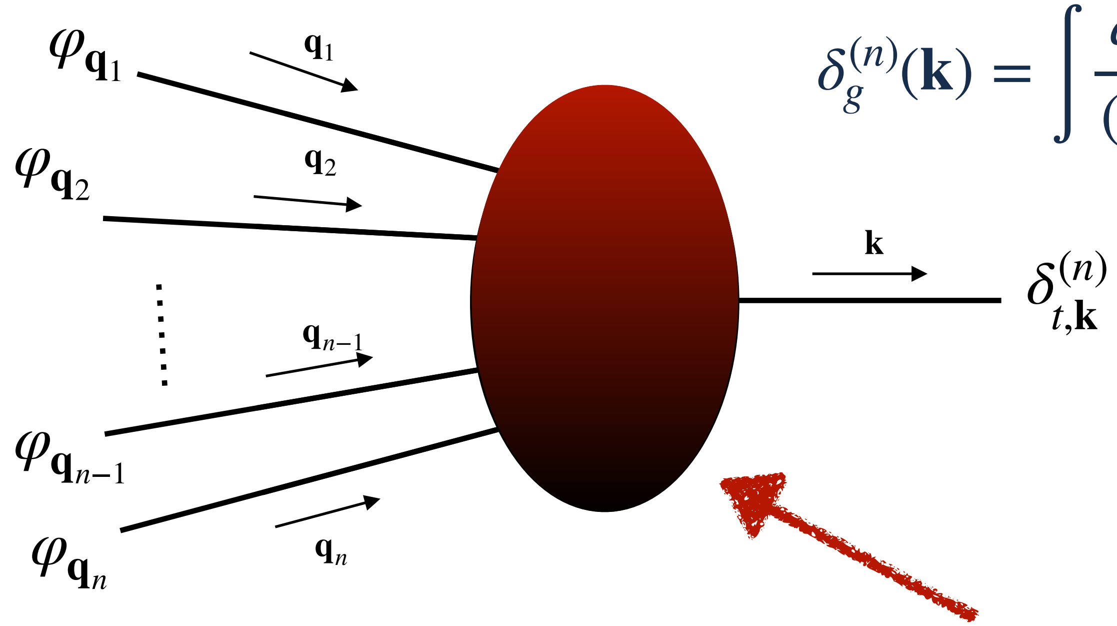


$$\delta_g^{(n)}(\mathbf{k}) = \int \frac{d^3 q_1}{(2\pi)^3} \dots \int \frac{d^3 q_n}{(2\pi)^3} K_n\left(\mathbf{q}_1, \dots, \mathbf{q}_n\right) \delta^{(1)}(\mathbf{q_1}) \dots \delta^{(1)}(\mathbf{q_n})$$

SYMMETRIES

LSS Bootstrap

- D'Amico G., **MM**, Pietroni M., Vernizzi F., JCAP 10 (2021)



$$\delta_g^{(n)}(\mathbf{k}) = \int \frac{d^3 q_1}{(2\pi)^3} \ldots \int \frac{d^3 q_n}{(2\pi)^3} K_n\left(\mathbf{q}_1, \ldots, \mathbf{q}_n\right) \delta^{(1)}(\mathbf{q_1}) \ldots \delta^{(1)}(\mathbf{q_n})$$

SYMMETRIES

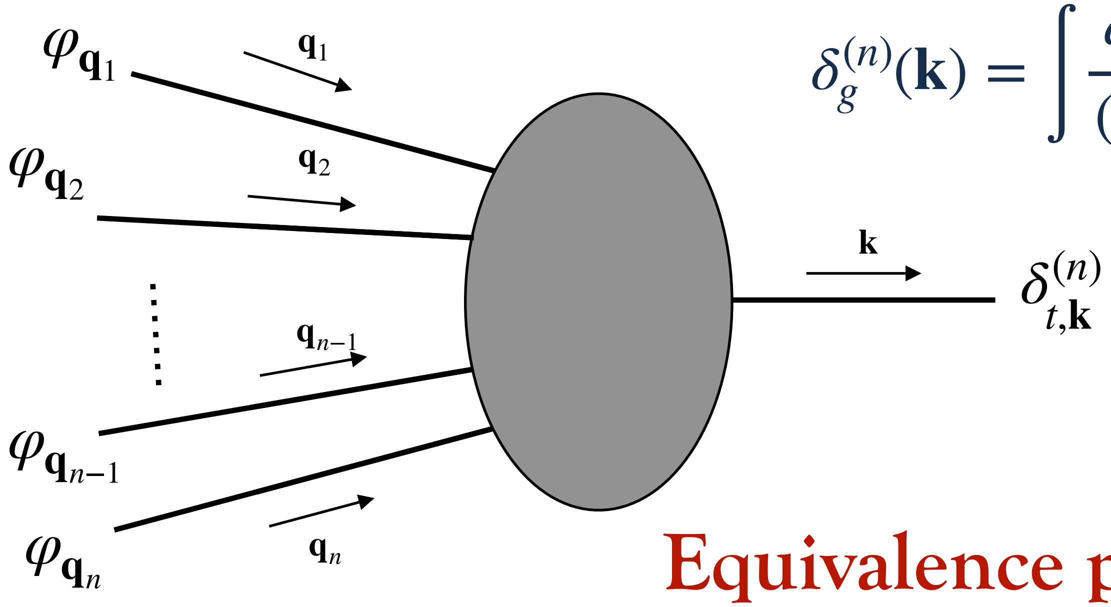
Homogeneity and isotropy

Mass and momentum conservation (only for dark matter)

Equivalence principle

LSS Bootstrap

- D'Amico G., **MM**, Pietroni M., Vernizzi F., JCAP 10 (2021)



