# CONTEMPORARY TOPICS IN CONDENSED MATTER PHYSICS


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BALKAN SCHOOL-TIRANA-JAN 25, 2024
(8:40AM TX TIME)

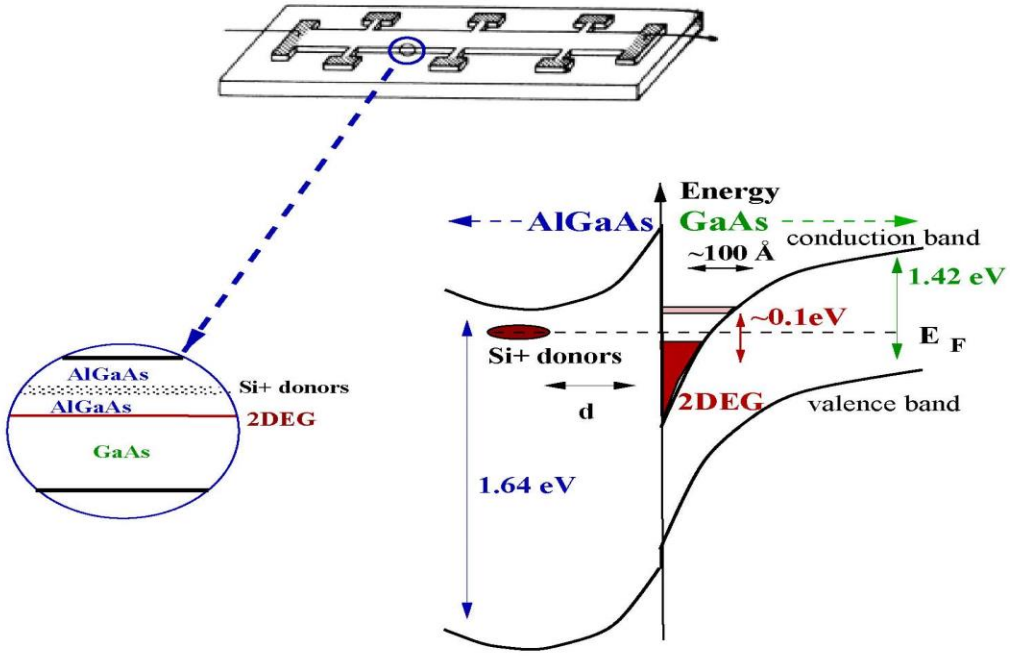
# OVERVIEW OF FEW TOPICS

- Low-dimensional strongly correlated electronic systems*
- Two-dimensional (2D) quantum Hall systems (in magnetic field)* (Monte Carlo simulations)
- Anisotropic 2D quantum systems and phase transitions*
- Confined nanoscale semiconductor quantum dot systems and materials
- Spin-based systems and spintronic devices

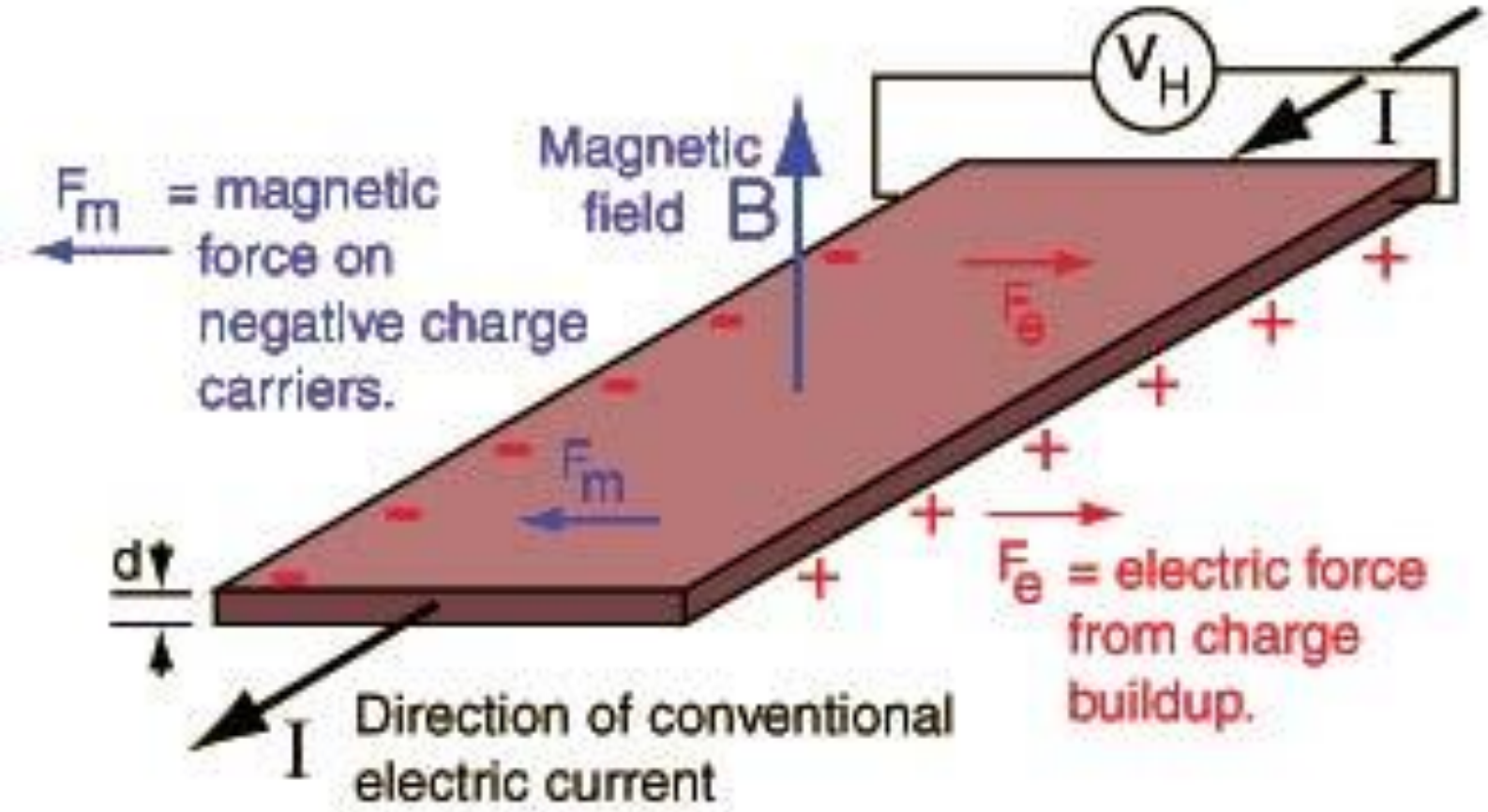
Note: Those denoted with asterisk (*) will be covered!
Those in red, may be covered depending on the available time!