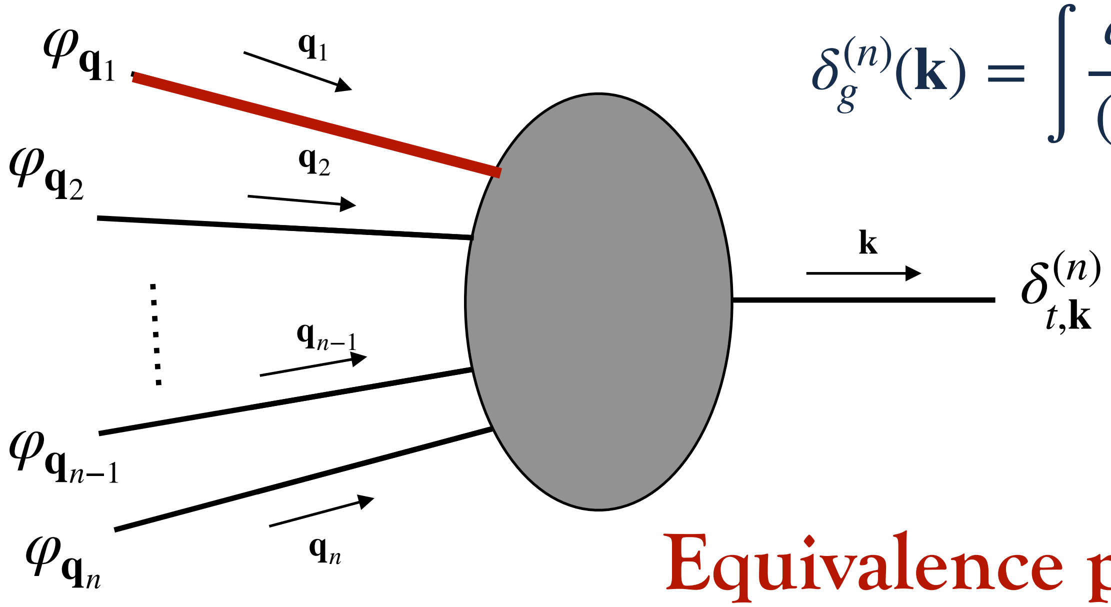
$$\delta_g^{(n)}(\mathbf{k}) = \int \frac{d^3q_1}{(2\pi)^3} \dots \int \frac{d^3q_n}{(2\pi)^3} K_n\left(\mathbf{q}_1, \dots, \mathbf{q}_n\right) \delta^{(1)}(\mathbf{q_1}) \dots \delta^{(1)}(\mathbf{q_n})$$

Equivalence principle

Leading Order: single momentum going $\to 0$

LSS Bootstrap

- D'Amico G., **MM**, Pietroni M., Vernizzi F., JCAP 10 (2021)



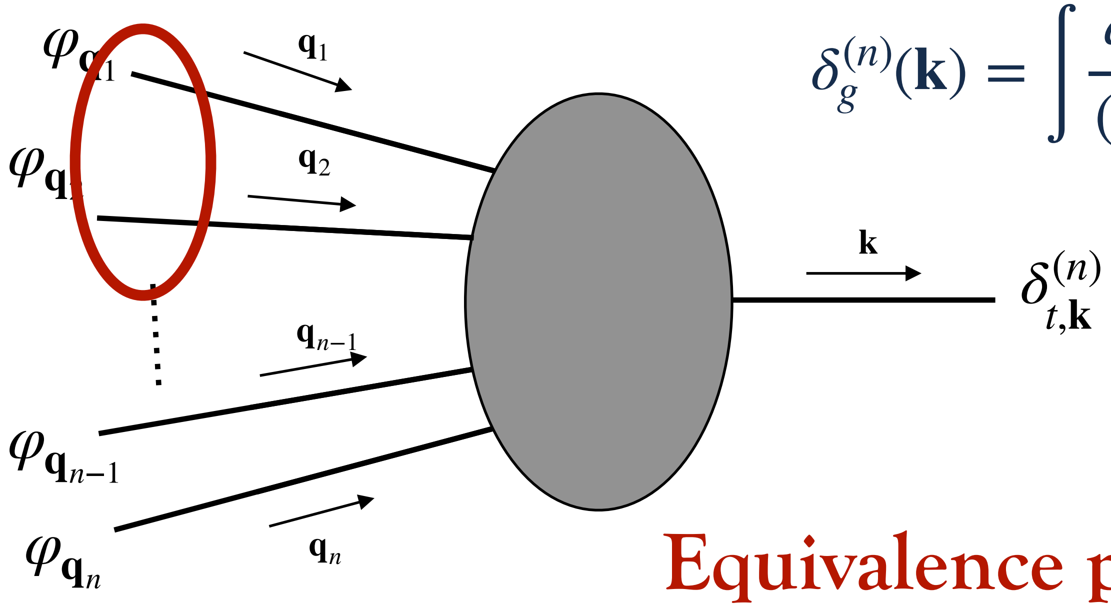
$$\delta_g^{(n)}(\mathbf{k}) = \int \frac{d^3q_1}{(2\pi)^3} \dots \int \frac{d^3q_n}{(2\pi)^3} K_n\left(\mathbf{q}_1, \dots, \mathbf{q}_n\right) \delta^{(1)}(\mathbf{q_1}) \dots \delta^{(1)}(\mathbf{q_n})$$

Equivalence principle

Leading Order: single momentum going $\rightarrow 0$

LSS Bootstrap

- D'Amico G., **MM**, Pietroni M., Vernizzi F., JCAP 10 (2021)



$$\delta_g^{(n)}(\mathbf{k}) = \int \frac{d^3q_1}{(2\pi)^3} \cdots \int \frac{d^3q_n}{(2\pi)^3} K_n\left(\mathbf{q}_1, \ldots, \mathbf{q}_n\right) \delta^{(1)}(\mathbf{q_1}) \ldots \delta^{(1)}(\mathbf{q_n})$$

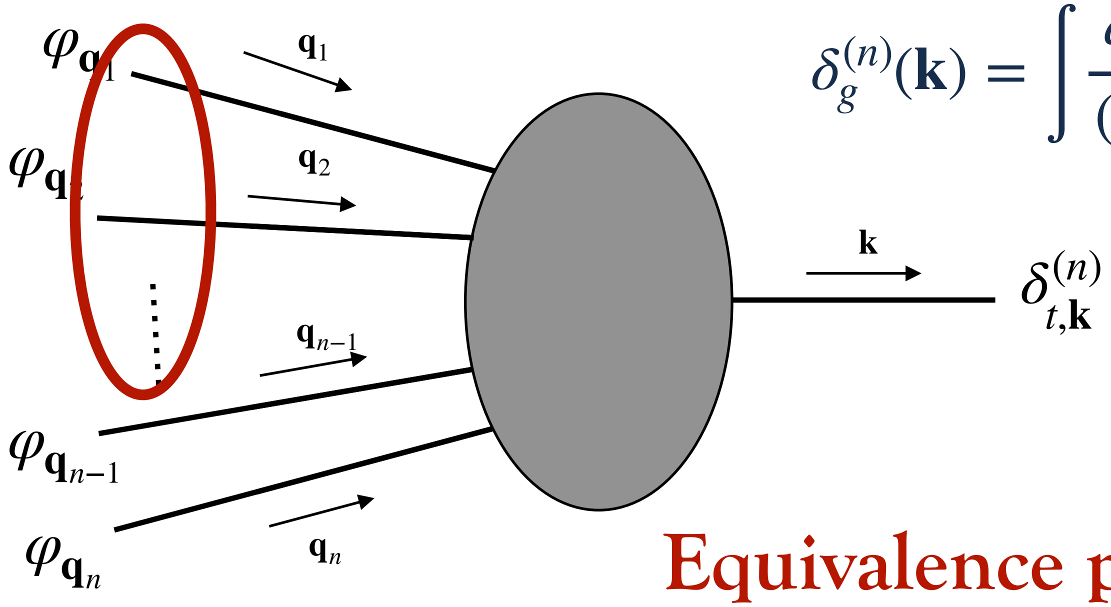
Equivalence principle

Leading Order: single momentum going $\to 0$

Next-to-Leading Order: sum of two momenta going $\to 0$

LSS Bootstrap

- D'Amico G., **MM**, Pietroni M., Vernizzi F., JCAP 10 (2021)



$$\delta_g^{(n)}(\mathbf{k}) = \int \frac{d^3q_1}{(2\pi)^3} \dots \int \frac{d^3q_n}{(2\pi)^3} K_n\left(\mathbf{q}_1, \dots, \mathbf{q}_n\right) \delta^{(1)}(\mathbf{q_1}) \dots \delta^{(1)}(\mathbf{q_n})$$

Equivalence principle

Leading Order: single momentum going $\rightarrow 0$

Next-to-Leading Order: sum of two momenta going $\rightarrow 0$

Next-to-Next-to-Leading Order: sum of three momenta going $\rightarrow 0$

LSS Bootstrap

Kernel at second order

$$K_1(\mathbf{q_1}) = c_0$$

$$K_2(\mathbf{q_1}, \mathbf{q_2}) = c_1 + c_\beta \beta(\mathbf{q}_1, \mathbf{q}_2) + c_\gamma \gamma(\mathbf{q}_1, \mathbf{q}_2)$$

$$\beta(\mathbf{q}_1, \mathbf{q}_2) = \frac{|\mathbf{q}_1 + \mathbf{q}_2|^2 \mathbf{q}_1 \cdot \mathbf{q}_2}{2 q_1^2 q_2^2}$$

$$\gamma(\mathbf{q}_1, \mathbf{q}_2) = 1 - \left(\frac{\mathbf{q}_1 \cdot \mathbf{q}_2}{q_1 \, q_2}\right)^2$$

$$\alpha_a(\mathbf{q}_1, \mathbf{q}_2) = \frac{\mathbf{q}_1 \cdot \mathbf{q}_2}{q_1^2} - \frac{\mathbf{q}_1 \cdot \mathbf{q}_2}{q_2^2}$$

$$\{ \cancel{c_0}, \cancel{c_1}, \cancel{c_\beta}, c_\gamma \}$$

Leading Order

Mass+momentum conservation (matter)

Only 3 parameters left!
(tracers)

Only 1 parameter left!
(matter)

LSS Bootstrap

Kernel at third order

$$K_3(\mathbf{q_1}, \mathbf{q_2}, \mathbf{q_3}) = c_2 + c_{\gamma 1}\gamma(\mathbf{q}_1, \mathbf{q}_2) + c_{\gamma 2}\gamma(\mathbf{q}_{12}, \mathbf{q}_3) + c_{\beta 1}\beta(\mathbf{q}_1, \mathbf{q}_2) + c_{\beta 2}\beta(\mathbf{q}_{12}, \mathbf{q}_3)$$

$$+c_{\gamma\gamma}\gamma(\mathbf{q}_1, \mathbf{q}_2)\gamma(\mathbf{q}_{12}, \mathbf{q}_3) + c_{\beta\beta}\beta(\mathbf{q}_1, \mathbf{q}_2)\beta(\mathbf{q}_{12}, \mathbf{q}_3) + c_{\gamma\beta}\gamma(\mathbf{q}_1, \mathbf{q}_2)\beta(\mathbf{q}_{12}, \mathbf{q}_3)$$

$$+c_{\beta\gamma}\beta(\mathbf{q}_1, \mathbf{q}_2)\gamma(\mathbf{q}_{12}, \mathbf{q}_3) + \left(c_\alpha + c_{\gamma\alpha}\gamma(\mathbf{q}_1, \mathbf{q}_2) + c_{\beta\alpha}\beta(\mathbf{q}_1, \mathbf{q}_2)\right)\alpha_a(\mathbf{q}_{12}, \mathbf{q}_3)$$

$$\{c_2, c_{\gamma 1}, c_{\gamma 2}, c_{\beta 1}, c_{\beta 2}, c_{\gamma\gamma}, c_{\beta\beta}, c_{\gamma\beta}, c_{\beta\gamma}, c_\alpha, c_{\gamma\alpha}, c_{\beta\alpha}\}$$

Leading Order

Next-to-Leading Order

Mass+mom. conservation

Only 4 parameters left!
(tracers)

Only 2 parameters left!
(matter)

LSS Bootstrap

In redshift space at 1-loop for the PS and tree-level for the BS we are mostly sensitive to the time dependent function that appears in the 2nd order kernel

$$B(\mathbf{q_1}, \mathbf{q_2}, \mathbf{q_3}) = 2Z_1(\mathbf{q_1})Z_1(\mathbf{q_2})Z_2(\mathbf{q_1}, \mathbf{q_2})P_L(q_1)P_L(q_2) + 2 \text{ perms.}$$