# THE 2-DIM ELECTRON SYSTEM

## Inversion layers



# QUANTUM HALL EFFECT (QHE)

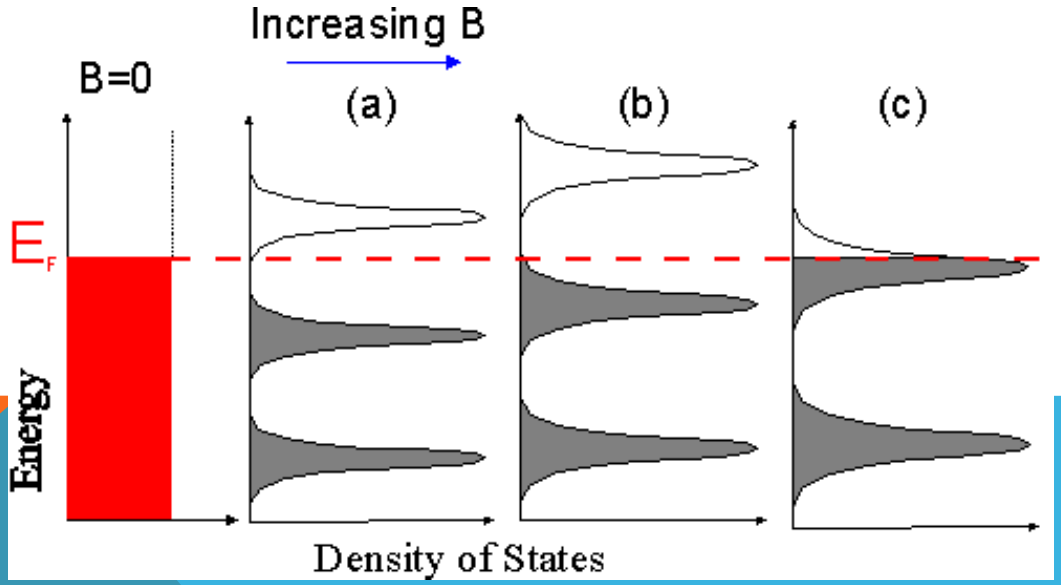


# LANDAU LEVELS

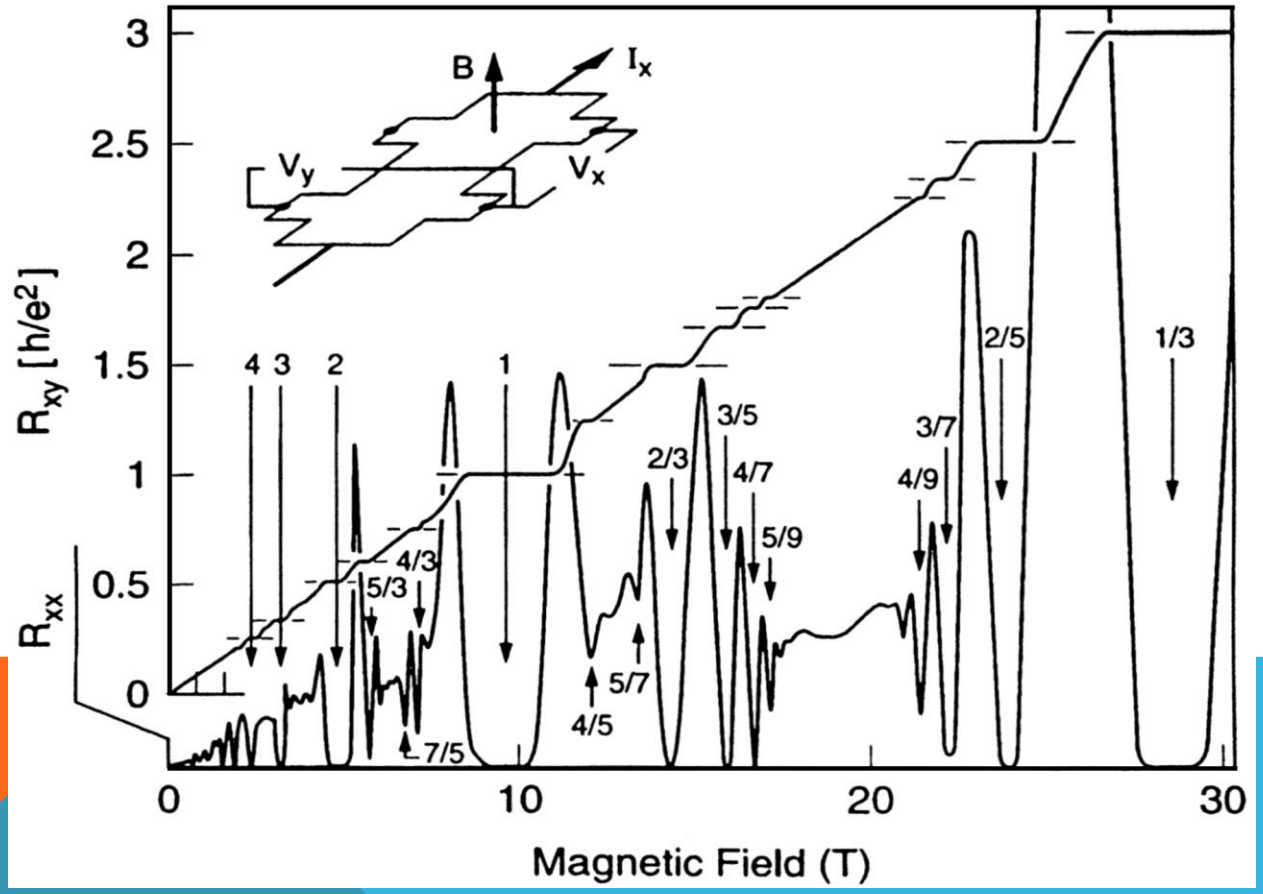
Landau levels ; Degeneracy ≈ B (magnetic field)

Energy, $E=\hbar\omega(L+1/2)$ ; L=0, 1, 2,...

Filling Factor, v=N/Degeneracy ; N=Number of electrons



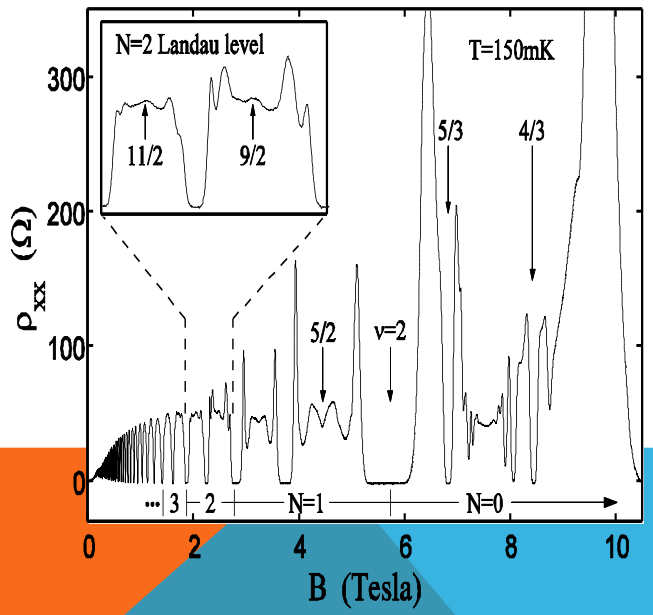
# QUANTIZATION OF HALL RESISTANCE

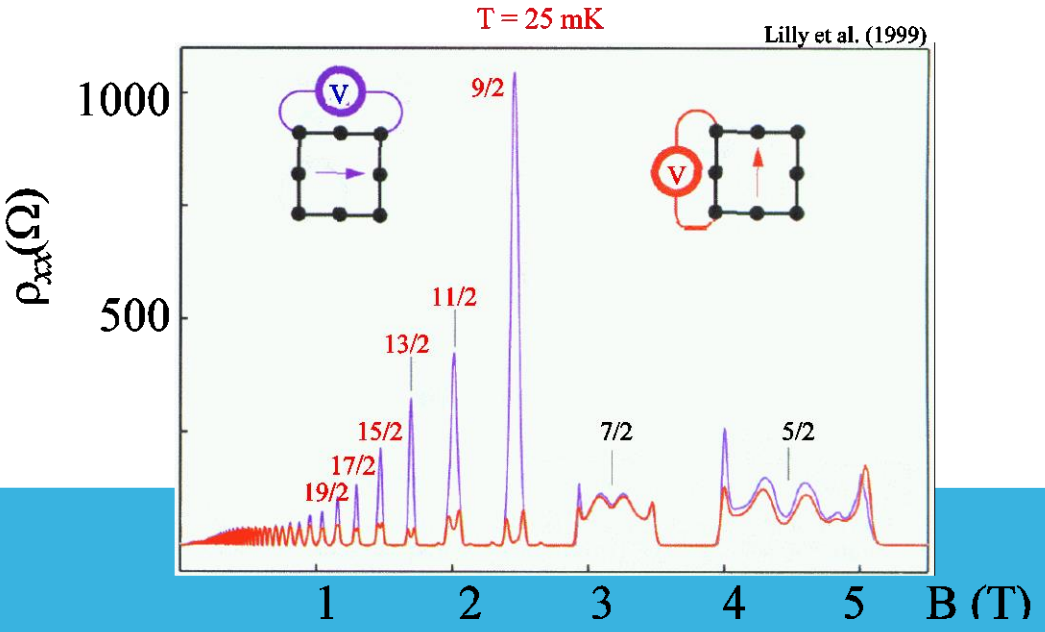


# ANISOTROPY: FILLING= 9/2, 11/2,...

High T (=150 mK) : Longitudinal resistance shows no anisotropy

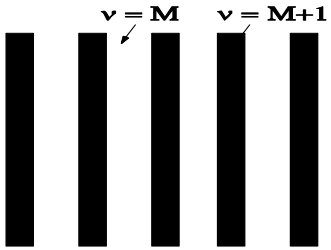
Low T (=25 mK) : Longitudinal resistance becomes anisotropic [M.P. Lilly et al. PRL 82, 394 (1999)]



# COMPETING THEORIES

Charge density wave (CDW ; stripe state):




CDW THEORY (FOR HIGH LANDAU LEVELS ; HARTREE-FOCK):

[Fogler et al. PRB 54, 1853 (1996) ; Moessner et al. PRB 54, 5006 (1996)].

Deficiencies of CDW approach:

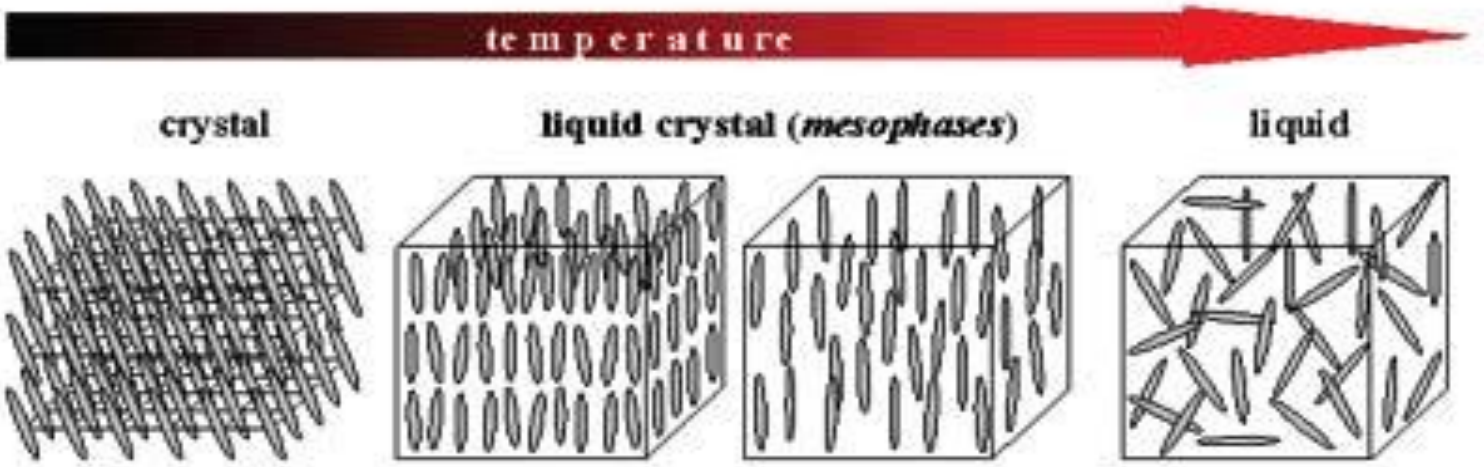
- Mean-Field theory (2D is critical)
- CDW formation energy: 5K >> Critical Temperature: 100 mK
- What is so special being around filling factor, 4+1/2
- Key issues: Why Anisotropy and why only at index L=2 and higher (so far!?)

LIQUID CRYSTAL THEORY (A LITTLE BIT MORE EXOTIC!):

- *Orientational order, **without positional order** can generate anisotropy!*
  Nematic liquid crystals have no positional order,have orientational order
  Phase transition at Tc indicates an isotropic liquid-anisotropic liquid transition
  Maybe Kosterlitz-Thouless type (?) [Wexler and Dorsey, PRB 64, 115312 (2001)]
  Quantum Signature: Wave Function with **Broken Rotational Symmetry (BRS)!**
  [Ciftja and Wexler, IJMPB 20, 747 (2006) ; PRB 65, 045306 (2002) ; 65, 205307 (2002)]

# CLASSICAL LIQUID CRYSTALS

## *thermotropic liquid crystals*



# BRS WAVE FUNCTION

BRS wave function (v=1/3) which can be generalized for any Laughlin-like filling factor at any Landau level [Ciftja et al, PRB 69, 125320 (2004)]

- $\Psi_{\alpha}=\prod_{j>k}(z_j-z_k)\,(z_j-z_k-\alpha)\,(z_j-z_k+\alpha)\,\exp[-(1/4)\sum_j|z_j|^2]$
- High school example of translational invariance without rotational invariance: $(z_j-z_k)\,\alpha$

BRS wave function (v=1/2 that can be extended to ¼ or 1/6) [Ciftja and Wexler, PRB 65, 205307 (2002)].

- $\Psi_{\alpha}=P_0\prod_{j>k}(z_j-z_k-\alpha)\,(z_j-z_k+\alpha)\,\exp[-(1/4)\sum_j|z_j|^2]\,\mathrm{Det}[\Phi_k]$

Which state, isotropic or BRS is stable at 1/3 and 1/2 ?!

What about the LLL (L=0) states themselves ?!

What about states in higher LL-s, 2L+1/3 and 2L+1/2 ?!

Monte Carlo method - Disk Geometry

Single LL approximation [ Frozen underlying LL-s]

# MODEL

H=K+V ; K = Kinetic Energy ; V = Potential (ee, eb, bb)

Electron-electron Coulomb pair interaction potential:

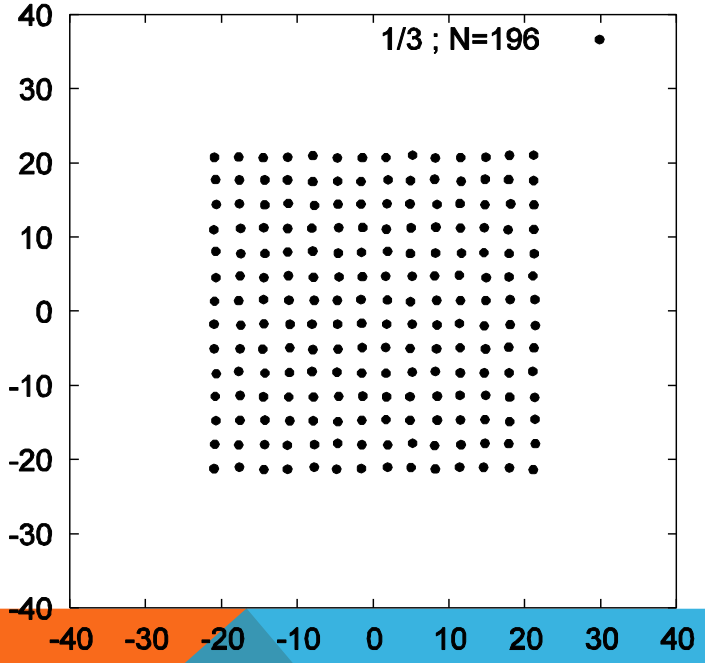
$V_{ee}(r) = e^2/\sqrt{(r^2 + \lambda^2)}$ ; $\lambda$ = layer thickness parameter

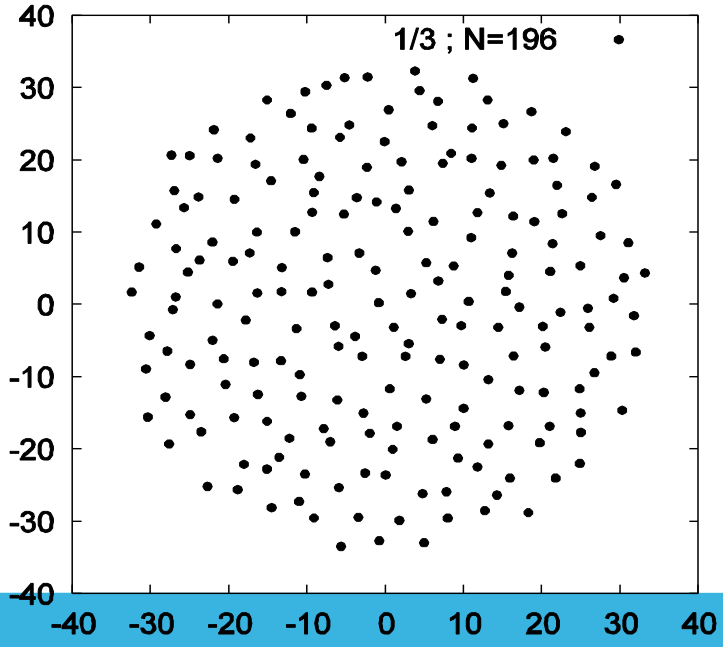
2L+1/3 (model state) ; L=0,1,2,.. ; 1/3 = FQHE Laughlin state
[anisotropy at (2L+0.5) +/- 0.2 range]

2L+1/2 (anisotropic for L>=2) ; 1/2 = Fermi liquid state

# ISOTROPIC LIQUID: N=1/3

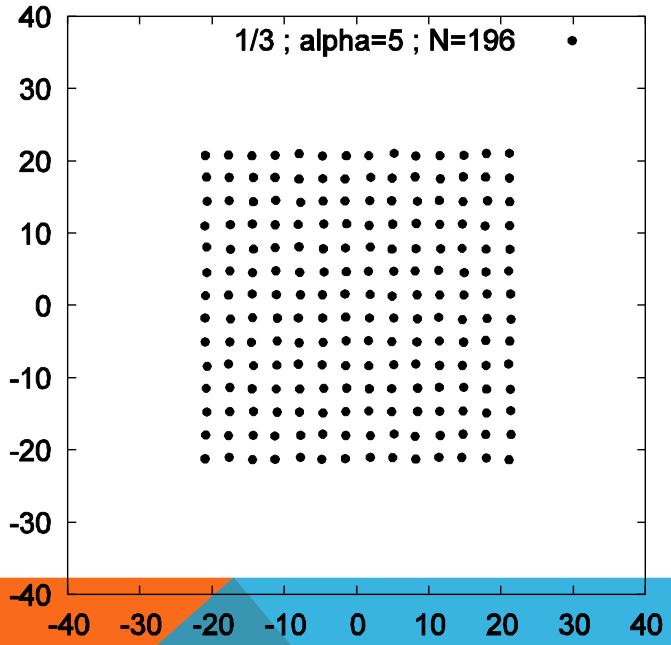
## MONTE CARLO SIMULATIONS WITH N=196 ELECTRONS

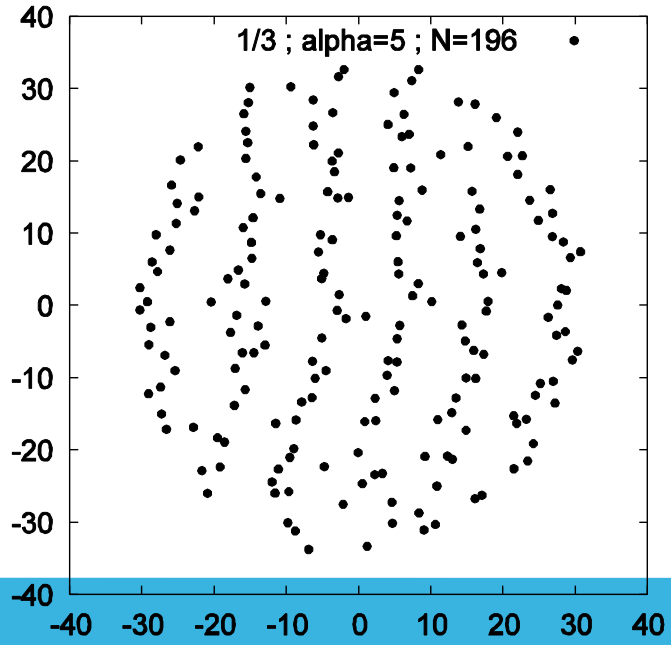


# ANISOTROPIC LIQUID CRYSTAL:N=1/3

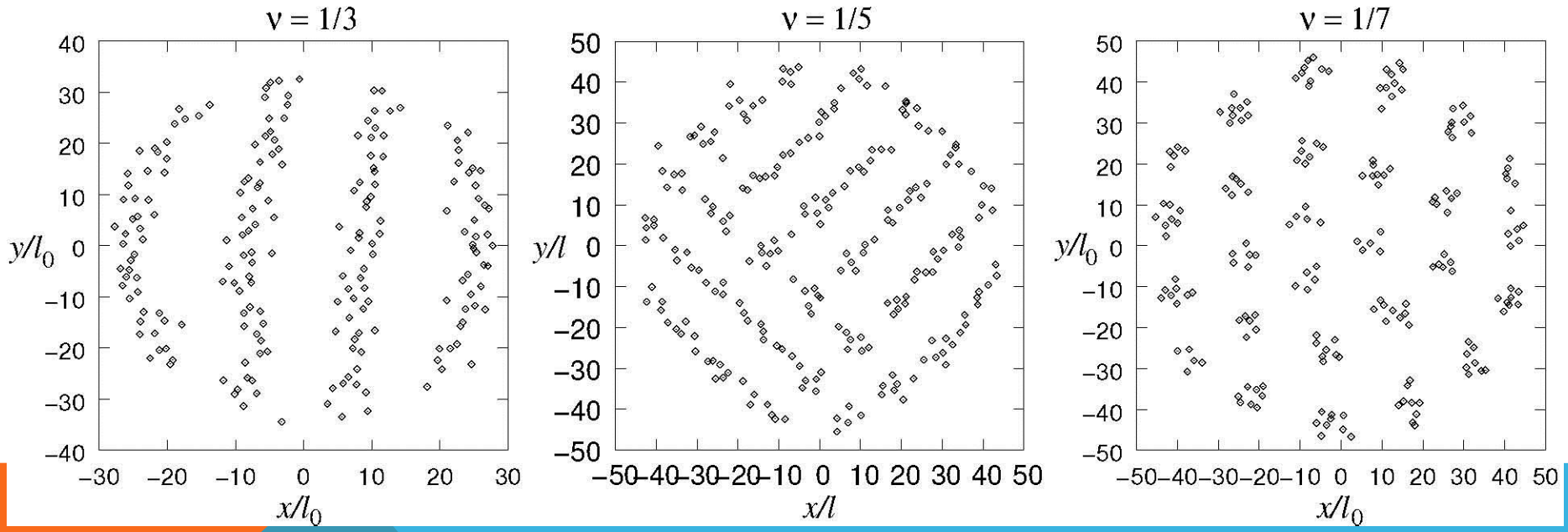
Monte Carlo simulations with N=196 electrons