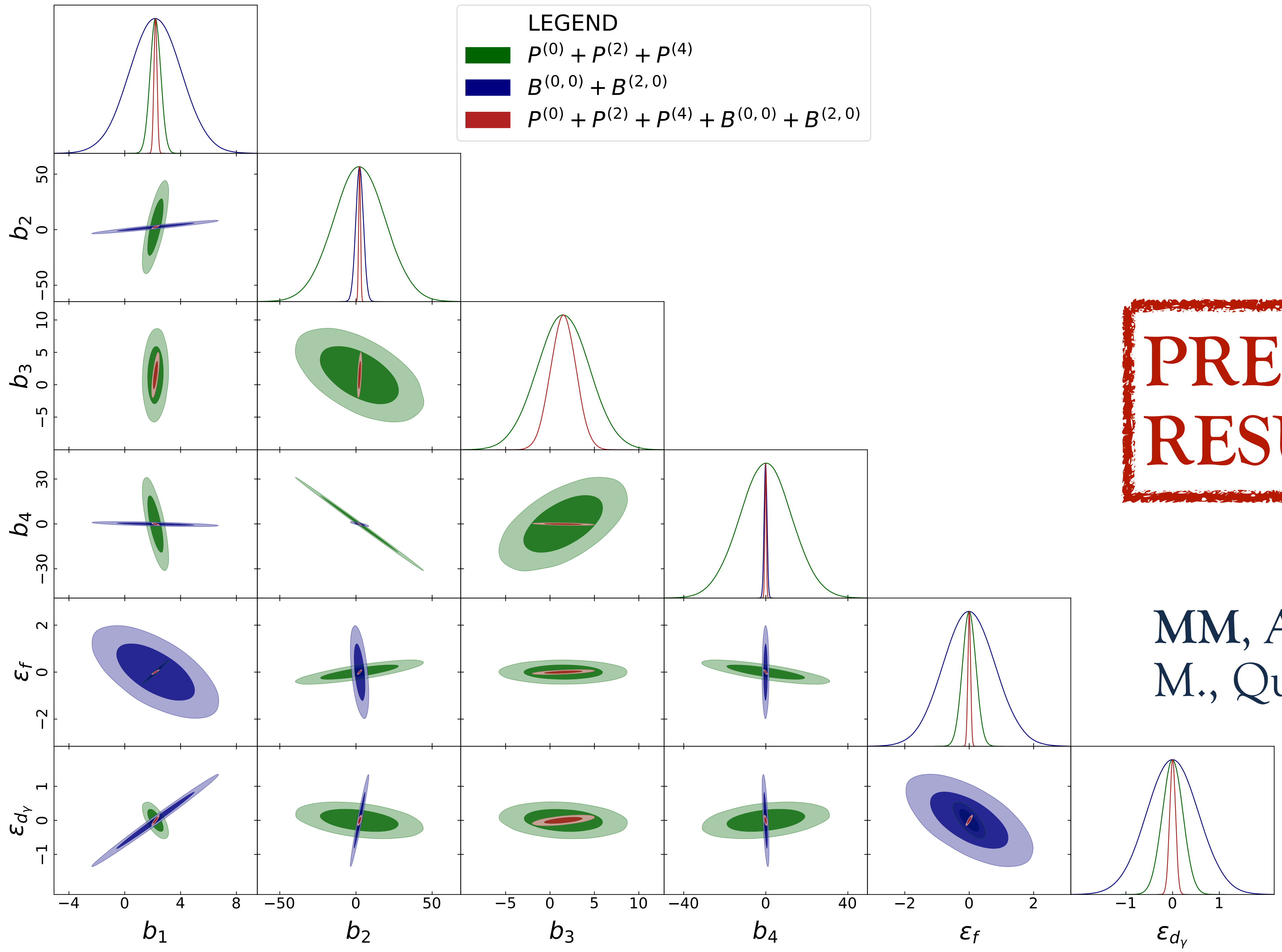
$$Z_2(\mathbf{q_1}, \mathbf{q_2}) = K_2(\mathbf{q_1}, \mathbf{q_2}) + f\mu_{12}^2 G_2(\mathbf{q_1}, \mathbf{q_2}) + \dots$$

$$G_2(\mathbf{q_1}, \mathbf{q_2}) = 2\beta(\mathbf{q}_1, \mathbf{q}_2) + d_\gamma \gamma(\mathbf{q}_1, \mathbf{q}_2)$$

$f = f^{\Lambda CDM}(1 + \varepsilon_f) \quad d_\gamma = d_\gamma^{\Lambda CDM}(1 + \varepsilon_{d_\gamma})$ ⟶ We can detect deviations from ΛCDM in a model-independent way!