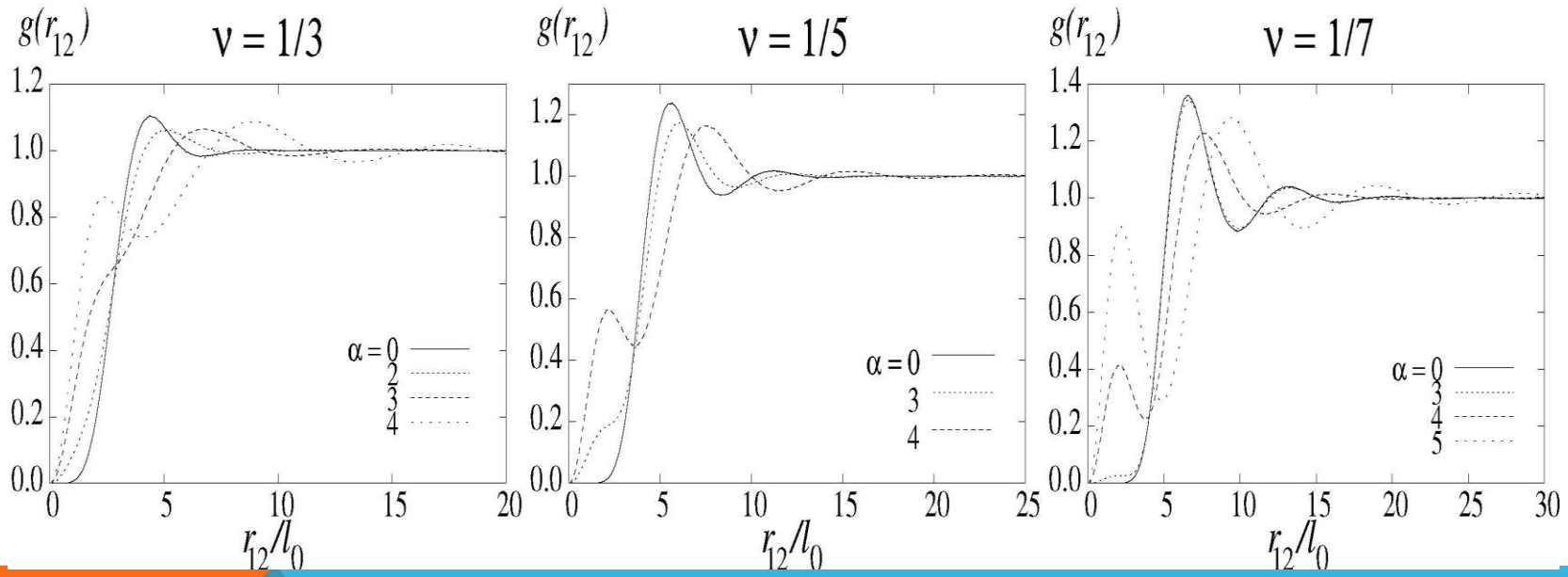



# WHAT ABOUT THE ANISOTROPY IN THE LLL (L=0)?

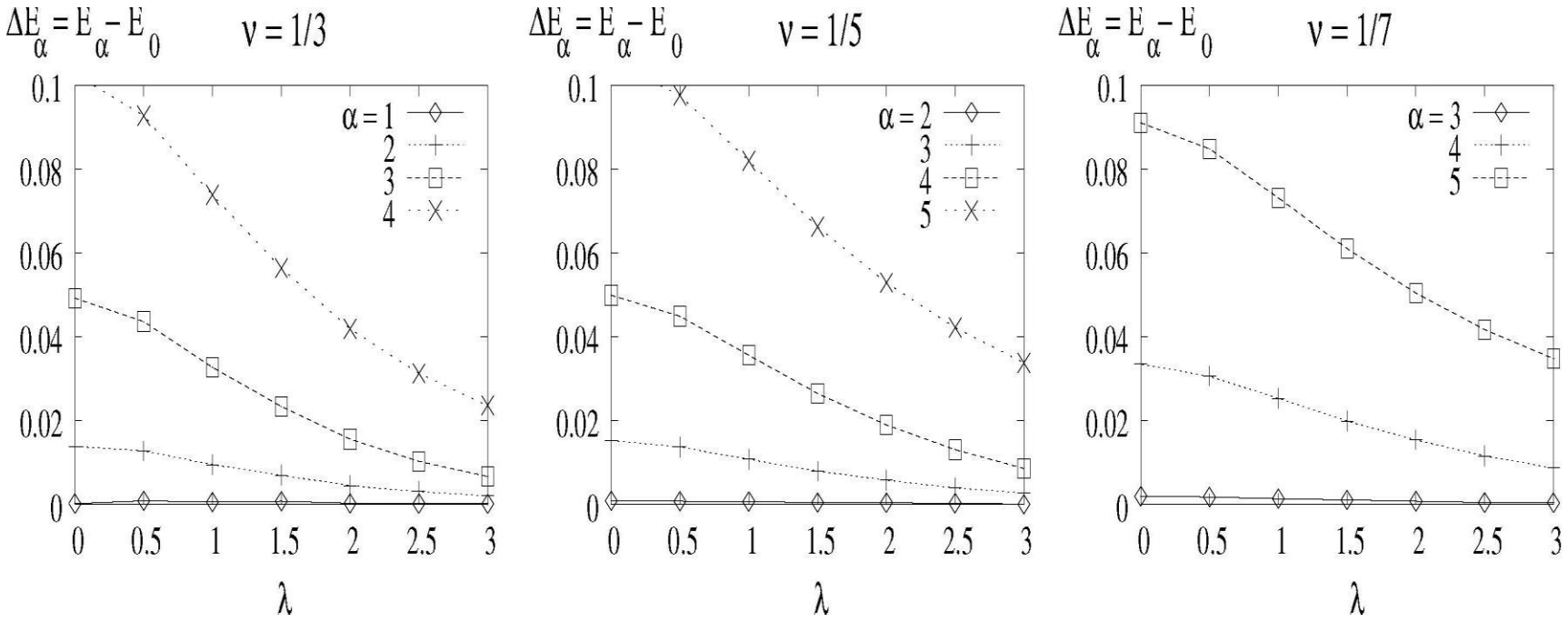
Maybe some hope at the intermediate regime before the onset of Wigner crystallization (Monte Carlo simulations)



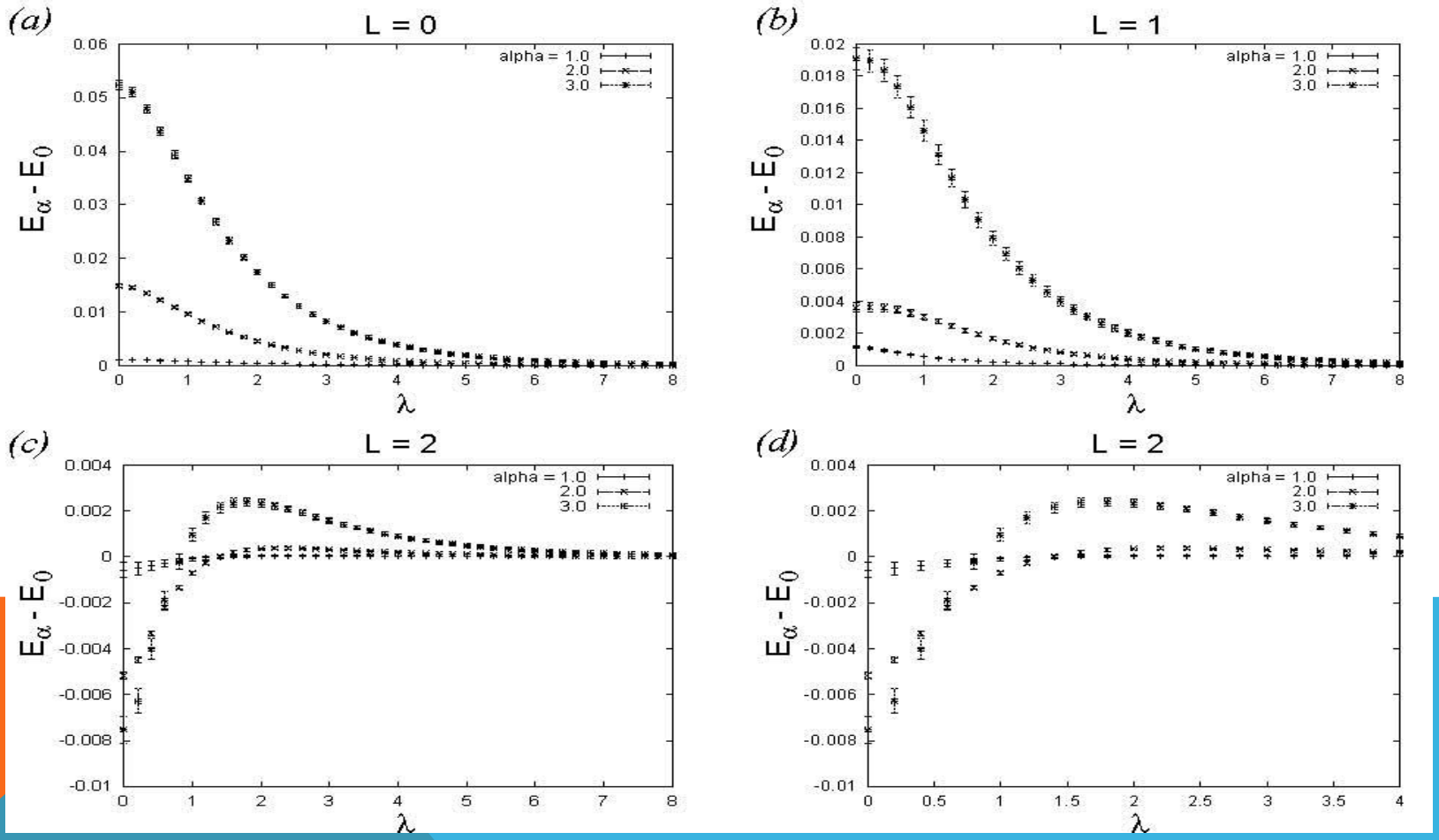
# ANGLE-AVERAGED PAIR DISTRIBUTION FUNCTIONS AT THE LLL (L=0)



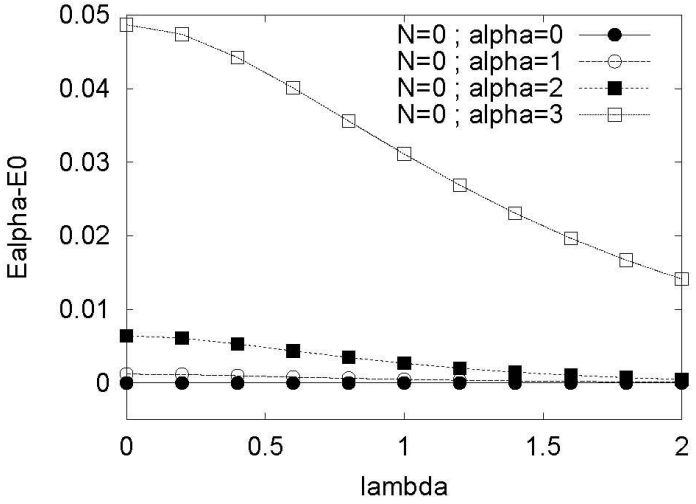
# ENERGY COMPARISON IN THE LLL (L=0)

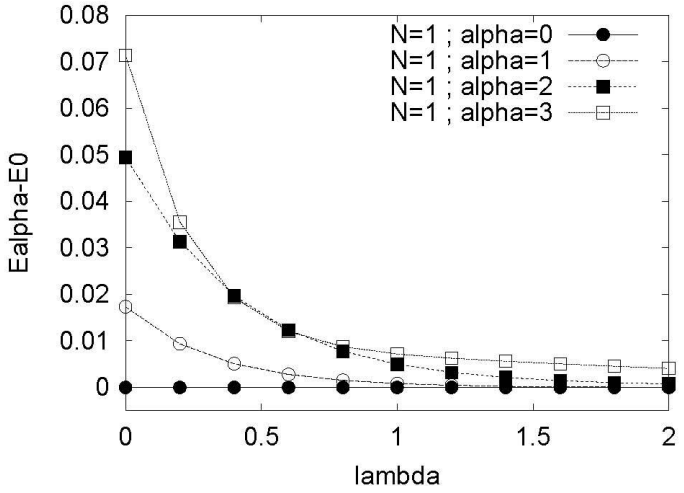


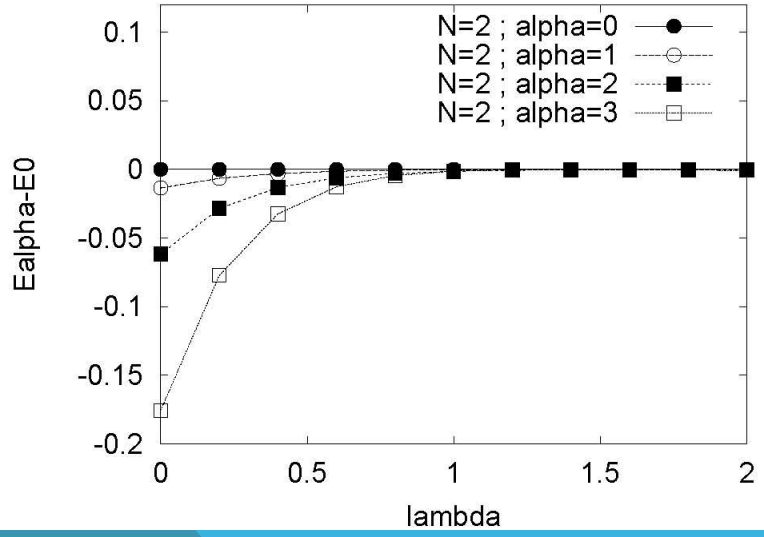
# BRS LAUGHLIN LIQUID IN HIGHER LANDAU LEVELS: N=2L+1/3



# BRS HALF-FILLED FERMI LIQUID IN HIGHER LANDAU LEVELS: N=2N+1/2



# MASS ANISOTROPY/ANISOTROPIC INTERACTION POTENTIAL

Should one be discouraged to search for anisotropy in the LLL, OR have a glimpse of hope if internal anisotropy is added?

## MASS ANISOTROPY IN AlAs (not GaAs) SYSTEMS?

Mass Anisotropy can be effectively included via an anisotropic Coulomb interaction potential:

$$V_\gamma(r_i, r_j) \cong e^2/(x_{ij}^2/\gamma^2 + \gamma^2\, y_{ij}^2)^{1/2}$$

When gamma=1, the potential becomes isotropic Coulomb

**Detailed Monte Carlo simulation results (gamma=2 ; nu=1/6)**

PHYSICAL REVIEW B **95**, 075410 (2017)

**Anisotropic magnetoresistance and piezoelectric effect in GaAs Hall samples**

Orion Ciftja

# SIMULATIONS WITH UP TO N=57 ELECTRONS

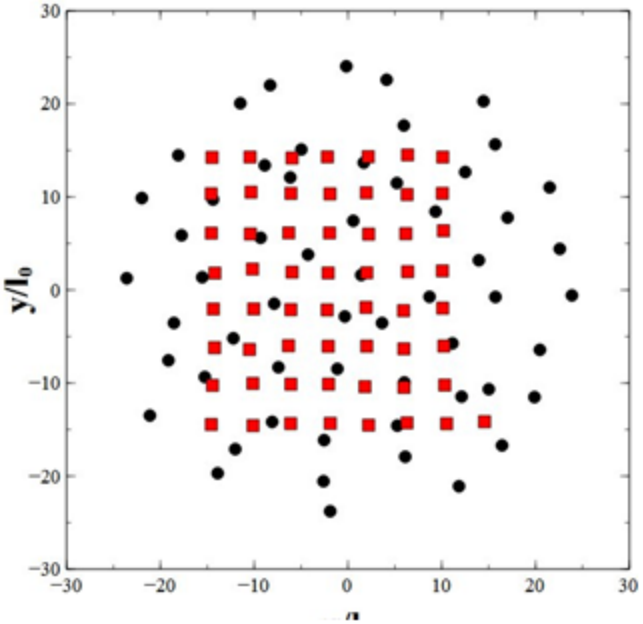



**Figure 1: Snapshot of the initial configuration (red squares) and final configuration of a system of $N = 57$ electrons (filled circles) after a full QMC run (disk geometry). The system of is described by an anisotropic Fermi liquid BRS wave function with $\alpha = 5$ and represents a state with filling factor $\nu = 1/6$ of the LLL.**

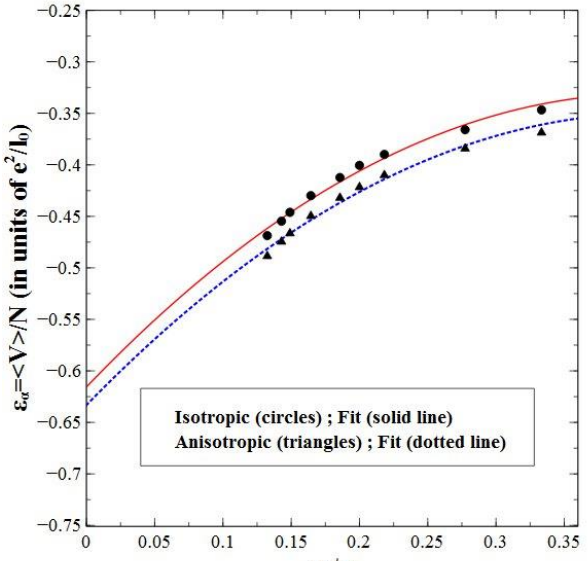



**Figure 3: Monte Carlo results in disk geometry corresponding to isotropic ($\alpha = 0$) and anisotropic ($\alpha$ _= 0) Fermi liquid states at filling factor $\nu = 1/6$ of the LLL. The interaction energy per electron is plotted as a function of $1/\sqrt{N}$ for systems with $N = 5$, 9, 13, 21, 25, 29, 37, 49, and 57 electrons.**