LSS Bootstrap

Euclid-like Forecast

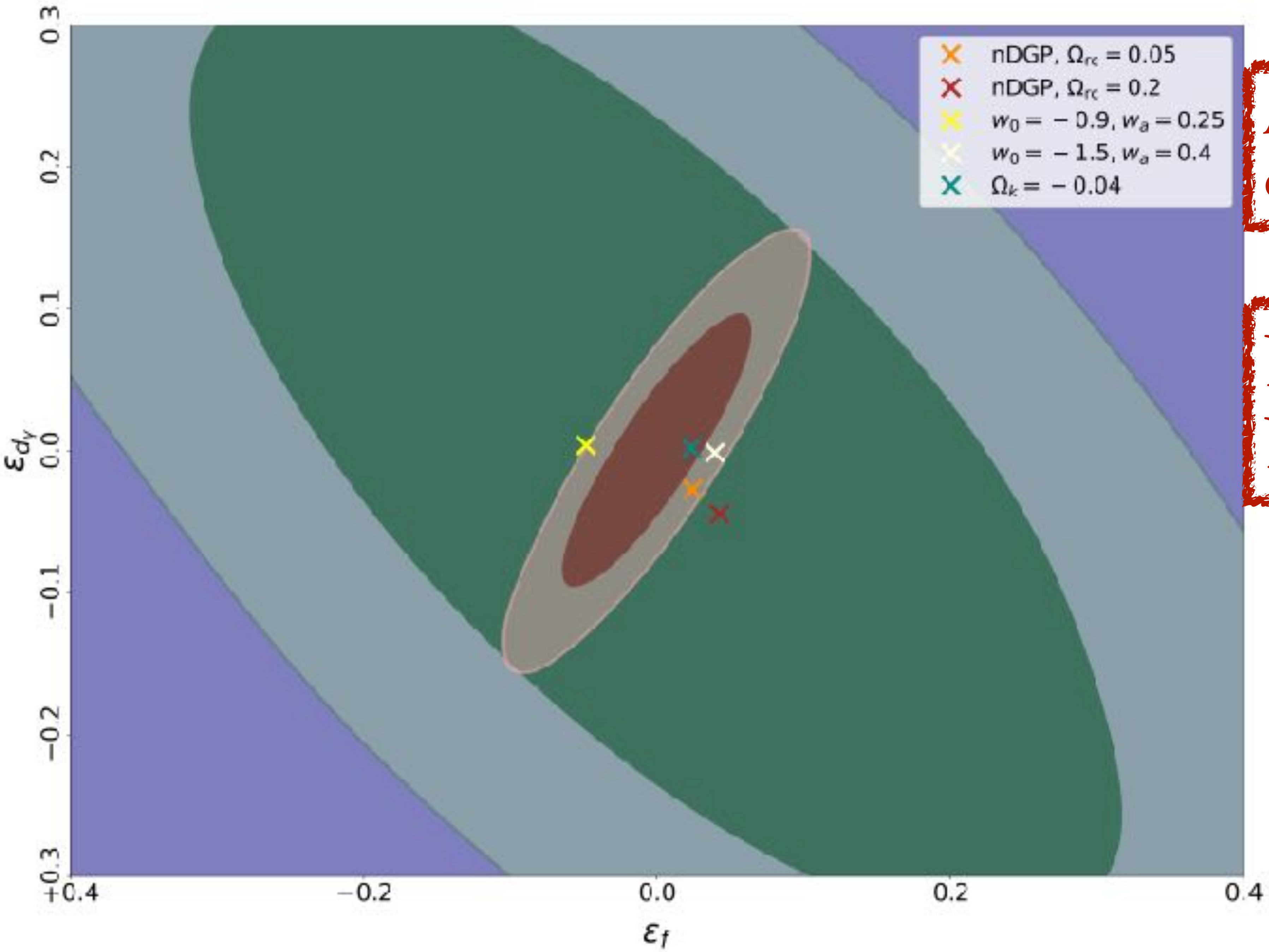


PRELIMINARY RESULTS!

MM, Amendola L., Pietroni M., Quartin M., in preparation

LSS Bootstrap

Euclid-like Forecast



A new dimension to constrain beyond-ΛCDM

PRELIMINARY RESULTS!

MM, Amendola L., Pietroni M., Quartin M., in preparation

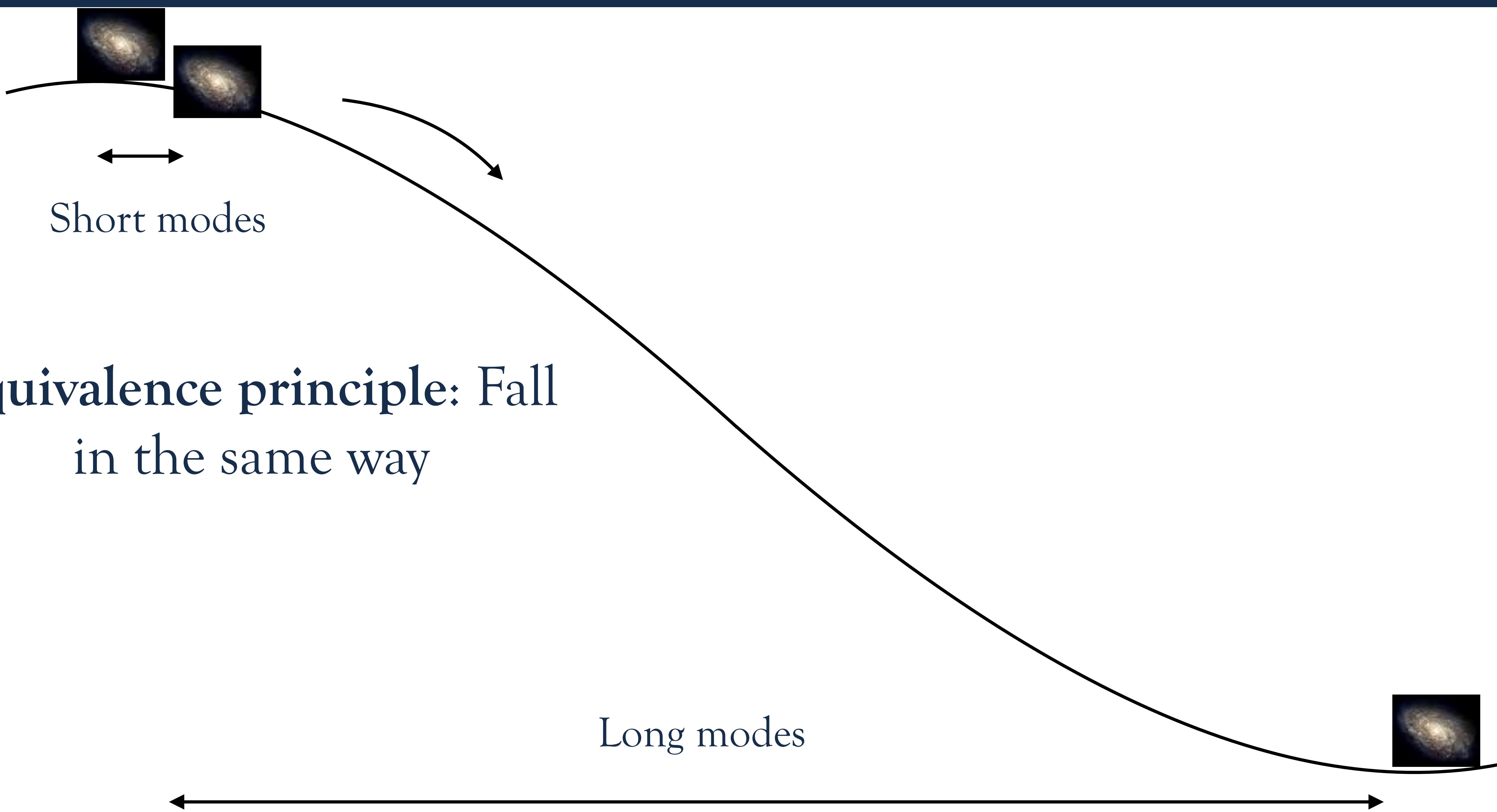
Conclusion and outlook

- Model-independent approaches cover a large variety of existing models
- Beyond-inflationary+ΛCDM physics (?)
- CR and BAO approach will be used within Euclid collaboration
- Bootstrap already implemented in PyBird (ongoing MCMC on sims)
- Implementing the Bispectrum (with M. Peron)


Thanks for your attention!