# ANISOTROPIC STATE IS MORE STABLE (LESS ENERGY) THAN THE ISOTROPIC ONE

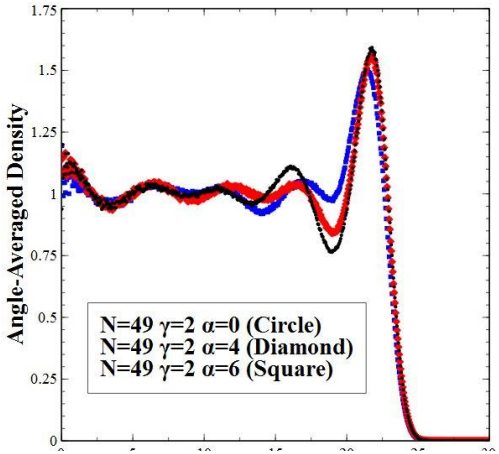
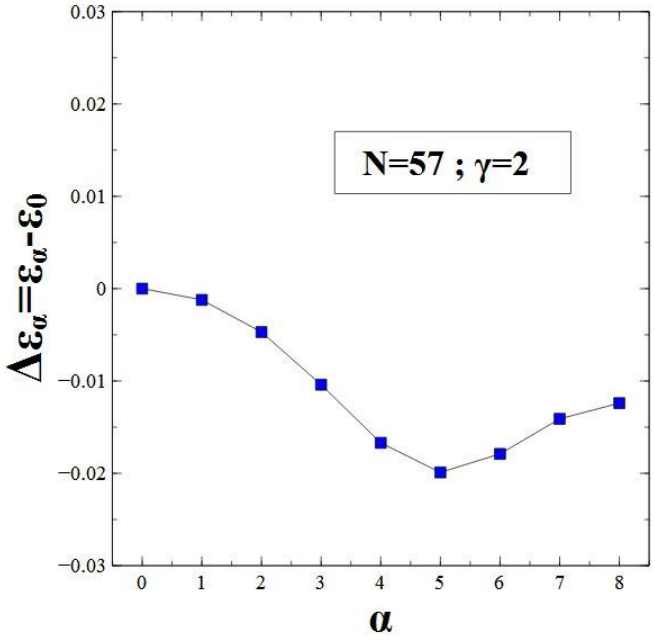



**Figure 4: Angle-averaged density function, $\rho(r)/\rho 0$, as a function of the dimensionless distance, $r/l0$, from the center of the disk for a system of $N$ = 49 electrons.**



# ANISOTROPIC FERMI LIQUID STATES WITH DEFORMED FERMI SURFACE DRIVEN BY A COMBINATION OF ANISOTROPIC EFFECTIVE MASS AND AN ANISOTROPIC INTERACTION POTENTIAL

Assume ka=alpha*kF AND kb=kF/alpha

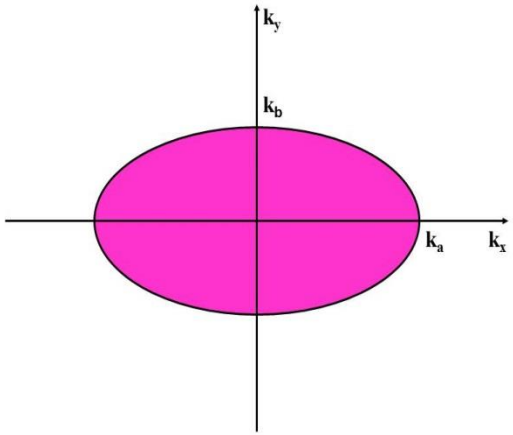



**Figure 5: Elliptical Fermi surface of a 2DEG.**

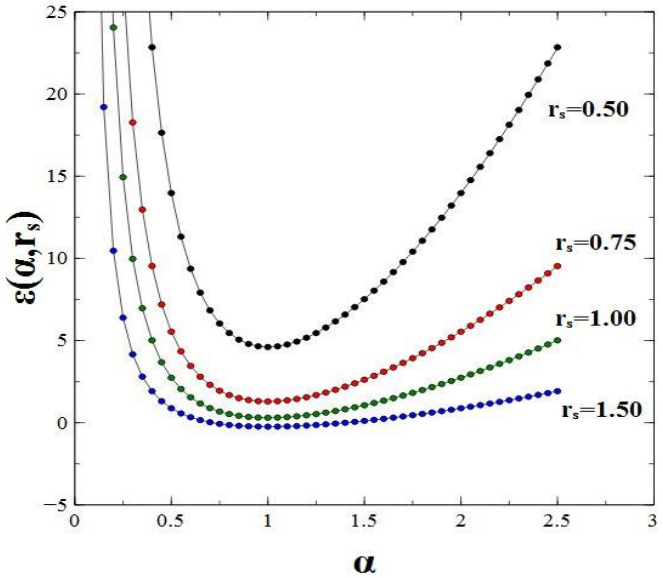



**Figure 6: Total energy per electron (in Rydbergs) as a function of parameter, $\alpha$ which measures the elliptical deformation of the Fermi surface at various densities. The case of an isotropic Coulomb potential, $\gamma$=1 (usual "isotropic" Coulomb).**

# INDUCED FERMI SURFACE DEFORMATIONS

1. O. Ciftja, *Deformation of the Fermi surface of a spinless two-dimensional electron gas in presence of an anisotropic Coulomb interaction potential*, **(Nature) Sci. Rep. 11, 3181 (2021)**. https://www.nature.com/articles/s41598-021-82564-y

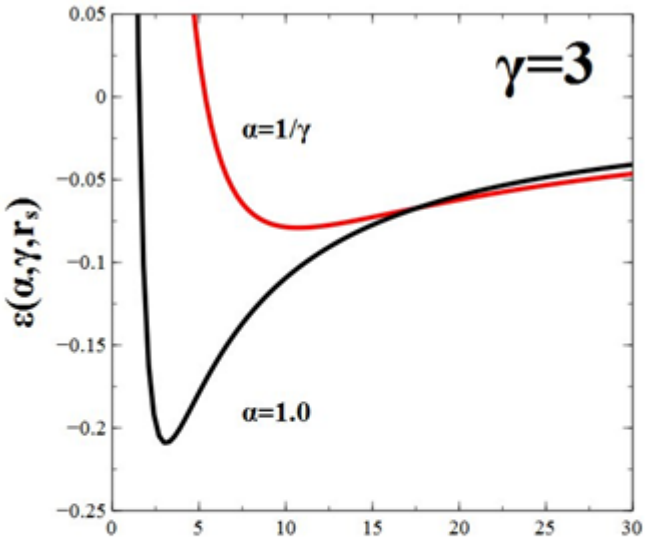



**Figure 7: Total energy per particle for the case alpha=1 (isotropic circular Fermi surface) and alpha=1/gamma (anisotropic elliptically deformed Fermi surface that gives minimum of potential energy) as a function of $r_s$ for a spinless 2DEG with anisotropic Coulomb interaction potential with gamma=3. Energies are given in Rydbergs.**

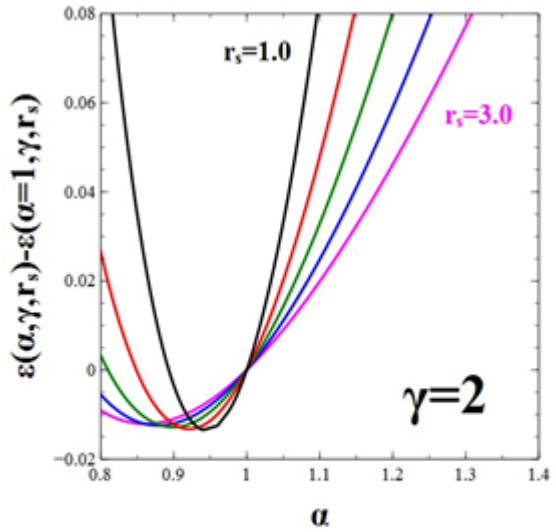



**Figure 8: The energy per particle difference as a function of elliptical deformation of the Fermi surface (alpha) for an anisotropic Coulomb Interaction potential with gamma=2 at various densities as measured by Wigner-Seitz parameter, $r_s$. Energies are measured in Rydbergs.**

# KEY POINTS TO TAKE HOME

A liquid crystal of electrons may represents a novel quantum phase

Isotropic quantum Hall liquid (well studied)

Wigner solid at very low density (very small filling factors) (expected)

Anisotropic liquid crystal (BRS) state missing in between (worth exploring)

Comparison between liquid crystal and isotropic liquid for systems with internal anisotropy (anisotropic mass) “forthcoming “

Quantum phases at finite T “not so forthcoming” (entropy, free energy)

Anisotropy in the LLL (around 1/7) ? (sensitive experiments might see a Liquid Crystal instead of a Wigner Solid? ; co-existance?; tilted field?)

**ANY TIME LEFT FOR NANOSTRUCTURES AND SPINTRONIC DEVICES?**

**If “YES”, let’s move on…..**

# CONFINED NANOSCALE SEMICONDUCTOR QUANTUM DOT SYSTEMS AND MATERIALS

# SPIN-BASED SYSTEMS AND SPINTRONIC DEVICES

# INTRODUCTION

What is a nanostructure? – Any structure so small that one or more of its dimensions has a length, L ~1 nano meter - Nanoscale

Quantum rules dictate that particles have dual nature with a de Broglie wavelength, $\lambda$ such that $p=h/\lambda$

Interesting physics when $L \approx \lambda$ (de Broglie wavelength) – This is the Nano World

At nanoscale, properties are size-dependent (unlike bulk properties)

The focus of this presentation is on the physics of confined electron systems in a semiconductor material at nanoscale ranging from a 2D electron gas down to "quantum dots" where electrons are confined in all three dimensions ("artificial atom")

# PARTICLE IN A BOX MODEL - A SIMPLE MODEL FOR QUANTUM DOTS

Quantum dot-s (QD-s) a.k.a. “quantum boxes” or “artificial atoms”

- Discrete energy levels ~ $1/L^2$ (where L is the length of the box)
- If there are N electrons inside the box, the Coulomb repulsion energy which scales ~1/L competes with the confining energy
- There are also constrains as dictated by quantum rules

Question: Will the electrons tend to separate rigidly from each other (like in a crystal state) or will they behave as free particles (like in a liquid state)?! Many Research Projects with Varying Levels of Difficulty

Even the properties of a quantum dot with N=2 electrons are very interesting [Ciftja et al. PRB 70, 205326 (04’), PRB 72, 205334 (05’)]

Potential for Novel Technological Applications in Devices as Well as Great Opportunity for Interesting Science – “Tunable Atom”

# MORE REALISTIC MODELS

Confinement potential felt by electrons in a quantum dot system is not realistically represented by “a particle in a box” model

The real confinement potential felt by electrons depends on the specific experimental setup used to fabricate the system

Many choices of research projects to involve undergraduate students

- Charged Disk Model
- Charged Square Model
- Many Other Models …

Other treatments considered

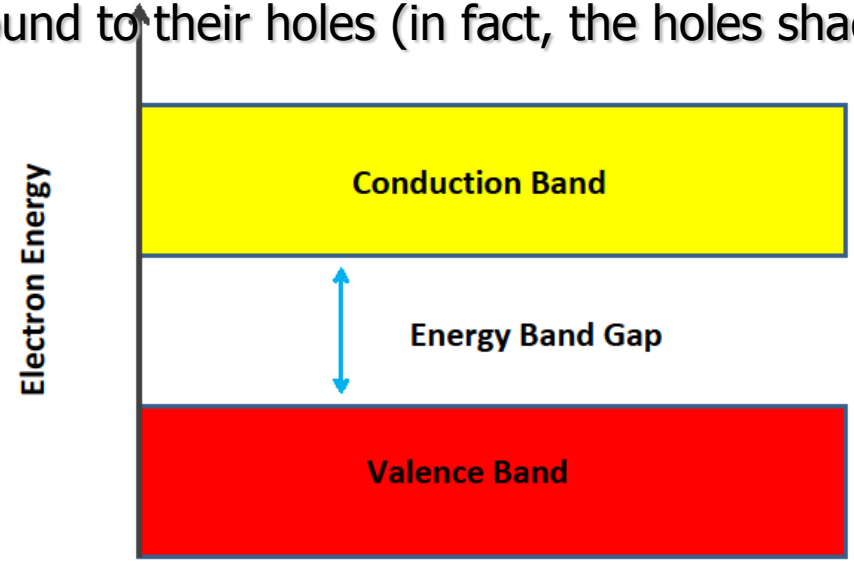
- Spherical confinement, harmonic oscillator (parabolic) potential
- Single electron properties
- Small systems with few electrons, etc

# HOW TO FABRICATE CONFINED ELECTRON SYSTEMS IN A SEMICONDUCTOR HOST

- You start with a 3D (bulk) semiconductor material like GaAs

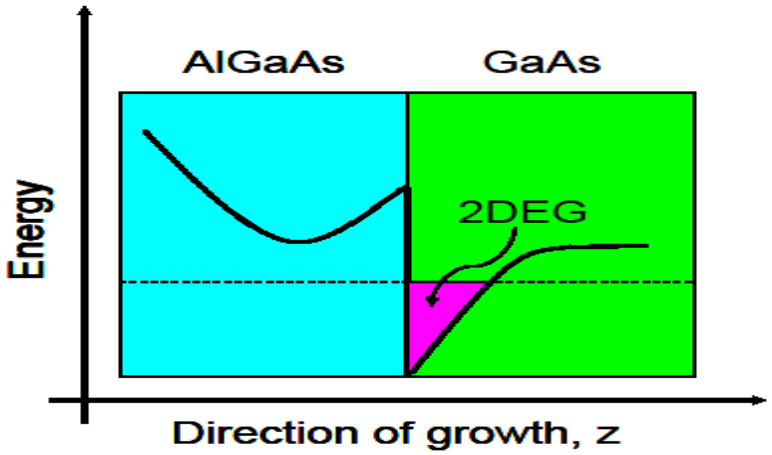
**BASIC PHYSICS**

- When excited to the Conduction Band, electrons in the conduction band behave as a typical 3D electron system (though with an effective mass)
- Holes (positive charge) are left behind in the Valence Band
- Electron-Hole pair = Exciton like an electron-proton in H atom (Bohr model)
- Electrons in the conduction band move in all three dimensions of space and are not tightly bound to their holes (in fact, the holes shadow them)



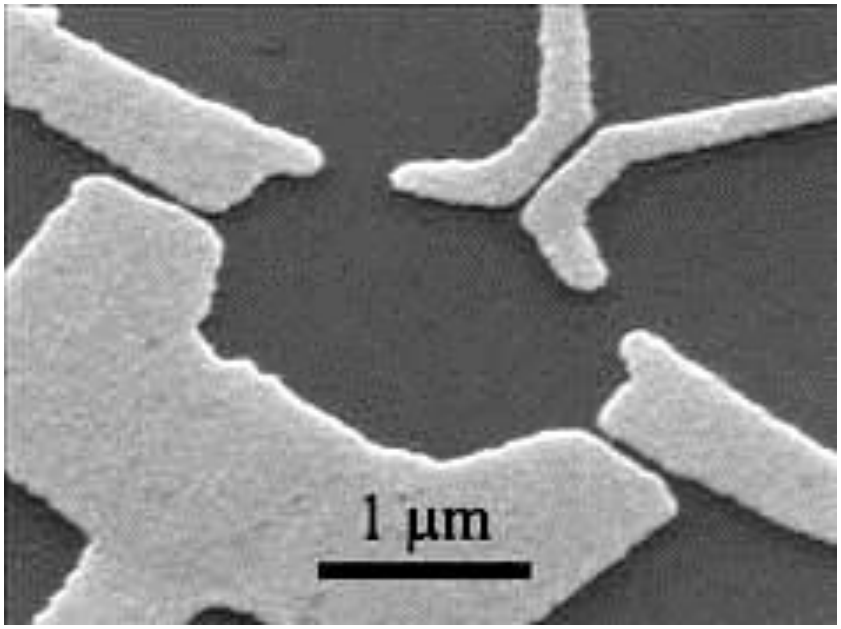
# FROM A 3D SYSTEM OF ELECTRONS IN THE CONDUCTION BAND TO A 2D ELECTRON GAS (2DEG)

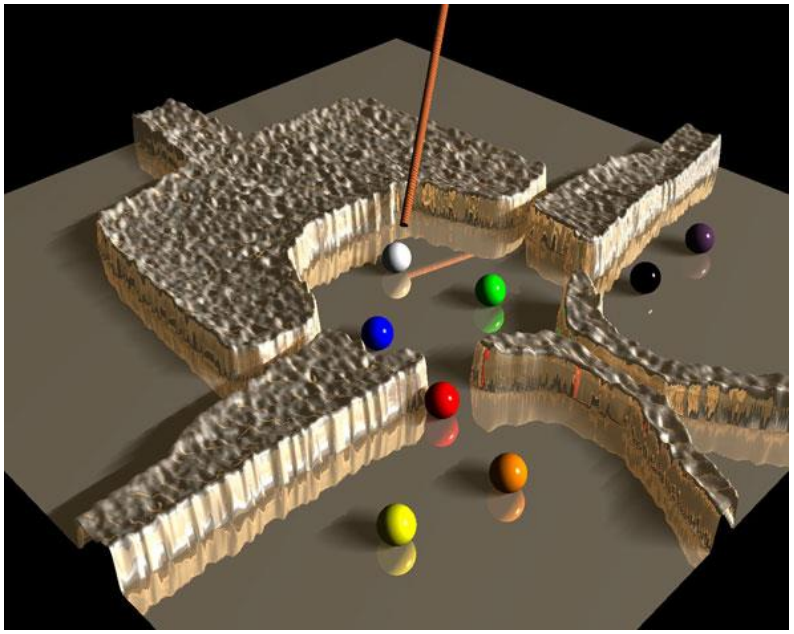
- Take a sample and dop GaAs with Al to form AlGaAs
- AlGaAs has a wider energy band gap than GaAs
- Place a sample of GaAs on top of AlGaAs – **heterojunction**
- Electrons on the AlGaAs side of the interface lower their energy by moving across the interface to the GaAs side



# 2D SEMICONDUCTOR QUANTUM DOTS

A 2D confined system patterned via gate electrodes



QDs as defined by 5 metallic gates fabricated on GaAs heterostructures

[http://pages.unibas.ch/phy-meso/Pictures/pictures.html]

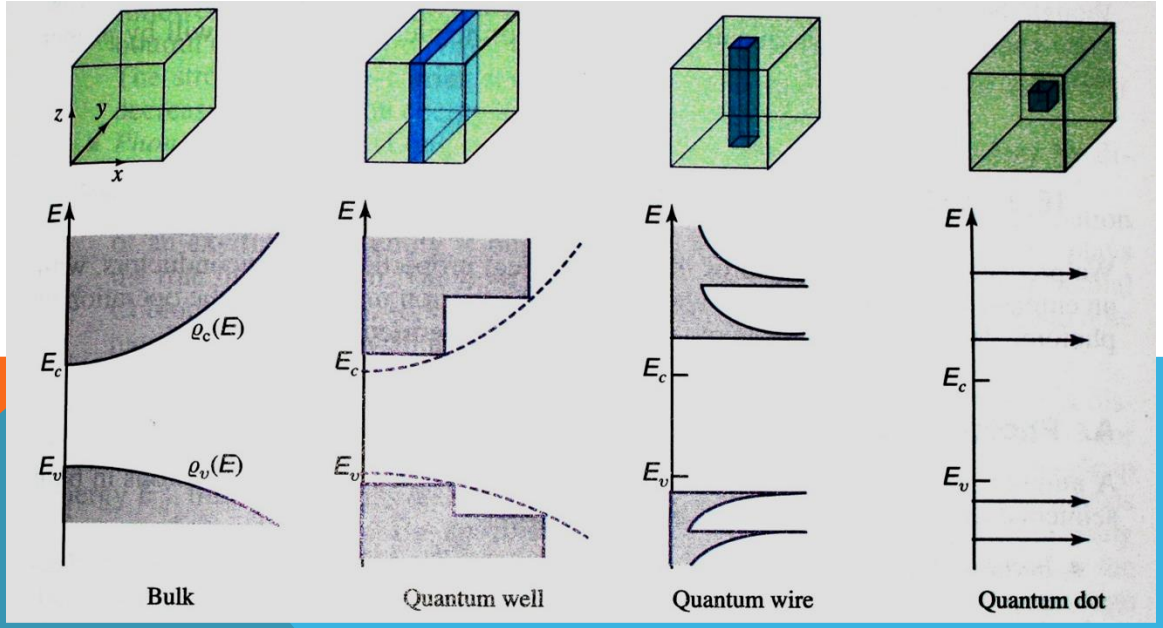
# CONFINEMENT AT THE NANOSCALE

In a semiconductor system, electrons and holes are confined in all three dimensions of space by a surrounding host material with a larger bandgap.