Backup slides

Equivalence principle



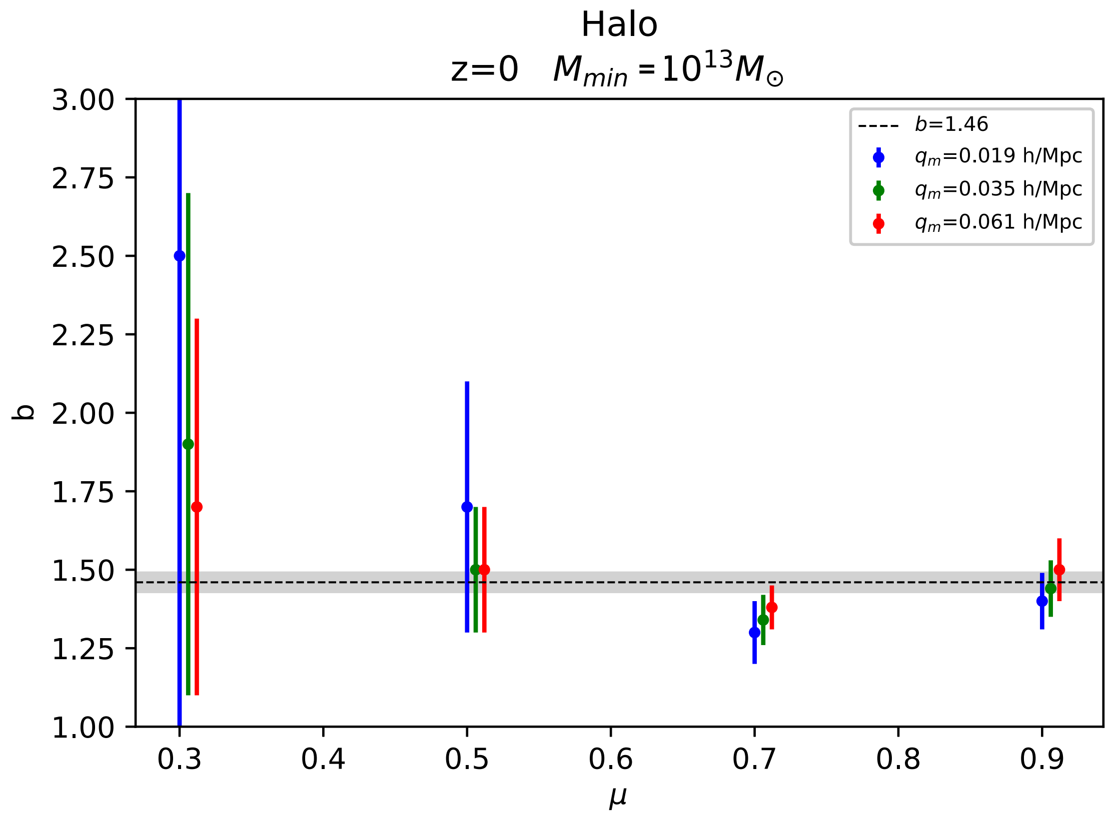
Equivalence principle: Fall in the same way

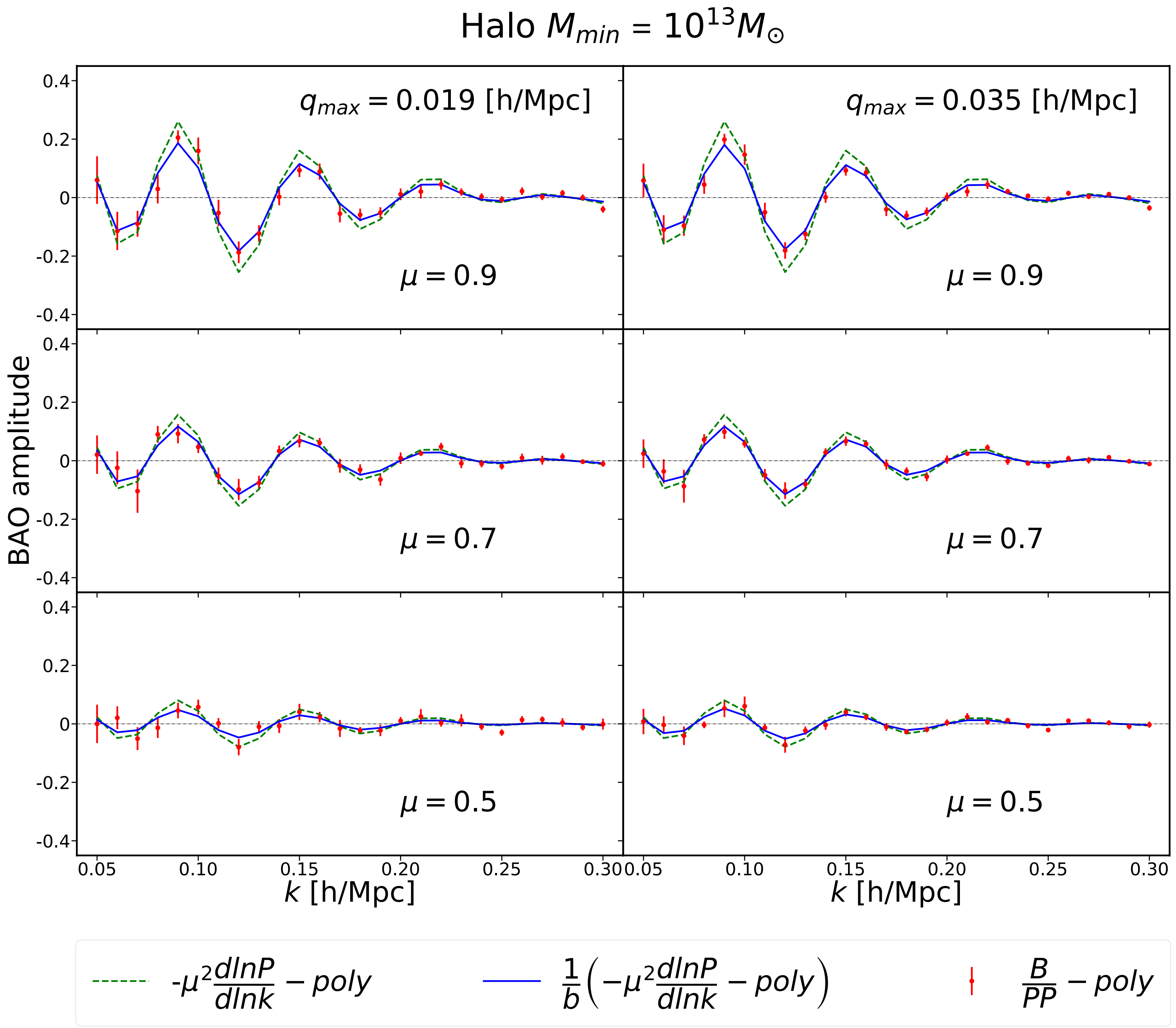
CR and BAO

N-body simulations: **real space w/ biased tracers,** MM+ (2019)

$$\lim_{q/k \to 0} \frac{B_t(q, k_+, k_-)}{P_t(q) P_t(k)} = -\frac{\mu^2}{b_t} \frac{d \log P(k)}{d \log k} + \dots$$