Quantum confinement leads to discrete energy levels (artificial atom).

Strength and geometry of the confining potential competes with the Coulomb repulsion energy of the electrons

Competing tendencies lead to very subtle phenomena




B.E.A. Saleh, M.C. Teich. Fundamentals of Photonics. fig. 16.1-29.

# LENGTH AND ENERGY SCALES

Typical semiconductor particles have a size comparable to Bohr's radius of the excitons (the electron – hole pair separated by some distance).

- Typical dimensions: 1 – 10 nm but can be as large as several μm
- Different shapes (2D/3D, circles, squares, cubes, spheres, pyramids, etc.)

Confinement energy tends to aggregate the electrons

Coulomb energy tends to separate the electrons (though, with quantum effects included, nothing is trivial)

Confining potentials with a given geometry want to impose a given shape

External factors (such as a magnetic field may change the whole physics)

Thus, there are many possible scenarios of interest from a theoretical and computational perspective

# SAMPLE PROJECT ON THE INFLUENCE OF SIZE ON THE ENERGY OF A FINITE SYSTEM

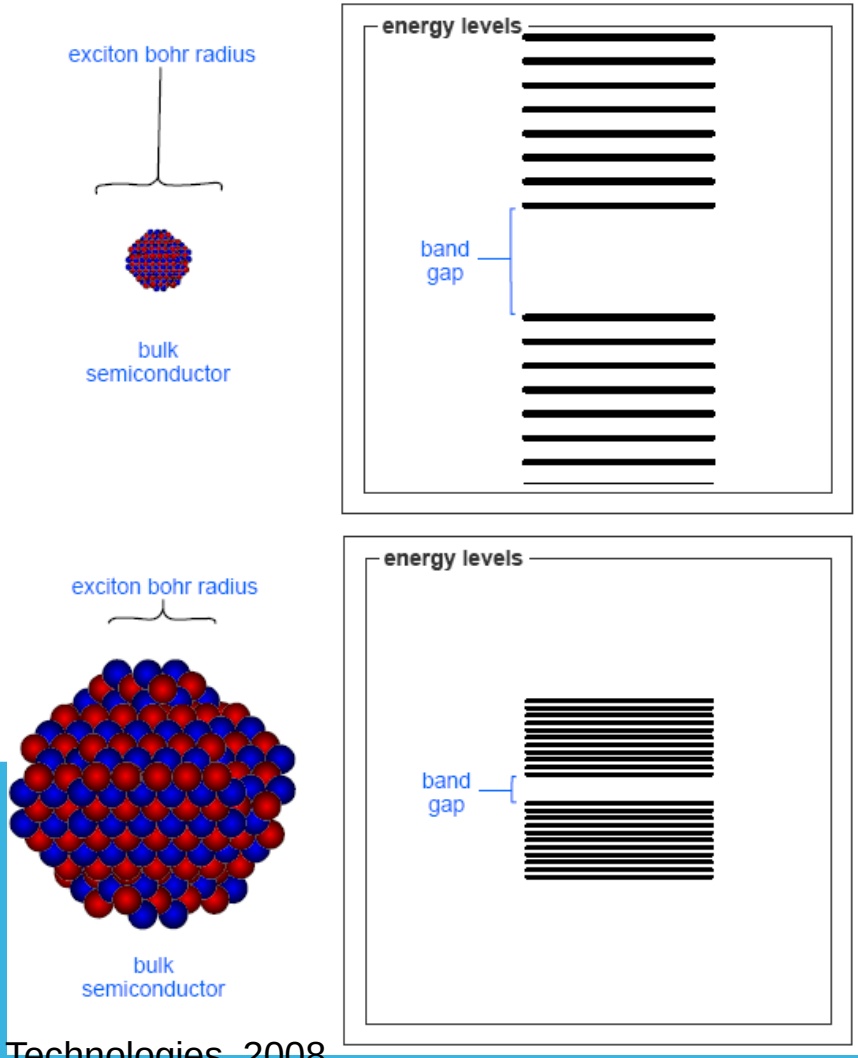
**Energy levels depend on size (and also the shape) of the quantum dot.**

**Smaller the quantum dot:**

- Energy levels increase in energy and spread out more.
- Higher band gap energy.

**There are many different scenarios, thus, many possibilities**

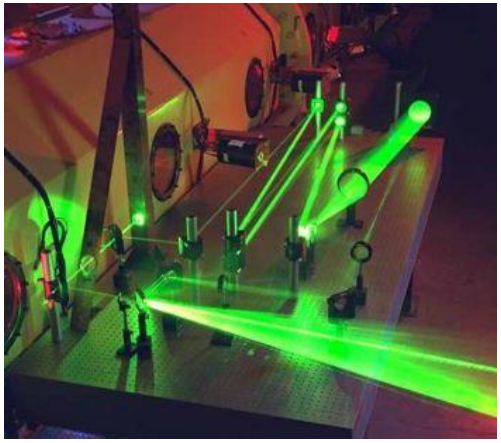
- Modeling
- Theory
- Mathematics




Figures are from “Quantum Dots Explained.” Evident Technologies. 2008.

# The Importance of Nanotechnology

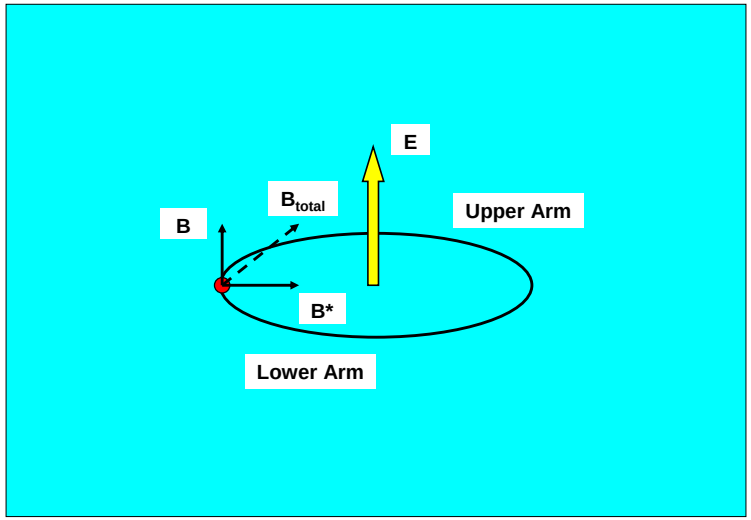
- Photovoltaic devices: solar cells
- Biology : biosensors, imaging
- Light emitting diodes: LEDs
- Spintronics (Spin-Based Electronics)
- Flat-panel displays
- Memory elements
- Photodetectors

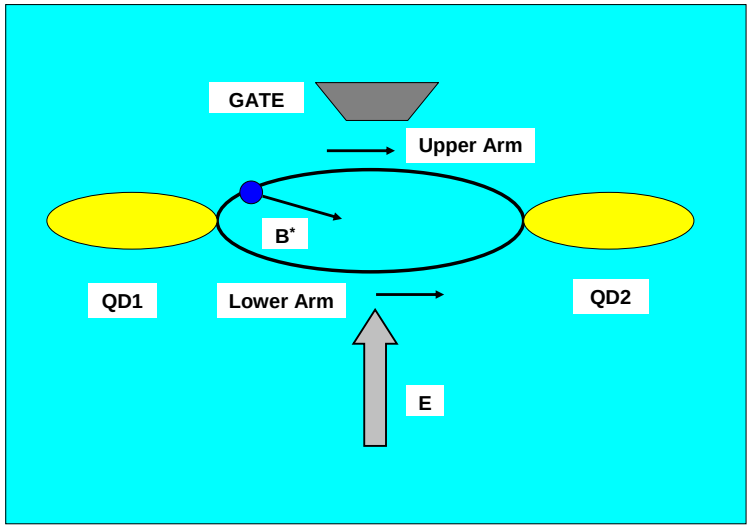


# Spin-based nano-devices

## (DOD-ARO Grant W911NF-13-1-0139)

- Novel spin-based (spintronic) devices where the spin (magnetic moment) of the electrons interacts with an electric and/or magnetic field – The idea is to use an electric field to control spin via spin-orbit coupling (which produces an effective magnetic field B*). **[O. Ciftja, Phys. Scr. 88, 058302 (2013)]**.



# Clusters and Thin Films of electrons

## (NSF Grants DMR-1104795 and DMR-1410350)

- Stabilization of rotating Wigner molecules ("dancing pairs of electrons") in a confined system of electrons
- Manipulation of spin by electric fields (similar to spin-orbit coupling in hydrogen atom)
- Finite systems of electrons confined in domains with sharp geometric features, thus, very sensitive to the environment -> **GOOD SENSORS**?
- Role of spin in confined nanoscale **systems**
  - Weakly confined molecular magnets/magnetic thin films
  - High-spin nanoscale magnets do "retain" magnetic information (at least at low temperatures) – **Biomedical Applications (ferrites)**

# CONCLUSIONS

Semiconductor Quantum Dot Systems - Systems of electrons confined in a semiconductor host material

Discrete energy levels ~ artificial atom

Interplay of confinement, electron-electron interaction, low dimensionality and external fields (a magnetic field)

Quantum electronics and spintronics

Role of electric field to manipulate spin (quantum computers)

Caution: Where are we going if Artificial Intelligence (AI) and Machine Learning (ML) can replace algorithms, human coding, software, etc???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THANKS FOR YOUR TIME!