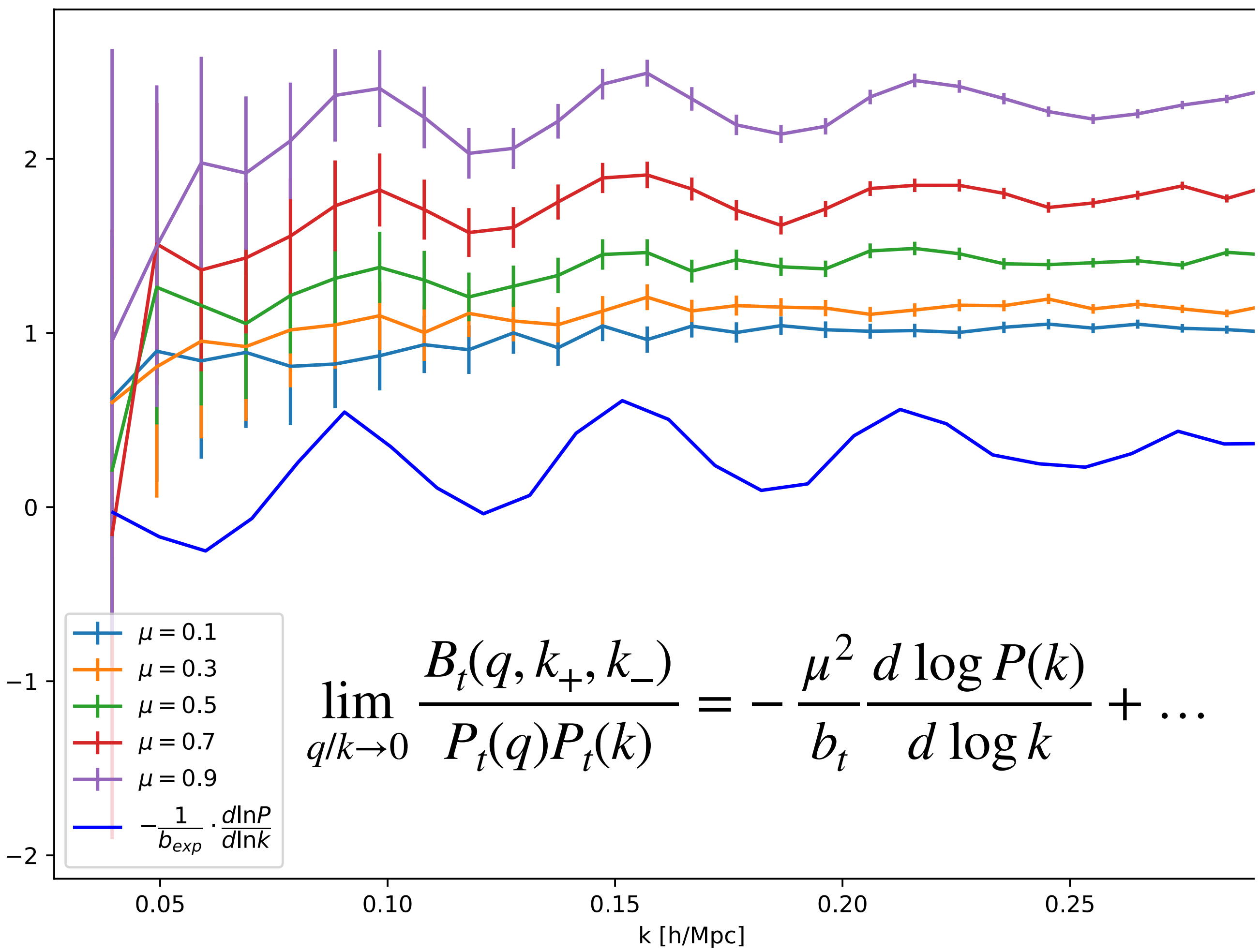
Bias parameter $\qquad b_t = \lim_{q \to 0} \frac{P_{tt}(q)}{P_{tm}(q)}$



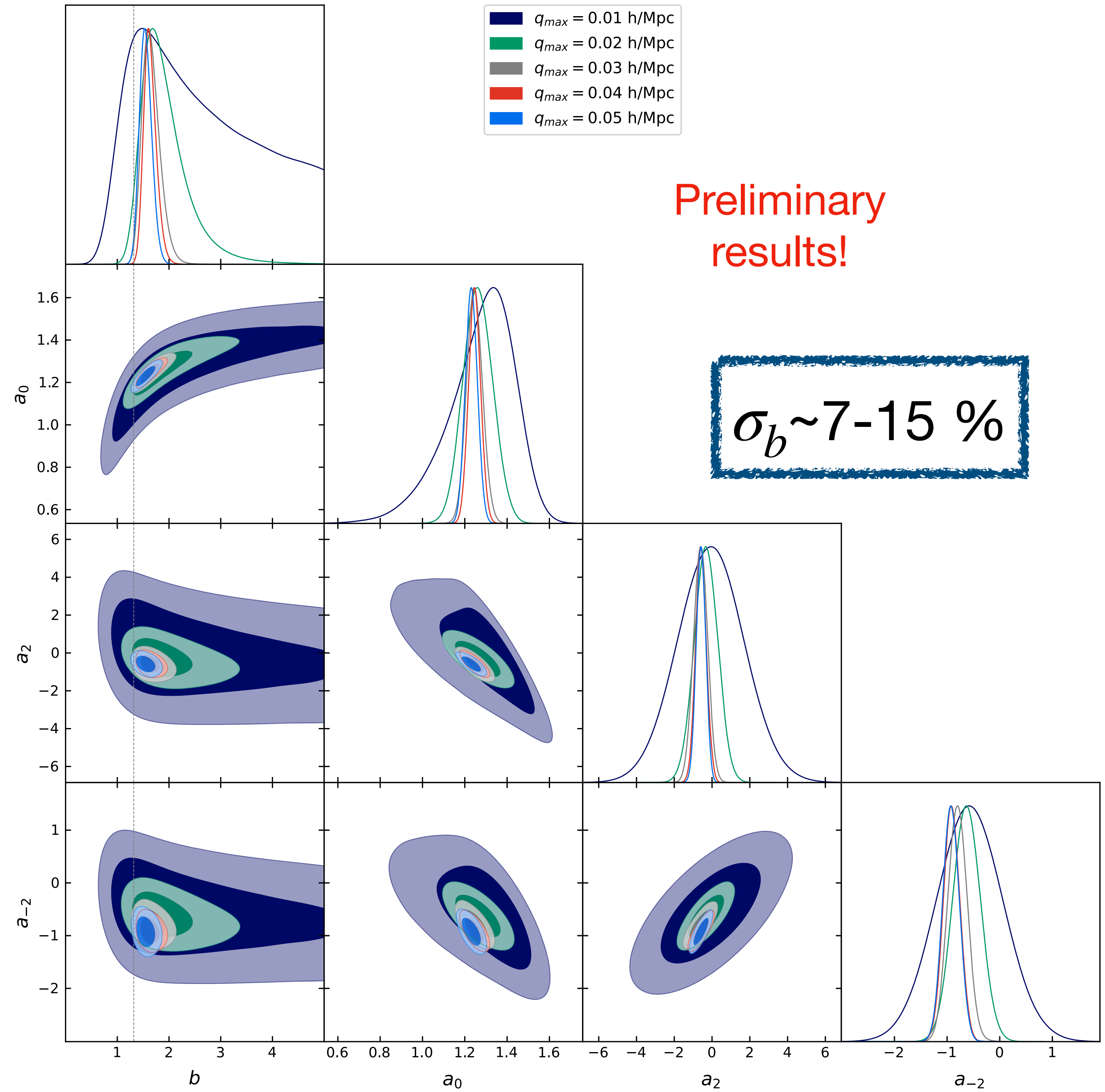


CR and BAO

Euclid Flagship simulations (Euclid coll.+, in prep.)



$$\lim_{q/k \to 0} \frac{B_t(q, k_+, k_-)}{P_t(q) P_t(k)} = -\frac{\mu^2}{b_t} \frac{d \log P(k)}{d \log k} + \ldots$$



Preliminary results!

σ_b~7-15 %

LSS Bootstrap

- D'Amico G., **MM**, Pietroni M., Vernizzi F., JCAP 10 (2021)

Equivalence principle

Leading Order: single momentum going $\rightarrow 0$

Next-to-Leading Order: sum of two momenta going $\rightarrow 0$

Next-to-Next-to-Leading Order: sum of three momenta going $\rightarrow 0$

LSS Bootstrap

- D'Amico G., **MM**, Pietroni M., Vernizzi F., JCAP 10 (2021)

Equivalence principle

Leading Order: single momentum going $\to 0$

Next-to-Leading Order: sum of two momenta going $\to 0$

N.$^{l-1}$-to-Leading Order: sum of $l-1$ momenta going $\to 0$

LSS Bootstrap

- D'Amico G., **MM**, Pietroni M., Vernizzi F., JCAP 10 (2021)

Equivalence principle

Leading Order: single momentum going $\to 0$

Next-to-Leading Order: sum of two momenta going $\to 0$

N.$^{l-1}$-to-Leading Order: sum of $l-1$ momenta going $\to 0$

$$\lim_{\mathbf{q}_1,\ldots,\mathbf{q}_m\to 0} K_n(\mathbf{q}_1,\ldots,\mathbf{q}_m,\mathbf{q}_{m+1},\ldots,\mathbf{q}_n) = \frac{\mathbf{q}_1\cdot\mathbf{Q}_{n,m}}{q_1^2}\cdots\frac{\mathbf{q}_m\cdot\mathbf{Q}_{n,m}}{q_m^2} K_{n-m}(\mathbf{q}_{m+1},\ldots,\mathbf{q}_n) + O\left(\left(\frac{1}{q}\right)^{m-1}\right)$$

LSS Bootstrap

- D'Amico G., **MM**, Pietroni M., Vernizzi F., JCAP 10 (2021)

Equivalence principle

Leading Order: single momentum going $\to 0$

Next-to-Leading Order: sum of two momenta going $\to 0$

N.$^{l-1}$-to-Leading Order: sum of $l-1$ momenta going $\to 0$

$$\lim_{\mathbf{q}_1+\mathbf{q}_2\to 0} K_n(\mathbf{q}_1, \mathbf{q}_2, \mathbf{q}_3, \ldots, \mathbf{q}_n) \supset \frac{\mathbf{k}\cdot\mathbf{q}_{12}}{q_{12}^2} K_{n-2}(\mathbf{q}_3, \ldots, \mathbf{q}_n) \int^{\eta} d\eta' \, f_+(\eta') \frac{D_+(\eta')^2}{D_+(\eta)^2} G_2(\mathbf{q}_1, \mathbf{q}_2; \eta')$$

LSS Bootstrap

- D'Amico G., **MM**, Pietroni M., Vernizzi F., JCAP 10 (2021)

Equivalence principle

Leading Order: single momentum going $\rightarrow 0$

Next-to-Leading Order: sum of two momenta going $\rightarrow 0$

N.$^{l-1}$-to-Leading Order: sum of l momenta going $\rightarrow 0$

$$\lim_{\mathbf{Q}_{l,0}\rightarrow 0} K_n(\mathbf{q}_1, \ldots, \mathbf{q}_l, \mathbf{q}_{l+1}, \ldots, \mathbf{q}_n) \supset \frac{\mathbf{k}\cdot\mathbf{Q}_{l,0}}{Q_{l,0}^2}\int^{\eta} d\eta'\, f_+(\eta')\left(\frac{D_+(\eta')}{D_+(\eta)}\right)^{l} G_l(\mathbf{q}_1, \ldots, \mathbf{q}_l; \eta')K_{n-l}(\mathbf{q}_{l+1}, \ldots, \mathbf{q}_n; \eta)$$